Customer Markets and the Real Effects of Monetary Policy Shocks

Inauguraldissertation
zur
Erlangung des Doktorgrades
der Wirtschaftswissenschaftlichen Fakultät
der Universität Augsburg

vorgelegt von

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Augsburg, im Dezember 2008

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Datum der mündlichen Prüfung: 25.03.2009

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Chapter 1

Introduction

1 Motivation

In an interview in the *Economic Dynamics Forum* given in 2004 Patrick Kehoe sharply criticizes existing sticky price models and points out their inability to account for the persistence observable in the data. He argues that the persistence generated by that models is due solely to the exogenously imposed unrealistically high degree of price stickiness, not consistent with the empirical observations. Kehoe summarizes his critique as follows:

"...Currently I see a large number of economists writing papers that take the existing sticky price models as they stand and tries to use them to address a number of issues, especially policy issues. I think that this is not a productive use of time. A better use of time for the sticky price enthusiasts is to go back to the drawing board and dream up another version of the model that has a chance at generating the patterns observed in the Great Depression. Doing so may be difficult, but the payoff is worth it."

There is substantial empirical evidence indicating that that monetary shocks induce highly persistent dynamic responses of inflation, output, consumption and investment, although nominal prices are very flexible, being adjusted every four months on average. As I show below, if a standard New Keynesian model is calibrated to match the most recent evidence on the frequency of price adjustment, it looses its ability to account for the persistence and the magnitude of the impulse responses to monetary policy shocks observable in the data. In addition, the implications of the model with respect to the second moments of the most important macroeconomic aggregates become completely at odds with what is found in the data.

In the light of the empirical evidence as well as Kehoe's critique, I take up the challenge formulated by him and take a first steps towards developing *"another version of the model"* which provides an endogenous explanation of

- the incomplete response of inflation to monetary shocks without resorting to exogenously imposed unrealistic degrees of price stickiness,
- the cyclical pattern of markups and
- the persistent reactions of most macroeconomic variables to demand and supply side disturbances.

Some of the most important building blocks of the models developed in this monograph are assumptions on the utility function. Many macroeconomists do not like theoretical frameworks in which the specification of the utility function plays a major role. Nonetheless, I would like to point out that every assumption can be regarded reasonable if, for whatever reason, we are not able to reject it, and at the same time, this assumption makes the implications of the model consistent with the relevant empirical evidence.

The monograph proceeds as follows: Chapter 1 reviews the empirical evidence on the effects of monetary policy shocks, the price setting behavior of firms, the cyclical pattern of average markups and discusses some of the major shortcomings of the standard New Keynesian model with Calvo pricing. Since the theoretical literature provides models which are only loosely related to the ones developed in chapters 2, 3 and 5, I close chapter 1 with a very brief review of existing theoretical studies. In chapter 2 I develop a model in which firms engage in dynamic market share competition and money enters non-additively the utility function. Market share competition substantially improves the qualitative predictions of the MIU[1] model. Several of its exact quantitative implications, however, remain inconsistent with the empirical evidence. Chapter 3 is devoted to the construction of a more *exotic* model combining dynamic market share competition and search activity in the goods market with the assumption that agents are characterized by inflation aversion. For a broad range of empirically plausible parameter combinations the Inflation Aversion model performs better (or at least as good as) the New Keynesian model with regard to the properties of the impulse responses to monetary shocks as well as the usual business cycles moments. Chapter 4 presents a GMM estimation of important parameters of the Inflation Aversion model. Chapter 5 develops a model with static market share competition which rationalizes some of the features of the Inflation Aversion model. Chapter 6 concludes.

[1]MIU - Money In the Utility function.

2 Impulse Responses to Monetary Policy Shocks

In a very comprehensive and exhaustive survey of the empirical literature on the effects of monetary policy shocks Christiano *et al* (1999) point out that there are mainly three strategies for identifying these shocks: First, by specifying a statistical model consisting of one or more equations, at least one of which can be interpreted as a central bank policy rule, and then assuming that the variables appearing in the policy rule do not respond contemporaneously to changes in the policy instrument. In so doing, the statistical residual in the policy rule can be interpreted as an estimator of the monetary shock. The second strategy is based on the assumption that monetary policy shocks do not affect the real economic aggregates in the long run. The third approach involves looking at the data as well as various publications of the central bank in order to find signals for unsystematic (exogenous) policy changes.[2] The last two approaches do not explicitly specify the policy rule of the central bank. In this section I provide a brief discussion of the results obtained by each of the three identification strategies.

2.1 The SVAR-Approach

The most popular approach to quantify the effects triggered off by the unsystematic component of monetary policy is the estimation and simulation of a so called *Structural Vector Autoregression Model (SVAR)*. The first step of this procedure is the estimation of a standard VAR reflecting the dynamic interactions between two or more macroeconomic variables. The VAR can be written in the following *reduced form*:

$$Y_t = c + \sum_{i=1}^{n} A_i Y_{t-i} + u_t,$$

where Y_t is an $m \times 1$-vector of observable variables while u_t denotes the $m \times 1$-vector containing the observable (or reduced-form) residuals. Under certain conditions the $m \times 1$-vector c, the $m \times m$-matrices A_i and the contemporaneous covariance matrix of u_t, Ω, can be estimated by Ordinary Least Squares. Ω is symmetric and in general non-diagonal. The latter implies that the residuals u_t are not orthogonal to each other. Therefore, they can not be assigned a meaningful economic interpretation. However, econometricians often assume that the observable data has been generated by the following unobservable or *structural* model:

$$AY_t = \tilde{c} + \sum_{i=1}^{n} \tilde{A}_i Y_{t-i} + B\epsilon_t, \tag{I.2.1}$$

[2] For example some decisions and announcements of the Federal Open Market Committee in the United States.

where the covariance matrix of the structural residuals ϵ_t is diagonal and the matrices A and B are unknown. The following relation between u_t and ϵ_t holds:[3]

$$u_t = A^{-1}B\epsilon_t.$$

Usually, the elements of ϵ_t possess a concrete economic interpretation. Since, however, the contemporaneous covariance matrix of the reduced-form residuals is symmetric, it does not provide enough information to identify the elements of A and B, and thus, the structural residuals. This is true even if we assume that one of these two matrices is an identity matrix. To solve the problem of recovering ϵ_t, econometricians usually resort to different kinds of restrictions on the elements of A and B. Such restrictions are based either on economic theory or on *ad hoc* assumptions both of which are subject to debate.

The first attempts to identify the effects of the unsystematic component of monetary policy within the VAR framework were made in the second half of the 70's and the early 80's. Examples are Sims (1972, 1977, 1978, 1980, 1986), Litterman and Weiss (1983), Leamer (1985) and many others. In spite of the large differences with respect to the specification of the VAR, the identification scheme employed as well as the assumption about the policy instrument[4] across these studies, they deliver strikingly similar qualitative predictions regarding the real effects of monetary policy shocks. Expansionary monetary shocks induce delayed, positive and *hump-shaped* responses of the measures of real activity used. The deviations of the latter from their respective initial levels is in most cases largest after about a year and a half.

Sims (1980): Sims (1980) considers two VAR models both covering the interwar period 1920-1941 and the postwar one 1948-1978. The first one is a three-dimensional VAR consisting of the log of the monetary aggregate M 1, the log of the level of real industrial production Y and the log of the wholesale price index P. The second model also includes the short term nominal interest rate R (the rate on 4-6 month prime commercial paper). Monthly data is used. Sims (1980) assumes that the money stock M 1 is a good single index of monetary policy and interprets the lags of the endogenous variables appearing in the equation determining M 1 as the systematic part of the policy rule. The monetary shock is identified as the residual of the money supply equation obtained by assuming that the matrix B in the

[3] c and the A_i-s relate to $\tilde{c}$ and the $\tilde{A}_i$-s according to:

$$c = A^{-1}\tilde{c}, \qquad A_i = A^{-1}\tilde{A}_i.$$

[4] Most of these studies approximate the behavior of the central bank by a money supply rule.

notation of (I.2.1) is lower triangular while A equals the identity matrix.[5] The endogenous variables are ordered as follows: M 1, Y, P and R, M 1, Y, P in the three- and the four-dimensional VAR respectively. Placing M 1 first in the former is motivated by the results of Granger-causality tests indicating that the money stock is causally prior to Y and P. The ordering of the latter SVAR is chosen to maximize the differences in innovation variances between the two periods and has a negligible effect on the impulse responses.[6] Consider first the three dimensional SVAR. In the interwar as well as the postwar period monetary policy innovations induce hump-shaped increases in industrial production Y which peaks at about 18 months and remains for more than six years above its initial level. The response of prices P is also hump-shaped but less persistent, disappearing after about 4 years. The monetary shock explains 66% (37%) of the 48-month forecast error of industrial production for the interwar (postwar) period. The same numbers for prices are 38% and 14%. The inclusion of the short term interest rate substantially reduces the fraction of forecast error variance, (FEV), in real activity Y attributable to monetary shocks: 54% and 32% for the two periods.[7] The reactions of industrial production and prices induced by these shocks are much weaker, much less persistent,[8] and for the postwar period they are no more hump-shaped. In contrast, interest rate innovations induce a strong, hump-shaped and persistent impulse response of industrial production and a similarly persistent but weaker response of prices which the more recent SVAR literature also discovers and interprets as reactions to monetary policy shocks. Sims (1980), however, interprets the shocks to the interest rate equation as capturing anticipated movements in money supply.

Sims (1986): In another paper on the same issue Sims (1986) assumes that B in equation (I.2.1) is a diagonal matrix and imposes two different sets of restrictions on A which are based partly on economic theory and partly on institutional characteristics of the Fed's behavior. The restrictions on A are such that each shock has a contemporaneous effect on all variables except investment which is predetermined. A zero restriction on , say, the element $a_{i,j}$ of A implies that there is only an indirect contemporaneous link between the j^{th} shock and the i^{th} endogenous variable. Sims (1986) uses quarterly U.S. data over the period 1948:1 - 1979:3. The variables are real GNP Y, real business fixed investment I, the GNP price deflator P, the monetary aggregate M 1, unemployment U and the Treasury-bill rate R. To separate the demand and supply of money from each other, the author assumes that the log of M 1,

[5] Such a specification of A and B represents a specific short run restriction on the *contemporaneous* effect of the shocks on the endogenous variables. It is implemented by using the Cholesky factorization of the covariance matrix of the nonstructural VAR residuals.

[6] See Sims(2008).

[7] Monetary shocks are again identified as the residuals to the equation determining M 1.

[8] The response of industrial production for example lasts for only two years (two quarters) in the interwar (postwar) period.

m_t, and the nominal interest rate R are contemporaneously related as follows:

$$R_t = a_{R,m} m_t + \epsilon_{1,t},$$

where $a_{R,m}$ is a coefficient to be estimated while $\epsilon_{1,t}$ denotes the monetary policy (money supply) shock. This equation rests on the idea that the monetary authority and the private banks are able to see short term interest rates and indicators of changes in monetary aggregates immediately. As a results, money supply innovations $\epsilon_{1,t}$ are immediately reflected in interest rate changes, while the remaining variables are only indirectly affected by $\epsilon_{1,t}$. Furthermore, according to the money supply equation R_t can only respond to the remaining variables with a delay.[9] The money demand equation is assumed to take either the form

$$m_t = a_{m,y} y_t + a_{m,p} p_t + a_{m,R} R_t + \epsilon_{2,t},$$

or

$$m_t = a_{m,y} y_t + a_{m,i} i_t + a_{m,p} p_t + a_{m,R} R_t + \epsilon_{2,t},$$

where y, p and i are the logs of GNP, prices and investment respectively. Both money demand schedules as well as the remaining equations determining output, unemployment and investment are based on versions of the AS-IS-LM model.[10] The system is specified so that the shocks to the monetary sector $\epsilon_{1,t}$ and $\epsilon_{2,t}$ can be transmitted into the reminder of the model only *via* interest rates. As Sims (1986) shows, both specifications of money demand imply virtually the same quantiataive and qualitative predictions. A money supply shock $\epsilon_{1,t} > 0$ leads to a sharp temporary increase in nominal interest rates and a persistent decline of M 1. Output and investment respond negatively in a hump-shaped manner, peaking after two years and returning to their pre-shock levels after about four years. The peak response of output (investment) is equal to 0.0086% (0.032%). Furthermore, there is a substantial delay in the response of prices which is much more persistent than that of output. The reaction of unemployment is also characterized by a time lag peaking after about two years and returning to its pre-shock value after about three years. The peak response of unemployment is equal to 0.27%. Unfortunately, due to their low quality, the figures provided by Sims (1986) do not allow to measure the delay in the responses of the real variables exactly.

A major shortcoming of these papers, as Sims (1986) also argues, is the use of the money stock (usually M 1) as a measure of monetary policy instead of some other variable more closely controlled by the central bank, such as the federal funds rate or the level of non-borrowed reserves. As is well known, the monetary aggregate M 1 arises as an equilibrium

[9] The delay is shorter than one quarter since the restrictions are imposed on the matrix A in (I.2.1).

[10] The equations for output, unemployment and investment are not discussed further in this chapter. The interested reader is referred to Sims (1986).

result of the interplay between money supply and money demand. Also problematic is the assumption made by most of the early SVAR-studies that M 1 (or the policy instrument in general) is causally prior to the other variables in the model. Since central banks have large sets of extremely frequently updated information (even real data) at their disposal, it is hard to imagine that a monetary authority will postpone the adjustment of the policy instrument until next month or even until next quarter instead of reacting contemporaneously to the flows of new information in order to avoid deviations from its stabilization targets. The statistical treatment of the time series used in these studies should be also subject to sharp criticism: The bulk of the early SVAR literature uses the levels or the log-levels of aggregates such as M 1, GNP or investment which are known to be non-stationary.

To at least partly overcome these shortcomings, the more resent SVAR literature dealing with the effects of monetary shocks tries to more precisely distinguish between different monetary policy instruments and their relevance, to more precisely capture the timing structure of information flows and adjustments of the policy instruments, and to more precisely handle the statistical properties of the data used. Institutional arguments for using the federal funds rate FF as a measure of the policy instrument can be found in Bernanke and Blinder[11] (1992) and Sims (1992). Christiano and Eichenbaum (1992) provide an institutional motivation[12] for equating the policy instrument to the level of non-borrowed reserves while Strongin (1995) suggests using the ratio of non-borrowed reserves to total reserves as a measure of the policy instrument.[13] In the following, I review some of the most interesting recent studies.[14]

Christiano *et al.* (1999): Christiano *et al.* (CEE) (1999) consider a seven-dimensional VAR consisting of the log of real GDP Y, the log of the implicit price deflator P, the log of the

[11]Bernanke and Blinder (1992) provide two theoretical and one institutional argument for using the federal funds rate as a measure of the monetary policy instrument. First, if the federal funds rate is a measure of policy and at the same time monetary policy matters, then FF should be a good predictor of major macroeconomic variables. The authors show that FF is a better forecaster of the economy than other interest rates or the monetary aggregates. Second, if the federal funds rate measures monetary policy, then it should respond to the Federal Reserve's perception of the state of the economy. Bernanke and Blinder (1992) show that the latter is true by estimating monetary policy reaction functions explaining movements in the funds rate by lagged target variables. Third, the authors find support of the view that FF does reflect policy changes by showing that the supply curve of non-borrowed reserves between Federal Open Market Committee (FOMC) meetings is extremely elastic at the target funds rate.

[12]In their view non-borrowed reserves is the monetary aggregate most closely controlled by the Fed, so that it is plausible to assume that its movements are attributable to monetary policy shocks only. Higher order aggregates such as M 1 and M 2 also react to money demand disturbances.

[13]Strongin (1995) argues that the demand for total reserves is completely interest inelastic in the short run, so that initially policy shocks only affect the composition of total reserves between borrowed and non-borrowed reserves.

[14]The exposition is partly based on Christiano *et al.* (1999).

smoothed change in an index of sensitive commodity prices[15] $PCOM$, the federal funds rate FF, the log of total reserves TR, the log of non-borrowed reserves NBR and the log of either M 1 or M 2 denoted by M. The data used by CEE (1999) is quarterly, detrended and covers the period 1965:3 - 1995:2. The authors identify the monetary policy shock *via* three alternative *benchmark* specifications. In the first one the monetary policy instrument is measured by the federal funds rate FF while in the second and third ones this is done by non-borrowed reserves and the ratio of non-borrowed to total reserves respectively. Each of these specifications involves the assumption that the matrix A in (I.2.1) governing the contemporaneous links between the endogenous variables of the model is lower triangular while the matrix B is an identity matrix. The causal priority assumed is as follows: $[Y, P, PCOM, FF, NBR, TR, M]$, $[Y, P, PCOM, NBR, FF, TR, M]$ and $[Y, P, PCOM, TR, NBR, FF, M]$[16] within the first, second and third SVARs respectively. According to these three orderings output Y, prices P and the index of sensitive commodity prices $PCOM$ are predetermined with respect to the policy instrument and can react to movements in it only with a lag. In other words, the central bank sets the policy instrument as a function of current and lagged values of Y, P and $PCOM$ and lagged values of the remaining two variables. The monetary policy shock is identified as the disturbance to the equation determining the evolution of the monetary policy instrument. In all three cases a contractionary policy shock leads to a persistent decline of output, and commodity prices, both displaying a hump-shaped pattern. The peak-response of output (commodity prices) is reached after about four (two) quarters. The FF-model (NBR-model) implies the strongest (weakest) peak-response of output Y, equal to -0.5% (-0.25%). By construction Y does not react in the impact period but according to all three SVARs its response remains insignificant until the end of the third quarter. In all three cases there is virtually no reaction of the GDP deflator P in the first 4-6 quarters. Then the *point estimate* of P smoothly decreases and remains below average for more than twenty quarters. However, the 95%-confidence bounds indicate that irrespective of the policy measure the response of P is insignificant at all horizons. What about the other variables? A contractionary policy shock in the FF-case induces a persistent rise in the federal funds rate and a persistent decline in non-borrowed reserves. The response of total reserves TR is at all horizons insignificant. This prediction is consistent with the arguments in Strongin (1995). M responds negatively with a one quarter delay and returns quickly to its pre-shock level.[17] If non-borrowed reserves NBR are used as the policy instrument there is a significant decline of total reserves for about 3 quarters. Consistent with this reaction the monetary aggregate

[15]This variable is a component in the Bureau of Economic Analysis' index of leading indicators.

[16]The assumption that the policy instrument is identical with the ratio of non-borrowed to total reserves is implemented by making the current value of total reserves TR an element of the information set of the central bank. The monetary policy shock is then the innovation to the NBR equation.

[17]This result holds irrespective of wether M 1 or M 2 is used.

M contemporaneously falls and remains below average for about a year. When the policy instrument is measured by the ratio NBR/TR, TR and M display similar impulse responses as in the FF-model. CEE (1999) also find that different assumptions with regard to which variables are predetermined when the policy instrument is set have a negligible effect on the predictions of each of the three SVARs. CEE (1999) also emphasize that the importance of monetary socks for output fluctuations depends crucially on the assumptions with regard to the policy instrument. In the FF-case the monetary shock accounts for 21%, 44% and 38% of the forecast error variance of output at the 4,8 and 12 quarter horizon respectively. In contrast, the NBR policy shock accounts for only 7%, 10% and 8% at the 4,8 and 12 quarter horizon. Examples for papers using similar measures of the monetary policy instrument, similar identification schemes and thus, reaching similar results are Christiano and Eichenbaum (1992), focusing on the quantification of the liquidity effect of monetary shocks, and Christiano *et al.* (1996a), focusing on the responses of firm's and household's assets and liabilities and other financial variables. CEE (1999) further examine the effects of a monetary disturbance when the policy instrument is measured by by M0, M 1 or M 2. Irrespective of which of the three monetary aggregates is used to approximate the policy rule, the implied responses of all variables are much weaker (in most cases even insignificant) and of lower persistence. As CEE (1999) note, due to the high imprecision in the estimated impulse responses, it would be a difficult task to reject the hypothesis that monetary policy has no real effects. While the point estimates of the reactions to a M 2 policy shock are similar to that obtained in the FF, NBR and NBR/TR models the M 0 and M 1 models deliver predictions which are quite different. As a reaction to contractionary M 0 shock output increases and remains above average for about 3-4 quarters. When the policy instrument is approximated by M 1, initially there is a slight drop in output followed by a persistent increase exhibiting a hump-shaped pattern. The GDP deflator P falls below average for about 4 to 6 quarters before returning to its pre-shock level (in the M 0-case) or reaching a slightly above average level (in the M 1-case). CEE (1999) point out that while the results obtained with the policy measures FF, NBR, NBR/TR and M 2 are at least partly consistent with some New Keynesian Models, the predictions of the M 0 and M 1 models can be characterized as more or less consistent with simple DSGE[18] models with flexible prices, motivating money demand by a simple cash-in-advance constraint or a transactions technology.[19]

Fisher (1997) and Gertler and Gilchrist (1994): Fisher (1997) examines how different components of aggregate investment respond to monetary policy shocks. He concludes that all components of investment decline after a contractionary intervention of the central bank.

[18]DSGE - Dynamic Stochastic General Equilibrium

[19]For example Cooley and Hansen (1989), Jovanovich (1982), Romer (1986), Lucas and Stokey (1987).

However, Fisher (1997) finds important differences with respect to the magnitude of the responses between the different types of investment. According to his results, residential investment exhibits the strongest response followed by equipment, durable goods expenditure and structures. Furthermore, residential investment responds most rapidly to monetary policy shocks, reaching its peak several quarters before the other variables do. Gertler and Gilchrist (1994) analyse the effects of monetary policy on the sales and inventories of large as well as small firms. According to their results, as a consequence of monetary contraction small firms' sales drop much more sharply than it is the case for large firms. Furthermore, the inventories of small firms decline immediately while that of large firms initially increase before falling below their pre-shock levels.

Christiano *et al.* **(2005):** One of the most influential SVAR studies for monetary macroeconomics is the one performed by Christiano *et al.* (CEE) (2005). To identify the monetary shock as the one associated with the equation for the federal funds rate R_t, the authors resort to the following recursive ordering of a set of macroeconomic variables. [20]

$$Y_t = [Y_{1,t}, R_t, Y_{2,t}],$$

where Y_t denotes the vector of endogenous variables. $Y_{1,t}$ contains the variables whose time-t values do not respond contemporaneously to monetary policy shocks. $Y_{2,t}$ is the vector of variables which can be contemporaneously affected by monetary shocks. $Y_{1,t}$ consists of the logs of real gross domestic product, real consumption, the GDP deflator, real investment, the real wage and labor productivity while $Y_{2,t}$ consists of the log of real profits and the growth rate of M2. In other words the variables in $Y_{1,t}$ are assumed to respond with a one period lag to monetary disturbances.[21] CEE (2005) obtain the following results: Output, consumption and investment respond in a hump-shaped fashion, peaking after about one and a half years and returning to their initial levels after about three years. The peak-responses for output, consumption and investment are about 0.6%, 0.2% and 1% respectively. The impulse response of inflation is also hump-shaped, but the peak (0.2%) is reached after about two years. Profits and labor productivity also rise but their reactions are much less persistent. The response of the real wage is positive but insignificant.

However, it should be noted that the particular recursive ordering underlying the results and implying that output, consumption or investment are contemporaneously unaffected by the federal funds rate, is a very strong assumption. The authors are aware of the problem and, therefore, develop a New Keynesian model incorporating a set of very specific (even unusual)

[20] I use the same notation as Christiano *et al.* (2005)

[21] This again is an example of an identification by short run restrictions. More formally, the matrix A in (I.2.1) is assumed to be lower diagonal.

timing assumptions so that it is at least partly consistent with the VAR-ordering chosen. The impulse responses implied by that model are then compared with the estimated ones. However, most monetary DSGE models with sticky prices suggest that the nominal interest rate set by the central bank is causal for the *current* values of the major macroeconomic aggregates and *vice versa*. For example, consider a sudden increase in the nominal interest rate which generates interest income in the next period. If prices are expected to remain (almost) unchanged, the expected real interest rate will rise. As a results, households, behaving according to their individual Euler equations, will have an incentive to reduce current consumption. Thus, current aggregate demand will tend to fall.

Altig *et al.* **(2005):** Altig *et al.* (2005) estimate a larger SVAR containing ten variables two of which are thought to capture important cointegration relations: the common trend in the log of labor productivity and the log of the real wage and the stationarity of the velocity of transaction balances with respect to GDP.[22] The monetary shock is identified by a recursiveness assumption similar to the one used by CEE (2005): The policy instrument, the federal funds rate, responds not only to lagged but also to current values of capacity utilization, the log of working hours, the difference between the log of labor productivity and the log of the real wage, the log of the consumption-output ratio, the log of the investment output ratio and the growth rates of output, the GDP deflator and the relative price of investment.[23] The innovations to a *neutral technology shock* and the one to *capital embodied technology* are identified *via* long run restrictions on the matrix A based on implications of the theoretical model Altig *et al.* (2005) develop in the same paper. The remaining disturbances are not given a particular economic interpretation. Altig *et al.* (2005) obtain the following impulse responses to a monetary policy shock: The reactions of the money growth rate and the interest rate are of limited persistence and are completed within roughly one year. The other variables respond over a longer period of time. Output, consumption, investment, working hours and capacity utilization all display hump-shaped responses, which peak after about a year. After an initial fall, inflation rises before reaching its peak response in roughly two years. The SVAR also predicts the existence of a significant liquidity effect, i.e. the interest rate and money growth move in opposite directions after a policy shock. Finally, the real wage and the price of investment do not respond significantly to a monetary policy shock. The quantitative results in Altig *et al.* (2005), too, are consistent with the findings in CEE (2005).

[22]See Altig *et al.* (2005) for details regarding the construction of the variable called "Transaction Balances".

[23]Formally this is a restriction on the the row of the matrix A in (I.2.1) corresponding to the federal funds rate.

Biovin and Giannoni (2008): Biovin and Giannoni (2008) focus on how the increasing importance of global forces over the last twenty years has altered key business cycles characteristics and the transmission of monetary policy shocks in the USA. The authors employ a so called Factor-Augmented VAR (FAVAR), containing unobservable U.S.-specific and global components (latent factors) reflecting the current state of the economy. To estimate the unobservable states as well as the parameters of the model, Biovin and Giannoni (2008) resort to a two-stage procedure described in Biovin and Giannoni (2006). The monetary policy instrument is measured by the federal funds rate. It is assumed to respond contemporaneously to the domestic and international factors reflecting the state of the economy, while the latter can respond to movements in the federal funds rate only with a lag. In other words, the latent factors are predetermined within the period with respect to monetary policy. The authors motivate this specification of the policy rule by pointing out that when conducting monetary policy the central bank is forced to react to variables which are either measured with error or are unobservable, such as potential output. A perhaps more convincing rationale for the inclusion of unobservable components, would be to interpret them as subjective weights attached by the policy maker to the different signals, leading indicators and variables he observes. These weights can not be observed by the econometrician, and in many cases they are, probably, also unobservable for the policy maker. Such subjective weights may change over time as a result of political pressure or deeper, cognitive factors. Biovin and Giannoni consider two subsamples, 1984:1 to 1999:4 and 2000:1 to 2005:4. The responses of output, different price indexes, investment and consumption obtained, are qualitatively and quantitatively similar to that provided by most of the other studies. The authors conclude that there is no evidence of a significant change in the transmission mechanism of monetary policy due to global forces. The point estimates, however, suggest that the higher importance of global factors in the second subsample might have contributed to reducing the persistence in the responses of the main macroeconomic aggregates. Biovin *et al.* (2007) also use the FAVAR-technique but consider only U.S.-variables. Nevertheless, the magnitude, shape and persistence of the monetary policy-induced reactions of key aggregates are very similar.

Fully Simultaneous Systems: The studies discussed so far assume a particular recursive ordering of the variables[24] or at least that some of them are predetermined.[25] In contrast, Sims and Zha (2006) specify a fully simultaneous system in which each variable can contemporaneously affect each other variable. To extract the monetary shock, Sims and Zha assume that monetary policy affects only a small subset of the endogenous variables directly. The remaining variables are only indirectly linked to the monetary policy instrument. In particular,

[24] Sims (1980), Christiano *et al.* (1999, 2005), Fisher (1997), Gertler and Gilchrist (1994)

[25] Sims (1986), Altig *et al.* (2005), Biovin and Giannoni (2008)

the authors use the federal funds rate as a policy measure and assume that the central bank only sees the current values of the price index of crude materials and a monetary aggregate when setting the current level of the interest rate. Nevertheless, movements in, say, aggregate output, too, have a contemporaneous effect on the federal funds rate since the monetary aggregate as well as the price of crude materials are directly linked to the current level of output. When the monetary aggregate is measured by M 2, the SVAR by Sims and Zha (2006) delivers results which are consistent with that obtained by CEE (1999): output and real wages display a persistent and significant decline, even in the period of the shock. The GDP deflator also responds negatively, but with a substantial delay. If, however, total reserves are used as the monetary aggregate, the responses of output and real wages become insignificant. The response of the GDP deflator remains unchanged. Examples for further SVAR studies which do not assume a recursive structure are Gordon and Leeper (1994) and Leeper *et al.* (1996). However, the models constructed in these papers can not be characterized as fully simultaneous since they contain predetermined variables. Thus, these models are more similar to the one proposed by Sims (1986) rather than to that of Sims and Zha (2006).

CEE (1999) reproduce the results obtained by Sims and Zha (1995), which is the working-paper version of Sims and Zha (2006). CEE (1999) use the same economic variables but their time series include more observations and are partly taken from data sources different from that used by Sims and Zha (1995). CEE (1999) use M 2 as a measure of the monetary aggregate and claim to find support for the results obtained with their FF, NBR and NBR/TR models. However, a more careful look at the impulse responses implied by the Sims-Zha SVAR reproduced by CEE (1999) should have led to a quite different interpretation! As a reaction to a contractionary monetary policy shock output initially increases. This is the only significant deviation of output from its long-run level. The point estimate of its response in the second quarter is also slightly positive but insignificant. In the following quarters output displays small, almost negligible fluctuations around its initial level. They are all insignificant. The only significant response of M 2 is a decrease in the quarter *after* the shock. The GDP deflator exhibits a persistent but insignificant decrease. The price of crude materials falls on impact and remains significantly lower than its initial level for about 3 quarters. The real wage takes an above average value for 1 to 2 quarters, but the deviation is insignificant. In sum, these results are neither consistent with most of the SVARs presented in CEE (1999) nor can they be reconciled with most of the modern monetary models of the new keynesian type. The Sims-Zha responses of output, prices, real wages and M 2 reproduced by CEE (1999) rather indicate that flexible-price models including cash-in-advance constraints or a plausibly specified transactions technology should be considered good candidates for explaining the cyclical patterns induced by monetary shocks.

2.2 Long-Run Restrictions

The SVAR studies discussed so far identify the unsystematic component of the monetary policy rule by restricting the behavior of the system within the period of the shock.[26] The long-run behavior of the system, however, is left unrestricted.

View authors have chosen a different approach for identifying monetary policy shocks. They impose restrictions on the limiting or *long-run* effects of particular shocks, in order to recover the relation between the observable and the structural shocks. To state it in more formal terms, consider the moving average representation of the VAR defined in (I.2.1):

$$Y_t = \sum_{j=0}^{\infty} C_j A^{-1} B \epsilon_{t-j} = C(L) A^{-1} B \epsilon_t = (1 - \sum_{i=1}^{n} A_i L^i)^{-1} A^{-1} B \epsilon_t,$$

where constant terms were dropped and $C(L)$ denotes the infinite lag polynomial implied by the VAR. L is the lag operator. The long-run effects of the shocks on the endogenous variables are obtained by setting $L = 1$:

$$C(1) A^{-1} B = (1 - \sum_{i=1}^{n} A_i)^{-1} A^{-1} B.$$

Long-run restrictions are imposed by setting some of the elements of the matrix $C(1)A^{-1}B$ at particular values, e.g. zero. Usually, A or B is assumed to be an identity matrix. Examples for such studies are Lastrapes and Selgin (1995), Pagan and Robertson (1995) and Cochrane (1994).

Lastrapes and Selgin (1995) measure the policy instrument by the monetary aggregate M 0. Therefore, in their study policy shocks are identical with money supply innovations. To identify the latter, Lastrapes and Selgin (1995) impose the restriction that a unit money supply shock causes prices to rise by a unit in the long run. In other words, in the long run real balances do not change. At the same time, the monetary policy shock is restricted to have a zero long-run effect on output and interest rates. The remaining shocks are also identified *via* long-run restrictions.[27] The responses obtained by Lastrapes and Selgin (1995) are not summarized here since they are similar, although slightly more pronounced, to that provided by CEE (1999). Pagan and Robertson (1995) examine the sensitivity of the specification proposed by Lastrapes and Selgin (1995) with respect to different types of restrictions. In particular, Pagan and Robertson (1995) retain the long-run restriction identifying the monetary policy shock but substitute some or all of the other restrictions by different short-run constraints, similar to that used by, e.g., CEE (1999, 2005). They show that if a subset of the Lastrapes-and-Selgin restrictions is combined with some short-run restrictions, the shape of the impulse responses remains unchanged but their magnitude becomes substantially smaller.

[26] As already mentioned, such constraints are termed *short-run restrictions*.

[27] See Lastrapes and Selgin (1995) for details.

Cochrane (1994) estimates a five dimensional VECM[28] containing the log of a monetary aggregate M, the federal funds rate FF, the log of output Y, the log of consumption C, the log of the GDP deflator P and the real interest rate R. Cochrane imposes two cointegration relations. The first one stems from the stationarity of the consumption-output ratio $Y - C$. When M is set equal to the aggregate M 2, the second cointegration relation is based on the observation that the velocity $Y + P$-M 2 does not exhibit a long run trend. In the case of M=M 1 it is postulated that

$$\text{M } 1 - P - Y - \alpha FF$$

is stationary. α is an element of the cointegration vector. In both cases the monetary policy shock is identified as that combination of M and FF shocks that has exactly no long-run effect on output. Unfortunately, Cochrane (1994) does not present the mathematical implementation of this restriction explicitly. For the sake of comparison the author also performs some experiments with SVARs containing the same variables but including only *conventional* short-run restrictions. Cochrane (1994) shows that the imposition of the long-run restriction makes the impulse responses of output and consumption less persistent but increases their magnitude. The latter is partly due to the much stronger liquidity effect[29] arising. The peak responses of output and consumption in the case of the long-run restriction are reached about 2 quarters after the shock. However, when M 1 is used as a measure of the monetary aggregate the reactions of Y and C can hardly be characterized as hump-shaped.

2.3 The Non-Econometric Approach

The econometric approach discussed above involve many assumptions which can be problematic or are at least subject to question. For example, the literature proposes different measures of the policy instrument, each of which may be wrong. Even more debatable is the specification of the policy rule: Which variables should appear in it? Is it linear at all and if not, what is the appropriate functional form? To avoid this difficulties some authors have chosen a non-mathematical but more direct way for identifying the unobservable component(s) of monetary policy.

Romer and Romer (RR) (1989, 1994) attempt to identify episodes in which the Fed tried to create a recession to dampen inflation by using the tools available to it. To do that, the authors analyse the records related to policy meetings of the Federal Reserve. RR interpret output movements in the immediate aftermath of such episodes as reflecting reactions to monetary policy rather then responses to other factors. RR motivate their interpretation by showing that on the one hand, in these episodes inflation did not have a direct effect on

[28] Vector Error Correction Model

[29] The decline in nominal interest rates induced by a monetary loosening is referred as the *liquidity effect*.

output and on the other, in these episodes the inflation movements were not induced by shocks which also affected output.

Similar work is done by Boshen and Mills (1991). They construct a monetary policy index based on their reading of the FOMC[30] minutes. The authors rate monetary policy on a discrete scale: $\{-2, -1, 0, 1, 2\}$ ranging from very tight (-2) to very loose (2).

To assess the impact of a Romer-and-Romer episode as well as that of a shift in the Boshen-Mills index on key macroeconomic variables, CEE (1999) modify their SVAR described above as follows: In the RR-case they include current and lagged values of a dymmy variable, assumed to be one during an RR episode and zero otherwise.[31] In the Boshen-Mills-case their policy measure is included as an endogenous variable placed first in the SVAR. The shape of the impulse responses induced by a Romer-and-Romer and a Boshen-and-Mills shock is similar to that obtained by CEE (1999) by using FF, NBR or NBR/TR as a policy measure. However, there are some differences with respect to the magnitude and the delay of the responses. The reactions induced by the RR shock tend to be about 3 to 10 times larger than that triggered off by a shock to the federal funds rate FF. CEE (1999) attribute this difference to the fact, that the Romer and Romer episodes coincide with periods characterized by large increases in the federal funds rate which, in turn, tends to induce more pronounced changes in the key macroeconomic variables. The responses to a Boshen-and-Mills shock have the same magnitude as that to a federal funds shock. The former, however, display much larger delays (about 20 quarters in the case of output) and are in most cases insignificant.

Critique of the approach adopted by Romer and Romer is found in Leeper (1996). His econometric results indicate that the RR episodes reflect endogenous responses to changes in economic conditions and thus, are a poor measure of monetary policy shocks.

2.4 Summary of the Results

The results provided by the SVAR literature, focusing on the dynamic responses of key variables to monetary policy shocks reviewed in this section, can be summarized as follows: The bulk of the evidence indicates that *contractionary* monetary policy shocks trigger off

[30] Fed's Open Market Committee

[31] The *observable* VAR takes the form:

$$Y_t = c + \sum_{i=1}^{n} A_i Y_{t-i} + \sum_{j=0}^{m} \beta_j d_{t-j} + u_t,$$

where d_t is the value of the dymmy variable in period t. The parameters of the model are estimated by OLS.

- *significant* negative reactions of the real aggregates output, consumption, investment and employment, delayed by 1 to 3 quarters
- *significant* negative reactions of the key price measures, such as the GDP deflator, the CPI and various indexes of commodity prices, delayed by 6 to about 20 quarters,
- a negative but *insignificant* decrease in real wages,
- a *significant* increase in various short-run interest rates,
- a *significant* decrease in the monetary aggregates M 0, M 1, M 2.

The impulse responses of prices and real aggregates are very persistent and display a hump-shaped pattern whereas the development of prices over time is much smoother. These results can be reconciled with the predictions of some of the New Keynesian models, including various real rigidities.

However, the empirical literature also provides some important exceptions. For example, the SVARS run by Christiano *et al.* (1999) in which the policy instrument is measured by the monetary aggregates M 0 or M 1, or the Sims-Zha VAR reestimated by Christiano *et al.* (1999). These models imply that a contractionary policy shock leads to:

- a *temporary* increase in output and wages, the latter being insignificant,
- a decrease in prices which is in most cases *insignificant*,
- a temporary drop in the aggregates M 0 and M 1.

The response of prices is the only one that can be characterized as more or less persistent. However, as just mentioned, it is in most cases insignificant. These results can be reconciled with the predictions of flexible-price models including a cash-in-advance constraint or motivating money demand by a transaction technology.

In a number of experiments with different SVAR specifications Cochrane (1994) shows that the predictions regarding monetary shocks are not robust with respect to the number of variables included: The higher the dimension of the SVAR, the lower the magnitude as well as the persistence of the impulse responses, and the lower the importance of the monetary shock as a driving force of the business cycle. The importance of a shock is measured by the fraction of the forecast error variance, FEV, of each endogenous variable it is responsible for. By moving from a three dimensional SVAR to a seven dimensional one the FEV of output at the 1 (3) year horizon explained by the monetary shock drops form about 25% (45%) to about 5% (8%) on average.[32]

[32] The exact numbers vary slightly across different VAR specifications and identification schemes.

2.5 Critique of the VAR Approach

The Lucas-Stokey Critique: The conventional procedure for assessing the performance of competing theoretical models is by comparing their predictions with that of one or more SVARs. Lucas and Stokey (1987) disagree[33] with this approach because the SVAR implications and their interpretation depend crucially on a set of identifying restrictions which, in most cases, are not satisfied in the models under consideration. In other words, the common approach is subject to a statistical inconsistency. Lucas and Stokey (1987) claim that a particular SVAR can only guide the choice among a set of competing theories if each of them is consistent with the identifying restrictions imposed on the VAR. In the same paper Lucas and Stokey develop a cash-in-advance model with flexible prices which is neither consistent with any recursive ordering of the variables in the VAR nor with any of the non-recursive short-run restriction schemes. Inspired by the Lucas-Stokey critique, Kehoe (2006) ironically suggests that SVAR researchers should always include in an appendix a list of the theoretical models that satisfy the identifying restrictions used in the estimation. Kehoe (2006) guesses that in most cases this list will turn to be extremely short.

The Chari-Kehoe-McGrattan Approach: Chari *et al.* (2006) (henceforth, CKM) and Kehoe (2006) point to a problem related to the *common* approach of comparing empirical impulse responses obtained with an SVAR with the theoretical responses implied by the model under consideration. Since on the one hand there is only a finite number of observations and on the other hand for statistical reasons researchers usually use a very small number of lags, the resulting *small-sample bias* and *lag-truncation bias* are in many cases large enough to make the estimated finite order VAR a poor approximation of the model's infinite order VAR. Therefore, CKM suggest to compare the empirical impulse responses to that from identical structural VARs run on the theoretical data instead of simply comparing the empirical to the theoretical impulse responses.[34] CKM provide examples in which SVARs deliver similar results when applied to the empirical as well as the theoretical time series. However, the predictions obtained with the data from the model are quite different from the *true* predictions of the theory. According to CKM, the coefficients and the impulse responses estimated by using the empirical data are sample statistics which should be compared with the *same* statistics obtained from the model, irrespective of whether these statistics have some deep economic interpretation. As CKM note, this would be the same symmetric treatment of empirical and

[33] The critique can be found in the conclusion of Lucas and Stokey (1987).

[34] CKP refer to this approach as the *Sims-Cogley-Nason* approach because it has been advocated by Sims (1989) and applied by Cogley and Nason (1995). However, to the best of my knowledge, this technique was brought to the profession by Chari *et al.* 2006. Therefore, I refer to it as the CKM approach.

theoretical time series moments, as the comparison of actual and theoretical variances and correlations proposed by Kydland and Prescott (1982).

The Leeper-Walker-Yang Warning: Leeper *et al.* (2008) (henceforth, LWY) show that, under fairly general conditions, the equilibrium in an economy characterized by *fiscal foresight*, e.g. foreseen tax changes, has a non-invertible VARMA representation. The simplest form of fiscal foresight is the following law of motion for taxes:

$$\tau_t = \bar{\tau} \exp(u_t + \epsilon_{t-q}), \qquad q > 1$$

where τ_t is the tax rate at time t, u_t is the (unforeseen) tax disturbance, while ϵ_{t-q} denotes the tax disturbance which is known (or anticipated) at time $t-q$. u_t and ϵ_t, both, follow simple White Noise processes. Non-invertibility of the VARMA representation, in turn, implies that the fundamental (structural) shocks to fiscal policy can not be recovered from current or past observable data (e.g. by estimating a VAR), irrespective of how creative the identification scheme is and how many observations and lags are included. But as LWY point out, the same is true with regard to technological or monetary policy foresight. Thus, if agents receive signals about future innovations to monetary policy, then the SVAR technique will be unable to identify the monetary policy shock. Consequently, most of the results delivered by the SVARs will be misleading. The typical lag between when a signal about a future shift in monetary policy is received and when this policy actually gets implemented is probably much shorter than the typical lag between the signal about a tax change and its implementation. Nonetheless, it can not be *a priory* ruled out that there exist such lags with respect to monetary policy and thus, there is some degree of monetary foresight. Unfortunately, LWY do not provide a solution to the potential problem they warn of.

3 Evidence on the Frequency and Size of Price Adjustments

This section provides a brief review of the most recent evidence on the behavior of nominal goods prices obtained with micro-level data. Examples for earlier empirical studies dealing with the properties of the price adjustment process for particular goods are Cecchetti (1979, 1986), Kashyap (1995), Carlton (1986) and many others. They conclude that prices adjust on average once a year. In contrast McCallum (1979), Domberger (1979), Rotemberg (1982), Benabou (1992) and many others use aggregated data and conclude that prices are even stickier.

Bils and Klenow (2004): Bils and Klenow (BK) (2004) use unpublished monthly and bimonthly data on prices for 350 categories of consumer goods and services from the CPI

Research Data Base provided by the Bureau of Labor Statistics (BLS). The sample covers the period from 1995 to 1997. BK distinguish between two types of price changes: regular price changes and transient price changes or *sales*, which are defined as temporary negative deviations from the regular price. When sales are included in the sample, the estimated median frequency of price adjustment equals 20.9%. This figure corresponds to a median duration of prices equal to 4.3 months which is slightly longer than one quarter. Excluding sales implies a median frequency of price changes equal to 16.9%, while the implied median duration of prices is about 5.5 months. A further interesting finding of the BK-study is the substantial dispersion in the frequency of price changes across product categories. It ranges from 54.3% for *raw goods* to 9.4% for *medical care*. To examine the behavior of product specific inflation, BK match the 350 categories to available NIPA time series on prices covering the period from January 1959 to June 2000. The number of resulting categories is 123.[35] BK show that for nearly all 123 product categories, inflation is far more volatile and far less persistent than implied by almost all New Keynesian Models assuming Calvo pricing. The authors adjust an AR(1) process to the inflation series for each category. They find that he mean of the autocorrelation coefficient across categories is close to zero at -0.05 (standard error 0.02), while the mean of the standard deviation of the innovation to the AR(1) process is 0.83 (standard error 0.08). Furthermore, the average correlation between the autocorrelation coefficient of the AR(1) process and the frequency of price adjustment is positive which is at odds with the prediction of the Calvo model. The latter implies that a higher fraction of firms which are not able to adjust their prices within a period and, thus, a lower average frequency of price adjustments, leads to a higher inflation persistence.

Klenow and Kryvstov (2005): Klenow and Kryvstov (KK) (2005) also use the detailed monthly CPI data provided by the BLS. The KK panel covers 123 product categories over the period from January 1988 through December 2003. KK, too, distinguish between periods with regular prices and periods with sales. The findings of KK with regard to the frequency of price changes can be summarized as follows: The average frequency of price adjustments on monthly basis equals 29.3%, when sales are included, and 23.3%, when sales are excluded. The corresponding average price durations are 3.41 months and 4.29 months respectively. KK further decompose the variance of inflation into two components: changes in the fraction of items adjusting their price (extensive margin) and changes in the average size of price adjustments (intensive margin). The authors find that about 95% of the variance of monthly inflation is due to the intensive margin, while the fraction of items changing prices fluctuate much less and are, thus, responsible for only 5% of inflation volatility.

[35] See Bils and Klenow (2004) for details.

Nakamura and Steinsson (2008): Nakamura and Steinsson (NS) (2008) use monthly data on individual products' prices from the CPI and PPI Research Data Bases provided by the BLS. The data covers the period from 1988 to 2005. NS distinguish between three types of price changes: regular price changes, sales and price changes associated with product substitutions, which are defined as price changes due to the introduction of new products, e.g. when switching from the spring to the fall clothing seasons. The authors hold the view that sales and product substitutions should be assumed orthogonal to macroeconomic conditions and thus *"fundamentally different from regular price changes typically emphasized by macroeconomists" (p. 1417)*. The results obtained with CPI data can be summarized as follows: Including sales and product substitutions implies a median frequency of price changes between 19.4% and 20.3% with a corresponding median duration of prices lying between 4.4 and 4.6 months. If sales and product substitutions are excluded from the sample, the respective numbers are between 8.7% and 11.1% for the median frequency of price changes and between 8.5 and 11 months for the median duration. There is an extremely large dispersion in the results obtained with PPI data. If product substitutions are excluded, the median frequency of price adjustments (median duration) are 10.8% (9.3 months) for finished goods, 13.3% (7.5 months) for intermediate goods and 98.9% (1.01 months) for crude materials. If product substitutions are included, the respective numbers are 12.1% (8.26 months) for finished goods, 14.9% (6.7 months) for intermediate goods and 98.9% (1.01 months) for crude materials. The median size of price changes within the CPI data equals 10.7% (of the initial level), with a high degree of heterogeneity across product categories. The corresponding number within the PPI data is 7.7%.

Examples of further studies providing similar evidence are Burstein and Hellwig (2007) and Chevalier *et al.* (2007) who use scanner data on retail sales provided by Dominick's Finer Food, a large supermarket chain with 86 stores in the Chicago area,[36] Gagnon's (2005) study of pricing behavior in Mexico between 1990 and 2000 and Dhyne *et al.* (2005) who investigate the pricing patterns arising in several European countries.

Kehoe and Midrigan (2008): Kehoe and Midrigan (2008) (henceforth, KM) also use the scanner data provided by Dominick's Finer Food, and also distinguish between regular price changes and sales. The authors document the following six facts:

- *Prices change frequently, but most price changes are temporary, and after temporary changes, prices tend to return to the regular price.*[37] According to the calculations performed by KM prices change on average every three weeks (when sales are included). Excluding sales implies an average price duration of about one year.

[36] Chevalier *et al.* (2007) use only the data on the price of Triscuits.
[37] KM (2008), p. 13.

- *Most temporary changes are cuts, not increases.*[38]
- *During a year, prices stay at their annual modal value most of the time. When prices are not at their mode, they are much more likely to be below it than above it.*[39]
- *Price changes are large and dispersed.*[40] According to KM, the average size of price changes is 17% of the initial value. The average *regular* price change is 11%.
- *Periods of temporary price cuts account for a disproportionately large share of goods sold. Quantities sold are more sensitive to prices when prices decline temporarily than when they decline permanently.*[41]
- *Price changes are clustered in time.*[42] The clustering of prices are measured by the hazard rate for price changes. For example, if a store has changed the price of a given product last week, then the probability to change that price again this week is about 38%. KM show that the hazard rate sharply declines in the first two weeks after a price change and follows a slightly negative trend thereafter.

In sum, the most recent empirical evidence indicates that the degree of price stickiness is extremely low, or at least much lower than usually assumed in the New Keynesian Models with Calvo pricing. As we will see later, if the calibration of the standard Calvo model is based on one of these studies, then the degree of monetary non-neutrality becomes substantially lower. For example, if one assumes that the average price duration is about 4.3 months, implying that 70% of all firms adjust their prices within a period, then the magnitude of the impulse responses in the Calvo economy becomes about five times smaller than when only 25% of all firms are able to set their prices optimally. Finally, if we want to be slightly more aggressive when interpreting the empirical results presented in this section, then we should conclude that there is no price stickiness at all!

4 The Cyclical Behavior of Markups

In a comprehensive survey of the empirical studies on the cyclical behavior of prices and marginal costs Rotemberg and Woodford (1999) emphasized the great importance of markups for output fluctuations at business cycle frequencies. According to their results, the output fluctuations attributable to variations of markups, which are orthogonal to fluctuations induced by shifts in the marginal cost curve, account for about 90% of the variance of output

[38]KM (2008), p. 14.
[39]KM (2008), p. 14.
[40]KM (2008), p. 14.
[41]KM (2008), p. 14.
[42]KM (2008), p. 14.

growth in the short run.[43] In addition, it can be easily shown that endogenous markup variation on the aggregate level has the potential to substantially magnify (or dampen) business cycles or make them more (or less) persistent.[44] Consider for example a positive supply side shock, e.g. a favorable technology disturbance, in a symmetric equilibrium. If markups remain constant the shock will have a positive effect on output through shifting the marginal cost curve downwards. But if the disturbance generates strong enough an incentive for firms to lower (raise) markups then the output reaction will become stronger (weaker) than in the constant-markups case. A demand side shock which doesn't shift the marginal cost curve will have no impact on aggregate output if firms are unable or unwilling to change markups. But if firms do lower (rise) markups in response to the demand shock output will rise (fall).

Real marginal costs and, thus, markups are not directly observable on the macro level. Even if one were able to estimate the cost curve of each individual firm in the economy, aggregation across all firms would be intractable. For that reason most authors approximate aggregate marginal costs or aggregate markups by some simple function of the labor share of GDP, labor costs or labor productivity, the output or unemployment gaps, a fiance variable such as Tobin's q, material or energy input prices, inventories or by a combination of two or more of them.

Rotemberg and Woodford (1999): Rotemberg and Woodford (RW) (1999) use quarterly NIPA data on various macroeconomic aggregates and economy wide wages and prices over the period from 1969:1 trough 1993:1. The simplest measure of average markups considered by RW is based on the following observation. Consider the Cobb-Douglas technology:

$$Y_t = N_t^{\omega} K_t^{1-\omega}, \qquad \omega \in (0,1),$$

where Y_t, N_t and K_t denote output, labor input and capital input of a typical firm. If the goods market is monopolistically competitive and, in addition, the equilibrium is symmetric, then the first order condition with respect to labor input of the firm reads:

$$\mu_t \omega \frac{Y_t}{N_t} = \frac{W_t}{P_t},$$

where μ_t denotes real marginal costs. W_t and P_t are the nominal wage and the nominal price level respectively. Since the markup mu_t equals the inverse of real marginal costs, the last

[43]Rotemberg and Woodford (1999) decompose output into two components. The fluctuations of the first result solely from shifts in the marginal cost curve for a constant markup while the second component responds only to deviations of markups from their steady state values, and hence represents movements along the marginal cost curve. Rotemberg and Woodford (1999) use the *predicted declines of output* as measure of the cyclical component of output and compare it with the two components of output growth they identify.

[44]Rotemberg and Woodford (1999) provide a simple example.

equation implies:

$$mu_t = \frac{1}{\mu_t} = \omega \cdot \underbrace{\frac{Y_t}{(W_t/P_t)N_t}}_{:=(s_t)^{-1}}, \tag{I.4.1}$$

where s_t denotes the labor share in output. Thus, the variations in markups can be recovered from observable variations in the labor share. RW report a small, negative correlation between the inverse of the labor share in output and HP-filtered GDP, equal to -0.095. The authors point out that mu_t (I.4.1) was derived under very strong assumptions (Cobb-Douglas technology, identity between marginal and average wages) and, thus, should be considered a very unprecise approximation of the actual markup. Therefore, RW proceed by constructing more elaborate measures of the *true* markup. First, RW assume that the aggregate production technology is given by the less restrictive CES function:

$$Y_t = \{\omega N_t^{\frac{\epsilon-1}{\epsilon}} + (1-\omega) K_t^{\frac{\epsilon-1}{\epsilon}}\}^{\frac{\epsilon}{\epsilon-1}}.$$

The corresponding output elasticity with respect to labor is given by:

$$\nu_t = 1 - (1-\omega)\left(\frac{Y_t}{K_t}\right)^{-\frac{\epsilon-1}{\epsilon}}.$$

Defining

$$\vartheta = \left(\frac{\epsilon-1}{\epsilon}\right)\left(\frac{1-\nu}{\nu}\right),$$

where ν is the steady state value of ν_t and log-linearizing around the steady state yields:

$$\widehat{mu}_t = \vartheta \widehat{\left(\frac{Y_t}{K_t}\right)} - \hat{s}_t, \tag{I.4.2}$$

where hats denote relative deviations from the stationary equilibrium. RW set the elasticity of substitution ϵ at 0.5 and together with the average labor share, 0.7, are able to calibrate ϑ. The resulting value is $\vartheta = -4$. The correlation of the markup defined in (I.4.2) with the cyclical component of GDP[45] is equal to -0.402. As a further extension, RW consider *overhead labor*. In this case the production function is given by

$$Y_t = (N_t - \bar{N})^{\omega} K_t^{1-\omega},$$

where $\bar{N}$ is the amount of overhead labor which must be hired regardless of the quantity of output that is produced. The presence of overhead labor in an otherwise standard Cobb-Douglas

[45] The cyclical component of GDP is measured by the *"Expected Declines of GDP"*. See Rotemberg and Woodford (1999) for details.

production function implies increasing returns to scale, although marginal cost remains independent of scale. The resulting output elasticity with respect to labor is again time varying and given by:

$$\nu_t = \omega \left(\frac{N_t}{N_t - \bar{N}} \right).$$

With these definitions and results it can be shown that the log deviation of the markup from its steady state evolves according to:

$$\widehat{mu}_t = -\frac{\nu}{\omega}\hat{N}_t - \hat{s}_t \tag{I.4.3}$$

The correlation between output and the markup approximated according to (I.4.3) equals -0.212.[46] The next extension is the assumption that the wage a typical firm has to pay is an increasing function of labor input. As a consequence, the marginal wage will be higher than the average wage.[47] RW assume that the dependence of the marginal-to-average wage ratio Q_t on average hours per worker H_t is given by a function proposed by Bils (1987). RW show that in this case the following equation holds:

$$\widehat{mu}_t = -\kappa\hat{H}_t - \hat{s}_t, \tag{I.4.4}$$

κ is the elasticity of the marginal-to-average wage ratio Q_t with respect to H_t. The choice $\kappa = 1.4$ is based on estimation results by Bils (1987). The resulting correlation between the markup and the cyclical component of GDP is equal to -0.372. Finally, RW assume that there are labor adjustment costs, taking the form $\pi_t N_t \phi\left(\frac{N_t}{N_{t-1}}\right)$, where π_t is the price of the input required to make the adjustment. The total cost associated with hiring an additional worker for one period is then given by:

$$W_t + \pi_t \left(\phi\left(\frac{N_t}{N_{t-1}}\right) + \frac{N_t}{N_{t-1}}\phi'\left(\frac{N_t}{N_{t-1}}\right)\right) - W_t E_t \left\{ \rho_{t,t+1}\frac{\pi_{t+1}}{\pi_t}\frac{N_{t+1}^2}{N_t^2}\phi'\left(\frac{N_{t+1}}{N_t}\right)\right\},$$

where $\rho_{t,t+1}$ denotes the stochastic discount factor. Under the assumption that $\frac{\pi_t}{W_t}$ is stationary the economy wide markup evolves according to:

$$\widehat{mu}_t = -\phi''(1)\frac{\pi}{W}(\Delta\hat{N}_t - \rho g_\pi E_t \Delta\hat{N}_{t+1}) - \hat{s}_t, \tag{I.4.5}$$

where ρ and g_π denote the steady state values of $\rho_{t,t+1}$ and $\frac{\pi_{t+1}}{\pi_t}$ respectively. RW calibrate the relevant elasticities as follows: $\phi''(1)\frac{\pi}{W} = 4$ and $\rho g_\pi = 0.99$. The correlation of the markup measure defined in (I.4.5) with the cyclical component of GDP is -0.542. RW also argue that explicitly allowing for labor hoarding also results in markup measures which are more countercyclical than that defined in (I.4.1).

[46] See Rotemberg and Woodford (1999) for details on the calibration of overhead labor $\bar{N}$ and the corresponding ν.

[47] A typical example are the above average wages payed for overtime hours.

Boldrin and Horvath (1996), Gomme and Greenwood (1995) and Ambler and Cardia (1996) also provide negative estimates of the correlation between the output and the labor share. Bils (1987) also reports countercyclical behavior of markups after controlling for the fact that higher wages are paid for overtime hours. Using regression analysis he finds that markups fall by 0.33% for each one-percent increase in employment. Basu's (1995) measure of markups based on data on energy and materials inputs is also countercyclical. Galeotti and Shiantarelli (1998), Bils and Kahn (1996) and Comin and Gertler (2003), too, provide evidence that markups are countercyclical. Also related to the short run fluctuations of marginal costs and markups is the VAR evidence provided by Christiano *et al.* (2005). They show that an expansionary monetary shock induces an increase in employment[48] and real wages. But if capital is fixed in the short run and there is diminishing marginal product of labor the positive response of employment can be associated with higher real wages only if markups fall. Hence, the impulse responses estimated by Christiano *et al.* (2005) can be seen as evidence supporting the hypothesis of countercyclical markups.

Gali, Gertler and Lopez-Salido (2002): Gali *et al.* (2002) (henceforth, GGL) also emphasize the importance of markup variations at business cycle frequencies. The authors focus on the welfare losses due to deviations from zero of the log gap, g_t, between the log of the marginal productivity of labor, mpl_t, and the log of the marginal rate of substitution between consumption and leisure, mrs_t,

$$g_t = mrs_t - mpl_t.$$

Under the assumption that households are wage setters engaging in monopolistic competition in the labor market, the following relation between the wage markup $mu_{w,t}$ and the marginal rate of substitution holds:[49]

$$mu_{w,t} = w_t - p_t - mrs_t,$$

where w_t and p_t denote the nominal wage and the nominal price level respectively. The economy wide price markup, $mu_{p,t}$, is defined as[50]

$$mu_{p,t} = mpl_t - (w_t - p_t).$$

Combining the last three equations yields:

$$g_t = -(mu_{p,t} + mu_{w,t}).$$

[48]In fact, Christiano *et al.* (2005) estimate the impulse responses of output to monetary shocks. But, as capital is a predetermined state variable, increases in output can occur only if hours increase.

[49]Precisely speaking, $mu_{w,t}$ is the markup rate. The wage markup is usually defined as $1 + mu_{w,t}$.

[50]Precisely speaking, $mu_{p,t}$ is the markup rate. The price markup is usually defined as $1 + mu_{p,t}$.

Hence, the deviations of the efficiency gap g_t from zero are solely due to deviations of $mu_{p,t}$ and/or $mu_{w,t}$ from zero. Under the assumption that labor is the only factor of production and at the same time technology is linear in it, the price markup can be written (up to an additive constant) as:

$$mu_{p,t} = y_t - n_t - (w_t - p_t),$$

where n_t is labor input. To measure mrs_t and thus the wage markup, GGL assume a fairly standard utility function, implying

$$mu_{w,t} = w_t - p_t - (\eta c_t + \theta n_t) + \xi_t,$$

where c_t denotes the log of consumption. $-\eta$ and θ denote the coefficient of relative risk aversion and the Frish-elasticity of labor supply. ξ_t is a low-frequency preference shifter, identified as the cubic trend in the marginal rate of substitution mrs_t. GGL use $\eta = 1$ and $\theta = 1$ as benchmark values but experiment by varying the two parameters. The production side and thus, the measurement of the price markup is also subject to a sensitivity analysis. GGL experiment by assuming a CES technology (including capital) or modeling overhead labor, or assuming a difference between the marginal and the average wage (e.g. due to overtime hours) or adjustment costs of labor. All these experiments deliver similar results which can be summarized as follows:[51] All specifications imply a strong negative correlation between the wage markup and GDP, ranging between -0.71 and -0.92. The correlation between the price markup and GDP is also negative but substantially lower in absolute value, ranging between -0.02 and -0.21. The lowest value, -0.02, is obtained for the benchmark case. All other specifications imply that the correlation between mu_p and GDP is larger than 0.10 in absolute value. Further, the wage markup is much more volatile than the price markup. The welfare metric constructed by GGL is proportional to the variance of the efficiency gap g_t and indicates that the average yearly welfare loss associated with post war U.S. business cycles equals 0.01% of average one year's consumption. The welfare costs associated with the two recessions of the mid 1970s and of the early 1980s as well as that of the early 1990s, however, are substantial: the loss due to the recessions in the 1970s and 1980s range between 4.5% and about 8% of average one year's per capita consumption, depending on the specification used to measure the gap. The corresponding numbers for the milder recession of the 1990s are 2% and 3%. These results support the findings in Rotemberg and Woodford (1999) regarding the importance of markup fluctuations.

[51] GGL use quarterly National Accounts data provided by the NIPA, covering the period 1960:1 to 2004:4.

It should be noted that there is much less empirical evidence indicating that markups are procyclical. For example Domowitz *et al.* (1986), Ramey (1991) and Kollman (1996) reach the conclusion that the correlation between markups and output is positive.[52]

5 The Standard New Keynesian Model with Calvo Price Setting

Consider the following standard New Keynesian Model.

5.1 The Model

Households

Let agents in this economy have preferences over consumption, real balancesand working hours given by

$$U = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\eta}}{1-\eta} + \phi \frac{(M_t/P_t)^{1-\chi}}{1-\chi} - \frac{b}{2} N_t^2 \right) \right\}, \qquad \phi, b, \eta, \chi > 0, \quad \beta \in (0,1),$$

where M_t/P_t and N_t denote real balances and working hours. In the above expression C_t is a composite good that includes all varieties:

$$C_t = \left\{ \frac{1}{n} \sum_{i=1}^{n} C_{i,t}^{\frac{\theta-1}{\theta}} \right\}^{\frac{\theta}{\theta-1}}, \qquad \theta > 1. \tag{I.5.1}$$

The demand function with respect to variety i is given by

$$C_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\theta} \cdot \frac{C_t}{n}, \tag{I.5.2}$$

where $P_{i,t}$ and P_t denote the price of variety i and the aggregate price level respectively. The corresponding utility-based price index is given by:

$$P_t = \left\{ \frac{1}{n} \sum_{i=1}^{n} P_{i,t}^{1-\theta} \right\}^{\frac{1}{1-\theta}}.$$

The budget restriction of the representative household is given by:

$$C_t + m_{t+1} - \frac{m_t}{\pi_t} = \frac{W_t}{P_t} N_t + \Pi_t + \frac{T_t}{P_t},$$

where W_t, π_t, Π_t, T_t and $m_t = \frac{M_t}{P_{t-1}}$ denote the nominal wage, the inflation factor, real profits, nominal net transfers form the government and real balances respectively.

[52] Domowitz *et al.* (1986) is an example of a cross-sectional study of the behavior of marginal costs. Ramey (1991) and Kollman (1996) use inventory data to construct their measures of marginal costs.

The first order conditions of the representative household read:

$$C_t^{-\eta} = \Lambda_t, \tag{I.5.3}$$

$$bN_t = \Lambda_t \frac{W_t}{P_t}, \tag{I.5.4}$$

$$\beta\phi m_{t+1}^{-\chi} E_t\{\pi_{t+1}^{\chi-1}\} = \Lambda_t - \beta E_t \left\{\frac{\Lambda_{t+1}}{\pi_{t+1}}\right\}, \tag{I.5.5}$$

$$C_t + m_{t+1} - \frac{m_t}{\pi_t} = \frac{W_t}{P_t} N_t + \Pi_t + \frac{T_t}{P_t}. \tag{I.5.6}$$

Firms

There are n product varieties, each produced by a profit maximizing monopolistic firm according to the linear production function

$$Y_{i,t} = Z_t N_{i,t},$$

where $N_{i,t}$ denotes labor input of firm i. Z_t denotes the total factor productivity which follows a stochastic process given by:

$$\ln(Z_t) = \rho_z \ln(Z_{t-1}) + \epsilon_t,$$

where ϵ_t follows a *White Noise Process* with variance σ_ϵ^2. Each period a typical firm i faces a constant probability $(1-\varphi) \in (0,1)$ of being able to choose its price $P_{i,t}$ optimally. With probability φ its price is mechanically adjusted according to

$$P_{i,t} = \pi P_{i,t-1},$$

where π is the steady state inflation factor. If at time t firm i is given the opportunity to adjust its price, it maximizes the following objective function:

$$E_t \sum_{j=0}^{\infty} \varphi^j \varrho_{t,t+j} \left(\frac{\pi^j P_{i,t}^o}{P_{t+j}} - \mu_{t+j}\right) \left(\frac{\pi^j P_{i,t}^o}{P_{t+j}}\right)^{-\theta} \frac{C_{t+j}}{n},$$

with respect to $P_{i,t}^o$, where $\mu_t = \frac{W_t/P_t}{Z_t}$ denotes real marginal costs and $\varrho_{t,t+j} = \beta^j \frac{C_{t+j}^{-\eta}}{C_t^{-\eta}}$ is the stochastic discount factor. The resulting first order condition can be written as:

$$\frac{P_{i,t}^o}{P_t} = \frac{\theta}{\theta-1} \frac{E_t \sum_{j=1}^{\infty} \varphi^j \beta^j C_{t+j}^{1-\eta} \mu_{t+j} \left(\frac{P_{t+j}}{\pi^j P_t}\right)^{\theta}}{E_t \sum_{j=1}^{\infty} \varphi^j \beta^j C_{t+j}^{1-\eta} \left(\frac{P_{t+j}}{\pi^j P_t}\right)^{\theta-1}}$$

After some tedious algebraic manipulations,[53] the behavior of the adjusting and non-adjusting firms can be aggregated to the following log-linear, forward looking *Phillips Curve*:

$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \frac{(1-\varphi)(1-\beta\varphi)}{\varphi}\hat{\mu}_t.$$

The markup over marginal costs is given by

$$mu_t = \frac{1}{\mu_t}.$$

Government

The central bank finances its lump-sum transfers to the public by changes in the nominal quantity of money:

$$M_{t+1} - M_t = T_t.$$

It is further assumed that in each period transfers constitute a fraction of current money supply:

$$T_t = (\tau_t - 1)M_t,$$

where the percentage deviation of τ_t from its steady state $\hat{\tau}_t$ follows a first order autoregressive process

$$\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + u_t, \qquad \rho_\tau \in [0,1).$$

u_t is assumed to be a *White Noise Process* with variance σ_u^2.

Equilibrium

In equilibrium, real wages and profits are given by

$$\frac{W_t}{P_t} = \frac{Z_t}{mu_t} \qquad \text{and} \qquad \Pi_t = \left(\frac{mu_t - 1}{mu_t}\right) Z_t N_t$$

respectively. These two results, together with the households first order conditions, (I.5.3) through (I.5.6) and the forward looking *Phillips Curve* describe the evolution of the economy.

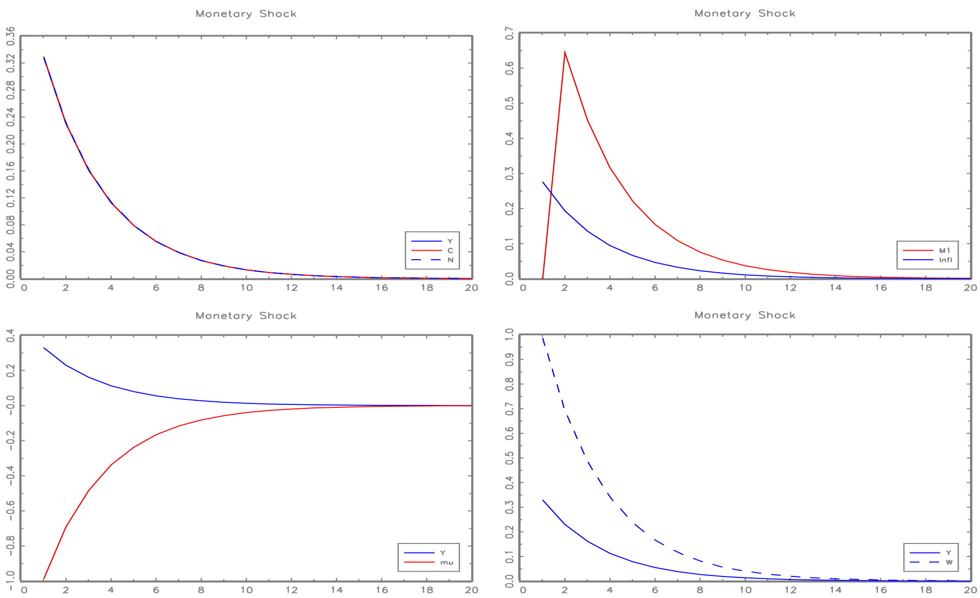
The model is parameterized on a quarterly basis as follows: $\beta = 0.991$, $N^* = 0.1386$, $\eta = 2$, $\chi = 2$, $\rho_\tau = 0$ or $\rho_\tau = 0.5$, $\sigma_u = 0.0092$, $\rho_z = 0.9641$, $\sigma_\epsilon = 0.0082$ and $mu^* = 1.2$. Note, that in this model the choice of mu^* affects only the impulse responses of profits.

[53] See for example Walsh (2003), p. 232 - 240 and p. 263 - 266.

5.2 Results

Most authors set the fraction of firms not able to adjust their prices φ at a value between 0.7 and 0.8, implying an average duration of prices lying between 3.33 and 5 quarters. Setting $\varphi = 0.75$ leads to the impulse responses to a temporary monetary shock without serial correlation, depicted in figure I.1.

Figure I.1: New Keynesian Model. Impulse responses to a monetary shock in $t = 1$, $mu^* = 1.2$, $\rho_\tau = 0$, $\varphi = 0.75$.



Percentage deviations from the long run mean. Y-Output, C-Consumption, N-Hours, W-Real Wage, M_1-Real Balances, $Infl$-Inflation, mu-Markup.

The responses of output, consumption, employment and inflation are not hump-shaped as suggested by the SVAR literature, but there are substantial real effects of monetary policy (output increases by 0.33%) and a relatively large degree of persistence - all real variables remain for more than 7 quarters well above average. Markups respond negatively to the innovation in money supply. Note, that the peak-response of output implied by the model (0.33%) is weaker than what is found by the bulk of the SVAR studies (between 0.5% and 0.7%). For example, Christiano *et al.* (2005) estimate the peak-response of output at 0.6%.

Christiano *et al.* (1997, 2005) suggest representing the monetary policy rule within a theoretical model as the following moving average process for the deviation of the growth factor of money supply from its steady state level:

$$\hat{\tau}_t = \theta(L)u_t = \sum_{i=0}^{\infty} \theta_i u_{t-i},$$

where u_t is the monetary shock and the θ_is are the impulse response coefficients implied by the SVARs run in Christiano *et al.* (1997, 2005). Based on the estimated θ_i, $i = \{0, 1, 2\}$,[54]

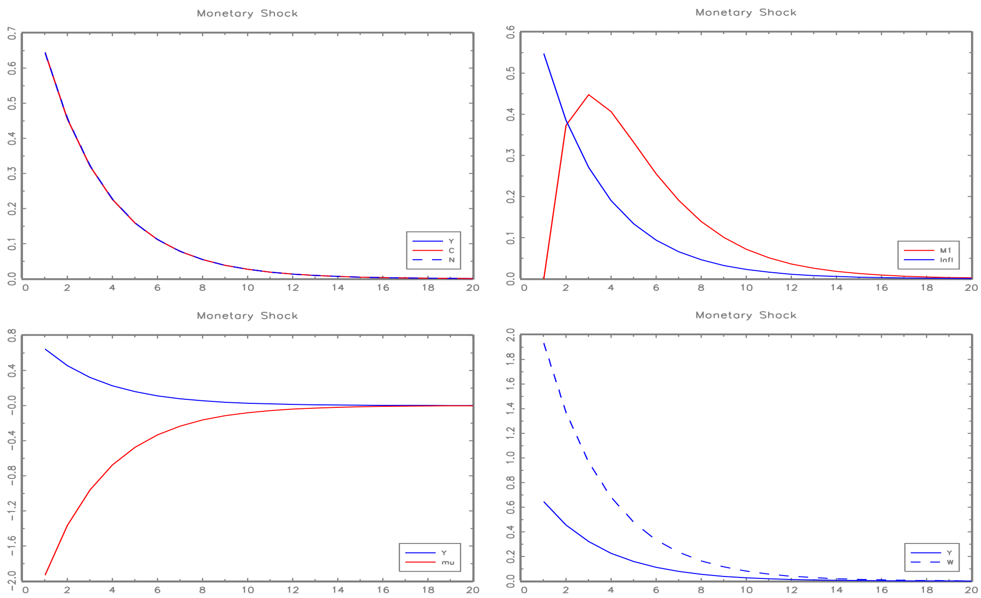
[54] The θ_i for $i = \{3, 4, \ldots\}$ are statistically indistinguishable from zero (see Christiano *et al.* (1997)).

Christiano *et al.* (1997) show that the monetary policy rule is well approximated by the AR(1) process:

$$\hat{\tau}_t = 0.5\hat{\tau}_{t-1} + u_t.$$

Setting ρ_τ at 0.5 in the New Keynesian model implies the impulse responses displayed in figure I.2. The degree of persistence is virtually unchanged, while the reactions of the real variables are of much larger magnitude. The peak-response of output is now consistent with most of the empirical findings. Figure I.3 depicts the reactions to an autocorrelated technology shock.

Figure I.2: New Keynesian Model. Impulse responses to a monetary shock in $t = 1$, $mu^* = 1.2$, $\rho_\tau = 0.5$, $\varphi = 0.75$.

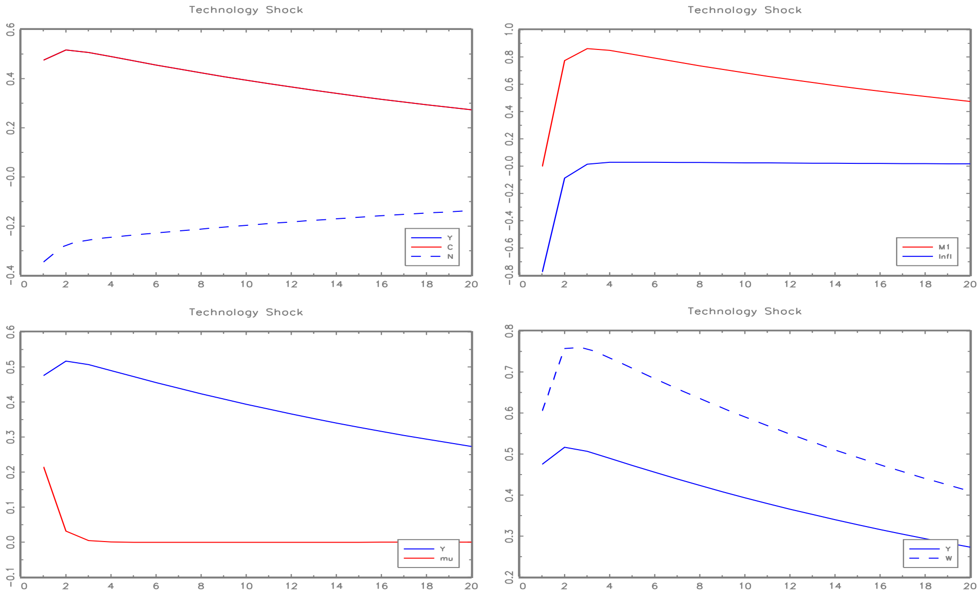


Percentage deviations from the long run mean. Y-Output, C-Consumption, N-Hours, W-Real Wage, M_1-Real Balances, $Infl$-Inflation, mu-Markup.

As can be seen, markups respond positively. Hence, technology shocks make markups less countercyclical. However, since the markup responses induced by productivity disturbances are much weaker and substantially less persistent than that induced by monetary policy, the latter is the dominant driving force of the cyclical variations in mu_t. Not surprisingly, the implied correlation between output and the markup is equal to -0.64 and -0.26 for $\rho_\tau = 0.5$ and $\rho_\tau = 0$ respectively. These numbers are consistent with the empirical evidence surveyed in section 4.

Let us now base the calibration of φ on the recent evidence on nominal price adjustment presented in section 3. For example, according to Bils and Klenow (2004), if temporary sales are taken into account, the average duration of prices is 4.3 months. These number, in turn, implies that, on quarterly basis, about 70% of all firms are able to adjust their prices each period, corresponding to $\varphi = 0.3$. The impulse responses associated with $\varphi = 0.3$, $\rho_\tau = 0$ and $\varphi = 0.3$, $\rho_\tau = 0.5$ are depicted in figures I.4 and I.5. The real effects of monetary policy

Figure I.3: New Keynesian Model. Impulse responses to a technology shock in $t = 1$, $mu^* = 1.2$, $\rho_z = 0.9641$, $\varphi = 0.75$.



Percentage deviations from the long run mean. Y-Output, C-Consumption, N-Hours, W-Real Wage, M_1-Real Balances, $Infl$-Inflation, mu-Markup.

are now about 5 times (for $\rho_\tau = 0$) and 3 times (for $\rho_\tau = 0.5$) weaker than what is found empirically. Further, the reactions of Y_t, W_t/P_t and mu_t are much less persistent than in the case of $\varphi = 0.75$, and are virtually complete after about two quarters. At the same time, $\varphi = 0.3$ implies a slightly stronger response of mu_t to technology shocks, while that of Y_t and N_t remain virtually unchanged. As a consequence, markups become almost acyclical in the case of $\rho_\tau = 0.5$ and even procyclical for $\rho_\tau = 0$. In the latter case the correlation between Y_t and mu_t is about 0.18. Further, irrespective of the value of φ, the model contains the counterfactual implication that the reaction of the real wage to monetary shocks is much stronger than that of output. The reverse is found in the data.[55]

What about capital accumulation? The standard New Keynesian model with adjustment costs of capital[56] delivers similar results as when there is no capital at all: As long as the fraction of firms not able to adjust their prices within the period is large ($\varphi = 0.75$) the predictions of the model with respect to the magnitude of the impulse responses to monetary shocks and the cyclical behavior of markups are well in line with the empirical evidence presented in the previous sections. The responses of the real variables, again, do not display a hump-

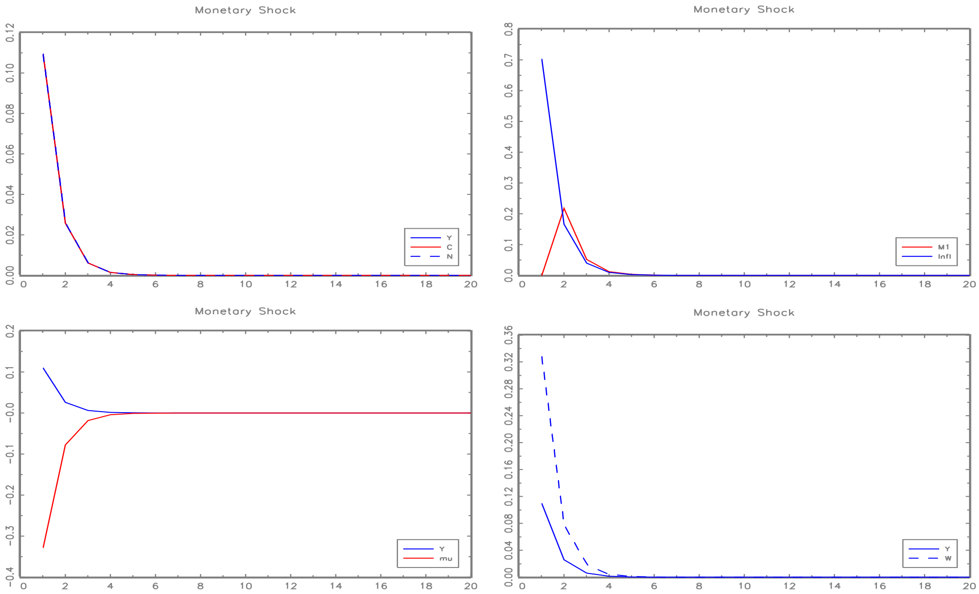
[55] See for example Christiano *et al.* (2005), Altig *et al.* (2005), Kydland and Prescott (1982).

[56] In this case capital evolves according to

$$K_{t+1} = \phi\left(\frac{I_t}{K_t}\right) K_t + (1-\upsilon)K_t, \qquad \upsilon \in (0,1),$$

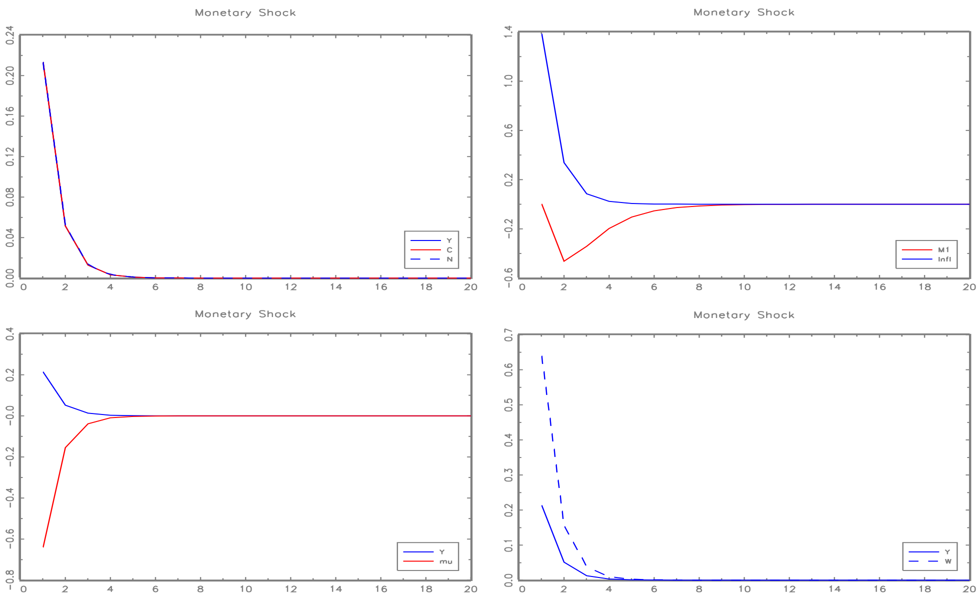
where in each simulation the elasticity of $\phi'\left(\frac{I_t}{K_t}\right)$ with respect to I_t/K_t is calibrated to ensure that investment is about 4.6 times as volatile as output.

Figure I.4: New Keynesian Model. Impulse responses to a monetary shock in $t = 1$, $mu^* = 1.2$, $\rho_\tau = 0$, $\varphi = 0.3$.



Percentage deviations from the long run mean. Y-Output, C-Consumption, N-Hours, W-Real Wage, M_1-Real Balances, $Infl$-Inflation, mu-Markup.

Figure I.5: New Keynesian Model. Impulse responses to a monetary shock in $t = 1$, $mu^* = 1.2$, $\rho_\tau = 0.5$, $\varphi = 0.3$.



Percentage deviations from the long run mean. Y-Output, C-Consumption, N-Hours, W-Real Wage, M_1-Real Balances, $Infl$-Inflation, mu-Markup.

shaped pattern, but remain for about 7 quarters significantly above average. A reduction of φ from 0.75 to 0.3 in the New Keynesian economy with adjustment costs of capital has similar consequences as in the model without capital - there is a substantial reduction of the degree of monetary non-neutrality. In addition, the reactions to policy shocks become purely temporary. The correlation between Y_t and mu_t remains negative but becomes very small in absolute value for $\rho_\tau = 0.5$, while it becomes large and positive in the case of $\rho_\tau = 0$.

Irrespective of the value of ρ_τ, if there are no adjustment costs of capital, the reactions of output, investment and labor to monetary shocks are purely temporary, with all three variables returning to their initial levels in the period after the shock. The response of consumption is virtually zero. In the case of $\rho_\tau = 0$ the jump of investment triggered off by the monetary shock is between 5 times (for $\varphi = 0.3$) and 40 times (for $\varphi = 0.75$) larger than what is reported by Christiano *et al.* (2005). $\rho_\tau = 0.5$ leads to even more pronounced responses of investment. Such results are completely at odds with the empirical evidence.

In sum, the standard New Keynesian model needs a very high degree of price stickiness (high φ) combined with substantially autocorrelated monetary shocks in order to be able to (at least partly) account for the observable magnitude and persistence of the effects of these shocks and the observable correlation between output and average markups. Equipped with a much lower and, thus, more realistic value of φ, the model fails to replicate these empirical facts. In addition, even if φ is high the New Keynesian model is unable to generate hump-shaped responses of output, consumption and inflation to monetary policy innovations. To improve the performance of the New Keynesian model, many authors combine a high degree of price stickiness with a full battery of additional real and nominal rigidities (e.g. backward indexation of the prices of the non-adjusting firms,[57] Calvo-type nominal wage setting, adjustment costs of capital and labor, habit persistence in consumption and leisure, matching frictions and job destruction in the labor market and even bounded rationality of part of the firms) as well as various further shocks (different kinds of preference shocks, wage-markup and price-markup shocks, investment specific shocks and even risk-premium shocks). Examples are Yun (1996), Woodford (1996), Rotemberg and Woodford (1997), Gali and Gertler (1999), Fuhrer (2000), Erceg *et al.* (2000), Sbordone (2002), Gali *et al.* (2001), Christiano *et al.* (2005), Walsh (2005), Trigari (2004), Altig *et al.* (2005), Smets and Wouters (2003, 2007) and many others. Most of these extensions are subject to debate. However, the assumptions most sharply criticized in the literature, are also the most crucial ones - the nature of the price setting behavior, the high degree of price stickiness assumed and the backward indexation of the prices of non-adjusting firms.

[57] Assume that firm i is not able to adjust its price in t. Ten $P_{i,t}$ is given by

$$P_{i,t} = \pi_{t-1} P_{i,t-1}.$$

5.3 Further Critique of the New Keynesian Model with Calvo Pricing

The Lack of Microeconomic Foundation of the Price-Setting Behavior

The major shortcoming of the New Keynesian model with price stickiness of the Calvo-type is that the latter is imposed exogenously rather than being derived from the interaction of profit and/or utility maximizing behavior and the market structure. In other words, the heart of the Calvo model - its pricing assumption - lacks microeconomic foundation. Many New-Keynesians argue that allowing firms to freely adjust their prices at any point in time, but assuming *small* menu costs (fixed costs of changing prices), would provide a rationale for the Calvo specification. Accordingly, the latter should be viewed as a computationally convenient short-cut of the analytically more complicated menu-cost model. In menu-cost models it depends on the current state of the economy wether particular firms will find it optimal to adjust prices or not. Therefore the price setting in these models is referred to as *state dependent*. In contrast, the Calvo specification is an example for *time dependent* pricing. The recent menu-cost literature, however, seems to reach the conclusion that the New Keynesian model with Calvo pricing *can not* be viewed as an approximation of an otherwise identical economy with state dependent pricing. Caplin and Spulber (1987) is one of the first studies emphasizing this point, and showing that the implications of the menu-cost model can differ dramatically from the predictions of theories with time dependent pricing. Caplin and Spulber (1987) construct a model in which in each period only a small (nearly constant) fraction of firms adjust prices. In other words, prices are extremely rigid. Nevertheless, there is no price stickiness on the aggregate level, and money is neutral. The intuition for this result is the following: In the model of Caplin and Spulber there is a stationary distribution of relative prices. Because of the menu costs, when a monetary shock hits the economy, only the firms with the lowest relative prices find it optimal to change prices. Since at the same time, there are no pricing complementarities[58] in the model, the adjusters adjust prices by a very large amount,[59] implying an increase in the aggregate price level just sufficient to offset the effects of the monetary expansion. Golosov and Lucas (2007) extend the Caplin-Spulber model by including idiosyncratic marginal cost shocks. On the one hand, this assumption makes the model more flexible, enabling the authors to calibrate it on the basis of the evidence provided by Bils and Klenow (2004). On the other hand, the idiosyncratic productivity shocks distort the mechanism described above, leading to monetary non-neutrality in the Caplin-Spulber model. Similar to the latter, the model of Golosov and Lucas (2007) also implies a virtually constant frequency of price adjustments, easing the

[58]For example, there is a high degree of *pricing complementarity* if the following holds: A higher fraction of firms that do not adjust their prices, implies a lower incentive for each individual firm to change its price, or a lower desired amount of adjustment.

[59]Precisely speaking, each adjusting firm sets its relative price equal to the highest relative price.

comparison with the standard New Keynesian model with Calvo pricing. Both models, the menu-cost and the Calvo one, are calibrated so that the implied average frequency of price adjustment is about 23% per month. The simulations performed by Golosov and Lucas reveal substantial qualitative and quantitative differences between the menu-cost and the Calvo model. The former implies weaker and much less persistent reactions of output and inflation to a one-time monetary expansion. While output in the Calvo model remains above average for about 2.5 quarters, the menu-cost model implies that output returns after 0.5 quarters (= 1.5 months) to its pre-shock level. Golosov and Lucas (2007) conclude that since the two models deliver substantially different predictions about the effects of monetary policy, the menu-cost model is *not* the microeconomic foundation of the New Keynesian model with Calvo pricing. Similar work, with similar results is done by Burstein and Hellwig (2007), Dotsey, King and Wolman (1999), Gertler and Leahy (2006) and Gorodnichenko (2008). These studies try to increase the degree of monetary non-neutrality in the menu-cost model by incorporating building blocks which amplify the pricing complementarities. However, as Burstein and Hellwig (2007) argue, to generate strong and persistent effects of monetary policy, these models need parameter values which are inconsistent with the micro evidence on the level of menu costs and the typical magnitude of price adjustments.

Golosov and Lucas (2007) and Bustrein and Hellwig (2007) also point out that macro models with continuous, convex costs of price adjustment[60] are subject to the same critique as the Calvo one since, in order to generate substantial monetary non-neutrality, these models need average adjustment costs which are much larger than the empirically observable magnitude of menu costs.[61]

Kehoe and Midrigan (2008) extend the Golosov-Lucas model by assuming that firms are subject to two types of idiosyncratic shocks, a persistent productivity disturbance and a temporary shock to the elasticity of demand for the firm's product. There are also two types of menu-costs - the lower one must be paid when the price is changed for only one period (temporary sale), while the higher one has to be paid when the price is changed permanently (adjustment of the regular price). The two types of idiosyncratic shocks combined with the two types of menu costs generate an explicit incentive for firms to carry out two different types of price changes and, thus, enables the model to better match the empirical facts regarding the pattern of regular prices and that of temporary sales. Kehoe and Midrigan (2008) then calibrate their menu-cost model as well as the Calvo model based on the scanner data provided by the Dominick's Finer Foods retail chain. The calibration of the Calvo model is performed in two different ways - by *including* and by *excluding* temporary sales when

[60] Prominent examples are Rotemberg (1982), Hairault and Portier (1995).

[61] See Burstein and Hellwig (2007) for an estimation of the magnitude of menu costs as well as for a survey of related literature.

computing the frequency of price adjustment. These are the two approaches usually found in the literature. Kehoe and Midrigan (2008) then argue that neither of the resulting two Calvo models is a good approximation of their menu-cost model. If temporary sales are included, the average frequency of price adjustment is too high, so that the Calvo model implies a lower degree of monetary non-neutrality than the Kehoe-Midrigen model does. If temporary sales are not taken into account, the resulting average frequency of price adjustment is too low, so that prices in the Calvo model are too sticky, with the consequence of much stronger and more persistent impulse responses than predicted by the Kehoe-Midrigan model. Kehoe and Midrigan (2008) then propose the following solution: If one does not want to work with the more complicated menu-cost model but rather with a simple short-cut, then he should set the frequency of price adjustment in the Calvo model at the *(intermediate)* value implying that the impulse responses implied by the Calvo model are exactly the same as that predicted by the menu-cost model. The *intermediate* value of the average price duration obtained by Kehoe and Midrigan (2008) is 17 weeks. This number corresponds to $\varphi \approx 0.3$ in the quarterly Calvo model. The corresponding impulse responses to a monetary shock are displayed in figures I.4 and I.5. Hence, if we want to reconcile the Calvo model with the menu-cost model, the former becomes completely inconsistent with the evidence on the cyclical behavior of markups and the effects of monetary policy shocks.

I disagree with the approach proposed by Kehoe and Midrigan (2008) for two reasons. First, every time a new building block is incorporated into the New Keynesian model one will have to carry out the same extension in the menu-cost model in order to determine the value of φ reconciling the two models. But if the cumbersome analytical and numerical analysis of the menu-cost model have already been done, then it would be extremely irrational to resort to an approximation such as the Calvo model, instead of using the *exact* menu-cost model to perform positive and normative analysis. Second, if we calibrate the Calvo model in the way proposed by Kehoe and Midrigan (2008), then the price setting in that model will still lack a microeconomic foundation. Simply because we do not set φ at a value consistent with, say, the Bils-Klenow evidence but rather at a so called *"intermediate"* value, in order to ensure that the impulse responses of the Calvo model are consistent with that of another theoretical model. But why should the impulse responses predicted by the menu-cost model be more plausible or more relevant than that estimated by, say, Christiano, Eichenbaum and Evans (2005)? Why should it be less correct to set φ at the value making the Calvo model consistent with the SVAR evidence, as is actually done by many New Keynesians? After all, the *a priori* belief that the SVAR evidence, obtained with empirical data, is closer to reality than the predictions of the manu-cost model are, appears extremely plausible. In other words, the right conclusion of the Kehoe-Midrigan paper should be similar to that drawn by Golosov and Lucas (2007) and many others: The model developed in Kehoe and Midrigan (2008) does not provide the microeconomic foundation of the price setting behavior assumed in the New Keynesian model.

The Chari-Kehoe-McGrattan Critique

Chari *et al.* (2008b) (henceforth, CKMc) criticize some features of the most recent New Keynesian models. CKMc focus on a model developed by Smets and Wouters (2007) because on the one hand it is fairly representative and on the other hand, the model is now being used to guide policy makers at the European Central Bank.

The performance of the Smets and Wouters (2007) hinges critically on the following four shocks: the wage-markup shock, the price-markup shock, the exogenous-spending shock and the risk-premium shock. CKMc argue that they are most likely to be non-structural and, thus, not invariant to monetary policy in general.

- The wage-markup shock in Smets and Wouters (2007) stems from fluctuations in the elasticity of substitution across different types of labor. CKMc argue that when expressed in terms of markup, the shock has a mean of 50% and a standard deviation of 2500% (!!!). The latter is completely unrealistic.
- CKMc argue that the Smets and Wouters (2007) model needs the wage-markup and the price-markup shocks in order to be able to account for the empirically observable labor wedge.[62] However, CKMc show that the labor wedge can be interpreted as arising from fluctuations in the bargaining power of unions or shifts in the utility of leisure. In both cases the labor wedge will be endogenous and, thus, not invariant to monetary policy. Therefore, the two shocks driving the labor wedge can be hardly interpreted as structural.
- CKMc also disagree with the interpretation of the expenditure spending shock as a structural shock to government spending. First, the shock has 3.5 the variance of government spending. Second, the shock emerges as a residual from the national income identity, and, thus, captures net exports and other endogenous variables not explicitly included into the Smets-Wouters model.
- CKMc also stress that the risk-premium shock has an unrealistically high volatility, having 6 times the variance of short-term nominal rates. Smets and Wouters (2007) provide only very rough and incomplete interpretation of this shock. CKMc believe that

[62] Roughly speaking, the labor wedge is the deviation between the marginal product of labor and the marginal rate of substitution between consumption and leisure. According to CKMc, there is a consensus in modern macroeconomics on the high importance of the labor wedge for business cycles fluctuations.

a more detailed investigation of the sources of such a shock, will reveal that there are endogenous forces driving the fluctuations of the risk premium, which are, in general, not invariant to monetary policy.

Two other features common for most modern New Keynesian models are also subject to critique by CKMc - the backward indexation of the prices of the non-adjusting firms and the specification of the Taylor rule as an approximation of the central bank's policy. The backward indexation is a way to make inflation more persistent in the model. However, this feature is inconsistent with the recent panel evidence on price setting, part of which is reviewed in section 3. According to that evidence, firms maintain a so called regular price for a year or more, frequently deviating from it by temporary setting a lower price. If actual price setting behavior were characterized by backward indexation, then it would be impossible to empirically observe such thing as the regular price. CKMc point to the empirical finance literature indicating that when the short rate changes (or is altered by the central bank) private agents significantly adjust their *long-run* expectations of the future short rate. CKMc interpret these results as indicating that interest rate policy must have a unit-root component.[63] The Taylor rules usually assumed in the macro literature imply that the short term nominal rate is stationary and ergodic. As a consequence, altering the short-run nominal rate has no effect on the long-run expectation of future short-run nominal rates.

Chari, Kehoe and McGrattan conclude:[64]

"We have argued here that New Keynesian models are not yet useful for policy analysis. Our basic reason is that macroeconomists working in this tradition have added so many free parameters to their models that those models are dubiously structural."

The purpose of the following chapters is to develop alternatives to the New Keynesian model which do not rely on questionable exogenous assumptions on the price setting behavior of private firms yet are (at least partly) able to account for the magnitude and persistence of the reactions to monetary policy shocks, the cyclical behavior of markups as well as various other business cycles facts.

6 Related Theoretical Studies

Since the models constructed in the following chapters a re only loosely related to the existing literature, this section is extremely short, only briefly pointing to some recent developments.

[63] The argument is made precise in Atkeson and Kehoe (2008).

[64] Chari *et al.* (2008b), p. 24.

Endogenous Price Rigidity: Haubrich and King (1991) develop a model providing an endogenous explanation of price stickiness. In that model firms are able to insure against idiosyncratic monetary shocks by signing nominal contracts. However, as the authors point out, the price-rigidity equilibrium is only one of the possible outcomes under the specific assumptions on the parameters made. The parameterization of their model, too, is only one of many plausible ones. Nakamura and Steinsson (2007) construct a model with good-specific habit persistence in which price stickiness arises as an equilibrium outcome. Price stickiness can be sustained because it helps firms to solve a time-inconsistency problem. However, there are again many further equilibria characterized by fully flexible prices. In addition, the results in Nakamura and Steinsson (2007) should be interpreted with caution because they are derived within a partial equilibrium framework. Thus, it can not be ruled out, that the general equilibrium predictions of the model (after incorporating the money market) with respect to the degree of monetary non-neutrality turn to be rather disappointing. The *sticky-information* literature proposes an approach slightly different from that adopted in the Calvo model. It assumes that nominal prices are fully flexible but the flow of information to particular agents is not. In particular, it is assumed that each firm faces a constant probability φ per period to be able to obtain the most recent information about the state of the economy. The remaining firms, constituting a fraction equal to $1-\varphi$ percent, are not able to update their information within the period and, thus, base production and pricing plans on old (outdated) information. For example, at time t a firm that was last able to update its information in $t-3$ build expectations about future economic conditions based on the information set available to the economy at the end of period $t-3$. In other words, the sticky-information models replace the Calvo-type price setting by a Calvo-type updating of information. Some of the most important studies in this area are Mankiw and Reis (2001, 2006a, 2006b) and Ball *et al.* (2003). These authors claim that their models are able to generate endogenous price-sluggishness and persistent and (at lest partly) hump-shaped impulse responses to monetary policy shocks. A major shortcoming of the sticky-information models is the fact that their most crucial component - the process of obtaining information - is exogenously given. To the best of my knowledge, the Calvo-type updating of information still lacks a microeconomic foundation.[65] Rotemberg (2002, 2004a, 2004b, 2008) assumes that customers become *angry* if they see the price chosen by a given firm as *unfair*. Anger, in turn, forces them to punish the unfair firm by reducing the demand for its product more strongly than it would be the case without anger. Rotemberg derives conditions ensuring that in order to avoid anger, firms

[65] Mackowiak and Wiederholt (2007) can be interpreted as an attempt in this direction. In their model managers can process only a limited amount of information because of cognitive constraints. As a result, in each period they are forced to decide whether to pay attention to aggregate or to idiosyncratic signals when setting prices.

find it optimal to partly reduce the flexibility of prices. Unfortunately, Rotemberg performs the analysis within a simple, partial equilibrium framework.

Real Business Cycles Models of Endogenous Markups: A prominent example is the model developed by Phelps and Winter (1970) which builds the basis of the models constructed in the next two chapters. In that model markups are endogenized by the assumption of a particular form of dynamic market share competition in continuous time. The discrete time version of that structure is used in the models presented below.[66] In a series of real business cycles models based on the partial equilibrium model proposed by Rotemberg and Saloner (1986), Rotemberg and Woodford[67] show that countercyclical markups may arise if firms are able to collude implicitly. In their models, markups respond negatively to demand side as well as to supply side shocks. Ravn *et al.* (2006, 2007) are able to generate countercyclical markups by introducing good-specific habit formation, the so called *deep habits*, into a standard RBC-model with a monopolistically competitive goods market. Edmond and Veldkamp (2008) show how countercyclical income dispersion can generate countercyclical markups, even without any price-setting frictions such as nominal price rigidity or slow reactions of the customer stock. Froot and Klemperer (1989), Klemperer (1987, 1995) and Kleshchelski and Vincent (2007) develop static models of the goods market in which customers face fixed costs of switching suppliers. All these models have in common the implication that firm's current pricing behavior has an influence on its future profits.

[66] Rotemberg and Woodford (1992, 1995) also use the discrete-time version of the model for comparison purposes.

[67] Rotemberg and Woodford (1992), (1995), (1996)

Chapter 2

A Monetary Customer Markets Model

1 Introduction

The empirical evidence reviewed in the previous chapter indicate that positive monetary shocks are expansionary and induce highly persistent dynamic responses of inflation, output, consumption and investment. Many economists try to explain this pattern by monetary business cycles models in which they assume some kind of exogenously given price stickiness combined with a whole battery of real rigidities and additional structural assumptions.[1] These models are extensively used to evaluate monetary and fiscal policy as well as to derive normative conclusions and suggestions on how monetary policy should be conducted. Unfortunately, most of these models perform rather poorly with respect to phenomena other than the impulse responses to monetary innovations such as the sample moments at business cycle frequencies of many macroeconomic variables or the reactions to real supply side shocks. This casts doubt on the appropriateness of the sticky price models for analyzing normative issues. The sticky-price models also do not provide any endogenous explanation of their most crucial component - the high degree of price or wage rigidity. The current chapter addresses the question of the propagation of monetary shocks as well as that of explaining a set of sample moments of the data by taking a different approach. I employ a model with fully flexible prices in which monetary *nonneutrality* is due to the assumption that the utility function of the representative household is non-additively separable in money and consumption. As shown below, the intrinsic mechanisms propagating nominal shocks in this *baseline* model are pretty weak and lead to predictions which are in many respects counterfactual. However, introducing market share competition in the goods market as proposed by Phelps and Winter (1970), and thus, making the markups of prices over marginal costs endogenous, substantially alters the qualitative as well as quantitative predictions of the model. In particular,

[1] The most widely used model framework in modern macroeconomics is the so called *"New Keynesian Model"* with *Calvo price setting*. Examples are Christiano *et al.* (2005), Walsh (2005) and many others.

for a broad range of empirically plausible values of the short run price elasticity of demand, the average markup and the degree of flexibility of capital accumulation the nonneutrality of money can be made arbitrarily strong. Furthermore, in these cases the model implies impulse responses of output, employment and wages displaying a one period delay and a substantial degree of persistence.

What are the main mechanisms at work in the model developed in this chapter? If the current utility function of the representative household in an otherwise standard monetary business cycles model is non-additively separable in money and consumption, and is given by the following CES-aggregator:

$$\left(aC_t^{1-b} + (1-a)\left(\frac{M_t}{P_t}\right)^{1-b}\right)^{\frac{1}{1-b}}, \quad a \in (0,1), \quad b > 0,$$

where C, M and P denote consumption, nominal cash balances and the price level respectively, then monetary *expansions* tend to be *contractionary*: They induce a sharp increase in current inflation which reduces the marginal utility of consumption *"today"* relative to its value in the future. The result is a relatively large negative deviation of labor supply and thus a drop of output and consumption in the period of the shock. Only in the case of a very high degree of flexibility of capital accumulation these variables reach above average values in the period after the shock. Otherwise, they return almost immediately, from below, to their respective long run levels. This prediction is counterfactual. At the same time, however, the disturbance of the time path of the marginal utility of consumption just described implies an increase in the stochastic discount factor which, in turn, leads to an increase in the present value of firms' future profits. If the goods market is characterized by the usual static monopolistic competition, future profits don't matter for the current pricing decisions of the firms. In contrast, if firms also engage in market share competition as suggested by Phelps and Winter (1970), future profits become a crucial determinant of firms' behavior. If expected future revenues increase relative to their current level, each firm will tend to make additional *"investments"* in future market shares by lowering its current price and thus by lowering its current markup. The decrease in markups will have a positive effect on the real wage and thus on the labor supply decision made by households. As a result, in the economy characterized by market share competition employment, consumption and output will tend to be procyclical or at lest less countercyclical than in the case of static monopolistic competition. As shown below, the lower the short run price elasticity of demand and the higher the steady state markup, the larger the fall in markups and thus, the more pronounced the increase in real wages, employment, and output. The persistence generated by the model is due to the interaction between capital accumulation and markup fluctuations and is described in sections 4 and 5.

In summary, the purpose of this chapter is threefold. First, to stress the importance of markup fluctuations for the qualitative and quantitative predictions of monetary models with non-additively separable utility functions. Second, to propose a theoretical framework alternative to that of the *New Keynesian Model*, which is able to explain a bunch of business cycles facts. And third, to understand and explain the model mechanisms leading to the most interesting and relevant implications and highlight their advantages and disadvantages in a much more detailed and explicit manner than usually done in the literature.

The chapter is organized as follows. Section 2 describes the baseline monetary model without capital accumulation while section 3 extends it by the assumption of market share competition in the goods market. Capital accumulation and adjustment costs of capital are introduced in sections 4 and 5. In section 7 I evaluate the performance of the model with adjustment costs of capital with respect to a subset of stylized business cycles facts and compare it with the performance of the *New Keynesian Model*. Section 8 concludes.

2 A Model with Fixed Capital and Static Monopolistic Competition

This section provides a short sketch of the baseline *Money in the Utility Function Model*.

2.1 The Theoretical Framework

Firms

There are n product varieties, each produced by a profit maximizing monopolistic firm according to the linear production function

$$Y_{i,t} = Z_t N_{i,t},$$

where $N_{i,t}$ denotes labor input of firm i. Z_t denotes the total factor productivity which follows a stochastic process given by:

$$\ln(Z_t) = \rho_z \ln(Z_{t-1}) + \epsilon_t,$$

where ϵ_t follows a *White Noise Process* with variance σ_ϵ^2.

The demand function faced by the producer of variety i is given by

$$C_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\theta} \cdot \frac{C_t}{n}, \qquad \theta > 0, \tag{II.2.1}$$

where C_t denotes aggregate consumption expenditure.

The profit maximizing relative price satisfies the equation

$$\frac{P_{i,t}}{P_t} = \frac{\theta}{\theta - 1} \frac{W_t/P_t}{Z_t},$$

where $\frac{W_t}{P_t}$ denotes the real wage and $mu = \frac{\theta}{\theta-1}$ the markup.

Households

Let agents in this economy have preferences over consumption, real balances and working hours given by[2]

$$U = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left(\left(aC_t^{1-b} + (1-a) \left(\frac{M_t}{P_t} \right)^{1-b} \right)^{\frac{1}{1-b}} - \frac{\phi}{2} N_t^2 \right) \right\}, \qquad \phi, b > 0, \quad \beta, a \in (0,1),$$

where M_t/P_t and N_t denote real balances and working hours. In the above expression C_t is a composite good that includes all varieties:

$$C_t = \left\{ \frac{1}{n} \sum_{i=1}^{n} C_{i,t}^{\frac{\theta-1}{\theta}} \right\}^{\frac{\theta}{\theta-1}}. \tag{II.2.2}$$

The corresponding utility-based price index is given by:

$$P_t = \left\{ \frac{1}{n} \sum_{i=1}^{n} P_{i,t}^{1-\theta} \right\}^{\frac{1}{1-\theta}}.$$

For $b \to 1$ the *current* utility function which I denote by u_t reduces to

$$u_t = C_t^a \left(\frac{M_t}{P_t} \right)^{1-a} - \frac{\phi}{2} N_t^2.$$

The budget restriction of the representative household is given by:

$$C_t + m_{t+1} - \frac{m_t}{\pi_t} + b_{t+1} - \frac{b_t}{\pi_t} = \frac{W_t}{P_t} N_t + \Pi_t + i_t \frac{b_t}{\pi_t} + \frac{T_t}{P_t},$$

where W_t, Π_t, T_t, $b_t = \frac{B_t}{P_{t-1}}$ and $m_t = \frac{M_t}{P_{t-1}}$ denote the nominal wage, real profits, nominal net transfers form the government, the real value of nominal bonds and real balances respectively. i_t is the one-period risk free nominal interest rate.

[2]A similar specification of the utility function is used by Maussner (2004).

Key Assumption I: Non-Separable Utility

The monetary general equilibrium models developed in the last ten years usually assume that the utility function of the representative agent is *separable* with respect to money and consumption, e.g.

$$\frac{C_t^{1-\eta}}{1-\eta} + \frac{\phi}{1-\chi}\left(\frac{M_t}{P_t}\right)^{1-\chi}, \qquad \text{for } 0 < \eta, \chi \neq 1, \quad \phi > 0,$$

$$\ln(C_t) + \tilde{\phi}\ln\left(\frac{M_t}{P_t}\right), \qquad \text{for } \eta = \chi = 1, \quad \tilde{\phi} > 0.$$

Nonetheless, it is quite well known that almost all *separable* specifications are just special cases of more general *non-separable*, (nested[3]) Cobb-Douglas or CES[4] aggregators combining consumption and real balances. Furthermore, economic theory does not provide any convincing reason for preferring the separable to the non-separable formulation *et vice versa*. The only comparative advantage of the former is perhaps its analytical simplicity. Indeed, in his seminal paper Sidrauski (1967) assumes that money and consumption enter the utility function non-separabely, through a Cobb-Douglas aggregator. The early literature inspired by Sidrauski (1967), e.g. Brock (1974, 1975), Fisher (1979), Asako (1983) and others, dealing with the stability and the steady state properties of monetary general equilibrium models, also consider the non-separable utility function to be more important while the separable specification is only treated as a special case.

Finally, the empirical evidence supports the assumption that utility is non-separable in consumption and real balances: in a more recent study Holman (1998) performs a GMM estimation of the Euler equation for optimal money holdings under different specifications of the utility function - Cobb-Douglas, CES and nested Cobb-Douglas or CES.[5] Based on a series of tests the author rejects the separable form while the Cobb-Douglas, the CES (used here) and the nested CES formulation can not be rejected.

[3]The nested Cobb-Douglas specification of the utility function is given by:

$$\frac{\left(C_t^{\alpha}\left(\frac{M_t}{P_t}\right)^{1-\alpha}\right)^{1-\rho}}{1-\rho}, \quad \alpha \in (0,1), \quad \rho > 0,$$

while the non-nested case is obtained by setting $\rho = 0$.

[4]CES - Constant Elasticity of Substitution.

[5]The nested CES specification of the utility function is given by:

$$\frac{\left(aC_t^{1-b} + (1-a)\left(\frac{M_t}{P_t}\right)^{1-b}\right)^{\frac{1-\rho}{1-b}}}{1-\rho}, \quad a \in (0,1), \quad b, \rho > 0,$$

while the non-nested case is obtained by setting $\rho = 0$.

All in all, the assumption that the utility function is non-separable in money and consumption seems to be at least as plausible as the opposite one. At the same time, the non-nested CES specification chosen in the current paper, although arbitrary, is not rejected by the data. As is well known, the non-separability with respect to consumption and real balances can be interpreted as a short-cut of an environment in which the transaction costs associated with the purchase of a given amount of consumption goods can be reduced by a higher level of real cash holdings.

First Order Conditions

The first order conditions of the representative household evaluated at the symmetric equilibrium read:

$$aC_t^{-b}\left(aC_t^{1-b}+(1-a)\left(\frac{m_t}{\pi_t}\right)^{1-b}\right)^{\frac{b}{1-b}}=\Lambda_t, \tag{II.2.3}$$

$$\phi N_t=\Lambda_t\frac{W_t}{P_t}, \tag{II.2.4}$$

$$\frac{1}{1+i_t}=\beta E_t\left\{\frac{\Lambda_{t+1}}{\Lambda_t}\frac{1}{\pi_{t+1}}\right\}, \tag{II.2.5}$$

$$\Lambda_t=\beta E_t\left\{(1-a)\frac{m_{t+1}^{-b}}{\pi_{t+1}^{1-b}}\left(aC_{t+1}^{1-b}+(1-a)\left(\frac{m_{t+1}}{\pi_{t+1}}\right)^{1-b}\right)^{\frac{b}{1-b}}+\frac{\Lambda_{t+1}}{\pi_{t+1}}\right\}, \tag{II.2.6}$$

$$C_t+m_{t+1}-\frac{m_t}{\pi_t}+b_{t+1}-\frac{b_t}{\pi_t}=\frac{W_t}{P_t}N_t+\Pi_t+i_t\frac{b_t}{\pi_t}+\frac{T_t}{P_t}. \tag{II.2.7}$$

(II.2.5) is the bond euler equation and (II.2.6) is the euler equation with respect to money balances.

Government

The central bank finances its lump-sum transfers to the public by changes in the nominal quantity of money:

$$M_{t+1}-M_t=T_t.$$

It is further assumed that in each period transfers constitute a fraction of current money supply:

$$T_t=(\tau_t-1)M_t,$$

where the percentage deviation of τ_t from its steady state $\hat{\tau}_t$ follows a first order autoregressive process

$$\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + u_t, \qquad \rho_\tau \in [0,1).$$

u_t is assumed to be a *White Noise Process* with variance σ_u^2.

Equilibrium

In equilibrium, real wages and profits are given by

$$\frac{W_t}{P_t} = \frac{Z_t}{mu} \qquad \text{and} \qquad \Pi_t = \left(\frac{mu-1}{mu}\right) Z_t N_t$$

respectively. These two results, together with the households first order conditions, (II.2.3) through (II.2.7) describe the evolution of the economy.

2.2 Understanding Key Features of the Model

Figure II.1 depicts the impulse responses to a monetary shock in $t=3$ without serial correlation, occurring in the third period.[6] The reactions of output, employment, consumption, real balances and the discount factor can be characterized as purely temporary one-time negative deviations from the steady state. Hence, the the positive monetary transfer is contractionary. The only variable deviating for more than one period from its long run level is the rate of inflation. Its value is above average for two quarters. To understand why there is no persistence in the reactions to monetary expansions, it is instructive to restate the household's optimality conditions under the assumption that $b=1$:[7]

$$N_t = aC_t^{a-1}\left(\frac{m_t}{\pi_t}\right)^{1-a}\frac{W_t}{P_t} \quad \Rightarrow \quad C_t^{2-a} = a\left(\frac{m_t}{\pi_t}\right)^{1-a}\frac{W_t}{P_t}$$

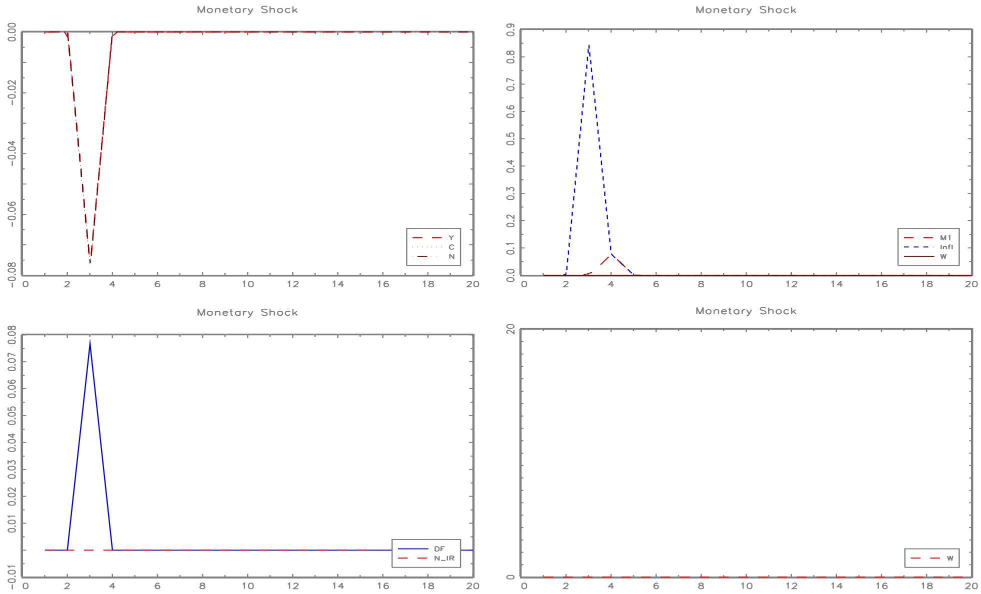
$$m_{t+1} = \beta\frac{1-a}{a}E_t\left\{C_{t+1}\frac{C_{t+1}^{a-1}\left(\frac{m_{t+1}}{\pi_{t+1}}\right)^{1-a}}{C_t^{a-1}\left(\frac{m_t}{\pi_t}\right)^{1-a}}\right\} + \beta E_t\left\{\frac{C_{t+1}^{a-1}\left(\frac{m_{t+1}}{\pi_{t+1}}\right)^{1-a}}{C_t^{a-1}\left(\frac{m_t}{\pi_t}\right)^{1-a}}\frac{1}{\pi_{t+1}}\right\}. \tag{II.2.1}$$

[6] See section 6 for details on the calibration. The corresponding programm is **"MIU_sim_cm_2d5d_1_i.g"**.

[7] In that case the first term in the utility function becomes a Cobb-Douglas aggregator assembling consumption and real balances:

$$u_t = C_t^a\left(\frac{m_t}{\pi_t}\right)^{1-a} - \frac{d}{2}N_t^2.$$

Figure II.1: MIU-model with fixed capital and no market share competition. Impulse responses to a monetary shock, $\rho_T = 0$, $a = 0.9$, $b = 1$. Percentage deviations from steady state.



Y - output, N - hours, C - consumption, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, DF - discount factor, $N-ir$ - nominal interest rate, W - real wage.

The second equation in (II.2.1) is a conventional forward looking condition for optimal money demand. The first term on the rhs stems from the increase in next-period utility induced by a marginal increase in money holdings. Loosely speaking, for a given stochastic discount factor an increase in expected future consumption increases the expected marginal utility of real balances m_{t+1} and thus rises the demand for that asset.[8] The second term embodies the link between the demand for money and its *real* interest rate $\frac{1}{\pi_{t+1}}$. An increase in expected inflation lowers the expected real rate of return on real balances, making it a less attractive asset. On the other hand current and future consumption can affect money demand *via* the discount factor. According to that link, m_{t+1} depends negatively on C_{t+1} and positively on C_t: since agents want to smooth consumption over time, they will try to avoid large deviations of the ratio $\frac{C_{t+1}}{C_t}$ from one by investing or desinvesting in real balances. Too low a ratio of future to current consumption $\frac{C_{t+1}}{C_t}$ will force households to raise their real cash holdings in order to be able to increase C_{t+1} and thus, to shift that ratio closer to unity.

To gain more intuition about the sign and shape of the impulse responses, let us take a more detailed look at the underlying economic mechanisms. The monetary expansion is seen by households as a positive income shock generating *for given prices* the usual income effect: agents try to raise each period's consumption demand by the same amount and lower each period's labor supply also by the same amount. To achieve this, each household tries to invest

[8] The positive link between expected or planned future consumption and the future marginal utility of money can be interpreted as follows: To achieve a higher level of consumption, agents need to make more transactions in the goods market which, in turn, requires larger *real* holdings of the medium of exchange - cash.

the same portion of his additional income in risk free bonds. But since the aggregate supply of bonds remains unchanged and equal to zero, the desire to increase bond holdings causes the nominal interest rate to fall just enough to force households to change the composition of their portfolios by leaving bond holdings unchanged and rising the desired amount of the other asset available-real balances. Since the marginal utility of consumption depends positively on real balances, the desired path of consumption expenditure gets altered with future consumption and labor supply being increased relative to their respective current values.[9] At the same time, as the current Z_t is given, the planned increase in current consumption demand accompanied by a lowering of current labor supply leads to an increase in current and expected nominal wages which, under constant markups, is completely passed through to nominal prices, leading to an increase in *current* inflation π_t. Expected future nominal wages and prices will also tend to rise since planned next-period consumption expenditure(labor supply) also get larger(smaller). But since the reactions of that two variables are affected by the increase in money holdings in the way just described, the steady state deviation of expected future inflation might turn to be stronger or weaker than that of present inflation, or even be negative, depending on whether desired next-period consumption or labor supply responds more strongly to changes in real money holdings. Note that real balances have two opposing effects on labor supply: An increase in m_t has a positive direct effect on N_t (see the first equation in the first line of (II.2.1)). At the same time, everything else given, the increase in labor supply increases labor income and makes a higher level of consumption possible. On the aggregate level consumption depends on m_t according to the second equation in the first line of (II.2.1). But a higher level of consumption implies a lower marginal utility of that variable and thus, creates an incentive for households to decrease labor supply. It turns out that the overall effect of changes in $\frac{m_t}{\pi_t}$ on N_t is positive and can be described by the following equation:

$$N_t = a^{\frac{1}{2-a}} \left(\frac{m_t}{\pi_t}\right)^{\frac{1-a}{2-a}} \left(\frac{W_t}{P_t}\right)^{\frac{1}{2-a}} .$$

The increase in current inflation and the reaction of future inflation should be such that in each period the disparity between aggregate consumption and labor supply and thus production, is eliminated. Depending on how the inflation path adjusts, both equilibrium consumption and labor in the present as well as in the future can fall, increase or remain unchanged.[10] If $E_t\pi_{t+1}$ actually changes, the incentive to build up money balances gets altered, starting a new loop of adjustments in desired current and future consumption demand, labor supply and inflation, until the new intertemporal equilibrium is reached.

[9]See the first line in (II.2.1).

[10]Note that only changes in expected future inflation affect the real interest rate of money balances.

Now observe that consumption depends on current inflation through the ratio $\frac{m_t}{\pi_t}$ and thus money demand m_{t+1} is affected by π_t only *via* terms of the form $\frac{m_{t+1}\pi_t}{m_t\pi_{t+1}}$. Log-linearizing one of that terms and taking expectations as of time t yields:

$$\hat{\tau}_t - E_t\hat{\pi}_{t+1}$$

which does not depend on current inflation π_t. Further, the fact that C_{t+1} and π_{t+1} appear in the second equation in (II.2.1) just induces an additional, *indirect*, log-linear link between $\hat{m}_{t+1}$ and $\hat{m}_{t+1}$, $E_t\hat{\pi}_{t+1}$. To see that, eliminate consumption from the system (II.2.1) and log-linearize around the nonstochastic steady state under the assumption $\hat{Z}_t = 0 \ \forall t$. The resulting equation reads:

$$\hat{m}_{t+1} = -\left(\frac{2\pi^*(1-a)+\beta}{\pi^*-\beta}\right) E_t\hat{\pi}_{t+1} + \frac{(1-a)\pi^*}{\pi^*-\beta}\hat{\tau}_t. \tag{II.2.2}$$

The link between current money demand and expected inflation is the result of the overlapping effects on m_{t+1} arising through the interdependence between m_{t+1} and $E_t\pi_{t+1}$ and current and next period consumption as well as real balances mentioned above. (II.2.2) is a reduced form, forward looking money demand equation. Money supply evolves according to

$$\hat{m}_{t+1} = \hat{m}_t - \hat{\pi}_t + \hat{\tau}_t. \tag{II.2.3}$$

(II.2.2) and (II.2.3) constitute a dynamic *supply-demand* system with an expected time path of prices given by $\hat{\pi}_t, E_t\hat{\pi}_{t+1}, E_t\hat{\pi}_{t+2}, \ldots$. In equilibrium the price expectations of money suppliers should be equal to that of the agents demanding money. So, one can shift (II.2.3) one period forward in time, take conditional expectations as of time t, take into account that $\hat{\tau}_t$ is a White Noise process and then eliminate expected inflation from (II.2.2). The resulting relationship

$$\hat{m}_{t+1} = \underbrace{\frac{2\pi^*(1-a)+\beta}{2\pi^*(1-a)+\pi^*}}_{\in(0,1)} E_t\hat{m}_{t+2} + \frac{(1-a)\pi^*}{2\pi^*(1-a)+\pi^*}\hat{\tau}_t$$

is a stochastic forward looking difference equation which should be solved forward as its root lies outside the unit circle. The solution is very simple and is computed by each agent in forming her rational expectations about inflation and other variables:

$$\hat{m}_{t+1} = \frac{(1-a)\pi^*}{2\pi^*(1-a)+\pi^*}\hat{\tau}_t.$$

Hence, the relative deviation of money balances from its steady state level follows a White Noise process. Now it is straightforward to show that

$$E_t\hat{m}_{t+2} = E_t\hat{\tau}_{t+1} = 0 = E_t\hat{m}_{t+3} = E_t\hat{m}_{t+4} = \ldots$$

and therefore

$$E_t\hat{\pi}_{t+1} = \hat{m}_{t+1}, \quad E_t\hat{\pi}_{t+2} = 0, \quad E_t\hat{\pi}_{t+3} = 0, \ldots$$

But if agents expect next-period inflation to rise by the same amount as their real balances, their expected wealth in $t+1$ will remain unchanged implying that they will have no incentive to change next-period consumption which, in turn, leaves the marginal utility of consumption and thus labor supply in $t+1$ unchanged.[11] Since the expected steady state deviations of real balances and inflation in all future periods are equal to zero, nobody will expect any positive or negative wealth effects of real money holdings in that periods.

What happens in the period in which the shock occurs? For a given level of consumption the large increase in inflation induces a sharp decline in the current marginal utility of consumption. As a result, labor supply falls even more and leads to a further decline in labor as well as dividend income. Since at the same time the positive income effect of the monetary shock is almost offset by the rise of the inflation rate, households have to reduce current consumption. Since the time path of the marginal utility of consumption is altered agents face an additional, utility based incentive to lower current consumption relative to its future level. However, the reduction of C_t is not sufficient to compensate (offset) the negative effect of inflation on the marginal utility of consumption. Therefore labor supply and thus, output unambiguously fall. One can see the overall effect of inflation on consumption formally by inspecting the second equation in the first line of (II.2.1): Since m_t is given, the increase in π_t lowers consumption demand. Observe that π_t affects C_t through three different channels. The direct one is negative *via* the dampening of the income effect of the monetary transfer. The second one is positive: the lower C_t caused by a higher π_t increases the marginal utility of consumption and the incentive to work. This, in turn, leads to a higher labor income and thus increases consumption demand. The third one is again negative and results from the direct negative dependance of the marginal utility of consumption on the rate of inflation.

If utility were additively separable in money and consumption the increase in real balances or *current* inflation won't alter the marginal utility of consumption or that of labor. Therefore current and future nominal wages will jump by the same amount with the consequence that in the entire future inflation remains at its steady state level while current inflation rises by an amount just sufficient to offset the positive income effect of the monetary transfer. As a result, nothing except current inflation would change in that economy. With utility non-additively separable in consumption and real balances the pattern of both, desired consumption and labor supply, are altered by changes in real balances $m_{t+1}, m_{t+2}, \ldots$ and current and expected inflation $\pi_t, E_t\pi_{t+1}, E_t\pi_{t+2}, \ldots$.

[11] See the first equation in (II.2.1).

Higher values of b as well as lower values of a imply that changes in real balances or inflation have a stronger impact on the marginal utility of consumption. For that reason changing the two parameters in that way magnifies the impulse responses to a one time monetary expansion without changing the qualitative predictions of the model.

The implication that positive monetary disturbances are contractionary is at odds with the conventional thinking as well as the empirical evidence[12] about the effects of monetary policy. To make the theory more realistic in the next section I extend it by the the assumption of market share competition in the goods market *á la* Phelps and Winter (1970).

3 A Model with Fixed Capital and Market Share Competition

I refer to this model as the *Customer Markets Model* with fixed capital.

3.1 The Theoretical Framework

Key Assumption II: Market Share Competition

Let us assume that the consumption index is given by

$$C_t = \left\{ \frac{1}{n} \sum_{i=1}^{n} x_{i,t}^{\frac{1}{\theta}} C_{i,t}^{\frac{\theta-1}{\theta}} \right\}^{\frac{\theta}{\theta-1}}, \tag{II.3.1}$$

where $x_{i,t}$ is exogenous to the consumer and evolves according to

$$x_{i,t+1} = g\left(\frac{P_{i,t}}{P_t}\right) \cdot x_{i,t} \tag{II.3.2}$$

The demand function for good i resulting from the above consumption aggregator is given by:

$$C_{i,t} = x_{i,t} \cdot \left(\frac{P_{i,t}}{P_t}\right)^{-\theta} \cdot \frac{C_t}{n}. \tag{II.3.3}$$

This is basically the assumption underlying the *"Customer Markets Model"* developed by Phelps and Winter (1970).

Phelps and Winter (1970) depart from the frictionless specification of the goods market by assuming that customers can not respond instantaneously to differences in firm specific prices. As the authors note, there are various rationales for this assumption - information imperfections, habits as well as costs of decision-making. An immediate consequence of such

[12] See Christiano *et al.* (2005).

frictions is that in the (very) short run each firm has some monopoly power over a fraction of all consumers. This fraction equals the firm's market share. In particular, Phelps and Winter (1970) assume that the transmission of information about prices evolves through random encounters among customers in which they compare recent demand experience. Under this assumption the probability with which a comparison between any two firms i and j is made will be approximately proportional to the product of their respective market shares $x_{i,t}$ and $x_{j,t}$. Therefore, one would expect that the time rate of net customer flow from all other firms to firm i will also be proportional to the product $x_{i,t}(1-x_{i,t})$. Under the assumption $1-x_{i,t}\approx 1$. Phelps and Winter formalize this as follows:

$$z_{t,i,*} = g(P_{i,t}, P_t)x_{i,t}(1-x_{i,t}) \approx g(P_{i,t}, P_t)x_i,$$

where $z_{t,i,*}$ is the net flow of customers to firm i from all its competitors and P_t denotes the average price in the market under consideration. The properties of the function $g()$ are specified below. Section 9 at the end of this chapter provides more formal details regarding the last equation and the underlying assumptions. $x_{i,t}$ can be also interpreted as an indicator of customers' satisfaction with the pricing behavior of firm i, or as a measure of the subjective weight assigned to good i within the consumption bundle. In the current chapter $x_{i,t}$ is called market share. I assume that the function $g(.)$ governing its law of motion has the properties:

$$g\left(1\right) = 1, \qquad g'\left(\frac{P_{i,t}}{P_t}\right) < 0,$$

and assume the following functional form for it

$$g\left(\frac{P_{i,t}}{P_t}\right) = \exp\left(\gamma\left(1-\frac{P_{i,t}}{P_t}\right)\right),$$

where $\gamma > 0$ is to be calibrated *via* the steady state of the economy. Because $x_{i,t}$ depends on the past pricing behavior of the firm, its profit maximization problem becomes dynamic: In this economy each firm faces a trade off between maximizing its current profits and maximizing its future market share.

The dependence of the market share in $t+1$ on past pricing behavior introduces a dynamic aspect into the profit maximization problem of the individual firm. Under dynamic market share competition the price set in the current period by a monopolistically competitive firm does not only affect its current profits but also its next-period market share and thus the expected present value of its future profits. Consequently, the monopolistically competitive firm faces a trade-off between charging the price that maximizes its current profits but will probably induce a decline in its next-period market share and charging a lower price, which does not maximize current profits, but leads to an increase in next-period market share and, thus, to higher future profits. As a result of this trade-off, markups turn to be lower on average than if there were no market share competition. There is also a further channel by which dynamic market share competition affects markups, and tends to make them countercyclical: An increase in current consumption demand caused by a positive monetary or technology shock leads to higher *stochastic discount factors* and thus lower interest rates. The lower in absolute value the intertemporal elasticity of substitution, the higher the rise in the expected present value of future profits caused by the higher discount factor and thus the stronger the incentive for firms to make additional *"investments"* in future market shares by choosing lower current markups.

Profit Maximization and Markups

Each firm maximizes

$$\max_{P_{i,t}} \left\{ x_{i,t} \left(\frac{P_{i,t}}{P_t} \right)^{-\theta} \frac{C_t}{n} \left(\frac{P_{i,t}}{P_t} - \mu_t \right) + E_t \left\{ \sum_{j=1}^{\infty} DF_{t,t+j} x_{i,t+j} \left(\frac{P_{i,t+j}}{P_{t+j}} - \mu_{t+j} \right) \left(\frac{P_{i,t+j}}{P_{t+j}} \right)^{-\theta} C_{t+j} \right\} \right\}$$

s. t.

$$x_{i,t+1} = g\left(\frac{P_{i,t}}{P_t} \right) x_{i,t},$$

where $DF_{t,t+j} = \beta^j \frac{\Lambda_{t+j}}{\Lambda_t}$ denotes the stochastic discount factor between periods t and $t+j$ which is given to the firm. μ_t denotes marginal costs. The first order condition of an arbitrary firm with respect to its relative price reads:

$$\left(\frac{P_{i,t}}{P_t} \right)^{-\theta} x_{i,t} C_t - \theta \left(\frac{P_{i,t}}{P_t} - \mu_t \right) \left(\frac{P_{i,t}}{P_t} \right)^{-\theta-1} x_{i,t} C_t + \frac{g'\left(\frac{P_{i,t}}{P_t}\right)}{g\left(\frac{P_{i,t}}{P_t}\right)} \Omega_t = 0,$$

where μ_t denotes marginal costs and

$$\Omega_t = E_t \left\{ \sum_{j=1}^{\infty} \beta^j \frac{\Lambda_{t+j}}{\Lambda_t} x_{i,t+j} \left(\frac{P_{i,t+j}}{P_{t+j}} - \mu_{t+j} \right) \left(\frac{P_{i,t+j}}{P_{t+j}} \right)^{-\theta} C_{t+j} \right\} = \tag{II.3.4}$$

$$= E_t \left\{ \beta \frac{\Lambda_{t+1}}{\Lambda_t} x_{i,t+1} \left(\frac{P_{i,t+1}}{P_{t+1}} - \mu_{t+1} \right) \left(\frac{P_{i,t+1}}{P_{t+1}} \right)^{-\theta} C_{t+1} \right\} + E_t \left\{ \beta \frac{\Lambda_{t+1}}{\Lambda_t} \Omega_{t+1} \right\}$$

is the expected present value of future profits. Defining the markup over marginal costs as

$$mu_{i,t} = \frac{P_{i,t}}{P_t \mu_t}, \qquad mu_t = \frac{1}{\mu_t},$$

one can write the FOC, evaluated at the symmetric equilibrium, as

$$mu_t = \frac{-\theta}{1 - \theta - \gamma \frac{\Omega_t}{C_t}} \tag{II.3.5}$$

In a symmetric intertemporal equilibrium in each period each firm sets the same price as all other firms. The most important implication regarding market shares is that $x_{i,t}$ equals one for all t and all i. According to equation (II.3.5) the equilibrium markup depends positively on current demand and negatively on the present value of future profits. In the static monopolistic competition model markups are given by

$$mu_t = \frac{\theta}{\theta - 1} \tag{II.3.6}$$

implying that at any point in time and in any given state of the economy pass-through of marginal cost changes to prices is complete. Unlike that model, in an environment characterized by market share competition markups will be generally time varying. Wether pass-through of marginal costs to prices will turn to be greater, lower or equal to one depends on the relative strength of the reactions of C_t and Ω_t to exogenous shocks. In the present model the discount factor is endogenous and strongly linked to current and next-period consumption, real balances and inflation - as shown above for $b = 1$ the discount factor is given by:

$$DF_t = \beta E_t \left\{ \frac{C_{t+1}^{a-1} \left(\frac{m_{t+1}}{\pi_{t+1}} \right)^{1-a}}{C_t^{a-1} \left(\frac{m_t}{\pi_t} \right)^{1-a}} \right\}.$$

For example, consider a positive monetary shock which *at given prices* increases current consumption *via* the positive income effect but also puts an upward pressure on current inflation as explained in the previous section. Obviously, the temporary (or even an one time) increase in current consumption will have a positive *direct* effect on markups but if at the same time the increase in current inflation π_t and/or next period cash balances m_{t+1} is *sufficiently*[13] large relative to the increase in C_t then the increase in the discount factor will be larger than that of current consumption, probably causing the term $\frac{\Omega_t}{C_t}$ to rise and thus markups to fall.

The equilibrium in this economy is described by the household's optimality conditions (II.2.3) through (II.2.7), the laws of motion of markups and the present value of future profits (II.3.5) and (II.3.4) respectively, and the equation specifying monetary policy.

3.2 Understanding Key Features of the Model

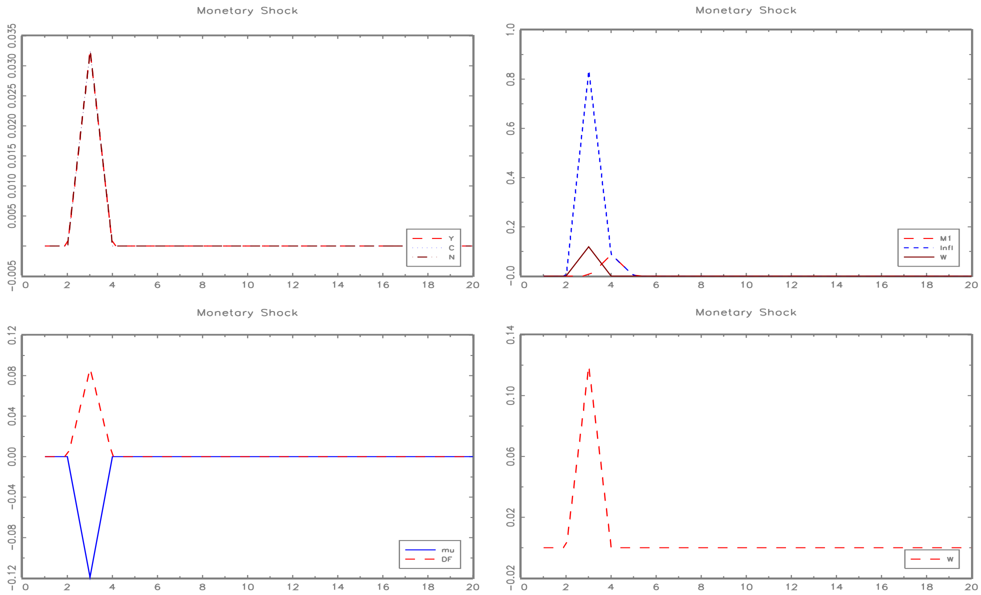
Figures (II.2) and (II.3) depict the impulse responses to a one time monetary expansion in $t = 3$ for $\theta = 0.6$ and $\theta = 1.4$ respectively.[14] In this economy inflation increases by 0.8322 (for $\theta = 0.5$) percent and 0.8382 percent (for $\theta = 1.4$) on impact, compared to 0.8433 percent in

[13] Actually one must compare the responses of C_t^a and $m_{t+1}^{1-a}\pi_{t+1}^{1-a}$.

[14] The corresponding program is **"MIU_sim_cm_2d5d_1_i.g"**. The calibration of the model is discussed in section 6.

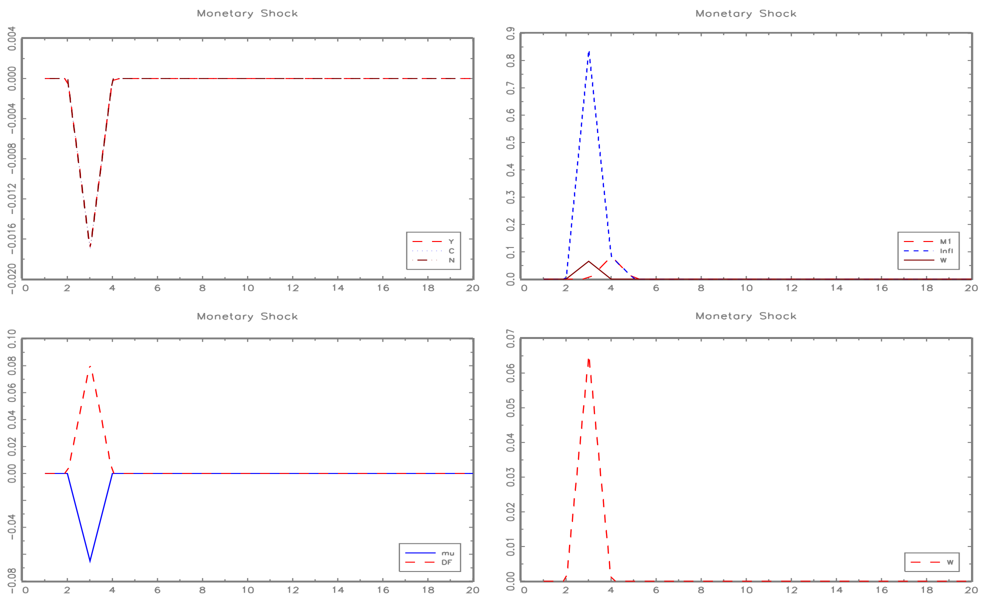
the model presented in the previous section. Hence the inclusion of market share competition strengthens the non-neutrality of money. Unfortunately, the change is quantitatively very small.

Figure II.2: MIU-model with fixed capital and market share competition. Impulse responses to a monetary shock, $\rho_T = 0$, $\theta = 0.5$, $a = 0.9$, $b = 1$. Percentage deviations from steady state.



Y - output, N - hours, C - consumption, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, DF - discount factor, W - real wage, mu - markup, $Omega$ - expected present value of firm's profits.

Figure II.3: MIU-model with fixed capital and market share competition. Impulse responses to a monetary shock, $\rho_T = 0$, $\theta = 1.4$, $a = 0.9$, $b = 1$. Percentage deviations from steady state.



Y - output, N - hours, C - consumption, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, DF - discount factor, W - real wage, mu - markup, $Omega$ - expected present value of firm's profits.

What's the intuition behind these results? Since households expect next period inflation to exactly offset any positive wealth effects stemming from the increase in real balances m_{t+1} and at the same time all future inflation rates, markups and productivity levels to remain constant they will have no incentive to set consumption, labor supply and savings at values different from their respective steady state values. As a consequence, the expected discounted present value of firm's profits Ω_t changes only because the discount factor DF_t changes, while the latter deviates from its steady state level only because the product $C_t^{1-a}\pi_t^{1-a}$ does. Hence, the log-deviation of the markup from its steady state level can be represented as:

$$\hat{mu}_t = -\xi((1-a)\hat{C}_t + (1-a)\hat{\pi}_t - \hat{C}_t) = \xi a\hat{C}_t - \xi(1-a)\hat{\pi}_t,$$

where $\xi = \frac{\gamma\frac{\Omega^*}{C^*}}{\gamma\frac{\Omega^*}{C^*}+\theta-1}$. With $a = 0.9$ the difference between the log-deviation of the discount factor and that of current consumption

$$\hat{DF}_t - \hat{C}_t = -a\hat{C}_t + (1-a)\hat{\pi}_t$$

will be positive as long as the increase in inflation is sufficiently large relative to the reaction of consumption. The latter is the case in all simulations performed. The optimal reaction of firms to an increase in Ω_t relative to C_t is to lower markups. As a result real wages (profits) rise (fall) forcing households to increase labor supply. But at the same time, the higher labor enables the economy to produce and therefore consume more. The potential to increase consumption as well as the above average inflation reduce the marginal utility of consumption, generating an incentive for households to reduce labor supply. Whether working hours will rise or fall depends on the relative strength of the positive effect of the markup and the negative effect of the fall in the marginal utility of consumption. Which of this two effects dominates depends on the short run elasticity of demand θ. Why? Optimal labor supply is given by

$$N_t = aC_t^{a-1}\left(\frac{m_t}{\pi_t}\right)^{1-a}\frac{W_t}{P_t}.$$

Its relative deviation from the steady state can be written as

$$\hat{N}_t = -\hat{C}_t + (\xi-1)(\underbrace{(1-a)\hat{\pi}_t - a\hat{C}_t}_{:=-\hat{mu}_t}),$$

and by imposing the equilibrium condition $N_t = C_t$ we get:

$$\hat{N}_t = \frac{(\xi-1)(1-a)}{2+a(\xi-1)}\hat{\pi}_t. \tag{II.3.1}$$

Since for $\theta \in (0,1)$ $\xi > 1$, while $\theta \geq 1$ implies $\xi \in (0,1]$, working hours respond positively (for $\theta < 1$) and negatively (for $\theta > 1$) to fluctuations of the inflation rate. In the case of $\theta \in (0,1)$ and thus $\xi > 1$ the slope of the first derivative of the current profit function is *relatively* small in absolute value. As a result, when changes of current inflation and/or current consumption occur firms need a *relatively* large adjustment of the markup in order to ensure that their respective Euler equations are still satisfied. Put differently, if current demand is relatively inelastic (the case of a low θ) the economy needs a larger adjustment of the markup to restore equilibrium after a monetary shock. In this case, for a given level of consumption, the fall of the markup is stronger than the decrease of the marginal utility of consumption, both caused by the increase in inflation. As a consequence, working hours increase. The resulting higher labor income for any given real wage as well as higher profits for any given markup level enable households to increase consumption. The latter, in turn, dampens the reactions of the markup and the marginal utility of consumption slightly. For a given level of C_t $\theta > 1$ and thus $\xi \in (0, 1)$ implies that the fall in the marginal utility of consumption is stronger than the increase in the real wage, both caused by the jump of the inflation rate. Therefore, in that case hours fall shifting income and consumption down. The reaction of consumption, again, implies a slight weakening of the effects induced by the rise in π_t.

Another way to gain intuition about the key mechanism in this model is as follows: Suppose, initially firms miss the occurence of the monetary shock and do not adjust the markup. Then consumption and inflation will react in exactly the same way as in the previous section - there will be a drop in current consumption and a large jump in current inflation. But can this situation be an equilibrium? The negative (positive) reaction of consumption (inflation) will induce an unambiguous[15] increase in

$$\hat{\Omega}_t - \hat{C}_t = \hat{DF}_t - \hat{C}_t = -a\hat{C}_t + (1-a)\hat{\pi}_t.$$

Hence, each firm will find it optimal to lower its markup. As a results the real wage will rise generating an incentive for households to increase labor supply. Thus, in this model for any level of consumption, labor supply will be higher than in the one developed in the previous section. For $\theta < 1$ labor, output and consumption actually increase, otherwise they fall.

According to figures (II.2) and (II.3) the major shortcoming of the model is that the one-time monetary disturbance induces purely temporary, one-time reactions of the main economic aggregates. This absence of any persistence is at odds with the empirical evidence provided by a vast number of studies employing structural VARs.[16] In the following sections I introduce different forms of capital accumulation and show that aside from making the production side of the economy more realistic, the inclusion of capital as a second state variable also substantially increases the persistence as well as magnitude of the responses to monetary shocks. The latter result contrasts with the results obtained by Heer and Maussner (2007) who show that extending Walsh's (2005) New Keynesian model by including capital accumulation substantially reduces the persistence of the real effects of monetary policy shocks.

[15] $\frac{m_{t+1}}{\pi_{t+1}}$ as well as all other future variables are expected to remain unchanged.

[16] See for example Christiano *et al.* (2005).

4 Capital Accumulation and Static Monopolistic Competition

4.1 The Theoretical Framework

Let us extend the model presented in section 2 by assuming that there is a second factor of production called capital. It is completely owned by households and in each period it is supplied to the firms at the rental rate R_t. The production function of an arbitrary firm i is of the Cobb-Douglas type and exhibits constant returns to scale:

$$Y_{i,t} = C_{i,t} + I_{i,t} = Z_t N_{i,t}^{\omega} K_{i,t}^{1-\omega}, \qquad \omega \in (0,1),$$

where Y_i is the sum of the production of the i-th type of consumption good C_i and the i-th type of investment good over which firm i has monopoly power. Capital is accumulated according to the law of motion

$$K_{t+1} = I_t + (1-\upsilon)K_t, \tag{II.4.1}$$

where K_t and $I_t = \left(\frac{1}{n}\sum_{i=1}^{n} I_{i,t}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}$ denote the aggregate stock of capital and aggregate investment respectively. $\upsilon \in (0,1)$ is the depreciation rate. Assuming that investment and consumption are perfect substitutes the budget restriction of the representative household reads:

$$C_t + I_t + m_{t+1} - \frac{m_t}{\pi_t} + b_{t+1} - \frac{b_t}{\pi_t} = \frac{W_t}{P_t}N_t + R_t K_t + \Pi_t + i_t\frac{b_t}{\pi_t} + \frac{T_t}{P_t}. \tag{II.4.2}$$

The Euler equation with respect to the stock of capital takes the form

$$\Lambda_t = \beta E_t \left\{\Lambda_{t+1}\left(1 + R_{t+1} - \upsilon\right)\right\}, \tag{II.4.3}$$

where Λ_t is the Lagrangean multiplier corresponding to the household's budget constraint.

In an environment characterized by monopolistic competition the equilibrium real rental rate of capital is smaller than the marginal product of capital and is given by

$$R_t = \frac{1-\omega}{mu_t}\left(\frac{N_t}{K_t}\right)^{\omega}.$$

Since in this version of the model there is only static monopolistic competition in the goods market the markup will be constant and tightly related to the short run elasticity of demand θ according to :

$$mu_t = mu = \frac{\theta}{\theta-1}, \qquad \forall t.$$

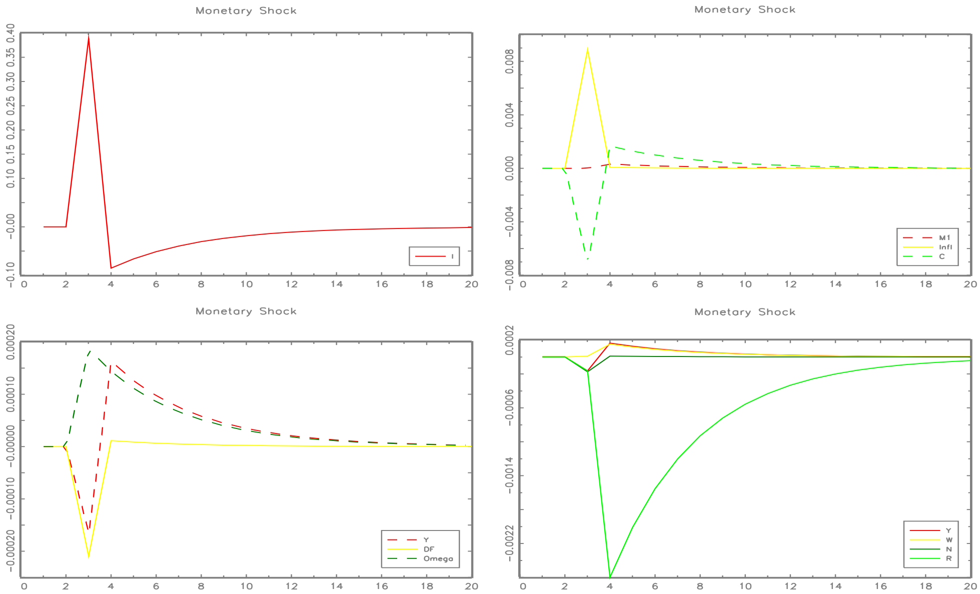
A description of the equilibrium in this economy is easily obtained - just add (II.4.1) and (II.4.3) to the equilibrium conditions of the model presented in section 2 and replace (II.2.7) by (II.4.2).

4.2 Understanding Key Features of the Model

Figure II.4 depicts the impulse responses to a non-autocorrelated monetary shock in $t = 3$. The initial sharp decline in working hours, output and consumption is due to the same mechanisms as that described in section 2: despite the positive income effect on consumption induced by the monetary transfer the jump of the inflation rate is sufficiently large to lower the marginal utility of current consumption. As a result the incentive to work *"today"* decreases relative to the incentive to supply labor in future periods, when the marginal utility of consumption is relatively high, because of the lower inflation and higher real balances. A similar reasoning governs the reaction of investment - it is optimal for households to accelerate capital accumulation at the expense of a lower consumption level *"today"* in order to be able to consume more *"tomorrow"*, when the marginal utility of consumption is relatively high. As can be seen, the increase in inflation alters the time path of the marginal utility of consumption sufficiently strongly, leading to a healthy increase in investment despite the fall in current labor and output. The additionally accumulated capital enables the economy to produce and consume more in the periods after the shock. The small positive deviation of the real wage from its steady state value in the period of the shock is due to the diminishing marginal productivity of labor. The above average wages in the aftermath of the shock are due to the increased stock of capital. Working hours are also above their long run level because of the increased marginal productivity of labor. In the period after the shock there is a large jump in output caused by the higher capital stock on the one hand and the above average hours on the other. The persistence in the impulse responses also results from the increase of the capital stock which is only slowly reduced to its initial level.

According to the shape of the reactions of the main economic aggregates displayed in figure II.4 monetary expansions are expansionary with an one-period delay - initially they induce a contraction but a long-lasting expansion in the aftermath of the shock. The impulse responses of some important macroeconomic variables to a positive monetary shock estimated by Christiano *et al.* (2005) also exhibit a delay of one or two periods. Nevertheless, there are several dimensions along which the predictions of the model presented in this section are at odds with the evidence provided by Christiano *et al.* (2005). For example, although output, consumption and working hours do not reach their respective highest levels initially (in the period of the shock), their impulse responses to the monetary disturbance are not *U-shaped* and of very limited magnitude. The only reaction which can be characterized as substantial is that of investment expenditure. The latter increases on impact only to fall below its long run level in the period after the shock. Christiano *et al.* (2005) find a similar, but more *U-shaped* response of investment in the data. Unfortunately, the deviation of investment from its steady state level is much stronger than what is observed empirically.

Figure II.4: MIU-model with endogenous capital without market share competition. Impulse responses to a monetary shock, $\rho_T = 0$, $a = 0.9$, $b = 1$, $\theta = 6 \Rightarrow mu^* = 1.2$. Relative deviations from steady state.



Y - output, I - investment, N - hours, C - consumption, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, DF - discount factor, $N - ir$ - nominal interest rate, W - real wage, $Omega$ - expected present value of firm's profits, R - real interest rate.

The magnitude as well as the persistence of the impulse responses predicted by the model are virtually insensitive to variations in the inverse of the elasticity of substitution between consumption and real balances b.

In the next section I explore how the introduction of market share competition affects the predictions of the MIU-model with endogenous capital presented in this section.

5 Capital Accumulation and Market Share Competition

I refer to the model presented in this section as the *Customer Markets Model* with fully flexible capital. The version developed in subsection 5.3 is called the *Customer markets Model* with adjustment costs of capital.

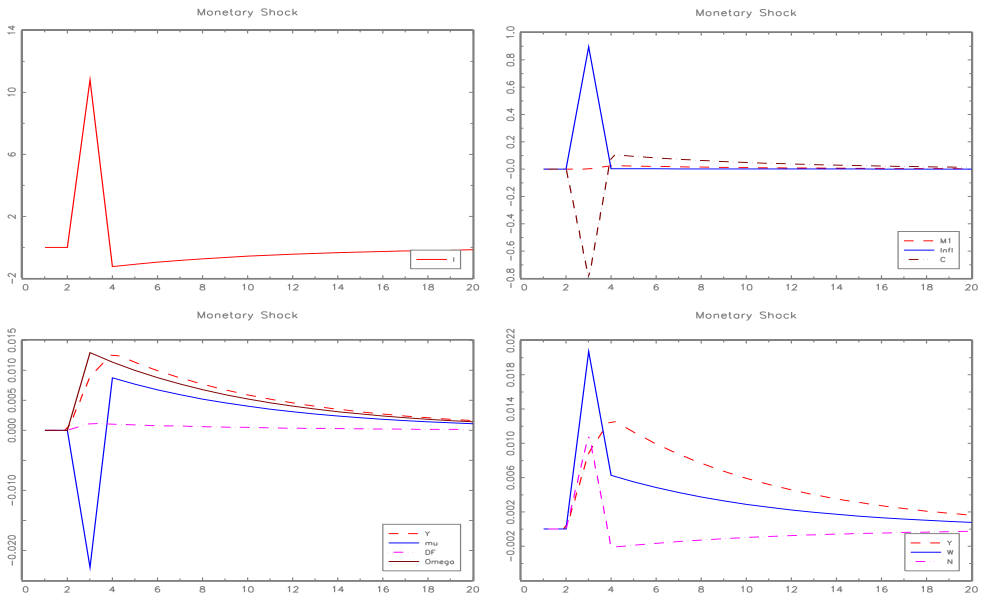
5.1 The Theoretical Framework

The introduction of market share competition just adds two further equilibrium conditions to the system describing the evolution of the economy presented in the previous section. Both of them result from the dynamic considerations arising within the firms' optimization problem in an environment characterized by market share competition. The two new equations are the laws of motion for markups and the present value of future profits (II.3.5) and (II.3.4) respectively. All other equilibrium conditions remain the same as in section 4.

5.2 Understanding Key Features of the Model

Figures II.5 through II.8 display the impulse responses to a one-time monetary expansion in $t = 3$ predicted by the model for different values of the short run elasticity of demand θ.[17]

Figure II.5: MIU-model with endogenous capital and market share competition. Impulse responses to a monetary shock, $\rho_T = 0$, $a = 0.9$, $b = 1$, $mu^* = 1.2$, $\theta = 0.2$. Percentage deviations from steady state.

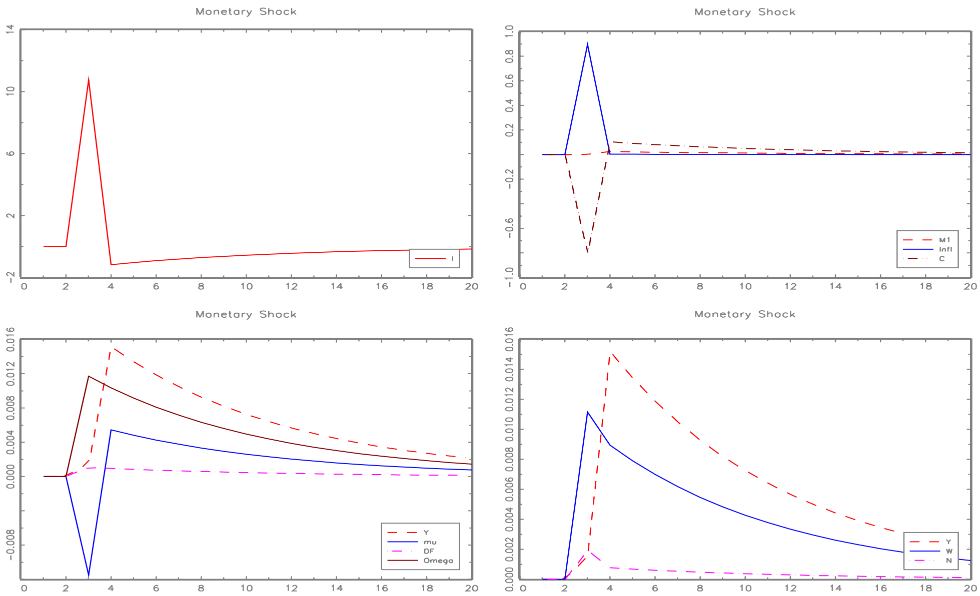


Y - output, I - investment, N - hours, C - consumption, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, DF - discount factor, $N - ir$ - nominal interest rate, W - real wage, $Omega$ - expected present value of firm's profits, mu - markup, R - real interest rate.

How does the inclusion of market share competition alter the qualitative predictions of the model with endogenous capital accumulation? Since markups become endogenous, the reactions of real wages and profits will turn to be different than the ones shown in figure II.4. For example, if firms reduce markups as a reaction to the monetary disturbance then for any given deviation of labor and output from their respective steady state levels the impact response of real wages (profits) will be stronger (weaker) than it was the case in the model presented in section 4. Hence, the income and substitution effects induced will force households to work more and therefore enable the economy to produce more than the one in the previous section. As figures II.5 through II.8 show, for a broad range of values of θ the impact reaction of the expected present value of future profits Ω_t is stronger than that of current demand $Y_t = C_t + I_t$ with the consequence that firms find it optimal to lower markups. In the *Customer Market Model* without capital the sole reason for the increase in Ω_t was the sharp jump in the discount factor caused by the increase in inflation while future profits remained unchanged. Unlike that model, in the economy presented in this section there is also a second force, beyond the increase of the discount factor, leading to an increase in the expected

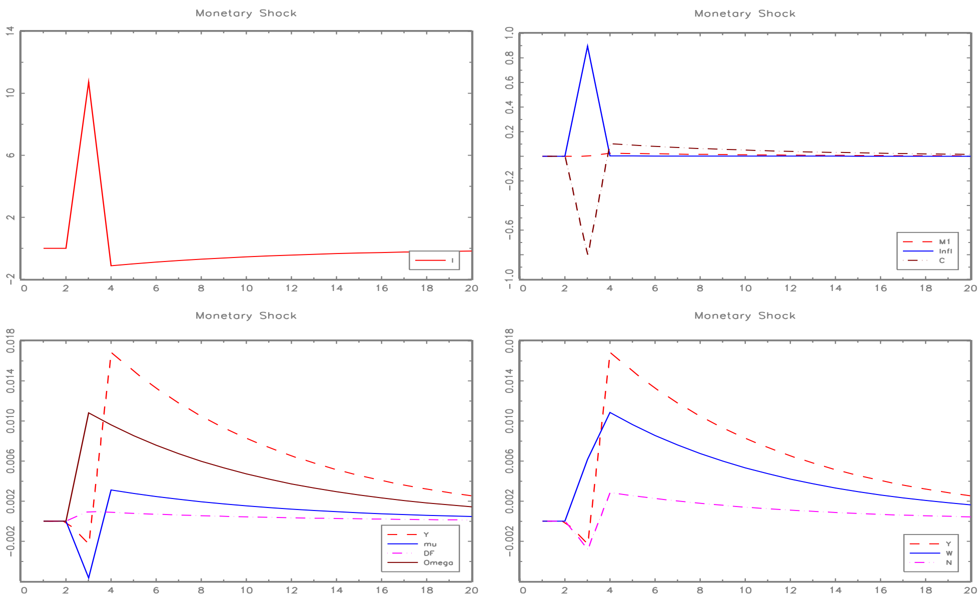
[17] The calibration of the model is described in section 6. The corresponding program is **"MIU_sim_cm_2d5c_1_i.g"**.

Figure II.6: MIU-model with endogenous capital and market share competition. Impulse responses to a monetary shock, $\rho_T = 0$, $a = 0.9$, $b = 1$,$mu^* = 1.2$, $\theta = 0.9$. Percentage deviations from steady state.



Y - output, I - investment, N - hours, C - consumption, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, DF - discount factor, $N - ir$ - nominal interest rate, W - real wage, $Omega$ - expected present value of firm's profits, mu - markup, R - real interest rate.

Figure II.7: MIU-model with endogenous capital and market share competition. Impulse responses to a monetary shock, $\rho_T = 0$, $a = 0.9$, $b = 1$, $mu^* = 1.2$, $\theta = 1.9$. Percentage deviations from steady state.

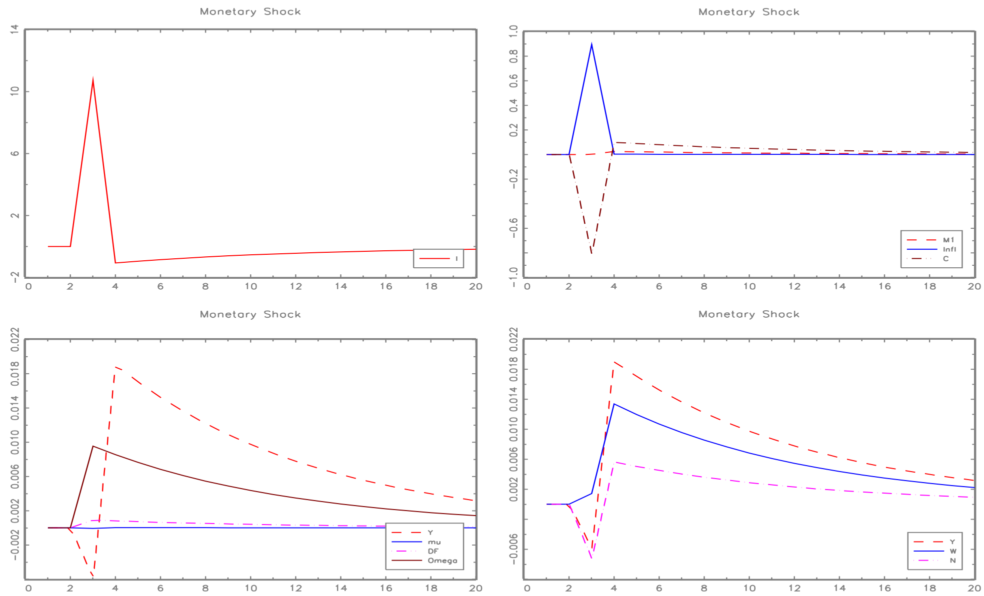


Y - output, I - investment, N - hours, C - consumption, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, DF - discount factor, $N - ir$ - nominal interest rate, W - real wage, $Omega$ - expected present value of firm's profits, mu - markup, R - real interest rate.

present value of profits: in the period of the shock households accumulate additional capital which increases their future income[18] with the consequence of a higher aggregate demand for

[18]For a given amount of hours worked and a given markup the additionally accumulated capital increases labor income by making working hours more productive, increases capital income despite the induced fall in the

Figure II.8: MIU-model with endogenous capital and market share competition. Impulse responses to a monetary shock, $\rho_\tau = 0$, $a = 0.9$, $b = 1$, $mu^* = 1.2$, $\theta = 5.9$. Percentage deviations from steady state.



Y - output, I - investment, N - hours, C - consumption, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, DF - discount factor, $N - ir$ - nominal interest rate, W - real wage, $Omega$ - expected present value of firm's profits, mu - markup, R - real interest rate.

goods in the aftermath of the shock. Everything else given, a higher level of future demand increases the expected present value of firms' profits. The decrease in the current markup level has a positive effect on the real wage and so, generates an incentive to supply more labor than in the economy without market share competition.[19] Figures II.5 through II.8 reveal that if the short run elasticity of demand is not too large[20] the drop in markups is sufficiently large to induce a large enough increase in the real wage which, in turn, more than compensates the negative effect on the labor supply decision induced by the large decline in the current marginal utility of consumption. Because of the large increase of investment in the period of the shock the economy accumulates substantial additional capital which leads to an an even sharper increase in output in the period after the shock. Since inflation returns to its long run level in the period after the shock, the deviation of the discount factor from its steady state value becomes very small, implying a smaller deviation from the long run level of Ω_t in that period. As can be seen, if θ lies in the empirically relevant range, the effect of capital accumulation is sufficiently strong to push output to a value higher than that of Ω_t in the aftermath of the shock. As a consequence, the markup rises to an above average level more or less sharply reducing (*via* the downward pressure on wages) the incentive to

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real interest rate *(diminishing marginal productivity of capital)* and it also has a positive effect on future profits. Note that the sum of the three income types does not depend on the markup mu_t. Hence, from the point of view of the individual household markup variations do only change the composition of the income stream but not its level.

[19]See section 4.

[20]Values of θ smaller than 1.1 imply a positive impact response of hours to monetary shocks.

work. According to the impulse responses shown, lower values of θ imply stronger reactions of markups which, in turn, induce a larger increase in working hours in the period of the shock and than a larger drop of that variable thereafter (compare figure II.5 with figure II.6).

Values of θ near one imply delayed responses of output and employment to monetary innovations with both variables reaching their highest values in the period after the shock. While such a prediction should be seen as more or less in line with the existing VAR evidence, the responses of several other variables to monetary disturbances are not consistent with the patterns found in the data. For example, in the model economy developed in this section a money supply loosening induces a large short-run contraction of consumption. In the periods after the shock consumption expenditure reaches an above average value but its deviation from the steady state level is very small. The responses of investment and real wages also do not exhibit an *U-shaped* form. Both variables reach their highest deviation from the stationary equilibrium in the period in which the monetary innovation occurs. Furthermore, the reaction of investment is many times larger than measured by any of the existing empirical studies. In addition, investment is the only variable the response of which can be characterized as a *substantial real effect* of monetary policy. The reactions of the remaining real variables are persistent but of limited magnitude. For example, in the case $\theta = 0.5$ the largest deviation of output from its steady state, reached in the second period, is equal to 0.014%, which is much less (about ten times smaller) than what a serially uncorrelated technology shock of similar size would induce.

Similar to the economy of section 4 the magnitude as well as the persistence of the impulse responses in the current model are virtually insensitive to variations in the elasticity of substitution between consumption and real balances $1/b$.

5.3 A Customer Markets Model with Adjustment Costs of Capital

How does the inclusion of adjustment costs of capital alter the dynamic properties of the model? The economic intuition suggests that if investment is sufficiently costly households will be reluctant to accelerate capital accumulation in such a dramatic manner as they do in the models developed in sections 4 and 5. As a consequence, there will be more resources left for consumption in the period of the shock, enabling the theory to get rid of the counterfactual sharp increase (decline) in investment (consumption). In addition, the presence of adjustment costs of capital will strengthen the incentive to raise future real money holdings m_{t+1} which, in turn, will induce a positive wealth effect in $t+1$, $t+2$, ... and so, probably, at least partly, preserve the persistence in the impulse responses. Note that the wealth effect in $t+1$ induced by a higher level of real balances triggers off qualitatively the same reactions as the positive income effect of the monetary disturbance in the period of the shock. Furthermore, because the introduction of capital adjustment costs shifts the properties of the model towards the fixed capital case, one could expect to be able to make the real effects of monetary shocks arbitrarily large by setting the short run elasticity of demand θ at a sufficiently low value and the elasticity of substitution between consumption and real balances b at a sufficiently high value. In other words, since the *Customer Markets Model* with adjustment costs of capital represents the intermediate case between the model developed in section 3 and the one presented in section 5 there is no *a priori* reason not to expect that it will be a combination of the favorable properties of the latter two models.

Formal Details: The flexibility of investment is reduced in an *ad hoc* manner by assuming that there is an additional adjustment cost of capital represented by the strict concavity of the strictly increasing function $\phi\left(\frac{I_t}{K_t}\right)$ in

$$K_{t+1} = \phi\left(\frac{I_t}{K_t}\right) K_t + (1-\upsilon)K_t. \tag{II.5.1}$$

Further, $\phi(.)$ has the properties:

$$\phi\left(\frac{I}{K}\right) = \phi(\upsilon) = \upsilon, \qquad \phi'(\upsilon) = 1,$$

where I and K are the steady state levels of investment and capital respectively.[21] The first assumption ensures that the steady state is characterized by the absence of adjustment costs while the second implies that in the stationary equilibrium *Tobin's q* is equal to one. Formally the equilibrium conditions (II.2.3) through (II.2.7) ought to be adjusted by including the household's first order condition with respect to investment

$$q_t = \frac{\Lambda_t}{\phi'\left(\frac{I_t}{K_t}\right)},$$

substituting the conventional transition equation for capital by (II.5.1) and replacing the first order condition with respect to next period's stock of capital (II.4.3) by

$$q_t = E_t\left\{\Lambda_{t+1}\frac{1-\omega}{mu_{t+1}}\frac{Y_{t+1}}{K_{t+1}} + q_{t+1}\left(1-\upsilon+\phi\left(\frac{I_{t+1}}{K_{t+1}}\right) - \phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}}{K_{t+1}}\right)\right\},$$

where q_t denotes the Lagrangean multiplier attached to (II.5.1) and q_t/Λ_t equals *Tobin's q*.

The individual firm faces the same optimization problem and thus, behaves according to the same optimality conditions as in the economy with market share competition and fully flexible capital described in the first part of section 5.

[21] The assumed law of motion of capital is also used by Correia *et al.* (1995), Jerman (1998) and others.

Impulse Responses to Monetary Shocks: In this model there is a further unobservable parameter besides a and b to be chosen - the elasticity of the adjustment cost function $\phi\left(\frac{I_t}{K_t}\right)$ with respect to the investment capital ratio I_t/K_t. Let ζ denote that elasticity. I first set ζ at the value which, combined with $\theta = 0.6$, implies that the theoretical model reproduces the empirically observable relation between the standard deviation of investment and that of output.[22] The parameters of the utility function a and b were again set at 0.9 and 1 respectively. First, I explore how the short run elasticity of demand θ affects the qualitative and quantitative predictions of the model. Figures II.9 through II.11 depict the impulse responses to a one-time monetary shock in $t = 3$, and reveal a rather disappointing picture.[23] As expected, by making investment more costly agents become unwilling to increase that variable by an amount as large as in the case of a fully flexible capital. However, at the same time the qualitative as well as quantitative properties of the model with respect to all other variables except consumption are dramatically shifted towards that of the *Customer Market Model* without capital. Much as in that model, output, wages, hours and markups reach their largest deviations from the stationary equilibrium in the period of the shock. The lower the value of θ the more pronounced the fall in markups and thus, the stronger the increase in wages. As a result, for relatively low (large) values of θ hours and thus output increase (fall). Unfortunately consumption reacts to monetary shocks in a similar way as it does in the *Customer Markets Model* with fully flexible capital presented in the first part of the current section - it decreases more or less sharply on impact and rises to an above average level in the periods after the shock. In other words, instead of assembling the favorable properties of the economy with fixed and that with flexible capital, the model with adjustment costs rather turns to be a combination of the undesirable features of that theories. By varying the parameters of the model it is possible to get arbitrarily close to the intermediate case which is characterized by impulse responses of significant magnitude[24] on the one hand and a more or less delayed and persistent deviations from the steady state[25] on the other. However, that intermediate case has an important drawback - the reactions of consumption to monetary shocks is negligible. The latter is at odds with the bulk of the empirical evidence.

A comparison with the New Keynesian model I:
For the sake of better comparability I use a version of the *New Keynesian Model* characterized by the same utility function, the same production technology and the same law of motion for capital as in the *Customer Markets Model* with adjustment costs of capital. From a technical point of view the only difference between the two models concerns the firm's condition for

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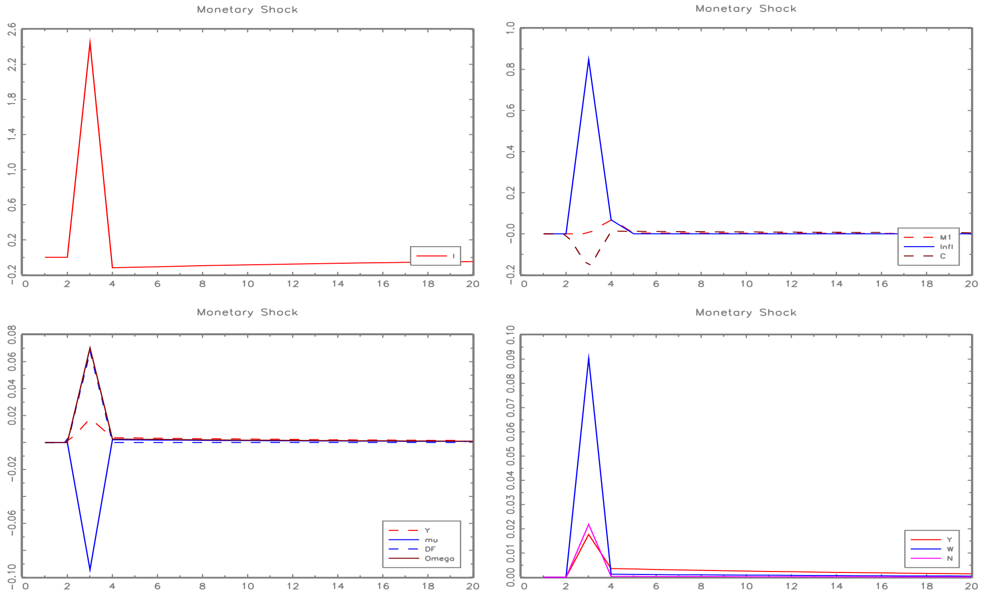
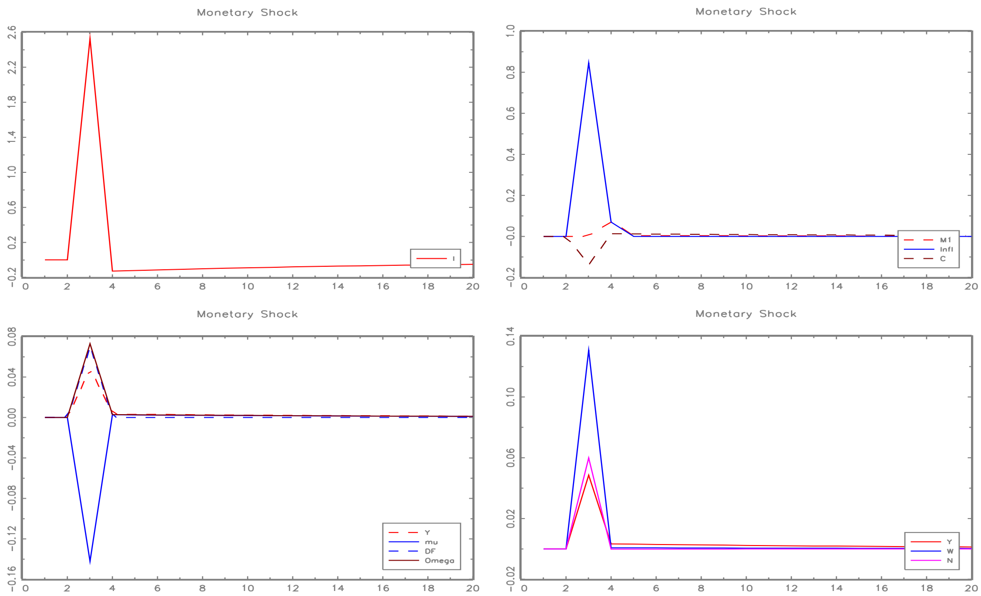
[22]The total factor productivity Z_t is assumed to be autocorrelated with coefficient of autocorrelation equal to 0.9641. (See Appendix 6 for calibration details.)
[23]The corresponding program is **"MIU_sim_cm2d5c_1_ac.g"**.
[24]As in the model with fixed capital.
[25]As in the model with flexible capital.

Figure II.9: MIU-model with adjustment costs of capital and market share competition. Impulse responses to a monetary shock, $\rho_T = 0$, $a = 0.9$, $b = 1$, $mu^* = 1.2$, $\theta = 0.6$, $\zeta = -0.02607$. Percentage deviations from steady state.



Y - output, I - investment, N - hours, C - consumption, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, DF - discount factor, $N - ir$ - nominal interest rate, W - real wage, $Omega$ - expected present value of firm's profits, mu - markup, R - real interest rate.

Figure II.10: MIU-model with adjustment costs of capital and market share competition. Impulse responses to a monetary shock, $\rho_T = 0$, $a = 0.9$, $b = 1$, $mu^* = 1.2$, $\theta = 0.2$, $\zeta = -0.02607$. Percentage deviations from steady state.

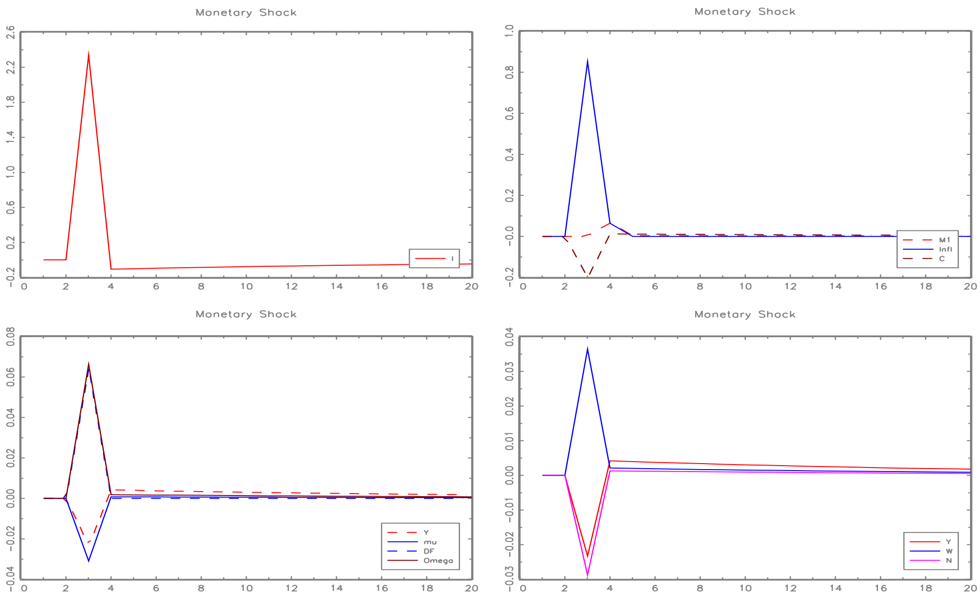


Y - output, I - investment, N - hours, C - consumption, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, DF - discount factor, $N - ir$ - nominal interest rate, W - real wage, $Omega$ - expected present value of firm's profits, mu - markup, R - real interest rate.

optimal price setting evaluated at the symmetric equilibrium. In the *New Keynesian Model* it reads:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} - \frac{(1-\varphi)(1-\varphi\beta)}{\varphi} \hat{mu}_t, \qquad \text{(II.5.2)}$$

Figure II.11: MIU-model with adjustment costs of capital and market share competition. Impulse responses to a monetary shock, $\rho_T = 0$, $a = 0.9$, $b = 1$, $mu^* = 1.2$, $\theta = 2.2$, $\zeta = -0.02607$. Percentage deviations from steady state.



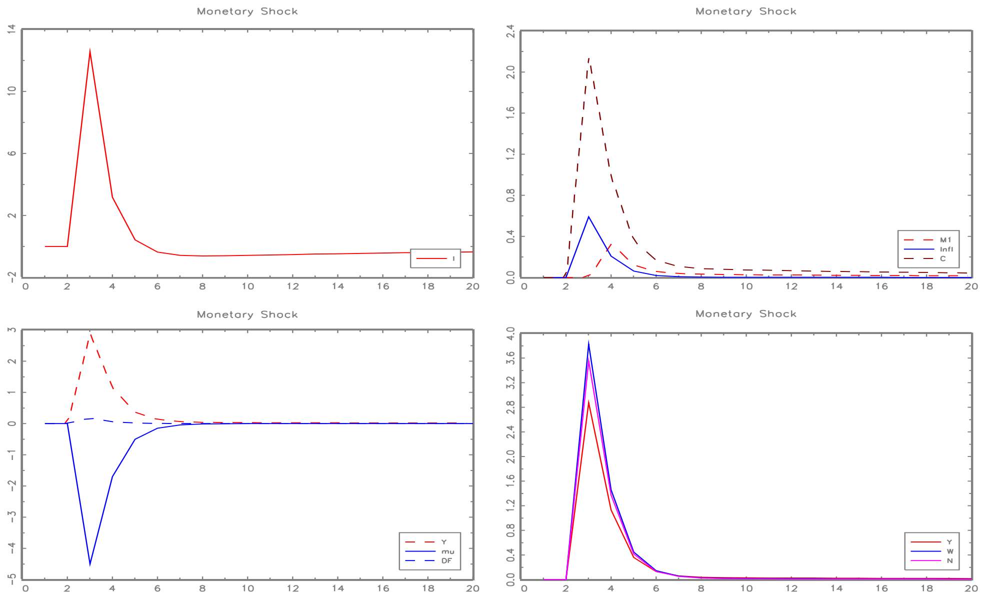
Y - output, I - investment, N - hours, C - consumption, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, DF - discount factor, $N - ir$ - nominal interest rate, W - real wage, $Omega$ - expected present value of firm's profits, mu - markup, R - real interest rate.

where φ denotes the fraction of firms which are not allowed to adjust their prices within a period. (II.5.2) is known as the *New Keynesian Phillipps Curve* and replaces (II.3.4) and (II.3.5). Figures II.12 and II.13 depict the impulse responses to an non-autocorrelated monetary shock for $\varphi = 0.75$ and $\varphi = 0.3$ respectively.[26] θ was set equal to 6 in order to ensure that the steady state markup equals 1.2. a and b again take the values 0.9 and 1. The value of $\zeta = -0.024$ implies that for $\varphi = 0.75$ investment is about 4.65 times as volatile as output. In both cases the impact reactions to the monetary innovation are about ten times stronger than in the *Customer Market Model*. Further, the New Keynesian Model implies more persistent deviations from the steady state[27] However, reducing the degree of price rigidity from $\varphi = 0.75$ to the value suggested by Bils and Klenow (2004) and Kehoe and Midrigan (2008), $\varphi = 0.3$, dramatically worsens the predictions of the model with respect to the duration of the impulse responses.[28] In summary, the New Keynesian Model ascribes much more relevance to monetary shocks than the *Customer Market Model* does. Furthermore, as figures II.9 through II.13 show, for intermediate values of capital adjustment costs the predictions of the former model are closer to the empirical evidence than that of the latter.

How does the degree of inflexibility of capital accumulation affect the results? As figure II.14 shows, setting ζ to the value estimated by Jerman (1998) and thus making investment

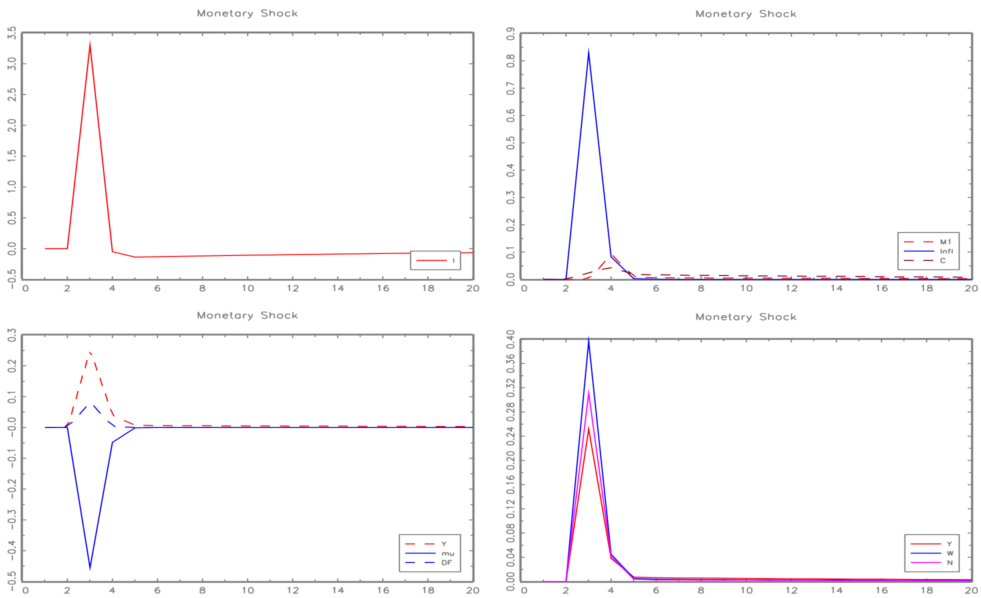
[26] See program **"keynes_ac.g"**
[27] See figure II.12.
[28] See figure II.13.

Figure II.12: The New Keynesian Model. Impulse responses to a monetary shock, $\rho_T = 0$, $a = 0.9$, $b = 1$, $mu^* = 1.2$, $\theta = 6$, $\varphi = 0.75$, $\zeta = -0.024$. Percentage deviations from steady state.



Y - output, I - investment, N - hours, C - consumption, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, DF - discount factor, $N - ir$ - nominal interest rate, W - real wage, $Omega$ - expected present value of firm's profits, mu - markup, R - real interest rate.

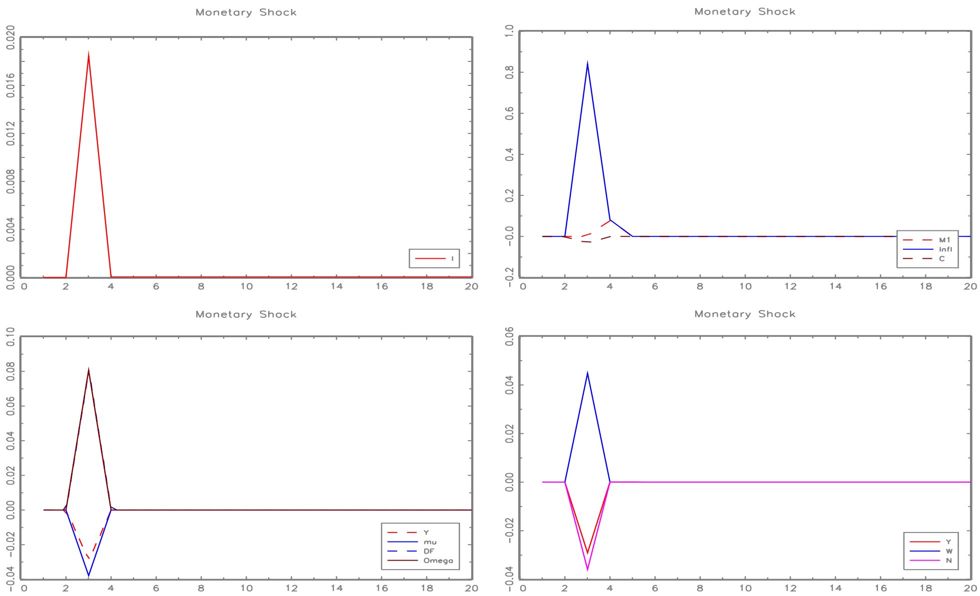
Figure II.13: The New Keynesian Model. Impulse responses to a monetary shock, $\rho_T = 0$, $a = 0.9$, $b = 1$, $mu^* = 1.2$, $\theta = 6$, $\varphi = 0.3$, $\zeta = -0.024$. Percentage deviations from steady state.



Y - output, I - investment, N - hours, C - consumption, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, DF - discount factor, $N - ir$ - nominal interest rate, W - real wage, $Omega$ - expected present value of firm's profits, mu - markup, R - real interest rate.

much more costly just makes the current version of the *Customer Markets Model* a close replication of the one with fixed capital. Hours, production and consumption increase by more on impact, while the reaction of investment is much weaker. Making investment less expensive, e.g. $\zeta = 0.00004347$, just leads to a new *Customer Markets Model* which mimics its version with fully flexible capital.

Figure II.14: MIU-model with adjustment costs of capital and market share competition. Impulse responses to a monetary shock, $\rho_{\tau} = 0$, $a = 0.9$, $b = 1$, $mu^{*} = 1.2$, $\theta = 0.2$, $\zeta = -4.3478$. Percentage deviations from steady state.



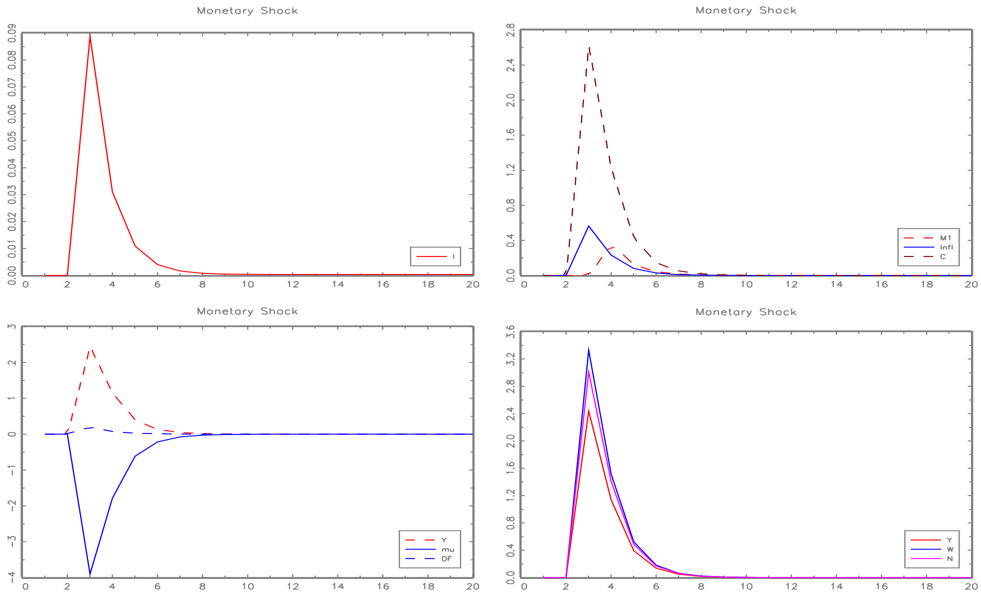
Y - output, I - investment, N - hours, C - consumption, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, DF - discount factor, $N - ir$ - nominal interest rate, W - real wage, $Omega$ - expected present value of firm's profits, mu - markup, R - real interest rate.

A comparison with the New Keynesian Model II:

In contrast, setting ζ at Jerman's value in the *New Keynesian Model* makes the impulse responses even more weaker with all macroeconomic aggregates except investment reaching their maximum in the period after the shock (figures II.15 and II.16). This kind of one period delay is consistent with the findings of Christiano *et al.* (1996, 2005). Setting ζ at 0.00004347 leads to a worsening of the prediction of the New Keynesian Model - the persistence in the impulse responses of all variables except consumption almost completely disappears (figure II.17). In addition, both models model provide the counterfactual implication of a large drop in consumption in the period of the shock.

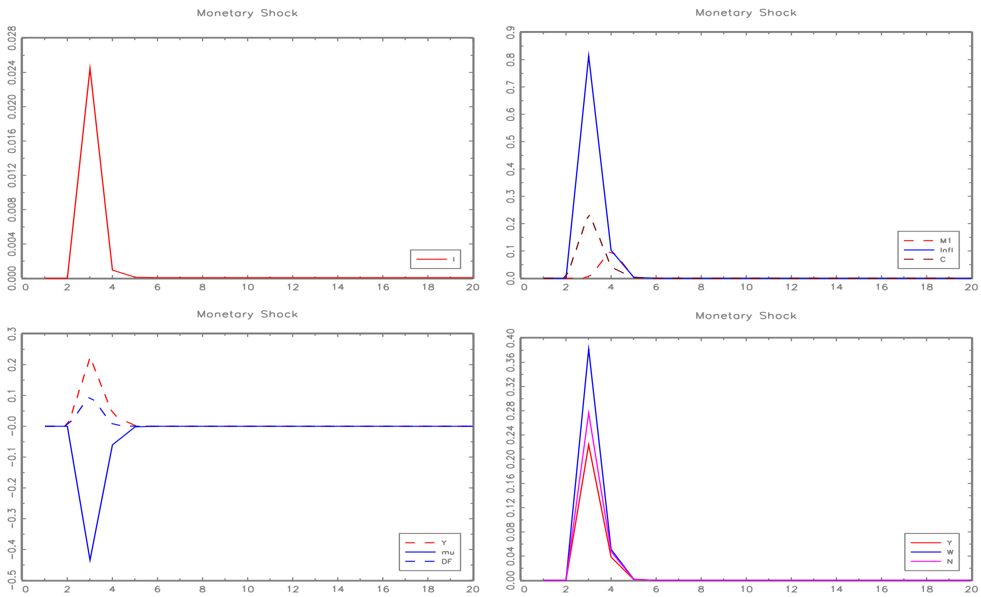
Variations in the elasticity of substitution between real balances and consumption $1/b$ have similar effects on the model's predictions as in the fixed-capital case. The higher the value of b and thus the lower the elasticity of substitution between C_t and $\frac{M_t}{P_t}$ the larger the magnitude of the impulse responses. Figure II.18 depicts the case $b = 20$. Relatively low values of b imply that $\frac{M_t}{P_t}$ and C_t are relatively close substitutes. Therefore from the household's point of view there is a higher incentive to compensate the drop in real balances induced by the increase in inflation in the period of the shock by choosing a higher level of consumption C_t. As a result, in the case of a low b the monetary disturbance alters the time path of the marginal utility of consumption less heavily than when b is high. Consequently, in the former case there are weaker incentives to adjust labor supply and the stock of capital. The impulse responses to a monetary shock in the case $b = 0.02$ are displayed in figure II.19. In the New

Figure II.15: The New Keynesian Model. Impulse responses to a monetary shock, $\rho_\tau = 0$, $a = 0.9$, $b = 1$, $mu^* = 1.2$, $\theta = 6$, $\varphi = 0.75$, $\zeta = -4.3478$. Percentage deviations from steady state.



Y - output, I - investment, N - hours, C - consumption, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, DF - discount factor, $N - ir$ - nominal interest rate, W - real wage, $Omega$ - expected present value of firm's profits, mu - markup, R - real interest rate.

Figure II.16: The New Keynesian Model. Impulse responses to a monetary shock, $\rho_\tau = 0$, $a = 0.9$, $b = 1$, $mu^* = 1.2$, $\theta = 6$, $\varphi = 0.3$, $\zeta = -4.3478$. Percentage deviations from steady state.

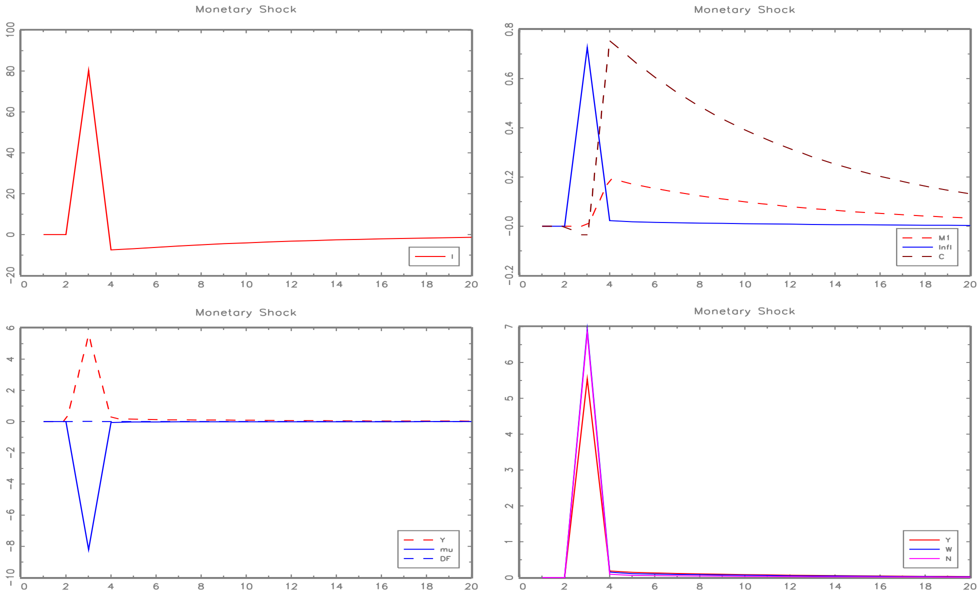


Y - output, I - investment, N - hours, C - consumption, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, DF - discount factor, $N - ir$ - nominal interest rate, W - real wage, $Omega$ - expected present value of firm's profits, mu - markup, R - real interest rate.

Keynesian Model higher values of b imply more persistent but less pronounced reactions to nominal disturbances (figures II.20 through II.23).

The results can be summarized as follows. If capital accumulation is relatively flexible the *Customer Markets Model* implies more realistic implications with respect to the duration and the shape of the effects of monetary shocks than the New Keynesian model does. Yet, in

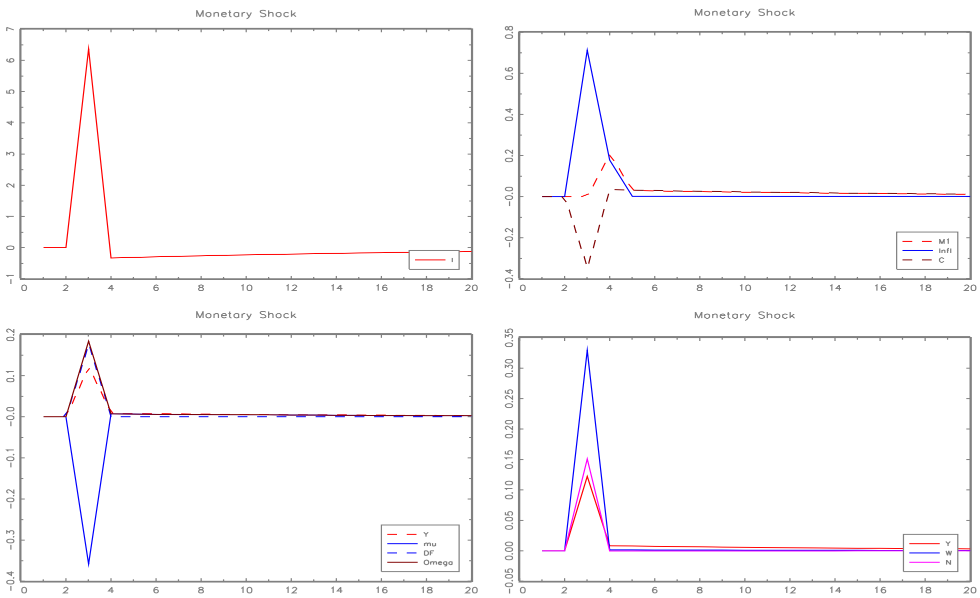
Figure II.17: The New Keynesian Model. Impulse responses to a monetary shock, $\rho_T = 0$, $a = 0.9$, $b = 1$, $mu^* = 1.2$, $\theta = 6$, $\varphi = 0.75$, $\zeta = -0.00004347$. Percentage deviations from steady state.



Y - output, I - investment, N - hours, C - consumption, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, DF - discount factor, $N - ir$ - nominal interest rate, W - real wage, $Omega$ - expected present value of firm's profits, mu - markup, R - real interest rate.

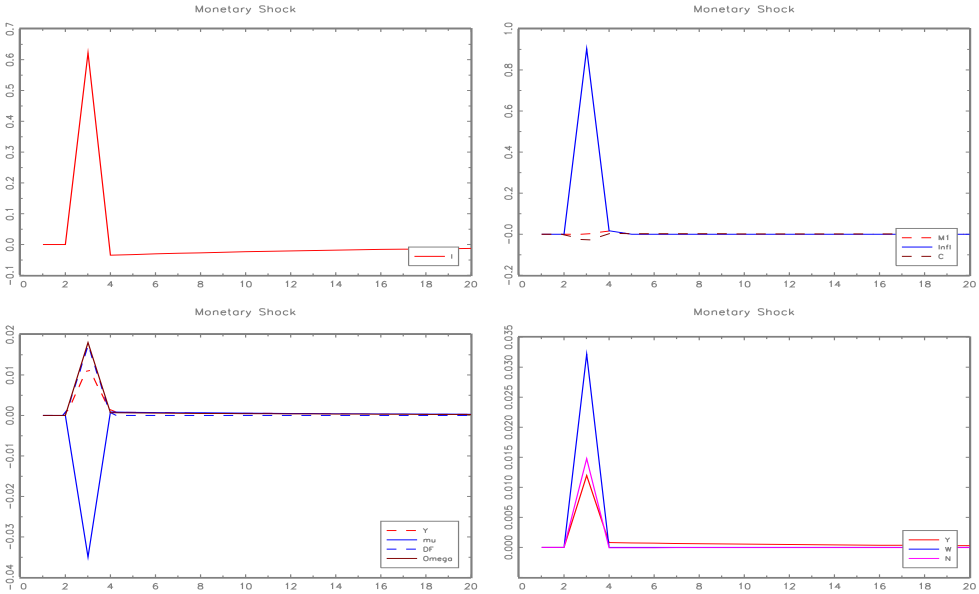
all cases the *Customer Markets Model* significantly understates the empirically observable magnitude of the impulse responses of output, consumption, labor and wages to monetary disturbances. In contrast, if capital accumulation is sufficiently costly or inflexible, then the New Keynesian model provides the more realistic predictions.

Figure II.18: MIU-model with adjustment costs of capital and market share competition. Impulse responses to a monetary shock, $\rho_T = 0$, $a = 0.9$, $b = 20$, $mu^* = 1.2$, $\theta = 0.2$, $\zeta = -0.02607$. Percentage deviations from steady state.



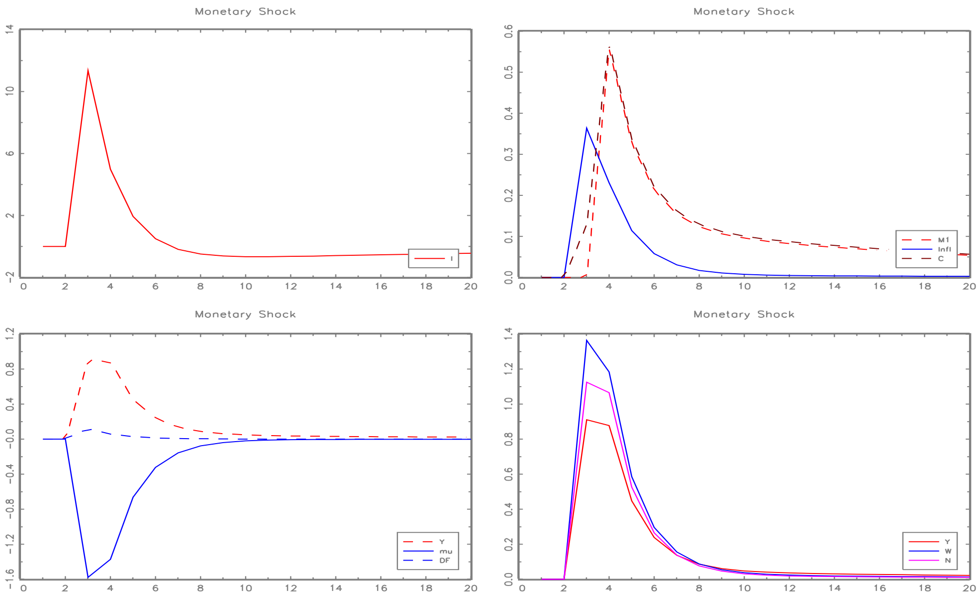
Y - output, I - investment, N - hours, C - consumption, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, DF - discount factor, $N - ir$ - nominal interest rate, W - real wage, $Omega$ - expected present value of firm's profits, mu - markup, R - real interest rate.

Figure II.19: MIU-model with adjustment costs of capital and market share competition. Impulse responses to a monetary shock, $\rho_\tau = 0$, $a = 0.9$, $b = 0.02$, $mu^* = 1.2$, $\theta = 0.2$, $\zeta = -0.02607$. Percentage deviations from steady state.



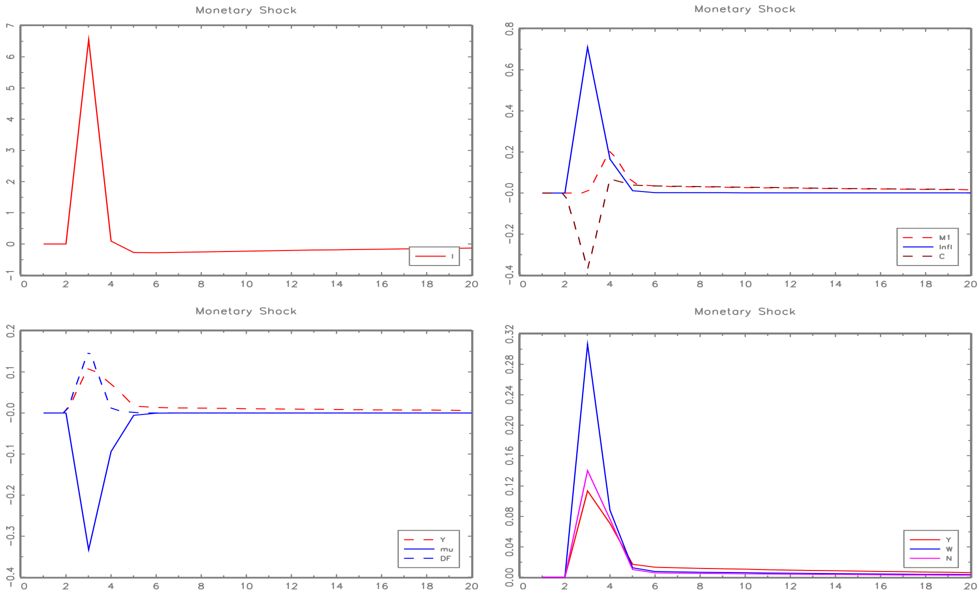
Y - output, I - investment, N - hours, C - consumption, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, DF - discount factor, $N - ir$ - nominal interest rate, W - real wage, $Omega$ - expected present value of firm's profits, mu - markup, R - real interest rate.

Figure II.20: The New Keynesian Model. Impulse responses to a monetary shock, $\rho_\tau = 0$, $a = 0.9$, $b = 20$, $mu^* = 1.2$, $\theta = 6$, $\varphi = 0.75$, $\zeta = -0.024$. Percentage deviations from steady state.



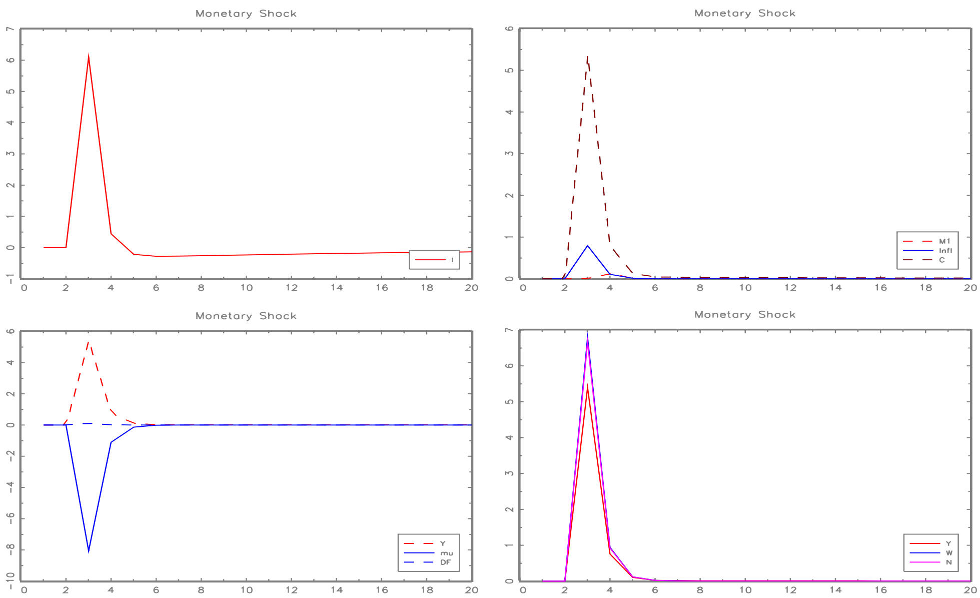
Y - output, I - investment, N - hours, C - consumption, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, DF - discount factor, $N - ir$ - nominal interest rate, W - real wage, $Omega$ - expected present value of firm's profits, mu - markup, R - real interest rate.

Figure II.21: The New Keynesian Model. Impulse responses to a monetary shock, $\rho_\tau = 0$, $a = 0.9$, $b = 20$, $mu^* = 1.2$, $\theta = 6$, $\varphi = 0.3$, $\zeta = -0.024$. Percentage deviations from steady state.



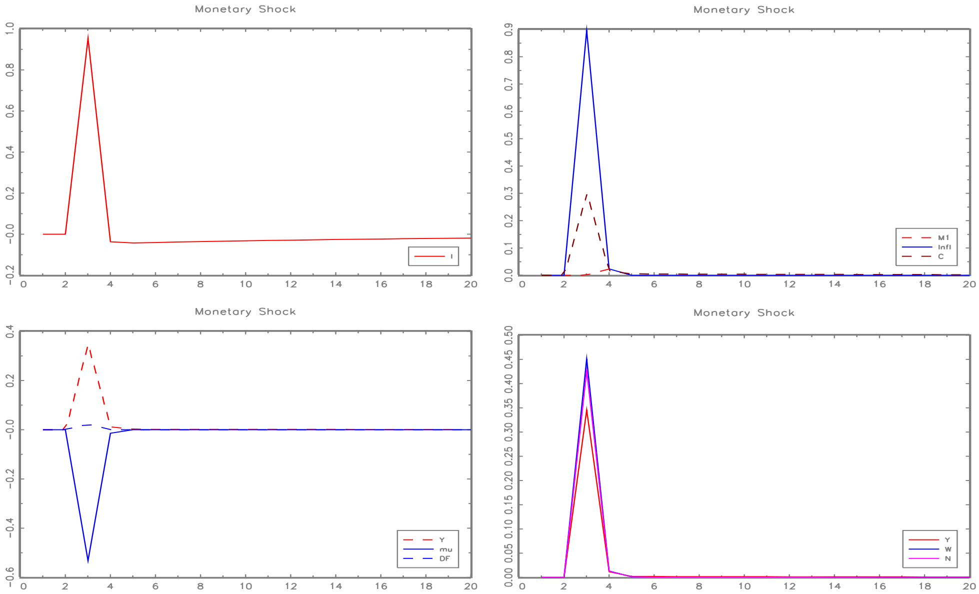
Y - output, I - investment, N - hours, C - consumption, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, DF - discount factor, $N-ir$ - nominal interest rate, W - real wage, $Omega$ - expected present value of firm's profits, mu - markup, R - real interest rate.

Figure II.22: The New Keynesian Model. Impulse responses to a monetary shock, $\rho_\tau = 0$, $a = 0.9$, $b = 0.02$, $mu^* = 1.2$, $\theta = 6$, $\varphi = 0.75$, $\zeta = -0.024$. Percentage deviations from steady state.



Y - output, I - investment, N - hours, C - consumption, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, DF - discount factor, $N-ir$ - nominal interest rate, W - real wage, $Omega$ - expected present value of firm's profits, mu - markup, R - real interest rate.

Figure II.23: The New Keynesian Model. Impulse responses to a monetary shock, $\rho_\tau = 0$, $a = 0.9$, $b = 0.02$, $mu^* = 1.2$, $\theta = 6$, $\varphi = 0.3$, $\zeta = -0.024$. Percentage deviations from steady state.



Y - output, I - investment, N - hours, C - consumption, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, DF - discount factor, $N - ir$ - nominal interest rate, W - real wage, $Omega$ - expected present value of firm's profits, mu - markup, R - real interest rate.

6 Calibration

In models featuring static monopolistic competition the short run price elasticity of demand for an individual good θ is restricted to be greater than unity in order to ensure that the markup of prices over marginal costs is greater than one and thus profits are positive. Usually θ is set to a value between 6 and 8 since empirically observable average markups are relatively low - according to most estimations they are smaller than 1.6. In contrast to the static monopolistic competition model in the economies featuring market share competition described above one does not need to impose the restriction $\theta > 1$ since θ is not the sole determinant of the steady state markup mu^*. In fact, as I show below, any value of θ smaller than $\frac{mu^*}{mu^*-1}$ is consistent with $mu^* > 1$ and a negative first derivative of the function $g\left(\frac{P_{i,t}}{P_t}\right)$. A large part of the empirical evidence suggests that the short run price elasticity of demand for nondurables is well below one. Carrasco *et al.* (2005) provide panel estimates of the price elasticities of the demand for *food*, *transport* and *services* in Spain which take the values -0.85, -0.78 and -0.82 respectively. According to the results in Bryant and Wang (1990) based on aggregate US time series the price elasticity of *total demand for nondurables* is equal to -0.2078. Blanciforti *et al.* (1986) estimate an *Almost Ideal Demand System (AIDS)* based on aggregate US time series. Their results with respect to the own-price elasticities of nondurables can be summarized as follows: *food* - between -0.21 and -0.51; *alcohol and tobacco* - between -0.8 and -0.25; *utilities* - between -0.20 and -0.67; *transportation* - between -0.38 and -0.66; *medical care* - between -0.57 and -0.70; *other nondurable goods* - between -0.29 and -1.26; *other services* - between -0.20 and -0.36. There is also evidence supporting a short run price elasticity of demand greater than one. For example, using Finish time series Mellin and Viren (1982) come to the conclusion that the own-price elasticity of nondurables takes a value slightly below -5. However, their estimates should be interpreted with caution, since they are most likely subject to a simultaneity bias. In a more recent paper Tellis (1988) surveys the estimates of the price elasticity of demand in the marketing literature. He provides a skewed distribution of the results found in that literature with mean, mode and standard deviation equal to -1.76, -1.5 and 1.74 respectively. The bulk of the estimated elasticities take values in the range [-2,0]. In light of the empirical evidence it appears more reasonable to set θ at a value lower than one. However, for the sake of completeness and better comparability with models featuring static monopolistic competition, I decide to carry out a sensitivity analysis with respect to θ by simulating the model for several values of θ below and several values above one.

Most authors set the steady state markup at a value in the range suggested by Rotemberg and Woodford (1993) - between 1.2 and 1.4. The same is done here - $mu^* = 1.2$ is chosen as a baseline value.

The distribution parameter appearing in the utility function a is set at 0.9 assigning a very high weight to consumption C_t and a very low one to money balances $\frac{M_t}{P_t}$. I vary the inverse of the elasticity of substitution between C_t and $\frac{M_t}{P_t}$, b, by setting it at values in the range $[0.02; 20]$.

The second part of the calibration involves finding the parameter values of γ and υ as well as the steady state values C^*, K^*, I^* and π^* satisfying the economy's non-stochastic stationary equilibrium.

$\tau^* = 1$ implies that the steady state value of the gross rate of inflation is equal to one. To be able to determine the value of γ one needs to compute $\frac{\Omega^*}{D^*}$ first. To find the value of Ω^* just observe that the steady state is characterized by the following relationships $\Lambda_{t+1} = \Lambda_t$, $\left(\frac{P_i}{P}\right)^* = 1$, $x_i^* = 1$ and $\frac{P_i}{P} - \mu^* = \frac{mu^*-1}{mu^*}$, and then insert them into the definition of Ω_t. After some algebraic manipulations one arrives at

$$\frac{\Omega^*}{D^*} = \frac{\beta}{1-\beta}\frac{mu^*-1}{mu^*}.$$

γ can then be derived from (II.3.5) evaluated at the steady state. This equation is reproduced here for convenience:

$$mu^* = \frac{-\theta}{1-\theta-\gamma\frac{\Omega^*}{D^*}}.$$

For γ to be positive θ should be smaller than $\frac{mu^*}{mu^*-1}$ which in the case $mu^* = 1.2$ is equivalent to the restriction $\theta < 6$. Next, in the models without capital, for a given value of N^*, C^* can be derived from the goods market equilibrium condition

$$Y^* = N^* = C^*.$$

The model featuring capital accumulation involves few additional calibration steps. The production elasticity of labor ω is chosen to satisfy the restriction:

$$\omega = mu^*\overline{\left(\frac{(W/P)N}{Y}\right)},$$

where $\overline{\left(\frac{(W/P)N}{Y}\right)}$ denotes the average actual labor share. The empirical estimates for this variable using US- as well as data for other industrialized countries vary between slightly below 0.6 and slightly below 0.8. I set labor share at the value estimated in chapter 4, where based on the methods proposed by Cooleey and Prescott (2005) I obtain the value 0.6747. The euler equation for optimal investment in capital (II.4.3) evaluated at the stationary equilibrium then implies

$$\frac{Y^*}{K^*} = mu^*\frac{1-\beta(1-\upsilon)}{\beta(1-\omega)}.$$

By using this result together with the definition of the production function and a given N^* one arrives at:

$$K^* = \frac{N^*}{\left(\frac{Y^*}{K^*}\right)^{\frac{1}{\omega}}}.$$

The depreciation rate of capital υ is also set at the value found in Chapter 4, 0.00708. In the next step the steady state value of investment I^* is easily derived via the low of motion for capital:

$$I^* = \upsilon K^*.$$

C^* star then follows from the aggregate resource constraint

$$C^* = Y^* - I^*.$$

There is only one additional parameter to be calibrated in the model with adjustment costs of capital[29] - the elasticity of $\phi\left(\frac{I_t}{K_t}\right)$ with respect to its argument I_t/K_t, denoted by ζ. Jerman (1998) provides a GMM estimate of ζ equal to -1/0.23. Unfortunately, in the models developed in the current chapter that value implies that investment is less volatile than output while the reverse is true in the industrialized world: For example, according to the computations performed chapter 4 in the USA investment is about 4,7 times as volatile as output. Therefore in subsection 5.3 ζ is chosen so that for a broad range of values of θ the model is able to replicate the empirically observable relation between the variability of output and that of investment.

I use the AR(1) process for the Solow-residual estimated by Gomme and Rupert (2006) with U.S.-data:

$$\ln(Z_t) = 0.9641 \ln(Z_{t-1}) + \epsilon_t, \tag{II.6.3}$$

where ϵ_t follows a White Noise process with standard deviation σ_ϵ equal to 0.0082. The implied unconditional standard deviation of the Solow-residual, σ_z, is given by

$$\sigma_z = \frac{\sigma_\epsilon}{\sqrt{1 - 0.9641^2}} = 0.03088.$$

The properties of the money supply process were estimated by fitting an AR(p) process to the growth rate of the monetary aggregate M1. The process chosen by minimizing the *Akaike information criterion* is given by:[30]

$$g_{M1,t} = 0.0037^{**} + 0.5097^{**} g_{M1,t-1} + 0.2251^{**} g_{M1,t-2} + \tilde{u}_t, \tag{II.6.4}$$

[29] See subsection 5.3.

[30] I used quarterly data from 1970:Q1 through 2003:Q3. According to the Ljung-Box-Q statistic and White's heteroscedasticity test the estimated residuals display neither serial correlation nor heteroscedasticity.

where $g_{M1,t}$ denotes the growth rate of M1,[31] $\tilde{u}_t$ the residual term and ** indicates significance at the 5% level. The estimated standard deviation of the unsystematic component of money supply σ_u equals 0.0092. The unconditional mean and standard deviation of $g_{M1,t}$ take the values 0.0138 and 0.0125 respectively.

The subjective discount factor is set at 0.991 which is a *standard value* often found in the literature. ϕ is chosen to be consistent with the observable average fraction of time spent working N^*.[32] Table II.1 summarizes the calibration of the model.

Table II.1:
Calibration

Households/Preferences	Firms/Technology	Central Bank
$a = 0.9$	$Z^* = 1$	$\tau^* = 1$
$b \in [0.02, 20]$	$\rho_z = 0.9641$	$\rho_\tau = 0$
$\beta = 0.991$	$\sigma_\epsilon = 0.0082$	$\sigma_u = 0.0092$
$\theta \in [0.2, 2.2]$	$mu^* \in [1.1; 1.4]$	
$N^* = 0.1386$	$\zeta > 0$, sensitivity analysisi	

7 Business Cycles Moments

In order to evaluate the goodness of a particular business cycle model, it has become a common practice to compare its quantitative predictions with respect to a set of second moments with the same set of moments found in empirical data. The same strategy is chosen in the current chapter. Since the goal of such an exercise is not the examination of the qualitative properties of the model, but rather the computation of its exact quantitative predictions, it is desirable to calibrate it in as sophisticated as possible. This is done in Appendix 6.

I perform two simulation experiments - one with $b = 20$ and the other with $b = 0.02$. As shown in sections 3 and 5 higher values of b magnify the impulse responses to monetary disturbances. In both simulations the elasticity of the first derivative of the adjustment cost of capital function ζ is set to the value implying the empirically observable relation between the volatilities of output and investment.

[31] Note that the stochastic process generating $\tau_t = M_{t+1}/M_t$ introduced in section 2 can be identified as the AR(2) process in (II.6.4) since

$$g_{M1,t} = \ln(M_t) - \ln(M_{t-1}) = \ln(\tau_{t-1}).$$

[32] See chapter 4 for details about the calibration of N^*.

I ignore the autocorrelation structure of the money supply process and assume that the percentage deviation of the growth factor of M_t from its long run level $\hat{\tau}_t$ follows a pure White Noise process whose standard deviation is identical with that of the unsystematic component of the money supply process, $\sigma_\tau = \sigma_u = 0.0092$.

Table II.2 summarizes the results obtained from the *Customer Market Model*. The computation of the empirical second moments is described in chapter 4. It is readily seen that the model developed in this chapter has two important shortcomings. First, it implies a large positive correlation between output and the markup. While as a reaction to a monetary innovation production and output tend to move in opposite directions, technological disturbances induce strong and persistent comovements of these variables. Obviously, the effects triggered off by technology shocks dominate. Second, the model performs very poorly with regard to the autocorrelation of inflation as well as its cross correlation with output. Nevertheless, the overall performance of the model should be seen as average with a slight tendency to understate the autocorrelations of most variables. After a large set of simulation exercises I concluded that the reactions to technology shocks are almost unaffected by variations in the parameter b. Hence any differences between the two specifications $b = 20$ and $b = 0.02$ present in Table II.2 are largely due to the fact that the propagation of monetary disturbances is substantially affected by changes in b.[33]

A comparison with the New Keynesian Model:
Table II.3 contains the results from the New Keynesian Model obtained under the assumption of a relatively large degree of price stickiness, $\varphi = 0.75$. Values of b lower than 18 lead to very unrealistic model implications with regard to the volatilities of most macroeconomic variables. For that reason the comparison between the two models only refers to the case $b = 20$. The New Keynesian Model performs better than the one developed here with respect to the cross correlations with output of inflation and the markup. The latter equals 0.52 and is not far from its empirical counterpart, 0.317. The correlation between output and the markup is too high in magnitude but has the correct sign. In contrast the *Customer Market Model* implies a negative correlation between output and inflation, -0.24, and a large positive one between output and the markup. Further, the predictions of the New Keynesian Model regarding the autocorrelations of the individual variables are *on average* closer to their respective empirical counterparts than it is the case in the *Customer Markets Model*. Nevertheless, the latter performs better with respect to the relative standard deviations of most variables as well as the cross correlations with output of all variables except inflation and the markup. Furthermore, the predictions of the *Customer Markets Model* with regard to the cyclical properties of the real wage match much better the empirical evidence than it is the case in the New Keynesian Model. The latter exhibits a drawback typical for most sticky price models - it substantially

[33]See also sections 3 and 5.

overstates the *relative* standard deviation of the real wage. If the fraction of firms that are not able to adjust their prices within the period is reduced to $\varphi = 0.3$, for both values of b the autocorrelations predicted by the New Keynesian model slightly decrease and, what is more important, the correlation between output and the markup becomes positive.

Unfortunately, the careful inspection of the second moments of the two models does only reveal that each of them has as many important advantages as significant shortcomings. Therefore, neither model can be considered better than the other one.

Table II.2:

Theoretical and Empirical Second Moments *(Adjustment Costs of Capital Model)*

Variable	$sd(x)$	$sd(x)/sd(y)$	$acorr(x)$	$corr(x,y)$
Output				
$\zeta = -0.0348,\ b = 20$	1.20	1.00	0.68	1.00
$\zeta = -0.0070,\ b = 0.02$	1.22	1.00	0.69	1.00
US Data	**1.547**	**1.000**	**0.863**	**1.000**
Consumption				
$\zeta = -0.0348,\ b = 20$	1.16	0.96	0.65	0.94
$\zeta = -0.0070,\ b = 0.02$	0.95	0.78	0.70	0.98
US Data	**0.697**	**0.451**	**0.889**	**0.735**
Hours				
$\zeta = -0.0348,\ b = 20$	0.26	0.21	0.36	0.82
$\zeta = -0.0070,\ b = 0.02$	0.21	0.17	0.67	0.99
US Data	**1.329**	**0.859**	**0.874**	**0.898**
Real Wage				
$\zeta = -0.0348,\ b = 20$	0.43	0.35	0.13	0.61
$\zeta = -0.0070,\ b = 0.02$	0.23	0.19	0.67	0.99
US Data	**0.815**	**0.527**	**0.637**	**0.472**
Investment				
$\zeta = -0.0348,\ b = 20$	5.88	4.88	0.01	0.46
$\zeta = -0.0070,\ b = 0.02$	5.74	4.72	0.56	0.89
US Data	**7.168**	**4.634**	**0.733**	**0.367**
Real Balances				
$\zeta = -0.0348,\ b = 20$	1.08	0.90	0.66	0.66
$\zeta = -0.0070,\ b = 0.02$	0.04	0.03	0.58	0.61
US Data	**3.222**	**2.083**	**0.941**	**0.280**
Inflation				
$\zeta = -0.0348,\ b = 20$	1.06	0.88	0.01	-0.24
$\zeta = -0.0070,\ b = 0.02$	0.87	0.72	-0.06	0.01
US Data	**0.387**	**0.250**	**0.497**	**0.317**
Markups				
$\zeta = -0.0348,\ b = 20$	0.88	0.73	0.52	0.83
$\zeta = -0.0070,\ b = 0.02$	0.78	0.64	0.69	0.99
US Data	**0.538**	**0.348**	**0.727**	**-0.058**

$mu^* = 1.2$, $\theta = 0.2$, serially uncorrelated monetary shock $\sigma_\tau = \sigma_u = 0.0092$. ζ denotes the elasticity of $\phi'\left(\frac{I_t}{K_t}\right)$ with respect to I_t/K_t. $sd(x)$ - standard deviation of x; $sd(x)/sd(y)$ - ratio of the standard deviation of x to that of output; $acorr(x)$ - first order autocorrelation of x; $corr(x,y)$ - contemporaneous correlation between x and output. The second moments refer to HP-filtered empirical and simulated data. The second moments implied by the model refer to averages over 300 simulations. Each simulated series consists of 135 observations.

Table II.3:

Theoretical and Empirical Second Moments *(New Keynesian Model with Adjustment Costs of Capital)*

Variable	$sd(x)$	$sd(x)/sd(y)$	$acorr(x)$	$corr(x,y)$
Output				
$\zeta=-0.057,\ b=20$	1.56	1.00	0.80	1.00
$\zeta=-0.0085,\ b=0.02$	5.61	1.00	0.10	1.00
US Data	**1.547**	**1.000**	**0.863**	**1.000**
Consumption				
$\zeta=-0.057,\ b=20$	1.21	0.78	0.79	0.98
$\zeta=-0.0085,\ b=0.02$	4.68	0.84	0.06	0.96
US Data	**0.697**	**0.451**	**0.889**	**0.735**
Hours				
$\zeta=-0.057,\ b=20$	1.81	1.16	0.55	0.77
$\zeta=-0.0085,\ b=0.02$	6.65	1.19	0.04	0.98
US Data	**1.329**	**0.859**	**0.874**	**0.898**
Real Wage				
$\zeta=-0.057,\ b=20$	2.15	1.38	0.54	0.74
$\zeta=-0.0085,\ b=0.02$	6.76	1.21	0.04	0.98
US Data	**0.815**	**0.527**	**0.637**	**0.472**
Investment				
$\zeta=-0.057,\ b=20$	7.31	4.69	0.60	0.89
$\zeta=-0.0085,\ b=0.02$	26.23	4.68	0.46	0.77
US Data	**7.168**	**4.634**	**0.733**	**0.367**
Real Balances				
$\zeta=-0.057,\ b=20$	1.16	0.74	0.75	0.94
$\zeta=-0.0085,\ b=0.02$	0.11	0.02	0.11	0.09
US Data	**3.222**	**2.083**	**0.941**	**0.280**
Inflation				
$\zeta=-0.057,\ b=20$	0.60	0.39	0.46	0.52
$\zeta=-0.0085,\ b=0.02$	0.78	0.14	0.03	0.95
US Data	**0.387**	**0.250**	**0.497**	**0.317**
Markups				
$\zeta=-0.057,\ b=20$	2.97	1.91	0.48	-0.48
$\zeta=-0.0085,\ b=0.02$	7.98	1.42	0.03	-0.95
US Data	**0.538**	**0.348**	**0.727**	**-0.058**

$\varphi = 0.75$, $mu^* = 1.2$, $\theta = 6$, serially uncorrelated monetary shock $\sigma_{\mathcal{T}} = \sigma_u = 0.0092$. ζ denotes the elasticity of $\phi'\left(\frac{I_t}{K_t}\right)$ with respect to I_t/K_t. $sd(x)$ - standard deviation of x; $sd(x)/sd(y)$ - ratio of the standard deviation of x to that of output; $acorr(x)$ - first order autocorrelation of x; $corr(x,y)$ - contemporaneous correlation between x and output. The second moments refer to HP-filtered empirical and simulated data. The second moments implied by the model refer to averages over 300 simulations. Each simulated series consists of 135 observations.

8 Conclusion

The model presented in this chapter extends the standard monetary business cycles model with non-additively separable utility function and fully flexible prices by introducing market share competition and thus endogenizing markups. This new feature substantially approves the quantitative and qualitative properties of the model. In particular, positive monetary shocks become expansionary while the reactions of output, employment and real wages become delayed by one period, much as indicated by many VAR studies.

I also evaluate the theoretical framework developed in this chapter by comparing its implications with that of the New Keynesian Model with Calvo pricing. I conclude that the former should be considered a useful alternative to the latter for analyzing positive as well as normative issues. The model also provides many dimensions along which it can be extended. For example by taking an explicit account of capital accumulation or labor market frictions. Nevertheless, the framework presented here is by no means better than the New Keynesian model. Therefore, in search for a theory superior to the New Keynesian one, in what follows, I sharply deviate from the existing literature.

9 Supplement to Chapter 2: Market Share Competition

Phelps and Winter (1970) depart from the frictionless specification of the goods market by assuming that customers can not respond instantaneously to differences in firm specific prices. As the authors note, there are various rationales for this assumption - information imperfections, habits as well as costs of decision-making, none of which is explicitly modeled in their paper. An immediate consequence of such frictions is that in the (very) short run each firm has some monopoly power over a fraction of all consumers. This fraction equals the firm's market share. In particular, Phelps and Winter (1970) assume that the transmission of information about prices evolves through random encounters among customers in which they compare recent demand experience. Under this assumption the probability with which a comparison between any two firms i and j is made will be approximately proportional to the product of their respective market shares x_i and x_j. Therefore, one would expect that the time rate of net customer flow from firm j to firm i will also be proportional to the product $x_i x_j$. Phelps and Winter formalize this in continuous time as follows:

$$z_{i,j} = \delta(p_i, p_j) x_i x_j,$$

where $z_{i,j}$ is the net flow of customers from j to i. The time indexes were dropped for convenience. The function $\delta(p_i, p_j)$ has the properties:

$$sgn(\delta(p_i, p_j)) = sgn(p_j - p_i), \quad \delta(p_i, p_j) = -\delta(p_j, p_i), \quad \delta_1 < 0, \quad \delta_2 > 0.$$

The market share x_i then evolves according to:

$$\dot{x}_i = \sum_{j=1}^{m} z_{i,j} = x_i \sum_{j=1}^{m} \delta(p_i, p_j) x_j = x_i \sum_{j=1, j\neq i}^{m} \delta(p_i, p_j) x_j,$$

where m is the number of firms. Defining the customer-weighted mean of other firms' prices $\bar{p}_i$ by

$$\bar{p}_i = \frac{\sum_{j\neq i}^{m} p_j x_j}{\sum_{j\neq i}^{m} x_j} = \frac{\sum_{j\neq i}^{m} p_j x_j}{1 - x_i}$$

and expanding $\delta(p_i, p_j)$, $\forall j \neq i$ in a first order Taylor's series with respect to its second argument one obtains:

$$\dot{x}_i \approx x_i(1 - x_i)\delta(p_i, \bar{p}_i) + x_i \delta_2(p_i, \bar{p}_i) \left(\underbrace{\sum_{j\neq i}^{m} p_j x_j}_{:=\bar{p}_i(1-x_i)} - \bar{p}_i(1 - x_i) \right) = x_i(1 - x_i)\delta(p_i, \bar{p}_i).$$

Assuming that each supplier is small enough, so that the following relations hold:

$$1 - x_i \approx 1 \quad \Rightarrow \quad \bar{p}_i \approx \sum_{j\neq i}^{m} p_j x_j = \bar{p},$$

where $\bar{p}$ is the overall mean price in the goods market, the law of motion of x_i reduces to

$$\dot{x}_i \approx \delta(p_i, \bar{p})x_i. \tag{II.9.5}$$

The discrete-time version of (II.9.5) used in the following sections reads:

$$x_{i,t+1} = g\left(\frac{p_{i,t}}{\bar{p}_t}\right)x_{i,t},$$

where $\delta(p_{i,t}, \bar{p}_t) = g\left(\frac{p_{i,t}}{\bar{p}_t}\right) - 1$.[34] Now assume that the demand of each individual belonging to the customer stock of firm i is given by $D\left(\frac{p_{i,t}}{\bar{p}_t}\right)$. Then the *demand curve* faced by firm i is given by:

$$x_{i,t}D\left(\frac{p_{i,t}}{\bar{p}_t}\right) = g\left(\frac{p_{i,t-1}}{\bar{p}_{t-1}}\right)x_{i,t-1}D\left(\frac{p_{i,t}}{\bar{p}_t}\right).$$

[34] To see this, write the discrete-time version of (II.9.5) in the more general form

$$x_{i,t} - x_{i,t-h} = \left(g\left(\frac{p_{i,t}}{\bar{p}_t}\right) - 1\right)h \cdot x_{i,t-h},$$

where $\left(g\left(\frac{p_{i,t}}{\bar{p}_t}\right) - 1\right)h$ measures the net customer flow to firm i over a time interval of length h. Divide both sides of the last equation by h, let h go to zero and assume that $x_{i,t}$ is differentiable with respect to t. The resulting equation is:

$$\dot{x}_{i,t} = \left(g\left(\frac{p_{i,t}}{\bar{p}_t}\right) - 1\right)x_{i,t}.$$

Chapter 3

Inflation Aversion and Monetary Policy

1 Introduction

In the light of the fact that on average the monetary customer markets model presented in the previous chapter does not represent an improvement relative to the New Keynesian model, I take a second step in developing *"another version of the model"* which provides an endogenous explanation of the incomplete or rigid response of nominal prices to monetary and interest rate shocks, the cyclical pattern of markups and the strong and persistent reactions of most macroeconomic variables to demand and supply side disturbances. To achieve these three goals, I extend a standard monetary business cycles model with monopolistic competition in the goods market and additively separable utility function along several dimensions.

First, based on the empirical evidence accumulated in research areas closely related to psychology and dramatically deviating from any tradition in the real and monetary business cycles theory I assume that agents' behavior is characterized by *inflation aversion* - current inflation has a direct negative effect on utility. By intensifying search and switching efforts in the goods market and thus switching from more expensive to cheaper products an individual household is able to reduce the direct disutility caused by inflation. The emerging positive relationship between search efforts and inflation crates a new channel by which nominal disturbances can be transmitted into the real economy. If an increase in search activity caused by an upward pressure on current inflation forces firms to pass through to prices only a fraction of the increase in nominal marginal costs, markups will tend to fall and thus real wages, hours and output will tend to increase. To ensure such an outcome the structure of the goods market should be altered in some way. I propose the following extensions of the *static* monopolistic competition framework.

First, as in the previous chapter, I assume that firms do not only engage in price competition but also in *dynamic market share competition* as proposed by Phelps and Winter (1970) in their *customer market model*. Second, I assume that the law of motion of the individual market share is governed by a *matching function* depending on the pricing behavior of the firm and the intensity of the search and switching activities households engage in. How does each individual assumption as well as the combination of them alter the characteristics and the implications of the model?

Further, I extend the law of motion of market share defined by Phelps and Winter (1970) and used by Rotemberg and Woodford (1993) by assuming that the function describing the growth factor of the firm-specific market share does not only depend on the firm's pricing decision but also on the intensity of search and switching efforts chosen by consumers. This function can be interpreted as a matching mechanism in the goods market assigning customers to suppliers and has the following key implication: If an individual firm charges a lower (higher) price relative to the overall price level, a higher search activity by households will induce a stronger increase (decrease) of that firm's customer stock. Therefore, in times of high search activity firms will tend to charge lower prices and thus choose lower markups. In other words, a higher overall search activity implies that households reallocate their demand more *aggressively* as a reaction to price differences, therefore firms with relatively high current prices suffer more severe losses in future market share. From the point of view of an individual household the intensification of her search for cheaper suppliers as well as the efforts aimed at the redistribution of her demand by adjusting the weights attached to the individual goods within the consumption bundle, the representative household is able to respond in a utility maximizing way to changes in relative goods prices. The costs induced by search activity are measured in real terms and reduce directly the resources available for consumption.

The assumption that households' behavior is characterized by *inflation aversion* and that by spending more resources on search and switching activities the disutility caused by inflation is reduced creates a new link between the real and the monetary side of the economy which is the crucial new feature of the model. As I show below, search activity depends positively on current inflation. The latter implies that nominal shocks will not be fully absorbed by changes in nominal prices: A monetary expansion will induce two opposing effects on current inflation - the usual positive one *via* the positive income effect on current demand and a *new* negative one *via* the positive effect on search and switching activity. The higher search activity then forces firms to choose lower current prices and thus lower markups than they would do if search were independent of inflation. As a consequence, on the one hand overall inflation rises by less than if there were no dependence between search and inflation and on the other real wages, hours and output rise. If search efforts were independent on current inflation, the positive and negative pressures on current markups induced by a monetary disturbance will exactly offset each other. As a consequence, markups and therefore real wages, hours and output will remain unchanged and the increase in inflation will be exactly sufficient to offset the income effect of the monetary shock. In other words, the *neutrality of money* will hold.

The main findings of this chapter can be summarized as follows. After extending the standard monetary business cycles model along the three dimensions described above it becomes able to generate endogenous countercyclical markups which react negatively to monetary as well as technology shocks, and endogenous sluggishness in nominal prices. Furthermore, the model provides an endogenous explanation of the persistence in actual business cycles. Hence, the model of this chapter should be considered a useful alternative to the *New Keynesian Model* for analyzing and evaluating monetary and interest rate policy.

The chapter is organized as follows. Section 2 motivates the main assumption of the model. Section 3 presents the benchmark model and its implications. Section 4 extends it by incorporating capital accumulation. Section 5 closes the chapter with a comparison between the model developed here and the New Keynesian model.

2 Inflation Aversion

According to most economic theories and the view of many economists inflation affects only indirectly the well-being. Either by reducing the real value of the wealth and income components denominated in nominal terms or by exacerbating already existing inefficiencies and therefore leading to a suboptimal distribution of private expenditure.[1] Absent these valuation or expenditure distribution effects, inflation would be of no relevance for the private economy and therefore of no relevance for policy makers and of no interest for economic theory. There is a similar consensus about how households' and firms' behavior depends on the variability of the inflation rate - in a purely indirect manner: A higher volatility of the inflation rate in most cases makes many income components more variable. As a consequence consumption tends to fluctuate more and hence reduces utility if agents are risk averse.

[1] In the *New Keynesian Models* with Calvo-type price setting, for example, higher inflation induces a larger dispersion of prices for the individual goods produced (and consumed) in the economy. The latter results in an inefficient dispersion of output and demand among the individual goods. As there is diminishing marginal utility of consumption of every individual good, the utility derived from consuming more of the cheaper goods is less than the utility loss from consuming less of the more expensive ones. If there is increasing marginal disutility of producing an individual good, the utility loss from producing more of the cheaper goods will be higher than the gain in utility from producing less of the more expensive ones. In these models it is often assumed that the central bank designs monetary or interest rate policy so as to minimize a quadratic loss function depending on some measure of the output gap and the inflation rate. As the loss function is just an approximation of the utility function of the representative agent in the economy, the presence of the inflation rate stems from the indirect effects on utility just described. (See Woodford (2003), pp. 383-405 and Walsh (2003) pp. 517-558 on this and related issues.)

In contrast to this traditional or mainstream way of thinking about inflation the empirical evidence accumulated in some research areas closely related to psychology indicates that consumption (or income) is by no means the only variable directly affecting people's subjective well-being.[2] Other variables such as the overall rate of inflation, the personal unemployment status, the general unemployment rate as well as the institutional and political framework seem to have highly significant direct effects on people's life satisfaction. Frey and Stutzer (2002) provide an exhaustive survey of *Happiness Research*, a discipline directly stemming from psychology. Some of the papers reviewed by the authors provide stylized facts on the correlation between income and happiness. According to the results, despite the sharp increase of per capita income in all industrial countries after World War II, in most of them average subjective well-being has remained unchanged or has even declined. The more elaborate econometric studies on this issue cited by Frey and Stutzer (2002) go beyond the purely descriptive analysis by controlling for individual characteristics of households as well as the effects of inflation, unemployment, institutional and other factors. They all come to the conclusion that the income level has no, or in rare cases a very limited explanatory power with respect to private agents' happiness.[3] At the same time inflation tends to have significant direct effects on life satisfaction. Di Tella *et. al.*, also cited by Frey and Stutzer (2002), show that after controlling for individual socioeconomic characteristics and the unemployment rate, an increase of inflation by five percentage points reduces average happiness by 0.05 *"units of satisfaction"* which is equivalent to a shift of five percent of the population from one life-satisfaction level to the next lower one.[4] Walton (1979) provides evidence based on a sample taken by the American Council of Life Insurance (AICL)[5] that consumers are often *"disturbed"*, *"frustrated"* and *"angry"* when confronted with higher prices for particular products or a higher overall price level.

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[2]Frey and Stutzer (2002) define *subjective well-being* as follows: It is the scientific term in psychology for individual's evaluation of her experienced *positive and negative affect*, *happiness* or *satisfaction with life*. In psychology they are separable constructs, whereas, to my knowledge, in modern macroeconomics only the abstract concept of *agent's utility* is used. The latter is a cardinal measure summerizing all three psychological concepts, allowing no precise distinction between them. Therefore the terms *subjective well-being*, *happiness* and *life satisfaction* are used interchangeably, as synonyms of *utility*.

[3]As many studies show, the *income* variable that does have a significant positive effect on subjective well-being is the *relative* rather than the *absolute* income level. The *relative* income of a person is defined as the quotient of his own income and a weighted average over all households. There is also for people in richer countries to be happier.

[4]In Di Tella *et. al.* (2001) satisfaction is measured by a 4-point scale ranging from *"not at all satisfied"* through *"very satisfied"*. To transform the ordinal scale into a cardinal one the numbers 1,2,3 and 4 are attached to the levels *"not at all satisfied"*,... and *"very satisfied"* respectively.

[5]To take a sample of people's opinion about inflation, ACLI placed advertisements in major newspapers and news magazines urging Americans to express their views on the subject in letters addressed to the author.

In a study inspired by the question wether Okun's *Economic Discomfort Index*[6] can be regarded as a good measure of agents' *disutility* Lovell and Tien (1999) come to the conclusion that current inflation has greater influence on people's well-being than current unemployment does, as measured by the elasticities of these two variables with respect to the *Index of Consumer Sentiment (ICS)*.[7] Furthermore, once the *change* in the unemployment rate and/or the growth rate of GDP are included in the regression equation the coefficient of the rate of unemployment tends to get insignificant. The latter indicates that by the inclusion of current unemployment in the *disutility index* one just approximates the effect of the cyclical component of income on agents' happiness. The relative strength of this effect compared with that of inflation is consistent with the findings in the papers reviewed by Frey and Stutzer (2002).

Much in line with the evidence provided by Lovell and Tien (1999) as well as that cited by Frey and Stutzer (2002) are the results in Hymans (1970). He also examines the dependence of the *Index of Consumer Sentiment (ICS)* on income, stock price changes and inflation. His results indicate that all three variables have highly significant direct effects on consumer sentiment. Since the long run trend as well as recent changes of stock prices provide a fairly good approximation of the changes in the value of households' wealth, it is less surprising that these variables have explanatory power with respect to consumer sentiment. Much more surprising is the role of inflation as an independent determinant of ICS. The fact that income is not the sole determinant of consumer sentiment explains why the latter usually has a significant coefficient in demand regressions in which income was already included. In a more recent empirical study Franses (2006) also provides evidence supporting the existence of substantial direct effects of inflation on consumer confidence.

The empirical studies discussed so far examine the relationship between subjective well-being and the *current* values of income, inflation, unemployment and other variables. But as most agents are to some extent forward looking, it can not be ruled out on *a priori* grounds that average happiness depends on expected rather than current inflation. Smyth *et. al.* (1994) apply a battery of *non-nested* tests in order to give an answer to the question wether agents are on average *forward* or *backward looking* when deciding on how satisfied they are with the current economic situation. According to their results, the hypothesis that agents' satisfaction with the current economic situation depends on expected future inflation and unemployment is rejected in favor of the one that current inflation and unemployment are the only significant determinants of people's subjective well-being.[8]

[6] Okun's *Economic Discomfort Index* is simply defined as the sum of the inflation and the unemployment rate.

[7] Lovell and Tien (1999) use the *Index of Consumer Sentiment* as an approximation of consumers' subjective well-being.

[8] Agents' satisfaction with current economic conditions is approximated by the Gallup's *Index of Presidential Popularity*.

The empirical findings on the determinants of agents' well-being can be summarized as follows: The current inflation rate has significant direct effects on utility and is at least as important as real income. Based on these results, I assume that the utility function of the representative agent depends directly on the overall rate of inflation. The pain caused by inflation can be reduced by spending some part of the currently available resources on search and switching efforts aimed at the reallocation of demand from suppliers charging relatively high to suppliers charging relatively low prices. As documented by Frey and Stutzer (2002), the other factors mentioned above e.g. the political situation and many institutional factors also have greater importance for individual well-being than income has. As none of them is explicitly taken into account in the model presented below, I do not reproduce the empirical findings on the effects of these variables on happiness.

Why does utility depend directly on inflation? It is the job of psychologists to give answer to this question and I do not provide any suggestions or even speculations about the possible reasons for the direct link between inflation and subjective well-being. In the model described below I simply assume that households' behavior is characterized by a property called *inflation aversion*.

3 The Model

I refer to this model as the *"Benchmark Model"* or the *"CM-Model"*.[9]

3.1 Theoretical Framework

Firms

There are n product varieties, each produced by a profit maximizing monopolistic firm according to the linear production function

$$Y_{i,t} = Z_t N_{i,t},$$

where $N_{i,t}$ denotes labor input of firm i. Z_t denotes the total factor productivity which follows a stochastic process given by:

$$\ln(Z_t) = \rho_z \ln(Z_{t-1}) + \epsilon_t,$$

where ϵ_t follows a *White Noise Process* with variance σ_ϵ^2.

[9] CM - Customer Market.

The demand function faced by the producer of variety i is given by

$$C_{i,t} = x_{i,t} \cdot \left(\frac{P_{i,t}}{P_t}\right)^{-\theta} \cdot \frac{D_t}{n}, \qquad \theta > 0, \tag{III.3.1}$$

where $x_{i,t}$ is a measure of firm i's market share. $x_{i,t}$ can be also interpreted as a measure of the subjective relative weight within the consumption bundle attached by households to product variety i. More precisely, $x_{i,t}$ is the fraction of aggregate demand firm i would face if all firms were to choose the same price. D_t and P_t denote, respectively, aggregate demand and the aggregate price level.[10]

Firm-specific market share evolves according to

$$x_{i,t+1} = g\left(\frac{P_{i,t}}{P_t}, s_t\right) \cdot x_{i,t} \tag{III.3.2}$$

where s_t are aggregate households' search and switching efforts. Each firm treats s_t as an exogenous variable. I assume that the function $g(.,.)$ governing the law of motion of market share has the following properties:

$$g(1, s_t) = 1, \qquad \frac{\partial g\left(\frac{P_{i,t}}{P_t}, s_t\right)}{\partial P_{i,t}/P_t} = g_1\left(\frac{P_{i,t}}{P_t}, s_t\right) < 0,$$

$$\frac{\partial g\left(\frac{P_{i,t}}{P_t}, s_t\right)}{\partial s_t} = g_2\left(\frac{P_{i,t}}{P_t}, s_t\right) < 0 (> 0) \quad \text{for} \quad \frac{P_{i,t}}{P_t} > 1 (< 1).$$

According to these assumptions, higher search activity today leads to a fall (rise) in next-period market share if the price the firm charges is higher (lower) than the overall price level. Market shares are bounded by 0 from below and by 1, n or some other positive value from above. Generally, to ensure that x_i remains $\forall t$ within these bounds, one should try to find a reasonable normalization of x_i. This issue will be one of the most important in models with heterogeneous firms but it does not arise here, since in equilibrium all firms charge the same price.[11] I assume the following functional form of $g(.,.)$:

$$g\left(\frac{P_{i,t}}{P_t}, s_t\right) = \exp\left(\left(1 - \frac{P_{i,t}}{P_t}\right) \cdot s_t\right).$$

Since, as shown below, s_t will be positive even though the equilibrium is symmetric, it would be perhaps more reasonable to interpret this variable as a kind of potential search activity or a kind of alertness with respect to firms' behavior, reflecting how well households are informed

[10] The demand function in (III.3.1) is very similar to that proposed by Phelps and Winters (1970). Rotemberg and Woodford (1993) apply the discrete time version of the *"Customer Market Model"* used here to the analysis of the effects of government spending on private consumption and investment.

[11] See below.

about the goods market. If a consumer is better informed about the price distribution and the behavior of individual suppliers, she will tend to need less time to discover deviations from the average price and, thus, will be able to react to them faster by punishing (rewarding) positive (negative) deviations from the average price level. A consumer who is less informed about the price distribution, will need more time to infer from the pricing behavior she observes whether the prices charged for particular goods are actually too high (or too low) or not. Such a customer will react more slowly (or with a larger lag) to any given deviation from the average price. Expressing the degree of alertness with respect to the goods market by the value of s_t is a convenient short-cut, by which I avoid cumbersome technical details of which I expect to be of extremely limited importance for the *macroeconomic* predictions of the model. In what follows, s_t is termed search activity or switching efforts.

The dependence of the market share in $t+1$ on past pricing behavior introduces a dynamic aspect into the profit maximization problem of the individual firm. Each firm maximizes

$$\max_{P_{i,t}} \left\{ x_{i,t} \left(\frac{P_{i,t}}{P_t}\right)^{-\theta} \frac{D_t}{n} \left(\frac{P_{i,t}}{P_t} - \mu_t\right) + \right.$$
$$\left. + E_t \left\{ \sum_{j=1}^{\infty} \beta^j \frac{\Lambda_{t+j}}{\Lambda_t} x_{i,t+j} \left(\frac{P_{i,t+j}}{P_{t+j}} - \mu_{t+j}\right) \left(\frac{P_{i,t+j}}{P_{t+j}}\right)^{-\theta} D_{t+j} \right\} \right.$$

s. t.

$$x_{i,t+1} = g\left(\frac{P_{i,t}}{P_t}, s_t\right) x_{i,t},$$

where $DF_{t,t+j} = \left\{\beta^j \frac{\Lambda_{t+j}}{\Lambda_t}\right\}$ denotes the stochastic discount factor between periods t and $t+j$ which is given to the firm. μ_t denotes marginal costs. The corresponding first order condition reads:

$$\left(\frac{P_{i,t}}{P_t}\right)^{-\theta} x_{i,t} D_t - \theta \left(\frac{P_{i,t}}{P_t} - \mu_t\right) \left(\frac{P_{i,t}}{P_t}\right)^{-\theta-1} x_{i,t} D_t + \frac{g_1\left(\frac{P_{i,t}}{P_t}, s_t\right)}{g\left(\frac{P_{i,t}}{P_t}, s_t\right)} \Omega_t = 0,$$

where

$$\Omega_t = E_t \left\{ \sum_{j=1}^{\infty} \beta^j \frac{\Lambda_{t+j}}{\Lambda_t} x_{i,t+j} \left(\frac{P_{i,t+j}}{P_{t+j}} - \mu_{t+j}\right) \left(\frac{P_{i,t+j}}{P_{t+j}}\right)^{-\theta} D_{t+j} \right\} =$$

$$= E_t \left\{ \beta \frac{\Lambda_{t+1}}{\Lambda_t} x_{i,t+1} \left(\frac{P_{i,t+1}}{P_{t+1}} - \mu_{t+1}\right) \left(\frac{P_{i,t+1}}{P_{t+1}}\right)^{-\theta} D_{t+1} \right\} + E_t \left\{ \beta \frac{\Lambda_{t+1}}{\Lambda_t} \Omega_{t+1} \right\}.$$

Defining the markup over marginal costs as

$$mu_{i,t} = \frac{P_{i,t}}{P_t \mu_t}, \qquad mu_t = \frac{1}{\mu_t},$$

one can write the FOC, evaluated at the symmetric equilibrium, as

$$mu_t = \frac{-\theta}{1-\theta+g_1(1,s_t)\frac{\Omega_t}{D_t}} \tag{III.3.3}$$

In a symmetric intertemporal equilibrium in each period each firm sets the same price as all other firms. The most important implication regarding market shares is that $x_{i,t}$ equals one for all t and all i. According to equation (III.3.3) the equilibrium markup depends positively on current demand and negatively on current search efforts as well as the present value of future profits. In the static monopolistic competition model markups are given by

$$mu_t = \frac{\theta}{\theta-1} \tag{III.3.4}$$

implying that at any point in time and in any given state of the economy pass-through of marginal cost changes to prices is complete. Unlike that model, in an environment characterized by market share competition markups will be generally time varying. Wether pass-through of marginal costs to prices will turn to be greater, lower or equal to one depends on the relative strength of the reactions of D_t, Ω_t and s_t to exogenous shocks. For example, consider a positive exogenous shock which increases current consumption. The temporary (or even an one time) increase in current consumption will have a positive *direct* effect on markups through the induced increase in aggregate demand D_t. Based on this result, many microeconomic models assuming a constant discount factor reach the conclusion that markups are procyclical. In the present model, however, the discount factor is endogenous and strongly linked to current consumption - as shown below the Lagrange-multiplier Λ_t is given by

$$\Lambda_t = C_t^{-\eta}.$$

Other things equal, if η is sufficiently large an increase in current consumption will cause larger an increase in Ω_t *via* the rise in the discount factor. As a consequence, the markup will tend to be countercyclical. Further, if search activity depends positively on consumption, as is the case in this model,[12] consumption will also have a second indirect negative effect on markups *via* s_t. Further, equation (III.3.3) implies that markups in this model are always lower than they would be if there were static monopolistic competition in the goods market.

Households

Let agents in this economy have preferences over consumption, real balances, working hours, search activity and overall inflation given by

$$U = E_t\left\{\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\eta}}{1-\eta}+\phi_m\frac{(M_t/P_t)^{1-\chi}}{1-\chi}-\frac{b}{2}N_t^2-\frac{\varrho}{\alpha}\frac{\pi_t}{s_t^{\alpha}}\right)\right\}, \qquad \phi_m, b, \varrho, \eta, \chi > 0, \quad \beta\in(0,1),$$

[12]See below.

where M_t/P_t, N_t and π_t denote real balances, working hours and the gross rate of overall inflation respectively. In the above expression C_t is a composite good that includes all varieties:

$$C_t = \left\{ \frac{1}{n} \sum_{i=1}^{n} x_{i,t}^{\frac{1}{\theta}} C_{i,t}^{\frac{\theta-1}{\theta}} \right\}^{\frac{\theta}{\theta-1}},$$

$$s_t = \left\{ \frac{1}{n} \sum_{i=1}^{n} x_{i,t}^{\frac{1}{\theta}} s_{i,t}^{\frac{\theta-1}{\theta}} \right\}^{\frac{\theta}{\theta-1}}. \tag{III.3.5}$$

$x_{i,t}$ evolves according to (III.3.2). The corresponding utility-based price index is given by:

$$P_t = \left\{ \frac{1}{n} \sum_{i=1}^{n} x_{i,t} P_{i,t}^{1-\theta} \right\}^{\frac{1}{1-\theta}}.$$

The last term in the period utility function reflects the assumption that since search activity usually leads to a reduction (rise) of demand for relatively expensive (cheap) goods, a positive psychological effect arises, which takes the form of a reduction of the subjective negative effect on household's well-being induced by inflation. Such a cognitive effect may arise as a result of the satisfaction with the fact that by increasing s_t one is able to more heavily punish the firms most intensively contributing to the increase in the aggregate price level - the firms with above average prices. A similar interpretation can be derived from one of the assumptions underlying the most recent theoretical models developed by Rotemberg[13]. Based on experimental evidence he assumes that agents become angry when the prices of the goods they desire increase by a sufficiently large amount. Similarly, the last term in the utility function of the current model can be motivated as follows: Inflation makes households angry. Since anger is an unpleasant feeling, it reduces utility. But the pain induced by anger can be at least partly dampened by *having revenge* on firms with relatively high prices *via* a more intense search activity s_t. Unfortunately, as to my knowledge, there is no empirical evidence supporting or rejecting the assumed dependence of current utility on search activity. Nevertheless, the good performance of the current model relative to the standard New Keynesian Model discussed below as well as the GMM estimations presented in the next chapter can be seen as an *indirect* evidence in favor (or at least not against) the last assumption on the structure of the utility function.

The budget restriction of the representative household is given by:

$$C_t + s_t + m_{t+1} - \frac{m_t}{\pi_t} = \frac{W_t}{P_t} N_t + \Pi_t + \frac{T_t}{P_t},$$

where W_t, Π_t, T_t and $m_t = \frac{M_t}{P_{t-1}}$ denote the nominal wage, real profits, nominal net transfers form the government and real balances respectively.

[13] Rotemberg (2002, 2004a, 2004b, 2008)

Some Reinterpretations of the Model

An Interpretation of $x_{i,t}$ as subjective weights: Let $x_{i,t}$ be a measure of the subjective weight within the consumption bundle an agent assigns to good i. According to equation (III.3.1) this subjective weight affects the demand for good i for any given level of the relative price $P_{i,t}/P_t$ and household's total demand D_t. A higher s_t enables the household to respond stronger to relative price differences between individual suppliers - the next-period weight of firms with relatively high (low) prices in the current period are reduced (increased) by a higher amount. This is a kind of gradual switching from suppliers with high current prices to suppliers charging lower current prices. It does not take place immediately, but with a one period lag. In other words, firms are punished (rewarded) in the next period for choosing relatively high (low) prices today. To be able to undertake such a switch, the household has to reduce the resources available for consumption. The costs in terms of real resources and the time lag are an approximation of the fact that it is (or may be) costly, time consuming and even painful to switch between goods or suppliers. In many cases it is not immediately obvious whether two goods are perfect substitutes or to what extent the one can be substituted for the other. For example many services such es consulting, banking as well as educational services contain components which are not directly observable. That makes comparisons between individual products costly, as they usually involve the time and resource consuming process of analyzing, tasting, testing and trying different products. Often it is not an easy task to find the firm supplying the desired product. The service sector again, provides a vast number of examples. Habits, too, play an important role in this context, since the switch from one good or supplier to another one may require a painful break of some habits. Since the same frictions which make switching between products costly, are also among the most important determinants making search efforts in the goods market necessary, s_t can be interpreted as search activity as well.

An Interpretation of $x_{i,t}$ and $g\left(\frac{P_{i,t}}{P_t}, s_t\right)$ as a measure of probability and a matching function respectively: Assume that at the beginning of each period t each household is *randomly* matched with one of the n firms and remains a customer of that firm until the end of the period. At the beginning of the next period a new round of assigning households to firms takes place with the result that the household either remains a customer of the same firm, or is matched with another supplier and so on. Each household faces a probability equal to $x_{i,t}$ to become a customer of firm i at the beginning of period t. Assume that this probability is independent of the firm she was matched to in the previous period. By intensifying search and switching efforts s_t the household increases the probability to become a next-period customer of a firm with a relatively low price in the current period. Accordingly, a higher s_t reduces the probability to be attached to the next-period customer base of a producer charging a relatively high price in t. In other words, firms are punished (rewarded) with a one period lag for choosing relatively high (low) prices. To be able to increase s_t, the household has to reduce the resources available for consumption in the same period. Given the distribution of prices and other properties of the products supplied, a higher search activity enables the household to achieve a more desirable allocation of her resources to individual goods. Under this interpretation of the model each of the two composite goods C_t and s_t represents a nonlinear risk aggregator as suggested by Chew and Dekel for valuating state dependent consumption. To avoid any heterogeneity across households, it can be assumed that there are complete *Arrow-Debreu*-markets allowing each household to insure against idiosyncratic consumption risk. Since there is a continuum of identical households, the law of large numbers implies that the probability for a household to be matched with firm i, $x_{i,t}$, is identical with the fraction of households actually becoming customers of firm i in period t. Alternatively, one can assume that each household is a family consisting of a large number of members. Each period each member receives the same amount of resources for purchasing goods as well as engaging in search as any other. The goods purchased are then pooled and distributed equally among members by the head of the family.

First Order Conditions

The first order conditions of the representative household evaluated at the symmetric equilibrium read:

$$C_t^{-\eta} = \Lambda_t, \tag{III.3.6}$$

$$bN_t = \Lambda_t \frac{W_t}{P_t}, \tag{III.3.7}$$

$$\varrho s_t = \pi_t^{\frac{1}{1+\alpha}} C_t^{\frac{\eta}{1+\alpha}}, \tag{III.3.8}$$

$$\beta\phi_m m_{t+1}^{-\chi} E_t\{\pi_{t+1}^{\chi-1}\} = \Lambda_t - \beta E_t\left\{\frac{\Lambda_{t+1}}{\pi_{t+1}}\right\}, \tag{III.3.9}$$

$$C_t + s_t + m_{t+1} - \frac{m_t}{\pi_t} = \frac{W_t}{P_t}N_t + \Pi_t + \frac{T_t}{P_t}. \tag{III.3.10}$$

Note that the optimality condition with respect to search activity will take exactly the same form as in (III.3.8) even when there were price dispersion in equilibrium. (III.3.8) states that at the optimum the marginal utility of consumption should be equal to the marginal utility of search. The latter is given by $\varrho\pi_t s_t^{-(1+\alpha)}$. For a given rate of inflation, an increase in consumption lowers its marginal utility and so, makes a lower marginal utility of search and thus a higher s_t and a stronger reduction of the disutility of inflation desirable. For a given consumption level a higher overall inflation increases the marginal utility of search. As a consequence households find it optimal to intensify search and switching efforts s_t. Although there is a vast number of game theoretic partial equilibrium models assuming costly search or switching in the goods market in order to explain equilibrium price dispersion, to the best of my knowledge, there are no empirical investigations of the cyclical properties of households' search and switching efforts in the goods market. Therefor I am not able to tell if the predictions of the model are consistent with the patterns of households' search efforts in actual economies.

Government

The central bank finances its lump-sum transfers to the public by changes in the nominal quantity of money:

$$M_{t+1} - M_t = T_t.$$

It is further assumed that in each period transfers constitute a fraction of current money supply:

$$T_t = (\tau_t - 1)M_t,$$

where the percentage deviation of τ_t from its steady state $\hat{\tau}_t$ follows a first order autoregressive process

$$\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + u_t, \qquad \rho_\tau \in [0, 1).$$

u_t is assumed to be a *White Noise Process* with variance σ_u^2.

Equilibrium

In equilibrium, real wages and profits are given by

$$\frac{W_t}{P_t} = \frac{Z_t}{mu_t} \qquad \text{and} \qquad \Pi_t = \left(\frac{mu_t - 1}{mu_t}\right) Z_t N_t$$

respectively. These two results, together with the households first order conditions, (III.3.6) through (III.3.10), the firm's first order condition (III.3.3) evaluated at the symmetric equilibrium and the definition of Ω_t, describe the evolution of the economy.

The inclusion of search activity s_t as an argument of the function describing the evolution of firm-specific market share introduces an externality from the point of view of the individual firm, since s_t depends on overall inflation and consumption.

3.2 Calibration

The short run elasticity of demand θ, the markup mu^* and the stochastic processes are calibrated in the same way as in section 6 of the previous chapter. η and χ are both set at 2 which are so called standard values, usually found in the literature.

The second part of the calibration involves finding the parameter values of b and ϱ as well as the steady state values s^*, C^* and π^* satisfying the economy's nonstochastic stationary equilibrium.

The estimation of $\tau^* = 1.0138$ is described in section 6 in chapter 2 and implies that the steady state value of the gross rate of inflation is equal to 1.0138. To be able to determine the value of search and switching efforts in the stationary equilibrium, s^*, one needs to compute $\frac{\Omega^*}{D^*}$ first. To find the value of Ω^* just observe that the steady state is characterized by the following relationships $\Lambda_{t+1} = \Lambda_t$, $\left(\frac{P_i}{P}\right)^* = 1$, $x_i^* = 1$ and $\frac{P_i}{P} - \mu^* = \frac{mu^*-1}{mu^*}$, and then insert them into the definition of Ω_t. After some algebraic manipulations one arrives at

$$\frac{\Omega^*}{D^*} = \frac{\beta}{1-\beta}\frac{mu^*-1}{mu^*}.$$

s^* can then be derived from (III.3.3) evaluated at the steady state. This equation is reproduced here for convenience:

$$mu^* = \frac{-\theta}{1-\theta-s^*\frac{\Omega^*}{D^*}}.$$

For s^* to be positive θ should be smaller than $\frac{mu^*}{mu^*-1}$ which in the case $mu^* = 1.2$ is equivalent to the restriction $\theta < 6$. Next, C^* can be derived from the goods market equilibrium condition

$$Y^* = N^* = s^* + C^*.$$

The last step involves solving equation (III.3.8) evaluated at the steady state

$$\varrho s^* = (C^*)^{\frac{\eta}{1+\alpha}}(\pi^*)^{\frac{1}{1+\alpha}}$$

with respect to the parameter ϱ for a given α.

Unfortunately, the empirical literature provides neither evidence with respect to the value α nor there is enough model information to determine this parameter. Therefore I investigate the implications of the model for different values of α by performing a sensitivity analysis. An attempt to estimate α by GMM, based on macroeconomic data, is made in the next chapter.

Table III.1 summarizes the calibration of the model:

Table III.1:
Calibration

Households/Preferences	Firms/Technology	Central Bank
$\eta = 2$	$Z^* = 1$	$\tau^* = 1.0138$
$\chi = 2$	$\rho_z = \{0, 0.95\}$	$\rho_\tau = 0$
$\beta = 0.991$	$\sigma_\epsilon = 0.0082$	$\sigma_u = 0.0092$
$\theta \in (0, 2)$	$mu^* \in [1.1; 1.6]$	
$N^* = 0.1386$		

3.3 Results

Monetary Shocks

The effects of θ: Figures III.1 through III.7 illustrate the impact of a positive monetary shock without serial correlation, $\rho_\tau = 0$ for different choices of θ.[14] mu^* and α are equal to 1.4 and 0.5 respectively. The responses are measured in relative (not percentage) deviations from the steady state. Let t denote the time index of the period in which the shock occurs. $t + 1$ is the time index of the period after the shock. For a given price level the rise in the nominal money supply induces a positive income effect encouraging households to increase consumption, money demand and search activity and reduce labor supply. These reactions generate an upward pressure on current nominal wages and prices as well as expected inflation. Since all nominal variables are fully flexible, they will rise. The increase in inflation weakens the positive income effect of the monetary impulse. But whether hours, output and consumption will actually rise, fall or remain constant depends on how do firms react to the increase in nominal wages and the changes in current consumption and search efforts. First, note that an increase in current consumption does not only have a positive effect on current demand and current profits but also on the discount factor. A higher discount factor in turn generates a stronger incentive for firms to invest more in future market share. Thus for any given level of current demand and households' search efforts firms will set lower prices than they would do if the discount factor remained constant. Second, if all firms rise their prices current inflation will rise. Assume for simplicity that the resulting equilibrium is symmetric. The higher inflation will induce households to intensify search and switching efforts. From the point of view of an individual firm the higher search activity creates an incentive to choose a lower than average price. Since all firms will do the same, the average price level and the resulting overall inflation will be lower than they would be in an environment in which search does not depend on π_t. In other words, if there is an upward pressure on inflation the externality arising from the positive relationship between s_t and π_t lowers pass-through and leads to a lower equilibrium inflation.

[14] The corresponding programs are **"sim_cm2d2_1.g"** and **"sim_cm2d2_1_i.g"**.

Figure III.1: Benchmark model *(CM-model)*. Impulse responses to a monetary shock, $mu = 1.4$, $\alpha = 0.5$, $\rho_T = 0$. Relative deviations from steady state.

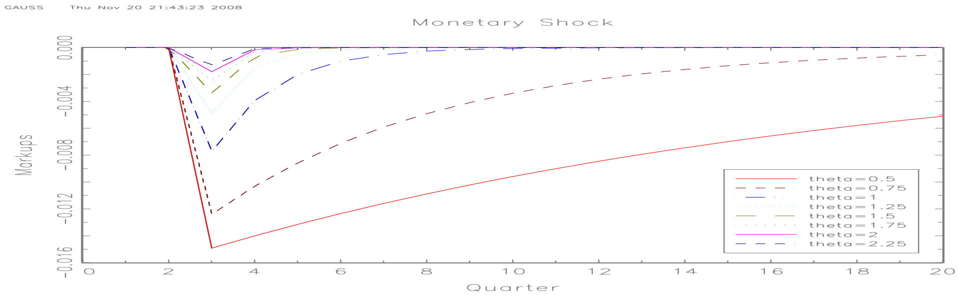


Figure III.2: Benchmark model *(CM-model)*. Impulse responses to a monetary shock, $mu = 1.4$, $\alpha = 0.5$, $\rho_T = 0$. Relative deviations from steady state.

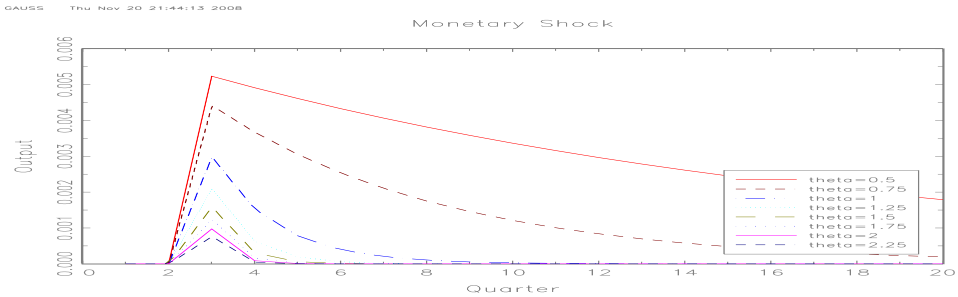


Figure III.3: Benchmark model *(CM-model)*. Impulse responses to a monetary shock, $mu = 1.4$, $\alpha = 0.5$, $\rho_T = 0$. Relative deviations from steady state.

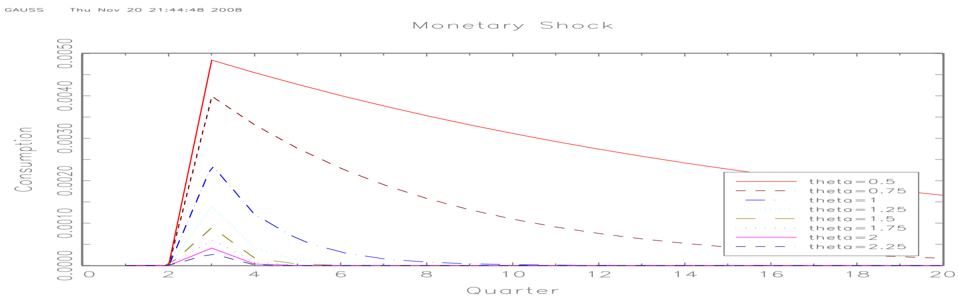
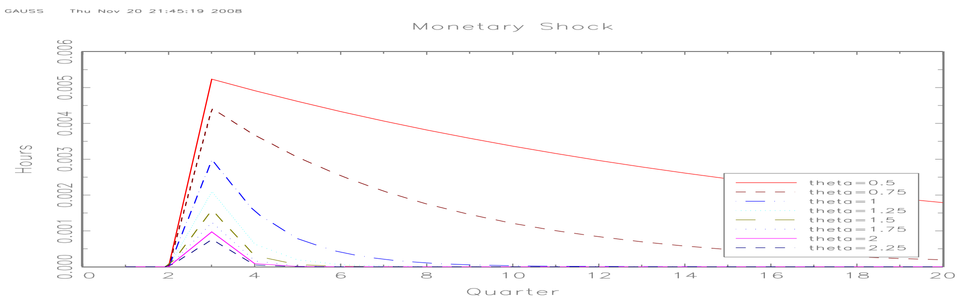


Figure III.4: Benchmark model *(CM-model)*. Impulse responses to a monetary shock, $mu = 1.4$, $\alpha = 0.5$, $\rho_T = 0$. Relative deviations from steady state.



For the reasons just described, firms pass through to prices only a fraction of the increase in nominal marginal costs. As a consequence, markups fall and real wages rise. As figures III.1 through III.7 show the increase in real wages $\left(\widehat{\frac{W_t}{P_t}}\right)$ is sufficient to induce working hours to

Figure III.5: Benchmark model *(CM-model)*. Impulse responses to a monetary shock, $mu = 1.4$, $\alpha = 0.5$, $\rho_T = 0$. Relative deviations from steady state.

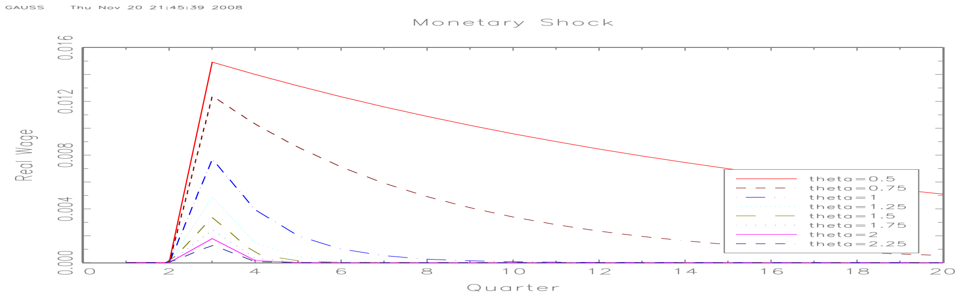


Figure III.6: Benchmark model *(CM-model)*. Impulse responses to a monetary shock, $mu = 1.4$, $\alpha = 0.5$, $\rho_T = 0$. Relative deviations from steady state.

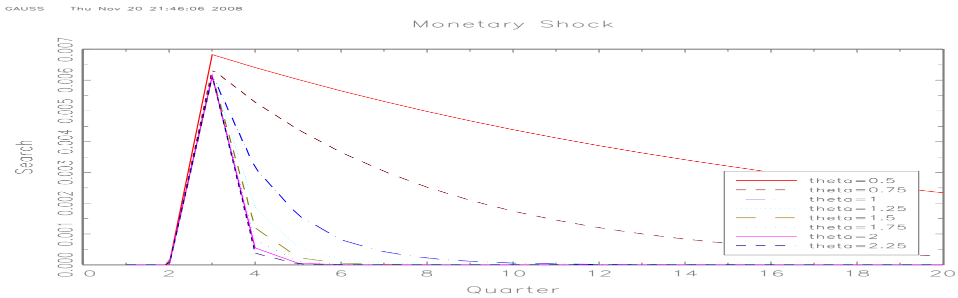
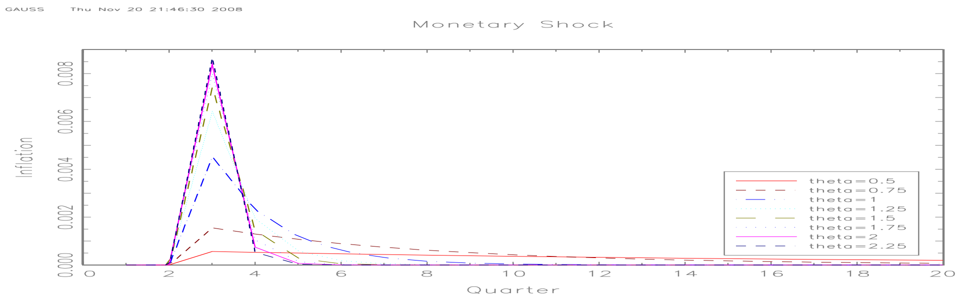


Figure III.7: Benchmark model *(CM-model)*. Impulse responses to a monetary shock, $mu = 1.4$, $\alpha = 0.5$, $\rho_T = 0$. Relative deviations from steady state.



rise, despite the increase in consumption. The higher output makes it possible to rise both, C_t and s_t.

It is important to note, that the key mechanism, making it possible for nominal disturbances to have real effects, is the direct link between search efforts and current inflation, which is due to the assumed form of inflation aversion. To understand that, assume that before search could be adjusted the rise in inflation induced by a monetary shock is just sufficient to force consumption and hours to remain constant. Is the situation just described an equilibrium allocation? No, it isn't for the following reason: Since inflation is above average search efforts will be above average too. Therefore firms will have an incentive to lower their prices and so equilibrium inflation will fall. As a result markups will deviate negatively and real wages, hours and output positively from their respective steady state levels. Absent the dependence between s_t and π_t, the positive pressures put on consumption C_t, expected future profits

Ω_t and search s_t by the monetary shock will exactly offset each other and the increase in inflation will be sufficient to offset the income effect of the monetary disturbance. In that case money will be *neutral*.

The impulse responses indicate that lower values of θ make the reactions to a one time monetary disturbance more pronounced in the period of the shock on the one hand and more persistent on the other. What is the intuition behind the stronger reactions when θ takes a relatively low value? The higher the value of θ, the higher in absolute value the slope of the *current profit function* with respect to mu_t at the symmetric equilibrium. Therefore, if θ is relatively high, firms will need a smaller adjustment of mu_t as a reaction to changes in the term $\frac{g(1,s_t)\Omega_t}{D_t}$, in order to ensure that their respective optimality conditions remain satisfied. The stronger deviations of markups for low values of θ then imply more pronounced reactions of real wages and thus hours and output. The higher output makes it possible to increase consumption which in turn has a positive effect on search activity. In fact, the initial reaction of s_t is virtually the same for all values of θ. For higher values of θ the rise in search is induced by the healthy increase of inflation, whereas for lower values of θ it results from the relatively strong positive reaction of consumption. Hence, pass-through in the low-θ-case is further decreased relative to that in the high-θ-case by the larger increase of the discount factor in the former.

Where does the higher persistence come from? Since it is optimal for households to smooth consumption over the entire future, the higher consumption in the period of the shock will force them to increase their investment in real balances. Since the positive deviation of real balances in $t+1$ is larger than that of inflation, the term $\frac{m_{t+1}}{\pi_{t+1}}$ increases. Hence, at the beginning of $t+1$ households start with above average real value of wealth, or in other words, are subject to a positive wealth effect. The latter induces qualitatively the same reactions as did the positive monetary shock in the previous period. The lower the value of θ, the higher the increase in the real value of wealth $\frac{m_{t+1}}{\pi_{t+1}}$, and thus the stronger the induced positive wealth effect in the period after the shock, $t+1$. As a result, the increase in $t+1$-consumption in the low-θ-economy will be larger than that in the high-θ-economy. The higher the increase in $t+1$-consumption relative to its average future level, the stronger the additional investment in money balances[15] m_{t+2} and therefore, the stronger the positive wealth effect in $t+2$ and so on.

The effects of ***mu^******:** Figures III.8 through III.10 depict the impulse responses to the same monetary shock for different values of mu^*. θ and α are set to 0.5 and 0.5 respectively. It can be shown that there is a positive relationship between the absolute value of the slope of the

[15] Because of consumption smoothing.

current profit function evaluated at the symmetric equilibrium and the steady state markup mu^*. Therefore, if mu^* is relatively high, firms will need a larger adjustment of mu_t as a reaction to changes in the term $\frac{g(1,s_t)\Omega_t}{D_t}$, in order to ensure that their respective optimality conditions remain satisfied. The impulse responses displayed in figure III.8 confirm that: A higher steady state markup implies a lower pass-through of marginal costs to prices and leads to more pronounced and more persistent impulse responses in the same way as low values of θ do.

Figure III.8: Benchmark model *(CM-model)*. Impulse responses to a monetary shock, $\theta = 0.5$, $\alpha = 0.5$, $\rho_T = 0$. Relative deviations from steady state.

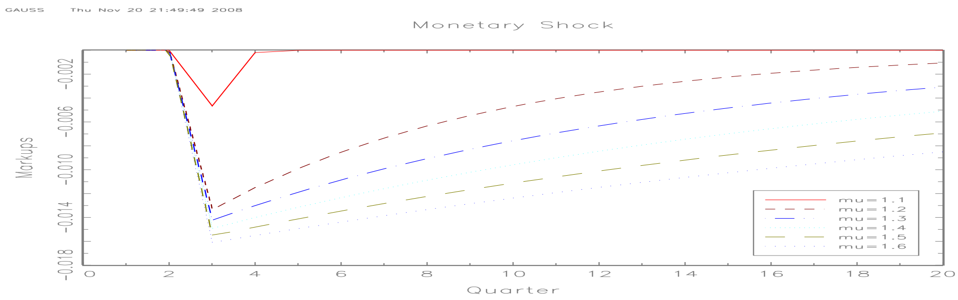


Figure III.9: Benchmark model *(CM-model)*. Impulse responses to a monetary shock, $\theta = 0.5$, $\alpha = 0.5$, $\rho_T = 0$. Relative deviations from steady state.

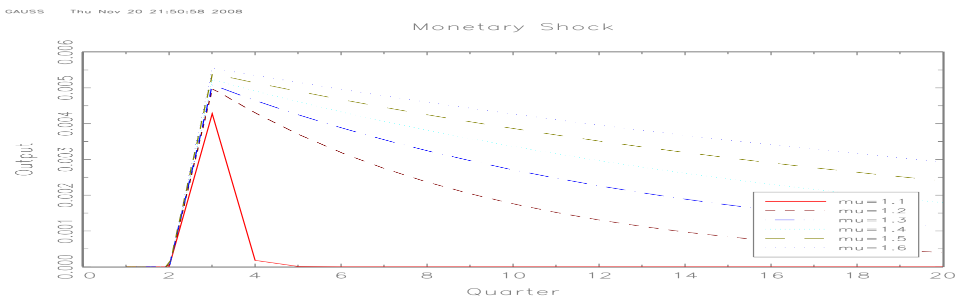
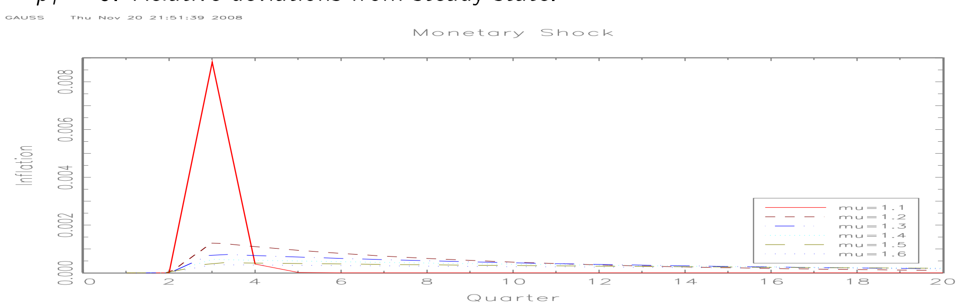


Figure III.10: Benchmark model *(CM-model)*. Impulse responses to a monetary shock, $\theta = 0.5$, $\alpha = 0.5$, $\rho_T = 0$. Relative deviations from steady state.



The effects of α: Figures III.11 through III.13 illustrate the impulse responses of markups, output and inflation to a one time monetary shock for different values of α. θ and μ are set to 0.5 and 1.4 respectively. As can be seen, lower values of α induce stronger and more persistent responses to monetary shocks. The reason is that lower values of α imply stronger positive reactions of search activity to changes in inflation and thus a tendency for firms to choose a lower (higher) pass-through as a reaction to a positive (negative) monetary disturbance. If α takes a very high value money is almost *neutral* in this model.

Figure III.11: Benchmark model *(CM-model)*. Impulse responses to a monetary shock, $\theta = 0.5$, $mu^* = 1.4$, $\rho_T = 0$.

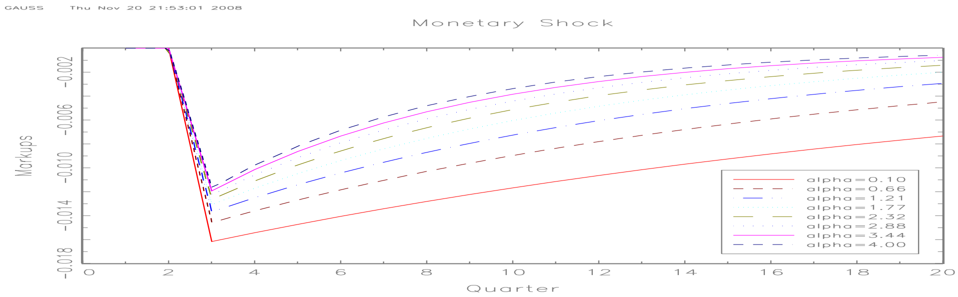


Figure III.12: Benchmark model *(CM-model)*. Impulse responses to a monetary shock, $\theta = 0.5$, $mu^* = 1.4$, $\rho_T = 0$.

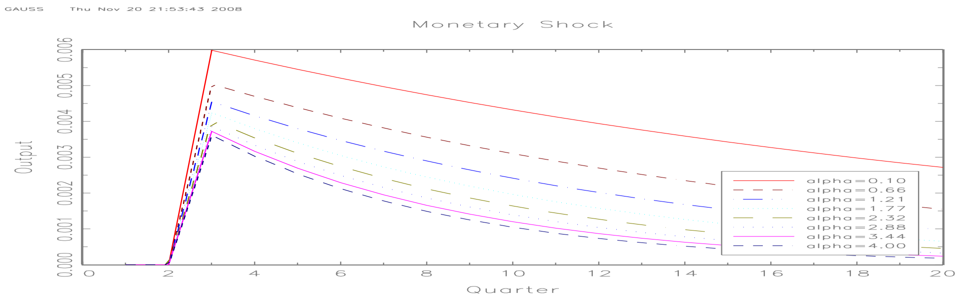
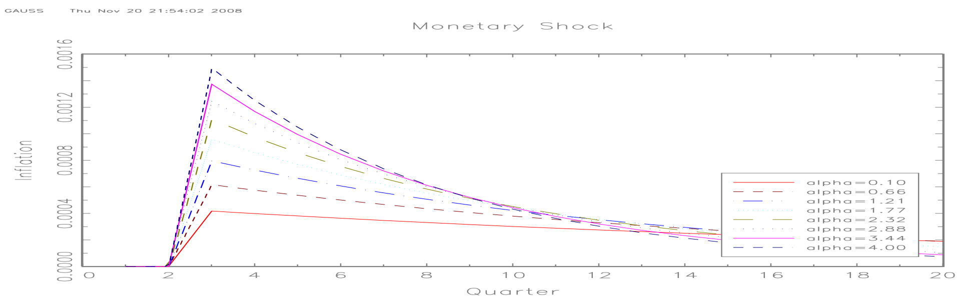


Figure III.13: Benchmark model *(CM-model)*. Impulse responses to a monetary shock, $\theta = 0.5$, $mu^* = 1.4$, $\rho_T = 0$.



A comparison with the New Keynesian Model: For the purpose of comparison I assume that technology and monetary policy are identical with that in the benchmark model. Further, in the New Keynesian model there are no market share competition and no search or switching activity. The consumption aggregator is modified as follows:

$$C_t = \left\{ \frac{1}{n} \sum_{i=1}^{n} C_{i,t}^{\frac{\theta-1}{\theta}} \right\}^{\frac{\theta}{\theta-1}},$$

and the utility function reads:

$$U = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\eta}}{1-\eta} + \phi_m \frac{(M_t/P_t)^{1-\chi}}{1-\chi} - \frac{b}{2} N_t^2 \right) \right\}, \qquad \phi_m, b, \eta, \chi > 0, \quad \beta \in (0,1).$$

The pricing decisions of individual firms can then be aggregated to the following log-linear, forward looking *Phillips Curve*:

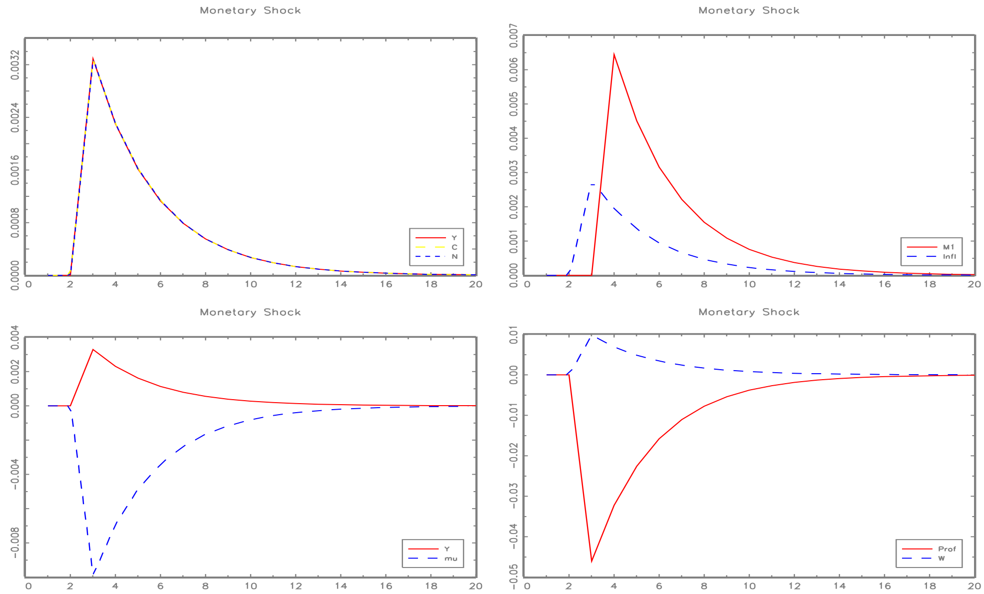
$$\hat{\pi}_t = \beta E_t(\hat{\pi}_{t+1}) + \frac{(1-\varphi)(1-\beta\varphi)}{\varphi} \hat{\mu}_t,$$

where μ_t denotes marginal costs. The model is parameterized as follows: $\beta = 0.991$, $N^* = 0.1386$, $\eta = 2$, $\chi = 2$, $\rho_\tau = 0$, $\sigma_u = 0.0092$, $\rho_z = 0$ or $\rho_z = 0.964$, $\sigma_\epsilon = 0.0082$, $\varphi = 0.75$ and $mu^* = 1.4$. Note, that in this model the choice of mu^* affects only the impulse responses of profits. Figure III.14 depicts the impulse responses to a purely temporary monetary shock in the third quarter.[16] As can be seen, the effect of the shock is largest on impact, dies out gradually and disappears completely after about 11 quarters. The peak-response of output equals about 0.33% which is much less than the 0.6% estimated by Christiano *et al.* (2005). Markups and profits respond negatively to the monetary impulse. Since, according to the empirical evidence, steady state markups in range between 1.2 and 1.4 as well as values of θ lower than one are economically plausible, for a fairly large range of parameter values the benchmark model presented in section 3 implies stronger and more persistent responses to monetary shocks than the New Keynesian model does. It is easy to find combinations of θ, mu^* and α implying that the peak-response of output is exactly 0.6%, while its duration equals 3.5 years (14 quarters), as found by Christiano *et al.* (2005), e.g. $\theta = 0.5$, $mu^* = 1.2$ and $\alpha = 0.3$. Note, further, that the degree of monetary nonneutrality in the benchmark model does not hinge on unrealistic assumptions with respect to the frequency of price adjustment. In particular, in the benchmark model prices are flexible and are adjusted each quarter. Thus, firstly, the benchmark model provides an alternative explanation of the observable real effects of nominal disturbances and secondly, for a broad range of parameter values it matches better the empirical evidence provided by Christiano *et al.* (2005) with respect to the magnitude and persistence of the responses to nominal shocks than the New Keynesian model does.

It is also important to note that the New Keynesian model implies some persistence of the impulse responses only for relatively high levels of price rigidity (high values of φ). For instance $\varphi = 0.5$ ($\varphi = 0.3$) imply that the effects of the monetary shock completely disappear after 5 (2) quarters.

[16] The corresponding program is **"new_keynes.g"**.

Figure III.14: New Keynesian Model. Impulse responses to a monetary shock, $mu^* = 1.2$, $\rho_\tau = 0$, $\varphi = 0.75$. Relative deviations from steady state.



Y-Output, C-Consumption, N-Hours, W-Real Wage, S-Search Effort, M_1-Real Balances, $Infl$-Inflation, $Prof$-Profits.

Technology Shocks

Figures III.15 through III.18 depict the impulse responses to a non-autocorrelated technology shock $\rho_z = 0$. Combinations of relatively low values of θ, $\theta \leq 1$ and a relatively high steady state markup, $\mu^* \geq 1.2$ imply highly persistent reactions to a one time increase of total factor productivity. According to these results, the benchmark model of section 3 provides an endogenous explanation of the persistence of *technolgy-shock-driven* business cycles, since it does not impose the exogenous assumption that the coefficient of serial correlation of total factor productivity ρ_z is greater than zero. In other words, large scale real- or monetary business cycle models incorporating a complex combination of assumptions e.g. *high degree of wage and price rigidity, and adjustment costs of capital, and matching frictions in the labor market, and habit persistence, and...* or simpler models assuming $\rho_z \in (0.99, 1)$ are not the only theories able to account for the observed duration of business cycles. There are also much simpler models, like the one presented in section 3, which are able to do that. For instance, the parameter combination $\theta = 0.7(1.2)$, $\delta = 1$, $\alpha = 0.5$ and $\mu^* = 1.4(1.2)$ imply the following first-order autocorrelations, $corr(x_t, x_{t-1})$: output: 0.68 (0.42), consumption: 0.67 (0.31), hours: 0.63 (0.24), real wages: 0.69 (0.42), markups: 0.67 (0.41).

Note, that the original version of the customer market model proposed by Phelps and Winter (1970) does not generate persistent responses to one-time technology shocks! The discrete time version of that model is presented in an appendix available upon request.

Figure III.15: Benchmark model *(CM-model)*. Impulse responses to a technology shock, *mu* $= 1.4$, $\alpha = 0.5$, $\rho_z = 0$. Relative deviations from steady state.

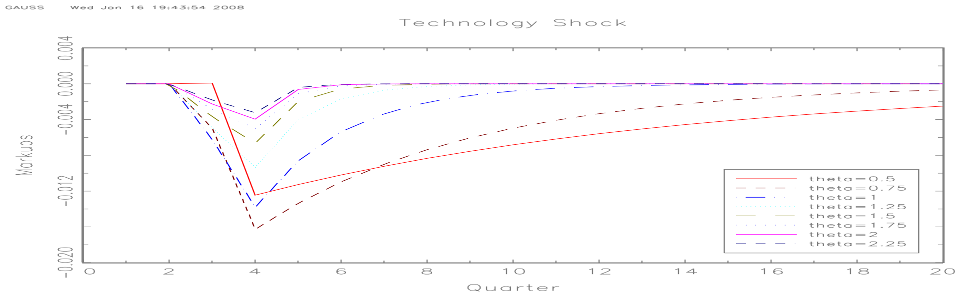


Figure III.16: Benchmark model *(CM-model)*. Impulse responses to a technology shock, *mu* $= 1.4$, $\alpha = 0.5$, $\rho_z = 0$. Relative deviations from steady state.

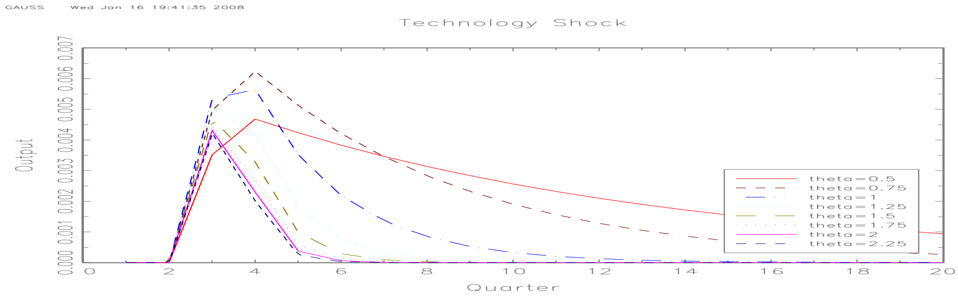


Figure III.17: Benchmark model *(CM-model)*. Impulse responses to a technology shock, $\theta = 0.7$, $\alpha = 0.5$, $\rho_z = 0$. Relative deviations from steady state.

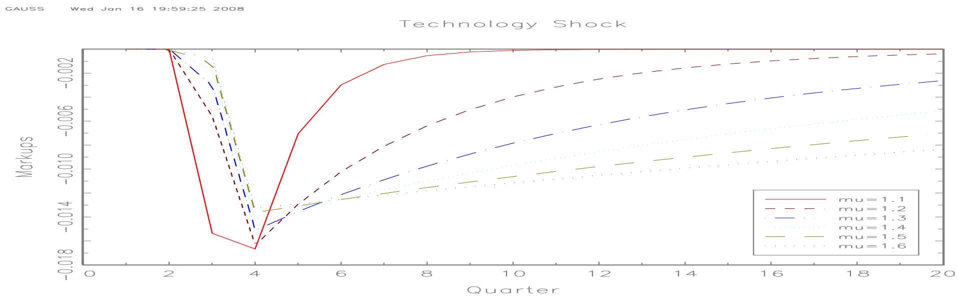
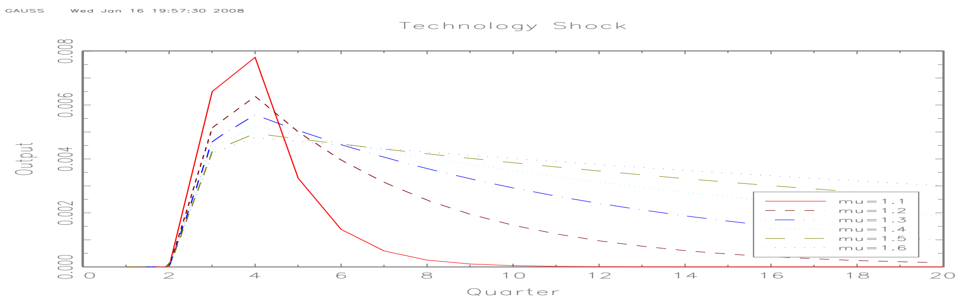
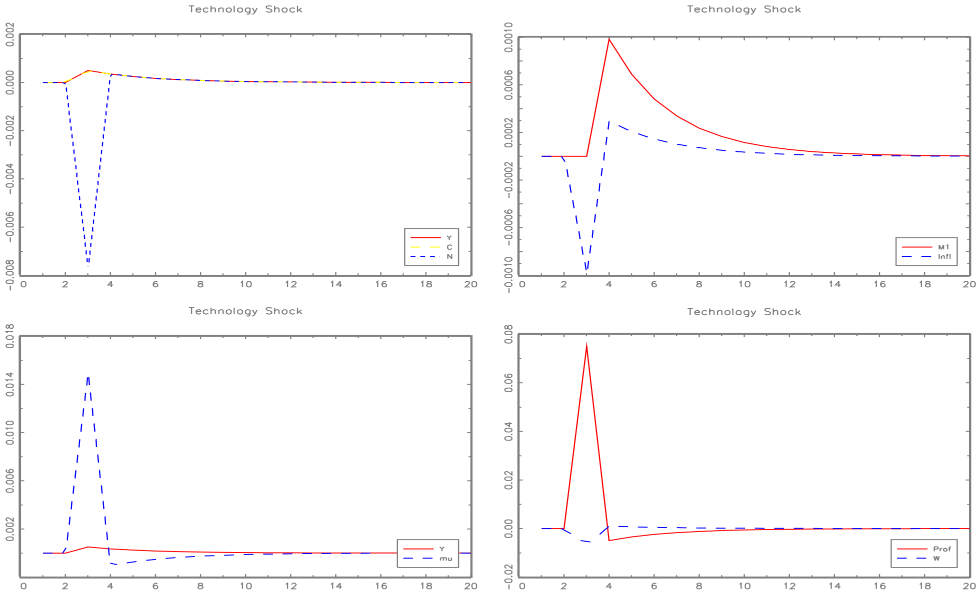


Figure III.18: Benchmark model *(CM-model)*. Impulse responses to a technology shock, $\theta = 0.7$, $\alpha = 0.5$, $\rho_z = 0$. Relative deviations from steady state.



A comparison with the New Keynesian Model: Figure III.19 displays the impulse responses to a technology shock without serial correlation implied by the *New Keynesian Moedl*. There is no such thing as persistence in the responses of hours and markups and the reactions of output and consumption are almost indiscernible.

Figure III.19: New Keynesian Model. Impulse responses to a technology shock, $mu^* = 1.2$, $\rho_z = 0$, $\varphi = 0.75$. Relative deviations from steady state.



Y-Output, C-Consumption, N-Hours, W-Real Wage, S-Search Effort, M_1-Real Balances, $Infl$-Inflation, $Prof$-Profits.

3.4 Summary of the Results

The model presented in section 3 extends the standard monetary business cycles model along three dimensions: market share competition, search and switching activity in the goods market and inflation aversion. The interplay of these new features of the model with the old ones enables it to better account for the cyclical properties of markups and the significant and persistent real effects of monetary impulses than the standard New Keynesian model does. Furthermore, the theory provides an endogenous explanation of the empirically observable persistent reactions to technology shocks without resorting to the assumption that total factor productivity follows an autoregressive process with a coefficient of autocorrelation near one.

An important challenge for future empirical research will be the attempt to quantify the cyclical properties of s_t and to estimate α. Chapter 4 describes such an attempt. From theoretical point of view, the model provides many dimensions along which it can be extended. In the following section I make the production side of the model more realistic by introducing capital as a second factor of production.

4 Capital Accumulation

4.1 The Model

Let us extend the model by assuming that there are two production factors - capital and labor. The production function of firm i exhibits constant returns to scale and is given by

$$Y_{i,t} = Z_t N_{i,t}^{\omega} K_{i,t}^{1-\omega}, \qquad \omega \in (0,1),$$

where $K_{i,t}$ denotes capital input and Z_t represents total factor productivity following the same stochastic process as in section 3. The aggregate stock of capital evolves according to

$$K_{t+1} = I_t + (1-\upsilon)K_t, \qquad \upsilon \in (0,1), \tag{III.4.1}$$

where aggregate Investment I_t is given by

$$I_t = \left\{ \frac{1}{n} \sum_{i=1}^{n} x_{i,t}^{\frac{1}{\theta}} I_{i,t}^{\frac{\theta-1}{\theta}} \right\}^{\frac{\theta}{\theta-1}}.$$

$x_{i,t}$ and θ are subject to the same assumptions as in section 3. All other assumptions remain the same.[17] The equilibrium in this economy is described by the following set of conditions:

$$C_t^{-\eta} = \Lambda_t, \tag{III.4.2}$$

$$bN_t = \Lambda_t \frac{W_t}{P_t}, \tag{III.4.3}$$

$$\varrho s_t = \pi_t^{\frac{\delta}{1+\alpha}} C_t^{\frac{\eta}{1+\alpha}}, \tag{III.4.4}$$

$$1 = E_t \left\{ \beta \frac{\Lambda_{t+1}}{\Lambda_t} \left(1 + \frac{1-\omega}{mu_t} \frac{Y_{t+1}}{K_{t+1}} - \delta \right) \right\}, \tag{III.4.5}$$

$$\beta \phi_m m_{t+1}^{-\chi} E_t\{\pi_{t+1}^{\chi-1}\} = \Lambda_t - \beta E_t \left\{ \frac{\Lambda_{t+1}}{\pi_{t+1}} \right\}, \tag{III.4.6}$$

$$K_{t+1} = Z_t N_t^{\omega} K_t^{1-\omega} + (1-\upsilon)K_t - C_t - s_t, \tag{III.4.7}$$

[17] See section 3.

$$Y_t = Z_t N_t^{\omega} K_t^{1-\omega}, \tag{III.4.8}$$

$$mu_t \frac{W_t}{P_t} = \omega \frac{Y_t}{N_t}, \tag{III.4.9}$$

$$mu_t = \frac{-\theta}{1-\theta-s_t\frac{\Omega_t}{Y_t}}, \tag{III.4.10}$$

$$\Omega_t = E_t\left\{\beta\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{mu_{t+1}-1}{mu_{t+1}}\right)Y_{t+1}\right\} + E_t\left\{\beta\frac{\Lambda_{t+1}}{\Lambda_t}\Omega_{t+1}\right\}, \tag{III.4.11}$$

$$m_{t+1} = \tau_t \frac{m_t}{\pi_t}. \tag{III.4.12}$$

The model featuring capital accumulation involves few additional calibration steps. The production elasticity of labor ω is chosen to satisfy the restriction:

$$\omega = mu^* \overline{\left(\frac{(W/P)N}{Y}\right)},$$

where $\overline{\left(\frac{(W/P)N}{Y}\right)}$ denotes the average actual labor share. The empirical estimates for this variable using U.S.- as well as data for other industrialized countries vary between slightly below 0.6 and slightly below 0.8. I set labor share at 0.6 since values larger than 0.625 combined with a steady state markup equal to 1.6 imply $\omega > 1$. The euler equation for optimal investment in capital (III.4.5) evaluated at the stationary equilibrium then implies

$$\frac{Y^*}{K^*} = mu^* \frac{1-\beta(1-\upsilon)}{\beta(1-\omega)}.$$

By using this result together with the definition of the production function and a given N^* one arrives at:

$$K^* = \frac{N^*}{\left(\frac{Y^*}{K^*}\right)^{\frac{1}{\omega}}}.$$

The depreciation rate υ is set at 0.0071.[18] In the next step the steady state value of investment I^* is easily derived via the law of motion for capital:

$$I^* = \upsilon K^*.$$

s^* is calibrated in the same way as in the benchmark model. C^* star then follows from the aggregate resource constraint

$$C^* = Y^* - I^* - s^*.$$

The remaining parameters are calibrated in the same way as in section 3.

[18] Details are provided in chapter 4.

4.2 Impulse Responses to Monetary Shocks

The impulse responses of output, markups, inflation and the stock of capital for different values of α, θ and mu^* are depicted in figures III.20 through III.28.[19] Similarly to the benchmark model the smaller θ and α and the larger mu^*, the stronger the responses of all real variables to the monetary disturbance. In contrast to the benchmark model, its effect on output, search, markups, hours and inflation lasts for only one period. Only the real wage remains slightly above average for a longer period of time and returns slowly to its pre-shock level. The responses of consumption, real money holdings and the stock of capital can be characterized as persistent but of limited magnitude. What is the intuition behind this results?

In the benchmark model a monetary shock induces households to transfer more real money holdings into the next period. The resulting positive wealth effect forces them to desire a higher consumption and a lower labor supply. As a result there is an upward pressure on nominal wages and prices in the period after the shock. As there are no shifts in labor productivity, the potential disparity between aggregate supply and demand in the goods and labor markets are large enough to induce an increase in inflation, despite the incomplete pass-through. The higher inflation then leads to a higher search activity which, in turn, causes a lower pass-through and therefore lower equilibrium markups (higher real wages). As a result of the strong positive reaction of real wages in the period after the shock, employment and thus output are above average. The same mechanisms are responsible for the steady state deviations in the next period and so on.

In the model with capital accumulation households are not restricted to invest only in real balances as there is a second channel of intertemporal substitution. Unfortunately, as a reaction to a monetary shock, this second dynamic link between *"today and tomorrow"* makes future labor more productive and so alleviates the disparity between supply and demand in the goods and labor markets: To understand why, first note that in the period after the shock the additionally accumulated capital enables firms to produce a higher amount of goods by using less labor. Thus, for a given level of aggregate demand and a given relative factor price $\frac{R}{W/P}$ the demand for labor will be lower than before the shock. This dampens the pressure on nominal wages and prices. As a consequence, in the period after the shock the deviation of inflation from its steady state level is almost zero. Therefore the positive reaction of search activity present in the benchmark model is absent in the model with capital accumulation. Absent the high level of search and switching efforts pass-through and markups remain relatively high and real wages relatively low in the period after the shock. In such a situation households see no incentive to work more than average (or as much as in the benchmark model). The additionally accumulated capital enables the economy to finance a

[19] The corresponding programs are **"sim_cm2d5a_1cap.g"** and **"sim_cm2d5a_1cap_i.g"**.

slightly above average post-shock consumption. The small positive (negative) deviations of real wages (the real interest rate) from the steady state in the aftermath of the shock also result from the higher capital stock.

Obviously the introduction of capital accumulation as a further channel for intertemporal substitution, while making the production side of the model more realistic, endows agents with a powerful tool for efficiently avoiding monetary pressure, reducing the effects of monetary disturbances to relatively weak, purely temporary deviations of some macroeconomic aggregates from their respective long run levels. At the end of this section I conclude that either investment is made too flexible, much more than it is in the real world, or monetary shocks are unimportant with regard to economic fluctuations, or the general structure of the model is at odds with reality. Disregarding the third alternative as implausible and leaving the second for future research, in the next section I assume that it is much more costly to adjust the stock of capital and so reduce the flexibility provided by investment without neglecting capital as a second factor of production.

Figure III.20: Endogenous Capital Model. Impulse responses to a monetary shock, $mu = 1.2$, $\theta = 0.8$, $\rho_T = 0$. Relative deviations from steady state.

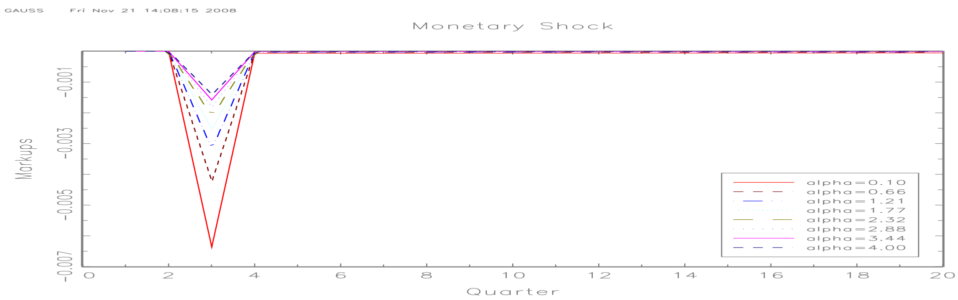


Figure III.21: Endogenous Capital Model. Impulse responses to a monetary shock, $mu = 1.2$, $\theta = 0.8$, $\rho_T = 0$. Relative deviations from steady state.

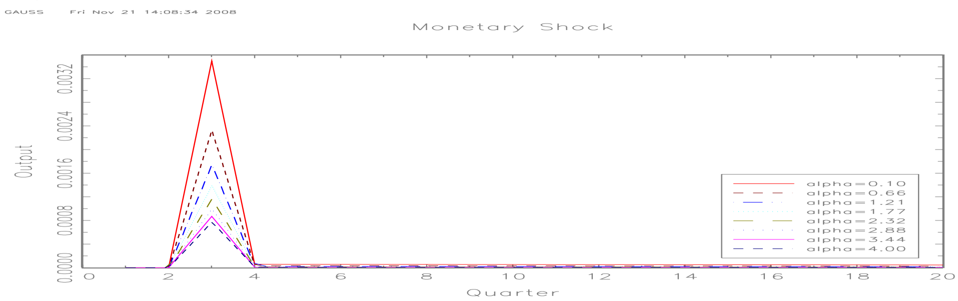


Figure III.22: Endogenous Capital Model. Impulse responses to a monetary shock, $mu = 1.2$, $\theta = 0.8$, $\rho_\tau = 0$. Relative deviations from steady state.

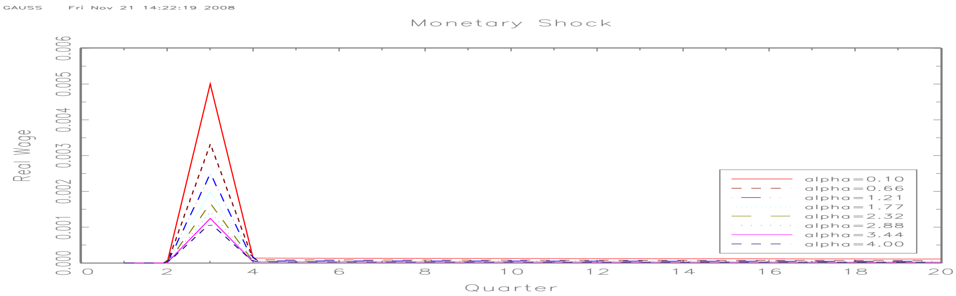


Figure III.23: Endogenous Capital Model. Impulse responses to a monetary shock, $mu = 1.2$, $\theta = 0.8$, $\rho_\tau = 0$. Relative deviations from steady state.

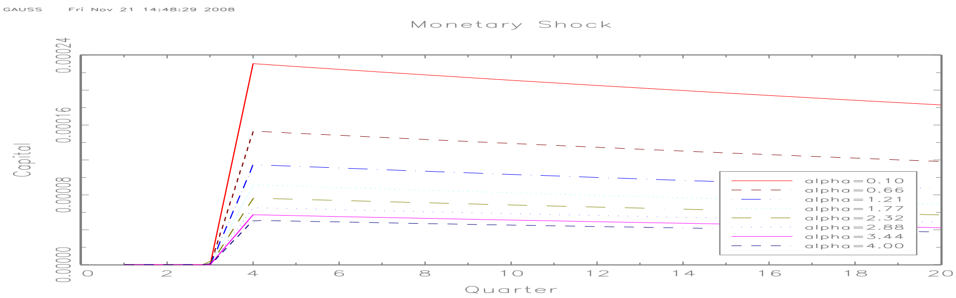


Figure III.24: Endogenous Capital Model. Impulse responses to a monetary shock, $mu = 1.2$, $\theta = 0.8$, $\rho_\tau = 0$. Relative deviations from steady state.

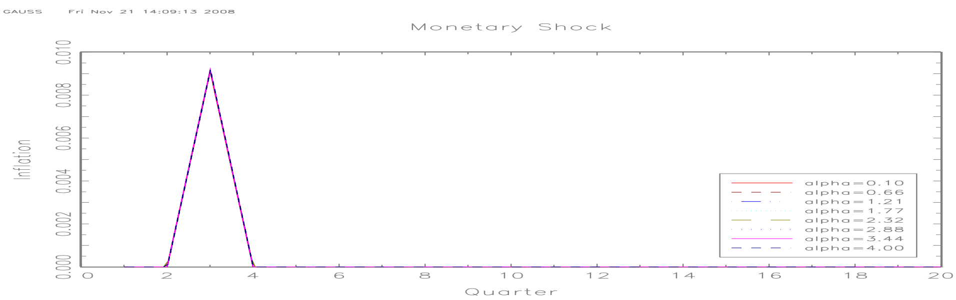


Figure III.25: Endogenous Capital Model. Impulse responses to a monetary shock, $mu = 1.2$, $\alpha = 0.5$, $\rho_\tau = 0$. Relative deviations from steady state.

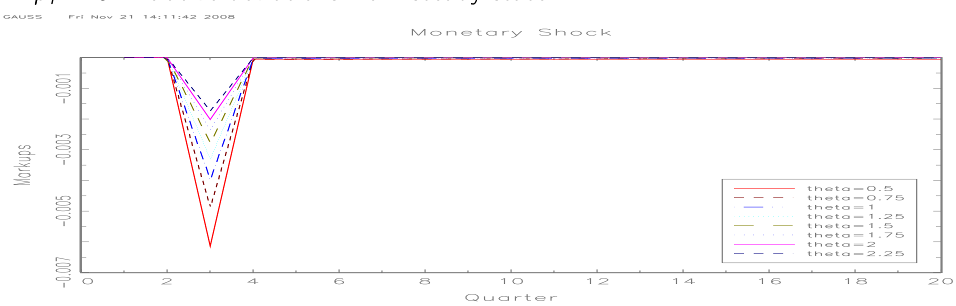


Figure III.26: Endogenous Capital Model. Impulse responses to a monetary shock, $mu = 1.2$, $\alpha = 0.5$, $\rho_T = 0$. Relative deviations from steady state.

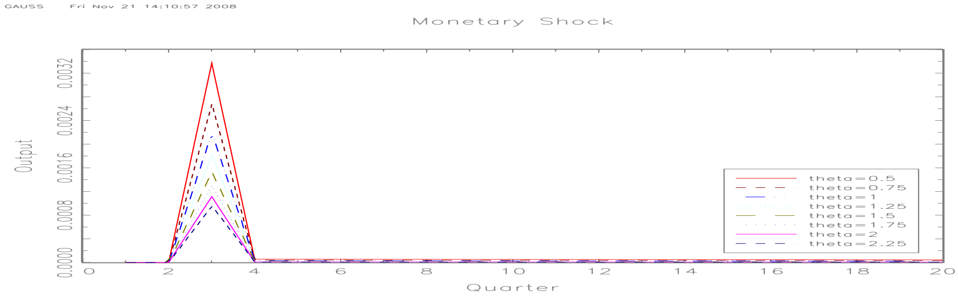


Figure III.27: Endogenous Capital Model. Impulse responses to a monetary shock, $\theta = 0.8$, $\alpha = 0.5$, $\rho_T = 0$. Relative deviations from steady state.

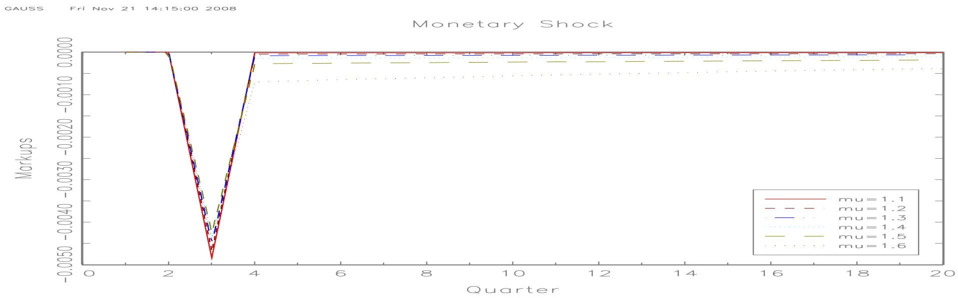
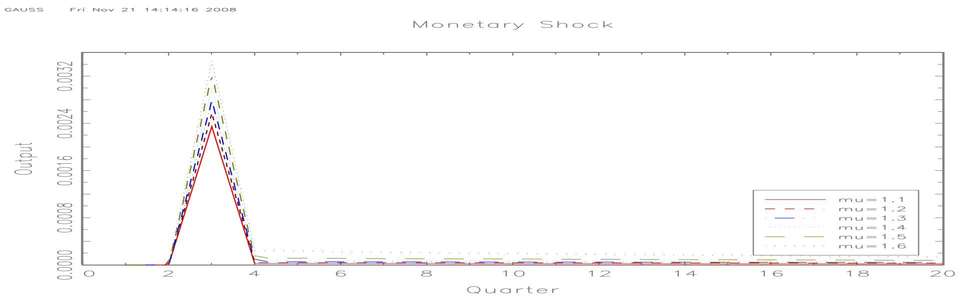


Figure III.28: Endogenous Capital Model. Impulse responses to a monetary shock, $\theta = 0.8$, $\alpha = 0.5$, $\rho_T = 0$. Relative deviations from steady state.



4.3 Adjustment Costs of Capital

The flexibility of investment is reduced in an *ad hoc* manner by assuming that there is an additional adjustment cost of capital represented by the strict concavity of the strictly increasing function $\phi\left(\frac{I_t}{K_t}\right)$ in

$$K_{t+1} = \phi\left(\frac{I_t}{K_t}\right) K_t + (1-\upsilon)K_t. \tag{III.4.1}$$

Further, $\phi(.)$ has the properties:

$$\phi\left(\frac{I}{K}\right) = \phi(\upsilon) = \upsilon, \qquad \phi'(\upsilon) = 1,$$

where I and K are the steady state levels of investment and capital respectively. The first assumption ensures that the steady state is characterized by the absence of adjustment costs while the second implies that in the stationary equilibrium *Tobin's q* is equal to one. Formally the equilibrium conditions (III.4.2) through (III.4.12) ought to be adjusted by including the household's first order condition with respect to investment

$$q_t = \frac{\Lambda_t}{\phi'\left(\frac{I_t}{K_t}\right)},$$

substituting the conventional transition equation for capital by (III.4.1) and replacing the first order condition with respect to next period's stock of capital (III.4.5) by

$$q_t = E_t\left\{\Lambda_{t+1}\frac{1-\omega}{mu_{t+1}}\frac{Y_{t+1}}{K_{t+1}} + q_{t+1}\left(1-\upsilon+\phi\left(\frac{I_{t+1}}{K_{t+1}}\right) - \phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}}{K_{t+1}}\right)\right\},$$

where q_t denotes the Lagrangean multiplier attached to (III.4.1) and q_t/Λ_t equals *Tobin's q*.

There is only one additional parameter to be calibrated in the model with adjustment costs of capital - the elasticity of $\phi\left(\frac{I_t}{K_t}\right)$ with respect to its argument I_t/K_t, denoted by ς. Jerman (1998) provides a GMM estimate of ς equal to -1/0.23. The same value is used in the computation of impulse responses. However, as shown in chapter 4 this $\varsigma = 1/0.23$ leads to the counterfactual implication that investment is half as volatile as output. For that reason in chapter 4 I also compute business cycle moments based on the value ς implying the empirically observable relation between the volatility of output and inflation.

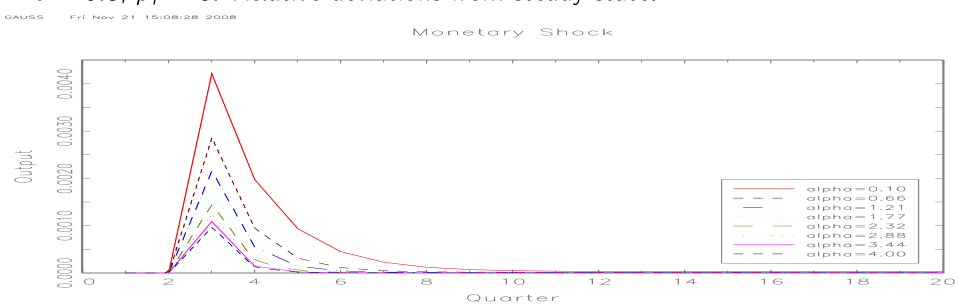
Impulse Responses to Monetary and Technology Shocks: The impulse responses to a serially uncorrelated monetary shock for different values of α, θ and mu^* are depicted in figures III.29 through III.36.[20] ς is set to the value implying that for $\alpha = 0.5$, $\theta = 0.8$

[20] The programs used are **"sim_cm2d6a_1cap.g"**, **"sim_cm2d6a_1cap_i.g"** and **"sim_cm2d6a_1cap_ii.g"** .

and $mu = 1.2$ investment is about 4.63 times as volatile as output. The reactions to a one-time monetary expansion are of similar magnitude and persistence as that implied by the benchmark model of section 3. There is also a similar pattern regarding variations of α θ and mu^*: Lower values of α and θ and higher steady state markups mu^* strengthen and prolong the impulse responses. A comparison between figures III.23 and III.32 reveals that the introduction of the adjustment cost mechanism makes investment sufficiently expensive and thus, capital accumulation a less desirable channel for intertemporal substitution. As a consequence in the model characterized by adjustment costs capital increases by a much smaller amount in the period after the shock than it is the case in the no-adjustment-costs economy of section 4. Hence, as expected, the less flexible technique for capital accumulation leads to an economic structure which is an intermediate case between the fixed capital model of section 3 and the one with fully flexible capital presented in section 4. Again, a notable feature to be emphasized is that the intrinsic mechanisms of the model are strong enough to generate substantial autocorrelation in all macroeconomic aggregates, even though the exogenous driving force follows a White Noise process.

Figures III.37 through III.42 display the impulse responses of output and the markup to a serially uncorrelated technology shock for different values of α, θ and mu^*. The pattern is again qualitatively similar to that implied by the benchmark model presented in section 3 - lower values of α and θ and larger values of mu^* cause stronger and more persistent reactions. Unlike most business cycles models which have to assume highly autocorrelated exogenous processes in order to be able to generate long-lasting impulse responses, in the current model the one-time technological disturbance is propagated in a very persistent manner by the endogenous mechanisms of the model. The responses of the remaining variables, not shown here,[21] are also similar to that implied by the benchmark model.

Figure III.29: Adjustment Costs of Capital Model. Impulse responses to a monetary shock, $mu = 1.2$, $\theta = 0.8$, $\rho_T = 0$. Relative deviations from steady state.



[21]The plots are available upon request.

Figure III.30: Adjustment Costs of Capital Model. Impulse responses to a monetary shock, $mu = 1.2$, $\theta = 0.8$, $\rho_\tau = 0$. Relative deviations from steady state.

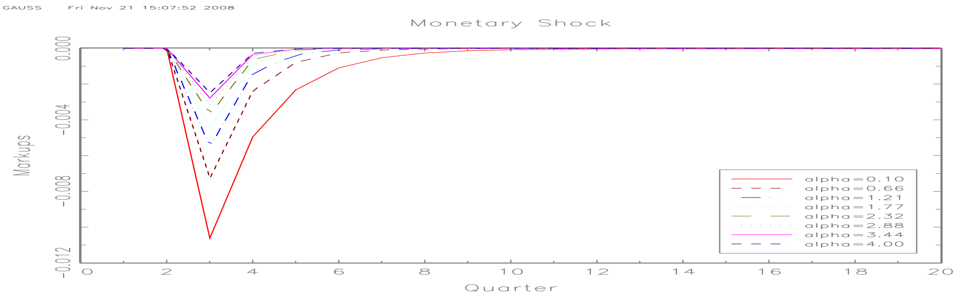


Figure III.31: Adjustment Costs of Capital Model. Impulse responses to a monetary shock, $mu = 1.2$, $\theta = 0.8$, $\rho_\tau = 0$. Relative deviations from steady state.

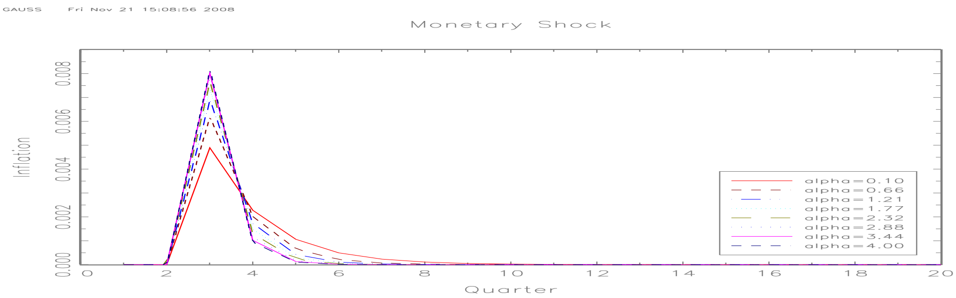


Figure III.32: Adjustment Costs of Capital Model. Impulse responses to a monetary shock, $mu = 1.2$, $\theta = 0.8$, $\rho_\tau = 0$. Relative deviations from steady state.

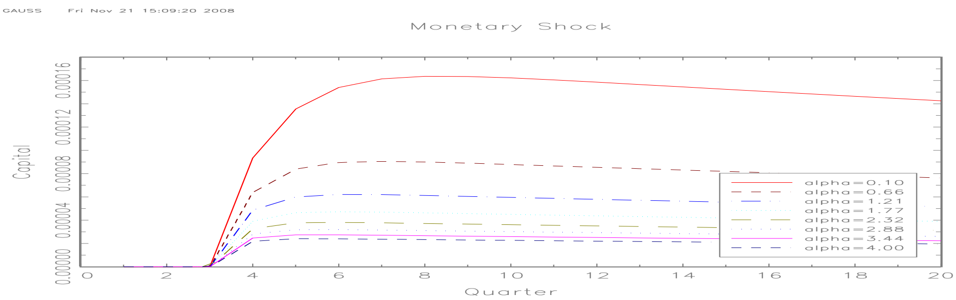


Figure III.33: Adjustment Costs of Capital Model. Impulse responses to a monetary shock, $mu = 1.2$, $\alpha = 0.5$, $\rho_\tau = 0$. Relative deviations from steady state.

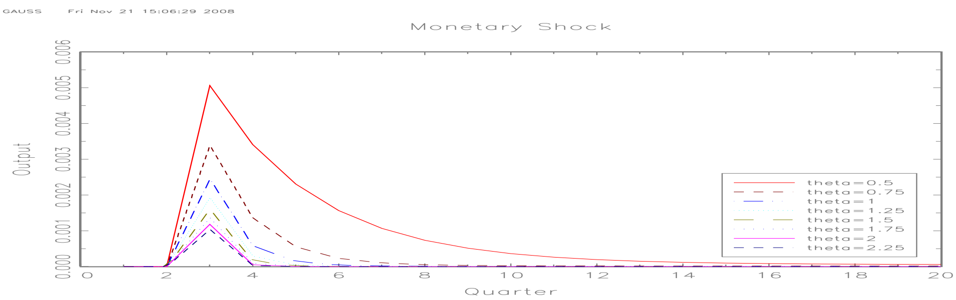


Figure III.34: Adjustment Costs of Capital Model. Impulse responses to a monetary shock, $mu = 1.2$, $\alpha = 0.5$, $\rho_{\tau} = 0$. Relative deviations from steady state.

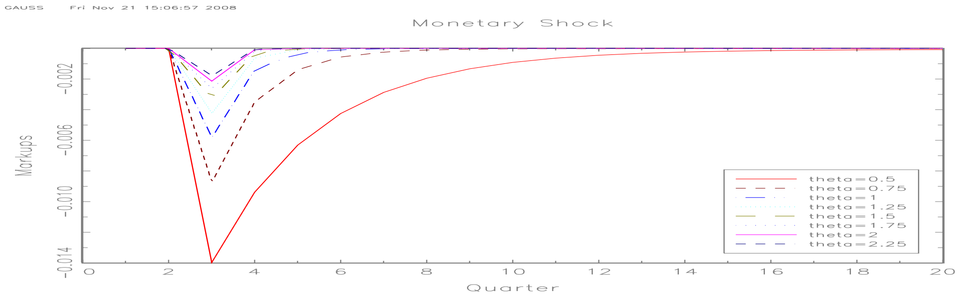


Figure III.35: Adjustment Costs of Capital Model. Impulse responses to a monetary shock, $\theta = 0.8$, $\alpha = 0.5$, $\rho_{\tau} = 0$. Relative deviations from steady state.

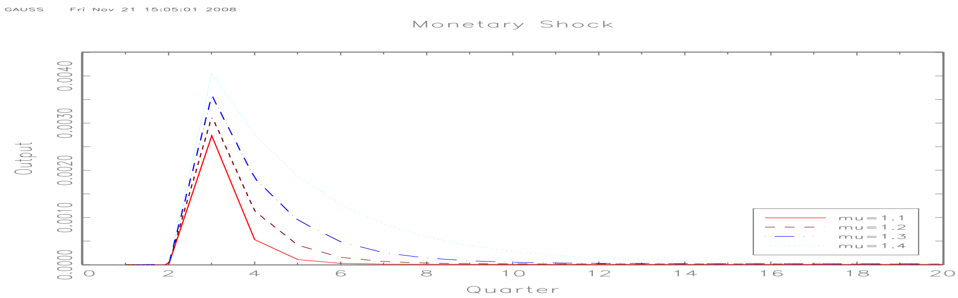


Figure III.36: Adjustment Costs of Capital Model. Impulse responses to a monetary shock, $\theta = 0.8$, $\alpha = 0.5$, $\rho_{\tau} = 0$. Relative deviations from steady state.

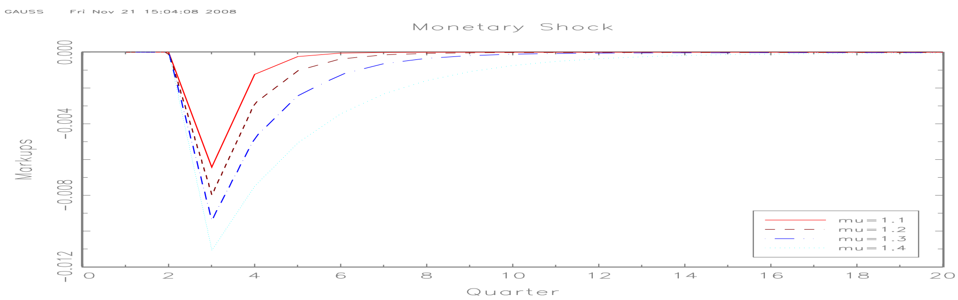


Figure III.37: Adjustment Costs of Capital Model. Impulse responses to a technology shock, $mu = 1.2$, $\alpha = 0.5$, $\rho_z = 0$. Relative deviations from steady state.

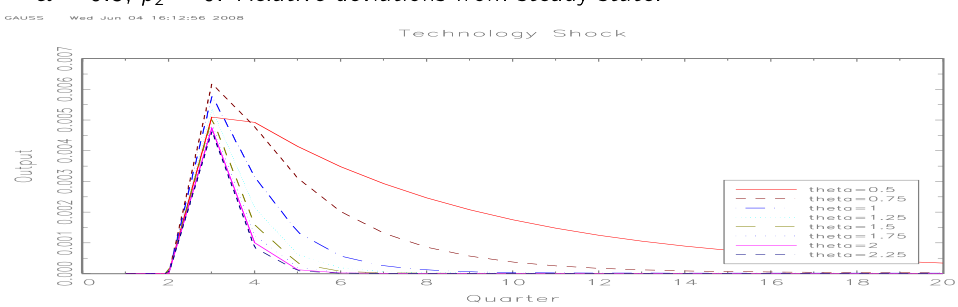


Figure III.38: Adjustment Costs of Capital Model. Impulse responses to a technology shock, $mu = 1.2$, $\alpha = 0.5$, $\rho_z = 0$. Relative deviations from steady state.

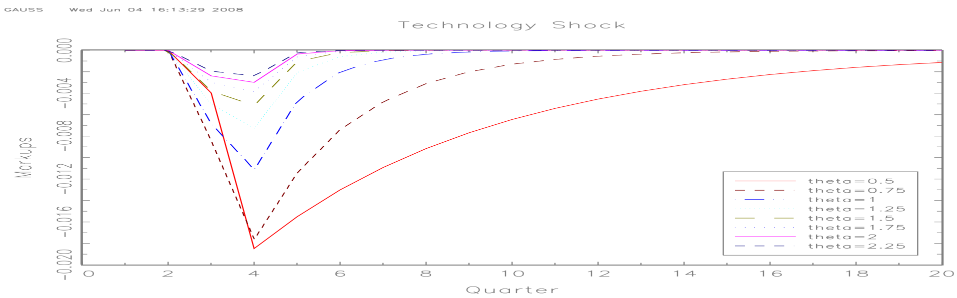


Figure III.39: Adjustment Costs of Capital Model. Impulse responses to a technology shock, $mu = 1.2$, $\theta = 0.8$, $\rho_z = 0$. Relative deviations from steady state.

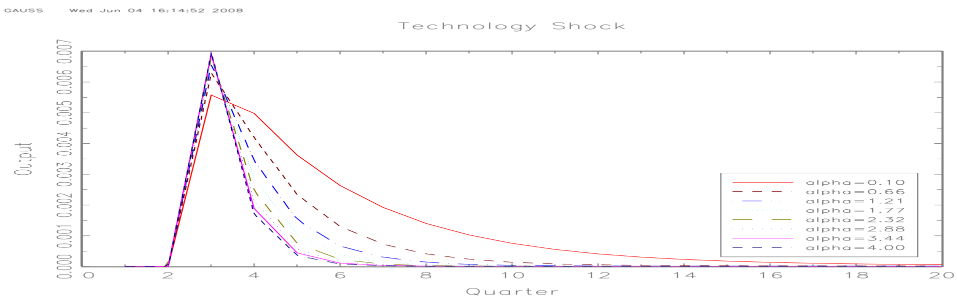


Figure III.40: Adjustment Costs of Capital Model. Impulse responses to a technology shock, $mu = 1.2$, $\theta = 0.8$, $\rho_z = 0$. Relative deviations from steady state.

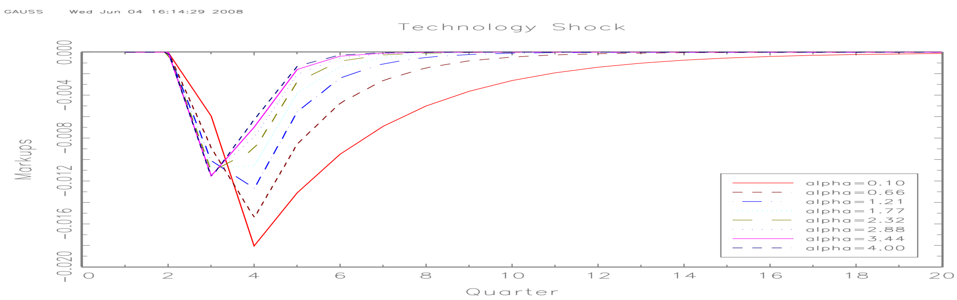


Figure III.41: Adjustment Costs of Capital Model. Impulse responses to a technology shock, $\alpha = 0.5$, $\theta = 0.8$, $\rho_z = 0$. Relative deviations from steady state.

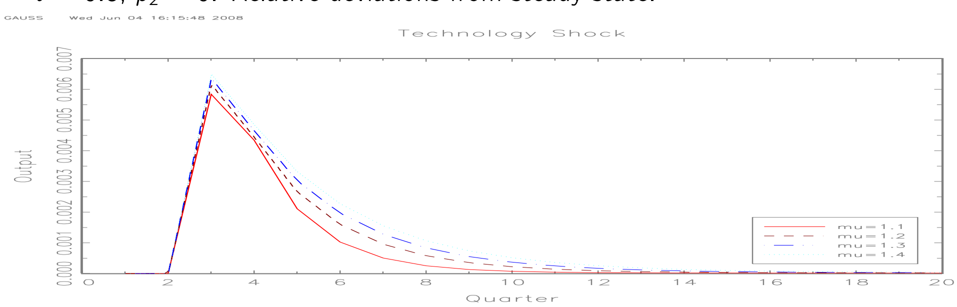
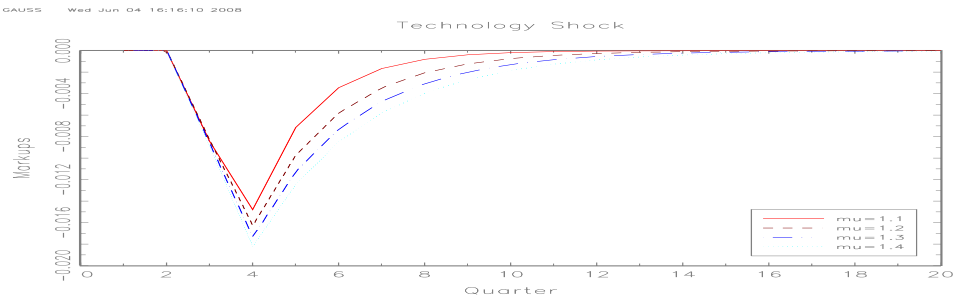


Figure III.42: Adjustment Costs of Capital Model. Impulse responses to a technology shock, $\alpha = 0.5$, $\theta = 0.8$, $\rho_z = 0$. Relative deviations from steady state.



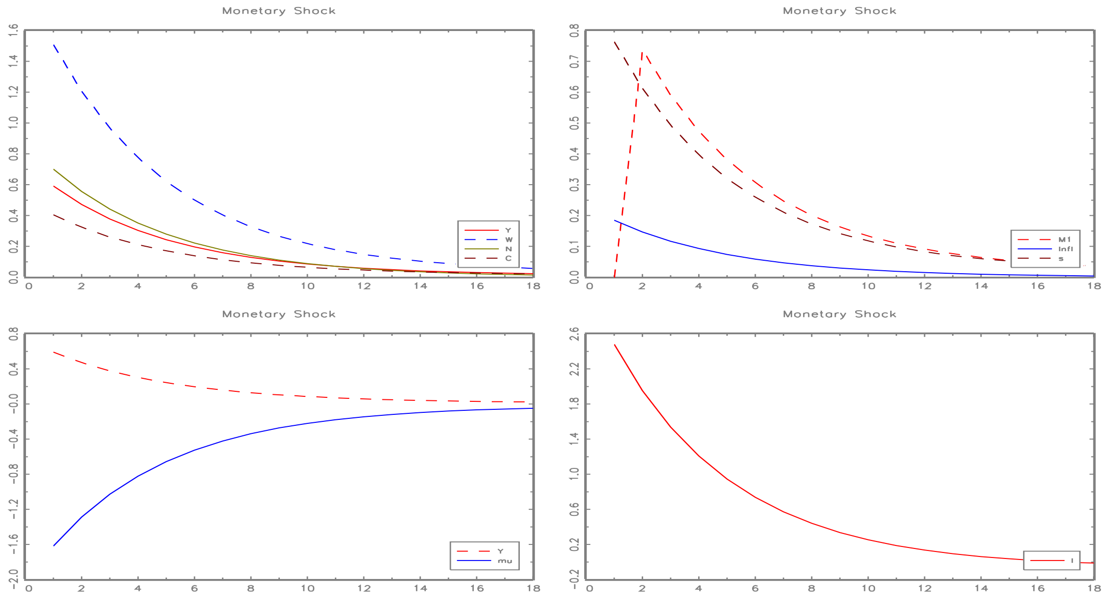
5 A Comparison with the New Keynesian Model

Let us now compare the model developed in the current section with the New Keynesian model with adjustment costs of capital accumulation.[22]. To bias the results towards a better performance of the New Keynesian model, I assume that that the fraction of firms which do not adjust their prices φ equals 0.75. The resulting impulse responses to a non-autocorrelated monetary shock are shown in figure III.44. The peak-response of output is 0.4% which is less than the point estimate of the peak-response of the same variable (0.6%) provided by Christiano *et al.* (2005). The peak-response of inflation predicted by the New Keynesian model, 0.3%, is also higher than Christiano *et al.*'s point estimate, 0.2%. Furthermore the New Keynesian model implies that all variables return to their respective long-run values after about 7 to 8 quarters as opposed to 12 (or even more) quarters estimated by Christiano *et al.* (2005). In contrast, as figure III.43 shows, in the model presented in this section it is easy to find a plausible combination of α, θ and mu^* which implies a peak-response and a persistence of output and inflation exactly identical with that estimated by Christiano *et al.* (2005). However, the two models share a common shortcoming - they both overstate the reactions of the real wage and investment to monetary disturbances.

In sum, there are parameter values for which the model developed in this chapter performs better than or at least as good as the New Keynesian model. Therefore, the former could be considered a useful alternative to the latter for analyzing business cycles phenomena and advising policy makers. As pointed in this as well as in previous sections there are empirical facts (the volatility of wages, the shape of the impulse responses) which can not be reconciled with the model constructed here. The literature provides various suggestions for possible modifications which may improve the performance of the model. They are left for future research. In the next chapter I turn to the estimation of one of the most important parameters of the model - α.

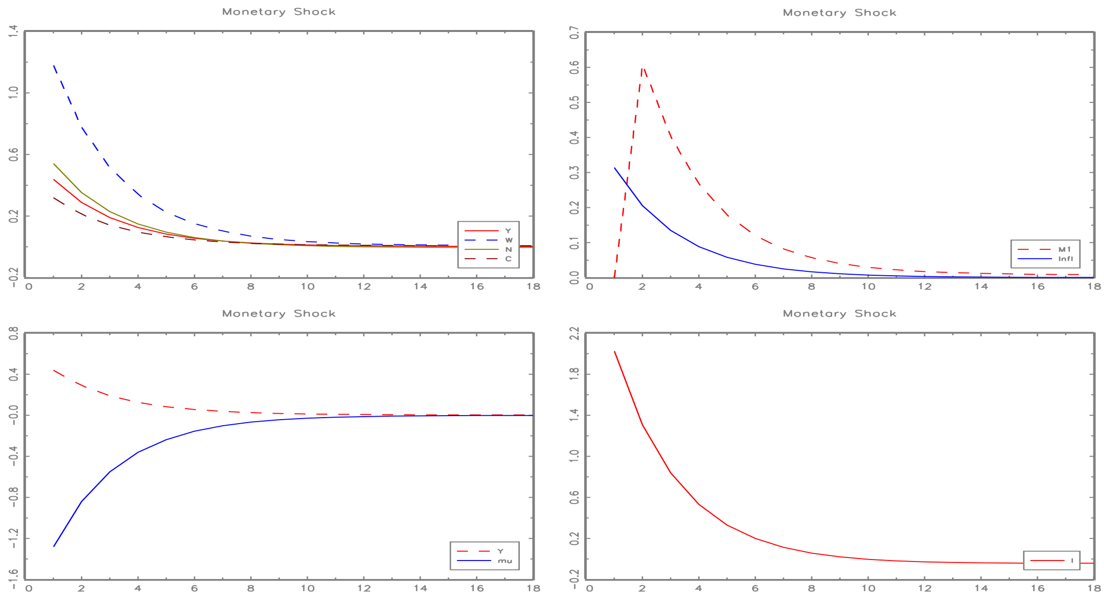
[22]The programs used are **"sim_cm2d6a_1cap_i.g"** and **"keynes_ac_as.g"**

Figure III.43: Adjustment Costs of Capital Model. Impulse responses to a monetary shock, $\alpha = 0.3$, $\theta = 0.5$, $mu^{*} = 1.25$, $\rho_z = 0$, $\rho_\tau = 0$. Percentage deviations from steady state.



Percentage deviations from the long run mean. Y-Output, C-Consumption, N-Hours, W-Real Wage, S-Search Effort, M_1-Real Balances, $Infl$-Inflation, $Prof$-Profits.

Figure III.44: New Keynesian Model with Adjustment Costs of Capital. Impulse responses to a monetary shock, $\varphi = 0.75$, $\rho_z = 0$, $\rho\tau = 0$. Percentage deviations from steady state.



Percentage deviations from the long run mean. Y-Output, C-Consumption, N-Hours, W-Real Wage, M_1-Real Balances, $Infl$-Inflation, $Prof$-Profits.

6 Supplement to Section 3. Understanding Key Features of the Model

In this section I investigate the role and the importance of the new building blocks in the following way. First, I look at each of them separately and provide detailed interpretation of the underlying economic mechanisms. Then I analyze the interaction between the various assumptions by starting with a very simple model including neither market share competition nor any form of search activity and then stepwise extending it by introducing new building blocks, one at a time.

6.1 Only Search Activity

Search Activity Depends Only on Current Consumption:

Let us call this version of the model the *OCC Model*.[23] In this version of the model it is assumed that the market share of an arbitrary firm evolves according to the same difference equation

$$x_{i,t+1} = \exp\left(\left(1 - \frac{P_{i,t}}{P_t}\right) \cdot s_t\right) \cdot x_{i,t}$$

as in section 3 but now firms take this law of motion as exogenously given and thus, do not take account of it when deciding on their respective optimal prices. Hence, each firm solves a purely static optimization problem leading to the purely static rule for optimal pricing given below. At the same time households do take the dependence between $x_{i,t+1}$ and their search effort s_t into consideration. It is easily recognized that this version of the model is identical with a flexible price money-in-utility model in which the utility function is additively separable with respect to consumption and real balances. Nevertheless, this model provides a simple framework for highlighting some important economic mechanisms in a tractable way.

The impulse responses to a purely temporary technology shock, with a no serial correlation, $\rho_z = 0$, are displayed in figure III.45.[24] All variables except inflation react purely temporary returning to their long run levels in the period after the shock. The responses of output, consumption, profits and real wages are positive, whereas that of hours negative. Only the response of inflation lasts for two periods. There is a sharp decline of the price level on impact followed by an increase in the next period. To get some intuition about the economic forces driving these impulse responses, note that after inserting the optimality condition for labor

[23] OCC-stands for Only Current Consumption.

[24] The corresponding program is **"sim_cm2d8_1a.g"**. The model is calibrated in the same way as in section 3.

supply into the household's budget restriction, one gets the following consumption function on the individual household's level

$$\left(\frac{c}{y}+\eta\frac{\theta-1}{\theta}+\frac{s}{y}\frac{\eta}{1+\alpha}\right)\hat{C}_t=2\frac{\theta-1}{\theta}\left(\widehat{\frac{W_t}{P_t}}\right)+\frac{1}{\theta}\widehat{Div}_t-\frac{m}{y}\left(\hat{\pi}_t+\hat{m}_{t+1}\right)+\frac{m}{y}\hat{m}_t, \quad \text{(III.6.1)}$$

where $c/y, s/y$, and m/y are the steady state values of the consumption-output, search-output and real balances-output ratios respectively. Div_t denotes dividend payments. All *"hat"*-variables denote percentage deviations from the respective values at the stationary equilibrium. The log-linear approximation of the optimality condition with respect to money demand reads[25]

$$\chi\hat{m}_{t+1}=\frac{\eta}{1-\beta}\hat{C}_t-\eta\frac{\beta}{1-\beta}E_t\hat{C}_{t+1}-\left(\frac{\beta}{1-\beta}+1-\chi\right)E_t\hat{\pi}_{t+1}. \quad \text{(III.6.2)}$$

In a symmetric equilibrium every firm sets its price according to the rule:

$$P_t=\frac{\theta}{\theta-1}\cdot\frac{W_t}{Z_t}, \quad \text{(III.6.3)}$$

or equivalently

$$\frac{W_t}{P_t}=\frac{\theta-1}{\theta}\cdot Z_t.$$

For a given nominal wage a positive technology shock reduces nominal marginal costs[26] making it optimal for every individual firm to charge a lower price. As a consequence there is a tendency for real wages to rise and for inflation to fall. As can be seen from equation (III.6.1), for given dividend payments $\widehat{Div}_t$ and money demand $\hat{m}_{t+1}$ both price changes create an incentive for the private household to increase her consumption expenditure and hence intensify search activity. As long as η is greater than $\frac{c}{y}+\frac{s}{y}\cdot\frac{\eta}{1+\alpha}$, the income effect of higher real wages on leisure is stronger than the corresponding *intratemporal* substitution effect between consumption and leisure implying a lower labor supply for any given real wage.[27] Since there are identical households, aggregate labor supply declines, generating additional (further) pressure on real wages. Note that the negative effect of declining labor supply on consumption was already substituted out in equation (III.6.1). The static monopolistic competitive structure of the goods market implies that current profits are directly proportional to output and thus to aggregate demand. Hence, if each household increases her consumption expenditure as well

[25] π^* is set to one for simplicity.

[26] Marginal costs are given by W_t/Z_t.

[27] If η is high enough, the decline in the marginal utility of consumption induced by an increase of the real wage is stronger than the increase of the real wage itself. As a consequence the utility gain of a marginal increase of hours worked gets lower than the corresponding utility loss, creating an incentive for households to lower labor supply. The utility gain of an additional unit of labor supply is given by $(W_t/P_t)\cdot C_t^{-\eta}$. The utility loss is given by N_t.

as her switching effort, aggregate profits will rise, forcing individual consumption (labor supply) to rise (fall) even more. As long as the elasticity of the marginal disutility of hours worked with respect to hours is positive[28] the decline of labor supply is weaker than the increase of total factor productivity, implying that equilibrium output also becomes higher. Since the technology shock is purely temporary and there is no capital accumulation, households anticipate lower future labor and dividend income. Therefore, in order to smooth consumption, each household will try to transfer some part of her additional current period income into the future. There are two ways she can do that - by investing in bonds or by building up real balances. Since all households are identical and the aggregate supply of one period riskless bonds is zero, the desire to smooth consumption by buying bonds results in a sharp increase of the real interest rate, just sufficient to discourage any incentives to invest in (or sell) bonds. The accumulation of real balances provides a second channel for intertemporal substitution of consumption. According to equation (III.6.2) the demand for real money holdings depends positively on the gap between current and next-period consumption. The larger the gap, the stronger the incentive to transfer resources from the current into future periods by increasing money holdings. A higher expected next-period inflation implies higher opportunity costs of holding money, reducing the demand for real balances. Since each household will try to increase her money holdings and since the nominal money supply remains unchanged, the current price level P_t should fall or expected inflation $E_t(\hat{\pi}_{t+1})$ should rise, or both. Intuitively speaking, P_t should fall relative to $E_t(P_{t+1})$ in order to encourage current and lower desired next-period consumption, and hence discourage money demand. More formally, a decline of P_t is needed for real money supply to increase. A rise in $E_t(\pi_{t+1})$ is needed for money demand to be weakened. If both effects are sufficiently strong, for given C_t and $E_t(C_{t+1})$ the money market will remain (stay) in equilibrium.

But how can expected future inflation $E_t(\pi_{t+1})$ rise though the money supply M_{t+2} and the level of technology Z_{t+1} are expected to remain unchanged? To answer this question, observe that, if households were able to accumulate additional *real* money holdings in the current period and at the same time there would be no inflation in $t+1$, $\pi_{t+1} = 0$, next-period consumption demand and switching effort will tend to be higher due to the positive wealth effect induced by the additionally accumulated real balances. Since aggregate output is expected to remain constant, there will be a disequilibrium in the goods market, with demand being higher than supply, putting *positive* pressure on the price level P_{t+1}. Households will expect that the increase of next-period prices P_{t+1} will be just sufficient to offset the positive wealth effect of the additional money holdings and thus restore equilibrium on the goods market. In other words households will expect next-period inflation to increase by $\hat{m}_{t+1}$

[28] It is equal to one in our case.

because

$$E_t\left(\frac{\hat{M}_{t+1}}{P_{t+1}}\right) = E_t\left(\frac{\hat{m}_{t+1}}{\pi_{t+1}}\right) = 0$$

implies

$$\hat{m}_{t+1} = E_t(\hat{\pi}_{t+1}).$$

It turns out that the increase in expected inflation $E_t(\hat{\pi}_{t+1})$ is noth strong enough, to make it optimal for households to leave their real money holdings in period t unchanged. To see this note that each household builds expectations about next-period inflation by exploring the general equilibrium structure of the economy.[29] First, she will realize that aggregate consumption depends only upon total factor productivity,

$$\hat{C}_t = \frac{2}{\frac{c}{y} + \frac{s}{y}\frac{\eta}{1+\alpha} + \eta} \cdot \hat{Z}_t, \tag{III.6.4}$$

and hence, being informed about the stochastic structure of Z_t, will expect that the deviation of aggregate consumption from its steady state level will be zero. Then, after aggregating the money demand equation (III.6.2) over all households and eliminating expected inflation by inserting the low of motion for money supply, she will arrive at the following relation between $\hat{m}_{t+1}$, $E_t(\hat{m}_{t+2})$ and $\hat{C}_t$:

$$\hat{m}_{t+1} = \frac{\eta}{\chi(1-\beta)(1+\varsigma)} \cdot \hat{C}_t + \frac{\varsigma}{1+\varsigma} \cdot E_t(\hat{m}_{t+2}),$$

where ς is given by $(1/\chi)\cdot(\beta/(1-\beta)+1-\chi)$ which is positive for $\chi \geq 1$ and $\beta \in \left(\frac{\chi-1}{\chi}, 1\right)$. By solving this equation forward the household will arrive at

$$\hat{m}_{t+1} = \frac{\eta}{\chi(1-\beta)(1+\varsigma)} \cdot \hat{C}_t.$$

By inserting this result into the low of motion for money supply and taking into account that $E_t(\hat{C}_{t+1}) = 0$ and $\hat{m}_t = 0$ she will conclude that

$$E_t(\hat{m}_{t+2}) = 0$$

and hence

$$E_t(\hat{\pi}_{t+1}) = \frac{\eta}{\chi(1-\beta)(1+\varsigma)} \cdot \hat{C}_t.$$

Inserting this result into the optimality condition for money demand and then aggregating over all households implies:

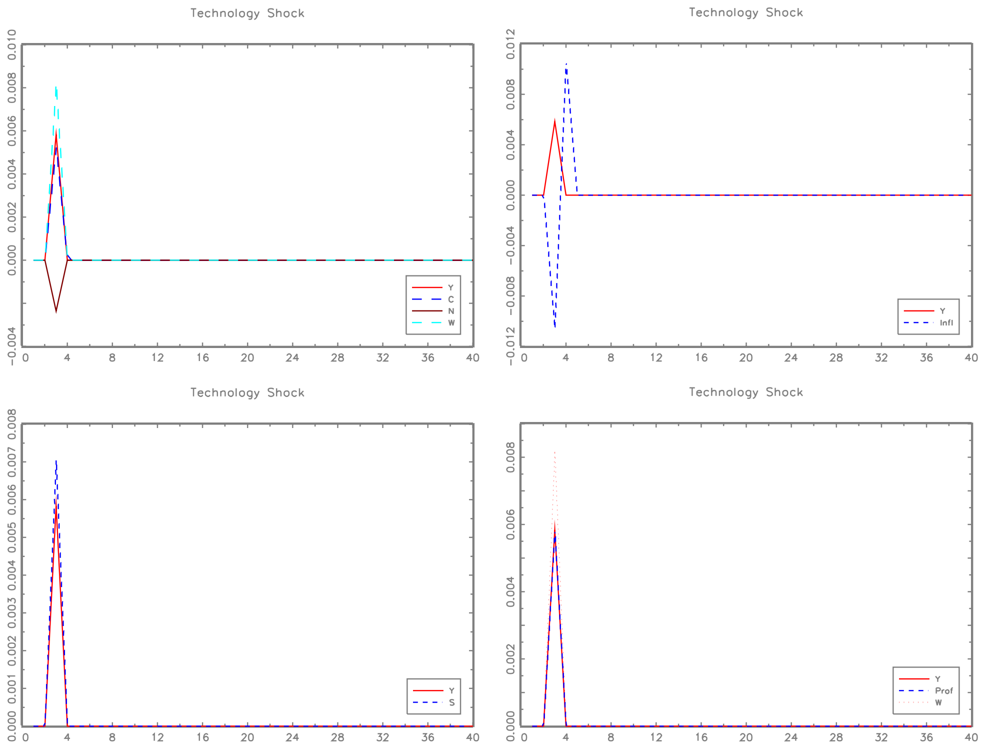
$$\hat{m}_{t+1} = \frac{\eta}{\chi(1-\beta)(1+\varsigma)} \cdot \hat{C}_t =$$

$$= \underbrace{\frac{2\eta}{\left(\frac{c}{y} + \frac{s}{y}\cdot\frac{\eta}{1+\alpha} + \eta\right)\chi(1-\beta)(1+\varsigma)}}_{\tilde{o}_1} \cdot \hat{Z}_t.$$

[29] It is assumed that all agents have *rational expectations*.

Hence, as long as $\chi \geq 1$ and $\beta \in \left(\frac{\chi-1}{\chi}, 1\right)$ the relation between real balances and the state of technology, both measured as percentage deviations from their respective steady state levels, is positive. The latter implies that the weighted sum of the increase of expected future consumption and inflation on the rhs of (III.6.2), is not sufficient to offset the positive effect of current consumption expenditure on real money demand. Therefore, as a reaction to a positive one-time technology shock, desired real balances $\hat{m}_{t+1}$ increase. To make that possible, the current price level should fall.

Figure III.45: Impulse Responses to a Technology Shock in $t = 3$, $\alpha = 0.5$, $\rho_z = 0$. Relative deviations from steady state.



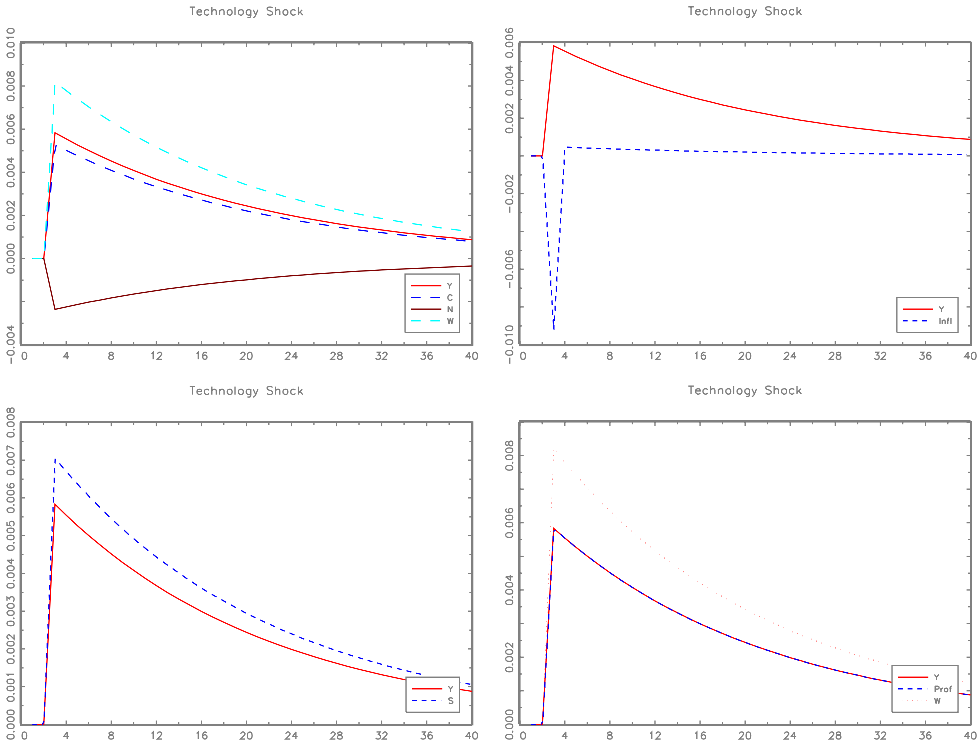
Percentage deviations from the long run mean. Y-Output, C-Consumption, N-Hours, W-Real Wage, S-Search Effort, M_1-Real Balances, $Infl$-Inflation, $Prof$-Profits. Search activity depends only on current consumption.

If the technology shock exhibits a positive autocorrelation, $\rho_z \in (0, 1)$, the reactions of all variables are more persistent than in the previous version of the model, see figure III.46. In this case the impulse response function of money demand takes the following form:

$$\hat{m}_{t+1} = \underbrace{\frac{1+\varsigma}{1+(1-\rho_z)\varsigma} \cdot \frac{2\eta(1-\rho_z\beta)}{\left(\frac{c}{y} + \frac{s}{y} \cdot \frac{\eta}{1+\alpha} + \eta\right)\chi(1-\beta)(1+\varsigma)}}_{:=\tilde{o}_2} \cdot \hat{Z}_t.$$

It is easy to show that the coefficient $\tilde{o}_2$ is positive but smaller than $\tilde{o}_1$. The reason is that with a positively autocorrelated level of technology, after a positive technology innovation in the current period, next-period income and thus next period consumption are also expected to be above average, which in turn implies a weaker incentive for additional consumption smoothing

Figure III.46: Impulse Responses to a Technology Shock in $t = 3$, $\alpha = 0.5$, $\rho_z = 0.95$. Relative deviations from steady state.



Percentage deviations from the long run mean. Y-Output, C-Consumption, N-Hours, W-Real Wage, S-Search Effort, M_1-Real Balances, $Infl$-Inflation, $Prof$-Profits. Search activity depends only on current consumption.

and hence, a weaker incentive to accumulate additional money holdings. As a consequence, the initial reaction of inflation is also weaker than in the "$\rho_z = 0$"-case. Furthermore, as a consequence of the positive autocorrelation of $\hat{Z}_t$, real balances do not return immediately after the shock to their long run level, but remain (for a long time) positive and converge asymptotically from above to their steady state value. Expected inflation is again higher than average because each household is expected to spend not only her current income but also a portion of the additionally accumulated money balances on goods and search efforts. Since, as explained above, households won't try to get rid off all the additional real money holdings, expected and actual next-period inflation, do not rise as much as in the "$\rho_z = 0$"-case. Actual inflation evolves according to

$$\hat{\pi}_{t+1} = (1 - \rho_z) \cdot \tilde{o}_2 \cdot \hat{Z}_t$$

and approaches asymptotically its steady state level from above.[30] Because of the static structure of the real part of the economy, the explanation for the impulse responses of output, consumption, hours, real wages and profits displayed in figure III.46 is the same as in the "$\rho_z = 0$"-case.

[30] In this model $(1 - \rho_z)$ equals 0.05.

Only Current Inflation

In the model version discussed in this subsection it is assumed that search effort reduces the disutility of inflation. The optimal amount of search activity is then given by an increasing function of inflation, taking the following form

$$\varrho s_t = (\pi_t)^{\frac{1}{1+\alpha}},$$

where $\alpha = 1$. As in the previous two sections, firms do not take into account the difference equation describing the evolution of market share.

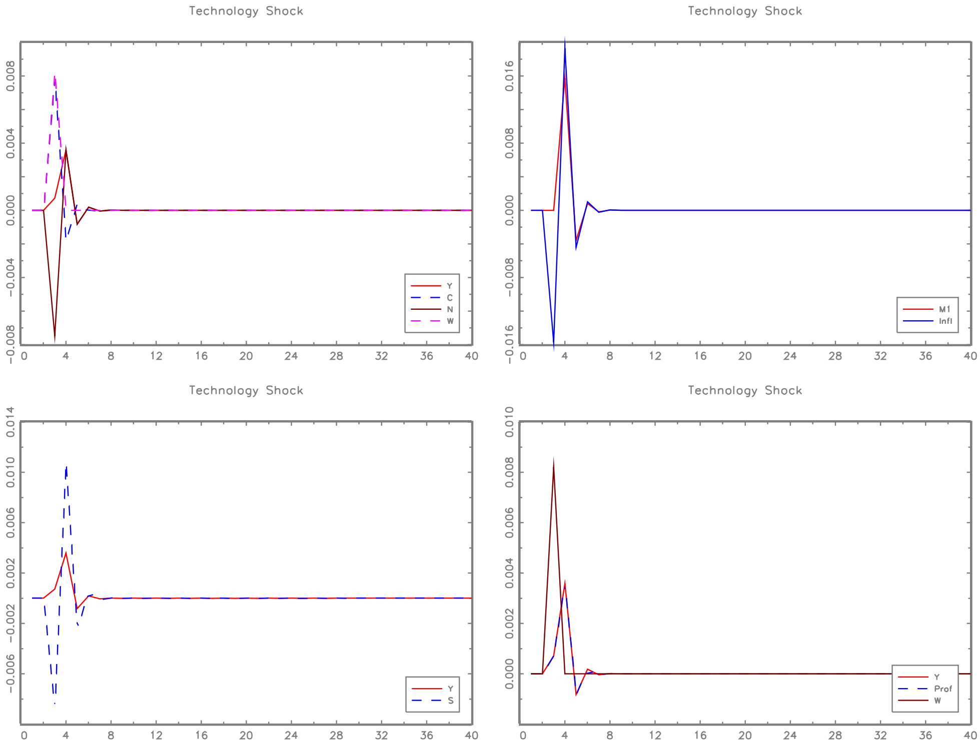
Technology Shocks: The impulse responses to a one-time technology shock, the $\rho_z = 0$ case, are displayed in figure III.47. The easiest way to gain some intuition about the economic forces leading to them is to compare the present model, to which I refer as the *OCI model*,[31] with the OCC model. If there were no dependence of search activity on inflation the economic forces at work in the OCI model would be exactly the same as in the OCC model. The decline in the current price level induced by the technology shock reduces search activity, making it possible (optimal) to increase consumption by more than in the OCC model for any given increase (decrease) in current income (inflation). In the OCC model there was a positive reaction of search activity to favorable technology shocks arising via the dependence of search on current consumption in that model. Thus, the reaction of search in the OCC model dampened that of consumption. In the OCI model a positive technology innovation has a negative impact on search effort via the link between s_t and π_t and strengthens the positive income effects already at work. The stronger reaction of current consumption leads to a more pronounced fall in working hours. In the period after the shock inflation is higher than average making it optimal to intensify search activity. The resulting negative income effect dampens consumption expenditure and increases labor supply and output. When the level of technology follows an AR(1) process with a positive coefficient of autocorrelation the main effects in the period of the shock and its immediate aftermath remain the same as in the "$\rho_z = 0$" case. The only thing that changes is that the impulse responses get more persistent replicating closely the autocorrelation structure of the shock, see figure III.48.

Monetary Shocks: The impulse responses to an unautocorrelated monetary shock are shown in figure III.49.[32] By temporary accelerating the growth of money supply the central bank induces a positive income effect which leads to a higher desired consumption as well as money demand and therefore to a lower desired labor supply. As the latter would

[31] OCI - Only Current Inflation.

[32] The corresponding program is **"sim_cm2d9_1a.g"**.

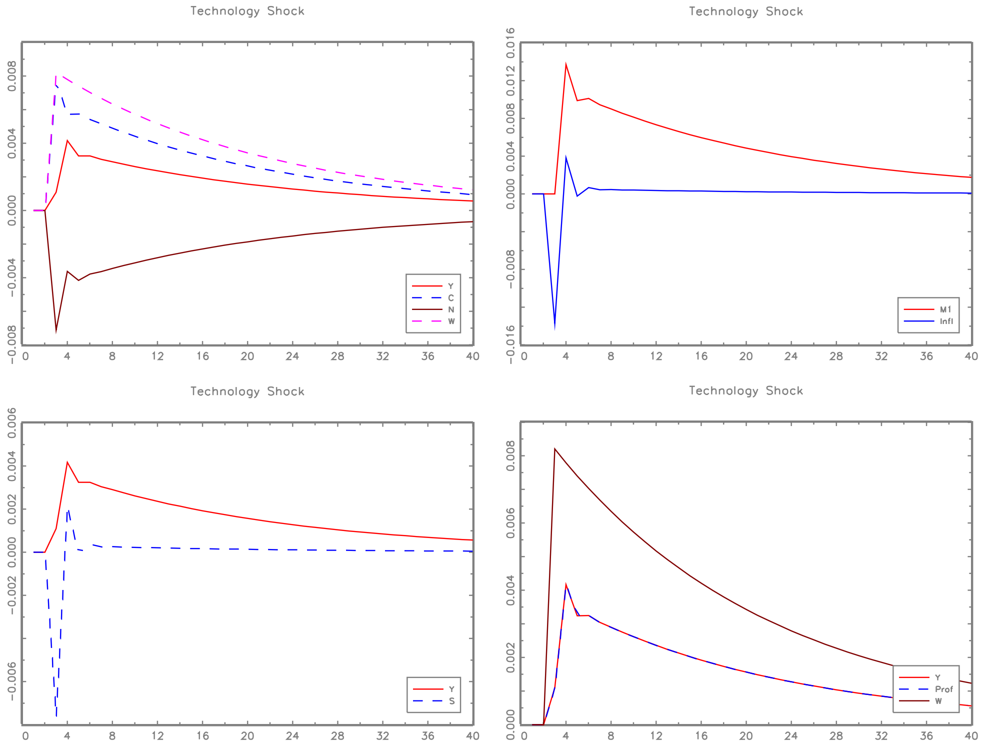
Figure III.47: Impulse Responses to a Technology Shock in $t = 3$, $\alpha = 0.8$, $\rho_z = 0$. Relative deviations from steady state.



Y-Output, C-Consumption, N-Hours, W-Real Wage, S-Search Effort, M_1-Real Balances, $Infl$-Inflation, $Prof$-Profits. Search activity depends only on current inflation.

have a negative effect on aggregate output and thus lead to a disequilibrium on the goods market, the price level rises. The price increase induces a negative wealth effect on current consumption by lowering the real value of money holdings. At the same time, for any given expected next period inflation, it discourages current in favor of future consumption and so lowers the demand for real balances. In the OCI model there is also a further transmission channel by which monetary policy can influence private agents' behavior: The increase in current inflation makes it optimal for households to allocate a larger part of their income to search effort. To achieve that, they must lower consumption demand. The higher the elasticity of search activity s_t with respect to current inflation, the larger the desired rise in s_t for any given increase in inflation and thus the lower the fraction of income that can be allocated to consumption. The fall in consumption has a positive effect on aggregate labor supply and production. Note that the whole additional as well as part of average output are absorbed by the positive deviation of search from its steady state level. To understand the behavior of real money holdings observe that for given expected next-period inflation and consumption a decline in current consumption expenditure weakens the incentive to transfer additional resources from the present into the future by investing in money. So the demand for money tends to fall leading to a negative wealth effect on consumption in the next period. Therefore, for a given level of production next period inflation will tend to fall. Households

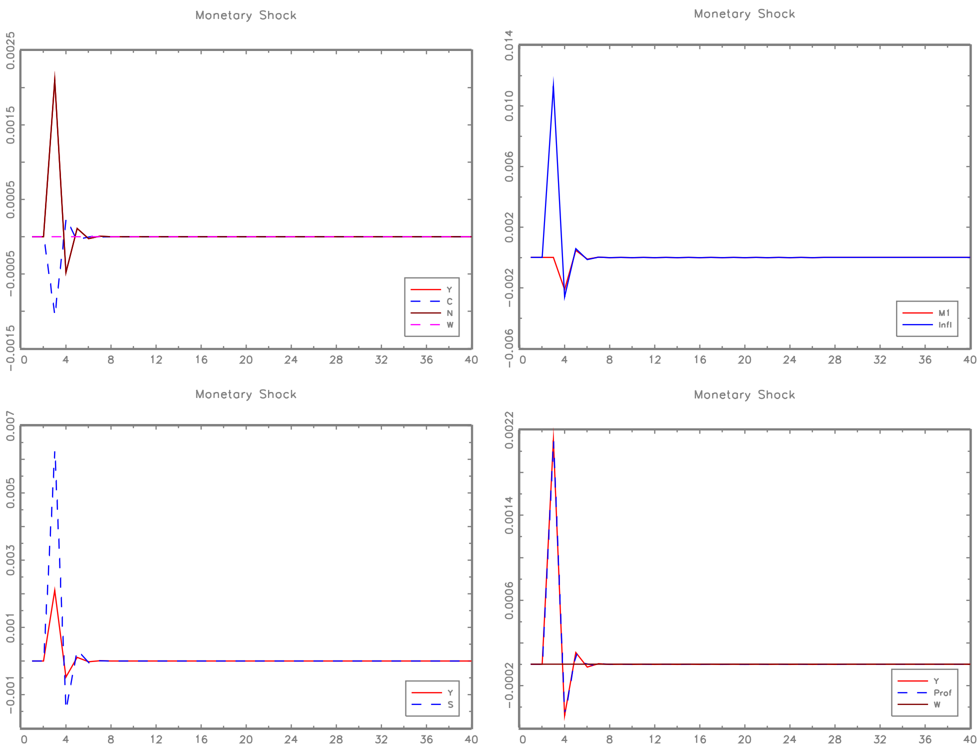
Figure III.48: Impulse Responses to a Technology Shock in $t = 3$, $\alpha = 0.8$, $\rho_z = 0.95$. Relative deviations from steady state.



Y-Output, C-Consumption, N-Hours, W-Real Wage, S-Search Effort, M_1-Real Balances, $Infl$-Inflation, $Prof$-Profits. Search activity depends only on current inflation.

are again able to compute their *rational* forecasts with respect to next period inflation by exploring the underlying structure of the economy. Under the chosen parametrization, for every α in the interval [0.1;4], expected next-period inflation falls. Hence, search activity in the period after the shock will be below average allowing for a higher consumption expenditure for any given level of real balances as well as real wage and dividend income. Both, a lower expected inflation $E_t(\hat{\pi}_{t+1})$ and a higher expected consumption $E_t(\hat{C}_{t+1})$ have a negative effect on the demand for real balances $\hat{m}_{t+1}$. As can be seen in figure III.49, the dependence of search activity on inflation is not sufficient to make the impulse responses of output, employment and consumption persistent, nor are they U-shaped as the VAR-evidence suggests. The effect of the monetary shock lasts for no more than 4 periods with all variables converging cyclically to their respective steady state levels. The damped oscillations result from the negative coefficient of autocorrelation in the feed-back rule for real balances $\hat{m}_t$ (see paragraph *Some Formal Details*").

Figure III.49: Impulse Responses to a Monetary Shock in $t = 3$, $\alpha = 0.8$. Relative deviations from steady state.



Y-Output, C-Consumption, N-Hours, W-Real Wage, S-Search Effort, M_1-Real Balances, $Infl$-Inflation, $Prof$-Profits. Search activity depends only on current inflation.

Some Formal Details: First note that the deviation of current consumption from its steady state value on the individual level evolves according to

$$\left(\frac{c}{y} + \eta\frac{\theta-1}{\theta}\right)\hat{C}_t = 2\hat{Z}_t - \frac{m}{y}(\hat{\pi}_t + \hat{m}_{t+1}) + \frac{m}{y}\hat{m}_t - \frac{s}{y}\underbrace{\frac{1}{1+\alpha}\hat{\pi}_t}_{:=\hat{s}_t}. \tag{III.6.5}$$

The last equation is obtained after inserting the optimality condition for labor supply and the definition of profits into the budget constraint of the household. If there are only monetary shocks one obtains from (III.6.6) the following relation between consumption and inflation on the aggregate level

$$\hat{C}_t = -\underbrace{\frac{\frac{s}{y}\frac{1}{1+\alpha}}{\frac{c}{y} + \eta\frac{\theta-1}{\theta}}}_{:=\xi}\hat{\pi}_t. \tag{III.6.6}$$

Hence, for expected next-period consumption one obtains:

$$E_t\hat{C}_{t+1} = -\xi E_t\hat{\pi}_{t+1}. \tag{III.6.7}$$

The log-linearized version of the money demand equation reads[33]

$$\hat{m}_{t+1} = \frac{\eta}{\chi(1-\beta)}\hat{C}_t - \frac{\eta\beta}{\chi(1-\beta)}E_t\hat{C}_{t+1} + \varsigma(E_t\hat{m}_{t+2} - \hat{m}_{t+1}). \tag{III.6.8}$$

[33] ς is defined as in the previous subsection.

Inserting of (III.6.6) and (III.6.7) as well as the law of motion of money supply

$$\hat{m}_{t+1} = \hat{m}_t - \hat{\pi}_t + \hat{\mu}_t$$

into (III.6.8) yields:

$$\hat{m}_{t+1} = \frac{\varsigma}{1+\varsigma} E_t \hat{m}_{t+2} - \underbrace{\frac{\eta}{\chi(1-\beta)} \cdot \frac{\xi}{1+\varsigma}}_{\xi_1} \hat{\pi}_t + \underbrace{\frac{\eta\beta}{\chi(1-\beta)} \cdot \frac{\xi}{1+\varsigma}}_{\xi_2} E_t \hat{\pi}_{t+1}.$$

Substituting $\hat{\pi}_t$ and $E_t\hat{\pi}_{t+1}$ away by the money supply equation and rearranging yields:

$$(1-\beta)(\chi(1+\varsigma)+\eta\xi)\hat{m}_{t+1} = (\varsigma\chi(1-\beta)-\eta\beta\xi)E_t\hat{m}_{t+2} - \eta\xi\hat{m}_t - \eta\xi\hat{\mu}_t. \qquad \text{(III.6.9)}$$

For

$$\frac{1}{1+\alpha} < \frac{(\beta-(\chi-1)(1-\beta))(\frac{c}{y}+\eta\frac{\theta-1}{\theta})}{\eta\beta\frac{s}{y}}$$

$\varsigma\chi(1-\beta)-\eta\beta\xi$ is positive and smaller than $(1-\beta)(\chi(1+\varsigma)+\eta\xi)$. For the parametrization chosen the latter inequality implies $\frac{1}{1+\alpha} < 2.15$, which is always the case for $\alpha > 0$. The easiest and at the same time the most general way to solve equation (III.6.9) is by using the *method of undetermined coefficients*. To start the procedure one can guess a solution of the form

$$\hat{m}_{t+1} = a\hat{m}_t + b\hat{\mu}_t.$$

Please note that this so called autoregressive solution is by no means the only one for equation (III.6.9). There is also a solution of the form $\hat{m}_{t+1} = c\hat{\mu}_t$ and also many others. The autoregressive solution proposed here belongs to the set of solutions consistent with the assumption that $\hat{m}_t$ is stationary (that the intertemporal budget restriction of the representative household holds as an equation). Inserting the last equation into (III.6.9) and equating coefficients yields:

$$a_{1,2} = \frac{(1-\beta)(\chi(1+\varsigma)+\eta\xi) \pm \sqrt{(1-\beta)^2(\chi(1+\varsigma)+\eta\xi)^2 + 4\eta\xi(\varsigma\chi(1-\beta)-\eta\beta\xi)}}{2(\varsigma\chi(1-\beta)-\eta\beta\xi)}.$$

Only the solution

$$a = \frac{(1-\beta)(\chi(1+\varsigma)+\eta\xi) - \sqrt{(1-\beta)^2(\chi(1+\varsigma)+\eta\xi)^2 + 4\eta\xi(\varsigma\chi(1-\beta)-\eta\beta\xi)}}{2(\varsigma\chi(1-\beta)-\eta\beta\xi)} < 0$$

is consistent with the assumption that $\hat{m}_t$ is stationary. The parametrization used in this model implies $a \in [-0.827, 0)$. For that values of a, b is negative:

$$b = -\frac{\eta\xi}{(1-\beta)(\chi(1+\varsigma)+\eta\xi) + |a|(\varsigma\chi(1-\beta)-\eta\beta\xi)} < 0.$$

Hence, the initial response of real money holdings to a positive monetary disturbance $\hat{\mu}_t > 0$ is given by

$$\hat{m}_{t+1} = b\hat{\mu}_t < 0,$$

where $\hat{m}_t = 0$ was taken into account. According to the law of motion for money supply, inflation in $t+1$ will be given by

$$\hat{\pi}_{t+1} = \hat{m}_{t+1} - \hat{m}_{t+2} + \hat{\mu}_{t+1}.$$

Inserting the obtained feed-back rule for $\hat{m}_t$ into this equation and taking expectations as of period t yields

$$E_t\hat{\pi}_{t+1} = \underbrace{(1-a)\cdot b}_{<0} \cdot \hat{\mu}_t,$$

since $1 - a > 0$ and $b < 0$.

6.2 Only Market Share Competition

This version of the model, to which I refer as the OMSC model, is identical to that of Phelps and Winter (1970). There is only dynamic market share competition between the monopolistically competitive firms as described in section 3. As explained in that section, an unexpected increase in total factor productivity rises current demand for every individual good creating an incentive for firms to rise their prices. At the same time however, the stochastic discount factor rises making future revenues and thus the investment in market share by lowering current prices more valuable. As I show below, for the parametrization chosen in this study, the negative effect of the discount factor dominates. Therefore markups fall. The decline in markups has a positive effect on the real wage. Its percentage deviation from the steady state can be written as

$$\frac{\hat{W}_t}{P_t} = \hat{Z}_t - \hat{mu}_t.$$

Since working hours evolve according to

$$\hat{N}_t = \frac{\hat{W}_t}{P_t} - \eta \hat{C}_t,$$

the sign of its response depends on the relative strength of the two opposing effects induced by the technology shock - the increase of the real wage and the increase of consumption. Together with the equilibrium condition on the goods market the last equation implies

$$\hat{N}_t = \frac{1-\eta}{1+\eta}\hat{Z}_t - \frac{1}{1+\eta}\hat{mu}_t. \tag{III.6.1}$$

The reactions of the variables of interest to a one time productivity disturbance are given in figure III.50.[34] Under the assumption of a short run elasticity of demand equal to one, $D'(1) = 1$, markups fall leading to an increase in real wages which turns out to be sufficient to offset the negative effect of rising consumption on hours. To gain some intuition about these reactions and to understand why the assumption on $D'(1)$ and η play an important role in this version of the model, it is instructive to take a look at the feed-back rules for $\hat{mu}_t$ and $\hat{N}_t$ first.

After some algebraic manipulations one arrives at the following forward-looking difference equation describing the behavior of the markup:

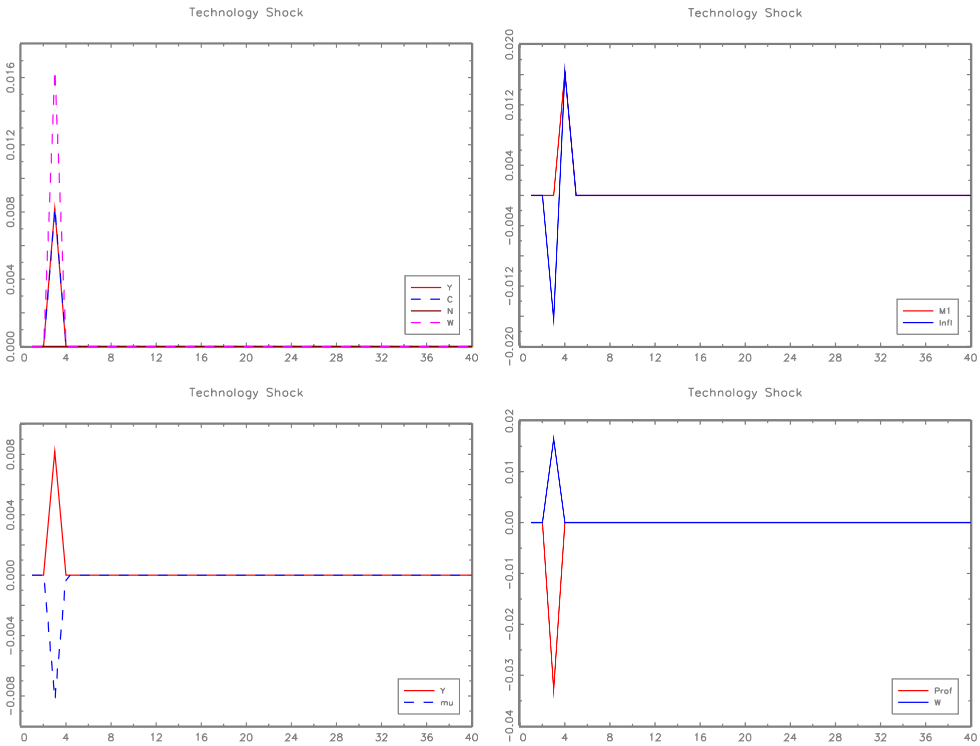
$$\hat{mu}_t = \vartheta E_t \hat{mu}_{t+1} - \frac{2\xi(1-\rho_z)(\eta-1)}{\xi(1-\eta)+1+\eta}\hat{Z}_t, \tag{III.6.2}$$

where

$$\xi = \frac{g'(1)\frac{\beta}{1-\beta}\frac{mu^*-1}{mu^*}}{1+D'(1)+g'(1)\frac{\beta}{1-\beta}\frac{mu^*-1}{mu^*}}$$

[34] The corresponding program is **"sim_cm2d_11.g"**.

Figure III.50: Impulse Responses to a Technology Shock in $t = 3$, $D'(1) = \theta$=1, $\rho_z = 0$. Relative deviations from steady state.



Y-Output, C-Consumption, N-Hours, W-Real Wage, S-Search Effort, M_1-Real Balances, $Infl$-Inflation, $Prof$-Profits. Search activity depends only on current inflation.

and

$$\vartheta = \frac{\xi(1+\eta)}{\xi(1-\eta)+1+\eta}\left(\frac{\beta}{\xi} - \frac{1-\beta}{mu^*-1} + \frac{1-\eta}{1+\eta}\right).$$

For markups to be always positive the following restriction should be satisfied:

$$1 + D'(1) < \frac{\beta(mu^*-1)}{mu^*(1-\beta)}.$$

There is a unique nonexploding solution of (III.6.2) only if ϑ is an element of the open interval $(-1, 1)$. A sufficient condition for $\vartheta > -1$ to be satisfied is

$$\theta > \frac{mu^*(1-\beta)}{2(mu^*-1)}$$

which is not very restrictive, for example for $mu^* \geq 1.05$, θ should be grater than 0.1. It is easy to show that there is a negative dependence between the steady state value of the markup and the lower bound for θ. A sufficient condition for $\vartheta < 1$ to be satisfied is given by

$$\xi > -\frac{(1+\eta)(mu^*-1)(1-\beta)}{(1-\beta)(1+\eta)+2(\eta-1)(mu^*-1)}.$$

Since ξ is positive and $\eta > 1$, the last condition imposes no further restrictions on the parameters of the model.

By solving equation (III.6.2) forward one obtains

$$\hat{mu}_t = -\underbrace{\frac{1}{1-\vartheta\rho_z}\cdot\frac{2\xi(\eta-1)(1-\rho_z)}{(1-\eta)\xi+\eta+1}}_{>0}\hat{Z}_t. \tag{III.6.3}$$

If the technology shock exhibits no autocorrelation, $\rho_z = 0$, the reaction coefficient in (III.6.3) reduces to

$$\hat{mu}_t = -\frac{2\xi\eta(\eta-1)}{(1-\eta)\xi+\eta+1}\hat{Z}_t.$$

If in addition the short run elasticity of demand for every individual good with respect to its price is equal to -1, $D'(1) = -1$, ξ takes the value of 1 and the feed-back rule for markups collapses to

$$\hat{mu}_t = -(\eta-1)\hat{Z}_t.$$

Inserting this result into (III.6.1) yields

$$\hat{N}_t = \frac{1-\eta}{1+\eta}\hat{Z}_t - \frac{1-\eta}{1+\eta}\hat{Z}_t = 0.$$

Hence, for $\rho_z = 0$ and $D'(1) = -1$ the two opposing effects on hours caused by a technology disturbance - the rise in real wages and the fall in the marginal valuation of wealth, exactly offset each other with the consequence that working hours do not respond to technology shocks. $|D'(1)| < 1$ (> 1) implies that $\xi > 1$ (< 1) and thus, there will be a positive (negative) reaction of working hours to technology shocks. The impulse responses obtained under the assumptions $\theta = 0.8$ and $\theta = 1.2$ are displayed in figures III.51 and III.52 respectively. What is the intuition behind these results?

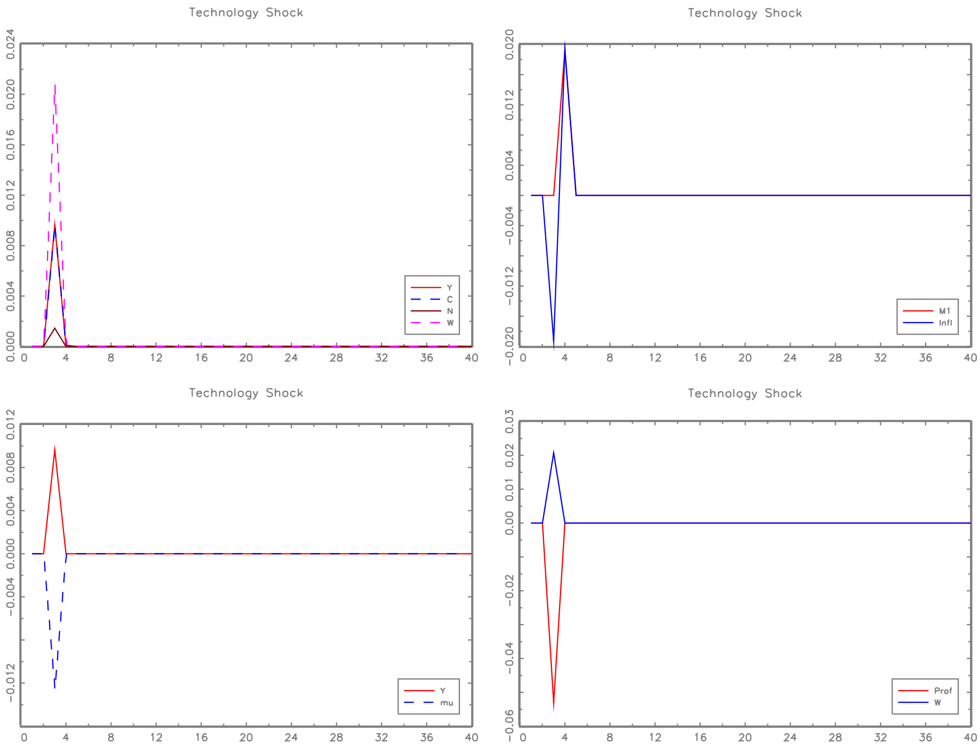
In contrast to the case of static monopolistic competition, in the economy presented in this section the optimal current relative price from the point of view of an individual firm is the one which implies that the sum of the *positive* first derivative of current profits and the *negative* first derivative of the present value of future profits, both with respect to $\frac{P_{i,t}}{P_t}$, equals zero.[35]

Let $\Pi_1\left(\frac{P_{i,t}}{P_t}, \mu_t, \theta\right)$ denote the first derivative of current profits with respect to its first argument. μ_t denotes marginal costs. As a reaction to a marginal cost shock, the firm will adjust its price in a way such that the sum of the *negative* effect on $\Pi_1\left(\frac{P_{i,t}}{P_t}, \mu_t, \theta\right)$ and the *positive* effect on the first derivative of expected future profits exactly offsets the impact on $\Pi_1\left(\frac{P_{i,t}}{P_t}, \mu_t, \theta\right)$ of the change in marginal costs. The first order condition with respect to the relative price $\frac{P_{i,t}}{P_t}$ may be written as follows:

$$\Pi_1\left(\frac{P_{i,t}}{P_t}, \mu_t, \theta\right) = -\frac{g'\left(\frac{P_{i,t}}{P_t}\right)}{g\left(\frac{P_{i,t}}{P_t}\right)}\Omega_t,$$

[35]Note that in this model $\frac{P_{i,t}}{P_t}$, $\forall i$ is always lower than the one which maximizes the strictly concave, unimodale *current* profit function. Hence, a relative price increase will have a positive effect on current profits.

Figure III.51: Impulse Responses to a Technology Shock in $t = 3$, $D'(1) = \theta$=0.8, $\rho_z = 0$. Relative deviations from steady state.



Y-Output, C-Consumption, N-Hours, W-Real Wage, S-Search Effort, M_1-Real Balances, $Infl$-Inflation, $Prof$-Profits. Search activity depends only on current inflation.

where Ω_t represents the expected present value of future profits which also depends on $g\left(\frac{P_{i,t}}{P_t}\right)$. The function $g\left(\frac{P_{i,t}}{P_t}\right) = \exp\left(\gamma - \gamma\frac{P_{i,t}}{P_t}\right)$, $\gamma > 0$ governs the evolution of the gross growth rate of firm i's market share. I assume that

$$g''(1) = (g'(1))^2. \tag{III.6.4}$$

In the paragraph *Some Formal Details* of this subsection I show that the absolute value of the slope of the first derivative of the profit function with respect to $\frac{P_{i,t}}{P_t}$ at the symmetric equilibrium depends positively on the demand elasticity θ:

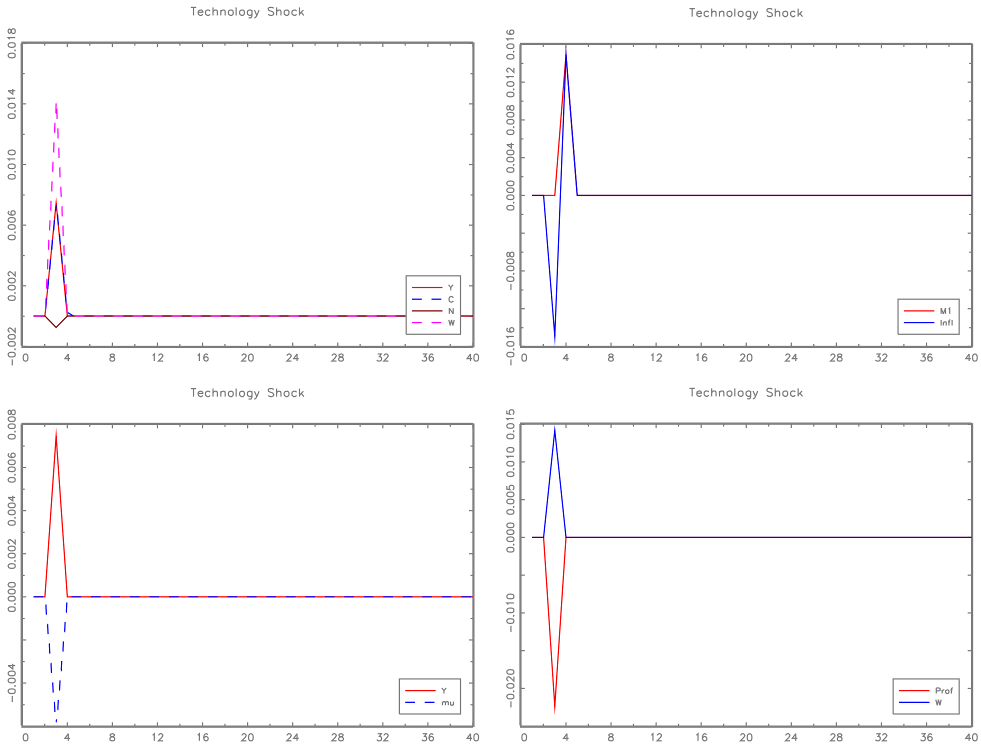
$$\left|\frac{\partial\Pi_{11}\left(1,\mu^*,\theta\right)}{\partial\theta}\right| > 0, \qquad \forall\theta < \frac{1+\mu^*}{2(1-\mu^*)}. \tag{III.6.5}$$

A higher θ also implies a stronger effect on the present value of future profits of any change in the current relative price. Under the assumption (III.6.4) this effect is approximately given by

$$\beta(1-\mu^*)\underbrace{g'(1)g'(1)}_{>0}\cdot\widehat{\left(\frac{P_{i,t}}{P_t}\right)}. \tag{III.6.6}$$

Under the calibration chosen, the steady state conditions imply that $|g'(1)|$ depends positively on θ. The effect of a change in μ_t on the slope of current profits can ba approximated as

Figure III.52: Impulse Responses to a Technology Shock in $t = 3$, $D'(1) = \theta$=1.2, $\rho_z = 0$. Relative deviations from steady state.



Y-Output, C-Consumption, N-Hours, W-Real Wage, S-Search Effort, M_1-Real Balances, $Infl$-Inflation, $Prof$-Profits. Search activity depends only on current inflation.

follows:

$$\underbrace{\Pi_{12}\left(1,\mu^*,\theta\right)}_{>0}\mu^*\hat{\mu}_t.$$

The impact of any percentage change in marginal costs on the partial derivative $\Pi_{12}(1,\mu^*,\theta)$ gets larger as θ increases. Under the assumption that C_{t+j} and μ_{t+j+1} remain constant for $j \geq 0$ the log-linear version of the optimality condition with respect to the relative price $\frac{P_{i,t}}{P_t}$ can be written as

$$\underbrace{\left(-\Pi_{11}\left(1,\mu^*,\theta\right)-\beta(1-\mu^*)g'(1)g'(1)\right)}_{>0 \text{ and depends ambiguously on } \theta}\cdot\widehat{\left(\frac{P_{i,t}}{P_t}\right)} = \underbrace{\Pi_{12}\left(1,\mu^*,\theta\right)}_{>0 \text{ and depends positively on } \theta}\mu^*\hat{\mu}_t. \quad \text{(III.6.7)}$$

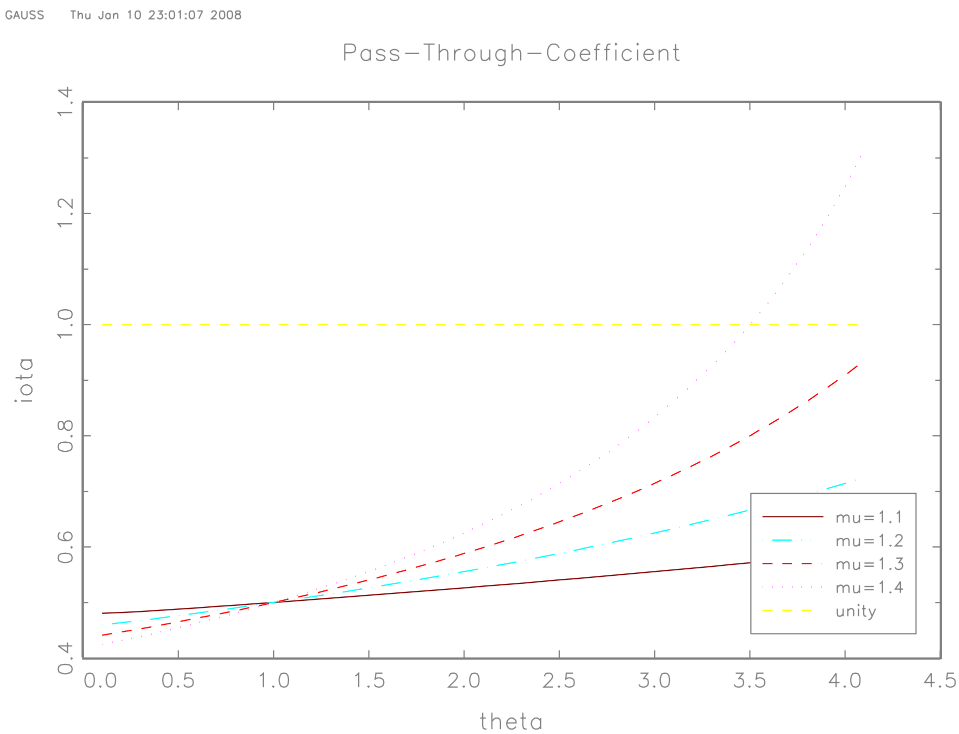
Equation (III.6.7) implies that there is an ambiguity with respect to the dependence between the coefficient of pass-through of marginal cost changes to prices ι

$$\iota = \frac{\Pi_{12}\left(1,\mu^*,\theta\right)}{-\Pi_{11}\left(1,\mu^*,\theta\right)-\beta(1-\mu^*)g'(1)g'(1))}$$

and the parameter θ. Figure III.53 depicts the behavior of ι as a function of θ. Since marginal costs are procyclical, for a given path of aggregate consumption a higher pass-through implies less anticyclical markups and thus, less procyclical real wages, with the consequence that for sufficiently large values of θ, in our case for $\theta > 1$, the response of real wages to technology shocks is no more sufficient to offset the effect on working hours induced by the reaction of current consumption. Figure III.53 also shows that on the individual firm's level pass-through is always positive and for a broad range of values of θ and $\mu^* = 1/mu^*$ lower than one. The latter implies that even without taking into account the effects of aggregate consumption on the stochastic discount factor and thus on the pricing decision of the firm, the correlation between the individual markup and marginal costs will be negative. Since marginal costs are procyclical, for constant current and future aggregate consumption the individual firm will find it optimal to let its own markup be countercyclical. If one also takes account of the fact that the stochastic discount factor is also procyclical, when a technology shock occurs, each firm will choose an even lower relative price than the one implied by (III.6.7). Consequently, its own markup will be even more countercyclical than implied by (III.6.7).

All other economic forces leading to the impulse responses displayed in figures III.50, III.52 and III.51 are the same as in the OCC-model.

Figure III.53: Pass-Through-Coefficient ι as a function of θ, no serial correlation in marginal costs



mu - denotes the steady state value of the markup.

If total factor productivity follows a first order autoregressive process with a positive coefficient of autocorrelation the implications of the *OMSC*-model change in a similar way as it was the case in the *OCC*-model. The impulse responses become more persistent and replicate closely the autocorrelation structure of the technology shock Z_t. Again, higher absolute values of the demand elasticity θ imply less countercyclical markups and thus less procyclical real wages and working hours.

Some Formal Details: It is assumed that the consumption bundle is given by the Dixit-Stiglitz-aggregator with a constant elasticity of substitution. The latter implies a constant elasticity of demand for every individual good with respect to its own price. I denote this elasticity by $\theta = -D'(1)$. The first order condition of firm i can be written as the following dynamic system:

$$\left(\frac{P_{i,t}}{P_t}\right)^{-\theta} - \theta\left(\frac{P_{i,t}}{P_t} - \mu_t\right)\left(\frac{P_{i,t}}{P_t}\right)^{-\theta-1} = \frac{-g'\left(\frac{P_{i,t}}{P_t}\right)}{g\left(\frac{P_{i,t}}{P_t}\right)}\frac{\Omega_t}{C_t},$$

$$\Omega_t = E_t\left(\beta\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{P_{i,t+1}}{P_{t+1}} - \mu_{t+1}\right)\left(\frac{P_{i,t+1}}{P_{t+1}}\right)^{-\theta} g\left(\frac{P_{i,t}}{P_t}\right) C_{t+1}\right) + E_t\left(\beta\frac{\Lambda_{t+1}}{\Lambda_t}\Omega_{t+1}\right), \tag{III.6.8}$$

where μ_t are real marginal costs and Ω_t is the expected present value of future profits. Using this two equations one may derive the steady state value of $g'(1)$ as a function of β, θ and μ^*:

$$g'(1) = \frac{(1-\beta)(\theta(1-\mu^*)-1)}{\beta(1-\mu^*)}.$$

It is easily seen that $g'(1)$ increases as θ becomes larger. The slope of the first derivative with respect to $P_{i,t}/P_t$ of current profits at the symmetric equilibrium in the steady state is given by

$$\Pi_{11}(1, \mu^*, \theta) = \theta^2(1-\mu^*) - \theta(1+\mu^*)$$

which is negative for

$$\theta < \frac{1+\mu^*}{1-\mu^*}.$$

The fraction $(1+\mu^*)/(1-\mu^*)$ is equal to $\{6; 7.66; 11; 21\}$ for $\mu^* = \{1.4^{-1}; 1.3^{-1}; 1.2^{-1}; 1.1^{-1}\}$. The first derivative of $\Pi_{11}(1, \mu^*, \theta)$ with respect to θ is given by:

$$\frac{d\Pi_{11}(1, \mu^*, \theta)}{d\theta} = 2\theta(1-\mu^*) - (1+\mu^*).$$

It is negative for

$$\theta < \frac{1+\mu^*}{2(1-\mu^*)}.$$

In the OMSC-model pass-through of marginal costs to prices is approximately given by the coefficient ι in the equation

$$\left(\frac{\hat{P_{i,t}}}{P_t}\right) = \underbrace{\frac{\Pi_{12}\left(1, \mu^*, \theta\right)}{-\Pi_{11}\left(1, \mu^*, \theta\right) - \beta(1-\mu^*)g'(1)g'(1)}}_{:=\iota} \hat{\mu}_t,$$

where

$$\iota = \frac{\theta\mu^*}{-\theta^2(1-\mu^*) + \theta(1+\mu^*) + \frac{(1-\beta)^2(\theta(1-\mu^*)-1)^2}{\beta(1-\mu^*)}}.$$

The denominator of the first derivative of ι with respect to θ is positive. The numerator of $\frac{d\iota}{d\theta}$ is given by

$$\frac{(1-(1-\beta)^2)\theta^2\mu^*(1-\mu^*)^2 + \mu^*(1-\beta)}{\beta(1-\mu^*)},$$

which is positive for all

$$\theta > 0.$$

Hence, the results provided so far in this subsection can be summarized as follows: A sufficient condition for the pass-through coefficient ι to depend positively on θ is given by

$$\theta \in \left(0, \frac{1+\mu^*}{1-\mu^*}\right).$$

In addition, there is a positive dependence between the pass-through coefficient ι and θ. If θ is an element of the open interval

$$\theta \in \left(\frac{1+\mu^*}{2(1-\mu^*)}, \frac{1+\mu^*}{1-\mu^*}\right)$$

ι grows as θ increases and there is *no ambiguity* with respect to the influence of θ on the sum

$$-\Pi_{11}\left(1, \mu^*, \theta\right) - \beta(1-\mu^*)g'(1)g'(1).$$

If marginal costs follow a first order autoregressive process with a positive coefficient of autocorrelation $\rho_\mu > 0$ the forward difference equation describing the dependence between firm i's relative price and marginal costs changes to

$$\frac{\hat{P_{i,t}}}{P_t} = \phi_1 E_t\left(\frac{\hat{P_{i,t}}}{P_t}\right) + \phi_2\hat{\mu}_t, \tag{III.6.9}$$

where

$$\phi_1 = \beta \frac{2\theta - (\theta^2+\theta)(1-\mu^*) + g'(1)(1-\theta(1-\mu^*))}{2\theta - (\theta^2+\theta)(1-\mu^*) - \beta(1-\mu^*)g'(1)g'(1)},$$

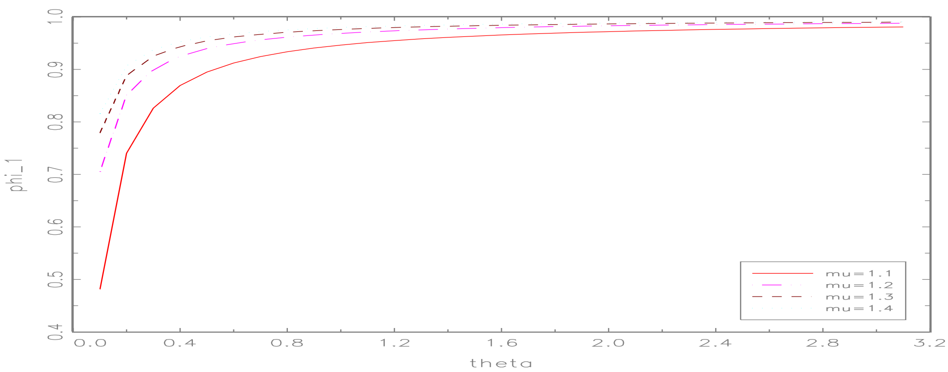
$$\phi_2 = \frac{\theta\mu^*(1-\rho_\mu) - \beta\mu^*\rho_\mu g'(1)}{2\theta - (\theta^2+\theta)(1-\mu^*) - \beta(1-\mu^*)g'(1)g'(1)}.$$

Figure III.54 shows the dependence between ϕ_1 and θ for different values of mu^*. As can be seen, ϕ_1 is always an element of the interval (0,1) implying the existence of the following non-exploding solution of the forward difference equation (III.6.9):

$$\frac{\hat{P_{i,t}}}{P_t} = \underbrace{\frac{1}{1-\phi_1\rho_\mu}\phi_2}_{:=\bar{\iota}}\hat{\mu}_t.$$

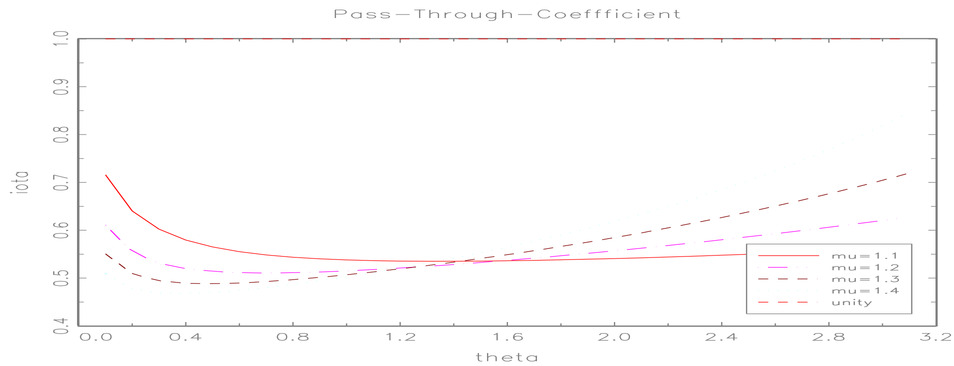
For $\rho_\mu = 0.7$ the pass-through coefficient $\bar{\iota}$ depends on θ as depicted in figure III.55. For each value of mu^* pass-through is an unimodal, strictly convex function of θ. The higher the equilibrium markup mu^* the lower the θ minimizing the pass-through coefficient. In figure III.55 it is assumed that ρ_μ takes the value of 0.72. Such a choice might seem arbitrary but it is not. I set ρ_μ to 0.72 for the following reasons: For the chosen calibration, the *OMSC*-model implies a first order autocorrelation of μ_t lower than 0.72 for all ρ_z-θ combinations. The same is true for the *CM*-model under the assumption that the monetary as well as the technology shock are not serially correlated. Lower values of ρ_μ shift the minimum of the coefficient $\bar{\iota}$ to the left, making the range in which there is a positive relationship between pass-through and θ even larger.

Figure III.54: ϕ_1 as a function of θ



mu denotes the steady state value of the markup.

Figure III.55: Pass-Through-Coefficient $\bar{\iota}$ as a function of θ, $\rho_{\mu} = 0.72$

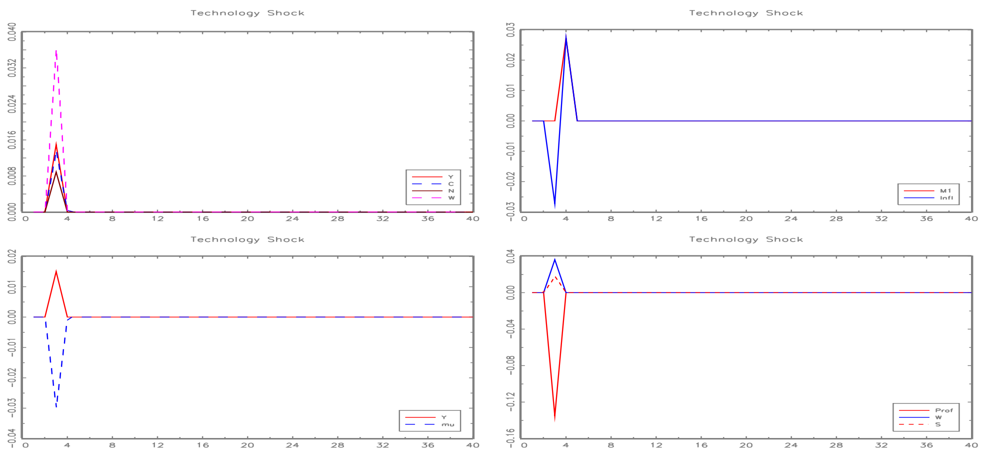


mu denotes the steady state value of the markup.

6.3 Market Share Competition And Search Activity Depending on Consumption

Search Activity Depends on Current Consumption Only: In the model version briefly described in this section it is assumed that in addition to market share competition there is a matching mechanism $g\left(\frac{P_{t,i}}{P_t}, s_t\right)$ combining the individual firm's pricing behavior with the search activity s_t households engage in. The matching function $g(.,.)$ then determines the gross growth rate of firm i's market share. In this version of the model, called the *MSCCC*-model, search costs are to be payed in the period in which they actually arise. As a result, there emerges a positive dependence between search effort s_t and aggregate consumption in t. The impulse responses to a non-autocorrelated technology shock in the *MSCCC*-model are displayed in figures III.56 through III.58.[36] The fact that the evolution of firm's market share now depends negatively on aggregate consumption via search activity makes markups more countercyclical than they are in the *OMSC*-model. In booms, as consumption increases, households intensify their search and switching efforts. That leads from the firm's point of view to more severe losses (gains) in market share for any given positive (negative) deviation from the average price level than it was the case in the *OMSC*-model. Hence, in order to avoid such losses each firm will have an incentive to choose a lower markup than it would do in the environment of the *OMSC*-economy. As figures III.56 through III.58 show, higher values of the demand elasticity θ lead to less countercyclical markups. The same is through if one assumes that the technology shock in the *MSCCC*-economy is positively autocorrelated.

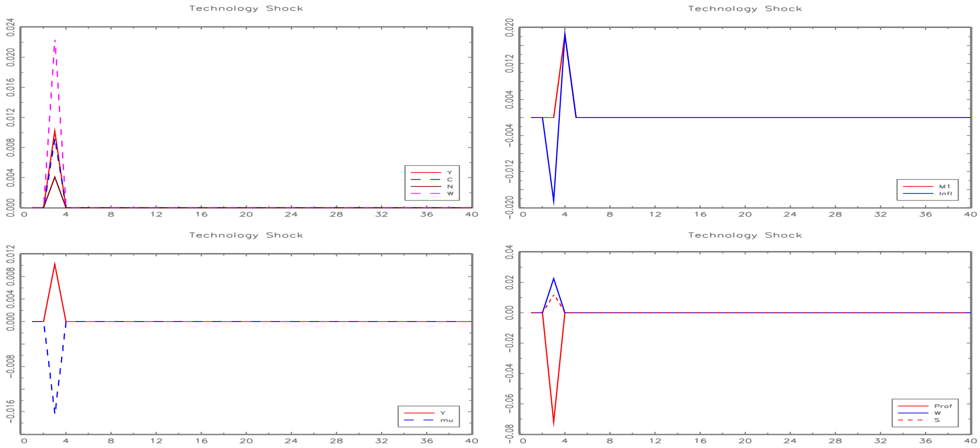
Figure III.56: Impulse Responses to a Technology Shock in $t = 3$, $D'(1) = \theta{=}1$, $mu^* = 1.2$, $\rho_z = 0$. Relative deviations from steady state.



Y-Output, C-Consumption, N-Hours, W-Real Wage, S-Search Effort, M_1-Real Balances, $Infl$-Inflation, $Prof$-Profits. Search activity depends only on current inflation.

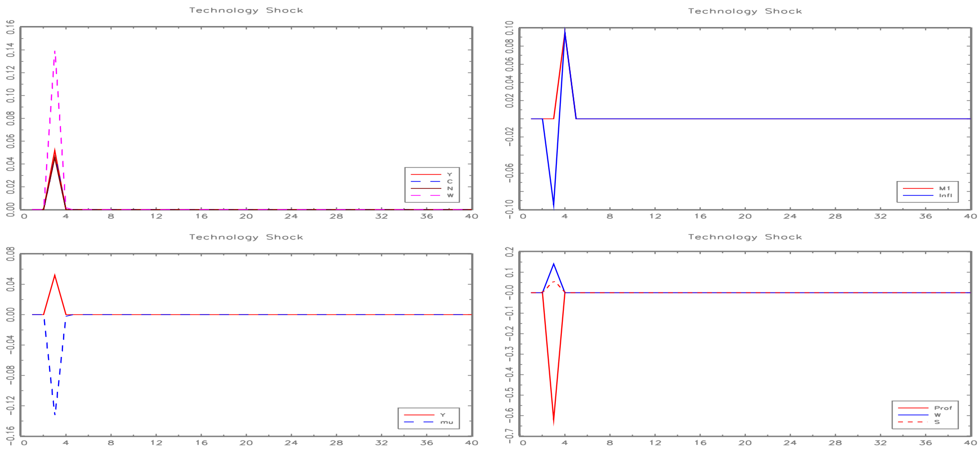
[36]The corresponding program is **"sim_cm2d_10a.g"**

Figure III.57: Impulse Responses to a Technology Shock in $t = 3$, $D'(1) = \theta$=1.2, $mu^{*} = 1.2$, $\rho_z = 0$. Relative deviations from steady state.



Y-Output, C-Consumption, N-Hours, W-Real Wage, S-Search Effort, M_1-Real Balances, $Infl$-Inflation, $Prof$-Profits. Search activity depends only on current inflation.

Figure III.58: Impulse Responses to a Technology Shock in $t = 3$, $D'(1) = \theta$=0.8, $mu^{*} = 1.2$, $\rho_z = 0$. Relative deviations from steady state.



Y-Output, C-Consumption, N-Hours, W-Real Wage, S-Search Effort, M_1-Real Balances, $Infl$-Inflation, $Prof$-Profits. Search activity depends only on current inflation.

Chapter 4

GMM Estimation

1 Introduction

In the previous chapter I developed a monetary business cycles model in which he extends the market share competition mechanism proposed in Phelps and Winter (1970) by introducing search in the goods market and then combining that structure with the assumption that the overall rate of inflation generates disutility which can be reduced by engaging in more intense and thus more expensive search activities. The direct dependence of the utility function on inflation was interpreted as *inflation aversion*. That dependence is the crucial building block of his theory. The model is able to generate endogenous countercyclical markups which react negatively to monetary as well as technology shocks, and endogenous sluggishness in nominal prices. Furthermore, for a fairly broad range of parameter values the intrinsic mechanisms of the model are strong enough to imply a substantial degree of persistence in the reactions to exogenous disturbances, even if the latter follow pure White Noise processes. Thus, his model provides an endogenous explanation of the observed persistence and autocorrelation in actual business cycles.

A major shortcoming of the analysis in the previous chapter is the absence of any empirical estimates with respect to several crucial components of his model: Due to the fact that search efforts in the goods market are not directly observable there is no evidence about their cyclical properties as well as with regard to the elasticity of the utility function with respect to search $-\alpha$. For that reason I performed a sensitivity analysis with respect to that parameter. I concluded that lower values of α lead to stronger and more persistent responses of the major aggregates to serially uncorrelated monetary and technology shocks. But wouldn't it be nice to know whether the data support the structure of such an *"exotic"* model or not? Wouldn't it be nice to know if α is positive at all and what is the empirically relevant range of that parameter?

The purpose of the current chapter is to provide some empirical evidence supporting or rejecting the model of the previous chapter as well as to shed light on the relevant range of one of its most important parameters, α. The version of the model including capital accumulation, to which I refer as the Inflation Aversion model, is reduced to a set of equations involving only observable variables. The sample moment restrictions derived from these equations are used to estimate a set of parameters by the *General Method of Moments (GMM)*. The main are that the data do not reject the general structure of the Inflation Aversion model and do not reject the assumption that α lies in the interval $(0, 1.5)$. The GMM estimate of α is then used to complete the calibration of the model with adjustment costs of capital. Its second moments, computed under different assumptions on the autocorrelation structure of the exogenous processes, are then confronted with their empirical counterparts. The model is able to account for several important features of the observable business cycles pattern in the U.S. economy. Furthermore, even without any serial correlation in the exogenous variables, the model explains a substantial part of the observed autocorrelation of the main macroeconomic aggregates.

The paper is organized as follows. Section 2 is devoted to the construction of the data set and the GMM estimation. Section 3 provides a comparison between the second moments of the data and that implied by the model. Section 5 concludes.

2 GMM-Estimation

2.1 Data

The model economy presented in the previous chapter is very abstract and does not take an explicit account of many features of the real world such as net exports, the government sector or consumer durables. The definitions of many variables in the model economy deviate substantially from the definitions used by institutions such as the NIPA or the OECD which provide data on the main macroeconomic aggregates. Therefore, before turning to parameter estimation, it should be a major task to transform the available data into measures and proxies of the variables of interest which are as close as possible to the definitions underlying the theoretical model. In doing so I follow the strategy recommended by Cooley and Prescott (1995). The computations presented in the following subsections are done in the File **"NIPA_UCapitalIncome_nominal.xls"** , Sheets **"CapitalStock_real"**, **"UnambCapInc"** and **"AmbCapInc_DEP"**.

Aggregate Output, Capital Stocks, Consumption

The theoretical model contains only one stock of capital used by the private production sector for generating the whole aggregate output. Each individual good is produced according to the same constant returns to scale technology as any other. The individual goods are then assembled to nondurable consumption, search efforts and investment, the latter three variables being perfect substitutes. The model economy does not distinguish between different kinds of capital, between private and government sector, and between durable and nondurable consumption goods. Accordingly, to make the available data consistent with the theory, one has to assume that the model's capital stock includes all kinds of capital accumulated in the actual economy. Similarly, the model's output should be assumed to include the output generated by all kinds of capital. Furthermore, a precise distinction between durables- and nondurables consumption expenditure should be made, with the former considered a part of aggregate investment. To transform the available data into the desired form, I proceed as follows. First, following Cooley and Prescott (1995) I assume that the government produces according to the same constant returns to scale technology as the private sector. Since in constructing the GNP-series the NIPA only takes account of the flow of services of household owned residential structures as well as the income generated by the net foreign position[1] the official series should be adjusted by the flow of services of government fixed capital as well as the stock of consumer durables. These two flows can be approximated as follows:

$$Y_{K,G,t} = (i_t + \upsilon_G)\tilde{K}_{G,t-1}, \qquad Y_{K,D,t} = (i_t + \upsilon_D)\tilde{K}_{D,t-1},$$

where $Y_{K,G,t}$ and $Y_{K,D,t}$ denote the income flow generated by government fixed capital and the stock of durables respectively. υ_G and υ_D are the corresponding rates of depreciation. i_t is the real rate of return on capital which is assumed to be identical across all kinds of capital. $\tilde{K}_{G,t-1}$ and $\tilde{K}_{D,t-1}$ are the quarterly end-of-period stocks of government fixed assets and durable goods respectively. According to the above formulas one needs to construct proxies of the unobservable variables i_t, υ_G and υ_D. $K_{G,t}$ and $K_{D,t}$ are provided by the NIPA. To be able to find these proxies, one has first to identify several other variables, not directly provided by the official institutions - the flow of services of the private stock of capital, the share of private capital in measured GNP and the *correctly measured* private stock of capital. Because, as Cooley and Prescott (1995) show, given the share of private capital income in measured GNP θ_P one can easily determine the flow of services of private capital $Y_{K,P,t}$:

$$Y_{K,P,t} = \theta_P GNP_t.$$

[1] These are are the net factor payments to (or from) the rest of the world.

Then given the private stock of capital $\tilde{K}_{Priv,t}$ and its depreciation $D_{P,t}$ one will be able to compute the real rate of return i_t:

$$i_t = \frac{Y_{K,P,t} - D_{P,t}}{\tilde{K}_{Priv,t}}.$$

Next, provided series for government fixed capital $\tilde{K}_{G,t}$, the stock of consumer durables $\tilde{K}_{D,t}$ and their respective depreciations $D_{G,t}$ and $D_{D,t}$, it will be possible to compute the depreciation rates

$$\upsilon_{G,t} = \frac{D_{G,t}}{\tilde{K}_{G,t-1}}, \qquad \upsilon_{D,t} = \frac{D_{D,t}}{\tilde{K}_{D,t-1}}.$$

The Share of Private Capital Income in Measured GNP and the Real Interest Rate: Cooley and Prescott (1995) show that the income share of private capital in measured GNP can be computed as

$$\theta_{P,t} = \frac{UKI_t + GNP_t - NNP_t}{GNP_t - AI_t},$$

with

- UKI_t - *Unambiguous Capital Income = Rental Income+Corporate Profits+Net Interest*,[2]
- AI_t - *Ambiguous Income =Proprietors Income+Net National Product -National Income*,[3]
- GNP_t - *Gross National Product* as provided by the NIPA,[4]
- NNP_t - *Net National Product* as provided by the NIPA.[5] [6] [7]

[2]See NIPA-Table 1.12 *National Income by Type of Income*

[3]See NIPA-Table 1.12 *National Income by Type of Income*

[4]See NIPA-Table 1.7.5 *Relation of Gross Domestic Product,Gross National Product, Net National Product, National Income, and Personal Income*

[5]The difference between GNP_t and NNP_t equals *Consumption of private fixed capital* or depreciation.

[6]See NIPA-Table 1.7.5 *Relation of Gross Domestic Product,Gross National Product, Net National Product, National Income, and Personal Income*

[7]Cooley and Prescott (1995) assume that the relation between $Y_{K,P}$ and measured GNP can be represented by the equation

$$Y_{K,P,t} = \theta_P GNP_t.$$

At the same time, by using the NIPA-data $Y_{K,P,t}$ can be constructed via

$$Y_{K,P,t} = UKI_t + \theta_P AI_t + GNP_t - NNP_t,$$

where again they assume that the share of the ambiguous component of private income AI_t attributable to the ownership of capital is equal to private capital's share in measured GNP θ_P. Equating the rhs of the last two equations and solving for θ_P delivers the expression for θ_K given in the main text.

I use the nominal quarterly series of these variables provided by the NIPA. The mean of $\theta_{P,t}$, denoted by θ_P corresponding to the time period 1964:Q1 - 2007:Q3 equals 0.2922. $Y_{K,P,t}$ at constant prices can then be computed as

$$Y_{K,P,t} = \theta_P GNP_t,$$

where GNP_t denotes Gross National Product at constant prices.

NIPA provides only annual, current price data for the stocks of private fixed capital[8] and durable goods[9]. Quarterly series for these two stocks are constructed according to the following interpolation formula:

$$\tilde{K}_{u,y+1,q} = K_{u,y} + \sum_{j=1}^{q} \phi_u I_{u,y+1,j}, \tag{IV.2.1}$$

where

$$\phi_u = \frac{K_{u,y+1} - K_{u,y}}{\sum_{j=1}^{4} \tilde{I}_{u,y+1,j}}.$$

$K_{u,y}$ and $\tilde{K}_{u,y,q}$ denote the real value of the capital stock of type $u = \{P, D, G\}$ at the end of year y and at the end of quarter q of year y respectively. $\tilde{I}_{u,y,q}$ denotes real net investment in the q-th quarter of year y in the stock of u. $\tilde{I}_{u,y,q}$ is constructed by subtracting real depreciation of the u-th type of capital $D_{u,y,q}$ form real gross investment in the u-th type of capital $I_{u,y,q}$. (IV.2.1) ensures that $\tilde{K}_{u,y+1,4} = K_{u,y+1}$ for all y.

The data used in these computations are:

- annual end of period stock of private fixed capital at current costs deflated by the yearly average of the implicit price deflator of gross private fixed investment,[10]
- annual end of period stock of government fixed capital at current costs deflated by the yearly average of the implicit price deflator of gross private fixed investment,[11]
- annual end of period stock of consumer durables at current costs deflated by the yearly average of the implicit price deflator of consumption durable expenditure,[12]

[8] NIPA Table 1.1. Current-Cost Net Stock of Fixed Assets and Consumer Durable Goods

[9] NIPA Table 1.1. Current-Cost Net Stock of Fixed Assets and Consumer Durable Goods

[10] NIPA, Table 1.1. *Current-Cost Net Stock of Fixed Assets and Consumer Durable Goods* and Table 1.1.9. *Implicit Price Deflators for Gross Domestic Product*

[11] NIPA, Table 1.1. *Current-Cost Net Stock of Fixed Assets and Consumer Durable Goods* and Table 1.1.9. *Implicit Price Deflators for Gross Domestic Product*

[12] NIPA, Table 1.1. *Current-Cost Net Stock of Fixed Assets and Consumer Durable Goods* and Table 1.1.9. *Implicit Price Deflators for Gross Domestic Product*

- nominal, quarterly gross private fixed investment and depreciation of private fixed capital (*Consumption of fixed private capital*) deflated by the implicit price deflator of gross private fixed investment,[13]
- nominal, quarterly gross government fixed investment (*Federal defense + Federal non-defense + State and local investment*) and depreciation of government fixed capital (*Consumption of fixed government capital*) deflated by the implicit price deflator of gross private fixed investment,[14]
- nominal, quarterly consumer durables expenditure and depreciation of consumer durables deflated by the implicit price deflator of consumer durables expenditure,[15]

The NIPA provides quarterly real data on the stock of inventories.[16] The last component of the private capital stock, the NIPA also doesn't take account of when computing the stock of private capital, is land. The quarterly current price value of *Real Estate* is provided by the Federal Funds Accounts. I deflate this series by the implicit price deflator of GDP. The correct quarterly real stock of private fixed capital $\tilde{K}_{Priv}$ can then be computed as the sum of the quarterly real values of *Private Fixed Capital (NIPA-Definition)* $\tilde{K}_P$, the stock of *Inventories* $\tilde{K}_{Inv}$ and the value of *Land* $\tilde{K}_L$:

$$\tilde{K}_{Priv,t} = \tilde{K}_{P,t} + \tilde{K}_{Inv,t} + \tilde{K}_{L,t}.$$

The quarterly real series for *Government Fixed Capital* $\tilde{K}_G$ and the *Stock of Consumer Durables* were also approximated according to (IV.2.1). The economy wide stock of capital $\tilde{K}_t$ is then given by:

$$\tilde{K}_t = \tilde{K}_{Priv,t} + K_{G,t} + K_{D,t}.$$

Given $\tilde{K}_{Priv}$ the *quarterly* real interest rate can be computed as

$$i_t = \frac{Y_{K,P,t} - D_{P,t}}{\tilde{K}_{Priv,t-1}},$$

where $D_{P,t}$ denotes depreciation of private fixed capital. I use data over the period 1970:Q1 through 2003:Q3. The resulting mean real rental rate of capital on quarterly basis i equals 0.0109.

[13] NIPA, Table 1.5.5. *Gross Domestic Product, Expanded Detail*, Table 5.1. *Saving and Investment* and Table 1.1.9. *Implicit Price Deflators for Gross Domestic Product*

[14] NIPA, Table 1.5.5. *Gross Domestic Product, Expanded Detail*, Table 5.1. *Saving and Investment* and Table 1.1.9. *Implicit Price Deflators for Gross Domestic Product*

[15] NIPA, Table 1.5.5. *Gross Domestic Product, Expanded Detail*, Table 5.1. *Saving and Investment* and Table 1.1.9. *Implicit Price Deflators for Gross Domestic Product*

[16] NIPA Table 1AU2. Real Manufacturing and Trade Inventories, Seasonally Adjusted, End of Period and Table 1BU. Real Manufacturing and Trade Inventories, Seasonally Adjusted, End of Period

Depreciation Rates: To derive the rate of depreciation of the u-th stock in period t, $\upsilon_{u,t}$ I just divide the real quarterly depreciation $D_{u,t}$ through the real quarterly end-of-period value of the corresponding stock *at the end of the previous period* $\tilde{K}_{u,t-1}$:

$$\upsilon_{u,t} = \frac{D_{u,t}}{\tilde{K}_{u,t-1}}.$$

The same data on depreciation is used as in the construction of the capital stocks. The average rates of depreciation in the period 1970:Q1 - 2003:Q3 of private fixed capital, government fixed capital, durables and Land equal 0.0104, 0.0089, 0.0509 and 0 respectively.[17] The overall rate of depreciation, computed as a weighted average of these three values, equals 0.00708.[18]

Economy Wide Capital Share, Aggregate Labor Income and Markups: Having approximated $\tilde{K}_{u,t}$, $\upsilon_{u,t}$ and i_t with $u = \{G, D\}$ we are in the position to compute the income flow of services of the government stock of capital $Y_{K,G,t}$ as well as the stock of consumer durables $Y_{K,D,t}$. The correct measure of aggregate production (income) $Y_{co,t}$ is:

$$Y_{co,t} = GNP_t + Y_{K,G,t} + Y_{K,D,t}.$$

The economy wide share of capital income in the *correctly measured* GNP is then readily computed as the ratio of overall capital income to $Y_{co,t}$:

$$\theta_{K,t} = \frac{Y_{K,P,t} + Y_{K,G,t} + Y_{K,D,t}}{GNP_t + Y_{K,G,t} + Y_{K,D,t}}.$$

The mean of $\theta_{K,t}$ for the period 1964:Q1 - 2007:Q3 equals 0.3253.

In an economy characterized by monopolistic competition in the goods market output equals the sum of labor income, capital income and pure profits:

$$Y_{co,t} = \frac{W_t}{P_t}N_t + R_t K_t + \Pi_t,$$

where R_t denotes the real rental rate of capital. Under the Cobb-Douglas production function assumed in the model the last equation can be written as

$$Y_{co,t} = \frac{\omega}{mu_t}Y_{co,t} + \frac{1-\omega}{mu_t}Y_{co,t} + \frac{mu_t - 1}{mu_t}Y_{co,t},$$

[17] Since there is no *"hard"* data available allowing the computation or the approximation of the depreciation of Land and since presumably it depreciates extremely slowly, the depreciation rate of this component of the capital stock (Land) is assumed to be zero. Nevertheless, there may be reasons for assuming a positive depreciation rate of Land, e.g. due to pollution, climate change or other forces worsening the environmental quality.

[18] If the stock of Land is excluded, the resulting average depreciation rate equals 0.0129.

where ω represents the production elasticity of labor and mu_t the markup of prices over marginal costs in the goods market. Since in the real world firm's profits either flow back as dividends to the owners of the capital stock or are retained and reinvested and hence increase the value or the amount of capital owned by the firm's shareholders, here profits are viewed as part of capital income. Thus, the sum of the rental income of capital $\frac{1-\omega}{mu_t}Y_{co,t}$ and pure profits $\frac{mu_t-1}{mu_t}Y_{co,t}$ equals total capital income and capital's share $\theta_{K,t}$ is given by

$$\theta_{K,t} = \frac{mu_t - \omega}{mu_t}.$$

Given the mean of $\theta_{K,t}$ and an estimate of the steady state value of the markup mu^*, one can use the last equation to compute the implied mean of the production elasticity of capital.

The only measure of aggregate labor income consistent with the theoretical model is that computed by multiplying the *correctly* measured Gross National Product $Y_{co,t}$ by labor share $\theta_L = 1 - \theta_K$:

$$LI_t = (1 - \theta_K)Y_{co,t},$$

where LI_t denotes aggregate labor income. Any other measure constructed by building linear combinations of several series reported by the NIPA such as different kinds of compensations of employees, labor income taxes and social contributions would be more time consuming to compute, would involve a higher approximation error and probably would be less consistent with the structure of the model.

To approximate the unobservable markup series, I again resort to economic theory. Observe that the model implies that the marginal productivity of labor and the real wage are related to each other as follows:

$$mu_t = \frac{W_t/P_t}{\omega Y_{co,t}/N_t}.$$

To substitute the ratio on the rhs of the last equation by observable variables, just multiply the nominator and the denominator by hours worked N_t:

$$mu_t = \frac{1}{\omega} \cdot \frac{LI_t}{Y_{co,t}} = \frac{1}{\omega}(1 - \theta_{K,t}).$$

Note that varying the mean of the markup mu^* only shifts the series $\{mu_t\}_{t=1970:Q1}^{2003:Q3}$ upwards or downwards but does not affect its cyclical properties.

Consumption and Inflation: The measure of consumption consistent with the theoretical model equals the sum of private expenditure on nondurable goods $C_{Nd,t}$ and services $C_{S,t}$ and government consumption expenditure $C_{G,t}$:

$$C_{co,t} = C_{Nd,t} + C_{S,t} + C_{G,t}.$$

The NIPA provides only nominal quarterly data on these variables. Therefore I deflate them by the respective implicit price deflators of nondurable goods, services and government consumption expenditure. The NIPA publishes separate implicit price deflators for private nondurables consumption expenditure and services but only a common price deflator for overall government expenditure, corresponding to the sum of government consumption and investment. Accordingly, I approximate the implicit price deflator of government consumption expenditure $P_{GC,t}$ as follows: First, I assume that the price deflator of government investment is the same as that of private fixed investment. Then $P_{GC,t}$ can be computed *via*:

$$P_{GC,t} = \left(\bar{P}_{G,t} - \frac{I_{G,t}}{G_t} P_{I,t} \right) \frac{G_t}{C_{G,t,nom}},$$

where $\bar{P}_{G,t}$ and $P_{I,t}$ denote the implicit price deflator of overall government expenditure and that of private fixed investment respectively. G_t and $I_{G,t}$ denote nominal overall government expenditure and nominal government investment respectively. $C_{G,t,nom}$ stands for nominal government consumption.

The price deflator consistent with $C_{co,t}$ is computed as a weighted average of the individual price deflators of $C_{Nd,t}, C_{S,t}$ and $C_{G,t}$:[19] [20]

$$P_{C,t} = \frac{C_{Nd,t}}{C_{co,t}} P_{Nd,t} + \frac{C_{S,t}}{C_{co,t}} P_{S,t} + \frac{C_{G,t}}{C_{co,t}} P_{G,t}.$$

Now, the inflation rate associated with aggregate consumption can be computed as:

$$\pi_t = \frac{P_{C,t}}{P_{C,t-1}} - 1.$$

[19] The deflators of $C_{Nd,t}$ and $C_{S,t}$ are taken from NIPA's Table 1.1.9. Implicit Price Deflators for Gross Domestic Product

[20] The following series were used:

- the nominal quarterly series, taken from NIPA's Table 1.5.5. *Gross Domestic Product, Expanded Detail*, for
  - *Personal consumption expenditures/Nondurable goods, Personal consumption expenditures/Services*,
  - *Government.../Federal/National defense/Consumption expenditure*,
  - *Government.../Federal/Nondefense/Consumption expenditure*,
  - *Governemnt.../State and Local/Consumption expenditure*.
- the implicit price deflators, taken from NIPA's Table 1.1.9. *Implicit Price Deflators for Gross Domestic Product*, for
  - *Personal consumption expenditures/Nondurable goods*,
  - *Personal consumption expenditures/Services*,
  - *Gross private domestic investment/Fixed investment*,
  - *Government consumption expenditures and gross investment*.

Hours: To construct the series for working hours I use quarterly data on *Total Employment*[21] and *Average Weekly Hours (Private Industry)*[22]. I assume that average weekly hours are the same across all sectors of the economy. Further, the individual's time endowment is normalized to *(90 days)*(16 hours)* per quarter. The measure of the economy wide time endowment per quarter consistent with the theoretical model is then given by *(90 days)*(16 hours)*population*. The population series is provided by the OECD and is also used to transform all the relevant variables into per capita terms. The fraction of time the *"representative"* individual in the actual economy spends working, N, is given by

$$N = \frac{(90\ days) * (\frac{Average\ weekly\ hours}{7}) * (Total\ Employment)}{(90\ days) * (16\ hours) * Population}.$$

The data used span the period from 1970:Q1 through 2003:Q3. The mean of N equals 0.1386.

Real Balances: I use the seasonally adjusted M1 series provided by the *Federal Funds Accounts*. Real balances M_t are constructed according to

$$M_t = \frac{M1_t}{P_{C,t-1}},$$

where $P_{C,t}$ is the *correctly* measured implicit price deflator of aggregate consumption.

Investment: Cooley and Prescott (1995) point out that since the measure of the economy's capital stock consistent with the theoretical model includes all kinds of capital the corresponding measure of investment should be also defined as the sum over all kinds of investment. Hence, aggregate investment I_t equals the sum of *Fixed Private Investment*[23] $I_{P,t}$, *Government Investment*[24] $I_{G,t}$, *Durable Goods Expenditure*[25] $I_{d,t}$, the *Change in Inventories*[26] $I_{Inv,t}$ and *Net Exports*[27] $I_{NEX,t}$:

$$I_t = I_{P,t} + I_{G,t} + I_{D,t} + I_{Inv,t} + I_{NEX,t}.$$

All series were deflated by the corresponding implicit price deflators also provided by the NIPA.

[21] Bureau of Labor Statistics, Series Id. LNS12000000Q

[22] Bureau of Labor Statistics, Series Id. CES0500000036

[23] NIPA, Table 1.5.5. *Gross Domestic Product, Expanded Detail*, *Gross domestic private investment/Fixed investment*

[24] NIPA, Table 1.5.5. *Gross Domestic Product, Expanded Detail*, *Government consumption expenditures and gross investment/National defense, Nondefense and State and Local investment*

[25] NIPA, Table 1.5.5. *Gross Domestic Product, Expanded Detail*, *Personal consumption expenditures/Durable goods*

[26] NIPA, Table 1.5.5. *Gross Domestic Product, Expanded Detail*, *Gross domestic private investment/Change in inventories*

[27] NIPA, Table 1.5.5. *Gross Domestic Product, Expanded Detail*, *Net Exports*

2.2 Reparameterizations of the Model

Sometimes, especially when estimating the parameters of nonlinear models, the range of possible values of particular parameters can be restricted by imposing a reparameterization of the model. For example suppose that the econometrician is interested in the estimation of some parameter β about which it is known from economic theory that it lies in the open interval $(0,1)$. One possible way to restrict β to that interval is to reformulate the model by replacing β by $\frac{\exp(\gamma)}{1+\exp(\gamma)}$ or $\frac{\gamma^2}{1+\gamma^2}$, or another function with range $(0,1)$ and then estimate the new parameter γ. Of course, this procedure does not always work, since the applied transformation introduces a (further) nonlinearity into the model, which may lead to (additional) numerical difficulties. There is also another feature of such nonlinear reparameterizations the researcher should be aware of. In most cases the transformation affects the values of the usually applied test statistics.[28] For example it may be the case that γ turns out to be highly significant despite the fact that β is insignificant, if estimated directly.

To see that, consider a statistical model consisting of only one equation with only one parameter β. Without loss of generality we can assume that the model can be represented by the *elementary zero function* $f(\beta, y_t)$, where $f(.,.)$ is the value corresponding to the t-th observation. The vector y_t consists of the period t (or earlier) observations of the model variables at least one of which is endogenous.[29] The model imposes the following *population* moment restriction on the data:

$$E\{f(\beta, y_t)\} = 0, \tag{IV.2.1}$$

where $E(.)$ denotes the unconditional expectations operator. The corresponding *sample* moment condition is given by

$$g(\beta, Y_T) = \frac{1}{T}\sum_{t=1}^{T} f(\beta, y_t) = 0,$$

where T denotes the sample size and $Y_T = y_1, y_2, ..., y_T$. The GMM estimator is the value $\hat{\beta}$ that makes $g(\hat{\beta}, Y_T)$ as close as possible to zero. Now assume that β is *asymptotically identified*[30] and that the assumptions underlying Proposition 14.1 in Hamilton (1994) are met. Then the GMM estimator $\hat{\beta}_T$ converges in probability to β and is asymptotically normally distributed:

$$\sqrt{T}(\hat{\beta}_T - \beta) \quad \xrightarrow{d} \quad \mathcal{N}(0, V), \tag{IV.2.2}$$

[28] Hamilton (1994) pp. 146 also points out that in general the standard errors derived from the reparameterized model will be different from the corresponding standard errors in the original model.

[29] If there are more than one endogenous variables y_t will also include the instruments corresponding to the t-th observation.

[30] See Davidson and MacKinnon (1993), Ch. 9.5, pp.367-380 for the notion of asymptotic identification.

where V denotes the asymptotic variance of $\hat{\beta}_T$.

Now assume that the model just described has been reparameterized by substituting β by a differentiable monotonic function $a^{-1}(\gamma)$ with the property $a(\beta) = \gamma$ and then γ was estimated by GMM. Lemma 2.5 in Hayashi (2000), (IV.2.2) together with the assumptions made on $a(.)$ implies

$$\sqrt{T}(\hat{\gamma}_T - \gamma) = \sqrt{T}(a(\hat{\beta}_T) - a(\beta)) \quad \overset{d}{\rightarrow} \quad \mathcal{N}(0, a'(\beta)Va'(\beta)^T). \tag{IV.2.3}$$

Assume that the sample size T is sufficiently large, so that $\hat{\gamma}_T \approx a(\hat{\beta}_T)$. Then, based on the asymptotic distributions of $\hat{\beta}_T$ and $\hat{\gamma}_T$ the t-ratios of both parameters can be approximated by:

$$t_\beta = \frac{\hat{\beta}_T}{\sqrt{\hat{V}_T}/\sqrt{T}}, \qquad t_\gamma = \frac{a(\hat{\beta}_T)}{a'(\hat{\beta}_T)\sqrt{\hat{V}_T}/\sqrt{T}},$$

where $\hat{V}_T$ is a consistent estimator of V. For example, if $a(\beta)$ is given by $\beta^{\frac{1}{k}}$ with $k > 1$ one obtains the following approximation of t_γ:

$$t_\gamma = \frac{k\hat{\beta}_T}{\sqrt{\hat{V}_T}/\sqrt{T}}.$$

Hence, if k is large enough the standard t-test will yield the result that $\hat{\gamma}_T$ is significant while $\hat{\beta}_T$ isn't.

2.3 Estimation

The econometric methodology adopted here is a variant of Hansen's (1992) GMM procedure extensively used in the RBC literature.[31] According to this technique the model's parameters or at least a subset of them are chosen so as to satisfy a bunch of moment restrictions implied by the equilibrium conditions of the theoretical economy characterized by inflation aversion, market share competition and adjustment costs of capital. Since the models presented in the previous chapter neglect the existence of long run growth trends in most macroeconomic aggregates, some modifications should be made to make the theoretical economy consistent with the long run behavior of these same variables in a typical industrialized economy.

Assume that on average consumption C_t, output Y_t, the real wage $\frac{W_t}{P_t}$, the stock of capital K_t, investment I_t, search activity s_t, real balances $\frac{M_t}{P_{t-1}}$ and the present value of firm's profits Ω_t each grow at the same rate as the exogenous and deterministic technological progress:

$$A_{t+1} = aA_t, \qquad a > 1.$$

[31] See for example Christiano and Eichenbaum (1992), Burnside *et. al.* (1993), Burnside and Eichenbaum (1994) and many others.

Thus, $y_t = \frac{Y_t}{A_t}$, $c_t = \frac{C_t}{A_t}$, $\tilde{w}_t = \frac{W_t/P_t}{A_t}$, $k_t = \frac{K_t}{A_t}$, $\tilde{s}_t = \frac{s_t}{A_t}$, $m_t = \frac{M_t/P_{t-1}}{A_t}$ and $\omega_t = \frac{\Omega_t}{A_t}$ are stationary. The real interest rate, the inflation factor π_t, hours N_t and the markup mu_t are assumed to be stationary. To ensure the existence of a balanced growth path, the utility function should be modified as follows

$$U = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\eta}}{1-\eta} + \phi_m \frac{(M_t/P_t)^{1-\chi}}{1-\chi} - A_t^{1-\eta} \frac{b}{2} N_t^2 - A_t^{\frac{\eta-\alpha-1}{1+\alpha}} \frac{\varrho}{\alpha} \frac{\pi_t}{s_t^{\alpha}} \right) \right\},$$

while the function $g(.,.)$ governing the evolution of market share should be redefined as

$$g\left(\frac{P_{i,t}}{P_t}, s_t\right) = \exp\left(\left(1 - \frac{P_{i,t}}{P_t}\right) \cdot \frac{s_t}{A_t}\right).$$

The former assumption ensures that both sides of the optimality conditions with respect to labor supply and switching efforts:

$$\begin{aligned} A_t^{\frac{\eta-\alpha-1}{1+\alpha}} \varrho s_t &= C_t^{\frac{\eta}{1+\alpha}} \pi_t^{\frac{1}{1+\alpha}}, \\ A_t^{1-\eta} b N_t &= C_t^{-\eta} \frac{W_t}{P_t} \end{aligned} \tag{IV.2.1}$$

exhibit the same long run trend. The latter assumption ensures that both sides of the equilibrium condition

$$mu_t = \frac{-\theta}{1-\theta+g_1(1,s_t)\frac{\Omega_t}{D_t}} = \frac{-\theta}{1-\theta-\frac{s_t}{A_t}\frac{\Omega_t}{D_t}} \tag{IV.2.2}$$

are stationary.

Two-Equation GMM Estimation of α, b and ϱ

An additional issue arising here regards the treatment of aggregate search efforts s_t within the economy wide income identity derived from the National Accounts in the previous paragraphs. Search activity is not directly observable but appears in the theoretical resource constraint. By construction, however, the National Accounts only consider private and public consumption as well as different kinds of private and public investment (see above). In other words, there aren't any components of the actual aggregate income identity which s_t can be (more or less) directly linked to. For this reason a further assumption regarding the relationship between search efforts and one or more of the variables appearing in the actual aggregate resource constraint is needed. I assume that observable private and government nondurables consumption expenditure equals the sum of the model's consumption and switching efforts. Search and switching efforts mainly consist of tasting and trying new products and comparing them to already known ones. In most cases these activities involve (or take the form of) using different kinds of services provided by firms. Therefore it is not implausible to assume that s_t

largely appears in measured consumption of nondurables while other observable variables are less strongly affected. Nevertheless, such an assumption should be viewed as a very rough approximation of the mechanisms actually at work, and it is left for future research to provide measures allowing a more precise identification of the search and switching components of consumption, investment and leisure. A consequence of this assumption is that aggregate consumption in the model does not correspond to the actual aggregate nondurables consumption expenditure in the U.S. economy. Therefore it is reasonable to view C_t as an unobservable variable. In sum, one ends up with two variables - consumption C_t and search efforts s_t, which are not directly observable and have to be replaced by functions of one or more observable ones before proceeding to the estimation step.

To do that, I use the first and the second equation in (IV.2.1) to eliminate s_t and C_t from the remaining equilibrium conditions. Then (IV.2.2) is used to eliminate Ω_t from the firms intertemporal optimality condition evaluated at the symmetric equilibrium:

$$\Omega_t = E_t\left\{\beta\frac{C_{t+1}^{-\eta}}{C_t^{-\eta}}\left(\frac{mu_{t+1}-1}{mu_{t+1}}\right)D_{t+1}\right\} + E_t\left\{\beta\frac{C_{t+1}^{-\eta}}{C_t^{-\eta}}\Omega_{t+1}\right\}, \tag{IV.2.3}$$

where $D_t = C_t + s_t + I_t = Y_t$ denotes overall demand. (IV.2.3) can then be transformed into the following equation depending only on observable macroeconomic variables:

$$E_t\left\{\beta\left(\frac{W_t/P_t}{W_{t+1}/P_{t+1}}\right)\left(\frac{A_{t+1}}{A_t}\right)^{1-\eta}\left(\frac{N_{t+1}}{N_t}\right)\left(\frac{mu_{t+1}-1}{mu_{t+1}}\right)\left(\frac{Y_{t+1}}{Y_t}\right)\left(\frac{\left(\frac{W_t/P_t}{A_t}\right)^{\frac{1}{1+\alpha}}\pi_t^{\frac{1}{1+\alpha}}}{N_t^{\frac{1}{1+\alpha}}(mu_t(1-\theta)+\theta)}\right)\frac{1}{\varrho b^{\frac{1}{1+\alpha}}}\right\}+$$

$$+E_t\left\{\beta\left(\frac{W_t/P_t}{W_{t+1}/P_{t+1}}\right)^{1+\frac{1}{1+\alpha}}\left(\frac{A_{t+1}}{A_t}\right)^{1-\eta+\frac{1}{1+\alpha}}\left(\frac{N_{t+1}}{N_t}\right)^{1+\frac{1}{1+\alpha}}\left(\frac{mu_{t+1}(1-\theta)+\theta}{mu_t(1-\theta)+\theta}\right)\left(\frac{Y_{t+1}}{Y_t}\right)\left(\frac{\pi_t}{\pi_{t+1}}\right)^{\frac{1}{1+\alpha}}-1\right\}=$$

$$= E_t\{\tilde{h}_{1,t}(\Psi_T)\} = 0,$$

where $\tilde{h}_{1,t}(.)$ represents the forecasting error corresponding to (IV.2.3).[32] $\Psi_T = (\psi_1, \psi_2, \psi_3)$ with $\psi_1 = (1+\alpha)^{-1}$, $\psi_2 = \varrho^{-1}b^{-\frac{1}{1+\alpha}}$ and $\psi_3 = b^{-\frac{1}{\eta}}$ denotes the vector of parameters to be

[32] $\frac{W_t/P_t}{A_t} = \left(\frac{W_t}{P_t}\right)^{cyc}$ is identified as the cyclical component of the real wage. Under the assumption that the real wage evolves according to

$$\frac{W_t}{P_t} = \frac{\tilde{W}}{P}a_{\frac{W}{P}}^t e^{\varepsilon_t}, \qquad a_{\frac{W}{P}} > 1$$

where $\frac{\tilde{W}}{P}$ is its starting value and ε_t is stationary, $\left(\frac{W_t}{P_t}\right)^{cyc}$ is defined by

$$\left(\frac{W_t}{P_t}\right)^{cyc} = \frac{\tilde{W}}{P}e^{\varepsilon_t}.$$

$\frac{\tilde{W}}{P}$ and $a_{\frac{W}{P}}$ were estimated by OLS. The cyclical component of any non-stationary variable in this chapter is defined and then estimated in a similar manner.

estimated. T is the sample size. Under rational expectations it should be the case that

$$E_t\{\tilde{h}_{1,t}(\Psi_{1,T})\mathbf{z}_{1,t}\} = 0,$$

where $\mathbf{z}_{1,t}$ is a vector of instruments. From a theoretical point of view any variable known at the time of forecasting (time t) or earlier can be considered a valid instrument. However, if $\tilde{h}_{1,t}(\Psi_{1,T})$ is serially correlated and at the same time endogenous variables are included in the vector $\mathbf{z}_{1,t}$, it is more appropriate to use values of $\mathbf{z}_{1,t}$ lagged by one ore more periods in order to ensure the validity of the instruments. Since there are plenty of potential instrument with respect to (IV.2.3) but no theoretical or *a priori* guidance on which of them should be used, I choose the instruments implying the most plausible and robust results. In the case of exact identification these are a constant and the markup mu_t. Thus the baseline estimation is based on:

$$E_t\{\tilde{h}_{1,t}(\Psi_{1,T})\mathbf{z}_{1,t}\} = \begin{pmatrix} E_t\{\tilde{h}_{1,t}(\Psi_T)\} \\ E_t\{\tilde{h}_{1,t}(\Psi_T)mu_t\} \end{pmatrix} = \begin{pmatrix} E_t\{h_{1,t}(\Psi_T)\} \\ E_t\{h_{2,t}(\Psi_T)\} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \text{(IV.2.4)}$$

where the purpose of the second to last equation is just to saves on notation. Note that $\mathbf{z}_{1,t}$ consists of two instruments - a constant, corresponding to the first equation in IV.2.4, and the markup, corresponding to the second equation in IV.2.4. After applying the law of iterated expectations on the last two equations one can derive the following sample moment restrictions (resulting from (IV.2.3)):

$$\begin{pmatrix} E\{E_t\{\tilde{h}_{1,t}(\Psi_T)\}\} \\ E\{E_t\{\tilde{h}_{1,t}(\Psi_T)mu_t\}\} \end{pmatrix} = \begin{pmatrix} E\{h_{1,t}(\Psi_T)\} \\ E\{h_{2,t}(\Psi_T)\} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \quad \text{(IV.2.5)}$$

The last estimation equation stems from the aggregate resource constraint:

$$Y_t = C_t + s_t + I_t$$

and can be represented in the following convenient for estimation form:

$$\underbrace{\left(\frac{Y_t}{I_t}\frac{I_t}{A_t}\left(\frac{W_t/P_t}{A_t}\right)^{\frac{-1}{\eta}} N_t^{\frac{1}{\eta}} - \frac{1}{\varrho b^{\frac{1}{1+\alpha}}}\left(\frac{W_t/P_t}{A_t}\right)^{\frac{1}{1+\alpha}-\frac{1}{\eta}} N_t^{\frac{1}{\eta}-\frac{1}{1+\alpha}} \pi_t^{\frac{1}{1+\alpha}} - \left(\frac{W_t/P_t}{A_t}\right)^{-\frac{1}{\eta}} N_t^{\frac{1}{\eta}} \frac{I_t}{A_t}\right)}_{:=\tilde{h}_{2,t}} = \frac{1}{b^{\frac{1}{\eta}}}.$$

The parameter $\psi_3 = \frac{1}{b^{\frac{1}{\eta}}}$ is identified as the unconditional mean of $\tilde{h_{2,t}}$:

$$E\left\{\tilde{h}_{2,t} - \frac{1}{b^{\frac{1}{\eta}}}\right\} = E\{h_{3,t}(\Psi_T)\} = 0, \quad \text{(IV.2.6)}$$

where the second to last equation again economizes on notation.

(IV.2.5) and (IV.2.6) are the three equations used to estimate the parameter vector Ψ for given values of β, θ and η. It should be noted that, as Burnside *et al.* (1993) point out, ignoring part of the moment restrictions implied by the theoretical model affects the asymptotic efficiency of the GMM estimator but not its consistency. As I show below, by using only (IV.2.5) to estimate the first two elements of Ψ I arrive at similar results with regard to α and $\varrho^{-1}b^{-\frac{1}{1+\alpha}}$ as when (IV.2.6) is taken into account. The Euler equation resulting from the optimal decision with respect to next period money balances can be used to estimate the utility parameter ϕ_m, while the Euler equation for optimal savings can be used to estimate some other parameter e.g. the rate of depreciation or a parameter representing the initial level of technology, or parameters describing the adjustment costs of capital. Since I eliminate consumption via the optimality condition with respect to labor supply, the latter two Euler equations do not involve any of the parameters in Ψ. At the same time (IV.2.5) and (IV.2.6) are independent of ϕ_m, the depreciation rate υ and the parameters of the production as well as the adjustment cost function. Hence, the point estimator of Ψ_T as well as its variance are independent of whether the last two Euler equations are included into the GMM procedure or not.[33] To control for heteroscedasticity and autocorrelation in the moment restrictions I use the Newey-West estimator of the weighting matrix with the Bartlett lag window. To the best of my knowledge, the existing literature does not provide much consistent and useful guidance on how to choose the lag truncation. Therefore, since quarterly data is used, the Bartlett window is truncated after four lags. For the sake of completeness I also experiment with different assumptions on the autocorrelation structure of the moment restrictions. As shown below, the results are robust with regard to the lag truncation.

The model is reparameterized as follows: Instead of estimating ψ_1, ψ_2 and ψ_3 directly, the three coefficients are replaced by the continuous, strictly increasing functions $\tilde{\psi}_1^5$, $\tilde{\psi}_2^{15}$ and $\tilde{\psi}_3^{15}$ respectively. Then $\tilde{\psi}_1$, $\tilde{\psi}_2$ and $\tilde{\psi}_3$ are estimated by GMM *via* (IV.2.5) and (IV.2.6). As pointed in subsection (2.2) such a reparameterization is not only made for reasons of convenience but also affects the covariance matrix of the estimated parameters. However, from a scientific point of view it should not be seen as an attempt to artificially achieve some degree of significance but as a possible way to discover the parametric structure of the model

[33] To see that consider a GMM procedure involving only the following two restrictions

$$f_{1,T}(\gamma_1) = E\{h_{1,t}(\gamma_1)\} = 0, \qquad f_{2,T}(\gamma_2) = E\{h_{2,t}(\gamma_2)\} = 0,$$

where the scalars γ_1 and γ_2 are the unknown parameters and T is the sample size. Then it is easy to verify that in the case of no serial correlation the usually employed consistent estimator of the variances of the two parameters (see Christiano and Eichenbaum (1992)) is given by

$$Var_T(\gamma_1) = \frac{1}{T}\frac{f''_{1,T}}{f'_{1,T}f'_{1,T}}, \qquad Var_T(\gamma_2) = \frac{1}{T}\frac{f''_{2,T}}{f'_{2,T}f'_{2,T}},$$

where the derivatives involved are evaluated at the GMM estimates of γ_1 and γ_2 respectively. When $h_{1,t}(\gamma_1)$ and $h_{2,t}(\gamma_2)$ are autocorrelated, $Var_T(\gamma_1)$ and $Var_T(\gamma_2)$ only depend on the *own* serial correlation of $h_{1,t}(\gamma_1)$ and $h_{2,t}(\gamma_2)$ respectively but not on any cross correlation terms.

that is *(most)* consistent with the empirical data and so leads to *(the most)* significant and *(the most)* robust estimates. Moreover, to specify the last term in the utility function as $\frac{\varrho}{\alpha}\frac{\pi_t}{s_t^{\alpha}}$ is as arbitrary as the specification

$$\left(\frac{\varrho}{(1+\tilde{\alpha})^5-1}\right)\cdot\left(\frac{\pi_t}{s_t^{((1+\tilde{\alpha})^5-1)}}\right),$$

where $(1+\tilde{\alpha})$ is a free parameter that can be estimated by GMM. Therefore, there is no *a priori* reason not to experiment with different assumptions on the functional form of the model's building blocks. The same reasoning can be applied to any parameter in the theoretical model.

Results: Table (IV.1) displays the results of the GMM procedure in the exactly identified case characterized by the three moment restrictions (IV.2.5) and (IV.2.6). As can also be seen the statistical significance as measured by the t-ratios increases slightly as the truncation lag gets lower. The implied value of α is very close to that assumed in the numerical simulations in the previous chapter. Table IV.2 displays the implied values of α, b and ϱ for different values of the short run elasticity of demand θ and the average markup mu^*. The lower the former and the higher the latter, the higher the estimates of α. Note that any positive value of α implies the same *qualitative* predictions of the theoretical model with lower values of α leading to stronger responses of search activity to changes in consumption and inflation and thus to more pronounced deviations from trend of the macroeconomic aggregates. According to Table (IV.2) the parameter combinations ($mu^* = 1.2$; $\theta \in [0.2, 1]$), ($mu^* = 1.3$; $\theta \in [0.8, 1.4]$) and ($mu^* = 1.4$; $\theta \in [1, 1.4]$) correspond to point estimates of α lower than one. As shown in chapter 3, values of α in that range lead to strong and persistent reactions of the macroeconomic aggregates to non-autocorrelated monetary as well as technology shocks. For example with the parameter combination $mu^* = 1.2$, $\theta = 0.8$ and $\alpha = 0.3$ that model implies that an one-time monetary shock causes output to remain for more than twelve quarters above average, jumping by 0.6 per cent initially and then adjusting slowly to its long run level. Relatively high values of θ and at the same time relatively low markups mu^* tend to lead to negative estimates of α, which are at odds with the assumptions underlying the theoretical economy. The estimates of b and ϱ lie in all cases in the plausible range with b being almost insensitive to changes in θ and mu^*. Since for the production elasticity of capital to be positive, the average markup should be smaller than 1.48, the sensitivity analysis only covers the range $mu^* \in [1.1; 1.4]$ for the steady state markup. All the estimates shown in Table IV.2 are different from zero at least at the 10% level of significance.

Table IV.1:

Exactly identified GMM estimator of Ψ *via* (IV.2.5) and (IV.2.6). Sample 1964:3 - 2007:3

Parameter	t-ratio (4)	t-ratio (3)	t-ratio (2)	t-ratio (1)	Implied Values of:
Instruments in (IV.2.5): $constant$, mu_t					
$\psi_1^{\frac{1}{5}} = 0.9772$	2.35**	2.42**	2.56**	2.91**	$\alpha = 0.1224$
$\psi_2^{\frac{1}{15}} = 0.6271$	1.59	1.64*	1.73*	1.97**	$b = 5.40 \times 10^{-6}$
$\psi_3^{\frac{1}{15}} = 1.4982$	631.43**	726.06**	885.35**	1245.59**	$\varrho = 540 \times 10^5$
Instruments in (IV.2.5): $constant$, mu_{t-1}					
$\psi_1^{\frac{1}{5}} = 0.8973$	1.97*	2.02**	2.13**	2.45**	$\alpha = 0.7188$
$\psi_2^{\frac{1}{15}} = 0.6950$	2.05**	2.11**	2.22*	2.56**	$b = 5.40 \times 10^{-6}$
$\psi_3^{\frac{1}{15}} = 1.4982$	631.43**	726.05**	885.35**	1245.59**	$\varrho = 272 \times 10^3$
Instruments in (IV.2.5): $constant$, mu_{t-2}					
$\psi_1^{\frac{1}{5}} = 0.9001$	2.25**	2.31**	2.45**	2.82**	$\alpha = 0.6922$
$\psi_2^{\frac{1}{15}} = 0.6929$	2.31**	2.38**	2.52*	2.90**	$b = 5.40 \times 10^{-6}$
$\psi_3^{\frac{1}{15}} = 1.49$	631.43**	726.05**	885.35**	1245.59**	$\varrho = 318 \times 10^3$

"t-ratio (n)" := t-ratio computed by truncating the Bartlett window after n lags.

"**" := significance at the 5% level.

"*" := significance at the 10% level.

Benchmark specification: $\theta = 0.8$, $mu^* = 1.2$, $\eta = 2$, $\beta = 0.991$. The program used is **"delta_7_gmm_b.g"**.

Table IV.2:
Exactly identified GMM estimator of Ψ *via* (IV.2.5) and (IV.2.6). Sample 1964:3 - 2007:3

θ	impl. α	impl. b	impl. ϱ
		$mu^* = 1.1$	
0.2	-0.3359	5.40e-006	1.07e+012
0.4	-0.3688	5.40e-006	4.20e+012
0.6	-0.3995	5.40e-006	1.71e+013
0.8	-0.4282	5.40e-006	7.31e+013
1.0	-0.4552	5.40e-006	3.28e+014
1.2	-0.4805	5.40e-006	1.55e+015
1.4	-0.5045	5.40e-006	7.77e+015
1.6	-0.5270	5.40e-006	4.13e+016
1.8	-0.5484	5.40e-006	2.34e+017
2.0	-0.5687	5.40e-006	1.42e+018
		$mu^* = 1.2$	
0.2	0.5930	5.40e-006	5.41e+005
0.4	0.4093	5.40e-006	2.26e+006
0.6	0.2545	5.40e-006	1.04e+006
0.8	0.1224	5.40e-006	5.40e+006
1.0	0.0082	5.40e-006	3.15e+008
1.2	-0.0914	5.40e-006	2.10e+009
1.4	-0.1793	5.40e-006	1.64e+010
1.6	-0.2573	5.40e-006	1.53e+011
1.8	-0.3271	5.40e-006	1.75e+012
2.0	-0.3900	5.40e-006	2.52e+013
		$mu^* = 1.3$	
0.2	2.2791	5.40e-006	2809.99
0.4	1.5946	5.40e-006	11651.57
0.6	1.1091	5.40e-006	55795.64
0.8	0.7467	5.40e-006	316001.67
1.0	0.4658	5.40e-006	2.18e+006
1.2	0.2415	5.40e-006	1.90e+007
1.4	0.0583	5.40e-006	2.19e+008
1.6	-0.0942	5.40e-006	3.58e+009
1.8	-0.2235	5.40e-006	8.93e+010
2.0	-0.3345	5.40e-006	3.81e+012
		$mu^* = 1.4$	
0.2	6.3063	5.40e-006	184.65
0.4	3.6491	5.40e-006	746.52
0.6	2.2918	5.40e-006	3623.68
0.8	1.4678	5.40e-006	21972.23
1.0	0.9142	5.40e-006	175327.99
1.2	0.5165	5.40e-006	1.97e+006
1.4	0.2168	5.40e-006	3.46e+007
1.6	-0.0172	5.40e-006	1.08e+009
1.8	-0.2055	5.40e-006	7.46e+010
2.0	-0.3605	5.40e-006	1.55e+013

Robustness check of the estimates with respect to the choice of θ and mu^*. GMM estimation performed under the assumption: $\eta = 2$, $\beta = 0.991$. The program used is **"delta_7_gmm_b.g"**

Single Equation GMM

To check the robustness of the procedure one can also estimate a subset of Ψ, $\Psi_a = (\psi_1, \psi_2)$, based only on equation (IV.2.3) and the resulting moment restrictions (IV.2.5). In this case one do not need to assume that the observable consumption of nondurables equals the sum of the model's consumption and search activity.

Tables IV.3 and IV.4 display the results obtained by estimating the two free parameters of equation (IV.2.3). As can be seen they are very similar to that obtained by using the three moment restrictions defined by (IV.2.5) and (IV.2.6). Again there is a tendency for α to get larger when the short run elasticity of demand gets lower. Note that neither b nor ϱ can be identified since the estimated parameter ψ_2 is defined as $\varrho^{-1} b^{-\frac{1}{1+\alpha}}$.

Table IV.3:
Exactly identified GMM estimator of Ψ_a *via* (IV.2.3). Sample 1964:3 - 2007:3

Parameter	t-ratio (4)	t-ratio (3)	t-ratio (2)	t-ratio (1)	Implied Values of:
			Instruments: $constant$, mu_t		
$\psi_1^{\frac{1}{5}} = 0.9772$	2.34**	2.42**	2.55**	2.91**	$\alpha = 0.1224$
$\psi_2^{\frac{1}{15}} = 0.6271$	1.59*	1.64*	1.73*	1.97**	
			Instruments: $constant$, mu_{t-1}		
$\psi_1^{\frac{1}{5}} = 0.8973$	1.97**	2.02*	2.13**	2.45**	$\alpha = 0.7188$
$\psi_2^{\frac{1}{15}} = 0.6950$	2.05**	2.11**	2.22**	2.56**	
			Instruments: $constant$, mu_{t-2}		
$\psi_1^{\frac{1}{5}} = 0.9001$	2.25**	2.32**	2.44**	2.81**	$\alpha = 0.6922$
$\psi_2^{\frac{1}{15}} = 0.6929$	2.31**	2.38**	2.51**	2.90**	

"t-ratio (n)" := t-ratio computed by truncating the Bartlett window after n lags.

"**" := significance at the 5% level.

"*" := significance at the 10% level.

Benchmark specification: $\theta = 0.8$, $mu^* = 1.2$, $\eta = 2$, $\beta = 0.991$. The program used is **"delta_6_gmm_a1.g"**.

Table IV.4:
Exactly identified GMM estimator of Ψ_a *via* (IV.2.3). Sample 1964:3 - 2007:3

	impl. α			
θ	$mu^* = 1.1$	$mu^* = 1.2$	$mu^* = 1.3$	$mu^* = 1.4$
0.2	-0.3359	0.5930	2.2791	6.3063
0.4	-0.3688	0.4093	1.5946	3.6491
0.6	-0.3995	0.2545	1.1091	2.2918
0.8	-0.4282	0.1224	0.7467	1.4678
1.0	-0.4552	0.0082	0.4658	0.9142
1.2	-0.4805	-0.0914	0.2415	0.5165
1.4	-0.5045	-0.1793	0.0583	0.2168
1.6	-0.5270	-0.2573	-0.0942	-0.0172
1.8	-0.5484	-0.3271	-0.2235	-0.2055
2.0	-0.5687	-0.3900	-0.3345	-0.3605

Robustness check of the estimates with respect to the choice of θ and mu^*. GMM estimation performed under the assumption: $\eta = 2$, $\beta = 0.991$. The program used is **"delta_6_gmm_a1.g"** .

Overidentifying Restrictions

The exactly identified system (IV.2.5)-(IV.2.6) does not provide a formal test of the theoretical model. To fill this gap I consider a GMM procedure based on an overidentified system and perform the so called *J*-test of the validity of the overidentifying instruments developed by Hansen(1982). In such a system the number of restrictions is greater than the number of unknown parameters. According to economic theory any variable with time index less than or equal to t is a valid instrument for estimating parameters in (IV.2.3), as it belongs to the information set at the time agents build their expectations. The results obtained with different sets of instruments are displayed in Table (IV.5). x_t^{cyc} denotes the cyclical component of a variable x_t. In all three cases the *J*-statistic lies below the corresponding critical value. The experiments performed[34] with combinations of instruments as well as truncation lags different from that shown in Table (IV.5) also provided *J*-statistic-values which did not reject the overidentifying restrictions. For each set of instruments, varying θ and mu^*, implied a behavior of the estimates which was almost identical to that depicted in Table (IV.4). The inclusion of (IV.2.6) as a further moment restriction also had a negligible effect on the estimates, their significance as well as the value of the *J*-statistic. Perhaps for numerical

[34] The results are not reported in the here.

reasons and/or because of multicolinearity it is not possible to perform the estimation with *any arbitrary* combination of instruments. There were cases in which the algorithm just did not converge. But in the cases it did, the estimates were significant at least at the 10% level and the J-statistic lay below the corresponding critical value. Table (IV.5) also indicates that the estimate of α is relatively sensitive with respect to the choice of instruments, varying from slightly above zero to values above 10. Excluding the value of land from the definition of the economy wide stock of capital did not have a significant impact on the results. However, there were more cases in which the estimation algorithm failed to converge.

Table IV.5:
Overidentified GMM estimator of ψ_1 and ψ_2 *via* (IV.2.3). Sample 1964:3 - 2007:3

Instruments: $constant,\ mu_t,\ \left(\frac{M_{t-1}}{P_{t-2}}\right)^{cyc},\ \pi_{t-1},\ \left(\frac{W_{t-1}}{P_{t-1}}\right)^{cyc}$

Parameter	t-ratio (2)	Implied Values of:
$\psi_1^{\frac{1}{5}} = 0.9826$	3.74^{**}	$\alpha = 10.3050$
$\psi_2^{\frac{1}{15}} = 0.6151$	2.34^{**}	
J-statistic: 0.1769	df=3	5%-Critical Value: 7.81

Instruments: $constant,\ mu_{t-2},\ \left(\frac{M_{t-2}}{P_{t-3}}\right)^{cyc},\ \pi_{t-1},\ \left(\frac{W_{t-2}}{P_{t-2}}\right)^{cyc}$

Parameter	t-ratio (1)	Implied Values of:
$\psi_1^{\frac{1}{5}} = 0.7359$	1.67^{*}	$\alpha = 3.6340$
$\psi_2^{\frac{1}{15}} = 0.7842$	4.54^{**}	
J-statistic: 1.9558	df=3	5%-Critical Value: 7.81

Instruments: $constant,\ mu_t,\ \left(\frac{M_{t-2}}{P_{t-3}}\right)^{cyc},\ \pi_t,\ \frac{W_t/P_t}{W_{t-1}/P_{t-1}},\ I_t^{cyc},\ \frac{C_t}{C_{t-1}}$

Parameter	t-ratio (0)	Implied Values of:
$\psi_1^{\frac{1}{5}} = 0.7885$	2.77^{**}	$\alpha = 2.2800$
$\psi_2^{\frac{1}{15}} = 0.7597$	5.30^{**}	
J-statistic: 5.9586	df=5	5%-Critical Value: 11.07

"t-ratio (n)" := t-ratio computed by truncating the Bartlett window after n lags.

"**" := significance at the 5% level.

"*" := significance at the 10% level.

"df" := degrees of freedom.

"J-statistic" := proposed by Hansen(1982) and distributed according to the $\chi^2(m)$-distribution, where m denotes the number of overidentifying restrictions.

Benchmark specification: $\theta = 0.8$, $mu^* = 1.2$, $\eta = 2$, $\beta = 0.991$. The file used is **"delta_7_gmm_b.wf1"** .

GMM Estimation Based on the Fixed Capital Model

In the inflation-aversion model with fixed capital overall demand equals the sum of nondurables consumption expenditure C_t and switching costs s_t. In order to put the model in a form suitable for GMM estimation, one can use equations (IV.2.1) in the same manner as in the previous two subsections to eliminate the unobservable variables C_t and s_t.

The moment restrictions (IV.2.5) and (IV.2.6) were modified by replacing GNP as measure of aggregate output by the sum of private and government nondurables consumption expenditure. The appropriate measure of labor income is obtained by multiplying the labor income series used in the previous subsections by the ratio of aggregate consumption of nondurables to GNP. Since the estimated version of the model implies that the aggregate production function is given by

$$Y_t = Z_t N_t$$

the markup can be deduced from

$$mu_t = \frac{Z_t}{W_t/P_t} = \frac{Y_t}{Labor\ Income_t},$$

where $Y_t = C_t + s_t$ corresponds to aggregate consumption of nondurables and $\frac{W_t}{P_t}$ denotes the real wage.

The implied values of α and ϱ are most similar to that obtained in the model with endogenous capital under the assumption of a relatively high average markup, $mu^* = 1.4$. The results are not reported here but are available upon request.

3 Business Cycles Moments

In order to evaluate the goodness of a particular business cycle model, it has become a common practice to compare its quantitative predictions with respect to a set of second moments with the same set of moments found in empirical data. The same strategy is chosen in the current chapter. Since the goal of such an exercise is not the examination of the qualitative properties of the model, but rather the computation of its exact quantitative predictions, it is desirable to calibrate it in a more sophisticated manner. First, the overall rate of depreciation υ is set to 0.00708 which is the value computed in section 2 by using the *correct* measure of the economy wide physical stock of capital. Second, unlike the previous sections, I do no set α to an arbitrary positive value, but resort to the GMM estimates provided in section 2. In particular, I perform the simulations with all economically plausible parameter combinations of α, θ and mu^* as given in Table IV.2 but present only the results obtained with the one implying the best fit between the theoretical and the empirical standard deviations $sd(x)$, where $sd(x)$ denotes the standard deviation of x. The ranges for θ and mu^* considered, $(0, 2]$ and $(1, 2]$ respectively, correspond to the empirical estimates with respect to these two parameters found in the literature.[35] Third, I do not make an attempt to estimate the properties of the Solow-residual based on the current model since they would be strongly affected by the choice of the steady state markup mu^*, but borrow the process for estimates provided by Gomme and Rupert (2006) obtained with US-data. The process estimated by them takes the form

$$\ln(Z_t) = 0.9641 \ln(Z_{t-1}) + \epsilon_t, \tag{IV.3.7}$$

where ϵ_t follows a White Noise process with standard deviation σ_ϵ equal to 0.0082. The implied unconditional standard deviation of the Solow-residual, σ_z, is given by

$$\sigma_z = \frac{\sigma_\epsilon}{\sqrt{1 - 0.9641^2}} = 0.03088.$$

The properties of the money supply process were estimated by fitting an AR(p) process to the growth rate of the aggregate M1. The process chosen by minimizing the *Akaike information criterion* is given by:[36]

$$g_{M1,t} = 0.0037^{**} + 0.5097^{**} g_{M1,t-1} + 0.2251^{**} g_{M1,t-2} + \tilde{u}_t, \tag{IV.3.8}$$

where $g_{M1,t}$ denotes the growth rate of M1,[37] $\tilde{u}_t$ the residual term and ** indicates significance at the 5% level. The estimated standard deviation of the unsystematic component of money

[35] Chapter 2 provides a brief review of that literature.

[36] I used quarterly data from 1970:Q1 through 2003:Q3. According to the Ljung-Box-Q statistic and White's heteroscedasticity test the estimated residuals display neither serial correlation nor heteroscedasticity.

[37] Note that the stochastic process generating $\tau_t = M_{t+1}/M_t$ introduced in chapter 2 can be identified as the AR(2) process in (IV.3.8) since

$$g_{M1,t} = \ln(M_t) - \ln(M_{t-1}) = \ln(\tau_{t-1}).$$

supply σ_u equals 0.0092. The unconditional mean and standard deviation of $g_{M1,t}$ take the values 0.0138 and 0.0125 respectively.

The elasticity of the first derivative of the adjustment cost of capital function ς is set either to the GMM estimate provided by Jerman (1998) given by -1/0.23 or to the value implying the empirically observable relation between the volatilities of output and investment.

To investigate the ability of the purely intrinsic mechanisms of the model to reproduce the observed business cycle patterns I first assume that the logarithm of total factor productivity $\ln(Z_t)$ as well as that of the growth factor of money supply $\ln(\tau_t)$ both follow a serially uncorrelated process with standard deviation equal to $\sigma_z = \sigma_\epsilon = 0.0082$ and $\sigma_\tau = \sigma_u = 0.0092$ respectively. Table IV.6 summarizes the results obtained with the following parameter choice: $mu^* = 1.2$, $\theta = 0.6$, $\alpha = 0.2545$ and $\varsigma = -1/0.23$ or $\varsigma = -0.07/0.23$. In spite of being driven by pure White Noise processes, the model is able to account for a substantial part of the observed first-order autocorrelations, the latter being more strongly understated in the case of a higher ς. All implied autocorrelations as well as cross correlations with output except that of inflation have the correct sign. The predicted contemporaneous correlation between the markup and output in the case of a higher ς equals -0.866 and is much closer to the estimates of Rotemberg and Woodford (1999), lying in the interval $[-0.372, -0.542]$, than to the value of -0.058 computed in section 2. A possible explanation of the difference between the empirical estimate obtained in the current chpter and that of Rotemberg and Woodford (1999) is that the latter construct a more precise proxy of the markup than I do. As Rotemberg and Woodford (1999) also show, their less sophisticated measures of the markup lead to much lower correlations with output, lying in the range $[-0.188, -0.273]$. Further favorable features of the model in the case $\varsigma = -0.07/0.23$ are that consumption and working hours are less volatile than output while the opposite holds with regard to real balances. ς was chosen so as for the model to be able to reproduce the observable relative standard deviation of investment. Unfortunately, in common with most sticky price models, the current one clearly overstates the volatility of the real wage. The implied standard deviation of that variable is about two times (for $\varsigma = -0.07/0.23$) and almost three times (for $\varsigma = -1/0.23$) larger than that of output while the empirically observable relative standard deviation of the real wage is slightly above 0.5. The model also implies a cross correlation between output and the real wage which is about two times larger than the one found in the data. The model also overstates the relative standard deviations of inflation and the markup.

If one sets the standard deviations of $\ln(Z_t)$ and $\ln(\tau_t)$ given by $\sigma_z = 0.03088$ and $\sigma_\tau = 0.0125$, the predicted standard deviations just become about three times larger without any notable changes in the implied relative standard deviations, autocorrelations and cross correlations with output.

In the second exercise performed I assume that $\ln(Z_t)$ and $\ln(\tau_t)$ evolve according to the AR(1) process in (IV.3.7) and the AR(2) process in (IV.3.8) respectively. Table IV.7 provides the simulation results. The first noteworthy feature is that now all autocorrelations and cross correlations have the correct sign. Second, the predictions of the model with respect to real balances and the rate of inflation get better: The autocorrelation of real balances as well as its correlation with output get for both values of ς very close to their respective empirical counterparts. The slight increase in the autocorrelations of investment and consumption as well as that in the standard deviation of output should be also characterized as shifts in the right direction.

Unfortunately, there are also important dimensions with respect to which the performance of the model worsens. For both values of ς hours become more volatile than output. The autocorrelation of output and wages also decrease by amounts which should not be characterized as negligible. Nevertheless, by imposing the processes (IV.3.7) and (IV.3.8) the overall fit between the model's predictions and the empirical observations improves.

Doubtless, the model with adjustment costs of capital is better able to account for the stylized business cycle facts than do many modern real- and monetary business cycle models and exhibits stronger endogenous mechanisms than most of those models. Yet, it is by no means perfect and future research should try to eliminate its shortcomings.

Table IV.6:
Theoretical and Empirical Second Moments *(Adjustment Costs of Capital Model)*

Variable	$sd(x)$	$sd(x)/sd(y)$	$acorr(x)$	$corr(x,y)$
Output				
$\varsigma = -1/0.23$	1.316	1.000	0.720	1.000
$\varsigma = -0.07/0.23$	1.163	1.000	0.561	1.000
US Data	**1.547**	**1.000**	**0.863**	**1.000**
Consumption				
$\varsigma = -1/0.23$	1.282	0.974	0.607	0.974
$\varsigma = -0.07/0.23$	0.816	0.701	0.446	0.979
US Data	**0.697**	**0.451**	**0.889**	**0.735**
Hours				
$\varsigma = -1/0.23$	1.553	1.180	0.589	0.807
$\varsigma = -0.07/0.23$	1.162	0.999	0.462	0.737
US Data	**1.329**	**0.859**	**0.874**	**0.898**
Real Wage				
$\varsigma = -1/0.23$	3.776	2.871	0.742	0.996
$\varsigma = -0.07/0.23$	2.501	2.150	0.614	0.983
US Data	**0.815**	**0.527**	**0.637**	**0.472**
Investment				
$\varsigma = -1/0.23$	0.648	0.492	0.607	0.974
$\varsigma = -0.07/0.23$	5.408	4.649	0.445	0.976
US Data	**7.168**	**4.634**	**0.733**	**0.367**
Real Balances				
$\varsigma = -1/0.23$	2.355	1.790	0.608	0.752
$\varsigma = -0.07/0.23$	1.566	1.347	0.446	0.587
US Data	**3.222**	**2.083**	**0.941**	**0.280**
Inflation				
$\varsigma = -1/0.23$	1.914	1.455	-0.121	-0.143
$\varsigma = -0.07/0.23$	1.565	1.345	-0.167	-0.179
US Data	**0.387**	**0.250**	**0.497**	**0.317**
Markups				
$\varsigma = -1/0.23$	3.909	2.971	0.733	-0.947
$\varsigma = -0.07/0.23$	2.485	2.136	0.576	-0.866
US Data	**0.538**	**0.348**	**0.727**	**-0.058**

$mu^* = 1.2$, $\theta = 0.6$, $\alpha = 0.2545$, serially uncorrelated exogenous processes with $\sigma_z = \sigma_\epsilon = 0.0082$ and $\sigma_\tau = \sigma_u = 0.0092$. ς denotes the elasticity of $\phi'\left(\frac{I_t}{K_t}\right)$ with respect to I_t/K_t (see section 4.3 of chapter 3). $sd(x)$ - standard deviation of x; $sd(x)/sd(y)$ - ratio of the standard deviation of x to that of output; $acorr(x)$ - first order autocorrelation of x; $corr(x,y)$ - contemporaneous correlation between x and output. The second moments refer to HP-filtered empirical and simulated data. The second moments implied by the model refer to averages over 300 simulations. Each simulated series consists of 135 observations. The program used is **"sim_cm2d6a_1cap_i.g"**.

Table IV.7:

Theoretical and Empirical Second Moments *(Adjustment Costs of Capital Model)*

Variable	$sd(x)$	$sd(x)/sd(y)$	$acorr(x)$	$corr(x,y)$
Output				
$\varsigma = -1/0.23$	1.944	1.000	0.652	1.000
$\varsigma = -0.047/0.23$	1.867	1.000	0.498	1.000
US Data	**1.547**	**1.000**	**0.863**	**1.000**
Consumption				
$\varsigma = -1/0.23$	1.683	0.866	0.656	0.999
$\varsigma = -0.047/0.23$	1.014	0.543	0.552	0.985
US Data	**0.697**	**0.451**	**0.889**	**0.735**
Hours				
$\varsigma = -1/0.23$	2.362	1.215	0.615	0.848
$\varsigma = -0.047/0.23$	2.052	1.099	0.381	0.833
US Data	**1.329**	**0.859**	**0.874**	**0.898**
Real Wage				
$\varsigma = -1/0.23$	5.483	2.820	0.634	0.979
$\varsigma = -0.047/0.23$	3.788	2.029	0.440	0.978
US Data	**0.815**	**0.527**	**0.637**	**0.472**
Investment				
$\varsigma = -1/0.23$	0.866	0.446	0.659	0.997
$\varsigma = -0.047/0.23$	8.664	4.640	0.484	0.999
US Data	**7.168**	**4.634**	**0.733**	**0.367**
Real Balances				
$\varsigma = -1/0.23$	0.880	0.453	0.888	0.464
$\varsigma = -0.047/0.23$	1.546	0.828	0.851	0.189
US Data	**3.222**	**2.083**	**0.941**	**0.280**
Inflation				
$\varsigma = -1/0.23$	0.822	0.423	0.613	0.833
$\varsigma = -0.047/0.23$	1.622	0.869	0.387	0.797
US Data	**0.387**	**0.250**	**0.497**	**0.317**
Markups				
$\varsigma = -1/0.23$	5.921	3.045	0.621	-0.917
$\varsigma = -0.047/0.23$	4.035	2.161	0.388	-0.879
US Data	**0.538**	**0.348**	**0.727**	**-0.058**

$mu^* = 1.2$, $\theta = 0.6$, $\alpha = 0.2545$, exogenous processes given by (IV.3.7) and (IV.3.8). ς denotes the elasticity of $\phi'\left(\frac{I_t}{K_t}\right)$ with respect to I_t/K_t (see section 4.3 of chapter 3). $sd(x)$ - standard deviation of x; $sd(x)/sd(y)$ - ratio of the standard deviation of x to that of output; $acorr(x)$ - first order autocorrelation of x; $corr(x,y)$ - contemporaneous correlation between x and output. The second moments refer to HP-filtered empirical and simulated data. The second moments implied by the model refer to averages over 300 simulations. Each simulated series consists of 135 observations. The program used is **"sim_cm2d6a_1cap_ii.g"**.

4 A Comparison with the New Keynesian Model

Based on the suggestion of Christiano *et al.* (1997) I approximate monetary policy by an AR(1) process with a coefficient of autocorrelation equal to 0.5. The technology shocks follows the AR(1) process estimated by Gomme and Rupert (2006). To bias the results towards a better performance of the New Keynesian model with adjustment costs of capital, I set the fraction of firms that are not able to adjust their prices at $\varphi = 0.75$. In both models the parameter of the adjustment-cost function ς is chosen to approximately match the observable relative deviation of investment. The second moments implied by the two models as well as that found in the date are given in figure IV.8. The Inflation Aversion model accounts better for the standard deviations of output, consumption and hours as well as the relative standard deviations of the latter two variables. The New Keynesian model is better able to mach the cyclical properties of inflation. With respect to the remaining variables both models perform equally well. Nevertheless, recall that setting φ at the much more realistic value 0.3 dramatically worsens the performance of the New Keynesian model. For example, the correlation between hours and output become negative. The standard deviation of output drops to 0.84 while inflation becomes more volatile than output. Furthermore, if we leave φ at 0.75 but assume that both exogenous processes are not serially correlated, the New Keynesian model implies that the standard deviation of output is 0.49, the autocorrelation of hours, wages and the markup are 0.043, 0.277 and 0.086 respectively and that hours are more than twice as volatile as output. Combining a low autocorrelation in the shocks with a low degree of price stickiness $\varphi = 0.3$ makes the predictions of the New Keynesian model even worse. More precisely, its predictions become *completely* at odds with the empirical observations.

Table IV.8:
New Keynesian model (NK) vs. Inflation Aversion model (IA)

Variable	$sd(x)$	$sd(x)/sd(y)$	$acorr(x)$	$corr(x,y)$
Output				
NK, $\varsigma = -0.063/0.23$	*1.118*	*1.000*	*0.605*	*1.000*
IA, $\varsigma = -0.045/0.23$	1.345	1.000	0.605	1.000
US Data	**1.547**	**1.000**	**0.863**	**1.000**
Consumption				
NK, $\varsigma = -0.063/0.23$	*0.828*	*0.741*	*0.628*	*0.994*
IA, $\varsigma = -0.045/0.23$	0.790	0.587	0.663	0.981
US Data	**0.697**	**0.451**	**0.889**	**0.735**
Hours				
NK, $\varsigma = -0.063/0.23$	*1.392*	*1.245*	*0.507*	*0.563*
IA, $\varsigma = -0.045/0.23$	1.263	0.939	0.365	0.641
US Data	**1.329**	**0.859**	**0.874**	**0.898**
Real Wage				
NK, $\varsigma = -0.063/0.23$	*2.621*	*2.344*	*0.532*	*0.927*
IA, $\varsigma = -0.045/0.23$	2.456	1.826	0.512	0.961
US Data	**0.815**	**0.527**	**0.637**	**0.472**
Investment				
NK, $\varsigma = -0.063/0.23$	*5.188*	*4.639*	*0.557*	*0.975*
IA, $\varsigma = -0.045/0.23$	6.248	4.646	0.588	0.998
US Data	**7.168**	**4.634**	**0.733**	**0.367**
Real Balances				
NK, $\varsigma = -0.063/0.23$	*0.796*	*0.711*	*0.852*	*0.547*
IA, $\varsigma = -0.045/0.23$	0.822	0.611	0.862	0.583
US Data	**3.222**	**2.083**	**0.941**	**0.280**
Inflation				
NK, $\varsigma = -0.063/0.23$	*0.734*	*0.657*	*0.496*	*0.731*
IA, $\varsigma = -0.045/0.23$	1.015	0.755	0.362	0.560
US Data	**0.387**	**0.250**	**0.497**	**0.317**
Markups				
NK, $\varsigma = -0.063/0.23$	*2.989*	*2.673*	*0.499*	*-0.701*
IA, $\varsigma = -0.045/0.23$	2.456	1.826	0.386	-0.744
US Data	**0.538**	**0.348**	**0.727**	**-0.058**

Parameterization of the Inflation Aversion model: $mu^* = 1.2$, $\theta = 0.6$, $\alpha = 0.2425$. Parameterization of the New Keynesian model: $\varphi = 0.75$. ς denotes the elasticity of $\phi'\left(\frac{I_t}{K_t}\right)$ with respect to I_t/K_t (see section 4.3 of chapter 3). $sd(x)$ - standard deviation of x; $sd(x)/sd(y)$ - ratio of the standard deviation of x to that of output; $acorr(x)$ - first order autocorrelation of x; $corr(x,y)$ - contemporaneous correlation between x and output. The second moments refer to HP-filtered empirical and simulated data. The second moments implied by the model refer to averages over 300 simulations. Each simulated series consists of 135 observations. The programs used are **"sim_cm2d6a_1cap_i.g"** and **"keynes_ac_as.g"**.

5 Conclusion

A set of parameters of the model with inflation aversion, market share competition and capital accumulation are estimated *via* GMM based on moment restrictions derived from the model. Of particular interest is the elasticity of the utility function with respect to search activity α. The data do not reject one of the crucial assumptions in the model regarding the range of α. The second moments of the model with adjustment costs of capital, computed under different assumptions on the autocorrelation structure of the exogenous processes, are then confronted with their empirical counterparts. The model is able to account for several important features of the observable business cycles pattern in the U.S. economy. Furthermore, even without any serial correlation in the exogenous variables, the model explains a substantial part of the observed autocorrelation in the main macroeconomic aggregates. Thus, the intrinsic mechanisms in the model are stronger and explain a larger part of the observable cyclical behavior of the macroeconomic variables than it is the case in most other business cycle models, including the New Keynesian one. The quantitative explanatory power of the latter stems to a large degree from the assumed high serial correlation of the exogenous processes and the high degree of price stickiness.

Chapter 5

Price Dispersion, Search and Monetary Policy

1 Introduction

The purpose of this chapter to develop a monetary model in which search activity in the goods market is modeled in a more explicit manner than in chapter 3. In particular, the economy developed here is characterized by price dispersion introduced exogenously *via* heterogeneity in productivity across firms. The price dispersion, in turn, generates an incentive for households to engage in search activity. On the one hand search efforts lead to transaction costs which reduce the (should be financed by) real balances accumulated in the previous period. On the other hand, a more intense search increases the probability for becoming a customer of a supplier charging relatively low prices.

The model provides a further rationale for the positive dependence of search efforts an the current level of inflation and consumption found in the Inflation Aversion model. A higher inflation erodes the value of individual nominal balances. Therefore, it becomes more important for consumers to find suppliers charging lower prices in order to at least partly compensate the negative effects of the higher inflation rate. A higher level of desired current consumption simply increases the marginal benefit of finding a cheaper supplier.

The impulse responses implied by the model have the sign predicted by the bulk of the SVAR literature. Unfortunately, their persistence is not consistent with the empirical evidence presented in chapter 1.

2 The Model

Goods Market Structure

There are n firms, all producing the same homogeneous good. In each period each household is randomly assigned to one of the suppliers. However, by engaging in search activity in the current period the typical household indexed by i can influence the probability $\tilde{x}_{i,l,t}$, $l = \{1, 2, ..., n\}$ of becoming a customer of each of the individual firms. As a consequence of a higher search intensity, the probability to be assigned to a store with above(below) average price gets smaller(larger) than $1/n$. More formally, $\tilde{x}_{i,l,t}$ is defined as:

$$\tilde{x}_{i,l,t} = \frac{\exp\left(\left(1 - \frac{P_{l,t}}{P_t}\right) s_{i,t}^{\gamma}\right)}{\sum_{j=1}^{n} \exp\left(\left(1 - \frac{P_{j,t}}{P_t}\right) s_{i,t}^{\gamma}\right)}, \qquad \gamma > 0 \tag{V.2.1}$$

where P_t denotes the overall price level $s_{i,t}$ represents the individual level of search in the goods market. According to this definition, a higher search activity induces an increase (fall) in the probability for becoming a customer of a firm that charges an above average (a below average) current price. Furthermore, $\tilde{x}_{i,l,t}$ is bounded between 0 and 1. Note that the actual (or potential) price dispersion in this framework can be also seen as an approximation of differences in quality between *almost* homogeneous products selling at the same nominal price.

There are two important ideas underlying the definition of $\tilde{x}_{i,l,t}$. First, in spite of the fact that the household is informed about the average price P_t as well as the distribution of individual prices, she doesn't know which supplier offers her the lowest price or the best conditions. In many cases it is not immediately obvious whether two suppliers charging the same price offer the same quality. For example many services such es consulting, banking as well as educational services contain components which are not directly observable. That makes comparisons between individual products costly, as they usually involve the time and resource consuming process of analyzing, tasting, testing and trying different products. Often it is simply not an easy task to find out where the cheapest supplier is located. The service sector again, provides a vast number of examples. Second, I assume that at the end of each period firms randomly change their respective positions within the cross-sectional productivity and thus, the cross-sectional price distribution. As a results, agents are not able to infer from past information, especially from observed past pricing behavior, which firms charge low enough prices and which do not. In other words, firm's movements along the price scala make any knowledge about the past pricing behavior of particular firms worthless, so that at the beginning of an arbitrary period t households are as well informed as they were at the beginning of $t-1$ and thus, have to play the same game again. The assumption on the intra-distribution mobility of firms is based on the evidence provided by Lach (2002) and Lach and Tsiddon

(1993). After controlling for observed as well as unobserved heterogeneity between almost identical products they show that there is substantial intra-distribution mobility disabling, as the authors conclude, consumers to learn which store charges consistently low prices

As explained below, under the assumption of a continuum of *ex ante* symmetric households which are able to perfectly pool all idiosyncratic income as well as expenditure risks the mass of households served by an arbitrary firm l, $x_{l,t}$ will be equal to the probability to become a customer at store l faced by a typical household:[1]

$$x_{l,t} = \tilde{x}_{i,l,t}.$$

I refer to $x_{l,t}$ as the market share of firm l. Since, as assumed below, it is costly to search for cheaper suppliers, deviations from the average price do not translate into an immediate drop or increase of the individual market share to zero or hundred percent respectively. Thus, each firm enjoys a small, short-run monopoly power over the consumers belonging to its customer base when setting its price. Consequently, the market structure can be characterized as a form of monopolistic competition. According to (V.2.1), if all firms were to choose the same price the fraction of aggregate demand each firm faces would be equal to $1/n$, irrespective of the level of search activity. If households do not engage in search at all, $s_{i,t} = 0$, $\forall i$ then again each supplier will serve a fraction of $1/n$ of the market, irrespective of the degree of price dispersion.

In contrast to the versions of the *Customer Market Model* described in chapters 2 and 3 where the individual firm's market share follows a random walk process, in the model presented here the market share $x_{l,t}$ is modeled as a variable *without memory*, which is purely statically related to the level of search efforts and the firm's relative prices. The assumption that the process of search, the price adjustments and the reactions of the individual market shares take place simultaneously can be regarded as reasonable, since one period in the model corresponds to one quarter in the real world.

Firms

Each profit maximizing monopolistic firm produces according to the linear production function

$$Y_{l,t} = (Z_t + \iota_{l,t})N_{l,t},$$

where $N_{l,t}$ denotes labor input of firm l. Z_t denotes the total factor productivity which follows a stochastic process given by:

$$\ln(Z_t) = \rho_z \ln(Z_{t-1}) + \epsilon_t,$$

[1] That is an implication of the *Law of Large Numbers*.

where ϵ_t follows a *White Noise Process* with variance σ^2_ϵ. $\iota_{l,t}$ is an exogenous firm specific shift variable which is assumed to evolve according to

$$\iota_{l,t} = \iota_l + \epsilon_{\iota_l,t}.$$

$\epsilon_{\iota_l,t}$ follows a *White Noise Process* with variance $\sigma^2_{\iota_l}$.

Under the assumption that each agent chooses the same level of search activity,[2] the demand function faced by the producer l is given by

$$D_{l,t} = x_{l,t} \cdot D_t = \left(\frac{\exp\left(\left(1 - \frac{P_{l,t}}{P_t}\right) s_t^\gamma\right)}{\sum_{j=1}^n \exp\left(\left(1 - \frac{P_{j,t}}{P_t}\right) s_t^\gamma\right)} \right) \cdot D_t, \tag{V.2.2}$$

where D_t, s_t and P_t denote, respectively, aggregate demand, aggregate search efforts and the aggregate price level. For given marginal costs, $\mu_{l,t}$ the profit maximization problem of a typical firm reads:

$$\max_{\frac{P_{l,t}}{P_t}} \left\{ \left(\frac{\exp\left(\left(1 - \frac{P_{l,t}}{P_t}\right) s_t^\gamma\right)}{\sum_{j=1}^n \exp\left(\left(1 - \frac{P_{j,t}}{P_t}\right) s_t^\gamma\right)} \right) \cdot D_t \left(\frac{P_{l,t}}{P_t} - \mu_{l,t} \right) \right\}.$$

It yields the following first order condition for optimal price setting:

$$\frac{P_{l,t}}{P_t} = \frac{1}{s_t^\gamma (1 - x_{l,t})} + \mu_{l,t}, \tag{V.2.3}$$

where $x_{l,t}$ also depends on $\frac{P_{l,t}}{P_t}$. Everything else given, a higher level of search efforts makes the typical firm more reluctant to set too high a relative price, and leads to a lower markup. A higher market share $x_{l,t}$ makes it less likely for a searching customer to meet a supplier other than l and thus, makes it less likely for him to find a firm charging a price lower than $P_{l,t}$. Consequently, a higher $x_{l,t}$ reduces the magnitude of the negative effect of any given level of search on the relative price $\frac{P_{l,t}}{P_t}$ and so enables firm l to choose a higher markup.

However, by inspecting equation (V.2.3) one can only gain some very rough intuition about the mechanisms underlying the price setting behavior of the firms, because (V.2.3) defines an implicit relationship between the relative price $\frac{P_{l,t}}{P_t}$, marginal costs $\mu_{l,t}$ and search efforts s_t. Log-linearizing (V.2.3) around the steady state[3] yields:[4]

$$\begin{aligned} \widehat{\left(\frac{P_{l,t}}{P_t}\right)} &= \frac{\gamma}{s^\gamma \cdot \frac{P_l}{P}} \left(\frac{\left(1 - \frac{P_l}{P}\right) s^\gamma - (1 - x_l)}{1 - x_l} \right) \hat{s}_t + \\ &+ \frac{x_l}{(1 - x_l) \cdot \frac{P_l}{P}} \cdot \sum_{j=1, j \neq l}^n x_j \frac{P_j}{P} \widehat{\left(\frac{P_{j,t}}{P_t}\right)} + (1 - x_l) \frac{\mu_l}{\frac{P_l}{P}} \hat{\mu}_{l,t}, \end{aligned} \tag{V.2.4}$$

[2] See the discussion below.

[3] The properties and the computation of the steady state will be discussed later on.

[4] To arrive at the result one has just to take into account the definition of $x_{l,t}$, $\forall l$, the fact that

$$\sum_{j=1}^n x_{j,t} = 1$$

where variables without time index denote steady state values, while a "hat", " ^ ", over a variable denotes its percentage deviation from the stationary equilibrium. According to (V.2.4), if firm l has an above average steady state price, an increase of search activity will force it pass-through to its price a smaller fraction of any given increase in its marginal costs and thus, to lower its markup. Only in the case of a sufficiently low P_l/P combined with a sufficiently large market share, x_l will an increase in aggregate search activity enable firm l to choose a higher pass-through and increase its markup. Putting any general equilibrium effects aside, if all other firms increase their respective relative prices then firm l will also find it optimal to do that. Note that in the general equilibrium discussed in the current chapter it will be possible for all firms to simultaneously increase their respective relative prices, provided that such a reaction comes along with (is backed by) the *"correct"* adjustment of the individual market shares.

The more conventional representation of the first order condition for optimal price setting is:

$$\frac{P_{l,t}}{P_t} = mu_{l,t}\mu_{l,t} = \left(1 + \frac{1}{s_t^{\gamma}(1 - x_{l,t})\mu_{l,t}}\right)\mu_{l,t},$$

where $mu_{l,t}$ denotes the firm specific markup. As can be easily seen, it will be time varying. As stressed above, the markup will be only procyclical when P_l/P is sufficiently low and at the same time x_l sufficiently large. Otherwise, $mu_{l,t}$ will tend to be countercyclical.

Since labor is the only factor of production, the real marginal costs of firm l are given by

$$\mu_{l,t} = \frac{W_t/P_t}{Z_t + \iota_{l,t}}. \tag{V.2.5}$$

Households

The economy is populated by a continuum of *ex ante* identical agents of total mass equal to one, organized in m equally large units. Let us refer to these units as *families*. The family indexed by $j = \{1, 2, ..., m\}$ consists of the agents with an index i in the interval $i \in \left[\frac{j-1}{m}, \frac{j}{m}\right]$. Each family faces a two-stage maximization problem, in which both stages take place simultaneously. At the *"first"* stage, the head of the family, also called planner, chooses the level as well as the distribution across the family's members of next-period wealth, current income, leisure and current expenditure, given the level and the distribution of the

and the definition of the price index:

$$P_t = \sum_{j=1}^{n} x_{j,t}P_{j,t},$$

and then to log-linearize.

family's current wealth, average prices and the current level of search activity. The j^{th} planner maximizes the following utility function:

$$U = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \int_{i=\frac{j-1}{m}}^{\frac{j}{m}} \left(\frac{C_{i,t}^{1-\eta}}{1-\eta} + \phi_m \frac{(M_{i,t}/P_t)^{1-\chi}}{1-\chi} - \frac{b}{2} N_{i,t}^2 \right) di \right\}, \qquad \phi_m, b, \eta, \chi > 0, \quad \beta \in (0,1),$$

subject to the budget constraint:

$$\int_{i=\frac{j-1}{m}}^{\frac{j}{m}} \left(\sum_{l=1}^{n} \tilde{x}_{i,l,t} \frac{P_{l,t}}{P_t} D_{i,t} + \frac{M_{i,t+1}}{P_t} - \frac{M_{i,t}}{\pi_t P_{t-1}} \right) di = \int_{i=\frac{j-1}{m}}^{\frac{j}{m}} \left(\frac{W_t}{P_t} N_{i,t} + \Pi_t + \frac{T_t}{P_t} \right) di, \quad \forall t,$$

where $D_{i,t}$, $M_{i,t}$ and $N_{i,t}$ denote the agent specific total expenditure, nominal balances and working hours respectively. W_t and T_t denote the nominal wage and nominal net transfers form the government respectively. Π_t is the sum of the firm-specific real profits which are assumed to flow in a lump-sum manner to family mambers. π_t represents the overall inflation factor. The total expenditure of the i^{th} member of the family, $D_{i,t}$, equals the sum of her consumption expenditure $C_{i,t}$ and the transaction costs $g(s_{i,t})\frac{M_{i,t}}{\pi_t P_{t-1}}$ arising when setting search efforts at $s_{i,t}$.

At the *"second"* stage, given average prices as well as the distribution of the family's wealth, income, leisure and consumption expenditure, each member of the family chooses the level of its search efforts in order to buy the amount of goods chosen by the planner at the lowest possible cost. The corresponding maximization problem can be written as:

$$\max_{s_{i,t}} \left\{ - \sum_{l=1}^{n} \tilde{x}_{i,l,t}(s_{i,t}) \frac{P_{l,t}}{P_t} D_{i,t}(s_{i,t}) \right\}, \tag{V.2.6}$$

where $\tilde{x}_{i,l,t}$ is defined in (V.2.1).

The two-stage structure can be seen as an approximation of the process of decision-making in many families, corporations, public and private institutions and other economic units and is consistent with the approach chosen by a large part of the *New Home Economics* literature. The latter views the family as a social unit in which one of the members, usually the husband, acts as a benevolent planner, endowed with dictatorial power over the other members of the family, who pools the income streams of the individual members and seeks to maximize a kind of *"social welfare function"* defined as a weighted average of the utility functions of the family's members. Each of them then chooses a set of variables (e.g. the production of particular home goods) to maximize her individual objective function. Examples are Becker (1973, 1974), Killingsworth (1983), Lundberg and Pollak (1997) as well as the literature cited there. In most of those models the decision to become (remain) a member of the family is endogenous and shapes the behavior of the *"dictator"*. In the current chapter it is assumed that for reasons exogenous to the model neither agent has an incentive to leave the family she belongs to.

Many corporations and other public and private institutions are similarly organized: There is a lot of dictatorship and centralized planning taking place in them, with the managers of such institutions making almost all important decisions and delegating only the ones of limited importance to the individual departments of their institution. In a panel study including 300 large U.S. firms Rajan and Wulf (2006) do find support of the widespread view that the organizational hierarchy of the U.S. firms has become flatter over the last twenty years. The process has been characterized by the elimination of many intermediate layers of management and a declining organizational distance between the CEO and the division managers. However, the authors point out that a naive interpretation of the observed organizational flattering as a "decentralization" might be incorrect because on the one hand, decision-making authority is being delegated down to the individual division heads but on the other hand, the CEO is getting a more direct control over the lower levels of the organization, which is a form of centralization. Rajan and Wulf (2006) further conclude that despite the organizational flattering found in the data, it is still the case that the CEO and the members of the senior management are the ones *'...who make the resource allocation decisions that ultimately determine the firm's performance...'*. In an excellent essay Argyris (1998) argues that in spite of the observable organizational flattering, most companies are still dictatorially governed without much *empowerment* of division managers and other employees actually being done. The reasons are, as both, research and practice, indicate, that on the one hand the *'command-and-control model'*[5] is what CEOs know best and on the other, most employees find it disadvantageous when being held personally accountable. Not surprisingly, the author comes to the conclusion that decentralization and empowerment in firms are just *'the emperor's new clothes'*.[6] Further examples for theoretical studies discussing recent developments in the organizational structure of firms and deriving similar conclusions are Kaplan (1996), Holmstrom and Kaplan (2001), Rajan and Zingales (2000) and many others.

Transaction costs in this model constitute a fraction of nominal money balances accumulated in the previous period. By this specification I attempt to capture the fact that for performing different tasks the departments of many public and private institutions as well as the members of many families are funded in nominal terms *in advance*. Given that more or less fixed nominal (monthly, quarterly or annual) budget the departments and family members carry out different types of transactions. An alternative rationale for the specification chosen, can be found by assuming that there are autonomous business units (or firms) performing search in the goods market and selling the information obtained for money to the households.[7] Loosely speaking,

[5] See Argyris (1998), p. 98.

[6] See Argyris (1998), p. 104.

[7] Under this specification the overall price index will be a weighted average of the prices of the consumption goods and the price charged for performing search activity.

both interpretations can be viewed as a kind of a *cash in advance* constraint with respect to search activity.

Assume for now that each member chooses the same level of search activity, $s_{i,t} = s_t$, $\forall i$. It is then easy to show that the optimal allocation from the point of view of the planner will be symmetric in any respect across the family members. Given that symmetry and applying the law of large numbers implies the following relation between the market shares of the individual firms $x_{l,t}$ and the agent specific probabilities to become a customer of firm l, $\tilde{x}_{i,l,t}$:

$$x_{l,t} = \tilde{x}_{i,l,t}, \quad \forall i, l.$$

Then by using the definition of the overall price index:

$$\sum_{j=1}^{n} x_{j,t} \frac{P_{j,t}}{P_t} = 1$$

the budget constraint of family j can be written as

$$\int_{i=\frac{j-1}{m}}^{\frac{j}{m}} \left(C_{i,t} + \frac{M_{i,t+1}}{P_t} - (1 - g(s_{i,t})) \frac{M_{i,t}}{\pi_t P_{t-1}} \right) di = \int_{i=\frac{j-1}{m}}^{\frac{j}{m}} \left(\frac{W_t}{P_t} N_{i,t} + \Pi_t + \frac{T_t}{P_t} \right) di, \quad \forall t.$$

The last equation shows that, as a consequence of the symmetry assumption, the typical family does not face any idiosyncratic risks although each individual member is exposed to individual uncertainty with respect to her current expenditure. If in addition initial wealth is distributed uniformly across families, the latter will be homogeneous. As a result, one will be able to resort to the representative agent framework. The latter is certainly less realistic than a similar model with heterogeneous households would be, but since I am only interested in the cyclical behavior of the most important macroeconomic aggregates and not in that of family-specific or firm-specific variables, the loss of relevant information caused by the symmetry assumption will be negligible. Furthermore, the representative agent approach has the advantage of involving a much lower computational burden than the heterogeneous agent framework does. The empirical literature dealing with the extent to which there is income pooling within families provides mixed evidence. Lundberg and Pollak (1997) review that literature and reject in their own study the hypothesis of income pooling by exploiting a natural experiment found in the data.

First Order Conditions

The first order conditions resulting from the first stage of optimization performed by the planner, evaluated at the symmetric family specific equilibrium, take the form:

$$C_t^{-\eta} = \Lambda_t, \tag{V.2.7}$$

$$bN_t = \Lambda_t \frac{W_t}{P_t}, \tag{V.2.8}$$

$$\beta \phi_m m_{t+1}^{-\chi} E_t \{\pi_{t+1}^{\chi-1}\} = \Lambda_t (1 - g(s_t)) - \beta E_t \left\{ \frac{\Lambda_{t+1}(1 - g(s_{t+1}))}{\pi_{t+1}} \right\}, \tag{V.2.9}$$

$$C_t + m_{t+1} - (1 - g(s_t)) \frac{m_t}{\pi_t} = \frac{W_t}{P_t} N_t + \Pi_t + \frac{T_t}{P_t}. \tag{V.2.10}$$

The family index was dropped from C_t, N_t , $m_t = \frac{M_t}{P_{t-1}}$ and s_t because of the homogeneity across the families in this economy.

The second stage of maximization implies the following first order condition:[8]

$$\frac{g'(s_{i,t}) s_{i,t}^{1-\gamma}}{\gamma} = \left(\sum_{l=1}^{n} \tilde{x}_{i,l,t} \left(1 - \frac{P_{l,t}}{P_t} \right)^2 \right) \frac{\pi_t}{m_{i,t}} D_{i,t},$$

where i denotes the index of the family member. Note that $\tilde{x}_{i,l,t}$ and $D_{i,t}$ also depend on $s_{i,t}$. Evaluating the last equation at the symmetric equilibrium and specifying $g(s_{i,t})$ as

$$g(s_{i,t}) = \frac{s_{i,t}^{\alpha}}{a}, \quad \alpha, a > 0, \quad \alpha > \gamma,$$

yields the condition

$$\frac{\alpha}{a\gamma} s_t^{\alpha-\gamma} = \left(\sum_{l=1}^{n} x_{l,t} \left(1 - \frac{P_{l,t}}{P_t} \right)^2 \right) \frac{\pi_t}{m_t} D_t. \tag{V.2.11}$$

This equation embodies the optimal trade off between the additional increase in transaction costs and the corresponding additional reduction of the average goods price, both brought about by a marginal increase in search activity s_t. Recall that for given relative prices a higher s_t implies a lower (higher) probability to become a customer of a relatively expensive (cheap) supplier. According to (V.2.11), if there is no price dispersion, search activity will be zero. To avoid the mathematical and computational complications stemming from the possibility of such a corner solution, the exogenous productivity processes Z_t and $\iota_{l,t}$, $l = \{1, 2, ..., n\}$ are

[8] Details on the derivation of this equation are given in the supplement at the end of this chapter (Section 8).

calibrated so as to ensure that at each point in time and in each state of nature there is a non-degenerate distribution of goods prices.[9] For given total expenditure D_t and market shares $x_{l,t}$ a higher inflation π_t reduces the transaction costs *per unit of* s_t, thus, forcing households to increase their search efforts. It is not easy to provide a more general intuition about that dependance, an intuition which is sufficiently close to reality. Perhaps it is plausible to assume that in times the economic conditions are worsening, and an increase inflation does represent a worsening since it erodes the value of individual nominal balances, it becomes more important for consumers to find suppliers charging lower prices in order to at least partly compensate the negative effects of the higher inflation rate. Perhaps it is plausible to assume that in such episodes households get more sensitive to differences in prices and are willing to take a more careful look at the price setting behavior in the goods and other markets. A similar effect arises in the class of monetary models known as *Shopping Time Models*.[10] The latter motivate the demand for real balances by the desire to reduce the transaction costs coming about with the purchase of consumption goods. A higher inflation rate in that models necessitates a higher fraction of time spent in the production of transaction services (or a higher fraction of time spent carrying out transactions). Put differently, a higher inflation in the shopping time models reduces the disutility of any given amount of transaction time.

Government

The central bank finances its lump-sum transfers to the public by changes in the nominal quantity of money:

$$M_{t+1} - M_t = T_t.$$

[9]Note further that if there is no search activity $s_t = 0$, each firm will be able to set its price at infinity since:

$$\frac{P_{l,t}}{P_t} = \frac{1}{s_t^{\gamma}(1 - x_{l,t})} + \mu_{l,t}.$$

Such a reaction would lead to a collapse of the economy. To avoid this, one could replace s_t^{γ} in the definition of the market share $x_{l,t}$ by the function $1 + s_t^{\gamma}$. This modification, however, reduces the set of parameter combinations implying an economically meaningful steady state. I performed a large number of numerical experiments in order to compare the qualitative and quantitative implications of both specifications, the one with s_t^{γ} and the one with $1 + s_t^{\gamma}$. The result was that, provided the parameterization of the model leads to an economically interpretable steady state, the two specifications imply virtually identical results. Nevertheless, it should be noted that the model developed here is suitable only for the analysis of economies characterized by some price dispersion in each period and each state of nature. To understand the evolution of economies which can switch from an asymmetric to a symmetric equilibrium and *vice versa*, one should resort to another theoretical tools.

[10]Examples are Saving (1971), Brock (1974), Croushore (1993), Jovanovich (1982), Romer (1986) and many others.

It is further assumed that in each period transfers constitute a fraction of current money supply:

$$T_t = (\tau_t - 1)M_t,$$

where the percentage deviation of τ_t from its steady state $\hat{\tau}_t$ follows a first order autoregressive process

$$\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + u_t, \qquad \rho_\tau \in [0, 1).$$

u_t is assumed to be a *White Noise Process* with variance σ_u^2.

Equilibrium

The evolution of the economy is described by the definition of marginal costs (V.2.5), the first order condition for optimal price setting (V.2.3), the households first order conditions (V.2.7), (V.2.8), (V.2.9) and (V.2.11), the aggregate consistency conditions

$$x_{l,t} D_t = (Z_t + \iota_{l,t}) N_{l,t}, \qquad l = \{1, 2, ..., n\}, \tag{V.2.12}$$

and

$$\sum_{l=1}^{n} N_{l,t} = N_t \tag{V.2.13}$$

as well as the definitions of $x_{l,t}$ for $l = \{1, 2, ..., n\}$ and D_t. Note that if (V.7.4) are satisfied, then the family's budget constraint implies:[11]

$$D_t = Y_t = \sum_{l=1}^{n} \frac{P_{l,t}}{P_t} (Z_t + \iota_{l,t}) N_{l,t}.$$

To close the model, one also needs to specify monetary policy and the exogenous productivity processes Z_t and $\iota_{l,t}$.

The inclusion of search activity s_t as an argument of the function describing the evolution of firm-specific market share introduces an externality from the point of view of the individual firm, since s_t depends on overall inflation and consumption.

[11] Note that the goods market equilibrium together with the definition of the price index imply:

$$\sum_{l=1}^{n} \frac{P_{l,t}}{P_t} (Z_t + \iota_{l,t}) N_{l,t} = \sum_{l=1}^{n} \frac{P_{l,t}}{P_t} x_{l,t} D_t = D_t.$$

3 Technical Discussion

Without any symmetry assumptions the expenditure minimization carried out in the second stage of utility maximization delivers the following first order condition:

$$\frac{\alpha}{\gamma a} s_{i,t}^{\alpha-\gamma} = \left(\frac{\sum_{l=1}^{n} \left\{ \left(1 - \frac{P_{l,t}}{P_t}\right) \left(\Upsilon_{i,t} - \frac{P_{l,t}}{P_t}\right) \tilde{x}_{i,l,t} \right\}}{\Upsilon_{i,t}} \right) \frac{\pi_t}{m_{i,t}} D_{i,t}, \tag{V.3.14}$$

where $\Upsilon_{i,t}$, defined as

$$\Upsilon_{i,t} = \sum_{l=1}^{n} \tilde{x}_{i,l,t} \frac{P_{l,t}}{P_t},$$

is the price index perceived by member i of an arbitrary family. If equation (V.3.14) has a unique solution $s_{i,t}^{*}$, then it will be straightforward to show that, provided the family planner chooses a symmetric allocation $m_{i,t} = m_t$ and $C_{i,t} = C_t$, $\forall i \in [\frac{j-1}{m}, \frac{j}{m}]$, each family member will find the same level of search efforts optimal: $s_{i,t} = s_t$, $\forall i \in [\frac{j-1}{m}, \frac{j}{m}]$. Otherwise, symmetry of the planner's allocation won't necessary imply symmetry with respect to search activity. Thus, it is important to identify conditions ensuring that equation (V.3.14) has only one solution. Surely, a more challenging goal would be to characterize the whole set of necessary and sufficient conditions for uniqueness. Unfortunately, the latter is not possible due to the high degree of nonlinearity in the model.

(V.3.14) can be more explicitly written as:

$$\begin{aligned} &\left(\frac{\sum_{l=1}^{n} \left\{ \left(1 - \frac{P_{l,t}}{P_t}\right) \left(\Upsilon_{i,t} - \frac{P_{l,t}}{P_t}\right) \tilde{x}_{i,l,t} \right\}}{\Upsilon_{i,t}} \right) \frac{\pi_t}{m_{i,t}} C_{i,t} + \\ &+ \left(\frac{\sum_{l=1}^{n} \left\{ \left(1 - \frac{P_{l,t}}{P_t}\right) \left(\Upsilon_{i,t} - \frac{P_{l,t}}{P_t}\right) \tilde{x}_{i,l,t} \right\}}{\Upsilon_{i,t}} \right) \frac{s_{i,t}^{\alpha}}{a} = \frac{\alpha}{\gamma a} s_{i,t}^{\alpha-\gamma}. \end{aligned} \tag{V.3.15}$$

First observe that the first derivative of $\Upsilon_{i,t}$ with respect to the agent's search activity $s_{i,t}$ is negative. Then it is easy to see that an increase of $s_{i,t}$ will have a positive effect on the lhs of equation (V.3.15) *via* the common denominator of the two terms, $\Upsilon_{i,t}$, and $s_{i,t}^{\alpha}$ appearing in the second term. Unfortunately, there is an ambiguous effect of $s_{i,t}$ on the numerators of the expressions in brackets on the lhs of (V.3.15). To see that, note that the product $\left(1 - \frac{P_{l,t}}{P_t}\right) \tilde{x}_{i,l,t}$ is positive for some l and negative for others. Thus, the sign of $\left(1 - \frac{P_{l,t}}{P_t}\right) \tilde{x}_{i,l,t} \frac{\partial \Upsilon_{i,t}}{\partial s_{i,t}}$ will depend on l. Further, since the sign of $\frac{\partial \tilde{x}_{i,l,t}}{\partial s_{i,t}}$ also depends on the firm's index l, the sign of $\left(1 - \frac{P_{l,t}}{P_t}\right) \left(\Upsilon_{i,t} - \frac{P_{l,t}}{P_t}\right) \frac{\partial \tilde{x}_{i,l,t}}{\partial s_{i,t}}$ will also be ambiguous.[12] Because

[12] Note that in most cases the sign of the product $\left(1 - \frac{P_{l,t}}{P_t}\right) \left(\Upsilon_{i,t} - \frac{P_{l,t}}{P_t}\right)$ will be positive since $\Upsilon_{i,t}$ will tend to take a value near one.

of the nonlinearity of equation (V.3.15) it is almost impossible to derive any interpretable, general conditions on the derivatives $\frac{\partial \tilde{x}_{i,l,t}}{\partial s_{i,t}}$ and the degree of price dispersion ensuring a unique solution $s^*_{i,t}$. Therefore I restrict the analytical and numerical analysis to the case $n = 2$.

If there are only two suppliers in the goods market, $n = 2$, (V.3.15) simplifies to

$$0 = \left(\frac{P_{1,t}}{P_t} - \frac{P_{2,t}}{P_t}\right)^2 \frac{e_{i,1,t}e_{i,2,t}}{(e_{i,1,t} + e_{i,2,t})\left(e_{i,1,t}\frac{P_{1,t}}{P_t} + e_{i,2,t}\frac{P_{2,t}}{P_t}\right)} \left(C_{i,t} + \frac{s^\alpha_{i,t}}{a}\frac{m_{i,t}}{\pi_t}\right)\frac{\pi_t}{m_{i,t}} - \frac{\alpha}{\gamma a}s^{\alpha-\gamma}_{i,t}, \tag{V.3.16}$$

where

$$e_{i,1,t} = \exp\left(\left(1 - \frac{P_{1,t}}{P_t}\right)s^\gamma_{i,t}\right), \qquad e_{i,2,t} = \exp\left(\left(1 - \frac{P_{2,t}}{P_t}\right)s^\gamma_{i,t}\right).$$

Without loss of generality it can be assumed that $P_{2,t} > P_{1,t}$ holds. A sufficient condition for (V.3.16) to have at most one solution $s^*_{i,t}$ is that the first derivative of its lhs with respect to $s_{i,t}$ is negative for all $s_{i,t} > 0$. This first derivative is given by

$$\left(\frac{P_{1,t}}{P_t} - \frac{P_{2,t}}{P_t}\right)^2 \frac{e_{i,1,t}e_{i,2,t}}{(e_{i,1,t} + e_{i,2,t})\left(e_{i,1,t}\frac{P_{1,t}}{P_t} + e_{i,2,t}\frac{P_{2,t}}{P_t}\right)} \times$$

$$\times \left\{\gamma s^{\gamma-1}_{i,t} \frac{\frac{P_{1,t}}{P_t}\left(\frac{P_{1,t}}{P_t} - \frac{P_{2,t}}{P_t}\right)e^2_{i,1,t} + \frac{P_{2,t}}{P_t}\left(\frac{P_{2,t}}{P_t} - \frac{P_{1,t}}{P_t}\right)e^2_{i,2,t}}{(e_{i,1,t} + e_{i,2,t})\left(e_{i,1,t}\frac{P_{1,t}}{P_t} + e_{i,2,t}\frac{P_{2,t}}{P_t}\right)} \left(C_{i,t}\frac{\pi_t}{m_{i,t}} + \frac{s^\alpha_{i,t}}{a}\right) + \alpha\frac{s^{\alpha-1}_{i,t}}{a}\right\}, \tag{V.3.17}$$

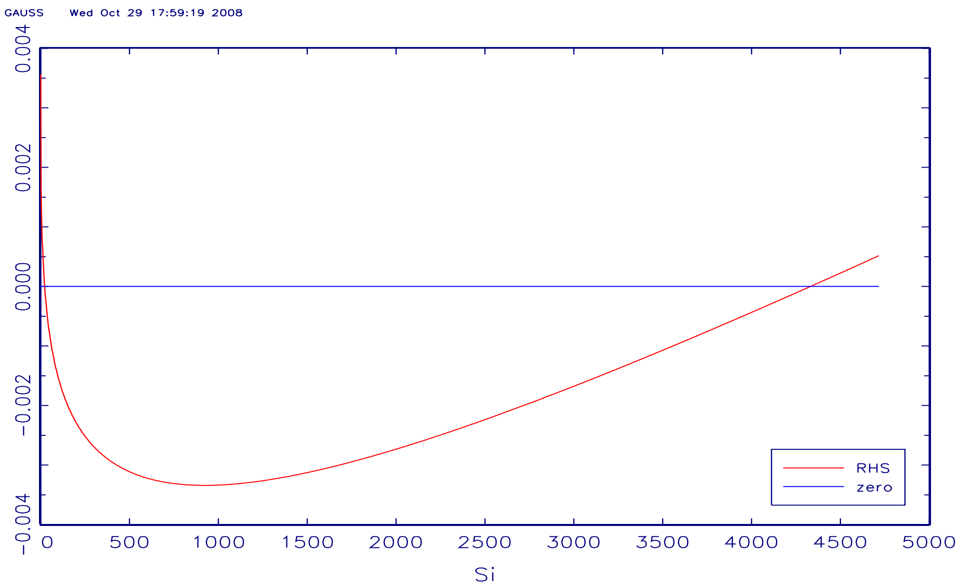
and its sign depends on the sign of the expression in curly brackets. A necessary condition for it to be negative is that the following inequality is satisfied:

$$\frac{P_{1,t}}{P_{2,t}} > \left(\frac{e_{i,2,t}}{e_{i,1,t}}\right)^2 = \frac{1}{\exp\left(2s^\gamma_{i,t}\left(\frac{P_{2,t}}{P_t} - \frac{P_{1,t}}{P_t}\right)\right)}, \quad \forall s_{i,t} > 0. \tag{V.3.18}$$

When (V.3.18) holds and at the same time the velocity of money with respect to consumption $C_{i,t}\frac{\pi_t}{m_{i,t}}$ is sufficiently large, then the sum in curly brackets in (V.3.17) will be negative. This requirements can be used to derive a sufficient condition on the parameters of the model provided one is able to find the solutions for the endogenous variables involved. However, as is readily confessed, due to the nonlinearity of this model, it is not possible to derive such a condition analytically. Alternatively, one can resort to numerical analysis and try to derive a restriction on the parameters and the steady state of the model heuristically. Such a restriction will ensure that in the stationary as well as in the relevant temporary equilibria equation (V.3.16) has a unique solution. If (V.3.17) turns to be positive, it will be possible for equation (V.3.16) to have multiple solutions $s^*_{i,t}$.

To examine the existence and the uniqueness of the solution $s^*_{i,t}$ of (V.3.16) numerically,[13] one needs to calibrate α, γ, a, the velocity of money with respect to output $v_y = Y\frac{\pi}{m}$, working hours N and both relative prices, P_1/P and P_2/P. The benchmark values for the velocity v_y, N, P_1/P and P_2/P are set to 2.15, 0.13, 0.97 and 1.08 respectively. Their computation is described in section 4. As also shown in section 4, given the values of α, γ, N, v_y and both relative prices one is in a position to compute the parameter a. In the numerical investigation of the properties of equation (V.3.16) I perform a sensitivity analysis by experimenting with values of α, γ, v_y as well as the difference $P_2/P - P_1/P$ different from their respective benchmark levels. v_y ranges between 0.5 and 4.3, α and γ take values in the intervals $[0.4, 20]$ and $[0.3, 19.8]$ respectively, whereas γ is always smaller than α. The values of the difference between P_2/P and P_1/P cover a bounded open interval the determination of which is described in section (4).

Figure V.1: Rhs of equation (V.3.16). $\frac{P_1}{P} = 0.97$, $\frac{P_2}{P} = 1.08$, $\frac{Y\pi}{m} = 2.15$, $\alpha = 0.9$, $\gamma = 0.7$.



RHS - rhs of equation (V.3.16), *Si* - level of individual search activity.

Fortunately, the results are readily summarized. It turns out that for all parameter combinations considered equation (V.3.16) has exactly two solutions $s^*_{i,t}$ and $s^{**}_{i,t}$ with $s^*_{i,t} < s^{**}_{i,t}$. Figures V.1 and V.2 depict the rhs of this equation for two sets of parameter values. The smaller solution always implies that the fraction of real balances used for transaction purposes $g(s^*_{i,t})$ lies in the interval $(0, 1)$ and is thus, consistent with the structure of the model. The larger one $s^{**}_{i,t}$ however, implies either $g(s^{**}_{i,t}) > 1$ or $g(s^{**}_{i,t}) \in (0, 1)$ depending on the particular parameter values. If the fraction $g(s^{**}_{i,t})$ turns to be larger than one, then we can disregard $s^{**}_{i,t}$ as a solution with no economic interpretation. In this case we are left with only one economically meaningful solution. As a consequence of this uniqueness, if the family

[13] The corresponding program is "**equilibrium_2a.g**".

planner allocates initial wealth $m_{i,t}$ and consumption expenditure $C_{i,t}$ symmetrically across family members, then each of them will choose the same level of search activity, $s_{i,t} = s_t$, $\forall i \in [\frac{j-1}{m}, \frac{j}{m}]$. If the following inequalities hold:

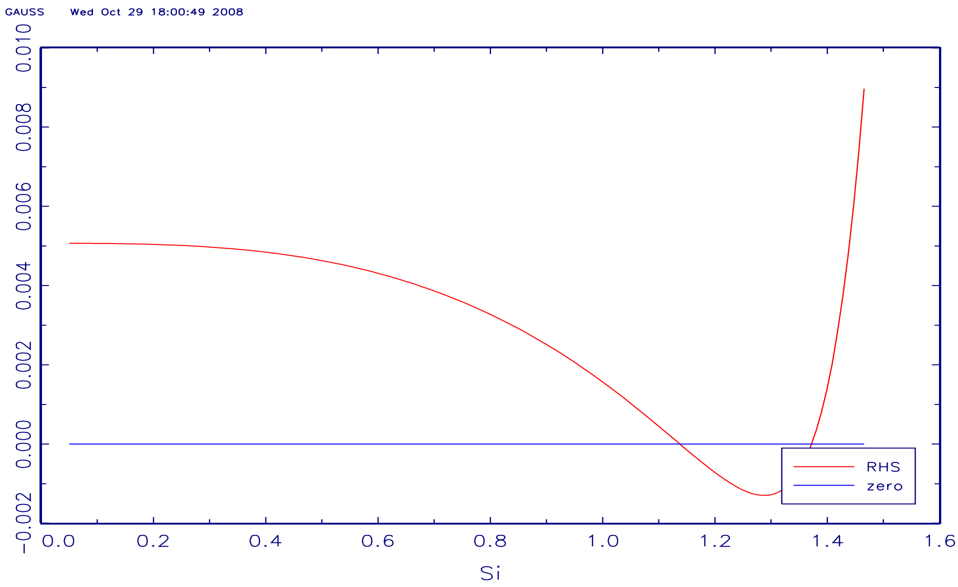
$$g(s^*_{i,t}) \in (0,1), \qquad g(s^{**}_{i,t}) \in (0,1),$$

there will be no economic reason for ignoring one of the solutions. This kind of multiplicity implies that in general agents will be heterogeneous with respect to the level of their search efforts even if the planner were to distribute money balances and consumption uniformly across the family members. It turns out that the higher the price dispersion $P_2/P - P_1/P$ and the smaller the difference between α and γ the more likely for multiple solutions to exist. In contrast, the higher the velocity of money v_y the larger the probability for $g(s^{**})$ to be grater than one and thus, the more likely for the economically relevant solution to (V.3.16) to be unique. The cases depicted in figures V.1 and V.2 are both characterized by the inequality

$$g(s^{**}_{i,t}) > 1$$

implying that $s^{**}_{i,t}$ can be ignored.

Figure V.2: Rhs of equation (V.3.16). $\frac{P_1}{P} = 0.97$, $\frac{P_2}{P} = 1.08$, $\frac{Y\pi}{m} = 2.15$, $\alpha = 20$, $\gamma = 17$.



RHS - rhs of equation (V.3.16), *Si* - level of individual search activity.

In the simulations presented below I use only calibrations of the model ensuring that (V.3.16) has only one interpretable solution. It is important to note, that the uniqueness of the solution of (V.3.16) does not necessary imply uniqueness of the stationary or any temporary equilibrium of the model. In the current chapter I concentrate on the symmetric equilibrium because in my view it is the most likely and most plausible one, provided that the families and their members are *ex ante* homogeneous. I do not make an attempt to prove the existence or non-existence of further equilibria, characterized by an asymmetric distribution of resources and heterogeneous levels of search activity, and leave this issue for future research. In this chapter I am only able to show numerically that for the calibrations chosen the symmetric equilibrium is *locally unique*.

4 Calibration

The calibration of this model is more involved than it was the case in the models already presented.[14] First, to reduce the computational burden arising in the approximation, the calibration and the simulation steps, I consider only the 2-firms case. I assume that the steady state of the economy is characterized by price dispersion with $\frac{P_1}{P}$ always being smaller than $\frac{P_2}{P}$. Alternatively one can assume that the difference between the two relative prices remains constant over time while in each period firms randomly switch their positions in the price distribution. The definition of the overall price index

$$x_1 \frac{P_1}{P} + x_2 \frac{P_2}{P} = 1, \qquad x_1 \in (0,1), \quad x_2 = 1 - x_1$$

implies that $\frac{P_1}{P} < 1$ and $\frac{P_2}{P} > 1$ hold. I start the calibration by setting the difference between both relative prices Δ as well as the lower one at particular values. Then the definition of the price index allows us to determine x_1:

$$x_1 \frac{P_1}{P} + (1 - x_1)\left(\frac{P_1}{P} + \Delta\right) = 1,$$
$$\Rightarrow$$
$$1 - x_1 = \frac{1 - P_1/P}{\Delta}.$$

To ensure that the market share of the cheaper supplier is larger then 50 percent, Δ should satisfy the following inequality

$$\Delta > \frac{1 - P_1/P}{0.5}. \tag{V.4.19}$$

Otherwise the firm charging the higher price will enjoy a larger market share. The level of search activity in the stationary equilibrium s is identified as the solution of the following equation:

$$\frac{\exp\left(\left(1 - \frac{P_1}{P} - \Delta\right) s^\gamma\right)}{\exp\left(\left(1 - \frac{P_1}{P}\right) s^\gamma\right) + \exp\left(\left(1 - \frac{P_1}{P} - \Delta\right) s^\gamma\right)} = \frac{1 - P_1/P}{\Delta},$$

[14] The corresponding programs are "**equilibrium_2.g**" for the general analysis of the sensitivity of the steady state with respect to the degree of price dispersion $\Delta = P_2/P - P_1/P$ and "**equilibrium_2a.g**" for the determination of the upper bound for Δ.

which implies

$$s = \left(\frac{\ln\left(\frac{\Delta + P_1/P - 1}{1 - P_1/P}\right)}{\Delta} \right)^{\frac{1}{\gamma}}.$$

I assume that the sum of the steady state levels of the firm specific productivity variables is equal to zero:

$$\iota_1 = \iota, \quad \iota_2 = -\iota.$$

Further, the steady state value Z of the economy wide productivity shock is set to one. Hence, the two total factor productivities in the stationary equilibrium are given by $1 + \iota$ and $1 - \iota$ for firm 1 and firm 2 respectively. To calibrate ι, I use the conditions for optimal price setting of the two firms:

$$\frac{P_1}{P} = \frac{1}{s^\gamma x_2} + \frac{W/P}{1 + \iota},$$

$$\frac{P_2}{P} = \frac{1}{s^\gamma x_1} + \frac{W/P}{1 - \iota}.$$

Combining these two equations yields:

$$\frac{1 + \iota}{1 - \iota} = \underbrace{\frac{\frac{P_2}{P} - \frac{1}{s^\gamma x_1}}{\frac{P_1}{P} - \frac{1}{s^\gamma x_2}}}_{:=\varphi}.$$

Thus ι can be computed as:

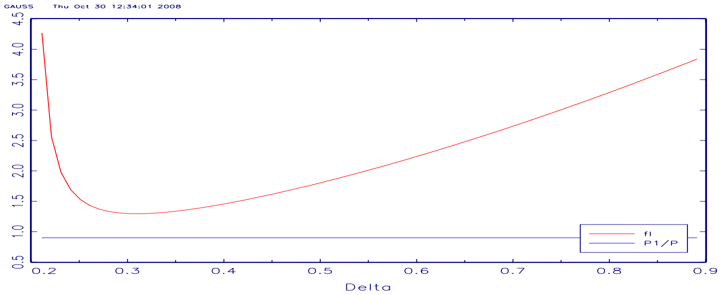
$$\iota = \frac{\varphi - 1}{\varphi + 1}.$$

Note that both the numerator and the denominator of φ must be grater than zero.[15] But since $P_2/P > P_1/P$ and $s^\gamma x_1 > s^\gamma x_2$ it suffices to ensure that the difference $\frac{P_1}{P} - \frac{1}{s^\gamma x_2}$ is positive. This requirement imposes an upper bound on Δ which can be approximated numerically. The numerical analysis[16] also allows us to find a lower bound for P_1/P. The lhs and the rhs of the inequality

$$\frac{P_1}{P} > \underbrace{\frac{1}{s^\gamma x_2}}_{:=f_l} \tag{V.4.20}$$

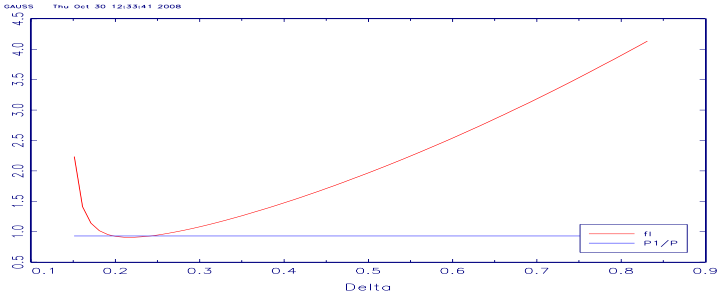
are depicted in figures V.3, V.4 and V.5 for different values of P_1/P and Δ. Recall that s as well as x_2 depend on Δ.

Figure V.3: Lhs of inequality (V.4.20) denoted by f_l. $\frac{P_1}{P} = 0.90$.



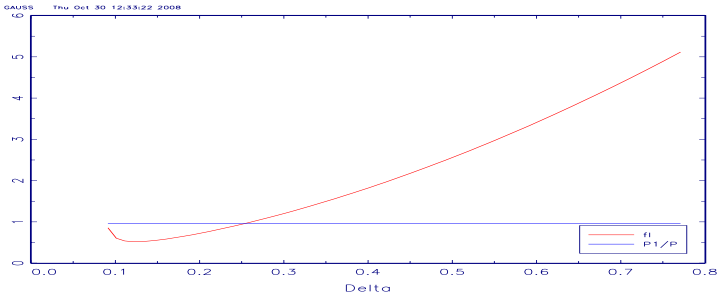
f_l - lhs of equation (V.4.20), *Delta* - Δ (degree of price dispersion).

Figure V.4: Lhs of inequality (V.4.20) denoted by f_l. $\frac{P_1}{P} = 0.93$.



f_l - lhs of equation (V.4.20), *Delta* - Δ (degree of price dispersion).

Figure V.5: Lhs of inequality (V.4.20) denoted by f_l. $\frac{P_1}{P} = 0.96$.



f_l - lhs of equation (V.4.20), *Delta* - Δ (degree of price dispersion).

As long as P_1/P is smaller than (or equal to) 0.92 the term $\frac{1}{s^{\gamma}x_2}$ will be larger than P_1/P irrespective of the measure of price dispersion Δ. If P_1/P takes values in the interval (0.92,0.95), there will be Δs satisfying (V.4.20) but their range will be very small (see figure V.4). In other words, for $P_1/P \in (0.92, 0.95)$ inequality (V.4.20) does not only define an upper but also a lower bound for Δ. The latter is even more restrictive than the one implied by (V.4.19). Accordingly, one needs a sufficiently large P_1/P in order for (V.4.20) to be as unrestrictive as possible with respect to the range of Δ. In particular, for $P_1/P \geq 0.96$ equation (V.4.20) only adds an upper bound for Δ to the restriction defined in (V.4.19). Provided that $\frac{P_1}{P} - \frac{1}{s^{\gamma}x_2} > 0$ is satisfied, it is easy to show that φ is greater than one. $\varphi > 1$ then implies that ι lies in the interval between zero and one.

[15]Otherwise marginal costs will have to be negative.

[16]The corresponding program is "**equilibrim_2a.g**".

An important question regarding the calibration of Δ is to what extent its range is consistent with the empirically observable price dispersion among homogeneous nondurable goods. For example $P_1/P = 0.97$ implies that Δ should range between 0.06 and 0.25, corresponding to a percentage difference between the two prices lying between 6 and 23 percent of P_1/P. Lach (2002) provides evidence on price dispersion for virtually homogeneous commodities based on a panel of stores in the USA. After controlling for observable as well as unobservable sources of heterogeneity[17] between physically homogeneous products he obtains the following estimates: The difference between the logarithms of the 95% and the 5% quantiles of the price distribution equals 0.10, 0.23, 0.22 and 0.16 for *Refrigerator*,[18] *Chicken*, *Coffee* and *Flour* respectively. The differences between the logs of the 75% and the 25% quantiles of the price distributions of the same commodities are equal to 0.03, 0.09, 0.05 and 0.04 respectively. Thus, choosing Δ to imply that $\ln(P_2/P) - \ln(P_1/P)$ lies between 0.10 and 0.20 can be seen as a compromise calibration. For the sake of completeness I also perform a sensitivity analysis with respect to Δ.

Next, by using the first order condition for optimal price setting of one of the firms one can compute the steady state value of the real wage:

$$\frac{W}{P} = (1+\iota)\left(\frac{P_1}{P} - \frac{1}{s^{\gamma}x_2}\right).$$

To calibrate the velocity of money with respect to output $v_y = Y\frac{\pi}{m}$ I use national accounts data provided by the NIPA. Real balances $\frac{M_t}{P_{t-1}}$ are measured as the ratio of the monetary aggregate M1 divided by the value of the nondurables consumption deflator in the previous period (base year 2000). The same deflator is also used to compute the inflation factor. Aggregate output is measured by the gross national product at constant prices (base year 2000) adjusted by the imputed product generated by the stock of durable goods and the government capital stock.[19] The mean of the velocity of money for the period from 1973:Q1 through 2003:Q4 equals 2.15. Given v_y one is able to compute the parameter a *via* the condition governing the optimal level of search efforts (V.2.11) evaluated at the symmetric steady state:

$$\frac{\alpha}{a\gamma}s^{\alpha-\gamma} = \left(\sum_{l=1}^{2} x_{l,t}\left(1-\frac{P_l}{P}\right)^2\right)\underbrace{\frac{\pi}{m}Y}_{:=v_y}.$$

The fraction of time spent working in the stationary equilibrium N is set to 0.1386. The labor inputs of the two firms, N_1 and N_2, can be computed as the solution to the following

[17] Lach (2002) controls for the store selling the particular product, the location of the store, the type of the store as well as for time effects.

[18] Note that refrigerators are durable goods.

[19] The computation procedure is described in Cooley and Prescott (1995).

system of equations:

$$\frac{N_1}{N_2} = \overbrace{\frac{x_1}{x_2}\frac{(1-\iota)}{(1+\iota)}}^{:=\varpi}$$
$$N_1 + N_2 = N.$$

The result reads:

$$N_1 = \frac{\varpi}{1+\varpi}N, \qquad N_2 = \frac{1}{1+\varpi}N.$$

Then the steady state levels of output and consumption are readily computed *via*:

$$Y = \frac{P_1}{P}(1+\iota)N_1 + \frac{P_2}{P}(1-\iota)N_2,$$

$$\frac{m}{\pi} = \frac{Y}{v_y},$$

$$C = Y - g(s)\frac{m}{\pi}.$$

The first of these equations reflects the definition of the economy's national product: the latter equals the sum of the two firm-specific products, both measured in units of a common numeraire. The second equation reflects the definition of the velocity of money with respect to output. The third equation stems from the requirement that in equilibrium aggregate supply Y equals aggregate demand $D = C + g(s)\frac{m}{\pi}$.

The remaining parameters are set to the so called *standard values* usually found in the literature. Table V.1 summarizes the calibration of the model.

Table V.1:
Calibration

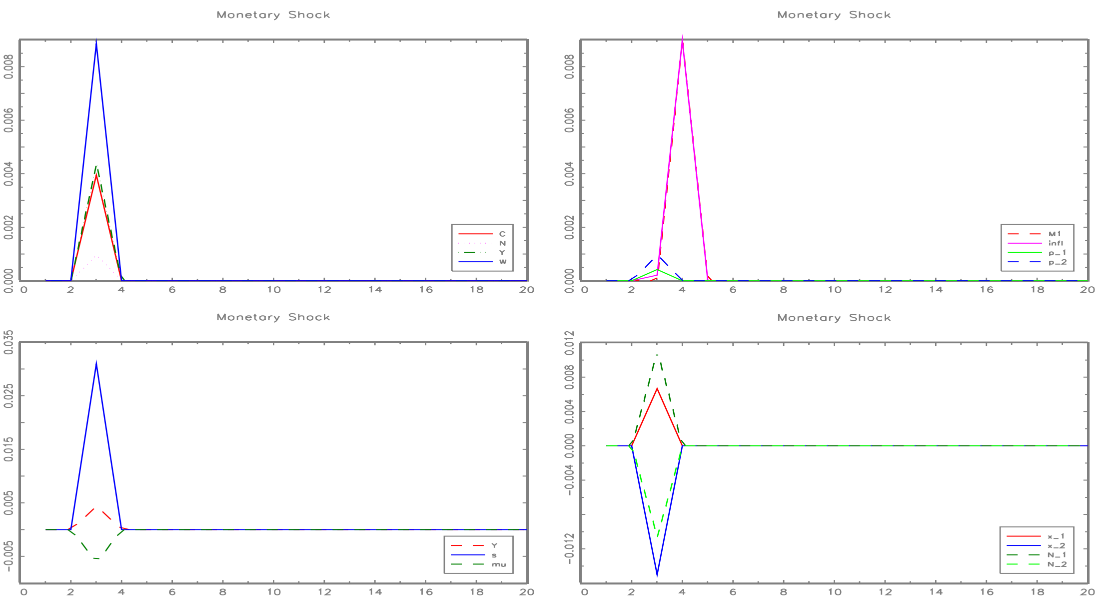
Households/Preferences	Firms/Technology	Central Bank
$\alpha > 0$ sensitivity analysis	$Z^* = 1$	$\tau^* = 1.0138$
$\gamma > 0$ sensitivity analysis	$\rho_z = \{0, 0.9641\}$	$\rho_\tau = 0$
$\beta = 0.991$	$\sigma_\epsilon = 0.0082$	$\sigma_u = 0.0092$
$\eta = 2$	$P_1/P \in (0.95, 0.99]$	$v_y = 2.15$
$\chi = 2$	Δ sensitivity analysis	

5 Results

5.1 Monetary Shocks

Figures V.6 through V.9 depict the impulse responses to a one time monetary expansion in the third period, computed with different sets of parameters.[20] As can be seen, such reparameterizations affect the quantitative predictions of the model but leave its qualitative properties almost unaffected. In particular, the larger difference between α and γ and/or the larger the degree of price dispersion Δ, and/or the higher the relative price[21] P_1/P, the lower the magnitude of the reactions to the monetary disturbance. At the same time, the difference $\alpha - \gamma$ ought to be sufficiently large in order to ensure the local uniqueness of the equilibrium.[22] The critical value of $\alpha - \gamma$, below which there are multiple equilibria, depends on the other parameters of the model.

Figure V.6: Impulse responses to a monetary shock, $\rho_T = 0$, $\alpha = 0.9$, $\gamma = 0.7$, $\Delta = 0.12$, $P_1/P = 0.97$. Relative deviations from steady state.



Y - output, N - hours, C - consumption, W - real wage, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, P_1 - relative price of firm 1, P_2 - relative price of firm 2, s - search activity, mu - average markup, N_1 - labor input of firm 1, N_2 - labor input of firm 2, x_1 - market share of firm 1, x_2 - market share of firm 2.

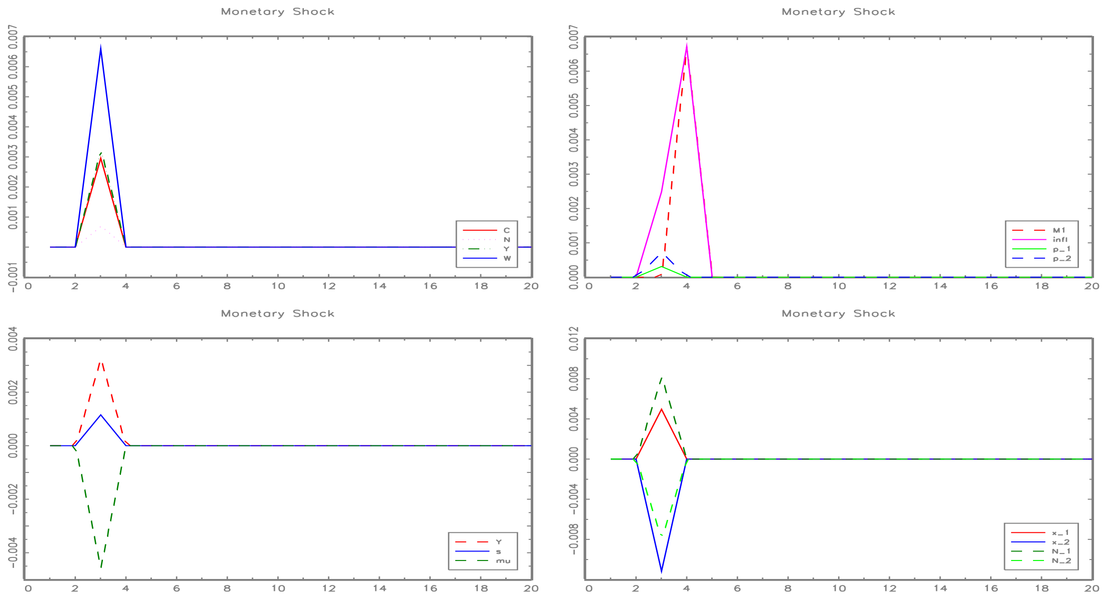
What's the intuition behind these results? For a given price level the monetary expansion induces a positive income effect which forces households to consume more and work less. As a consequence, there is a huge positive pressure on nominal prices and inflation. In a standard model with fully flexible prices and additively separable utility function the increase in the inflation rate will be just sufficient to offset the positive income effect of the monetary shock. In contrast, in the economy presented in this chapter a higher desired level of consumption

[20] The corresponding program is "**sim_cm2d6a.g**".

[21] Recall that P_1/P is the relative price of the firm charging the lower price.

[22] Otherwise, the log-linear version of the model has too many eigenvalues inside the unit circle.

Figure V.7: Impulse responses to a monetary shock, $\rho_\tau = 0$, $\alpha = 20$, $\gamma = 14$, $\Delta = 0.12$, $P_1/P = 0.97$. Relative deviations from steady state.

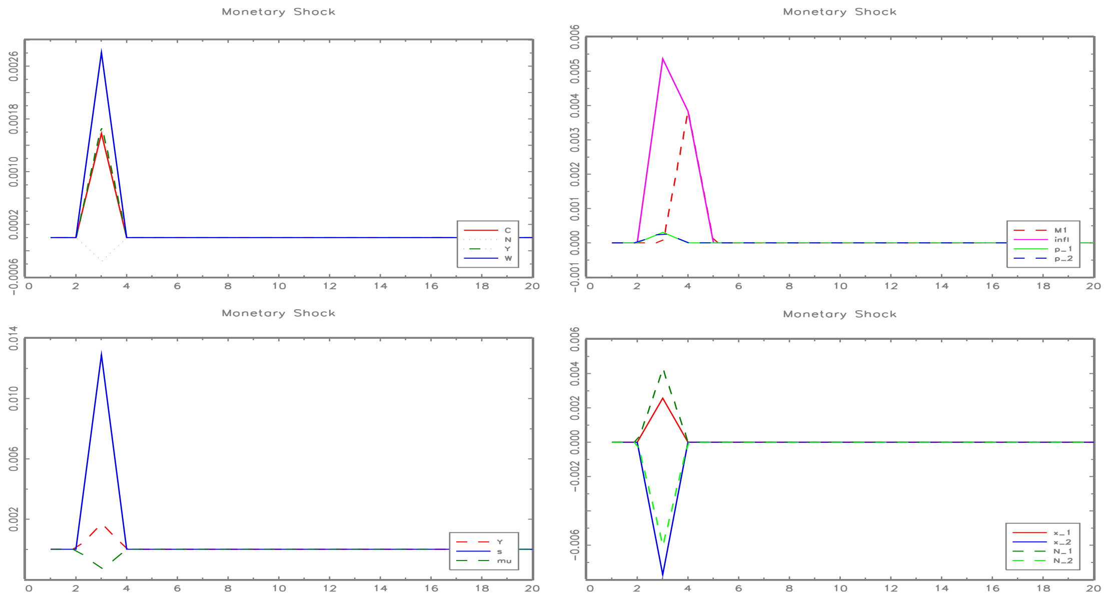


Y - output, N - hours, C - consumption, W - real wage, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, P_1 - relative price of firm 1, P_2 - relative price of firm 2, s - search activity, mu - average markup, N_1 - labor input of firm 1, N_2 - labor input of firm 2, x_1 - market share of firm 1, x_2 - market share of firm 2.

increases the benefit of additional search efforts. At the same time the higher inflation reduces the transaction costs per unit of search. Both effects create an incentive for agents to increase their search activity in the goods market. In an environment characterized by a more intense search each firm, fearing a decline in its market share, will be reluctant to pass through to prices the whole increase in marginal costs. Hence, there will be a fall in markups, leading to an increase in real wages. The latter effect induces households to work more which, in turn, enables the economy to produce more and dampens the positive pressure on current inflation. Indeed, if the difference between α and γ is not too large, there is virtually no reaction of inflation in the period of the shock (see for example figure V.6). In all simulations performed the monetary expansion leads to a drop in both firm-specific markups. However, depending on the calibration chosen, the reaction of firm 1's markup can be stronger or weaker than that of firm 2's one. Note that since search activity as well as the market share of firm 1 increase, there is an unambiguous negative effect on the markup of firm 2. In the case of firm 1, however, there are two effects working in opposite directions: while the increase in search efforts reduces firm 1's markup $mu_{1,t}$, the decline in the other firm's market share, x_2 has a positive effect on $mu_{1,t}$. As figures V.6 through V.9 show, the rise in search activity also leads to a large shift of demand towards the cheaper supplier, reflected by the healthy increase in his market share x_1.[23]

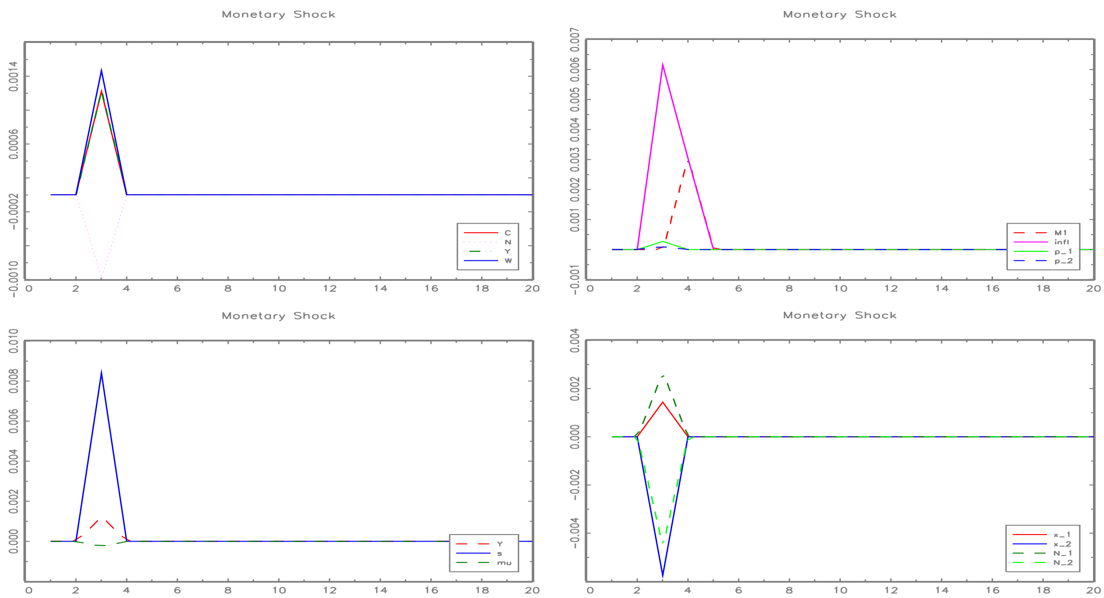
[23] See the fourth panel in figures V.6 through V.9.

Figure V.8: Impulse responses to a monetary shock, $\rho_\tau = 0$, $\alpha = 0.9$, $\gamma = 0.7$, $\Delta = 0.16$, $P_1/P = 0.97$. Relative deviations from steady state.



Y - output, N - hours, C - consumption, W - real wage, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, P_1 - relative price of firm 1, P_2 - relative price of firm 2, s - search activity, mu - average markup, N_1 - labor input of firm 1, N_2 - labor input of firm 2, x_1 - market share of firm 1, x_2 - market share of firm 2.

Figure V.9: Impulse responses to a monetary shock, $\rho_\tau = 0$, $\alpha = 0.9$, $\gamma = 0.7$, $\Delta = 0.20$, $P_1/P = 0.97$. Relative deviations from steady state.



Y - output, N - hours, C - consumption, W - real wage, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, P_1 - relative price of firm 1, P_2 - relative price of firm 2, s - search activity, mu - average markup, N_1 - labor input of firm 1, N_2 - labor input of firm 2, x_1 - market share of firm 1, x_2 - market share of firm 2.

According to V.6 through V.9, the real effects of the monetary shock disappear after one period. The reason for this absence of persistence is that in the period after the shock the increase in real balances is exactly offset by an equally strong reaction of inflation. Hence, in the period after the shock there are neither any positive wealth effects on consumption and leisure nor any positive (or negative) pressure on search activity *via* the term $\frac{m_t}{\pi_t}$. Thus, there are no incentives for agents to search more (or less) than in the stationary equilibrium.

A further interesting feature of the model is that as a reaction to a monetary expansion both relative prices deviate positively from their respective steady state values. However, there is no violation of the definition of the overall price index since the impulse responses of P_1/P and P_2/P are accompanied by suitable reactions of the market shares x_1 and x_2.

Summarizing the results, I would like to point out that even though the model presented here does not reproduce the empirically observable shape and persistence of the impulse responses to monetary shocks, it proposes a simple mechanism which substantially amplifies the real effects of monetary policy, making their magnitude consistent with the empirical estimates. Furthermore, as figure V.6 suggests, combining the current model with other theoretical building blocks could be a fruitful line of research when trying to explain the observable *delayed* response of inflation to monetary expansions.

5.2 Technology Shocks

Figures V.10 through V.12 depict the impulse responses to an economy wide technology shock with no serial correlation. In all cases there is a slight decrease in both relative prices, with the reaction of P_2/P being stronger. There is a large drop in search activity and perhaps surprisingly, an increase in the market share of the firm charging the higher price. The intuition behind these results is as follows: The technology shock enables the economy to produce more even by employing less labor and so puts a downward pressure on inflation. The fall in inflation has a strong negative effect on search activity. The lower level of search intensity in the goods market enables both firms to choose higher markups. At the same time, due to the technological improvement, for any given real wage marginal costs become lower. The firm with the lower steady state productivity (firm 2) faces a larger marginal cost decrease, which enables it to reduce its relative price P_2/P by a larger percentage amount than firm 1 does.[24] The stronger (weaker) decrease in the relative price of firm 2 (firm 1) combined with a lower search activity in the goods market, in turn, leads to an increase in the market share of the supplier charging the higher price (firm 2).

What happens to markups? The fall in both, overall search efforts and the market share of firm 1, unambiguously makes it possible for firm 2 to increase its markup. With regard to firm 1 there are again two opposing effects: The decline in search intensity allows firm 1 to set a higher markup but the increase in the market share of its competitor does the

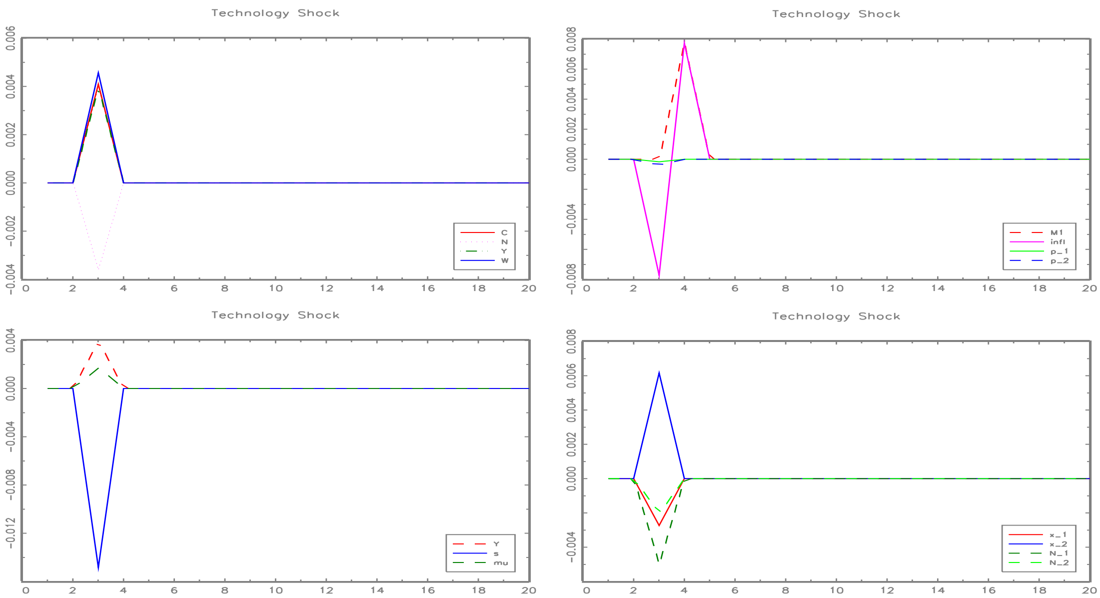
[24] The direct effect of the productivity disturbance on the relative price is measured by the term $\frac{W/P}{(P_1/P)(1+\iota)}\hat{Z}_t$ in the case of firm 1 and by $\frac{W/P}{(P_2/P)(1-\iota)}\hat{Z}_t$ for firm 2. In all numerical simulations performed $(P_1/P)(1+\iota)$ is larger than one while $(P_2/P)(1-\iota)$ takes a value below one.

opposite. Which of these two effects dominates depends on the value of γ. To see this, just log-linearize the first order condition for optimal price setting of firm 1 and take a look at the elasticities in front of s_t and $x_{2,t}$. The log-linear equation reads:

$$\widehat{\left(\frac{P_{1,t}}{P_t}\right)} = -\frac{\gamma}{(P_1/P)s^\gamma x_2}\hat{s}_t - \frac{1}{(P_1/P)s^\gamma x_2}\hat{x}_{2,t} + \frac{W/P}{(P_1/P)(1+\iota)}\widehat{\left(\frac{W_t}{P_t}\right)} - \frac{W/P}{(P_1/P)(1+\iota)}\hat{Z}_t.$$

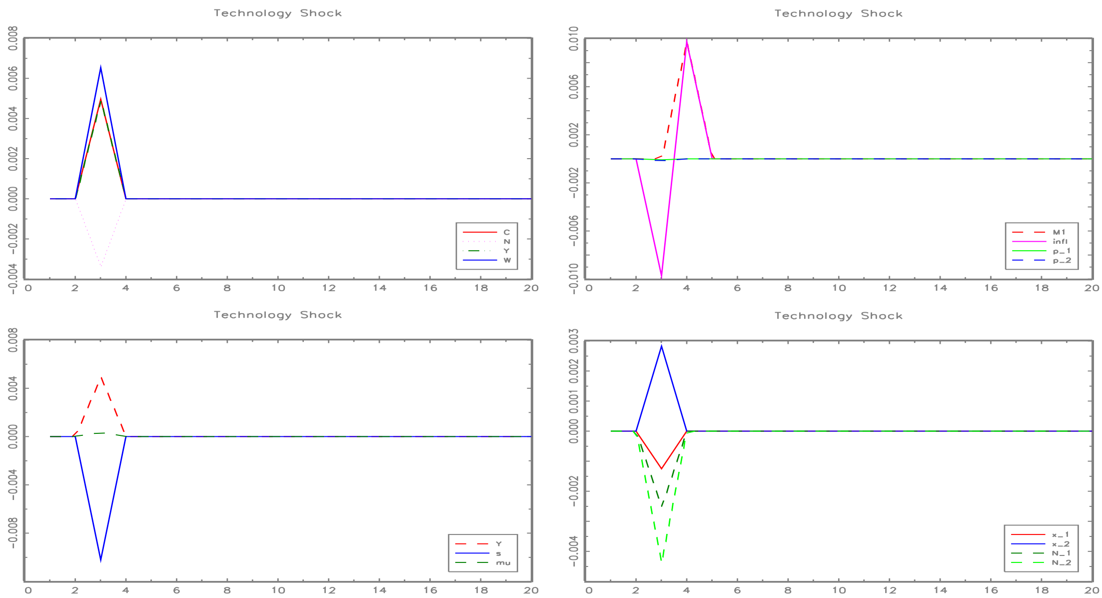
Now it is readily seen that the smaller the parameter γ, the lower the value of the elasticity $\frac{\gamma}{(P_1/P)s^\gamma x_2}$ and thus, the smaller the importance of changes in search activity for the determination of $P_{1,t}/P_t$. Hence, if γ is sufficiently low, firm 1 will reduce its markup as a reaction to a positive productivity shock. The latter is supported by figures V.13 through V.15 which show the impulse responses of the firm-specific markups for different values of γ. The other sets of parameters examined reveal qualitatively the same picture. Furthermore, almost all of the paramtereizations used imply that the average markup is countercyclical.

Figure V.10: Impulse responses to a technology shock, $\rho_z = 0$, $\alpha = 0.9$, $\gamma = 0.6$, $\Delta = 0.12$, $P_1/P = 0.97$. Relative deviations from steady state.



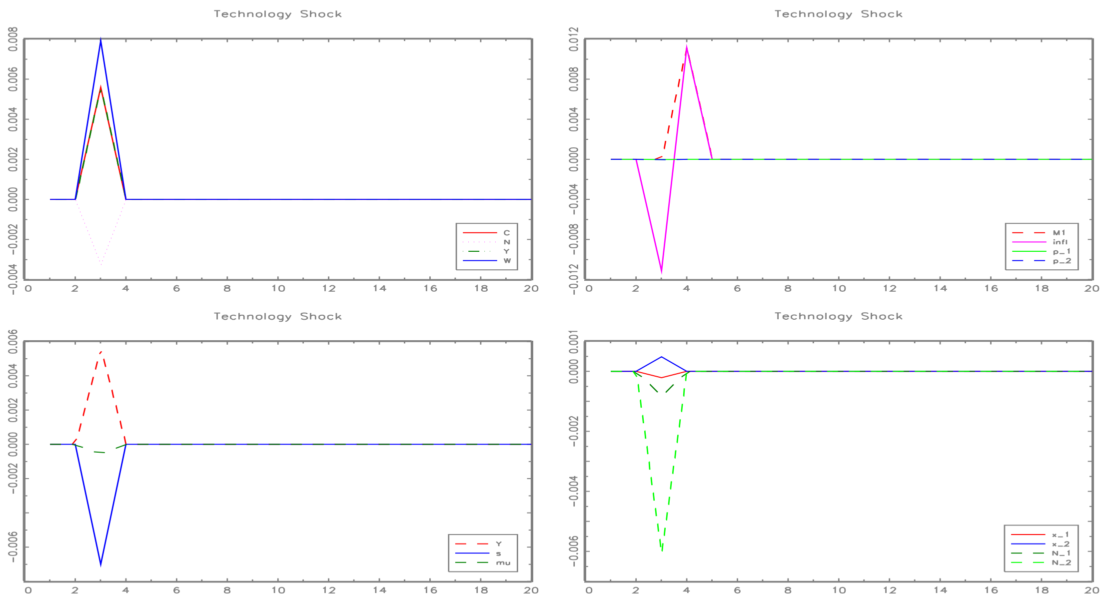
Y - output, N - hours, C - consumption, W - real wage, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, P_1 - relative price of firm 1, P_2 - relative price of firm 2, s - search activity, mu - average markup, N_1 - labor input of firm 1, N_2 - labor input of firm 2, x_1 - market share of firm 1, x_2 - market share of firm 2.

Figure V.11: Impulse responses to a technology shock, $\rho_z = 0$, $\alpha = 0.9$, $\gamma = 0.4$, $\Delta = 0.12$, $P_1/P = 0.97$. Relative deviations from steady state.



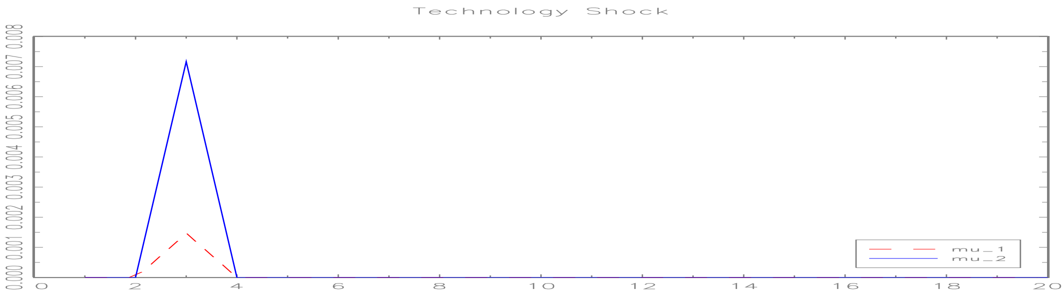
Y - output, N - hours, C - consumption, W - real wage, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, P_1 - relative price of firm 1, P_2 - relative price of firm 2, s - search activity, mu - average markup, N_1 - labor input of firm 1, N_2 - labor input of firm 2, x_1 - market share of firm 1, x_2 - market share of firm 2.

Figure V.12: Impulse responses to a technology shock, $\rho_z = 0$, $\alpha = 0.9$, $\gamma = 0.1$, $\Delta = 0.12$, $P_1/P = 0.97$. Relative deviations from steady state.



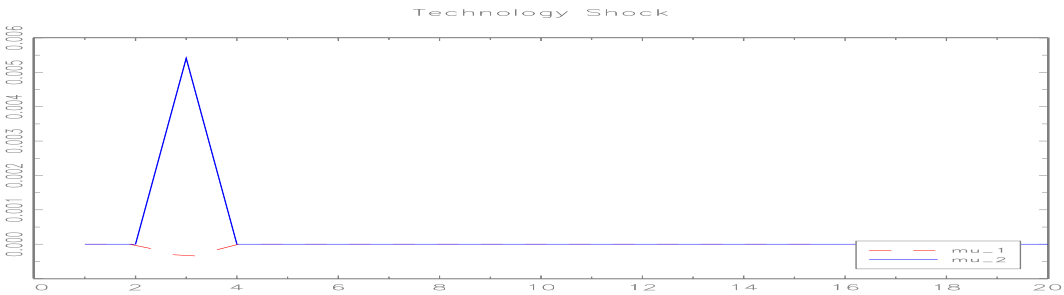
Y - output, N - hours, C - consumption, W - real wage, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, P_1 - relative price of firm 1, P_2 - relative price of firm 2, s - search activity, mu - average markup, N_1 - labor input of firm 1, N_2 - labor input of firm 2, x_1 - market share of firm 1, x_2 - market share of firm 2.

Figure V.13: Impulse responses to a technology shock, $\rho_z = 0$, $\alpha = 0.9$, $\gamma = 0.1$, $\Delta = 0.12$, $P_1/P = 0.97$. Relative deviations from steady state.



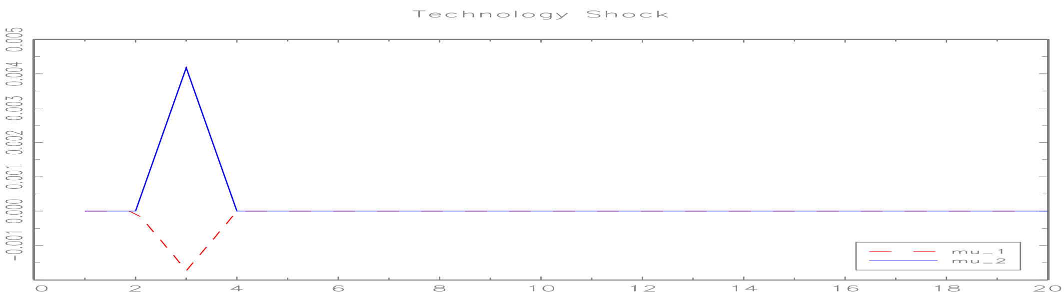
mu_1 - markup of firm 1, mu_2 - markup of firm 2.

Figure V.14: Impulse responses to a technology shock, $\rho_z = 0$, $\alpha = 0.9$, $\gamma = 0.6$, $\Delta = 0.12$, $P_1/P = 0.97$. Relative deviations from steady state.



mu_1 - markup of firm 1, mu_2 - markup of firm 2.

Figure V.15: Impulse responses to a technology shock, $\rho_z = 0$, $\alpha = 0.9$, $\gamma = 0.4$, $\Delta = 0.12$, $P_1/P = 0.97$. Relative deviations from steady state.



mu_1 - markup of firm 1, mu_2 - markup of firm 2.

6 Capital Accumulation

6.1 The Model

Let us extend the model by assuming that there are two production factors - capital and labor. The production function of firm l exhibits constant returns to scale and is given by

$$Y_{l,t} = (Z_t + \iota_{l,t}) N_{l,t}^{\omega} K_{l,t}^{1-\omega}, \qquad \omega \in (0,1),$$

where $K_{l,t}$ denotes capital input and $Z_t + \iota_{l,t}$ represents total factor productivity following the same stochastic process as in section 2. The aggregate stock of capital evolves according to

$$K_{t+1} = I_t + (1-\upsilon)K_t, \qquad \upsilon \in (0,1). \tag{V.6.1}$$

Marginal costs of firm l, $\mu_{l,t}$ are now given by

$$\mu_{l,t} = \frac{(W_t/P_t)^{\omega} R_t^{1-\omega}}{\omega^{\omega}(1-\omega)^{1-\omega}(Z_t + \iota_{l,t})}, \tag{V.6.2}$$

where R_t denotes the rental rate of capital.

Household's First Order Conditions

Retaining the notation used in section 2 the set of first order conditions describing the behavior of he typical household have to be extended by the following three equations:

$$\Lambda_t = \beta E_t\{\Lambda_{t+1}(1 + R_{t+1} - \upsilon)\},$$

$$\underbrace{C_t + I_t + g(s_t)\frac{m_t}{\pi_t}}_{:=D_t} + m_{t+1} - \frac{m_t}{\pi_t} = \frac{W_t}{P_t}N_t + R_t K_t + \Pi_t + \frac{T_t}{P_t}, \tag{V.6.3}$$

$$K_{t+1} = I_t + (1-\upsilon)K_t.$$

The first condition is the Euler equation governing optimal capital accumulation. The second one is the modified budget constraint. Note that in this version of the model aggregate demand D_t equals the sum of consumption, investment and the expenditure on transaction services. The last equation in (V.6.3) is the low of motion of the capital stock.

Equilibrium

The evolution of the economy is described by the *new* definition of marginal costs (V.6.2), the first order condition for optimal price setting (V.2.3), the households first order conditions (V.2.7), (V.2.8), (V.2.9) and (V.2.11) modified by (V.6.3), the aggregate consistency conditions

$$x_{l,t}D_t = (Z_t + \iota_{l,t})N_{l,t}^{\omega}K_{l,t}^{1-\omega} \qquad l = \{1, 2, ..., n\}, \tag{V.6.4}$$

and

$$\sum_{l=1}^{n} N_{l,t} = N_t, \qquad \sum_{l=1}^{n} K_{l,t} = K_t \tag{V.6.5}$$

as well as the definitions of $x_{l,t}$ for $l = \{1, 2, ..., n\}$ and D_t. Note that if (V.7.4) are satisfied, then the family's budget constraint implies:

$$D_t = Y_t = \sum_{l=1}^{n} \frac{P_{l,t}}{P_t}(Z_t + \iota_{l,t})N_{l,t}^{\omega}K_{l,t}^{1-\omega}.$$

To close the model, one again needs to specify monetary policy and the exogenous productivity processes Z_t and $\iota_{l,t}$.

Calibration

P_1/P. α, γ, the measure of price dispersion Δ, the velocity of money with respect to output v_y, ι, N and the parameter a are calibrated in the same way as in the fixed capital case.[25][26] Then the marginal costs of the two firms can be calibrated by using their price setting conditions:

$$\mu_1 = \frac{(W/P)^{\omega}R^{1-\omega}}{\omega^{\omega}(1-\omega)^{1-\omega}(1+\iota)} = \left(\frac{P_1}{P} - \frac{1}{s^{\gamma}x_2}\right), \qquad \mu_2 = \frac{(W/P)^{\omega}R^{1-\omega}}{\omega^{\omega}(1-\omega)^{1-\omega}(1-\iota)} = \left(\frac{P_2}{P} - \frac{1}{s^{\gamma}x_1}\right)$$

To calibrate the production elasticity of labor, first observe that the real wage and the two marginal costs are related as follows:

$$\frac{W}{P} = \mu_1(1+\iota)\omega N_1^{\omega-1}K_1^{1-\omega} = \mu_2(1+\iota)\omega N_2^{\omega-1}K_2^{1-\omega}.$$

Hence,

$$\frac{W}{P}N_1 = \omega\mu_1 \underbrace{(1+\iota)N_1^{\omega}K_1^{1-\omega}}_{=x_1Y}, \quad \frac{W}{P}N_2 = \omega\mu_2 \underbrace{(1+\iota)N_2^{\omega}K_2^{1-\omega}}_{=x_2Y}.$$

Adding the last two equations together and rearranging yields:

$$\omega = \underbrace{\frac{(W/P)N}{Y}}_{:=\text{ labor share}} \cdot \frac{1}{x_1\mu_1 + x_2\mu_2}.$$

[25] See section 4.

[26] The analysis of the sensitivity of the steady state with respect to Δ can be found in the program "**equilibrium_3.g**".

The labor share is computed on the basis of the national accounts data provided by the NIPA and transformed in the way suggested by Cooley and Prescott (1995). The value obtained equals 0.6748. Unfortunately, $P_1/P < 0.98$ implies that the range of values of Δ consistent with $\omega \in (0,1)$ is extremely small. When the relative price P_1/P is larger or equal to 0.98, Δ should be smaller than 0.09 in order for ω to lie in the range between zero and one.

The consumption-output ratio $\frac{C}{Y}$ is found by using the economy's resource constraint:

$$\frac{C}{Y} = \frac{1 - \frac{g(s)}{v_y}}{1 - \frac{I}{C}},$$

where the investment-consumption ratio $\frac{I}{C}$ is set at its empirical value 0.1982 obtained with date from NIPA's national accounts, transformed as in Cooley and Prescott (1995).

To determine the ratio K_1/K_2 I use the first order conditions for optimal capital input of both firms, evaluated at the stationary equilibrium:

$$\frac{R}{R} = 1 = \frac{\mu_1}{\mu_2}\frac{1+\iota}{1-\iota}\left(\frac{N_1}{N_2}\right)^{\omega}\left(\frac{K_1}{K_2}\right)^{-\omega}. \tag{V.6.6}$$

Multiplying the rhs of the last equation by $\frac{K_1}{K_2} \cdot \frac{K_2}{K_1}$ and rearranging yields:

$$1 = \frac{\mu_1}{\mu_2}\frac{x_1}{x_2}\frac{Y}{Y}\frac{K_2}{K_1}$$

which implies:

$$\frac{K_1/Y}{K_2/Y} = \underbrace{\frac{\mu_1}{\mu_2}\frac{x_1}{x_2}}_{:=\varphi_2}.$$

Then the aggregate consistency condition for capital input

$$\frac{K_1}{Y} + \frac{K_2}{Y} = \frac{K}{Y}$$

allows me to determine K_1/Y and K_2/Y:

$$\frac{K_1}{Y} = \frac{\varphi_2}{1+\varphi_2}\frac{K}{Y}, \qquad \frac{K_2}{Y} = \frac{1}{1+\varphi_2}\frac{K}{Y}.$$

The economy wide capital intensity K/Y is estimated with data provided by the NIPA. The value obtained is 17.44. Having found K_1/Y and K_2/Y one can use equation (V.6.6) and the aggregate consistency condition

$$N_1 + N_2 = N$$

to compute N_1 and N_2. The result is:

$$N_1 = \frac{\varphi_1}{1+\varphi_1}N, \qquad N_2 = \frac{1}{1+\varphi_1}N,$$

where

$$\varphi_1 = \frac{K_1}{K_2}\left(\frac{\mu_1}{\mu_2}\frac{1-\iota}{1+\iota}\right)^{\frac{1}{\omega}}.$$

The depreciation rate υ is calibrated as in the previous sections.

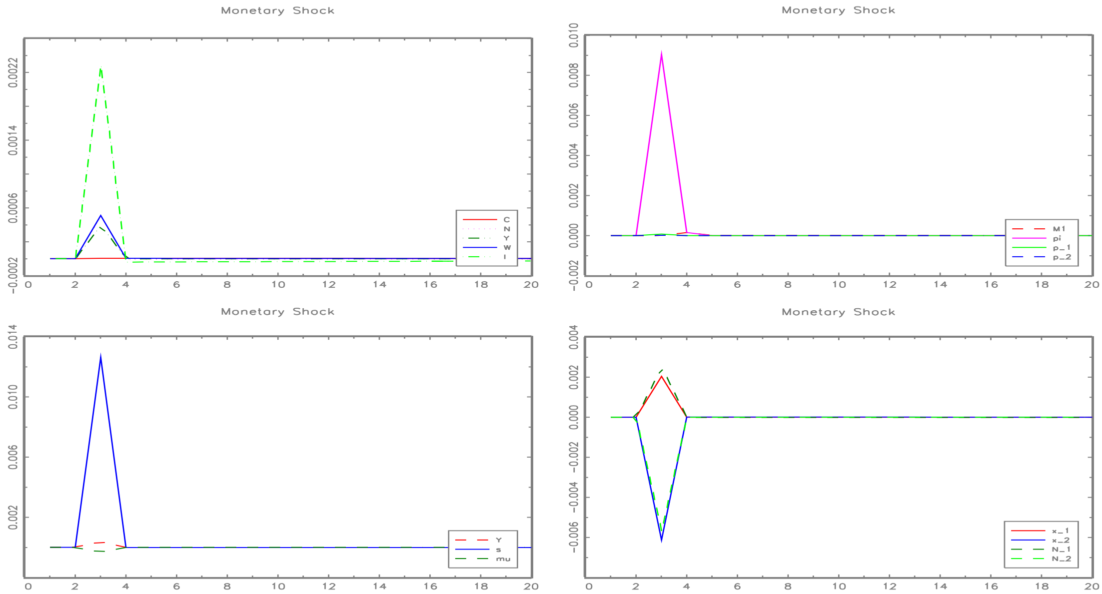
6.2 Results

Monetary Shocks: Figure V.16 depicts the impulse responses to a non-autocorrelated monetary shock.[27] Obviously, the model with flexible capital delivers the same qualitative predictions as the one without capital accumulation: The monetary shock is expansionary mainly due to the fall in markups and triggers off relatively weak reactions of the main economic aggregates. For example the peak-response of output (investment) is about 10 (3) times weaker than what is predicted by the SVAR of Chrstiano *et al.* (2005). The reactions implied by the current model again have the counterfactual property of being extremely short-lived. Similar to the no-capital case, the larger the difference between α and γ and/or the higher the degree of price dispersion measured by Δ, the weaker the real effects of the monetary shock. In general, in this version of the model the response of consumption is of much smaller magnitude than it was the case in the economy presented in section 2. The reason is that capital accumulation allows a more effective consumption smoothing. To take advantage of this possibility, households sharply increase investment in the period of the shock, absorbing in this way virtually the whole additional output. The resulting increase in the capital stock enables the economy to produce and consume more over a relatively long period of time. However, the deviations of output and consumption from their respective steady state values in the periods after the shock are very small.

Technology Shocks: The reactions to technology shocks predicted by the model with flexible capital are also similar to their counterparts implied by the model of section 2 (see figure V.17). For the bulk of the parameterizations examined markups respond negatively to improvements in productivity. As a result, in almost all cases markups are countercyclical on average. The incentive to smooth consumption over time is again strong enough to force households to substantially accelerate capital accumulation in the period of the shock. As a consequence, virtually the whole additional production is again absorbed by investment expenditure. In the periods after the shock there is a *very small* positive deviation of consumption from its steady state value financed by the additionally accumulated capital.

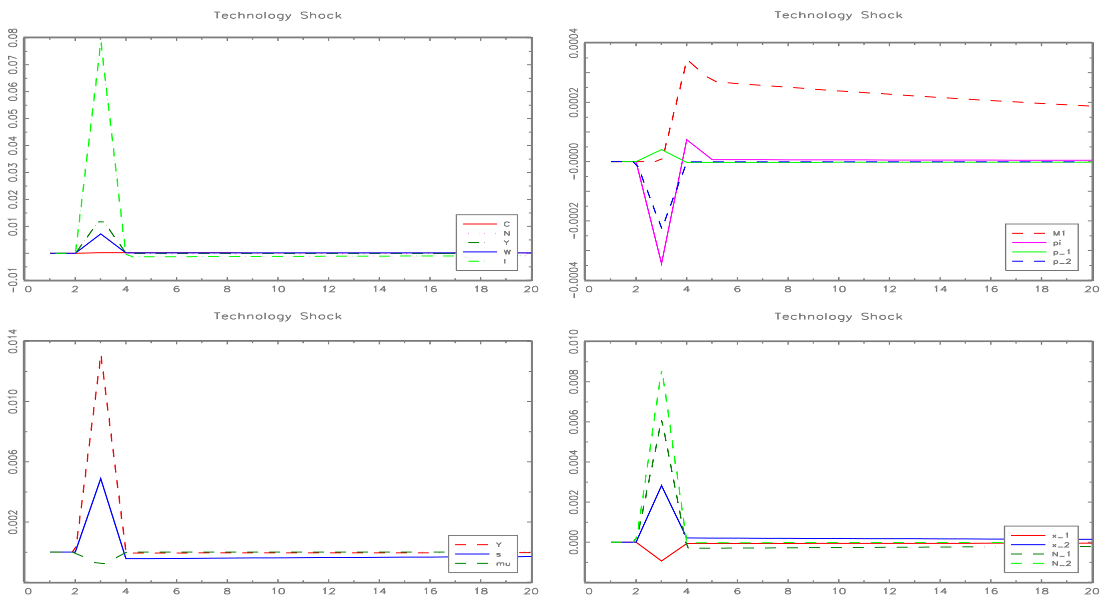
[27] The corresponding program is "**sim_cm2d7a.g**".

Figure V.16: Impulse responses to a monetary shock, $\rho_\tau = 0$, $\alpha = 0.9$, $\gamma = 0.7$, $\Delta = 0.04$, $P_1/P = 0.99$. Relative deviations from steady state.



Y - output, N - hours, C - consumption, W - real wage, I - investment, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, P_1 - relative price of firm 1, P_2 - relative price of firm 2, s - search activity, mu - average markup, N_1 - labor input of firm 1, N_2 - labor input of firm 2, x_1 - market share of firm 1, x_2 - market share of firm 2.

Figure V.17: Impulse responses to a technology shock, $\rho_z = 0$, $\alpha = 0.9$, $\gamma = 0.7$, $\Delta = 0.04$, $P_1/P = 0.99$. Relative deviations from steady state.



Y - output, N - hours, C - consumption, W - real wage, I - investment, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, P_1 - relative price of firm 1, P_2 - relative price of firm 2, s - search activity, mu - average markup, N_1 - labor input of firm 1, N_2 - labor input of firm 2, x_1 - market share of firm 1, x_2 - market share of firm 2.

Unfortunately, the inclusion of capital accumulation does not make the predictions of the model with respect to the persistence of the impulse responses to monetary and real shocks more realistic.

7 Shopping-Time Models

7.1 A Standard Shopping-Time Model

Consider an economy with fully flexible prices and perfectly competitive markets. The utility function of the representative household takes the form

$$U = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\eta}}{1-\eta} - \frac{b}{2}(1-L_t)^2 \right) \right\}, \qquad b, \eta > 0, \quad \beta \in (0,1),$$

where L_t denotes leisure. The corresponding budget constraint is given by

$$C_t + \frac{M_{t+1}}{P_t} - \frac{M_t}{\pi_t P_{t-1}} = \frac{W_t}{P_t} N_t + \frac{T_t}{P_t}, \quad \forall t.$$

A positive valuation of money arises through the following *"shopping-time"* constraint:

$$\frac{s_t^\alpha}{1+s_t^\alpha} = \kappa \frac{C_t}{m_t/\pi_t}, \tag{V.7.1}$$

where $\kappa > 0$ and $\frac{s_t^\alpha}{1+s_t^\alpha}$ is the time needed to carry out transactions in the goods market. According to (V.7.1) a higher real value of the money balances accumulated in the previous period, $\frac{m_t}{\pi_t}$ reduces he transaction time associated with a given desired level of consumption and thus, lowers the transaction costs. The shopping-time technology (V.7.1) originates from the idea that in a typical barter economy each agent faces extremely large search costs since she can only achieve the desired consumption bundle if she is able to find enough other individuals supplying exactly the goods our agent desires and at the same time, demanding exactly the good(s) she supplies. The search costs, however, can be substantially reduced by the introduction of money as a common medium of exchange and unit of account. The transaction cost motive for holding money dates back to Baumol (1952) and Tobin (1956). In more recent papers Saving (1971), Jovanovich (1982) and Romer (1986) develop general equilibrium versions of the shopping-time model. The model analysed in this subsection is very similar to the one presented in Walsh (2003), Ch. 3.

The time constraint of the household reads:

$$L_t + N_t + \frac{s_t^\alpha}{1+s_t^\alpha} = 1.$$

The representative firm produces according to the production function:

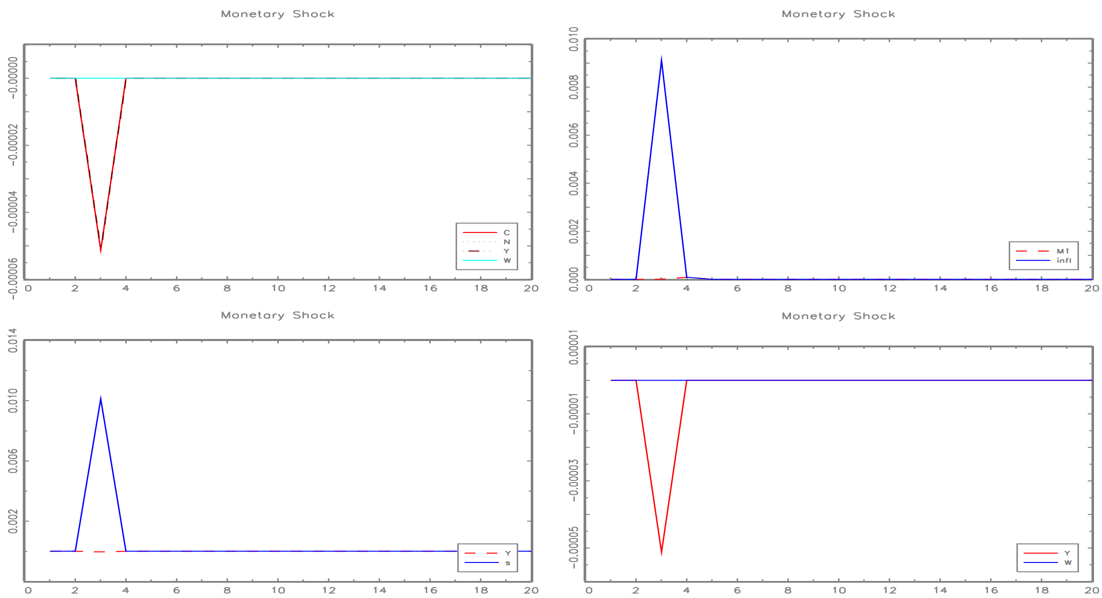
$$Y_t = Z_t N_t,$$

where Z_t evolves according to the same stochastic process as in the previous sections. The behavior of the central bank, too, is modeled as in section 2.

Impulse Responses: As figure V.18 shows the real effects of a one-time increase in money supply are negligible.[28] [29] Furthermore, the monetary expansion is *contractionary*. It leads to a decline of working hours, output and consumption, while the real wage remains unchanged.[30] The time spent shopping is the only variable which reacts positively to the monetary shock. In addition, the model os not able to reproduce the persistence observable in the data. Variations of the parameter α have a negligible effect on the quantitative implications of the model, with higher values of α making the real effects of the monetary disturbance even weaker.

How does the introduction of the kind of market share competition proposed in section 2 alter the predictions of the shopping-time model?

Figure V.18: Impulse responses to a monetary shock, $\rho_T = 0$, $\alpha = 0.9$. Relative deviations from steady state.



Y - output, N - hours, C - consumption, W - real wage, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, s - measure of transaction time.

7.2 Shopping-Time and Market Share Competition I

Let us assume that the structure of the goods market is the same as the one described in section 2. Assume further that the representative household solves the same problem as in subsection 7.1 but the fraction of time spent shopping affects the individual market shares of the firms in this economy. Assume that the household does not internalize this effect. The latter can be thus, characterized as an externality induced by shopping or as a by-product of shopping. The idea behind this assumption is that the more time households spend shopping

[28]The remaining parameters are set at their *standard values*.

[29]The corresponding program is "**sim_cm2d8c.g**".

[30]Real wages in this economy can only change if total factor productivity changes.

and thus, the more transactions they are involved in, the better informed they are about the current price distribution as well as the price setting behavior of the individual suppliers. This information allows households to at least partly shift their demand towards the relatively cheap suppliers.

The market share of firm l evolves according to:

$$x_{l,t} = \frac{\exp\left(\left(1 - \frac{P_{l,t}}{P_t}\right) s_t^{\gamma}\right)}{\sum_{j=1}^{n} \exp\left(\left(1 - \frac{P_{j,t}}{P_t}\right) s_t^{\gamma}\right)}, \qquad \gamma > 0. \tag{V.7.1}$$

Hence, firms with above average prices suffer larger losses in market share when households devote more time to transactions in the goods market. Note that in this model aggregate demand equals aggregate consumption expenditure.

Equilibrium: The evolution of this economy is described by the following set of equations. The utility maximization problem of the representative household delivers the following first order conditions:

$$\begin{aligned}
&C_t^{-\eta} = \Lambda_t + \kappa \frac{\Gamma_t \pi_t}{m_t}, \\
&b(1 - L_t) = \Lambda_t \frac{W_t}{P_t}, \\
&\Lambda_t \frac{W_t}{P_t} = \Gamma \frac{\alpha s_t^{\alpha-1}}{(1 + s_t^{\alpha})^2}, \\
&\frac{s_t^{\alpha}}{1 + s_t^{\alpha}} = \kappa \frac{C_t}{m_t / \pi_t}, \\
&\Lambda_t = \beta E_t \left\{ \kappa \frac{\Gamma_{t+1}}{m_{t+1}^2} C_{t+1} \pi_{t+1} + \frac{\Lambda_{t+1}}{\pi_{t+1}} \right\}, \\
&C_t + m_{t+1} - \frac{m_t}{\pi_t} = \frac{W_t}{P_t} N_t + \Pi_t + \frac{T_t}{P_t}, \\
&N_t + \frac{s_t^{\alpha}}{1 + s_t^{\alpha}} + L_t = 1,
\end{aligned} \tag{V.7.2}$$

where Λ_t and Γ_t are the lagrangean multipliers associated with the budget constraint and the shopping-time constraint respectively.

The conditions for optimal price setting of the two firms are again given by:

$$\frac{P_{1,t}}{P_t} = \frac{1}{s_t^{\gamma} x_{2,t}} + \frac{W_t/P_t}{Z_t + \iota_{1,t}},$$
$$\frac{P_{2,t}}{P_t} = \frac{1}{s_t^{\gamma} x_{1,t}} + \frac{W_t/P_t}{Z_t + \iota_{2,t}}. \tag{V.7.3}$$

Furthermore, the following aggregate consistency conditions hold:[31]

$$x_{l,t} D_t = (Z_t + \iota_{l,t}) N_{l,t}, \qquad l = \{1, 2\}, \tag{V.7.4}$$

and

$$\sum_{l=1}^{2} N_{l,t} = N_t. \tag{V.7.5}$$

Impulse Responses: Unfortunately, it turns out that there is only a relatively small range of parameter values implying an economically meaningful stationary equilibrium.[32] In particular, to ensure that the steady state value of leisure is positive, the relative price P_1/P should be lower than 0.96 and at the same time, the difference $\alpha - \gamma$ should be sufficiently low. The critical value for $\alpha - \gamma$ depends on the absolute values of the two parameters as well as on P_1/P. Figure V.19 depicts the impulse responses to a monetary disturbance without serial correlation. The increase in inflation necessitates a higher level of transaction time. As a result firms are forced to reduce their markups which in turn, leads to a real wage increase. However, the latter is not sufficient to induce households to work more and thus, the economy to produce more. The reason is that the jump in inflation generates too strong an incentive for agents to raise shopping time, so that labor supply and leisure ought to be reduced. As a consequence, the positive monetary shock again leads to an economic contraction in which the real wage and the time spent shopping are the only non-nominal aggregates deviating positively from their respective steady state values.

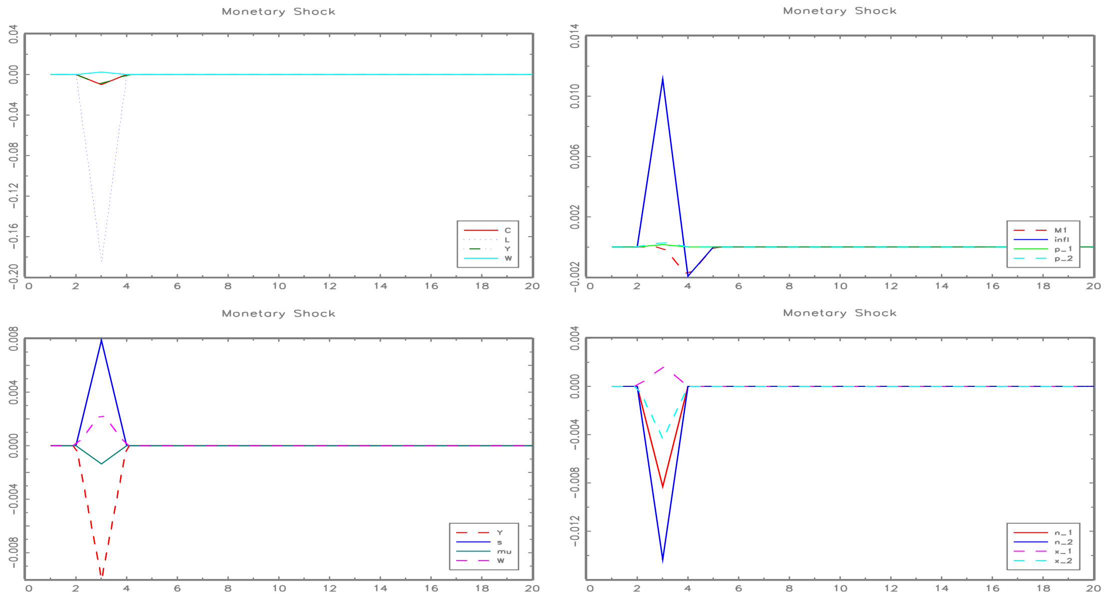
As can also be seen, the real effects of monetary policy are much more pronounced in this version of the shopping-time model than in the one described in subsection 7.1. The sensitivity analysis performed revealed that varying the model parameters within the economically meaningful range has a negligible effect on the quantitative predictions of the model.

[31] The overall price index is defined as:

$$\sum_{l=1}^{n} x_{l,t} \frac{P_{l,t}}{P_t} = 1.$$

[32] The corresponding programs are "**equilibrium_4.g**" for the analysis of the steady state and "**sim_cm2d8b.g**" for the computation of the impulse responses provided in this paragraph.

Figure V.19: Impulse responses to a monetary shock, $\rho_\tau = 0$, $\alpha = 0.9$, $\gamma = 0.8$, $P_1/P = 0.94$, $\Delta = 0.18$. Relative deviations from steady state.



Y - output, N - hours, C - consumption, W - real wage, L - leisure, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, p_1 - relative price of firm 1, p_2 - relative price of firm 2, s - measure of transaction time, mu - average markup, x_1 - market share of firm 1, x_2 - market share of firm 2, n_1 - labor input of firm 1, n_2 - labor input of firm 2.

7.3 Shopping-Time and Market Share Competition II

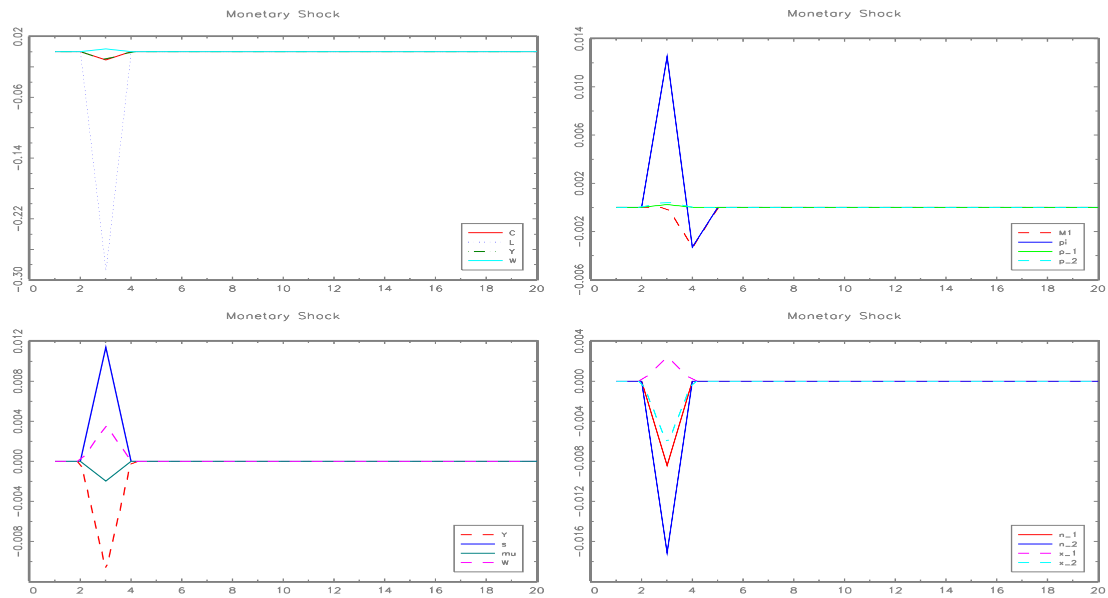
Now assume that households are aware of the link between the time they spent carrying out transactions and the probability to become a customer of a particular firm. Assume further that the household sector has the structure proposed in section 2, so that the utility maximization is performed in two stages and the equilibrium is symmetric. In the second stage of utility maximization, given the lagrangeans Λ_t and Γ_t, the real wage W_t/P_t and the level of consumption expenditure C_t the typical member of an arbitrary family chooses the optimal level of shopping time. The latter is set according to:

$$\Lambda_t \frac{W_t}{P_t} - \Gamma \frac{\alpha s_t^{\alpha-1}}{(1+s_t^\alpha)^2} = \frac{s_t^{\gamma-1}}{\gamma} \left(\sum_{l=1}^{n} x_{l,t} \left(1 - \frac{P_{l,t}}{P_t}\right)^2 \right) C_t.$$

Figure V.20 displays the impulse responses to the same one-time monetary shock.[33] It is easily seen, that the reactions implied by the modified model are virtually identical to that predicted by the previous version of the model. Thus, the more complicated and more sophisticated theoretical mechanisms underlying the theoretical framework of the current subsection do not eliminate the major weaknesses of the model presented in subsection 7.2.

[33]The corresponding program is "**sim_cm2d8a.g**".

Figure V.20: Impulse responses to a monetary shock, $\rho_\tau = 0$, $\alpha = 0.9$, $\gamma = 0.8$, $P_1/P = 0.94$, $\Delta = 0.18$. Relative deviations from steady state.



Y - output, N - hours, C - consumption, W - real wage, L - leisure, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, p_1 - relative price of firm 1, p_2 - relative price of firm 2, s - measure of transaction time, mu - average markup, x_1 - market share of firm 1, x_2 - market share of firm 2, n_1 - labor input of firm 1, n_2 - labor input of firm 2.

7.4 Steady State and Calibration

The purpose of this subsection is to summarize the most important calibration steps regarding the models presented in subsections 7.2 and 7.3.

The households' first order conditions with respect to s_t, M_{t+1} and C_t evaluated at the steady state imply:

$$C^{-\eta} = \Lambda + \kappa \frac{\Gamma \pi}{m},$$

$$\Lambda \frac{W}{P} = \Gamma \frac{\alpha s^{\alpha-1}}{(1+s^\alpha)^2} + \underbrace{\frac{s^{\gamma-1}}{\gamma} \left(\sum_{l=1}^{2} x_l \left(1 - \frac{P_l}{P}\right)^2 \right) C}_{:=\xi_5}, \tag{V.7.1}$$

$$\frac{\pi - \beta}{\pi \beta} = \kappa \frac{C\pi}{m} \frac{\Gamma}{\Lambda} \frac{1}{m}.$$

ξ_5, s and $C = \frac{P_1}{P}(1+\iota)N_1 + \frac{P_2}{P}(1-\iota)N_2$ are calibrated in the same way as in section 4.[34] With respect to α I perform a sensitivity analysis. Then take into account the definition

[34] s is given by:

$$s = \left(\frac{\ln\left(\frac{\Delta + P_1/P - 1}{1 - P_1/P}\right)}{\Delta} \right)^{\frac{1}{\gamma}}.$$

of the shopping-time technology, multiply both sides of the first equation in V.7.1 by C and solve the resulting system for Λ, Γ and m. The *relevant* results are:

$$\Lambda = \frac{\xi_5 + \frac{\alpha}{s(1+s^\alpha)} C^{1-\eta}}{\frac{W}{P} + \frac{\alpha}{s(1+s^\alpha)} C},$$

$$\Gamma = \frac{1+s^\alpha}{s^\alpha}(C^{1-\eta} - \Lambda C), \tag{V.7.2}$$

$$m = \frac{s^\alpha}{1+s^\alpha}\frac{\beta\pi}{\pi-\beta}\frac{\Gamma}{\Lambda}.$$

The steady state value of leisure L is determined by the time constraint with $N = 0.1386$:

$$L = 1 - N - \frac{s^\alpha}{1+s^\alpha}.$$

The calibration of the model presented in subsection 7.2 is obtained by setting ξ_5 equal to zero.

8 Supplement to Chapter V

The first order condition at the second stage reads:

$$\frac{g'(s_{i,t})s_{i,t}^{1-\gamma}}{\gamma} = \left(\sum_{l=1}^{n} \tilde{x}_{i,l,t}\left(1 - \frac{P_{l,t}}{P_t}\right)^2\right)\frac{\pi_t}{m_{i,t}} D_{i,t},$$

where i denotes the index of the family member. To derive this result, the following relationships and results were used: First, recall that

$$\tilde{x}_{i,l,t} = \frac{\exp\left(\left(1 - \frac{P_{l,t}}{P_t}\right) s_{i,t}^{\gamma}\right)}{\sum_{j=1}^{n} \exp\left(\left(1 - \frac{P_{j,t}}{P_t}\right) s_{i,t}^{\gamma}\right)} \quad \Rightarrow$$

$$\Rightarrow \quad \frac{\partial \tilde{x}_{i,l,t}}{\partial s_{i,t}} = \gamma s_{i,t}^{\gamma-1}\tilde{x}_{i,l,t}\left(1 - \frac{P_{l,t}}{P_t}\right) - \gamma s_{i,t}^{\gamma-1}\tilde{x}_{i,l,t}\left(\sum_{l=1}^{n}\tilde{x}_{i,l,t}\left(1 - \frac{P_{l,t}}{P_t}\right)\right).$$

Second, observe that

$$\sum_{l=1}^{n}\tilde{x}_{i,l,t}\left(1 - \frac{P_{l,t}}{P_t}\right) = 0,$$

so that

$$\frac{\partial \tilde{x}_{i,l,t}}{\partial s_{i,t}} = \gamma s_{i,t}^{\gamma-1} \tilde{x}_{i,l,t} \left(1 - \frac{P_{l,t}}{P_t}\right).$$

Hence

$$\sum_{l=1}^{n} \frac{\partial \tilde{x}_{i,l,t}}{\partial s_{i,t}} \frac{P_{l,t}}{P_t} = \gamma s_{i,t}^{\gamma-1} \sum_{l=1}^{n} \tilde{x}_{i,l,t} \left(\frac{P_{l,t}}{P_t} - \left(\frac{P_{l,t}}{P_t}\right)^2\right).$$

Now add to the last equation the following one:

$$0 = \gamma s_{i,t}^{\gamma-1} \sum_{l=1}^{n} \tilde{x}_{i,l,t} \left(\frac{P_{l,t}}{P_t} - 1\right)$$

and rearrange to obtain

$$\sum_{l=1}^{n} \frac{\partial \tilde{x}_{i,l,t}}{\partial s_{i,t}} \frac{P_{l,t}}{P_t} = \gamma s_{i,t}^{\gamma-1} \sum_{l=1}^{n} \tilde{x}_{i,l,t} \left(-1 + 2\frac{P_{l,t}}{P_t} - \left(\frac{P_{l,t}}{P_t}\right)^2\right) = -\gamma s_{i,t}^{\gamma-1} \sum_{l=1}^{n} \tilde{x}_{i,l,t} \left(1 - \frac{P_{l,t}}{P_t}\right)^2.$$

Now insert this result into the second-stage first order condition

$$-g'(s_{i,t}) \frac{m_{i,t}}{\pi_t} = D_{i,t} \sum_{l=1}^{n} \frac{\partial \tilde{x}_{i,l,t}}{\partial s_{i,t}} \frac{P_{l,t}}{P_t}$$

to obtain the result given in the main text.

Chapter 6

Conclusion

This monograph describes three different theoretical frameworks that can be used to address issues concerning business cycles phenomena or monetary policy. Not surprisingly, each framework has its advantages and drawbacks. But one of the theories emerges as better than or, if we want to be more modest, at least as good as the widely used New Keynesian model. It is the Inflation Aversion model. I conclude that the latter is a useful, or even better, alternative to the former. And because for a wide range of parameter values both models deliver similar results, it can not be ruled out that the *true* foundation of the New Keynesian framework is not a version of the menu-cost model, but rather the Inflation Aversion model.

The models developed in the current monograph are only baseline versions that can be extended in various directions. For example consider nominal exchange rate pass-through. According to the empirical evidence, movements in the nominal exchange rate are not fully passed through to import prices. Furthermore, the degree of pass-through is time varying. For example in a world economy characterized by search activity in the goods market and static market share competition as defined in the last chapter, the optimal pricing decision of a foreign firm selling part of its output in the home market will be governed by the following condition:

$$P_{i,t} = \left(1 + \frac{1}{s_t^{\gamma}(1 - x_{i,t})} \frac{Z_{i,t}^*}{\varepsilon_t \frac{P_t^*}{P_t} \frac{W_t^*}{P_t^*}}\right) \frac{\varepsilon_t W_t^*}{Z_{i,t}^*},$$

where ε_t denotes the nominal exchange rate. W_t^*, P_t^* and $Z_{i,t}^*$ are the foreign nominal wage, the foreign price level and the productivity of the foreign firm respectively. $P_{i,t}$ is the price it charges in the home goods market. $x_{i,t}$ is the respective market share. s_t denotes home search activity. It is easily seen that, everything else equal, an increase in the nominal exchange rate will not be completely passed through to the price $P_{i,t}$ since a higher ε_t implies a lower firm-specific markup. For values of the short run elasticity of demand lower than one the models with dynamic market share competition also imply incomplete exchange rate pass-through.

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Appendix A

New Keynesian Model

The log-linear equations are given in order of appearance in the program. Variables without time index denote steady state values.

1 A Model with Fixed Capital

The optimality condition with respect to consumption implies:

$$C_t^{-\eta} = \Lambda_t,$$

$$\Rightarrow$$

$$\underline{-\eta \hat{C}_t = \hat{\Lambda}_t}.$$

The optimality condition with respect to working hours implies:

$$bN_t = \Lambda_t \frac{W_t}{P_t}$$

$$\Rightarrow$$

$$\underline{\eta \hat{C}_t + \hat{N}_t - \widehat{\frac{W_t}{P_t}} = 0.}$$

The next equation reflects the definition of the production function:

$$Y_t = Z_t N_t \left((1-\varphi) \left(\frac{P_t^o}{P_t} \right)^{-\theta} + \varphi \left(\frac{\pi}{\pi_t} \right)^{-\theta} \right)^{-1}$$

$$\Rightarrow$$

$$\underline{\hat{N}_t - \hat{Y}_t = -\hat{Z}_t},$$

while the expression $\left((1-\varphi)\left(\frac{P_t^o}{P_t}\right)^{-\theta}+\varphi\left(\frac{\pi}{\pi_t}\right)^{-\theta}\right)^{-1}$ has no first order effects on the equilibrium dynamics:

$$\widehat{\left((1-\varphi)\left(\frac{P_t^o}{P_t}\right)^{-\theta}+\varphi\left(\frac{\pi}{\pi_t}\right)^{-\theta}\right)^{-1}}=0.$$

The equation determining the equilibrium real wage implies:

$$mu_t=\frac{(W_t/P_t)N_t}{Y_t}$$

$$\Rightarrow$$

$$\underline{-\hat{N}_t+\hat{Y}_t-\widehat{\frac{W_t}{P_t}}=\widehat{mu}_t.}$$

The aggregate consistency condition reads:

$$C_t=Y_t$$

$$\Rightarrow$$

$$\underline{\hat{C}_t-\hat{Y}_t=0}.$$

The next equation reflects the definition of the markup mu_t:

$$\frac{1}{\mu_t}=mu_t,$$

$$\Rightarrow$$

$$\underline{\hat{\mu}_t+\widehat{mu}_t=0},$$

where μ_t denotes marginal cost.

The optimality condition with respect to money demand implies:

$$\beta\phi_m m_{t+1}^{-\chi}E_t\{\pi_{t+1}^{\chi-1}\}=\Lambda_t-\beta E_t\left\{\frac{\Lambda_{t+1}}{\pi_{t+1}}\right\}$$

$$\Rightarrow$$

$$\underline{\chi\hat{m}_{t+1}+\left(\frac{\beta}{\pi-\beta}+1-\chi\right)E_t\hat{\pi}_{t+1}-\frac{\beta}{\pi-\beta}E_t\hat{\Lambda}_{t+1}+\frac{\pi}{\pi-\beta}\hat{\Lambda}_t=0}$$

The next equation results from the law of motion of money supply:

$$m_{t+1}=\frac{\tau_t}{\pi_t}m_t$$

$$\Rightarrow$$

$$\underline{\hat{m}_{t+1}-\hat{m}_t+\hat{\pi}_t=\hat{\tau}_t}.$$

The last dynamic equation id the New Keynesian Phillips Curve:

$$\underline{\beta E_t \hat{\pi}_{t+1} - \hat{\pi}_t = -\frac{(1-\varphi)(1-\varphi\beta)}{\varphi}\hat{\mu}_t.}$$

2 A Model with Endogenous Capital

The first two equations read:

$$\underline{-\eta \hat{C}_t = \hat{\Lambda}_t},$$

and

$$\underline{\hat{N}_t - \frac{\widehat{W_t}}{P_t} = \hat{\Lambda}_t.}$$

The third equation becomes:

$$Y_t = Z_t N_t^{\omega} K_t^{1-\omega} \left((1-\varphi) \left(\frac{P_t^o}{P_t} \right)^{-\theta} + \varphi \left(\frac{\pi}{\pi_t} \right)^{-\theta} \right)^{-1}$$

$\Rightarrow$

$$\underline{\omega \hat{N}_t - \hat{Y}_t = (\omega - 1)\hat{K}_t - \hat{Z}_t}.$$

The fourth log-linear equation does not change:

$$\underline{-\hat{N}_t + \hat{Y}_t - \frac{\widehat{W_t}}{P_t} = \widehat{mu}_t.}$$

The aggregate consistency condition becomes:

$$Y_t = C_t + I_t,$$

$\Rightarrow$

$$\underline{\frac{C}{Y}\hat{C}_t - \hat{Y}_t + \frac{I}{Y}\hat{I}_t = 0.}$$

The first three dynamic equations are the same:

$$\underline{\chi \hat{m}_{t+1} + \left(\frac{\beta}{\pi - \beta} + 1 - \chi \right) E_t \hat{\pi}_{t+1} - \frac{\beta}{\pi - \beta} E_t \hat{\Lambda}_{t+1} + \frac{\pi}{\pi - \beta} \hat{\Lambda}_t = 0}$$

$$\underline{\hat{m}_{t+1} - \hat{m}_t + \hat{\pi}_t = \hat{\tau}_t}.$$

$$\underline{\beta E_t \hat{\pi}_{t+1} - \hat{\pi}_t - \frac{(1-\varphi)(1-\varphi\beta)}{\varphi}\widehat{mu}_t = 0.}$$

The Euler equation with respect to next-period's stock of capital implies:

$$\Lambda_t = \beta E_t \left\{ \Lambda_{t+1} \left(1 + \frac{(1-\omega)}{mu_{t+1}} \frac{Y_{t+1}}{K_{t+1}} - \upsilon \right) \right\}$$

$$\Rightarrow$$

$$\underline{(1-\beta(1-\upsilon))\hat{K}_{t+1} - E_t\hat{\Lambda}_{t+1} + (1-\beta(1-\upsilon))E_t\widehat{mu}_{t+1} + \hat{\Lambda}_t = (1-\beta(1-\upsilon))E_t\hat{Y}_{t+1}},$$

where υ denotes the rate of depreciation. The last equation reflects the law of motion of capital:

$$K_{t+1} = I_t + (1-\upsilon)K_t,$$

$$\Rightarrow$$

$$\underline{\hat{K}_{t+1} - (1-\upsilon)\hat{K}_t = \upsilon\hat{I}_t}.$$

3 Adjustment Costs of Capital

The first five equations remain unchanged:

$$\underline{-\eta\hat{C}_t = \hat{\Lambda}_t},$$

$$\underline{\hat{N}_t - \frac{\widehat{W_t}}{P_t} = \hat{\Lambda}_t.}$$

$$\underline{\omega\hat{N}_t - \hat{Y}_t = (\omega-1)\hat{K}_t - \hat{Z}_t}.$$

$$\underline{-\hat{N}_t + \hat{Y}_t - \frac{\widehat{W_t}}{P_t} - \widehat{mu}_t = 0.}$$

$$\underline{\hat{Y}_t - \widehat{mu}_t = \hat{K}_t + \hat{R}_t}.$$

The next equation is the optimality condition with respect to investment:

$$q_t = \frac{\Lambda_t}{\phi'\left(\frac{I_t}{K_t}\right)}$$

$$\Rightarrow$$

$$\underline{\varsigma \hat{I}_t + \hat{q}_t = \varsigma \hat{K}_t} + \hat{\Lambda}_t,$$

where $\varsigma < 0$ is the elasticity of the first derivative of the adjustment-costs function $\phi'(.)$ with respect to its argument I/K, evaluated at the steady state. q_t is Tobin's q. The last static equation stems from the resource constraint:

$$\underline{\frac{C}{Y}\hat{C}_t - \hat{Y}_t + \frac{I}{Y}\hat{I}_t = 0.}$$

The first three dynamic equations are the same:

$$\underline{\chi \hat{m}_{t+1} + \left(\frac{\beta}{\pi - \beta} + 1 - \chi\right) E_t \hat{\pi}_{t+1} - \frac{\beta}{\pi - \beta} E_t \hat{\Lambda}_{t+1} + \frac{\pi}{\pi - \beta} \hat{\Lambda}_t = 0}$$

$$\underline{\hat{m}_{t+1} - \hat{m}_t + \hat{\pi}_t = \hat{\tau}_t.}$$

$$\underline{\beta E_t \hat{\pi}_{t+1} - \hat{\pi}_t = \frac{(1-\varphi)(1-\varphi\beta)}{\varphi} \widehat{mu}_t.}$$

The log-linear version of the Euler equation with respect to next-period's stock of capital becomes:

$$q_t = E_t \left\{ \Lambda_{t+1} R_{t+1} + q_{t+1} \left(1 - \upsilon + \phi\left(\frac{I_{t+1}}{K_{t+1}}\right) - \phi'\left(\frac{I_{t+1}}{K_{t+1}}\right) \frac{I_{t+1}}{K_{t+1}} \right) \right\},$$

$$\Rightarrow$$

$$\underline{-\varsigma \upsilon \hat{K}_{t+1} - R E_t \hat{R}_{t+1} - R E_t \hat{\Lambda}_{t+1} = -\varsigma E_t \hat{I}_{t+1} + (1-\upsilon) E_t \hat{q}_{t+1} - \hat{q}_t.}$$

The last equation corresponds to the law of motion of capital:

$$\underline{\hat{K}_{t+1} - (1-\upsilon)\hat{K}_t = \upsilon \hat{I}_t.}$$

Appendix B

Chapter 2

The log-linear equations are given in order of appearance in the program. Variables without time index denote steady state values.

1 A Model with Fixed Capital

$$aC_t^{-b}\left(aC_t^{1-b}+(1-a)\left(\frac{m_t}{\pi_t}\right)^{1-b}\right)^{\frac{b}{1-b}}=\Lambda_t,$$

$\Rightarrow$

$$\underline{b(\xi_1-1)\hat{C}_t-\hat{\Lambda}_t=-b\xi_2\hat{m}_t+b\xi_2\hat{\pi}_t},$$

where

$$\xi_1=\left(1+\frac{1-a}{a}\left(\frac{1-a}{a}\frac{\beta}{\pi-\beta}\right)^{\frac{1-b}{b}}\right)^{-1},\qquad \xi_2=\left(1+\frac{a}{1-a}\left(\frac{a}{1-a}\frac{\pi-\beta}{\beta}\right)^{\frac{1-b}{b}}\right)^{-1}.$$

$$\phi N_t=\Lambda_t\frac{W_t}{P_t}$$

$\Rightarrow$

$$\underline{\hat{N}_t-\widehat{\frac{W_t}{P_t}}-\hat{\Lambda}_t=0.}$$

$$Y_t=Z_tN_t$$

$\Rightarrow$

$$\underline{\hat{N}_t-\hat{Y}_t=-\hat{Z}_t}.$$

$$mu_t = \frac{(W_t/P_t)N_t}{Y_t}$$

$$\Rightarrow$$

$$\underline{-\hat{N}_t + \hat{Y}_t - \widehat{\frac{W_t}{P_t}} = \widehat{mu}_t.}$$

$$C_t = Y_t$$

$$\Rightarrow$$

$$\underline{\hat{C}_t - \hat{Y}_t = 0}.$$

$$\Lambda_t = \beta E_t \left\{ (1-a) \frac{m_{t+1}^{-b}}{\pi_{t+1}^{1-b}} \left(a C_{t+1}^{1-b} + (1-a) \left(\frac{m_{t+1}}{\pi_{t+1}} \right)^{1-b} \right)^{\frac{b}{1-b}} + \frac{\Lambda_{t+1}}{\pi_{t+1}} \right\}$$

$$\Rightarrow$$

$$\underline{b \frac{\pi - \beta}{\pi} \hat{m}_{t+1} + \frac{\pi - b\pi + b\beta}{\pi} E_t \hat{\pi}_{t+1} = b \frac{\pi - \beta}{\pi} E_t \hat{C}_{t+1} + E_t \hat{\Lambda}_{t+1} - \hat{\Lambda}_t.}$$

Note that the last equation, evaluated at the steady state, implies:

$$\Lambda \frac{\pi - \beta}{\pi} = (1-a) \left(\frac{m}{\pi} \right)^{-b} \left(a C^{1-b} + (1-a) \left(\frac{m}{\pi} \right)^{1-b} \right)^{\frac{b}{1-b}}.$$

Eliminating Λ by inserting the first optimality condition, also evaluated at the steady state, yields:

$$\frac{a}{1-a} \frac{\pi - \beta}{\beta} = \left(\frac{m/\pi}{C} \right)^{-b}.$$

$$m_{t+1} = \frac{\tau_t}{\pi_t} m_t$$

$$\Rightarrow$$

$$\underline{\hat{m}_{t+1} - \hat{m}_t + \hat{\pi}_t = \hat{\tau}_t}.$$

The optimality condition of the representative firm can be written as:

$$\left(\frac{P_{i,t}}{P_t} \right)^{-\theta} x_{i,t} Y_t - \theta \left(\frac{P_{i,t}}{P_t} - \mu_t \right) \left(\frac{P_{i,t}}{P_t} \right)^{-\theta - 1} x_{i,t} Y_t + \frac{g' \left(\frac{P_{i,t}}{P_t} \right)}{g \left(\frac{P_{i,t}}{P_t} \right)} \Omega_t = 0,$$

where μ_t denotes marginal costs and

$$\Omega_t = E_t \left\{ \sum_{j=1}^{\infty} \beta^j \frac{\Lambda_{t+j}}{\Lambda_t} x_{i,t+j} \left(\frac{P_{i,t+j}}{P_{t+j}} - \mu_{t+j} \right) \left(\frac{P_{i,t+j}}{P_{t+j}} \right)^{-\theta} Y_{t+j} \right\} = \tag{II.1.1}$$

$$= E_t \left\{ \beta \frac{\Lambda_{t+1}}{\Lambda_t} x_{i,t+1} \left(\frac{P_{i,t+1}}{P_{t+1}} - \mu_{t+1} \right) \left(\frac{P_{i,t+1}}{P_{t+1}} \right)^{-\theta} Y_{t+1} \right\} + E_t \left\{ \beta \frac{\Lambda_{t+1}}{\Lambda_t} \Omega_{t+1} \right\}$$

The first equation implies that in the symmetric equilibrium the markup mu_t is given by:

$$mu_t = \frac{-\theta}{1 - \theta - \gamma \frac{\Omega_t}{Y_t}} \tag{II.1.2}$$

Log-linearizing the last equation yields:

$$\widehat{mu}_t = \frac{\gamma \frac{\Omega}{Y}}{1 - \theta - \gamma \frac{\Omega}{Y}} (\hat{\Omega}_t - \hat{Y}_t) = -\xi (\hat{\Omega}_t - \hat{Y}_t).$$

The log-linear version of equation II.1.1 can be written as:

$$\hat{\Omega}_t = E_t \widehat{DF}_{t,t+1} + \frac{1 - \beta}{(mu - 1)} E_t \widehat{mu}_{t+1} + (1 - \beta) E_t \hat{Y}_{t+1} + \beta E_t \hat{\Omega}_{t+1},$$

where $DF_{t,t+1}$ denotes the stochastic discount factor between t and $t+1$. Combining the last two equations to eliminate $\hat{\Omega}_t$ yields:

$$\underline{\left(\frac{\beta}{\xi} - \frac{1 - \beta}{mu - 1} \right) E_t \widehat{mu}_{t+1} - \frac{1}{\xi} \widehat{mu}_t = E_t \hat{Y}_{t+1} + E_t \hat{\Lambda}_{t+1} - \hat{Y}_t - \hat{\Lambda}_t.}$$

To obtain the log-linear system for the model without market share competition, just neglect the last equation and replace the term $\widehat{mu}_t$ appearing in the fourth underlined equation (the definition of the real wage) by zero.

To obtain the corresponding New Keynesian model, just replace the optimality conditions for consumption and next period real balances in the standard New Keynesian model by the ones given in this section.

2 A Model with Endogenous Capital

The first two equations read:

$$\underline{b(\xi_1 - 1)\hat{C}_t = -b\xi_2 \hat{m}_t + b\xi_2 \hat{\pi}_t + \hat{\Lambda}_t,}$$

and

$$\underline{\hat{N}_t - \frac{\widehat{W_t}}{P_t} = \hat{\Lambda}_t.}$$

The third equation becomes:

$$Y_t = Z_t N_t^{\omega} K_t^{1-\omega}$$

$$\Rightarrow$$

$$\underline{\omega \hat{N}_t - \hat{Y}_t = (\omega - 1)\hat{K}_t - \hat{Z}_t}.$$

The fourth log-linear equation does not change:

$$\underline{-\hat{N}_t + \hat{Y}_t - \widehat{\frac{W_t}{P_t}} = \widehat{mu}_t.}$$

The definition of the real interest rate R_t implies:

$$\underline{\hat{Y}_t - \hat{R}_t = \hat{K}_t + \widehat{mu}_t}.$$

The first three dynamic equations are the same:

$$\underline{b\frac{\pi - \beta}{\pi}\hat{m}_{t+1} + \frac{\pi - b\pi + b\beta}{\pi}E_t\hat{\pi}_{t+1} - E_t\hat{\Lambda}_{t+1} + \hat{\Lambda}_t = b\frac{\pi - \beta}{\pi}E_t\hat{C}_{t+1}}.$$

$$\underline{\hat{m}_{t+1} - \hat{m}_t + \hat{\pi}_t = \hat{\tau}_t}.$$

$$\underline{\left(\frac{\beta}{\xi} - \frac{1-\beta}{mu - 1}\right)E_t\widehat{mu}_{t+1} - E_t\hat{\Lambda}_{t+1} - \frac{1}{\xi}\widehat{mu}_t + \hat{\Lambda}_t = E_t\hat{Y}_{t+1} - \hat{Y}_t}.$$

The log-linear version of the Euler equation with respect to next-period's stock of capital reads:

$$\underline{E_t\hat{\Lambda}_{t+1} - \hat{\Lambda}_t = -(1 - \beta(1 - \upsilon))E_t\hat{R}_{t+1}},$$

where υ denotes the rate of depreciation. The resource constraint becomes:

$$\underline{\hat{K}_{t+1} - (1 - \upsilon)\hat{K}_t = -\frac{C}{K}\hat{C}_t + \frac{Y}{K}\hat{Y}_t}.$$

3 Adjustment Costs of Capital

The first five equations remain unchanged:

$$\underline{b(\xi_1 - 1)\hat{C}_t = -b\xi_2\hat{m}_t + b\xi_2\hat{\pi}_t + \hat{\Lambda}_t},$$

$$\hat{N}_t - \frac{\widehat{W}_t}{P_t} = \hat{\Lambda}_t.$$

$$\omega \hat{N}_t - \hat{Y}_t = (\omega - 1)\hat{K}_t - \hat{Z}_t.$$

$$-\hat{N}_t + \hat{Y}_t - \frac{\widehat{W}_t}{P_t} = \widehat{mu}_t.$$

$$\hat{Y}_t - \hat{R}_t = \hat{K}_t + \widehat{mu}_t.$$

The next equation is the optimality condition with respect to investment:

$$q_t = \frac{\Lambda_t}{\phi'\left(\frac{I_t}{K_t}\right)}$$

$$\Rightarrow$$

$$\varsigma \hat{I}_t + \hat{q}_t = \varsigma \hat{K}_t + \hat{\Lambda}_t,$$

where $\varsigma < 0$ is the elasticity of the first derivative of the adjustment-costs function $\phi'(.)$ with respect to its argument I/K, evaluated at the steady state. q_t is Tobin's q. The last static equation stems from the resource constraint:

$$\frac{C}{Y}\hat{C}_t - \hat{Y}_t + \frac{I}{Y}\hat{I}_t = 0.$$

The first three dynamic equations are the same:

$$b\frac{\pi - \beta}{\pi}\hat{m}_{t+1} + \frac{\pi - b\pi + b\beta}{\pi}E_t\hat{\pi}_{t+1} - E_t\hat{\Lambda}_{t+1} + \hat{\Lambda}_t = b\frac{\pi - \beta}{\pi}E_t\hat{C}_{t+1}.$$

$$\hat{m}_{t+1} - \hat{m}_t + \hat{\pi}_t = \hat{\tau}_t.$$

$$\left(\frac{\beta}{\xi} - \frac{1-\beta}{mu - 1}\right)E_t\widehat{mu}_{t+1} - E_t\hat{\Lambda}_{t+1} - \frac{1}{\xi}\widehat{mu}_t + \hat{\Lambda}_t = E_t\hat{Y}_{t+1} - \hat{Y}_t.$$

The log-linear version of the Euler equation with respect to next-period's stock of capital becomes:

$$q_t = E_t\left\{\Lambda_{t+1}\frac{1-\omega}{mu_{t+1}}\frac{Y_{t+1}}{K_{t+1}} + q_{t+1}\left(1-\upsilon+\phi\left(\frac{I_{t+1}}{K_{t+1}}\right)-\phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}}{K_{t+1}}\right)\right\},$$

$\Rightarrow$

$$\underline{-\varsigma\upsilon\hat{K}_{t+1} - RE_t\hat{\Lambda}_{t+1} - RE_t\hat{R}_{t+1} = -\varsigma E_t\hat{I}_{t+1} + (1-\upsilon)E_t\hat{q}_{t+1} - \hat{q}_t}.$$

The last equation corresponds to the law of motion of capital:

$$\underline{\hat{K}_{t+1} - (1-\upsilon)\hat{K}_t = \upsilon\hat{I}_t}.$$

To obtain the corresponding New Keynesian model, just replace the optimality conditions for consumption and next period real balances in the standard New Keynesian model by the ones given in this section.

Appendix C

Chapter 3

The log-linear equations are given in order of appearance in the program. Variables without time index denote steady state values.

1 A Model with Fixed Capital

$$C_t^{-\eta} = \Lambda_t,$$

$\Rightarrow$

$$\underline{-\eta \hat{C}_t - \hat{\Lambda}_t = 0}.$$

$$bN_t = \Lambda_t \frac{W_t}{P_t}$$

$\Rightarrow$

$$\underline{\eta \hat{C}_t + \hat{N}_t - \widehat{\frac{W_t}{P_t}} = 0.}$$

$$Y_t = Z_t N_t$$

$\Rightarrow$

$$\underline{\hat{N}_t - \hat{Y}_t = -\hat{Z}_t}.$$

$$mu_t = \frac{(W_t/P_t)N_t}{Y_t}$$

$\Rightarrow$

$$\underline{-\hat{N}_t + \hat{Y}_t - \widehat{\frac{W_t}{P_t}} = \widehat{mu}_t.}$$

The optimality condition with respect to search activity implies:

$$\varrho s_t = C_t^{\frac{\eta}{1+\alpha}} \pi_t^{\frac{1}{1+\alpha}},$$

$\Rightarrow$

$$-\frac{\eta}{1+\alpha}\hat{C}_t + \hat{s}_t = \frac{1}{1+\alpha}\hat{\pi}_t.$$

$$C_t + s_t = Y_t$$

$\Rightarrow$

$$\frac{C}{Y}\hat{C}_t - \hat{Y}_t + \frac{s}{Y}\hat{s}_t = 0.$$

$$\beta\phi_m m_{t+1}^{-\chi} E_t\{\pi_{t+1}^{\chi-1}\} = \Lambda_t - \beta E_t\left\{\frac{\Lambda_{t+1}}{\pi_{t+1}}\right\}$$

$\Rightarrow$

$$\chi\hat{m}_{t+1} + \left(\frac{\beta}{\pi-\beta} + 1 - \chi\right) E_t\hat{\pi}_{t+1} = \frac{\beta}{\pi-\beta}E_t\hat{\Lambda}_{t+1} - \frac{\pi}{\pi-\beta}\hat{\Lambda}_t$$

$$m_{t+1} = \frac{\tau_t}{\pi_t}m_t$$

$\Rightarrow$

$$\hat{m}_{t+1} - \hat{m}_t + \hat{\pi}_t = \hat{\tau}_t.$$

The optimality condition of the representative firm can be written as:

$$\left(\frac{P_{i,t}}{P_t}\right)^{-\theta} x_{i,t}Y_t - \theta\left(\frac{P_{i,t}}{P_t} - \mu_t\right)\left(\frac{P_{i,t}}{P_t}\right)^{-\theta-1} x_{i,t}Y_t + \frac{g_1\left(\frac{P_{i,t}}{P_t}, s_t\right)}{g\left(\frac{P_{i,t}}{P_t}, s_t\right)}\Omega_t = 0,$$

where μ_t denotes marginal costs and

$$\Omega_t = E_t\left\{\sum_{j=1}^{\infty}\beta^j\frac{\Lambda_{t+j}}{\Lambda_t}x_{i,t+j}\left(\frac{P_{i,t+j}}{P_{t+j}} - \mu_{t+j}\right)\left(\frac{P_{i,t+j}}{P_{t+j}}\right)^{-\theta}Y_{t+j}\right\} = \tag{III.1.1}$$

$$= E_t\left\{\beta\frac{\Lambda_{t+1}}{\Lambda_t}x_{i,t+1}\left(\frac{P_{i,t+1}}{P_{t+1}} - \mu_{t+1}\right)\left(\frac{P_{i,t+1}}{P_{t+1}}\right)^{-\theta}Y_{t+1}\right\} + E_t\left\{\beta\frac{\Lambda_{t+1}}{\Lambda_t}\Omega_{t+1}\right\}$$

The first equation implies that in the symmetric equilibrium the markup mu_t is given by:

$$mu_t = \frac{-\theta}{1-\theta-s_t\frac{\Omega_t}{Y_t}} \tag{III.1.2}$$

Log-linearizing the last equation yields:

$$\widehat{mu}_t = \frac{s\frac{\Omega}{C}}{1-\theta-s\frac{\Omega}{C}}(\hat{\Omega}_t + \hat{s}_t - \hat{Y}_t) = -\xi(\hat{\Omega}_t - \hat{Y}_t).$$

The log-linear version of equation III.1.1 can be written as:

$$\hat{\Omega}_t = E_t\widehat{DF}_{t,t+1} + \frac{1-\beta}{(mu-1)}E_t\widehat{mu}_{t+1} + (1-\beta)E_t\hat{Y}_{t+1} + \beta E_t\hat{\Omega}_{t+1},$$

where $DF_{t,t+1}$ denotes the stochastic discount factor between t and $t+1$. Combining the last two equations to eliminate $\hat{\Omega}_t$ yields:

$$\underline{\left(\frac{\beta}{\xi} - \frac{1-\beta}{mu-1}\right)E_t\widehat{mu}_{t+1} - \frac{1}{\xi}\widehat{mu}_t = E_t\hat{Y}_{t+1} + E_t\hat{\Lambda}_{t+1} - \beta E_t\hat{s}_{t+1} - \hat{Y}_t - \hat{\Lambda}_t + \hat{s}_t}.$$

2 A Model with Endogenous Capital

The first two equations read:

$$\underline{-\eta\hat{C}_t = \hat{\Lambda}_t},$$

and

$$\underline{\hat{N}_t - \frac{\widehat{W_t}}{P_t} = \hat{\Lambda}_t}.$$

The third equation becomes:

$$Y_t = Z_t N_t^{\omega} K_t^{1-\omega}$$

$$\Rightarrow$$

$$\underline{\omega\hat{N}_t - \hat{Y}_t = (\omega-1)\hat{K}_t - \hat{Z}_t}.$$

The fourth log-linear equation does not change:

$$\underline{-\hat{N}_t + \hat{Y}_t - \frac{\widehat{W_t}}{P_t} = \widehat{mu}_t}.$$

The definition of the real interest rate R_t implies:

$$\underline{\hat{Y}_t - \hat{R}_t = \hat{K}_t + \widehat{mu}_t}.$$

$$\underline{-\frac{\eta}{1+\alpha}\hat{C}_t + \hat{s}_t = \frac{1}{1+\alpha}\hat{\pi}_t}.$$

The first three dynamic equations are the same:

$$\chi\hat{m}_{t+1} + \left(\frac{\beta}{\pi-\beta} + 1 - \chi\right) E_t\hat{\pi}_{t+1} - \frac{\beta}{\pi-\beta}E_t\hat{\Lambda}_{t+1} + \frac{\pi}{\pi-\beta}\hat{\Lambda}_t = 0$$

$$\hat{m}_{t+1} - \hat{m}_t + \hat{\pi}_t = \hat{\tau}_t.$$

$$\left(\frac{\beta}{\xi} - \frac{1-\beta}{mu-1}\right) E_t\widehat{mu}_{t+1} - E_t\hat{\Lambda}_{t+1} - \frac{1}{\xi}\widehat{mu}_t + \hat{\Lambda}_t = E_t\hat{Y}_{t+1} - \beta E_t\hat{s}_{t+1} - \hat{Y}_t + \hat{s}_t.$$

The log-linear version of the Euler equation with respect to next-period's stock of capital reads:

$$E_t\hat{\Lambda}_{t+1} - \hat{\Lambda}_t = -(1-\beta(1-\upsilon))E_t\hat{R}_{t+1},$$

where υ denotes the rate of depreciation. The resource constraint becomes:

$$\hat{K}_{t+1} - (1-\upsilon)\hat{K}_t = -\frac{C}{K}\hat{C}_t + \frac{Y}{K}\hat{Y}_t - \frac{s}{K}\hat{s}_t.$$

3 Adjustment Costs of Capital

The first six equations remain unchanged:

$$-\eta\hat{C}_t = \hat{\Lambda}_t,$$

$$\hat{N}_t - \frac{\widehat{W_t}}{P_t} = \hat{\Lambda}_t.$$

$$\omega\hat{N}_t - \hat{Y}_t = (\omega-1)\hat{K}_t - \hat{Z}_t.$$

$$-\hat{N}_t + \hat{Y}_t - \frac{\widehat{W_t}}{P_t} = \widehat{mu}_t.$$

$$\hat{Y}_t - \hat{R}_t = \hat{K}_t + \widehat{mu}_t.$$

$$-\frac{\eta}{1+\alpha}\hat{C}_t + \hat{s}_t = \frac{1}{1+\alpha}\hat{\pi}_t.$$

The next equation is the optimality condition with respect to investment:

$$q_t = \frac{\Lambda_t}{\phi'\left(\frac{I_t}{K_t}\right)}$$

$$\Rightarrow$$

$$\varsigma\hat{I}_t + \hat{q}_t = \varsigma\hat{K}_t + \hat{\Lambda}_t,$$

where $\varsigma < 0$ is the elasticity of the first derivative of the adjustment-costs function $\phi'(.)$ with respect to its argument I/K, evaluated at the steady state. q_t is Tobin's q. The last static equation stems from the resource constraint:

$$\frac{C}{Y}\hat{C}_t - \hat{Y}_t + \frac{s}{Y}\hat{s}_t + \frac{I}{Y}\hat{I}_t = 0.$$

The first three dynamic equations are the same:

$$\chi\hat{m}_{t+1} + \left(\frac{\beta}{\pi-\beta} + 1 - \chi\right)E_t\hat{\pi}_{t+1} - \frac{\beta}{\pi-\beta}E_t\hat{\Lambda}_{t+1} + \frac{\pi}{\pi-\beta}\hat{\Lambda}_t = 0$$

$$\hat{m}_{t+1} - \hat{m}_t + \hat{\pi}_t = \hat{\tau}_t.$$

$$\left(\frac{\beta}{\xi} - \frac{1-\beta}{mu-1}\right)E_t\widehat{mu}_{t+1} - \frac{1}{\xi}\widehat{mu}_t = E_t\hat{Y}_{t+1} + E_t\hat{\Lambda}_{t+1} - \hat{Y}_t - \hat{\Lambda}_t.$$

The log-linear version of the Euler equation with respect to next-period's stock of capital becomes:

$$q_t = E_t\left\{\Lambda_{t+1}\frac{1-\omega}{mu_{t+1}}\frac{Y_{t+1}}{K_{t+1}} + q_{t+1}\left(1 - \upsilon + \phi\left(\frac{I_{t+1}}{K_{t+1}}\right) - \phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}}{K_{t+1}}\right)\right\},$$

$$\Rightarrow$$

$$-\varsigma\upsilon\hat{K}_{t+1} - RE_t\hat{\Lambda}_{t+1} = RE_t\hat{R}_{t+1} - \varsigma E_t\hat{I}_{t+1} + (1-\upsilon)E_t\hat{q}_{t+1} - \hat{q}_t.$$

The last equation corresponds to the law of motion of capital:

$$\hat{K}_{t+1} - (1-\upsilon)\hat{K}_t = \upsilon\hat{I}_t.$$

Appendix D

Chapter 5

The log-linear equations are given in order of appearance in the program. Variables without time index denote steady state values.

1 A Model with Fixed Capital

$$C_t^{-\eta} = \Lambda_t,$$

$\Rightarrow$

$$\underline{-\eta \hat{C}_t = \hat{\Lambda}_t}.$$

$$bN_t = \Lambda_t \frac{W_t}{P_t},$$

$\Rightarrow$

$$\underline{\hat{N}_t - \widehat{\frac{W_t}{P_t}} = \hat{\Lambda}_t}.$$

The equations defining the firm-specific marginal costs imply:

$$\mu_{1,t} = \frac{W_t/P_t}{Z_t + \iota}, \qquad \mu_{2,t} = \frac{W_t/P_t}{Z_t - \iota},$$

$\Rightarrow$

$$\underline{\widehat{\frac{W_t}{P_t}} - \hat{\mu}_{1,t} = \frac{1}{1+\iota}\hat{Z}_t}, \qquad \underline{\widehat{\frac{W_t}{P_t}} - \hat{\mu}_{2,t} = \frac{1}{1-\iota}\hat{Z}_t}.$$

The next equation reflects the first aggregate consistency condition:

$$x_{1,t}Y_t = (Z_t + \iota)N_{1,t},$$

$$\Rightarrow$$

$$\hat{Y}_t + \hat{x}_{1,t} - \hat{N}_{1,t} = \frac{1}{1+\iota}\hat{Z}_t.$$

$$C_t + g(s_t)\frac{m_t}{\pi_t} = Y_t,$$

$$\Rightarrow$$

$$\frac{C}{Y}\hat{C}_t - \hat{Y}_t + \frac{m}{\pi Y}\frac{\alpha s^\alpha}{a}\hat{s}_t - \frac{m}{\pi Y}\frac{s^\alpha}{a}\hat{\pi}_t = -\frac{m}{\pi Y}\frac{s^\alpha}{a}\hat{m}_t,$$

where $\frac{m}{\pi Y}$ is calibrated by using the empirically observable velocity of money with respect to output.

$$\frac{P_{1,t}}{P_t} = \frac{1}{s_t^\gamma x_{2,t}} + \frac{W_t/P_t}{Z_t + \iota}, \qquad \frac{P_{2,t}}{P_t} = \frac{1}{s_t^\gamma x_{1,t}} + \frac{W_t/P_t}{Z_t - \iota},$$

$$\Rightarrow$$

$$\frac{W/P}{(P_1/P)(1+\iota)}\frac{\widehat{W}_t}{P_t} - \frac{\gamma}{(P_1/P)s^\gamma x_2}\hat{s}_t - \frac{\widehat{P}_{1,t}}{P_t} - \frac{1}{(P_1/P)s^\gamma x_2}\hat{x}_{2,t} = \frac{W/P}{(P_1/P)(1+\iota)}\frac{1}{1+\iota}\hat{Z}_t,$$

$$\frac{W/P}{(P_2/P)(1-\iota)}\frac{\widehat{W}_t}{P_t} - \frac{\gamma}{(P_2/P)s^\gamma x_1}\hat{s}_t - \frac{\widehat{P}_{2,t}}{P_t} - \frac{1}{(P_2/P)s^\gamma x_2}\hat{x}_{1,t} = \frac{W/P}{(P_2/P)(1-\iota)}\frac{1}{1-\iota}\hat{Z}_t.$$

The next equation results from the definition of the price index:

$$x_{1,t}\frac{P_{1,t}}{P_t} + x_{2,t}\frac{P_{2,t}}{P_t} = 1,$$

$$\Rightarrow$$

$$x_1\frac{P_1}{P}\frac{\widehat{P}_{1,t}}{P_t} + x_2\frac{P_2}{P}\frac{\widehat{P}_{2,t}}{P_t} + x_1\frac{P_1}{P}\hat{x}_{1,t} + x_2\frac{P_2}{P}\hat{x}_{2,t} = 0.$$

The following two equations are the definition of the variables $exp_{1,t}$ and $exp_{2,t}$:

$$exp_{1,t} = \exp\left(\left(1 - \frac{P_{1,t}}{P_t}\right)s_t^\gamma\right), \qquad exp_{2,t} = \exp\left(\left(1 - \frac{P_{2,t}}{P_t}\right)s_t^\gamma\right),$$

$$\Rightarrow$$

$$\gamma\left(1 - \frac{P_1}{P}\right)s^\gamma\hat{s}_t - s^\gamma\frac{P_1}{P}\frac{\widehat{P}_{1,t}}{P_t} - \widehat{exp}_{1,t} = 0,$$

$$\gamma\left(1 - \frac{P_2}{P}\right)s^\gamma\hat{s}_t - s^\gamma\frac{P_2}{P}\frac{\widehat{P}_{2,t}}{P_t} - \widehat{exp}_{2,t} = 0.$$

The definition of the market shares

$$x_{1,t} = \frac{exp_{1,t}}{exp_{1,t} + exp_{2,t}}, \qquad x_{2,t} = \frac{exp_{2,t}}{exp_{1,t} + exp_{2,t}}$$

can be represented as:

$$\underline{\hat{x}_{1,t} - (1 - x_1)\widehat{exp}_{1,t} + x_2\widehat{exp}_{2,t} = 0}, \quad \underline{\hat{x}_{2,t} + x_1\widehat{exp}_{1,t} - (1 - x_2)\widehat{exp}_{2,t} = 0}.$$

The optimality condition with respect to search activity implies:

$$\frac{\alpha}{a\gamma}s_t^{\alpha-\gamma} = \left(\sum_{l=1}^{2} x_{l,t}\left(1 - \frac{P_{l,t}}{P_t}\right)^2\right)\frac{\pi_t}{m_t}D_t,$$

$\Rightarrow$

$$\underline{-\hat{Y}_t + (\alpha - \gamma)\hat{s}_t + 2\frac{P_1/P}{1 - (P_1/P)}\xi_3\frac{\widehat{P}_{1,t}}{P_t} + 2\frac{P_2/P}{1 - (P_2/P)}\xi_4\frac{\widehat{P}_{2,t}}{P_t} - \xi_3\hat{x}_{1,t} - \xi_4\hat{x}_{2,t} - \hat{\pi}_t = -\hat{m}_t},$$

where

$$\xi_3 = \frac{\left(1 - \frac{P_1}{P}\right)^2 x_1}{\left(1 - \frac{P_1}{P}\right)^2 x_1 + \left(1 - \frac{P_2}{P}\right)^2 x_2}, \qquad \xi_4 = \frac{\left(1 - \frac{P_2}{P}\right)^2 x_2}{\left(1 - \frac{P_1}{P}\right)^2 x_1 + \left(1 - \frac{P_2}{P}\right)^2 x_2}.$$

The next equation reflects the second aggregate consistency condition:

$$x_{2,t}Y_t = (Z_t - \iota)N_{2,t},$$

$\Rightarrow$

$$\underline{\hat{Y}_t + \hat{x}_{2,t} - \hat{N}_{2,t} = \frac{1}{1 - \iota}\hat{Z}_t}.$$

The last static equation is the consistency condition with respect to working hours:

$$N_{1,t} + N_{2,t} = N_t,$$

$\Rightarrow$

$$\underline{\hat{N}_t - \frac{N_1}{N}\hat{N}_{1,t} - \frac{N_2}{N}\hat{N}_{2,t} = 0}.$$

In contrast, the dynamic block of the model is very simple. From the condition for optimal money demand we have:

$$\beta\phi_m m_{t+1}^{-\chi}E_t\{\pi_{t+1}^{\chi-1}\} = \Lambda_t(1 - g(s_t)) - \beta E_t\left\{\frac{\Lambda_{t+1}(1 - g(s_{t+1}))}{\pi_{t+1}}\right\},$$

$\Rightarrow$

$$\underline{\chi\hat{m}_{t+1} - \frac{\beta}{\pi - \beta}E_t\hat{\Lambda}_{t+1} + \frac{\pi}{\pi - \beta}\hat{\Lambda}_t =}$$

$$\underline{= -\frac{\beta}{\pi - \beta}\frac{\alpha s^\alpha}{a - s^\alpha}E_t\hat{s}_{t+1} + \left(\chi - 1 - \frac{\beta}{\pi - \beta}\right)E_t\hat{\pi}_{t+1} + \frac{\pi}{\pi - \beta}\frac{\alpha s^\alpha}{a - s^\alpha}\hat{s}_t}.$$

The law of motion of money supply implies:

$$\hat{m}_{t+1} - \hat{m}_t = -\hat{\pi}_t + \hat{\tau}_t.$$

2 A Model with Endogenous Capital

$$-\eta \hat{C}_t = \hat{\Lambda}_t.$$

$$\hat{N}_t - \widehat{\frac{W_t}{P_t}} = \hat{\Lambda}_t.$$

The equations defining the firm-specific marginal costs imply:

$$\omega \widehat{\frac{W_t}{P_t}} - \hat{\mu}_{1,t} = -(1-\omega)\hat{R}_t + \frac{1}{1+\iota}\hat{Z}_t, \qquad \omega \widehat{\frac{W_t}{P_t}} - \hat{\mu}_{2,t} = -(1-\omega)\hat{R}_t + \frac{1}{1-\iota}\hat{Z}_t.$$

The next equation reflects the first aggregate consistency condition:

$$x_{1,t} Y_t = (Z_t + \iota) N_{1,t}^{\omega} K_{1,t}^{1-\omega},$$

$$\Rightarrow$$

$$\omega \hat{N}_{1,t} - \hat{Y}_t - \hat{x}_{1,t} + (1-\omega)\hat{K}_{1,t} = -\frac{1}{1+\iota}\hat{Z}_t.$$

$$C_t + g(s_t)\frac{m_t}{\pi_t} + I_t = Y_t,$$

$$\Rightarrow$$

$$\frac{C}{Y}\hat{C}_t - \hat{Y}_t + \frac{I}{Y}\hat{I}_t + \frac{m}{\pi Y}\frac{\alpha s^{\alpha}}{a}\hat{s}_t - \frac{m}{\pi Y}\frac{s^{\alpha}}{a}\hat{\pi}_t = -\frac{m}{\pi Y}\frac{s^{\alpha}}{a}\hat{m}_t,$$

where $\frac{m}{\pi Y}$ is calibrated by using the empirically observable velocity of money with respect to output.

$$-\frac{\gamma}{(P_1/P)s^{\gamma}x_2}\hat{s}_t - \widehat{\frac{P_{1,t}}{P_t}} - \frac{1}{(P_1/P)s^{\gamma}x_2}\hat{x}_{2,t} + \frac{\mu_1}{P_1/P}\hat{\mu}_{1,t} = 0,$$

$$-\frac{\gamma}{(P_2/P)s^{\gamma}x_1}\hat{s}_t - \widehat{\frac{P_{2,t}}{P_t}} - \frac{1}{(P_2/P)s^{\gamma}x_2}\hat{x}_{1,t} + \frac{\mu_2}{P_2/P}\hat{\mu}_{2,t} = 0.$$

The next six equations are the same as before:

$$x_1\frac{P_1}{P}\frac{\widehat{P}_{1,t}}{P_t}+x_2\frac{P_2}{P}\frac{\widehat{P}_{2,t}}{P_t}+x_1\frac{P_1}{P}\hat{x}_{1,t}+x_2\frac{P_2}{P}\hat{x}_{2,t}=0.$$

$$\gamma\left(1-\frac{P_1}{P}\right)s^{\gamma}\hat{s}_t-s^{\gamma}\frac{P_1}{P}\frac{\widehat{P}_{1,t}}{P_t}-\widehat{exp}_{1,t}=0,$$

$$\gamma\left(1-\frac{P_2}{P}\right)s^{\gamma}\hat{s}_t-s^{\gamma}\frac{P_2}{P}\frac{\widehat{P}_{2,t}}{P_t}-\widehat{exp}_{2,t}=0.$$

$$\hat{x}_{1,t}-(1-x_1)\widehat{exp}_{1,t}+x_2\widehat{exp}_{2,t}=0,\qquad \hat{x}_{2,t}+x_1\widehat{exp}_{1,t}-(1-x_2)\widehat{exp}_{2,t}=0.$$

$$-\hat{Y}_t+(\alpha-\gamma)\hat{s}_t+2\frac{P_1/P}{1-(P_1/P)}\xi_3\frac{\widehat{P}_{1,t}}{P_t}+2\frac{P_2/P}{1-(P_2/P)}\xi_4\frac{\widehat{P}_{2,t}}{P_t}-\xi_3\hat{x}_{1,t}-\xi_4\hat{x}_{2,t}-\hat{\pi}_t=-\hat{m}_t,$$

where

$$\xi_3=\frac{\left(1-\frac{P_1}{P}\right)^2x_1}{\left(1-\frac{P_1}{P}\right)^2x_1+\left(1-\frac{P_2}{P}\right)^2x_2},\qquad \xi_4=\frac{\left(1-\frac{P_2}{P}\right)^2x_2}{\left(1-\frac{P_1}{P}\right)^2x_1+\left(1-\frac{P_2}{P}\right)^2x_2}.$$

The second aggregate consistency condition becomes:

$$x_{2,t}Y_t=(Z_t-\iota)N_{2,t}^{\omega}K_{2,t}^{1-\omega},$$

$$\Rightarrow$$

$$\omega\hat{N}_{2,t}-\hat{Y}_t-\hat{x}_{2,t}+(1-\omega)\hat{K}_{2,t}=-\frac{1}{1-\iota}\hat{Z}_t.$$

The next equation is the consistency condition with respect to working hours:

$$N_{1,t}+N_{2,t}=N_t,$$

$$\Rightarrow$$

$$\hat{N}_t-\frac{N_1}{N}\hat{N}_{1,t}-\frac{N_2}{N}\hat{N}_{2,t}=0.$$

The consistency condition with respect to capital implies:

$$K_{1,t}+K_{2,t}=K_t,$$

$$\Rightarrow$$

$$\frac{K_1}{Y}\hat{K}_{1,t}-\frac{K_2}{Y}\hat{K}_{2,t}=\frac{K}{Y}\hat{K}_t.$$

The next two equations result from the firm-specific optimality conditions with respect to capital input:

$$\mu_{1,t}(Z_t+\iota)N_{1,t}^{\omega}K_{1,t}^{-\omega}=R_t, \qquad \mu_{2,t}(Z_t-\iota)N_{2,t}^{\omega}K_{2,t}^{-\omega}=R_t$$

$\Rightarrow$

$$\underline{\hat{\mu}_{1,t}+\omega\hat{N}_{1,t}-\omega\hat{K}_{1,t}=\hat{R}_t-\frac{1}{1+\iota}\hat{Z}_t}, \qquad \underline{\hat{\mu}_{2,t}+\omega\hat{N}_{2,t}-\omega\hat{K}_{2,t}=\hat{R}_t-\frac{1}{1-\iota}\hat{Z}_t}.$$

In contrast, the dynamic block of the model is very simple. From the condition for optimal money demand we have:

$$\beta\phi_m m_{t+1}^{-\chi}E_t\{\pi_{t+1}^{\chi-1}\}=\Lambda_t(1-g(s_t))-\beta E_t\left\{\frac{\Lambda_{t+1}(1-g(s_{t+1}))}{\pi_{t+1}}\right\},$$

$\Rightarrow$

$$\underline{\chi\hat{m}_{t+1}-\frac{\beta}{\pi-\beta}E_t\hat{\Lambda}_{t+1}+\frac{\pi}{\pi-\beta}\hat{\Lambda}_t=}$$

$$\underline{=-\frac{\beta}{\pi-\beta}\frac{\alpha s^{\alpha}}{a-s^{\alpha}}E_t\hat{s}_{t+1}+\left(\chi-1-\frac{\beta}{\pi-\beta}\right)E_t\hat{\pi}_{t+1}+\frac{\pi}{\pi-\beta}\frac{\alpha s^{\alpha}}{a-s^{\alpha}}\hat{s}_t}.$$

The law of motion of money supply implies:

$$\underline{\hat{m}_{t+1}-\hat{m}_t=-\hat{\pi}_t+\hat{\tau}_t}.$$

The Euler equation with respect to next-period's stock of capital implies:

$$\underline{E_t\hat{\Lambda}_{t+1}+(1-\beta(1-\upsilon))E_t\hat{R}_{t+1}-\hat{\Lambda}_t=0},$$

where υ denotes the rate of depreciation. Finally, the law of motion of capital reads:

$$\underline{\hat{K}_{t+1}-(1-\upsilon)\hat{K}_t=\upsilon\hat{I}_t}.$$

3 A Shopping-Time Model

$$C_t^{-\eta}=\Lambda_t+\kappa\frac{\Gamma_t\pi_t}{m_tC_t},$$

$\Rightarrow$

$$\underline{-\eta\hat{C}_t-\kappa\Gamma\frac{\pi C}{m}C^{\eta-1}\hat{\pi}_t-\kappa\Gamma\frac{\pi C}{m}C^{\eta-1}\hat{\Gamma}_t=\kappa\Gamma\frac{\pi C}{m}C^{\eta-1}\hat{m}_t+C^{\eta-1}\hat{\Lambda}_t},$$

where Γ_t is the Lagrangean multiplier with respect to the transaction-time constraint.

$$b(1-L_t) = \Lambda_t \frac{W_t}{P_t},$$
$$\Rightarrow$$
$$\underline{\frac{L}{1-L}\hat{L}_t + \widehat{\frac{W_t}{P_t}} = -\hat{\Lambda}_t.}$$

The equations defining the firm-specific marginal costs imply:

$$\mu_{1,t} = \frac{W_t/P_t}{Z_t+\iota}, \qquad \mu_{2,t} = \frac{W_t/P_t}{Z_t-\iota},$$
$$\Rightarrow$$
$$\underline{\widehat{\frac{W_t}{P_t}} - \hat{\mu}_{1,t} = \frac{1}{1+\iota}\hat{Z}_t,} \qquad \underline{\widehat{\frac{W_t}{P_t}} - \hat{\mu}_{2,t} = \frac{1}{1-\iota}\hat{Z}_t.}$$

The next two equations are the aggregate consistency conditions:

$$x_{1,t}Y_t = (Z_t+\iota)N_{1,t},$$
$$\Rightarrow$$
$$\underline{\hat{Y}_t + \hat{x}_{1,t} - \hat{N}_{1,t} = \frac{1}{1+\iota}\hat{Z}_t.}$$

$$x_{2,t}Y_t = (Z_t-\iota)N_{2,t},$$
$$\Rightarrow$$
$$\underline{\hat{Y}_t + \hat{x}_{2,t} - \hat{N}_{2,t} = \frac{1}{1-\iota}\hat{Z}_t.}$$

$$C_t = Y_t,$$
$$\Rightarrow$$
$$\underline{\hat{C}_t - \hat{Y}_t = 0},$$

$$\underline{-\frac{\gamma}{(P_1/P)s^\gamma x_2}\hat{s}_t - \frac{\widehat{P_{1,t}}}{P_t} - \frac{1}{(P_1/P)s^\gamma x_2}\hat{x}_{2,t} + \frac{\mu_1}{P_1/P}\hat{\mu}_{1,t} = 0,}$$

$$\underline{-\frac{\gamma}{(P_2/P)s^\gamma x_1}\hat{s}_t - \frac{\widehat{P_{2,t}}}{P_t} - \frac{1}{(P_2/P)s^\gamma x_2}\hat{x}_{1,t} + \frac{\mu_2}{P_2/P}\hat{\mu}_{2,t} = 0.}$$

$$x_1\frac{P_1}{P}\frac{\widehat{P}_{1,t}}{P_t}+x_2\frac{P_2}{P}\frac{\widehat{P}_{2,t}}{P_t}+x_1\frac{P_1}{P}\hat{x}_{1,t}+x_2\frac{P_2}{P}\hat{x}_{2,t}=0.$$

$$\gamma\left(1-\frac{P_1}{P}\right)s^{\gamma}\hat{s}_t-s^{\gamma}\frac{P_1}{P}\frac{\widehat{P}_{1,t}}{P_t}-\widehat{exp}_{1,t}=0,$$

$$\gamma\left(1-\frac{P_2}{P}\right)s^{\gamma}\hat{s}_t-s^{\gamma}\frac{P_2}{P}\frac{\widehat{P}_{2,t}}{P_t}-\widehat{exp}_{2,t}=0.$$

$$\hat{x}_{1,t}-(1-x_1)\widehat{exp}_{1,t}+x_2\widehat{exp}_{2,t}=0,\qquad \hat{x}_{2,t}+x_1\widehat{exp}_{1,t}-(1-x_2)\widehat{exp}_{2,t}=0.$$

The optimality condition with respect to search (shopping time) s_t reads:

$$\Lambda_t\frac{W_t}{P_t}-\Gamma_t\frac{\alpha s_t^{\alpha-1}}{(1+s_t^{\alpha})^2}=\frac{s_t^{\gamma-1}}{\gamma}\left(\sum_{j=1}^{2}x_{j,t}\left(1-\frac{P_{j,t}}{P_t}\right)^2\right)Y_t,$$

$\Rightarrow$

$$\hat{Y}_t+\left(\gamma-1+\frac{\Gamma\alpha s^{\alpha-1}}{(1+s^{\alpha})^2\xi_5}\frac{(\alpha-1-s^{\alpha}(\alpha+1))}{(1+s^{\alpha})}\right)\hat{s}_t+-\frac{\Lambda(W/P)}{\xi_5}\frac{\widehat{W}_t}{P_t}-$$

$$-2\frac{P_1/P}{1-(P_1/P)}\xi_3\frac{\widehat{P}_{1,t}}{P_t}-2\frac{P_2/P}{1-(P_2/P)}\xi_4\frac{\widehat{P}_{2,t}}{P_t}+\xi_3\hat{x}_{1,t}+\xi_4\hat{x}_{2,t}+\frac{\Gamma\alpha s^{\alpha-1}}{(1+s^{\alpha})^2\xi_5}\hat{\Gamma}_t=\frac{\Lambda(W/P)}{\xi_5}\hat{\Lambda}_t,$$

where

$$\xi_3=\frac{\left(1-\frac{P_1}{P}\right)^2x_1}{\left(1-\frac{P_1}{P}\right)^2x_1+\left(1-\frac{P_2}{P}\right)^2x_2},\qquad \xi_4=\frac{\left(1-\frac{P_2}{P}\right)^2x_2}{\left(1-\frac{P_1}{P}\right)^2x_1+\left(1-\frac{P_2}{P}\right)^2x_2},$$

$$\xi_5=\frac{s^{\gamma-1}}{\gamma}\left(\sum_{j=1}^{2}x_j\left(1-\frac{P_j}{P}\right)^2\right).$$

The log-linear version of the transactions-time constraint reads:

$$\hat{C}_t-\frac{\alpha}{1+s^{\alpha}}\hat{s}_t+\hat{\pi}_t=\hat{m}_t.$$

The consistency condition with respect to hours, leisure and shopping time is:

$$L_t + \frac{s_t^\alpha}{1+s_t^\alpha} + N_{1,t} + N_{2,t} = 1,$$

$$\Rightarrow$$

$$\underline{L\hat{L}_t + \frac{\alpha s^\alpha}{(1+s^\alpha)^2}\hat{s}_t + N_1\hat{N}_1 + N_2\hat{N}_2 = 0.}$$

The dynamic block of the model is again very simple. The optimality condition for money demand can be log-linearized as follows:

$$\Lambda_t = \beta E_t\left\{\kappa\frac{\Gamma_{t+1}C_{t+1}\pi_{t+1}}{m_{t+1}^2} + \frac{\Lambda_{t+1}}{\pi_{t+1}}\right\},$$

$$\Rightarrow$$

$$\underline{\hat{m}_{t+1} - \frac{\pi}{\beta}E_t\hat{\Lambda}_{t+1} + \frac{\pi-\beta}{\beta}\hat{\Lambda}_t = E_t\hat{C}_{t+1} + \frac{\pi-\beta}{\beta}E_t\hat{\pi}_{t+1} + E_t\hat{\Gamma}_{t+1}.}$$

The money supply process implies:

$$\underline{\hat{m}_{t+1} - \hat{m}_t = -\hat{\pi}_t + \hat{\tau}_t.}$$

Curriculum Vitae

Education

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105th Primary School, Sofia, Bulgaria

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Areas of Concentration: Mathematics, English, German

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University of Augsburg
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Areas of Concentration: Empirical Macroeconomics; Mathematics and Statistics; Finance and Banking

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Student assistant at the chair of mathematical economics and econometrics (Prof. Dr. Ulrich K. Schittko)

10/2001 - 05/2003

Student assistant at the chair of empirical macroeconomics (Prof. Dr. Alfred Maussner)

12/2005 - 04/2009

University of Augsburg
Doctoral degree in Economics *(Doktor der Wirtschaftswissenschaften (Dr. rer. pol.))*
Title of Thesis: "Customer Markets and the Real Effects of Monetary Policy Shocks"

Employment

06/2003 - 10/2009

University of Augsburg
Research and teaching assistant at the chair of empirical macroeconomics (Prof. Dr. Alfred Maussner)

10/2009 - Present

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