Stochastic growth and regime shift risk in renewable resource management*

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Abstract

Renewable resources are affected by both environmental variability, which makes the year-to-year stock growth uncertain, and the risk of irreversible events (e.g., a stock collapse). Little is known about how a renewable resource harvester would optimally respond to the combined effects of both sources of risk. In this paper, we propose a simple dynamic resource model to investigate this issue. For some structures of the harvesting cost function, we find that anticipating a higher variability in biological growth induces a cautious management policy, but only when regime shift risk is accounted for. Accounting for the risk of regime shift may prescribe large changes in management responses to anticipated random changes in biological growth whereas ignoring such a risk prescribes small changes in management. Optimal escapement is not constant but varies across all periods when the planning horizon is finite and the regime shift risk is endogenous.

Keywords: Renewable resources; Dynamic analysis; Stochastic growth; Regime shifts

JEL Classification Codes: D41, D42, C61; K21, L12

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1 Introduction

Real world fisheries management is affected both by stochastic risk (stock growth uncertainty) and by catastrophic risk (regime shifts) that could be caused by natural or political forces. For example, recruitment to the North East Arctic cod fishery, which is currently the most valuable whitefish fishery in the world, varies up to sevenfold from one year to the next (ICES, 2018). At the same time, resource managers have to account for the risk of a sudden shutdown of the fishery caused by, e.g., an oil spill, a collapse caused by climatic events, or by a moratorium that is mandated by political pressure groups. In this paper, we ask how these two sources of risk interact and which consequences this has for optimal management strategies.

To address this issue, we develop a stylized renewable resource extraction model that accounts for the resource stock dynamics. Current harvest, biological growth, and environmental conditions determine future resource stocks. Following Reed (1979), Weitzman (2002), and Costello and Polasky (2008), we focus our attention on scenarios in which the resource price is constant, harvesting cost are (potentially) stock-dependent and consider a multiplicative, i.i.d., shock to the resource dynamics. We also allow for the possibility of a sudden, and irreversible regime shift that triggers a permanent closure of the fishery. We consider both exogenous regime shifts or endogenous regime shifts. In the former case, the probability of regime shift is time-dependent, but is not affected by harvest decisions. In the latter case, the probability of a regime shift endogenously depends on harvest decisions. With the help of this model, we characterize situations under which optimal resource extraction becomes more cautious or more aggressive.

Our analysis reveals that an exogenous regime shift threat prescribes an aggressive management policy, but when the threat of regime shift is sufficiently sensitive to extraction, management ought to be more cautious. For some structures of the harvesting cost function, we find that anticipating a greater variability in biological growth induces a cautious management policy, but only in the presence of regime shift risk. Ignoring regime shift risk prescribes small changes in escapement (the resource stock left in the water after harvesting) as a response to random changes in biological growth, whereas accounting for the regime shift risk may prescribe large changes in escapement. Furthermore, we find that optimal escapement declines stronger with the discount rate when the regime shift risk is sensitive to harvest decisions than when it is exogenous, or when it is not present. Hence, it is important to account for regime shift risk when designing management strategies to respond to multiple uncertainties (Crepin et al., 2012).

2 Literature review

Our paper connects two strands of the economic literature on renewable resource management. The first strand analyzes the effect of stochastic risk without considering the possibility of regime shift. The second strand of literature studies the effects of regime shift risk in an otherwise deterministic environment.

The seminal paper in the first strand of literature is by Reed (1979) who shows that the optimal escapement level (the resource stock left in the water after harvesting) is constant when the resource price does not depend on harvest and is constant. Several papers have hence refined and extended Reed's model by adding spatial structure (Costello and Polasky, 2008), or the choice of regulatory instrument (Weitzman, 2002). In particular, a number of papers have shown that the result that optimal escapement is constant does not hold when also stock measurements or harvest levels are uncertain (Sethi et al., 2005) or when there are capital- or policy adjustment costs (Singh et al., 2006; Boettiger et al., 2016). In this paper, we show that the threat of regime shift may also be a factor giving rise to a time dependent optimal escapement.

The pioneering contributions in the second strand of literature include the studies by Cropper (1976) and Kemp (1976), which investigate the effects of regime shift risks in a resource extraction context. In line with the growing realization of the importance of regime shift risk, there are by now a number of contributions that analyze a range of applications from saltwater intrusion (Tsur and Zemel, 1995) to the disintegration of the West-Antarctic ice sheet (Nævdal, 2006). Tsur and Zemel (2021) and Long (2021) provide a recent survey of these and related studies. Polasky et al. (2011) summarize and characterize the literature at hand of a simple fishery model with a linear objective function. The main distinctions in the literature are whether the regime shift implies a collapse of the resource or a reduction of its renewability, and whether the probability of a regime shift is exogenous or endogenous (i.e., depends on the state of the system). The resource manager should be cautious in cases in which the occurrence of regime shift entails a decline in biological growth and the regime shift risk is endogenous. When the regime shift risk is exogenous in these cases, there is no change in optimal extraction. In contrast, exploitation should be more aggressive in cases in which the occurrence of regime shift triggers a stock collapse and the regime shift risk is exogenous. In the collapse/endogenous-risk cases, there are two countervailing effects: The risk of future collapse incentivizes more aggressive extraction today, while the fact that the collapse risk can be influenced incentivizes more cautious extraction. Which of the two effects dominates depends on the likelihood that caution successfully avoids the regime shift.

Combining analytical with numerical methods, Sakamoto (2014) shows that the ambiguous result in the collapse/endogenous-risk cases is amplified in a non-cooperative setting. In simple terms, agents try to grab what they can before it is too late when catastrophe avoidance becomes unlikely, but cooperation and caution increases when the catastrophe may be avoided. Miller and Nkuiya (2016) analyze coalition formation in a fishery model and show that an endogenous regime shift risk increases coalition sizes and it allows the players, in some cases, to achieve full cooperation.

Ren and Polasky (2014) and de Zeeuw and He (2017) point out that the optimal management response to the threat of a sudden change in the renewability of the resource can also be more aggressive harvesting, rather than more cautious harvesting, when the objective function is not linear. While our model considers the case when the regime shift implies the collapse of the resource rather than a loss of renewability, the structure of the harvesting cost, and hence the properties of objective function, also play a central role in our paper. Utilizing numerical simulations and allowing for a reversible regime shift in both biological and economic conditions, Kvamsdal (2022) re-affirms the sensitivity of the management response to the curvature of the objective function. Moreover, Kvamsdal (2022) investigates whether regime shifts are observable or non-observable to the manager, finding only small differences in the respective management response.

The distinction between observable and non-observable regime shift also plays a key role in Baggio (2016) and Baggio and Fackler (2016). The latter two papers allow for two sources of uncertainty (stock growth uncertainty and regime shift risk). Baggio (2016) presents a calibrated numerical fishery model and shows that, compared to an uniformed situation, resource rents are doubled when the manager is informed about the (exogenous) regime shift risk. Baggio and Fackler (2016) similarly present a numerical model under different information structures, focussing on differences in the reactions to endogenous or exogenous regime shift risk. A key difference to our work is that Baggio (2016) and Baggio and Fackler (2016) consider reversible regime shifts that affect the growth dynamics. In this paper, we focus on irreversible regime shift issues that entail a closure of the fishery.

Our analysis is also related to a substantial body of economic papers that investigates the optimal management of a pollution stock in a setting where the system dynamic may randomly change over time. Two modelling approaches that rely on two ecological regimes (an "ecologically desirable regime" and an "ecologically undesirable regime") have been intensively used so far. In the first approach, the pollution stock dynamics shift between the "ecologically desirable regime" and the "ecologically undesirable regime" whenever a pollution stock is crossed (Brozović and Schlenker, 2011). In this context, the ecologically undesirable regime is modelled as a state in which pollution accumulates faster. The second approach models the "ecologically undesirable regime" through a penalty function with an exogenous or endogenous hazard rate. Interesting contributions in this category include the seminal paper by Clarke and Reed (1994), which models irreversible events like global warming as a permanent decline in the payoff function. Our study complements these contributions as in addition to considering the risk of abrupt regime shifts, the particular nature of our ecosystem requires the manager to account for the effects of random changes in the resource stock dynamics.

A number of papers investigate the optimal exploitation of various resources under environmental uncertainty and the risk of irreversible regime shifts.[1] In an early contribution within a framework that allows for the risk of extinction, Saphores (2003) proposes the management of a renewable resource population subject to stochastic growth due to random changes in environmental conditions. In contrast to this paper, he concentrates on the exogenous risk of extinction case only and relies on numerical simulations to show that the optimal management policy may change non-monotonically with the variance of the stochastic shock. Leizarowitz and Tsur (2012) extend the above paper to a sophisticated multiple species model. In contrast to our paper, this latter study concentrates on scenarios in which environmental uncertainty affects resource growth additively and does not address the effects of changes in the variance of the stochastic shock. In a setting where the pollution stock decays at a stochastic rate, Zemel (2012) addresses the management of polluting goods in a system prone to a climate tipping point. In contrast to our discrete-time fishery model case, his pollution control model reveals that an increase in the variance of the stochastic

[1]While they do not address the management of a natural resource stock, Cai and Lontzek (2019) examine economic growth in the presence of economic and climate risks, two different sources of uncertainty.

shock first increases and then decreases the response to the regime shift risk.[2] Sims and Finnoff (2016), at the hand of an invasive species example, illustrate how financial and environmental uncertainties can create opposing irreversibilities. The attempt to avoid bad financial outcomes (inefficient mitigation expenditures) counteracts the incentives to avoid bad environmental outcomes. The net effect depends on the size of the damages, and the variability of the different processes.

3 The model

The manager of a renewable resource makes inter-temporal harvest decisions to maximize her expected net present value. We consider a discrete time framework with $T+1$ time periods denoted by $t = 0, 1, 2, ..., T$. In addition to considering scenarios in which the planning horizon is infinite (i.e., $T = +\infty$), we also allow for cases in which $T < +\infty$. The resource stock at the beginning of period t is X_t and h_t represents period-t harvest. Variations in environmental conditions (e.g., temperature, upwelling, salinity) affect stock dynamics, which are given by:

$$X_{t+1} = Z_t g(y_t). \tag{1}$$

Z_t captures random changes in period-t environmental conditions and g represents the growth function, which is increasing and concave. The variable $y_t = X_t - h_t$ stands for period-t escapement (the resource stock after harvest). As in Reed (1979) and Costello and Polasky (2008), we assume that the mean of Z_t equals one and Z_t, $t = 0, 1, 2, ...$ are independent and identically distributed random variables. Moreover, any realization of Z_t falls within the interval $[\underline{Z}\ \bar{Z}]$, with $0 < \underline{Z} < \bar{Z} < \infty$.

[2]While they do not explicitly account for the risk of a potential future regime shift, Grass et al. (2015) rely on bifurcation theory to investigate a shallow lake system subject to a stochastic recharge rate.

The regime shift process operates as follows. At the beginning of the initial period, the manager anticipates that a regime shift may occur at the beginning of period $\tau \geq 1$. More precisely, the manager can derive value from exploiting the resource in periods 0, 1, 2, ..., $\tau-1$. However, in periods $\tau, \tau+1, \tau+2$,..., the manager cannot derive value from the resource anymore. As such, the post-event value function is set to zero. This does not necessarily mean that the resource itself is wiped out after the regime shift, it could also represent scenarios where the market of the resource collapses due to e.g. a drop in consumer demand after an oil spill. Similarly, our model could represent a situation where access to the fishery is closed due to a moratorium or the introduction of a marine reserve.

The occurrence date τ is a random variable. The possibility to access to the resource is captured by a Markovian process M_t with two states: O_p (for "open fishery") or C_ℓ (for "closed" or "collapsed fishery"). Changes from one state to the other work according to the transition probabilities

$$\begin{aligned} \Pr(M_{t+1}{=}C_\ell | M_t{=}O_p) &= \rho(y_t); \qquad \Pr(M_{t+1}{=}O_p | M_t{=}O_p) = 1-\rho(y_t); \\ \Pr(M_{t+1}{=}O_p | M_t{=}C_\ell) &= 0; \qquad \Pr(M_{t+1}{=}C_\ell | M_t{=}C_\ell) = 1, \end{aligned} \tag{2}$$

where $0 \leq \rho(y_t) \leq 1$ represents the hazard rate. We assume that $\rho(y_t)$ (weakly) decreases in y_t (i.e., $\rho'(y_t) \leq 0$). In some cases, the regime shift risk is exogenous and constant. Notable examples include scenarios in which the regime shift is triggered by abrupt climate change, or an unanticipated oil spill. In other cases, the probability of a regime shift may depend on the resource stock. This would be particularly appropriate when modelling a catastrophic trophic cascade that occurs once the resource stock falls below a certain level, or a moratorium that is politically mandated (e.g. due to environmental pressure groups) when resource extraction drives the stock to a low level.

At the beginning of period t, the manager learns the current resource stock (X_t). Thereafter, she chooses her harvest (h_t), which determines current escapement ($y_t = X_t - h_t$). Towards the end of the period, growth and the random shock determine the resource stock for the next period according to (1). In making her current harvest decision, the manager accounts for the possibility of a regime shift as well as the effects of current harvest on the regime shift risk and the evolution of the resource stock. Mathematically, the Bellman equation for the problem (formulated in terms of escapement) faced by the manager reads:

$$V_t(X_t) = \max_{y_t} \left\{ p(X_t - y_t) - \int_{y_t}^{X_t} c(s)ds + \beta(1 - \rho(y_t))\mathbb{E}\left[V_{t+1}(X_{t+1})\right] \right\} \quad (3)$$

$$\text{subject to (1)}$$

In the optimization problem (3), $\mathbb{E}$ stands for the expected value operator and $\beta \in (0, 1)$ is the discount factor. The term $p(X_t - y_t)$ is the revenue resulting from harvesting $X_t - y_t$ resource units and p is a positive constant representing the resource price. The third right-hand side term in (3) represents the continuation value of the problem. The second right-hand side term in (3) is the total cost function. $c(s)$ is the marginal harvesting cost function, which is differentiable and (weakly) decreasing in the fish stock. To cleanly expose harvest responses to environmental instability, we will separately consider two scenarios. First, we examine the constant marginal cost scenario in which $c'(s) = 0$ for all $s > 0$. In the second scenario, the "stock effect" prevails, that is, marginal harvesting cost strictly decline as the resource stock increases (i.e., $c'(s) < 0$).

Denote by X_∞ the resource stock defined as: $p = c(X_\infty)$ if $c(0) \geq p$ and $X_\infty = 0$ if $c(0) < p$. The variable X_∞ can be interpreted as the smallest resource stock that gives rise to non-negative economic profit. For the sake of tractability, we assume that X_∞ is self-sustaining. That is, even the worst realization of the random shock cannot prevent the smallest economically viable resource stock from growing (i.e., $\underline{Z} \times g(X_\infty) > X_\infty$). A clear implication of this assumption is that in periods $t = 0, 1, 2, ..., T-1$, it cannot be optimal to harvest the resource stock down to an escapement level smaller than or equal to X_∞. To see why this result holds, denote by $\varphi_t(y_t) = p(X_t - y_t) - \int_{y_t}^{X_t} c(s)ds + \beta(1-\rho(y_t))\mathbb{E}\big[V_{t+1}(Z_t g(y_t))\big]$, the objective function in (3). From this formula, we derive

$$\varphi_t'(y_t) = (-p + c(y_t)) + \beta(1-\rho(y_t))\mathbb{E}\big[Z_t g'(y_t)) V_{t+1}'(Z_t g(y_t))\big] - \beta\rho'(y_t)\mathbb{E}\big[V_{t+1}(Z_t g(y_t))\big].$$

The first right-hand side term of this expression is non-negative for $0 \le y_t \le X_\infty$ because X_∞ is self-sustaining and $c' \le 0$. The sum of the second and third right-hand side terms of the expression is positive for $0 \le y_t \le X_\infty$ because X_∞ is self-sustaining and $\rho'(y_t) \le 0$. Therefore, $\varphi'(y_t) > 0$ for all $0 \le y_t \le X_\infty$. This result reveals that it is suboptimal to choose any escapement smaller than or equal to X_∞. In other words, in periods $t = 0, 1, 2, ..., T-1$, optimal escapement must be strictly greater than X_∞.

To simplify the analysis, we restrict our attention to interior solutions in the remainder of this paper. The first-order condition for the maximization of the right-hand side of (3) can be written as

$$p - c(y_t) = \beta(1-\rho(y_t))\mathbb{E}\big[Z_t g'(y_t)) V_{t+1}'(X_{t+1})\big] - \beta\rho'(y_t)\mathbb{E}\big[V_{t+1}(X_{t+1})\big]. \tag{4}$$

This condition shows how the interplay between environmental, economic, and political conditions affects current escapement decisions. For an interior solution, (4) shows that the manager chooses current escapement so as to equate marginal revenue to the value forgone from harvesting today rather than saving the resource for future harvests. Using the above computations, we derive the following proposition.

Proposition 1. *Assuming that the planning horizon is finite (i.e., $T < +\infty$), the following results hold.*

(i) If $\rho(y_t)$ is strictly decreasing in y_t, then optimal escapement varies across periods $t =$

$0, 1, 2, ..., T$.

(ii) Assuming that $\rho(y_t)$ is a constant function, then optimal escapement remains unchanged across periods $t = 0, 1, 2, ..., T-1$.

Proof. See Appendix A.1.

The results of this proposition hold under both deterministic stock growth and stock growth uncertainty. Such results add to the seminal papers by Reed (1979) and Costello and Polasky (2008) that investigate optimal renewable resource management in a context where the resource price is constant. These papers do not consider the effects of regime shift risk and concentrate on scenarios in which resource growth is uncertain due to random changes in environmental conditions (e.g., temperature, nutrients). They find that, for an interior solution, optimal escapement does not change across periods prior to the last one when the planning horizon is finite. In this paper, we have shown in Proposition 1 that such conventional wisdom does not necessarily hold when the manager faces regime shift risk in addition to stock growth uncertainty.[3]

The intuition underlying this result can be gleaned from above derivations. Recall that the manager chooses period-t escapement so as to equate the marginal cost of increasing escapement ($p - c(y_t)$) and the marginal benefit of increasing escapement (i.e., the right-hand side of (4)). The marginal cost of increasing escapement ($p - c(y_t)$) does not depend on $\rho(y_t)$. As shown in Appendix A.1, $\mathbb{E}\big[V_{t+1}(X_{t+1})\big] = \mathbb{E}\big[p - c(X_{t+1})\big] = \mathbb{E}\big[p - c(Z_t g(y_t))\big]$ for $t = 0, 1, 2, ..., T-1$. As a result, in the case where $\rho(y_t)$ is constant, the marginal benefit of increasing escapement depends on y_t and does not explicitly depends on time. These results explain why optimal escapement is time independent across periods $t = 0, 1, 2, ..., T-1$ when $\rho(y_t)$ is a constant function. In the case where $\rho(y_t)$ is strictly decreasing in y_t, the second right-hand side term in (4) in addition to depending on y_t, explicitly depend on time

[3]As shown in Appendix A.2, this conclusion does not qualitatively change when $\rho(y)$ is neither strictly decreasing over the whole range of y nor constant.

because $V_{t+1}(X_{t+1})$ is time-dependent as shown in Appendix A.1. This result implies that, when $\rho'(y_t) < 0$, optimal escapement becomes time dependent because the marginal benefit of increasing escapement is time dependent in this scenario.

To further unveil implications of both sources of uncertainty, from now on, we restrict our attention to scenarios in which the planning horizon is infinite. Since X_{t+1} is a function of y_t, condition (4) suggests that our model may sustain an optimal escapement policy that does not depend on the current stock size. We formally examine this question in the following proposition.

Proposition 2. *(i) Period-t escapement is stock-independent, and (ii) escapement, denoted by y^*, is the solution to*

$$\begin{aligned} p - c(y) = \beta(1-\rho(y))\mathbb{E}\big[Z_t g'(y)(p - c(Z_t g(y)))\big] \\ - \frac{\beta\rho'(y)}{1-\beta(1-\rho(y))} \times \mathbb{E}\Big[p(Z_t g(y) - y) - \int_y^{Z_t g(y)} c(s)ds\Big]. \end{aligned} \tag{5}$$

Proof. See Appendix A.3.

Although the manager makes escapement decisions before observing the realization of Z_t, the optimal escapement policy y^* is deterministic. As shown in condition (5), y^* depends on the distribution of Z_t. Moreover, condition (5) illustrates how the harvesting cost structure, distribution of Z_t, resource growth, discount factor, resource price, and the probability of regime shift affect current escapement. For scenarios in which the marginal harvesting cost function is constant and there is no threat of regime shift (i.e., $\rho = 0$ and $\rho' = 0$), using the fact that $\mathbb{E}(Z_t) = 1$, condition (5) simplifies to

$$\frac{1}{\beta} = g'(y^*).$$

This formula represents the standard golden rule of growth stating that at the optimum, the expected biological return and the financial rate of return are equal. In the setting of this paper where in addition to stock growth uncertainty, the manager faces the threat of regime shift, such a golden rule modifies to

$$\frac{1}{\beta} = g'(y^*)(1-\rho(y^*)) - \frac{\rho'(y^*)}{1-\beta(1-\rho(y^*))}(g(y^*)-y^*).$$

This expression reveals that the standard golden rule of growth is adjusted to account for the possibility of regime shift. We next investigate the sensitivity of escapement incentives to changes in the distribution of random shocks.

4 Effects of uncertainty

Keeping fixed the probability of regime shift, this section concentrates on the manager's responses to random changes in environmental conditions. Specifically, we investigate whether changes in the distribution of Z_t intensifies or lowers extraction. We first discuss scenarios in which the function $xc(x)$ is concave in x, then when it is linear, and finally when it is convex.

We make use of the concept of second-order stochastic dominance defined as follows.

Definition 1. Denote by $\tilde{Z}$ and $\hat{Z}$ two random variables with the same mean (i.e., $\mathbb{E}(\tilde{Z}) = \mathbb{E}(\hat{Z})$). The variable $\hat{Z}$ is a mean preserving spread of $\tilde{Z}$ if the inequality $\mathbb{E}(U(\tilde{Z})) \geq \mathbb{E}(U(\hat{Z}))$ is valid for any concave utility function U.

Since $c(x)$ represents the unit cost of extraction, when marginal harvesting costs are constant, $xc(x)$ can be interpreted as the cost of completely depleting the resource stock. Our analysis suggests that the shape of $xc(x)$ critically affects the manager's attitude toward stock growth uncertainty as revealed in the following proposition.

Proposition 3. *Provided that the function $xc(x)$ is concave in x. A mean preserving spread of Z_t always increases current escapement.*

Proof. See Appendix A.4.

To shed light on forces driving the result of Proposition 3, it can be useful to first compare escapement under uncertain stock growth with escapement under deterministic stock growth. In the particular case where $Z_t = 1$, condition (5) retrieves optimal escapement from the deterministic setting (denoted by $\bar{y}$), which is the solution to

$$p - c(y) = \beta(1-\rho(y))g'(y)\left[p - c(g(y))\right] - \frac{\beta\rho'(y)}{1-\beta(1-\rho(y))} \times \left[p(g(y)-y) - \int_y^{g(y)} c(s)ds\right]. \quad (6)$$

In the case where $xc(x)$ is concave, we derive three important properties for the optimum.

First, for a given level of escapement and – consequently – a given level of regime shift risk, we call the first right-hand side term of (5) the "Direct Effect", as it represents the direct effect of uncertainty. Moreover, the function $\beta(1-\rho(y))Z_t g'(y)[p - c(Z_t g(y))]$ is convex in the random variable Z_t. As such, for a fixed level of escapement, the Direct Effect is greater than the first right-hand side term of (6).

Second, since a lower level of escapement implies a higher probability of regime shift when $\rho'(.) < 0$, we call the second right-hand side term of (5) the "Risk Effect". Given that the marginal harvesting cost function is decreasing (i.e., $c' < 0$), the function $-\frac{\beta\rho'(y)}{1-\beta(1-\rho(y))} \times [p(Z_t g(y) - y) - \int_y^{Z_t g(y)} c(s)ds]$ is convex in the random variable Z_t. Consequently, holding escapement constant, the Risk Effect is greater than the second right-hand side term of (6).

Third, the left-hand side terms of (5) and (6) are identical and increasing in escapement. These three properties imply that optimal escapement under stock growth uncertainty is greater than optimal escapement under the deterministic stock growth scenario when $xc(x)$

is concave.

Holding escapement constant, the Direct Effect increases in response to a mean preserving spread as long as $xc(x)$ is concave. This pulls towards a higher escapement level. In addition, a mean preserving spread raises the Risk Effect, which also pulls towards a higher escapement. Therefore, the manager optimally increases current escapement in response to a mean preserving spread when $xc(x)$ is concave. This result is valid irrespective of whether or not the probability of regime shift is endogenous.

To further understand extraction responses to uncertainty, we next examine the scenario where $xc(x)$ is linear;[4] the results are summarized in the following proposition.

Proposition 4. *Provided that the function $xc(x)$ is linear in x.*

(i) A mean preserving spread of Z_t does not affect current escapement if $\rho'(.) = 0$ or marginal harvesting costs are constant.

(ii) A mean preserving spread of Z_t increases escapement if $\rho'(.) \neq 0$ and marginal harvesting costs are not constant.

Proof. See Appendix A.5.

The result (i) of Proposition 4 is driven by the fact that, for a given level of escapement, the Direct Effect and Risk Effect do not depend on the distribution of Z_t when $\rho'(.) = 0$ or marginal harvesting costs are constant. Consequently, in this context, the manager does not change current escapement in response to mean preserving spreads.

Result (ii) of Proposition 4 illustrates the importance of accounting for the threat of regime shift. An interesting body of economic papers examine how a renewable resource manager responds to stock growth uncertainty (Reed, 1979; Costello and Polasky, 2008), but in scenarios where regime shifts cannot occur (i.e., $\rho \equiv 0$). In this specific context, the Direct Effect does not depend on mean preserving spreads , when $xc(x)$ is linear, and

[4] There are two interesting scenarios in which the function $xc(x)$ is linear. First, if $c(x)$ is constant, then $xc(x)$ is linear in x. Second, if $c(x) = A/x$, then marginal harvesting costs are not constant and $xc(x)$ is linear in x.

the Risk Effect is of course nil. For these reasons, the manager does not modify its current harvest in response to any mean preserving spreads as long as $xc(x)$ is linear.

In this paper, we analyze the situation where the manager faces the threat of a regime shift, in addition to stock growth uncertainty. In this context, not yet explored, the Risk Effect emerges as a new channel in response to a mean preserving spread. We show that the Direct Effect does not change in response to mean preserving spread. Moreover, holding escapement constant, a mean preserving spread raises the Risk Effect if $\rho'(.) \neq 0$ and marginal harvesting costs are not constant. The increased Risk Effect implies that the manager optimally raises current escapement in response to mean preserving spreads.

For completeness, we next discuss extraction responses to stock growth uncertainty under scenarios in which $\rho' \neq 0$ and $xc(x)$ is convex. In this particular context, escapement responses to mean preserving spreads are still driven by the Direct Effect and the Risk Effect, which now work in opposite directions. The Direct Effect tends to lower escapement while the Risk Effect tends to increase escapement. Each of both forces may dominate the other depending on economic, environmental, and biological conditions.

Under scenarios where the ecosystem is not prone to a regime shift, a prominent class of economic papers (e.g., Reed, 1979) consider marginal harvesting cost of the form

$$c(X) = \frac{A}{X^{\theta}}, \quad \text{for all} \quad X > 0, \tag{7}$$

where $A > 0$ and $\theta \geq 0$ are parameters. It is important to notice that $xc(x)$ is concave as long as $0 < \theta < 1$, linear when $\theta = 0$ or $\theta = 1$, and convex for $\theta > 1$. To illustrate our contribution with respect to such papers, we next examine how the cost structure defined in (7) affects harvest responses to environmental instability. Findings are summarized in the following proposition.

Proposition 5. *Provided that the marginal harvesting cost function in defined* (7) *with*

$\theta > 0$.

A mean preserving spread increases current escapement if y^* satisfies

$$-\rho'(y^*) > (\theta - 1)(1 - \beta(1 - \rho(y^*)))(1 - \rho(y^*))g'(y^*). \quad (8)$$

Proof. See Appendix A.6.

In the case where $c(.)$ is defined in (7), this proposition provides three interesting properties of harvest responses to changes in environmental conditions. First, a mean preserving spread always increases current escapement when $\rho' \neq 0$ and $\theta = 1$. This finding is consistent with the results of Proposition 4. Second, a mean preserving spread increases escapement when $0 < \theta < 1$. This finding is consistent with the result of Proposition 3. Third, the finding sheds new light on prior economic papers (e.g., Reed, 1979; Costello and Polasky, 2008; Kapaun and Quaas, 2013), which analyze a risk neutral manager's responses to resource growth uncertainty in a system that is not prone to regime shifts and where the resource price is constant. In this context, when $xc(x)$ is convex, a mean preserving spread always diminishes the Direct Effect whereas the Risk Effect is nil. As a result, the manager optimally lowers current escapement in response to mean preserving spreads when $xc(x)$ is convex.

In our model, where in addition to stock growth uncertainty, the manager faces the threat of a regime shift, the Risk Effect counteracts with the Direct Effect when $xc(x)$ is convex. In the particular case where $c(.)$ is defined in (7), Proposition 5 suggests that in response to a mean preserving spread, the interplay between the Direct Effect and the Risk Effect may give rise to a novel result: The manager raises current escapement in response to mean preserving spreads when $xc(x)$ is convex and the biological growth of the resource stock is small.

5 Numerical example

This section proposes a numerical example to further illustrate harvest responses to potential changes in environmental conditions. To quantify the regime shift risk, we consider the probability function $\rho(y) = a \times e^{-\gamma y}$. The variable $\gamma \geq 0$ represents the elasticity of the regime shift risk with respect to escapement. Such a probability is more sensitive to changes in escapement as γ increases.[5] The marginal harvesting cost function is defined in (7) such that $X_\infty = (\frac{A}{p})^{\frac{1}{\theta}} > 0$.

We make use of the Beverton-Holt growth function defined as: $g(y) = \frac{y/\alpha}{1+(y/v)}$. We focus our attention on a binomial shock Z_t, which takes the value $Z_H = T_H + (1 - p_H)\lambda$ with probability $0 \leq p_H \leq 1$ and where T_H is a positive real number. Moreover, Z_t takes the value $Z_L = T_L - p_H\lambda > 0$ with probability $1 - p_H$, where $0 < T_L \leq T_H$ and $\lambda \in [0, T_L/p_H)$ are parameters. To ensure that X_∞ is self-sustaining (i.e., $Z_L g(X_\infty) > X_\infty$), we assume that $0 \leq \lambda < \bar{\lambda} \equiv [T_L - \alpha(1 + (X_\infty/v))]/p_H$.

To be consistent with the assumption $\mathbb{E}(Z_t) = 1$, we assume that the parameters of the distribution satisfy the equality $p_H T_H + (1 - p_H)T_L = 1$. For two arbitrary numbers $0 \leq \lambda_1 < \lambda_2 < \bar{\lambda}$, define $Z_{t|\lambda=\lambda_2} = Z_{t|\lambda=\lambda_1} + \varepsilon$ where ε is a random variable that takes the value $(1 - p_H)(\lambda_2 - \lambda_1)$ with probability p_H and $-p_H(\lambda_2 - \lambda_1)$ with probability $1 - p_H$. It can be shown that the equality $\mathbb{E}(\varepsilon/Z_{t|\lambda=\lambda_1}) = 0$ is always valid. Following Rothschild and Stiglitz (1970), these last two results reveal that $Z_{t|\lambda=\lambda_2}$ is a mean preserving spread of $Z_{t|\lambda=\lambda_1}$ for any λ_1 and λ_2 that satisfy $0 \leq \lambda_1 < \lambda_2 < \bar{\lambda}$. Consequently, any increase in λ over the interval $[0\ \bar{\lambda})$ represents a mean preserving spread. The set of parameters used in simulations is portrayed in Table 1. The values of λ used in our numerical analysis are produced following the sequence $\lambda_j = \lambda_{j-1} + h$, $j = 1, 2, ..., 99$ with $\lambda_0 = 0$ and $h = 0.0393$.

Keeping fixed the distribution of Z_t with $\lambda = 0.4$, we first examine how escapement under

[5] At $\gamma = 0$, the regime shift risk is exogenous and as $\gamma \to \infty$ the regime shift risk traces the abscissa and the ordinate, that is, ρ is practically zero everywhere, except close to the origin where it rises steeply.

Table 1: Parameters used in simulations

Hazard function parameter	$a = 0.8$ and $\gamma \in [0, 8]$
Growth function parameters	$\alpha = 0.1$ and $\upsilon = 0.3$
Random variable parameters	$T_H = T_L = 1$, $p_H = 0.1$, and $\lambda = 0.4$
Resource price	$p = 1$
Initial resource stock	$X_0 \geq 3$
Marginal cost function parameters	$A = 0.95$ and $\theta = 1.12$
Discount rate	$r = \frac{1}{\beta} - 1 = 0.052$

the no-regime shift risk scenario (thin horizontal line in Figure 1) changes in response to the introduction of regime shift risk (thick black line in Figure 1). Changes in γ obviously do not affect escapement under the no-regime shift risk scenario. When regime shift risk is positive, optimal escapement depends on γ. The relationship between escapement under the regime shift risk and γ is not monotonic.

When the probability of regime shift is exogenous ($\gamma = 0$), we find that escapement is lower than when there is no regime shift risk at all. In this case, and for our specific numerical example, optimal escapement is about 11% below optimal escapement when the regime shift risk is not present or is ignored. When the probability of regime shift risk is endogenous ($\gamma > 0$), escapement under the regime shift risk case initially increases as γ rises. For small values of γ, accounting for regime shift risk still implies a lower optimal escapement level. However, above a certain value of γ (0.151 in our example) this result is reversed.

Note that the distance between escapement under regime shift risk and escapement under the no-regime shift risk case may reach a peak. In our numerical example, the maximum escapement level is 1.3 times greater than escapement under the no-regime shift risk case. After this peak at $\gamma = 0.91$, the gap between optimal escapements under the no-regime shift risk and regime shift risk cases narrows again. As γ takes large values, optimal escapement under regime shift risk approaches the no-regime shift risk level from above.

Keeping fixed the probability of a regime shift, we have also investigated how escapement

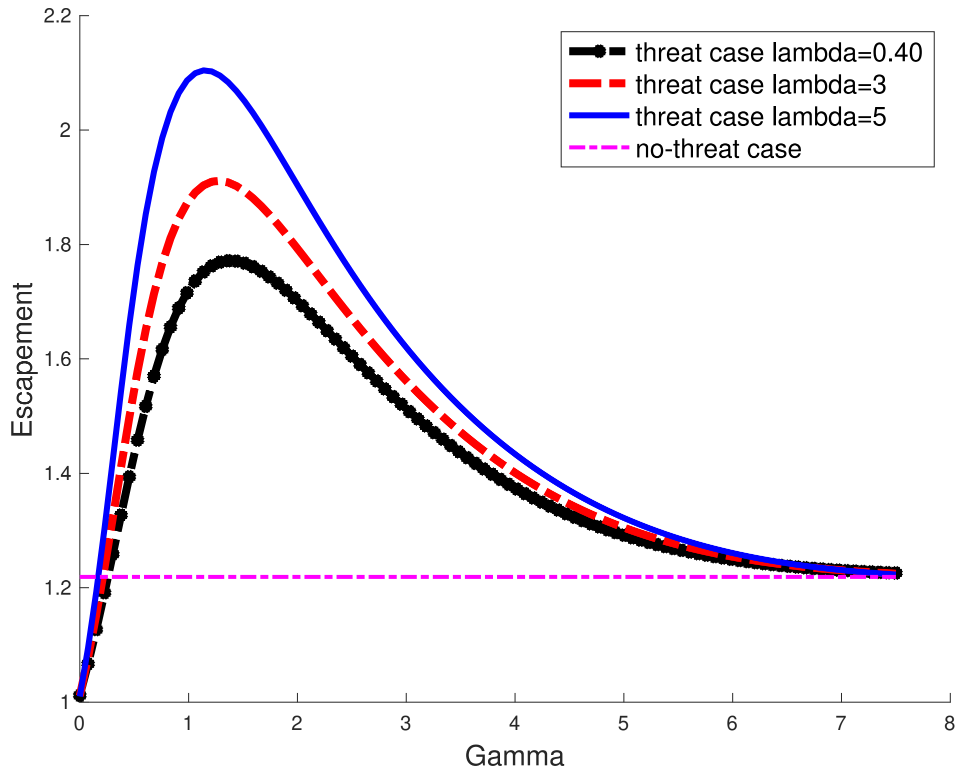


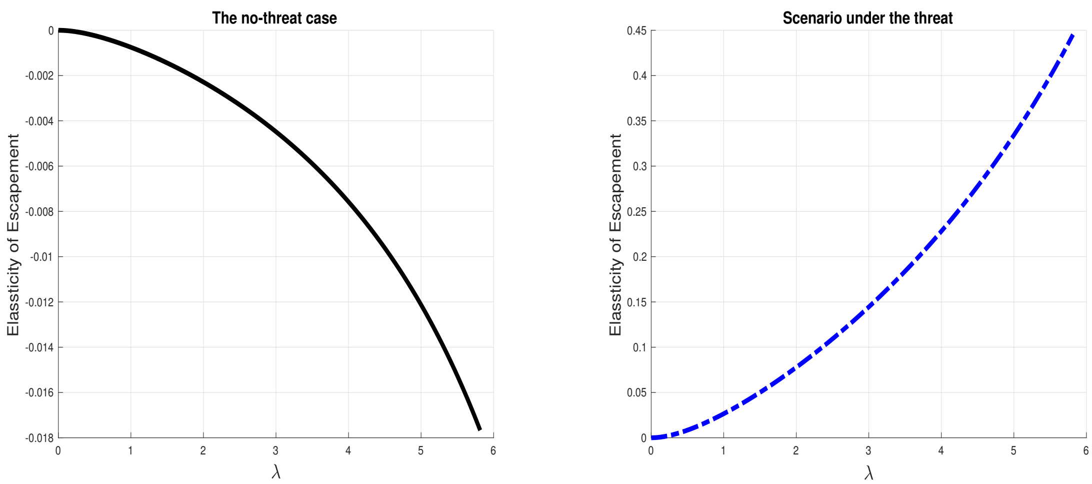
Figure 1: Escapements under the threat of regime shift and no-threat scenarios as a function of γ.

optimally changes in response to mean preserving spreads of stock growth uncertainty (increases in λ). Our baseline corresponds to cases in which stock growth is uncertain and the manager does not face the regime shift risk. As illustrated in Figure 2(a), any mean preserving spread always reduces optimal escapement under the baseline scenario.[6] Moreover, the sensitivity (elasticity) of escapement with respect to any mean preserving spreads is small.

When, in addition to stock growth uncertainty, the manager faces the regime shift risk, we have investigated implications of considering low and high values for the sensitivity of

[6]Note that the elasticity of escapement and the derivative of escapement with respect to λ have the same signs. As such, when the elasticity of escapement with respect to λ is always negative, escapement decreases in λ. However, when the elasticity of escapement with respect to λ is always positive, escapement increases in λ.

the regime shift risk with respect to escapement (i.e., γ). As shown in Appendix A.7, mean preserving spreads lower optimal escapement as long as $\theta > 1$, $0 \leq a < 1$, and γ is small or high. This analytical finding is in line with our numerical derivations. Indeed, results obtained under the baseline no-regime shift risk scenario remain qualitatively valid if the sensitivity of the regime shift risk with respect to escapement is sufficiently small or high (e.g., $0 \leq \gamma \leq 0.001$ or $\gamma \geq 8$, not shown). Specifically, in this setting, any mean preserving spread considered in our numerical analysis diminishes optimal escapement under regime shift risk, but not in a significant way.



(a) No threat of regime shift

(b) Regime shift threat, γ=0.85

Figure 2: Elasticity of optimal escapement with respect to λ when $\theta = 1.12$

When γ is neither very small nor sufficiently high (e.g., $\gamma \in [0.1, 7]$), simulations reveal that mean preserving spreads can have profound effects on optimal escapement. In particular, in contrast to the no-regime shift risk case, the manager actually increases optimal escapement under regime shift risk in response to mean preserving spreads. Moreover, Figure 2(b) illustrates that, for $\gamma = 0.85$, unlike the no-regime shift risk case, the mean preserving spread associated with any substantial increase in λ, significantly increases the optimal escapement level under regime shift risk.

Holding the risk of regime shift constant, changes in the discount rate further unveil harvest responses to uncertainties. The results depicted in Figure 3 suggest that both escapement under the regime shift risk and stock growth uncertainty and escapement under the baseline scenario decline as the discount rate is increased. Our sensitivity analysis suggests that this result remains valid for a wide array of values for υ, α, γ, and $p > 0$ in the relevant range. However, relative to the baseline scenario, escapement under stock growth uncertainty and regime shift risk declines faster (see Figure 3) when the sensitivity of ρ with respect to escapement is sufficiently high (e.g., $\gamma = 2, ..., 5$). This result is reversed when γ is sufficiently small (e.g., $\gamma = 0, ..., 0.1$).

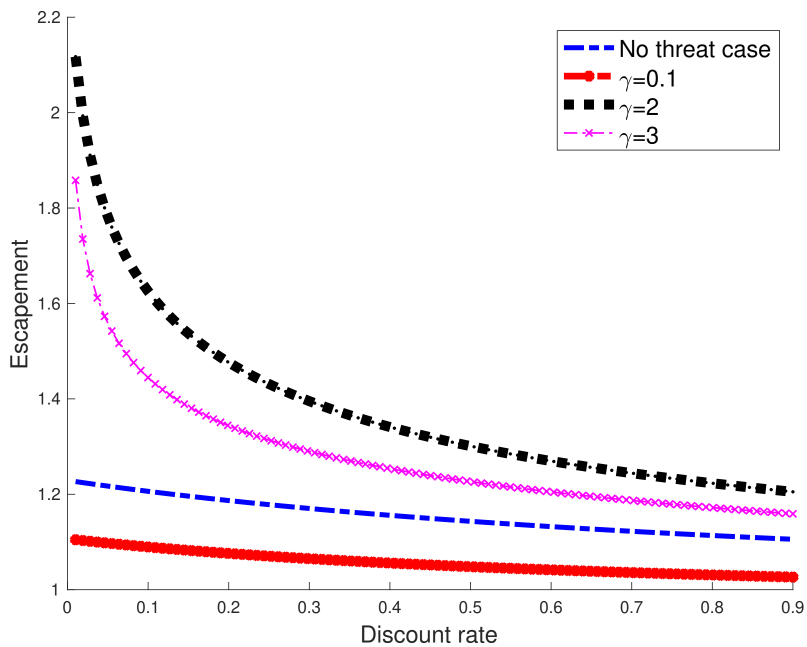
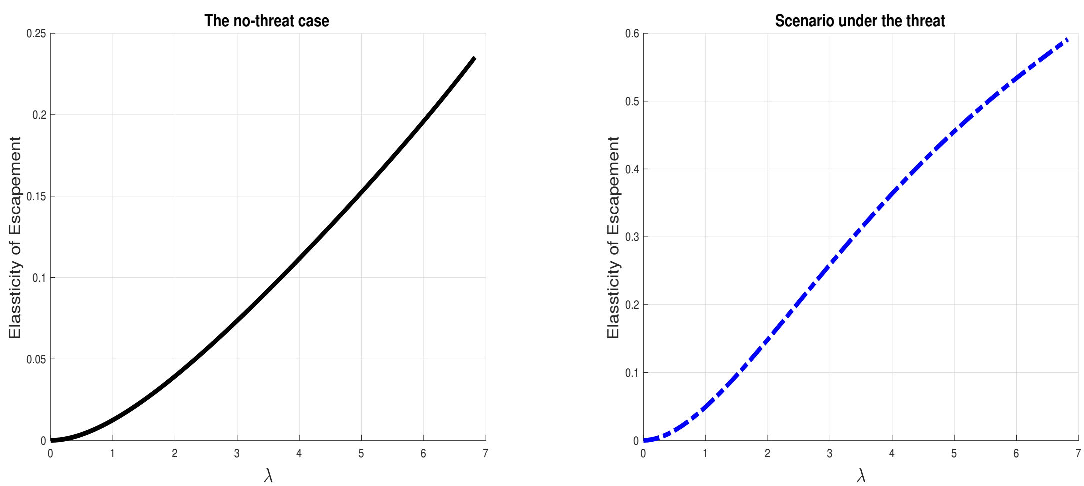


Figure 3: Escapements under the threat of regime shift and no-threat scenarios as a function of the discount rate.

Finally, holding the probability of regime shift constant, changing the marginal harvesting cost function parameter to $\theta = 0.12$ helps further unveil harvest responses to uncertainties. When γ is neither high nor small, simulations show that stock growth uncertainty can considerably change optimal escapement, but only in the presence of the threat. For example, as depicted in Figure 4 (right panel) for $\gamma = 0.85$, relative to the deterministic growth case (i.e., $\lambda = 0$), the level of stock growth uncertainty associated with $\lambda = 6.8251$ raises the optimal escapement level under regime shift risk by about 67.33%. However, relative to the deterministic growth case, such a level of uncertainty raises optimal escapement under the baseline scenario by 17.1% only (note the different scales of the y-axis of Figure 4). Moreover, in response to any mean preserving spread (i.e., any increase in λ), the manager optimally increases the escapement levels under both the regime shift risk and no-regime shift risk cases. This result accords with Proposition 5 and remains valid under the baseline scenario. Our sensitivity analysis reveals that this latter result is robust to changes in γ, v, α, and $p > 0$.



(a) No threat of regime shift

(b) Regime shift threat, γ=0.85

Figure 4: Elasticity of optimal escapement as a function of λ when $\theta = 0.12$

6 Conclusion

In this paper, we have examined how a renewable resource manager optimally responds to uncertainty. Such uncertainty results from two specific channels. First, resource growth is subject to natural uncertainty due to stochastic changes in ecological conditions. Second, the manager faces the threat of a possible future regime shift. We have designed a simple bio-economic framework to illustrate harvest incentives under such conditions.

We find that changes in the distribution of stock growth uncertainty affect optimal extraction through two channels. The 'Direct Effect' prevails and captures the fact that natural uncertainty, by affecting stock growth, alters the structure of harvesting costs. The 'Risk Effect' illustrates the idea that the manager has incentives to diminish current extraction. This second channel exists because the likelihood of regime shift increases as extraction intensifies. We find that in response to random changes in biological growth, the manager may increase, reduce, or not change her current extraction. Importantly, as portrayed in Table 2, we delineate conditions under which new behavioral responses to stock growth uncertainty emerge relative to prior economic papers (e.g., Reed, 1979; Costello and Polasky, 2008) that do not account for the possibility of regime shifts.

Table 2: Overview of key results, optimal escapement, denoted by y^*, responses to mean preserving spreads of stock growth uncertainty for different combinations of regime shift risk and structures of harvesting cost.

<table>
<tr><th></th><th>$xc(x)$ linear</th><th>$xc(x)$ convex</th><th>$xc(x)$ concave</th></tr>
<tr><td>no regime shift risk (ρ=0)</td><td>no effect on y^*</td><td>decrease y^*</td><td rowspan="3">always increases y^*
(Prop 3)</td></tr>
<tr><td>exogenous risk (ρ >0, ρ'=0)</td><td>no effect on y^*
(Prop 4-i)</td><td rowspan="2">ambigous; increase y^*
when cond (8) holds
(Prop 5)</td></tr>
<tr><td>endogenous risk (ρ >0, ρ' ≠0)</td><td>no effect on y^* when $c(x)$ constant;
increase y^* when $c(x)$ not constant
(Prop 4-ii)</td></tr>
</table>

Our results may shed new light on how a sole owner responds to uncertainty. For instance, relying on numerical simulations, prior economic papers (e.g., Clark and Kirkwood, 1986;

Sethi et al., 2005; Kapaun and Quaas, 2013) have examined how a risk neutral renewable resource manager adapts to stock growth uncertainty, but in a setting where there is no possibility of regime shift. They find that the manager does not significantly change optimal escapement in response to mean preserving spreads. Our findings, however, suggest that if the manager faces regime shift risk in addition to stock growth uncertainty (for example, fearing expropriation, an oil spill, or a biological collapse of the resource) such conventional wisdom does not necessarily hold.

The ecosystem considered in this paper is prone to an irreversible closure of the fishery. Baggio and Fackler (2016) address the optimal management of a fishery subject to two sources of uncertainty that affect the resource biological growth. Specifically, they consider random shocks along with the possibility of future reversible regime shift that entails a drop in biological growth. They find that optimal escapement in the low productivity regime is smaller relative to the high productivity regime when the probability of regime shift is exogenous.

While more work is needed, it seems fair to speculate that (i) the consequence of the regime shift (collapse or loss in renewability) is not decisive for whether the management response is cautious or aggressive, and (ii) that the irreversible set-up analyzed here is the limiting case of a setup with a reversible regime shift. While the irreversibility of the regime shift allowed a particularly tractable model formulation that enabled us to present analytical solutions, studying reversible regime shifts is a promising avenue for future work because it would naturally open to study issues of experimentation and learning about the tipping point at which the regime shift occurs (Groeneveld et al., 2014; Diekert, 2017).

Furthermore, our model may serve as a starting point for empirical case-studies or more detailed theoretical work that acknowledges that socio-ecological systems are complex adaptive systems (Levin, 2003; Crepin et al., 2011). The possibility of regime shift importantly shapes in situ resource stock (e.g., wild fish stocks) dynamics. Moreover, stock growth of such resources is often subject to random chocks triggered by sudden changes in environmental conditions. Our results would guide policy aimed at sustainably managing such socio-ecological systems (Crepin et al., 2012). Extensions of this paper could broaden our model applicability. For example, we have concentrated on a risk neutral sole owner case, assuming a deterministic resource price, and a stock independent environmental shock. In some contexts, the resource price may adjust to random or systemic changes in market conditions. A fishery (e.g., high sea) could also be exploited strategically by risk neutral and risk averse agents. The environmental shock may be stock-dependent because the resource stock may be more susceptible to changes in environmental conditions (e.g., drought) if it is near a minimum viable population. Incorporating these features into our model represents an important avenue for future research.

Appendix

A Mathematical derivations

A.1 Proof of Proposition 1

Recall that by assumption, the planning horizon is finite. In this context $V_{T+1}(X_{T+1}) = 0$ because T represents the last period and the manager does not value the resource stock in period $T+1$. This result combined with (3) reveals that $y_T^* = X_\infty$. Evaluating the maximization problem (3) at the optimum, we derive

$$V_T(X_T) = p(X_T - y_T^*) - \int_{y_T^*}^{X_T} c(s)ds \quad \text{and} \quad V_T'(X_T) = p - c(X_T). \tag{9}$$

Substituting (9) into (4) for $t = T-1$, we find that period $T-1$ optimal escapement, denoted by y_{T-1}^* is the solution to

$$\begin{aligned} p - c(y_{T-1}) = {} & \beta(1-\rho(y_{T-1}))\mathbb{E}\big[Z_{T-1}g'(y_{T-1}))\,(p - c(Z_{T-1}g(y_{T-1})))\,\big] \\ & - \beta\rho'(y_{T-1})\mathbb{E}\big[V_T\,(Z_{T-1}g(y_{T-1})))\,\big]. \end{aligned} \tag{10}$$

This formula shows that y_{T-1}^* does not depend on X_{T-1}. Evaluating the maximization problem in (3) at the optimum for $t = T-1$, we derive

$$\begin{aligned} V_{T-1}(X_{T-1}) = {} & p(X_{T-1} - y_{T-1}^*) - \int_{y_{T-1}^*}^{X_{T-1}} c(s)ds \\ & + \beta(1-\rho(y_{T-1}^*))\mathbb{E}\left[V_T(Z_{T-1}g(y_{T-1}^*))\right] \quad \text{and} \quad V_{T-1}'(X_{T-1}) = p - c(X_{T-1}). \end{aligned} \tag{11}$$

Substituting (11) into (4) for $t = T-2$, we find that period $T-2$ optimal escapement, denoted by y^*_{T-2} is the solution to

$$p - c(y_{T-2}) = \beta(1-\rho(y_{T-2}))\mathbb{E}\big[Z_{T-2}g'(y_{T-2}))\,(p - c(Z_{T-2}g(y_{T-2})))\big] \\ - \beta\rho'(y_{T-2})\mathbb{E}\big[V_{T-1}\,(Z_{T-2}g(y_{T-2})))\big]. \tag{12}$$

This expression reveals that y^*_{T-2} does not depend on X_{T-2}.

(i) Assume that $\rho(y)$ strictly declines as y is increased. In this case, the second right-hand side terms in (10) and (12) differ. As a result, y^*_{T-2}, y^*_{T-1}, and y^*_T are not identical. Using the above approach, it can be generally shown that optimal escapement differs across all periods.

(ii) Assume that $\rho(y)$ does not change as y is increased. In this case, the second right-hand side term in (4) vanishes. Moreover, the above derivations reveal that $V'_t(X_t) = p - c(X_t)$ for $t = 0, 1, 2, ..., T$. These last three results imply that for $t = 0, 1, 2, ...T-1$, y^*_t is the solution to

$$p - c(y_t) = \beta(1-\rho(y_t))\mathbb{E}\big[Z_t g'(y_t))\,(p - c(Z_t g(y_t)))\big]. \tag{13}$$

Hence, y^*_t does not change across periods $t = 0, 1, 2, ...T-1$.

A.2 Proof for the time-dependence issue

Here, our goal is to proof that optimal escapement can be time dependent prior the last period when $\rho(y)$ is neither strictly decreasing everywhere nor constant. Since by assumption, $\rho(y)$ is weakly decreasing in y, only three scenarios are possible. (S1) $\rho(y)$ is strictly decreasing in y; (S2) $\rho(y)$ is constant; and (S3) $\rho(y)$ is strictly decreasing in y over an interval and constant over another interval.

To shed light on scenario (S3), assume that the escapement space can be divided into two disjoint intervals. In the first interval, say (a_1, a_2), $\rho(y)$ is strictly decreasing in y. In the second interval, say $(a_3; a_4)$, $\rho(y)$ is constant. Denoting by y_t^*, $t = 0, 1, 2, ..., T$ period-t optimal escapement, it can be helpful to distinguish three cases.

-Case 1: $y_0^*, ..., y_{T-1}^* \in (a_1, a_2)$. Since this setting is similar to the one in which $\rho(y)$ strictly decreasing in y over the whole range of y, the result (i) of Proposition 1 holds in this case.

-Case 2: $y_0^*, ..., y_{T-1}^* \in (a_3, a_4)$. Since this setting is similar to the one in which $\rho(y)$ constant over the whole range of y, the result (ii) of Proposition 1 is valid in this case.

-Case 3: Only some of values of y_t^*, $t = 0, 1, 2, ..., T-1$ fall within the interval (a_1, a_2) and the others fall within the interval (a_3, a_4). In this setting, if $y_t^* \in (a_1, a_2)$ and $y_s^* \in (a_3, a_4)$, then we necessarily have $y_t^* \neq y_s^*$ because by assumption, (a_1, a_2) and (a_3, a_4) are disjoint intervals. This result suggests that optimal escapement is time-dependent prior the last period. Therefore, the conventional wisdom highlighted in the paragraph right below Proposition 1 does not hold in this case.

A.3 Proof of Proposition 2

Evaluating (3) at the optimum, we get

$$V(X_t) = p(X_t - y^*) - \int_{y^*}^{X_t} c(s)ds + \beta(1 - \rho(y^*))\mathbb{E}\left[V(X_{t+1})\right]. \tag{14}$$

Here, we restrict our attention to scenarios in which y^* does not depend on X_t. In this context, (1) reveals that X_{t+1} depends on y^* and is independent of X_t. Therefore, the third right-hand side term in (14) does not depend on X_t. Using this result, we differentiate both sides of (14), which leads to

$$V'(X_t) = p - c(X_t). \tag{15}$$

Since this expression holds for an arbitrary value of X_t, it implies that

$$V'(X_{t+1}) = p - c(X_{t+1}) = p - c(Z_t g(y_t)). \tag{16}$$

Integrating condition (15) gives rise to

$$V(X) = p(X - y^*) - \int_{y^*}^{X} c(s)ds + \xi, \quad \text{for all} \quad X, \tag{17}$$

where ξ represents a constant of integration. To determine ξ, we proceed as follows. Evaluating Conditions (14) and (17) at $X_t = y^*$, we get $V(y^*) = \xi = \beta(1-\rho(y^*))\mathbb{E}(V(X_{t+1}))$. Evaluating the function in (17) at X_{t+1}, we derive

$$\xi = \beta(1-\rho(y^*))\mathbb{E}(V(X_{t+1})) = \beta(1-\rho(y^*)) \times \mathbb{E}\left[p(X_{t+1} - y^*) - \int_{y^*}^{X_{t+1}} c(s)ds + \xi\right].$$

Solving this equation with respect to ξ, and using (1), we find that

$$\xi = \frac{\beta(1-\rho(y^*))}{1-\beta(1-\rho(y^*))} \times \mathbb{E}\left[p(Z_t g(y^*) - y^*) - \int_{y^*}^{Z_t g(y^*)} c(s)ds\right].$$

Combining this result with the fact that $\xi = \beta(1-\rho(y^*))\mathbb{E}(V(X_{t+1}))$, we derive

$$\mathbb{E}\{V(X_{t+1})\} = \frac{1}{1-\beta(1-\rho(y^*))} \times \mathbb{E}\left[p(Z_t g(y^*) - y^*) - \int_{y^*}^{Z_t g(y^*)} c(s)ds\right].$$

Substituting this finding and the formula in (16) into (4), the result follows.

A.4 Proof of Proposition 3

Holding escapement constant, it can be helpful to define the function

$$R(Z_t) = \beta(1-\rho(y))\{Z_t g'(y))(p - c(Z_t g(y)))\} \\ - \frac{\beta\rho'(y)}{1-\beta(1-\rho(y))} \times \{p(Z_t g(y) - y) - \int_y^{Z_t g(y)} c(s)ds\}. \tag{18}$$

Differentiating both sides of this equality with respect to Z_t, we obtain

$$R'(Z_t) = \beta(1-\rho(y))\{g'(y))(p - \ell'(Z_t))\} - \frac{\beta\rho'(y)}{1-\beta(1-\rho(y))} \times \{pg(y) - g(y)c(Z_t g(y))\},$$

where $\ell(Z_t) = Z_t c(Z_t g(y))$. Differentiating this formula with respect to Z_t, we get

$$R''(Z_t) = -\beta(1-\rho(y))g'(y)\ell''(Z_t)) + \frac{\beta\rho'(y)}{1-\beta(1-\rho(y))} \times (g(y))^2 c'(Z_t g(y)).$$

Since by assumption $xc(x)$ is concave, $\ell''(Z_t) < 0$ and the first right-hand side term of this expression is positive. It second right-hand side term is also positive because $\rho'(y) \leq 0$ and $c' < 0$. These results imply that $R''(Z_t) > 0$ for all Z_t. Therefore, $R(Z_t)$ is convex in Z_t. Since $\mathbb{E}\{R(Z_t)\}$ represents the right-hand side of (5), this result implies that a mean preserving spread increases the right-hand side of (5), holding fixed escapement. It is also important to note that the left-hand side of (5) increases in y. These findings show that a mean preserving spread increases optimal escapement whenever $xc(x)$ is concave.

A.5 Proof of Proposition 4

(i) Since $xc(x)$ is linear, the right-hand side of (5) does not depend on the distribution of Z_t when $\rho' = 0$ or $c' = 0$. In this context, any mean preserving spreads do not affect optimal escapement because the left hand side of (5) does not depend on Z_t.

(*ii*) Here we assume that $xc(x)$ is linear, $\rho' \neq 0$, and $c' \neq 0$. In this context, the first right-hand side of (5) does not depend on the distribution of Z_t. The second right-hand side of (5) increases as a result of a mean preserving spread because $-\frac{\beta\rho'(y)}{1-\beta(1-\rho(y))} \times [p(Z_t g(y) - y) - \int_y^{Z_t g(y)} c(s)ds]$ is convex in Z_t. Therefore, a mean preserving spread increases optimal escapement.

A.6 Proof of Proposition 5

Holding escapement constant, $R(Z_t)$ is the function defined in (18).

$$R''(Z_t) = \beta(1-\rho(y))g'(y)[-2g(y)c'(Z_t g(y)) - Z_t(g(y))^2 c''(Z_t g(y))] \\ - \frac{\beta\rho'(y)}{1-\beta(1-\rho(y))} \times \{-g(y)c'(Z_t g(y))\}.$$

Using $c(.)$ defined in (7), this expression simplifies to

$$R''(Z_t) = \beta A\theta \times \frac{g(y)^{-\theta}}{Z_t^{\theta+1}} \times [-(\theta-1)(1-\rho(y))g'(y) - \frac{\rho'(y)}{1-\beta(1-\rho(y))}]. \quad (19)$$

$R(.)$ is convex in Z_t if and only if the bracketed term is positive. Moreover, $\mathbb{E}\{R(Z_t)\}$ is equal to the right-hand side of (5) and y^* is the solution to (5). These results reveal that if the bracketed term in (19) evaluated at $y = y^*$ is positive, a mean preserving spread of Z_t optimally raises current escapement. That is, if y^* satisfies condition (8), then a mean preserving spread of Z_t optimally raises current escapement.

A.7 Proof that "mean preserving spreads lower y^* if $\theta > 1$, $0 \leq a < 1$, and γ is small or high"

For scenarios in which $\rho(y) = ae^{-\gamma y}$ and $g(y) = \frac{y/\alpha}{1+(y/v)}$, condition (8) simplifies to

$$a\gamma\alpha e^{-\gamma y^*} > \frac{(\theta - 1)}{(1 + (y^*/v))^2}[1 - ae^{-\gamma y^*}][1 - \beta(1 - ae^{-\gamma y^*})].$$

Notice that this inequality does not hold when $\theta > 1$, $0 \leq a < 1$, and γ is small or high. Therefore, by Proposition 5, mean preserving spreads reduce optimal escapement when $\theta > 1$, $0 \leq a < 1$, and γ is small or high.

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