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Three Essays about Dynamic Stochastic General Equilibrium Models with Overlapping Generations

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Contents

List of Publications

The following essays in my thesis are already published in academic journals:

A) "Aging, the Great Moderation and Business-Cycle Volatility in a Life-Cycle Model", with Burkhard Heer (University of Augsburg) and Stefan Rohrbacher (University of Augsburg), Macroeconomic Dynamics, 2017, vol. 21(2), p. 362-383, DOI: 10.1017/S136510051500053X.

B) "The Age-Specific Burdens of Short-Run Fluctuations in Government Spending", with Burkhard Heer (University of Augsburg), Journal of Economic Dynamics and Control, 2018, vol. 90, p. 45-75, DOI: 10.1016/j.jedc.2018.01.041.

Author's Contribution to the Essays

My contributions to the three essays in this thesis are:

A) "Aging, the Great Moderation and Business-Cycle Volatility in a Life-Cycle Model": I developed the model, performed the analytic calculations, and wrote all program codes. About 75 percent of the sections in this essay are written by me.

B) "The Age-specific Burdens of Short-run Fluctuations in Government Spending": I developed the model, performed the analytic calculations, and wrote all program codes. Moreover, about 50 percent of the sections in this essay are written by me.

C) "The Effects of Financing Rules in Pay-As-You-Go Pension Systems on the Life and the Business Cycle": I am the single author of this essay.

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Chapter 1

Introduction

1.1 Overview and Motivation

My cumulative dissertation consists of three essays about dynamic stochastic general equilibrium (DSGE) models with overlapping generations.[1] In particular, all three models in these essays have in common that they feature a large-scale overlapping generations structure, in the spirit of the seminal work by Auerbach and Kotlikoff (1987), which allows to introduce heterogeneous households in the economic analysis. The agents, therefore, differ by their age and, depending on the respective essay and the associated research questions, their age-specific productivity and their consumer type. Furthermore, the presented models also have in common that they methodically and numerically build on works by Ríos-Rull (1996), Heer and Maußner (2009), and Heer and Maußner (2012). They thereby implement overlapping generations into DSGE models, which became the standard workhorse of modern macroeconomics during the last decades and date back to the seminal work by Kydland and Prescott (1982). Therefore, it is also possible to study the associated short-run dynamics of age-specific and aggregate variables around the corresponding steady state of these models in numerical simulations, if an economy is hit, for example,

[1]I would like to thank Burkhard Heer (University of Augsburg), Johann Scharler (University of Innsbruck), Alfred Maußner (University of Augsburg), and Andreas Pollak (University of Saskatchewan) for helpful comments during my dissertation research. All remaining errors are mine.

by total factor productivity or government spending shocks. In particular, Ríos-Rull (1996) finds that economies which are populated by overlapping generations imply almost the same set of business cycle statistics in comparison to standard DSGE models with only one infinitely-lived representative agent. He concludes that "overlapping-generations economies can be used to study properties of the age distribution of various variables"[2] from a macroeconomic point of view. For these reasons mentioned above, this model framework is very well suited to study research questions about the (short-run) effects of demographic transitions and economic policies which focus on the age-specific effects on the behavior of households over the life cycle, aggregate variables, and the inequality of income and wealth across cohorts. Hence, large-scale DSGE models with overlapping generations are able to yield more detailed insights into some important policy issues which I address in three papers of my cumulative dissertation.

With respect to the associated research questions in my thesis, my dissertation consists of the following essays and is structured as follows: My first paper with the title "Aging, the Great Moderation and Business-Cycle Volatility in a Life-Cycle Model", which I present in Chapter 2, is a joint work with Burkhard Heer (University of Augsburg) and Stefan Rohrbacher (University of Augsburg). There, we study the impacts of the demographic transition and the downward shift of age-specific labor supply volatilities on the volatility of aggregate output during the Great Moderation, which describes the pronounced decline in the volatility of key macroeconomic variables, like the average growth rate of real gross domestic product (GDP), since the mid-1980s. We find that the aforementioned downward shift of the age-specific labor supply volatility curve is able to explain a reduction of output volatility by 23%. In contrast, the influence of changes in the age-composition itself is rather negligible since demographic composition effects almost cancel each other out at the aggregate level. This paper was published in the journal "Macroeconomic Dynamics" in the year 2017.[3]

[2]See Ríos-Rull (1996, page 486).

[3]See also Heer et al. (2017).

Moreover, my second essay with the title "the Age-Specific Burdens of Short-Run Fluctuations in Government Spending" in Chapter 3 is also a joint work with Burkhard Heer and was published in the "Journal of Economic Dynamics and Control" in the year 2018.[4] In this paper, we investigate the effects of short-run fluctuations of government spending and its financing form on the distribution of income and wealth between cohorts and the associated impacts on individual welfare over the life cycle. First, we show that an unexpected increase of one standard deviation in government consumption results in a slight decline of income and wealth inequality. Second, and contrary to the conventional wisdom that the financing of government expenditures by debt rather than taxes especially burdens young generations, we find that debt-financing also harms Ricardian retirees. If government spending shocks also result in endogenous adjustments of the price of capital, then this channel induces more pronounced fluctuations in their lifetime utility in contrast to tax-financed changes in government spending.

Chapter 4 contains the current version of my working paper with the title "the Effects of Financing Rules in Pay-As-You-Go Pension Systems on the Life and the Business Cycle", where I am the single author. In this essay, I investigate the impacts of financing rules for potential financial surpluses in pay-as-you-go pension systems on the business cycle and study the resulting effects on the consumption smoothing behavior of households over the life cycle. The results in this paper point out that the financing form has strong and very different age-specific effects on the consumption smoothing behavior over the life cycle. Moreover, I show that the impacts of higher fluctuations of aggregate variables on the volatility of individual lifetime utilities can rather be negligible.

In Chapter 5, I summarize the most important findings of the presented essays in my thesis, which are also intensively discussed in the respective section for each paper. Moreover, I also point to directions for further research in the final conclusion.

[4]See also Heer and Scharrer (2018).

1.A Bibliography

Auerbach, A. J. and L. J. Kotlikoff (1987). *Dynamic Fiscal Policy* (1st ed.). Cambridge: Cambridge University Press.

Heer, B. and A. Maußner (2009). *Dynamic General Equilibrium Modeling: Computational Methods and Applications* (2nd ed.). Heidelberg: Springer.

Heer, B. and A. Maußner (2012). The Burden of Unanticipated Inflation: Analysis of an Overlapping-Generations Model with Progressive Income Taxation and Staggered Prices. *Macroeconomic Dynamics 16*(2), 278–308.

Heer, B., S. Rohrbacher, and C. Scharrer (2017). Aging, the Great Moderation, and Business-Cycle Volatility in a Life-Cycle Model. *Macroeconomic Dynamics 21*(02), 362–383.

Heer, B. and C. Scharrer (2018). The Age-specific Burdens of Short-run Fluctuations in Government Spending. *Journal of Economic Dynamics and Control 90*, 45–75.

Kydland, F. E. and E. C. Prescott (1982). Time to Build and Aggregate Fluctuations. *Econometrica 50*(6), 1345–1370.

Ríos-Rull, J.-V. (1996). Life-Cycle Economies and Aggregate Fluctuations. *Review of Economic Studies 63*(3), 465–489.

Chapter 2

Aging, the Great Moderation and Business-Cycle Volatility in a Life-Cycle Model[1]

[1]This essay is a joint work with Burkhard Heer (University of Augsburg) and Stefan Rohrbacher (University of Augsburg) which was published in the journal "Macroeconomic Dynamics" (DOI: 10.1017/S136510051500053X), see Heer et al. (2017) in Section 1.A on page 4.

Chapter 3

The Age-Specific Burdens of Short-Run Fluctuations in Government Spending[1]

[1]This essay is a joint work with Burkhard Heer (University of Augsburg) which was published in the "Journal of Economic Dynamics and Control" (DOI: 10.1016/j.jedc.2018.01.041), see Heer and Scharrer (2018) in Section 1.A on page 4.

Chapter 4

The Effects of Financing Rules in Pay-As-You-Go Pension Systems on the Life and the Business Cycle

4.1 Introduction

Revenues and pension benefits in pay-as-you-go (PAYG) pension schemes depend mainly on labor earnings which fluctuate over the business cycle. Fig. 4.1 shows the associated development of the cyclical component of aggregate hours and real wages for Germany from 1991:1 to 2016:4. Both variables vary substantially and, as a consequence, contribution rates, pension benefits and/or the stock of financial assets in a PAYG system have to be adjusted so that its budget is balanced. However, these adjustments affect the intergenerational allocation of income and aggregate risk which in turn influences the consumption smoothing behavior of households over the life cycle. For example, the social security authority could only adjust the contribution rates. Such a financing rule shifts macroeconomic risks to younger generations since it increases the volatility of net wages of workers and holds pension benefits constant. Workers are, however, better able to deal with higher economic risks by changing their labor supply and savings rates in response to macroeconomic shocks, whereas retirees can only adjust their savings rate.

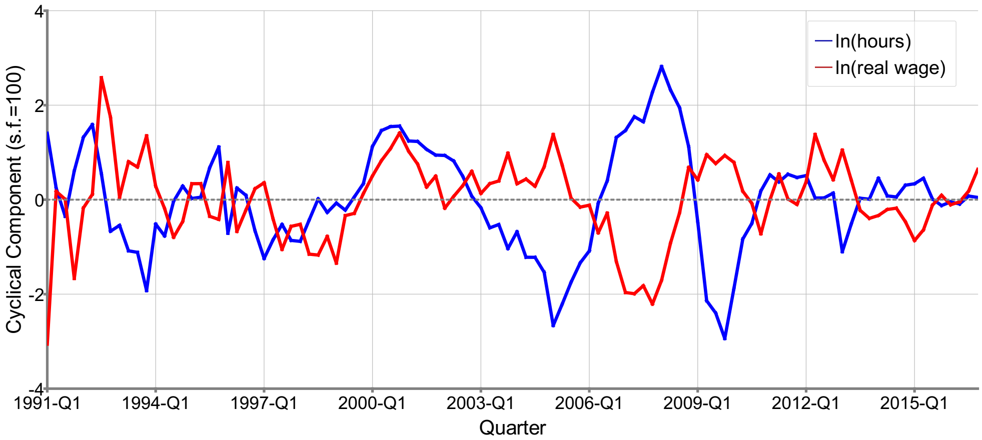


Figure 4.1: The Cyclical Component of Real Wages and Hours (hp-filtered with weight 1600, s.f. = scaling factor, soure: Destatis, own calculations).

In this paper, I study the effects of different financing rules for potential financial surpluses in PAYG systems on the business cycle and the age-specific consumption smoothing behavior of households in a large-scale real business cycle model with overlapping generations, where households take the inter-temporal link between contributions and pension benefits explicitly into account. In particular, I find that sluggish adjustments of contribution rates that are implemented by adjusting a buffer stock of financial assets of a PAYG system both stabilize an economy and help to decrease the volatility of (remaining) life-time utilities of retirees and workers close to retirement, in contrast to solely complete adjustments of contribution rates. Such a policy reduces the distortionary effects of labor taxation and also allows a more flexible accumulation of wealth over the life cycle, which helps future retirees to hedge better against macroeconomic shocks over the business cycle.

The most closely related papers to mine are Thøgersen (1998) and Wagener (2003). Thøgersen (1998) studies the effects of PAYG pensions programs on the intergenerational allocation of risk and welfare. He finds that defined contribution rates imply a lower income risk and higher ex-ante welfare across generations. In contrast, Wagener (2003) shows that different PAYG schemes are not comparable in a ex ante perspective due to different information sets and decisions over the life cycle. From an ex post perspective, he concludes that fixed replacement rates are preferable to defined contributions. They improve intergenerational risk-sharing and induce higher utility levels. Both studies, however, assume that labor supply is completely inelastic in their two-period overlapping generations models, where they exclude general equilibrium effects since all prices (respectively, their probability distributions) are fully exogenous. This paper extends the aforementioned research and presents a large-scale dynamic stochastic overlapping generations model that methodically builds on Ríos-Rull (1996) and Heer and Maußner (2012). In particular, the model features 240 generations, takes general equilibrium effects into account, and labor supply is endogenous so that workers can adjust their labor supply in response to changes in factor prices. This approach, therefore, allows to study in more detail the age-specific life cycle effects of different financing rules in PAYG systems which keep the social security budget balanced over the business cycle.

The rest of this paper is organized as follows. Section 2.2 describes and explains the model, which I calibrate in Section 2.3. The resulting steady state is discussed in Section 2.4, while Section 2.5 studies the effects of different financing rules in pay-as-you-go systems on aggregate variables and the consumption smoothing behavior of households over the business cycle. Section 2.6 concludes.

4.2 The Model

In this section, I present a model with overlapping generations and aggregate uncertainty, where the period length is set to one quarter. Households optimize their expected life-time utility, firms maximize profits, and a PAYG system transfers resources across generations.

4.2.1 Demographics

Each year, a new cohort is born and its size ψ_s is constant at age $s = 1$ (corresponding to a real life age of 21). Households live at most T quarters, where they work in the first T_w quarters and are retirees in the subsequent $T_r = T - T_w$ periods. In addition, each s-year old household survives from age s to $s+1$ with an exogenously given probability of ϕ_s, where $\phi_0 \equiv 1$. Thus, the mass of households ψ_{s+1} at age

$s+1$ evolves according to $\psi_{s+1} = \phi_s \psi_s$. For simplification, I normalize the total mass of living households $\sum_{s=1}^{T} \psi_s$ to one.

4.2.2 Households

A household at age $s = 1$ in period t maximizes the following discounted expected lifetime utility U_t with respect to consumption c_t^s and labor supply n_t^s:

$$U_t = E_t \sum_{s=1}^{T} \beta^{s-1} \left(\prod_{j=1}^{s} \phi_{j-1} \right) u\!\left(c_{t+s-1}^{s}, n_{t+s-1}^{s}\right), \tag{4.1}$$

where $n_t^s \in [0,1]$ for $s \leq T_w$ and $n_t^s \equiv 0$ for $s > T_w$. Moreover, the specification of the instantaneous utility function $u(c_t^s, n_t^s)$ follows Trabandt and Uhlig (2011),

$$u(c_t^s, n_t^s) = \begin{cases} ln(c_t^s) - \frac{\gamma_0}{1+1/\gamma_1} \left(n_t^s\right)^{1+1/\gamma_1}, & \text{for } \eta = 1, \\ \frac{1}{1-\eta} \left[\left(c_t^s\right)^{1-\eta} \left(1 - \frac{\gamma_0(1-\eta)}{1+1/\gamma_1} \left(n_t^s\right)^{1+1/\gamma_1}\right)^{\eta} - 1 \right], & \text{for } \eta \neq 1. \end{cases} \tag{4.2}$$

These preferences feature a constant Frisch elasticity of labor supply γ_1 and a constant intertemporal elasticity of substitution $1/\eta$. The parameter γ_0 controls the labor supply in the steady state of the model.

Households at age $s = 1$ are born without assets and accumulate a stock of capital $k_{t,j}^s$ over their life cycle. Their capital earns the real interest rate r_t and depreciates at the rate δ. Moreover, I assume that the oldest households at age $s = T$ leave no bequests and are not allowed to die indebted. The net labor income of workers depends on the real wage w_t, the age-specific productivity e^s, and the contribution rate τ_t^p for the PAYG system, where pensions $pens_t^s$ are only paid to retired agents. The government collects all accidental bequests and transfers them lump-sum back to the households in the form of tr_t. The respective budget constraint of a s-year old household in period t is given by

$$c_t^s + k_{t+1}^{s+1} = \begin{cases} \left(1 + r_t - \delta\right) k_t^s + \left(1 - \tau_t^p\right) w_t e^s n_t^s + tr_t, & \text{for } s \leq T_w, \\ \left(1 + r_t - \delta\right) k_t^s + pens_t^s + tr_t, & \text{for } s > T_w, \end{cases} \tag{4.3}$$

with $k_t^1 = k_t^{T+1} \equiv 0$, $n_t^s \equiv 0$ for $s > T_w$, and $pens_t^s \equiv 0$ for $s \leq T_w$.

Pension entitlements $pent_t^s$ depend on average lifetime labor earnings and an exogenously given replacement ratio ζ. For ease of notation, I also introduce the parameter θ_t which is equal to one in the steady state and can be adjusted by the social security authority such that it controls the effective replacement ratio $\zeta\theta_t$ in period t outside the steady state. Thus, pension benefits are represented by

$$pens_t^s = \theta_t \, pent_t^s, \tag{4.4}$$

where pension entitlements can be expressed as

$$pent_t^s = \begin{cases} \dfrac{\zeta}{T_w} \displaystyle\sum_{i=1}^{T_w} w_{t-i} e^{T_w - i + 1} n_{t-i}^{T_w - i + 1}, & \text{for } s = T_w + 1, \\ pent_{t-s+T_w}^{T_w+1}, & \text{for } s > T_w + 1. \end{cases} \tag{4.5}$$

The representative first-order conditions that solve the optimization problems of households consist of the aforementioned budget constraints (4.3) and

$$\lambda_t^s = \frac{\partial u(c_t^s, n_t^s)}{\partial c_t^s}, \tag{4.6}$$

$$\lambda_t^s = \beta \phi_s E_t \left\{ \lambda_{t+1}^{s+1} \left(1 + r_{t+1} - \delta\right) \right\}, \tag{4.7}$$

$$\begin{aligned} 0 = {} & \frac{\partial u(c_t^s, n_t^s)}{\partial n_t^s} + \left(1 - \tau_t^p\right) w_t e^s n_t^s \lambda_t^s + \\ & E_t \left\{ \sum_{a=T_w+1}^{T} \beta^{a-s} \left(\prod_{j=s+1}^{a} \phi_{j-1} \right) \lambda_{t+a-s}^a \frac{\zeta \theta_{t+a-s} w_t e^s}{T_w} \right\}. \end{aligned} \tag{4.8}$$

The variable λ_t^s denotes the Lagrange multiplier.

4.2.3 Production

Aggregate Output Y_t is characterized by a Cobb-Douglas production function,

$$Y_t = Z_t N_t^{1-\alpha} K_t^{\alpha}. \tag{4.9}$$

The variables N_t and K_t denote aggregate labor and capital, respectively. Moreover, the stochastic technology level Z_t follows a standard AR(1) process: $\ln Z_t = \rho \ln Z_{t-1} + \epsilon_t$, where $\epsilon_t \sim N(0, \sigma^2)$. The corresponding profit maximization under perfect competition implies zero profits and that factor rewards equal their marginal products,

$$w_t = (1-\alpha) Z_t \left(\frac{K_t}{N_t}\right)^{\alpha}, \tag{4.10}$$

$$r_t = \alpha Z_t \left(\frac{K_t}{N_t}\right)^{\alpha-1}. \tag{4.11}$$

4.2.4 Social Security & Government

The government, in form of a social security authority, collects contributions at the rate τ_t^p of gross labor incomes of workers and holds a buffer stock of financial assets F_t which invests in the capital market. Moreover, the age-specific public pension entitlements $pent_t^s$ adjust over time since they depend on gross pre-retirement earnings and the steady state replacement ratio ζ according to equation (4.5). These entitlements give in turn the pension benefits $pens_t^s = \theta_t pent_t^s$ that can be adjusted in the short run by the variable θ_t, as also described in equation (4.4). Thus, the budget of the PAYG system is given by

$$Pens_t + F_{t+1} = \tau_t^p w_t N_t + (1 + r_t - \delta) F_t, \tag{4.12}$$

where

$$Pens_t = \theta_t Pent_t, \tag{4.13}$$

$$Pent_t = \sum_{s=T_w+1}^{T} \psi_s \, pent_t^s, \tag{4.14}$$

$$N_t = \sum_{s=1}^{T_w} \psi_s e^s n_t^s. \tag{4.15}$$

In order to describe the dynamics of the variables τ_t^p, F_{t+1}, and θ_t around the steady state, I follow a similar approach as in Galí et al. (2007) for fiscal policy rules and specify the following financing rule in the PAYG scheme,

$$F_{t+1} - F = \omega_F S_t, \tag{4.16}$$

$$\theta_t Pent_t - Pent_t = \omega_R \left(1 - \omega_F\right) S_t, \tag{4.17}$$

with

$$S_t = \tau^p \left(w_t N_t - wN\right) + \left(R_t F_t - RF\right) - \left(Pent_t - Pent\right) \tag{4.18}$$

and $\omega_F, \omega_R \in [0, 1]$. The expression $R_t \equiv 1 + r_t - \delta$ in equation (4.18) defines the gross interest rate. Moreover, variables without a time index denote the corresponding steady state values. The term S_t describes potential surplusses in the PAYG system under the assumption that the PAYG administration keeps the effective replacement ratio and the contribution rate constant, $\zeta\theta_t = \zeta\theta$ and $\tau_t^p = \tau^p$. Hence, the exogenous parameters ω_F and ω_R in equation (4.16) and (4.17) control the adjustments of financial assets and effective replacement ratios over the business cycle. If, for example, both parameters are equal to zero, the budget constraint (4.12) implies that only the contribution rate τ_t^p is allowed to change in order to keep the budget balanced. In contrast, a high value for ω_F results in pronounced adjustments of financial assets which dampen the associated budget effects on the other variables.

Furthermore, I assume that all accidental bequests are collected by the government and transferred as lump-sums to the household sector. This implies

$$tr_t = \sum_{s=1}^{T} (1 - \Phi_{s-1})\psi_{s-1} \left[(1 + r_t - \delta)k_t^s \right]. \tag{4.19}$$

4.2.5 Equilibrium

In a general equilibrium, individual and aggregate behavior must be consistent. Thus, the following conditions have to be satisfied for all t,

$$N_t = \sum_{s=1}^{T_w} \psi_s n_t^s, \tag{4.20a}$$

$$K_t = \sum_{s=1}^{T} \psi_{s-1} k_t^s + F_t, \tag{4.20b}$$

$$C_t = \sum_{s=1}^{T} \psi_s c_t^s, \tag{4.20c}$$

such that the goods market clears:

$$Z_t N_t^{1-\alpha} K_t^{\alpha} = C_t + I_t, \tag{4.21}$$

where $I_t = K_{t+1} - (1 - \delta)K_t$.

4.3 Calibration

I calibrate the model on a quarterly basis for the German economy and linearize the model around the steady state.[1] Households live at most $T = 240$ quarters and work for $T_w = 160$ quarters such that they enter retirement at a real life age of 61 years and die with certainty at a real life age of 81 years. These numbers roughly correspond with the average age when households enter retirement and

[1]In particular, I use the solution methods described in Chapters 9 and 10 in Heer and Maußner (2009) and modified codes of the provided CoRRAM package (see `www.wiwi.uni-augsburg.de/vwl/maussner/dge_buch/dge_book_2ed/downloads_2nd`).

the average life expectancy for men and women for the year 2011 in Germany, see Deutsche Rentenversicherung Bund (2017). Moreover, I approximate the average survival probabilities ϕ_s with German life tables for the sample 1992 to 2012 which are provided by the Federal Statistical Office (Destatis). The smoothed productivity profiles e^s of s-year-old workers are taken from Heer (2019), who calculates these profiles with data of age-specific hourly wages during 1990 and 1997 for Germany.

With respect to the production technology, I use values estimated by Flor (2014) for the German economy for the sample 1991:1 to 2012:4. The production elasticity of capital is equal to $\alpha = 0.34$ and the depreciation rate δ equals 1.7 percent. Moreover, the autocorrelation parameter for technology shocks is set to $\rho = 0.83$ and the corresponding standard deviation of innovations is equal to $\sigma = 0.0082$, where Flor (2014) takes both capital and labor as factor inputs into account for the calculation of the Solow residual.

The parameters describing the PAYG system are chosen as follows: The replacement ratio ζ of pensions relative to average pre-retirement earnings is set to 42 percent and taken from DICE Database (2016) for the year 2011. Moreover, I assume that the stock of financial assets F is equal to aggregate (quarterly) pensions entitlements $Pent$ in the steady state and set the parameter θ, which controls the effective replacement ratio $\zeta\theta_t$ in period t, equal to one, respectively. The resulting stationary contribution rate amounts to $\tau^p = 16$ percent, which is a little bit lower than its empirical counterpart of 19.90 percent for the year 2011.[2] With respect to the parameters ω_F and ω_R, I distinguish between three cases that I will discuss in the following sections:

- **Case 1**: This is the benchmark case, where I assume that the PAYG authority seeks to keep the contribution rate τ_t^p as constant as possible and does not adjust the effective replacement ratio $\zeta\theta_t$ over the business cycle.[3] For that

[2] See Deutsche Rentenversicherung Bund (2017).

[3] In Germany, the PAYG administration adjusts the contribution rates when the size of the reserve fund (Nachhaltigkeitsrücklage) exceeds (undershoots) the monthly expenditures by 150 (20) percent, see §158 SGB VI.

reason, I set the parameter $\omega_P = 0$ and $\omega_F = 0.95$.[4]

- **Case 2**: In this case, the PAYG authority chooses $\omega_P = 1$ and $\omega_F = 0$ so that only the effective replacement ratio $\zeta\theta_t$ fluctuates over the business cycle.

- **Case 3**: Here, the PAYG system only adjusts the contribution rates, $\omega_F = \omega_R = 0$, where the stock of financial assets and the effective replacement ratio stay constant.

Regarding the preference parameters, I set the discount factor β equal to 1.00 such that the real rate of return on capital, $r_t - \delta$, equals a value of 4 percent which describes the long term average according to Busl and Seymen (2013). Furthermore, the parameter $\gamma_0 = 4.34$ implies an average labor supply in the steady state of 0.33. With respect to the Frisch labor supply elasticity, I choose $\gamma_1 = 2.15$ in order to roughly match a relative volatility of aggregate hours to aggregate output of 0.62 for the benchmark case according to Flor (2014). This value is in line with the macro-economic literature, which often uses Frisch elasticities between 2 and 4.[5] Furthermore, I choose a standard value of 2 for the parameter η implying an intertemporal elasticity of substitution of 0.5.

4.4 Steady State

Fig. 4.2 presents the behavior of households over the life-cycle in the steady state. The consumption profile in the upper left panel increases until an age of 60 years and displays a kink when households enter retirement due to the increase of leisure that in turn increases the marginal utility of consumption.[6] Their labor supply, as displayed in the upper right panel, increases during the first 15 years and falls monotonously

[4] This calibration still ensures local stability around the steady state and allows to approximate this form of financing. Then, the PAYG administration mainly changes its stock of financial assets and dampens the adjustments of contribution rates.

[5] See, for example, Peterman (2016).

[6] For the reader's convenience, I use the real life age in years in contrast to the quarterly age index s in the discussions and figures hereinafter.

thereafter when income and wealth effects start to dominate the substitution effects. Moreover, the lower left panel shows that households build up wealth for retirement until an age of about 55 years and start to decrease their stock of capital in the following periods in order to smooth their consumption over the life cycle. The age-specific efficiency profile also follows a hump-shaped pattern and is displayed in the lower right panel.

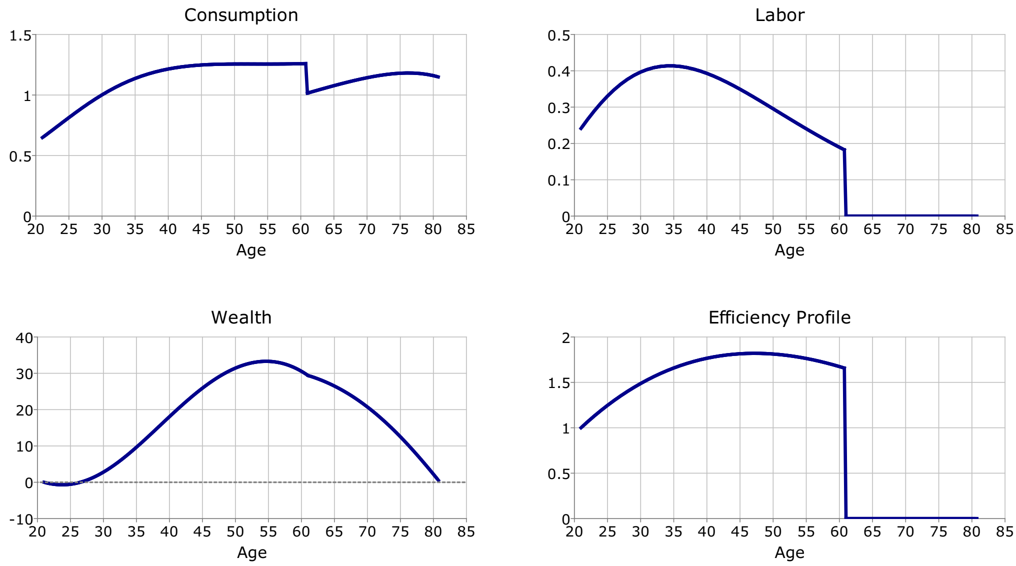


Figure 4.2: Steady-State Behavior of Households (abscissa: age in years).

4.5 Effects over the Business Cycle

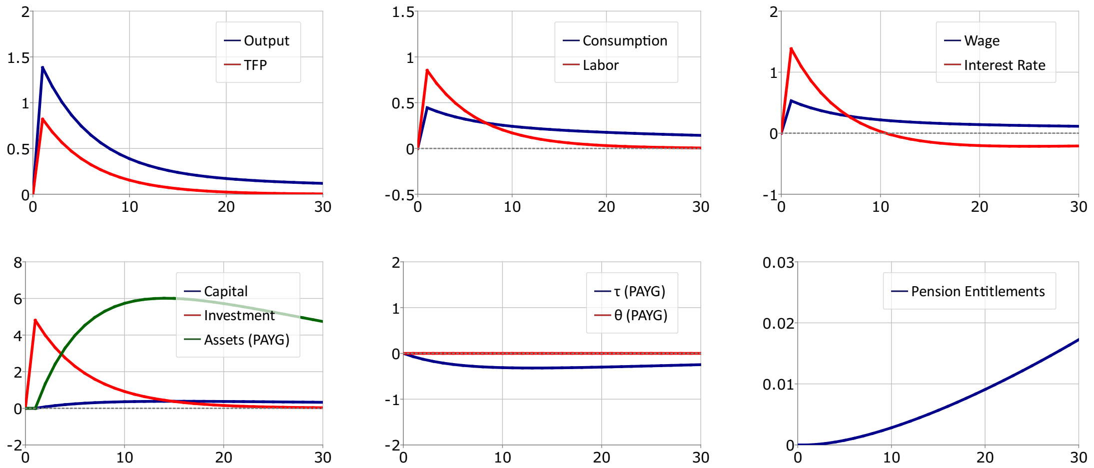
In this section, I study how financing rules in PAYG systems affect aggregate variables and the consumption smoothing behavior of households. Fig. 4.3 presents the associated impulse responses of aggregate variables to a positive one-time productivity shock of one standard deviation in period $t = 2$.

The first two rows in Fig. 4.3 show the benchmark case. A technology shock increases output, labor supply, consumption, and investment. The real interest rate rises due to the increase in productivity and labor supply. Moreover, both the rise of average productivity and the increase of the stock of capital in the subsequent periods dominate the negative effects of labor supply increases on the marginal product of labor so that the real wage rate also rises, as illustrated in the upper right panel of Fig. 4.3. Furthermore, the increase in labor incomes leads to financial surpluses in the PAYG system and growing pension entitlements. For Case 1, the panels of the second row show, in particular, that the PAYG administration mainly invests these surpluses in financial assets in order to dampen the reduction in contribution rates.

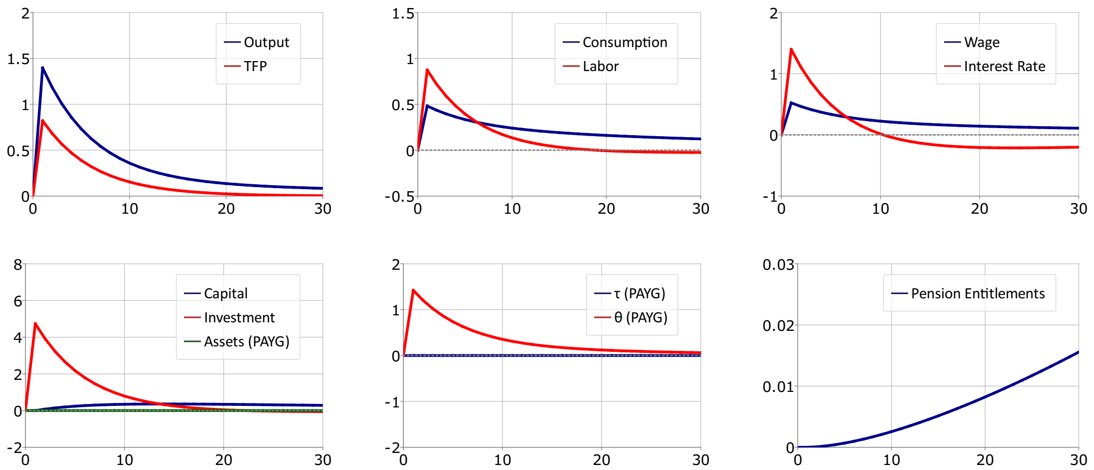
The impulse responses for Case 2 and 3 are plotted in the last four rows of Fig. 4.3. Overall, the behavior of aggregate variables in Case 2 is almost identical in comparison with Case 1, while the amplitudes of impulse responses in Case 3 are a little bit more pronounced. For example, output Y_t increases on impact by 1.38 percent in Case 1, 1.39 percent in Case 2, and by 1.60 percent with respect to Case 3, whereas the contribution rates decline by 0.07, 0, and 1.64 percent in Cases 1 to 3, respectively. The economic intuition for these results is straightforward. On the one hand, pronounced adjustments of contribution rates in Case 3 induce stronger distortionary effects on individual labor supply decisions and, therefore, result in larger fluctuations of aggregate labor supply and real output. On the other hand, the share of financial assets only amounts to 0.84 percent of aggregate capital in the steady state. For that reason, the associated distortionary impacts of asset changes in a PAYG system in Case 1 on real factor prices are rather negligible so that the results in Case 1 and 2 are very similar. Thus, if the PAYG authority solely adjusts the contribution rates, it increases these distortionary effects, while the other financing forms help to stabilize the economy by keeping the contribution rates (almost) constant.

Table 4.1, which displays the simulated standard deviations of aggregate variables and their empirical counterparts for the sample 1991:1 to 2012:4, also confirms the previous results. Financing rules, which try to keep the contributions rates mostly constant, imply lower volatilities of aggregate output, labor, and consumption. For example, the second column shows that the standard deviation of aggregate output amounts to 1.64 in Case 1 and increases by 16 percent to 1.91 in Case 3, whereas it almost stays constant with regard to Case 2. Comparing our benchmark model in the first two rows with empirical data in the last two rows in Table 4.1, we can, moreover, see that the benchmark model produces standard characteristics of

Case 1 - Adjustment of Assets and Contribution Rates ($\omega_F = 95\%$ and $\omega_R = 0\%$):



Case 2 - Adjustment of Replacement Ratios ($\omega_F = 0\%$ and $\omega_R = 100\%$):



Case 3 - Adjustment of Contribution Rates ($\omega_F = 0\%$ and $\omega_R = 0\%$):

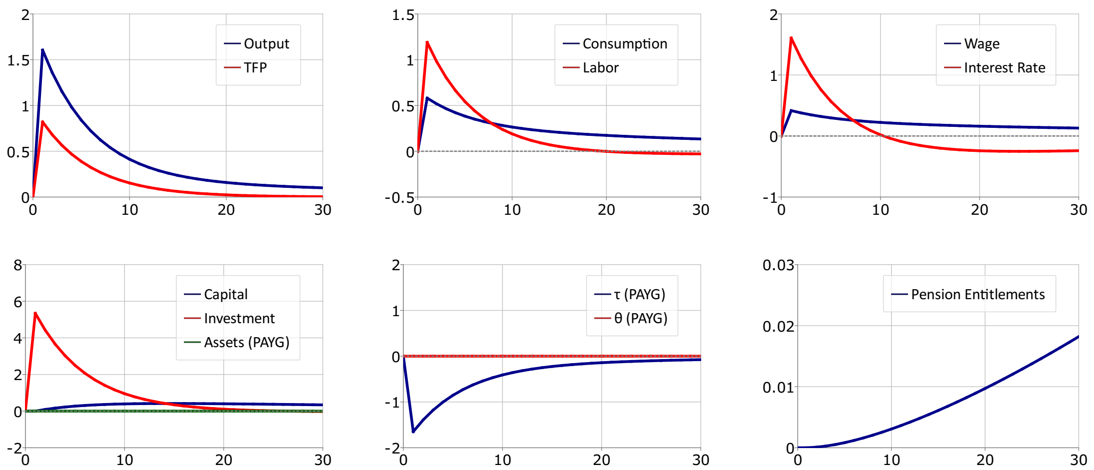


Figure 4.3: Impulse Responses of Aggregate Variables (ordinate: percent deviations, abscissa: periods).

business cycle volatilities which roughly match the data regarding output and labor. However, the standard deviations of investment, consumption, and the real wage are a little bit more different than their empirical values.

	Y	N	I	C	w	τ^p
Case 1:	1.64	1.02	5.79	0.53	0.63	0.25
	(1.00)	(0.62)	(3.52)	(0.32)	(0.38)	(0.15)
Case 2:	1.66	1.04	5.71	0.58	0.62	0.00
Case 3:	1.91	1.42	6.45	0.69	0.50	1.97
Data*:	1.51	0.93	4.35	0.77	0.79	·
	(1.00)	(0.62)	(2.88)	(0.51)	(0.52)	·

Table 4.1: Standard Deviations of Aggregate Variables (time series were hp-filtered using a parameter of 1600 over 100,000 simulations with a period lenghth of 88 quarters, in parentheses: relative deviations with respect to output, *sample: 1991:1-2012:4, source: Flor (2014)).

The previous findings clearly suggest that financing forms aiming to keep the contribution rates of a PAYG scheme (nearly) constant stabilize an economy. These financing rules, however, have different effects on the intergenerational allocation of risk and affect the consumption smoothing behavior of households over the life cycle. For this reason, I simulate 1,000,0000 periods with the same sequence of random total factor productivity shocks for Case 1 to 3 and compute the standard deviations of (remaining) ex-post lifetime utilities of all households at ages 21 to 80, which I express as consumption equivalent changes (CECs).[7]

By comparing the volatilities of consumption equivalent changes in Fig. 4.4 for each case, we can see that sole adjustments of replacement ratios in Case 2 are associated with very low volatilities in lifetime utility for workers and and very high volatilities in life time utility of retirees since this financing forms shifts the macroeconomic risk towards retirees. In contrast, the social security authority only adjusts the contributions rates in Case 3 and, therefore, burdens current working households with larger fluctuations in their net income, whereas the pronounced adjustments of

[7] The consumption equivalent change describes the percentage variation of steady state consumption that is equivalent to a given change in intertemporal welfare.

the stock of financial assets in Case 1 dampen these income fluctuations in Case 3. However, it is interesting that the standard devitions of lifetime utilities almost follow the same patterns in Case 1 and 3, where the volatility of consumption equivalent changes is slightly lower for households who are older than 30 years in Case 1. Thus, households are able to deal with both financing forms almost equally well by accordingly changing their labor supply and savings decisions over the life cyle, even though Case 3 causes more pronounced fluctuations of aggregate variables, as shown in Table 4.1. The effects of larger fluctuations of aggregate variables on the consumption smoothing behavior of households, therefore, seem to be negligible. Furthermore, in comparison to Case 2, the volatilities in Cases 1 and 3 are slightly higher up to an age of about 57 years and considerably lower for older age groups. For example, the standard deviation of the youngest (oldest) household amounts to 0.81 (0.59) and 0.78 (0.65) percent in Case 1 and 3, while it is equal to a value of 0.58 (2.38) percent with respect to Case 2.

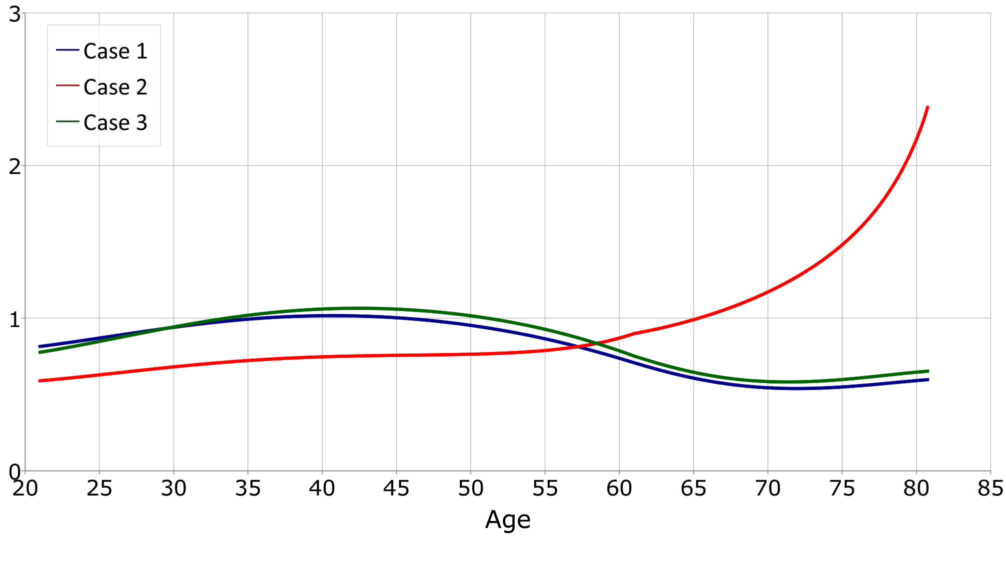


Figure 4.4: Volatilities of Consumption Equivalent Changes (ordinate: s.d. of CECs in percent, abscissa: age in years).

4.6 Conclusion

The analysis in this paper has shown how financing rules for additional surpluses in a PAYG system affect aggregate variables and the consumption smoothing behavior of households over the business cycle. Financing forms that keep the contribution rates in a PAYG system constant or almost constant imply in general lower absolute standard deviations of aggregate output, labor, investment, and consumption in comparison to complete adjustments of contribution rates. However, the effects on the volatility of (remaining) life-time utilities can be very different. On the one hand, sole adjustments of current replacement ratios, which mainly burden old generations due to constant contribution rates, result in very low fluctuations in lifetime utility of very young households and very high fluctuations for retirees. On the other hand, a financing rule that mainly adjusts the stock of financial assets of a PAYG scheme in order to avoid large fluctuations of contributions rates increases slightly the volatility of life-time utilities of young households, but also implies much lower volatilities for workers close to retirement and retirees. Moreover, complete adjustments of contribution rates yield only slightly higher standard deviations of lifetime utilities of households who are older than 30 years with respect to the aforementioned financing rule, despite much larger fluctuations in aggregate variables. Consequently, the impacts of higher standard deviations of aggregate variables on the volatility of individual welfare can rather be negligible. One should, however, be careful to use these welfare results for normative conclusions since no financing form studied in this paper strictly dominates the other.

4.A Bibliography

Busl, C. and A. Seymen (2013). The German Labour Market Reforms in a European Context: A DSGE Analysis. *ZEW Discussion Paper* (13-097).

Deutsche Rentenversicherung Bund (2017). *Rentenversicherung in Zeitreihen* (23rd ed.). Berlin: DRV-Schriften.

DICE Database (2016). *Gross / Net Pensions Replacement Rates from Public, Mandatory Private and Voluntary Private Pension Schemes, 2005 - 2015*. Munich: ifo Institute.

Flor, M. (2014). Post Reunification Economic Fluctuations in Germany: a Real Business Cycle Interpretation. *Review of Business and Economics Studies* (4), 5–17.

Galí, J., J. D. López-Salido, and J. Vallés (2007). Understanding the Effects of Government Spending on Consumption. *Journal of the European Economic Association 5*(1), 227–270.

Heer, B. (2019). *Public Economics: The Macroeconomic Perspective* (1st ed.). Heidelberg: Springer (forthcoming).

Heer, B. and A. Maußner (2009). *Dynamic General Equilibrium Modeling: Computational Methods and Applications* (2nd ed.). Heidelberg: Springer.

Heer, B. and A. Maußner (2012). The Burden of Unanticipated Inflation: Analysis of an Overlapping-Generations Model with Progressive Income Taxation and Staggered Prices. *Macroeconomic Dynamics 16*(2), 278–308.

Peterman, W. B. (2016). Reconciling Micro and Macro Estimates of the Frisch Labor Supply Elasticity. *Economic Inquiry 54*(1), 100–120.

Ríos-Rull, J.-V. (1996). Life-Cycle Economies and Aggregate Fluctuations. *Review of Economic Studies 63*(3), 465–489.

Thøgersen, Ø. (1998). A Note on Intergenerational Risk Sharing and the Design of Pay-As-You-Go Pension Programs. *Journal of Population Economics 11*(3), 373–378.

Trabandt, M. and H. Uhlig (2011). The Laffer Curve Revisited. *Journal of Monetary Economics 58*(4), 305–327.

Wagener, A. (2003). Pensions as a Portfolio Problem: Fixed Contribution Rates vs. Fixed Replacement Rates Reconsidered. *Journal of Population Economics 16*(1), 111–134.

Chapter 5

Conclusion

This section summarizes the overall findings of my cumulative dissertation and points to directions for future research. My thesis contains three essays that address important research questions in DSGE models with overlapping generations. This model framework thereby allows the introduction of heterogeneous households in the economic analysis such that these models are able to take into account the economic effects of demographic transitions or economic policies on age-specific decisions over the life cycle and the associated impacts on aggregate variables.

The first paper with the title "Aging, the Great Moderation, and Business-Cycle Volatility in a Life-Cycle Model", see Chapter 2, studies the contribution of demographic effects on the Great Moderation which describes the decrease in the volatility of output components in many advanced economies after the mid-1980s. For that reason, the model presented in this chapter incorporates overlapping generations into a relatively standard real business cycle model with technology shocks as the source of cyclical fluctuations. Moreover, this model empirically replicates the age-composition, the absolute level of age-specific labor supply, and the corresponding labor supply volatilities across all cohorts in the United States before and during the Great Moderation such that it is well suited to address the aforementioned research topic. The results, however, point out that demographic transitions play only a minor role since the effects of changes in the age-composition cancel each other out at the aggregate level. In contrast, the empirically observed downward shift of age-specific volatilities of labor supply across all cohorts during the Great Moderation explains about 23 percent of the decline in output's volatility. Hence, an ageing population does not imply much lower fluctuations in the (real) components of GDP as long as such a demographic transition is not accompanied by the aforementioned downward shift of labor supply volatilities across all age groups. Pure demographic effects are then rather negligible.

The second paper in my dissertation, titled "the Age-Specific Burdens of Short-Run Fluctuations in Government Spending" which I present in Chapter 3, investigates the distributional and age-specific welfare effects of short-run fluctuations in government spending and its associated financing form. This paper thereby contributes to the renewed research interest in fiscal policy after the Great Recession from 2007 to 2009, where the overlapping generations model for the economic analysis both replicates the empirically observed inequality of income and wealth across cohorts and accords well with empirical evidence from VAR studies for the United States. For these reasons, and in comparison to the first paper, the model in this chapter features a lot of heterogeneity at the individual level with three productivity types and two types of households, namely both Ricardian households and rule-of-thumb consumers. In addition, the model replicates the most important redistributive features of the tax and the pension system in the United States. Furthermore, it also incorporates New Keynesian rigidities, adds a second sector for capital goods, and features a fiscal policy rule which controls the adjustment of government debt and taxes in response to government spending shocks. In particular, the results of this paper show that tax financing of government expenditures results in lower fluctuations of (remaining) ex-post lifetime utilities across all generations if the price of capital is endogenous and adjusts in response to government spending shocks. Please note that retirees also face lower fluctuations in their lifetime utilities in the case of tax-financed adjustments of government expenditure. This interesting finding, therefore, runs contrary to the conventional wisdom that the financing of government expenditures by debt mainly burdens young generations. Moreover, government spending shocks result in a significant redistribution of wealth and income at the individual level. This redistribution is, however, not captured by aggregate measures of inequality like the Gini coefficient since the redistributive effects at the individual level cancel each other out at the aggregate level. For this reason, an increase of government consumption only slightly decreases the Gini coefficient of gross market income and wealth.

My third paper in Chapter 4, "the Effects of Financing Rules in Pay-As-You-Go Pension Systems on the Life and the Business Cycle", studies the impacts of financing rules for potential surpluses in pay-as-you-go pension systems on the business and the life cycle. Since revenues in pay-as-you-go systems fluctuate over the business cycle due to changes in aggregate labor supply and real wages, a balanced budget implies that either the contribution rate, pension benefits and/or the stock of financial assets have to be adjusted. The model framework in this paper consists of a standard real business cycle model with overlapping generations for the German economy, which is similar to the model used in Chapter 2. This model, however, incorporates financing rules for potential financial surpluses in pay-as-you-go pension systems, where households maximize a different utility function and explicitly take into account the intertemporal link between pension contributions and pension benefits. Therefore, the model allows for income and substitution effects on age-specific labor supply decisions which are caused by different financing rules. I find that financing forms that keep the contribution rate (almost) constant imply lower fluctuations of aggregate output, labor, investment, and consumption. However, no financing form strictly dominates the other since the resulting effects on the volatility of (remaining) life-time utilities across all cohorts are very different. In addition, the results also point out that pronounced changes in the standard deviations of aggregate variables can only marginally impact on the volatility of individual welfare.

In sum, all three papers mainly build on the seminal work by Ríos-Rull (1996, see Chapter 1.A), who was the first who implemented overlapping generations into a DSGE model. This framework allows researchers to study research questions about demographic changes, the allocation of resources across cohorts, the associated effects on age-specific decisions over the life cycle, and their economic importance; which are addressed in the aforementioned papers. Moreover, these models definitely provide a very promising framework for more future theoretical and empirical research. For example, since Ricardian Equivalence does not hold in these models, it would be interesting to study the effects of government debt on other countries by incorporating the overlapping generations structure into a two-country DSGE model. Moreover, it might also be very promising to implement second order approximation methods into DSGE models with overlapping generations. Such a model framework could, for example, study age-related research questions about monetary policy with ex-ante welfare comparisons and possible endogenous age-specific portfolio choices.