Comparative Analysis of different Wigner-Ville Distribution Implementations for the ECG-based Detection of Obstructive Sleep Apnea

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Abstract— Obstructive sleep apnea (OSA) syndrome is a common breathing abnormality. During apnea, the airflow is impeded or totally interrupted. The reaction of the autonomic nervous system terminates the apnea and also leads to changes in heart rate variability (HRV). As shown in previous studies, the spectral analysis of HRV allows for a diagnosis of apnea. Therefore, a high quality time-frequency distribution is of great significance. The Wigner-Ville-Distribution (WVD) offers a very high resolution in both, time and frequency. However, the proper handling of cross terms resulting in the calculation of the WVD is a crucial point using the WVD. To cope with this task, the presented work compares different methods regarding their ability of cross term suppression and applicability. Furthermore it is shown that using spectral information of ECG overnight recordings from the Physionet Apnea Data-base, these datasets can be separated.

Keywords— Apnea, ECG, Heart Rate Variability, Wigner-Ville-Distribution, Physionet.

I. INTRODUCTION

The obstructive sleep apnea syndrome has a prevalence of 4% in adult men and 2% in adult women [1]. The main reason for the occurrence of apnea is the reduction of the muscular tone in sleep. This causes an obstruction of the upper airway. In addition, a large tongue or obesity may encourage this process. During episodes of obstructive apnea, the respiratory effort is continued. The autonomic nervous system causes an arousal which terminates the apnea. The apnea is followed by a hyperventilation. The heart rate declines during the apnea and rises at the end of it. These cyclical variations in heart rate are typical for the apnea [2],[3]. There is a large research interest for detecting the OSA from ECG data. The Computers in Cardiology (CinC) Challenge 2000 aims at identifying apneic episodes from ECG overnight recordings. Various methods were applied to solve this problem [4]. Ambulant, portable health monitoring systems become more and more attractive. The Fraunhofer IPMS develops such Body Area Networks. These networks combine a number of different sensors and can even be integrated into clothes [5]. Using these technologies, effective screening is possible. The approach of ECG based apnea detection is to use data from screenings to select patients that show indications of apnea. Selected patients are then further examined in a sleep laboratory.

II. MATERIAL

The apnea ECG database from Physionet [6] is used to evaluate the implemented algorithms. The apnea database is divided into a training database and a test database. Each database contains 20 records of patients suffering from apnea (group A), 10 records from a control group (C) and 5 records (group B) that neither belong to group A nor to group C. The CinC Challenge was divided in two tasks. The first task was to assign the datasets from the test database to one of the groups A or C. The aim of the second task was to label each minute of the datasets apneic or non-apneic.

III. METHODS

A. Generating a Tachogram

The heart rate series are plotted against time in a tachogram. The instantaneous heart rate corresponds to the reciprocal of the distance between two consecutive QRS complexes. A Wavelet based approach is applied to detect the QRS complexes [7]. The RR series are cleared up from spikes and extrasystoles. A linear interpolation and a resampling with 10Hz is used to generate an equidistant NN series. After band-pass filtering (low-pass cutoff frequency: 0,4 Hz, high-pass cutoff frequency: 0,001 Hz) the NN series, it is resampled to 1 Hz. For filtering, cascaded Butterworth filters of second order are applied.



Fig. 1: Qualitative demonstration of the WVD, its modifications and of the STFT. (a) Each of the two signals consists of two frequency modulated Gaussian functions with frequencies 0,15 Hz (top) and 0,35 Hz (bottom). The sum of the signals in (a) is transformed according to Wigner and its modifications (b),(c),(d) and with the STFT (e).

B. Wigner-Ville Distribution

The WVD is used to analyze the frequency content of HRV. Its time-frequency resolution is twice as high as this of the Short Time Fourier Transform (STFT) [8].

The WVD is derived from the ambiguity function. For a signal z(t), the WVD is defined as:

$$WVD_{z}(t,f) = \int_{-\infty}^{\infty} \underbrace{z(t+\frac{\tau}{2})z^{*}(t-\frac{\tau}{2})}_{\Psi_{zz}(t,\tau)} exp(-j2\pi f\tau)d\tau \quad (1)$$

The expression $\Psi_{zz}(t,\tau)$ in (1) is the so called instantaneous autocorrelation function. Its calculation causes cross terms (or interference terms). These cross terms are the major disadvantage of the WVD.

C. Cross Term (CT) suppression

The most common method for CT suppression is the Smoothed Pseudo Wigner-Ville-Distribution (SPWVD). An-

other method uses a Time Frequency Distribution Series (TFDS) [12]. In the following, both methods are explained in detail. Further methods are given in Table 1.

SPWVD: The calculation of the SPWVD is separated in two steps. In the first step, a window function $h(\tau)$ is applied:

$$WVD_{z}^{P}(t,f) = \int_{-\infty}^{\infty} h(\tau)z(t+\frac{\tau}{2})z^{*}(t-\frac{\tau}{2})e^{-j2\pi f\tau}d\tau.$$
 (2)

Depending on the length of the window function $h(\tau)$ this suppresses, to some extent, the CT for multicomponent signals. The reason is that the window function makes the WVD local. But there are still CT left from different auto terms within the window. These CT can be reduced by convoluting the result from (2) with function g(t):

$$WVD_z^{SP}(t,f) = WVD_z^P(t,f) *_{t'} g(t)$$

= $\int_{-\infty}^{\infty} g(t-t')WVD_z^P(t',f)dt'$ (3)

By applying the function $h(\tau)$ and g(t), the resulting timefrequency distribution is smeared to some degree in time and frequency. Nevertheless, the resolution provided by the SP-WVD is still superior to the STFT.

TFDS: For the TFDS, the signal under investigation, z(t), is written in terms of linear combination of a set of elementary functions. These elementary functions should have a good localization in time and frequency. The Gabor elementary functions used in the TFDS satisfy these conditions.

The elementary functions are defined as:

$$h_{m,n}(t) = \left(\frac{\alpha}{\pi}\right)^{0.25} exp\left\{-\frac{\alpha}{2}(t-mT)^2 + jn\Omega t\right\}$$
(4)

and a signal is represented as:

Table 1: Further methods for CT suppression in the WVD

Method	Approach and Properties
TFDS, Wil-	TFDS using a Wilson base; (+): No re-
son base[9]	dundancy (-): High computational effort
Superposition	Knowledge of auto terms in STFT re-
of STFT and	duces CT in WVD (+): Simple; High res-
WVD [10]	olution (-): Auto terms have to have a
	minimum distance
Phase shifting	Shifted auto terms produce shifted CT;
of signal [11]	(+): High Resolution (-): A priori knowl-
	edge about auto terms



Fig. 2: Different Time-Frequency-Distributions for HRV from a Group A dataset (Record a03 apneadb)

$$z(t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} C_{m,n} h_{m,n}(t).$$
(5)

The Gabor elementary functions in 4 are time shifted and frequency modulated Gaussian functions. $C_{m,n}$ are the Gabor coefficients. The coefficients are calculated using the *Time Frequency Toolbox* [13]. The Cross WVD is applied to (5):

$$W_{z}(t,f) = \sum_{m,m'} \sum_{n,n'} C_{m,n} C_{m',n'}^{*} W_{h,h'}(t,f).$$
(6)

By setting (m, m', n, n') in (6), the distances |m - m'| and |n - n'| are determined. These two distances describe the

width of the instantaneous autocorrelation function. By manipulating them, a windowing can be achieved. This improves the resolution of the original Gabor spectrogram and simultaneously suppresses the CT effectively.

Figure 1 shows the interference terms in the WVD and the high time-frequency concentration of the SPWVD and the TFDS compared to the STFT.

D. Configuration of SPWVD and TFDS

Both, SPWVD and TFDS are programmed using MAT-LAB. Gaussian windows are used for SWPVD and TFDS.

SPWVD: To analyze frequencies down to 0,01 Hz and less, the minimum length of window function $h(\tau)$ is 100 s. The following window lengths are a compromise for a good time-frequency resolution: $h(\tau) = 181$ s, g(t) = 60 s.

TFDS: For the TFDS, the Gabor coefficients have to be calculated first. The Gabor elementary functions only form a basis of $L^2(\mathbb{R})$ if $T \cdot \Omega = 2\pi$ in (4). But the closer $T \cdot \Omega$ gets to 2π , the worse the dual function becomes. This circumstance requires some degree of oversampling, meaning $T \cdot \Omega > 2\pi$. The oversampling rate $Q = M \cdot N/L$ is set to 128. For a signal consisting of L = 8192 samples, N = 512 Gabor coefficients in time and M = 2048 in frequency are calculated. For the Cross WVD applied in the following step, the distance d = |m - m'| + |n - n'| is set to 4.

IV. RESULTS

WVD and methods for CT suppression: Using different test signals, the best results for CT suppression were achieved using the SPWVD. Compared to the TFDS, the SPWVD has less computational effort and the configuration is less complex. The TFDS requires some degree of oversampling resulting in a high redundancy.

A dataset from group A was analyzed using SPWVD, TFDS and STFT (Gaussian window; 181 s) (see Fig. 2(a) - 2(c)). Compared to the STFT, both the SPWVD and the TFDS provide a higher time-frequency resolution. The SP-WVD is chosen for further analysis.

Dataset Separation: In Fig. 2(a), the annotated apneic episodes (annotated by an expert) are marked on the time axis. There is an interrelation between these annotations and the rise of the energy in the frequency range between 0,02 Hz and 0,04 Hz. This interrelation was not noticeable in all of the datasets. Characteristic peaks arise when averaging the spectra in time. For datasets from group A, this peak lies at 0,021 \pm 0,006 Hz. For datasets from group C, it lies at 0,011 \pm 0,002 Hz. These values have been determined from the training dataset.



Fig. 3: Spectras for one dataset from group A and one from group C averaged in time.

Table 2: Results for classifying datasets by their averaged peak in the frequency domain

	Training Database	Test Database
Specificity	85 %	100 %
Sensitivity	90 %	100 %

The datasets are classified by setting a threshold value. This threshold is based on the previously mentioned features and is set to 0,015 Hz. Using this threshold, datasets from groups A and C are separated (see Fig. 3). The results for classification within the training database and the test database are summarized in Table 2.

V. DISCUSSION

Using SPWVD for HRV Analysis: To reduce CT artefacts in the WVD, a modification has to be applied. The SPWVD reduces the time frequency resolution of the WVD but is still superior to the STFT. Using the high resolution, a minute by minute analysis of HRV is possible.

Apnea detection: The spectral analysis of HRV has shown an interrelation between episodes containing apnea and energy changes in the frequency range between 0,02 Hz and 0,04 Hz. This result agrees with results from other works [3], [14]. For datasets from group C, the characteristic peak is in the range between 0,006 Hz and 0,014 Hz. This peak, detected in this work, is a further feature to separate the datasets from groups A and C. But the minute by minute analysis using the selected features does not work properly for all datasets. One reason is that the physiological causes for the very low frequencies in HRV are not entirely clear.

VI. CONCLUSION

The WVD is a powerful tool for the combined time frequency analysis. The CT artefacts in the WVD are reduced by using appropriate mathematical methods. The SPWVD was found to be the most efficient approach for CT suppression in this application.

Using the SPWVD to analyze HRV it was shown that it is possible to separate group A and C datasets from the Physionet apnea database. Good results were achieved using a simple threshold decision method.

Future work will use HRV to diagnose pathological changes. The ECG derived respiratory signal will be used in combination with the analysis of HRV to realize investigations with a higher temporal resolution.

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