Universität
Augsburg
University

Evidence on the resource scarcity risk of the German Energiewende

New frameworks to measure scarcity and (time-varying) cross-commodity interdependencies

Dissertation
zur Erlangung des akademischen Grades
Dr.-Ing.
eingereicht an der
Mathematisch-Naturwissenschaftlich-Technischen Fakultät
der Universität Augsburg

vorgelegt von

Amelie Schischke

Augsburg, July 2023

Erstgutachter: Prof. Dr. Andreas Rathgeber
Zweitgutachter: Prof. Dr. Yarema Okhrin

Termin der mündlichen Prüfung: 25.09.2023

Abstract

To keep the climate change under control, 196 parties signed the legally binding, international treaty on climate change, the Paris Agreement (2015), in which they committed themselves to limit global warming. A crucial part on the way to CO_2 neutrality is the decarbonization of the energy sector, whereby renewable energy technologies like wind power and photovoltaic systems, associated storage technologies as well as building renovations are key elements for the energy transition. However, the built-up of these technologies requires huge amounts of raw materials, see Valero et al. (2018), which is why the question arises whether enough resources are available.

While from a geological point of view, enough mineral raw materials seem to be available, see Federal Ministry for Economic Affairs and Climate Action (2022a), the increased resource requirements can lead to (short-term) shortages, and therefore to price peaks. Hereby, this thesis analyzes the resource scarcity risks of four potential transformation pathways of the German Energiewende. Therefore, we propose and apply a new framework to assess the resource scarcity risk under the consideration of the substitutability of commodities, the actual future required resource amounts of the project as well as the commodity market structure, using new commodity market models, reflecting the impact of fundamentals on - as well as the spillover effects between - commodity prices.

In the empirical application, we first apply the commodity market models on the industrial metals markets. The results indicate the framework is able to showcase the strong co-movement in commodity prices as well as the simultaneous impact of the economy on all commodity markets. Moreover, various spillover effects of commodity-specific supply and demand, both within and across commodity markets, as well as their impact on prices, underline the importance to account for fundamentals, but also to jointly model commodity markets.

Subsequently, we incorporate the commodity market models in the scarcity risk assessment framework to analyze the resource requirements of four transformation pathways for the German Energiewende. Hereby, the results indicate on commodity level cobalt, indium and nickel, followed by copper and lithium, mainly allocated to energy storage, solar photovoltaics (PV) technologies and wind parks, bear the highest scarcity risks, and therefore, will be the key commodities for the German Energiewende. The comparison of the four transformation paths, suggests the path which models the transition of the German energy system with full support by the society, shows the lowest scarcity risks, as an active support of the German population for the energy transition significantly reduces the required amounts of raw materials and therefore the scarcity risks. Various robustness analyses underline the main finding that a reduced energy demand combined with a resource-optimal energy system decreases the resource scarcity.

Contents

List of Figures

List of Tables

List of Abbreviations

List of Variables

$\mathbf{A}$	Parameter matrix used for the transformation of the (MS-)GVAR model
$\tilde{\mathbf{A}}$	Regime-dependent parameter matrix for the MS-GVAR model
$\mathbf{a}$	Intercept vector in the (MS)(G)VAR model
$\tilde{\mathbf{a}}$	Regime-dependent intercept vector in the MS-GVAR model
$\mathbf{a}_{exog}$	Intercept vector in the (MS-)VAR model of the exogenous variables
$adj_k^{\text{MS-GVAR}}$	Adjustment term of the mean squared prediction error (MSPE) of the unrestricted model in the test of Clark and West (2007)
$\boldsymbol{\alpha}$	Parameter vector for the endogenous variables in the regression equation of the measurement equation of the MS-VAR model
$\mathbf{B}$	Matrix of regression parameters in the measurement equation of the state space representation of the MS-VAR model
$\mathbf{b}$	Intercept vector in the reduced form of the (MS-)GVAR model
β	Parameters in the logistic regression model
$\hat{\beta}$	Estimator for the parameters in the logistic regression model
$\mathbf{C}$	Common weight matrix
$CoVaR$	Conditional value at risk
$c_{T,M,P,spec}$	Sequence indexed by the sample size, penalizing for the inclusion of redundant states and lags in the model selection for MS-VAR models
$cred$	Index for loans in a loan portfolio, $cred \in pf$
$\mathbf{D}$	Demand weight matrix
d	Parameter for the Hannah Quinn criterion (HQC^{MS}) of the penalty term $c_{T,M,P,spec}$ in the model selection for the MS-VAR model
$\mathbf{demand}$	Commodity-specific demand variable
$demand_{i,t}^*$	Commodity-specific, external demand variable
Δ	Convergence criteria for the EM algorithm
$\boldsymbol{\delta}$	Shock hitting the system for the calculation of GIRFs
$\boldsymbol{\delta}_{exog}$	Shock hitting the exogenous variables for the calculation of GIRFs
dim	Dimension of estimated parameters in the MS-VAR model
$\mathbf{e}$	Vector of exogenous variables in the (MS-)(G)VAR model
$\mathbf{e}^{n_{boot}}$	Bootstrap sample of exogenous variables
$\tilde{\mathbf{e}}$	Average of realizations of the exogenous variables for the Monte Carlo integration to calculate generalized impulse response functions
$\mathbf{En}_i$	Vector of lagged endogenous variables in the measurement equation of the MS-VAR model
$\mathbf{en}$	Vector of endogenous variables in the measurement equation of the MS-VAR model
$\mathbf{Ex}_i$	Vector of lagged exogenous variables in the measurement equation of the MS-VAR model
$\mathbf{ex}$	Vector of exogenous variables in the measurement equation of the MS-VAR model

EAD	Exposure at default of a loan
EAS	Exposure at scarcity in the risk assessment framework, representing the scaled resource amounts for a project
EL	Expected loss of a portfolio of loans
ES	Expected loss due to scarcity, the resulting scarcity measure on commodity as well as on project level of the risk assessment framework
$\boldsymbol{\eta}$	Vector of densities of the vector of endogenous variables conditional on the regime probabilities
$\boldsymbol{\varepsilon}$	Error term in the (MS-)(G)VAR model
$\hat{\boldsymbol{\varepsilon}}$	Error term based on the estimated (MS-)(G)VAR model
$\boldsymbol{\varepsilon}_{exog}$	Error term in the (MS-)VAR model of the exogenous variables
$\hat{\boldsymbol{\varepsilon}}_{exog}$	Error term of the estimated (MS-)VAR model of the exogenous variables
$\boldsymbol{\varepsilon}^{n_{boot}}$	Bootstrap sample of residuals
$\boldsymbol{\varepsilon}_{exog}^{n_{boot}}$	Bootstrap sample of residuals of the exogenous variables
ε	Error term in the logistic regression model
$\mathbf{F}$	Transpose of the transition probability matrix for the transition equation of the state space representation of the MS-VAR model, $\mathbf{F}_i = \mathbf{P}_i'$
$\mathbf{G}$	Parameter matrix in the (final) (MS-)GVAR model
GFEVD	Generalized forecast error variance decomposition for the GVAR model
GI	Generalized impulse response function for the (MS-)(G)VAR model
$\boldsymbol{\gamma}$	Vector of structural parameters in the MS-VAR model
$\hat{\boldsymbol{\gamma}}$	Estimator for the vector of structural parameters in the MS-VAR model
$\mathbf{H}$	Parameter matrix in the reduced form of the (MS-)GVAR model
HHI	Hirschman-Herfindal index
HQC^{MS}	Criterion of Hannan and Quinn (1979) in the model selection of the MS-VAR model
h	Index for the industry for the calculation of the demand weight matrix, $h \in \{\text{Automotive/Transportation, Chemistry/Pharmaceutics, Electrics, Construction, Mechanical Engineering}\}$
I	End of in-sample period
$\mathbb{I}$	Indicator function
$\mathbf{I}$	Identity matrix
$\boldsymbol{\iota}_m$	m-th column of the identity matrix
i	Index for the commodities, $i \in \{1, 2, \ldots, N\}$
$\tilde{\imath}$	Additional index for the commodities, $\tilde{\imath} \in \{1, 2, \ldots, N\}$
$ind_{h,i}$	Proportion of consumption of commodity i in industry h
IC	Criterion function in the model selection of the MS-VAR model
J	Number of observations in the out-of-sample period
j	Iteration index in the EM algorithm
K	Length of the global vector of all commodity-specific variables in the (MS-)GVAR model, $K = \sum_{i=1}^{N} K_i$
k	Index of the variables in the global vector of all commodity-specific variables, $\mathbf{x}_t$, in the (MS-)GVAR model, $k \in \{1, 2, \ldots, K\}$
$\tilde{k}$	Additional index of the variables in the global vector of all commodity-specific variables, $\mathbf{x}_t$, in the (MS-)GVAR model, $\tilde{k} \in \{1, 2, \ldots, K\}$
K_i	Number of the commodity-specific endogenous variables in the (MS-)(G)VAR model
k_i	Index of the commodity-specific endogenous variables in the (MS-)(G)VAR model, $k_i \in \{1, 2, \ldots, K_i\}$
K_i^*	Number of the commodity-specific external variables in the (MS-)(G)VAR model

K_{endog}	Number of the endogenous variables in the (MS-)VAR model
k_{endog}	Index of the endogenous variables in the (MS-)VAR model, $k_{endog} \in \{1, 2, \ldots, K_{endog}\}$
K_{exog}	Number of the exogenous variables in the (MS-)(G)VAR model
k_{exog}	Index of the exogenous variables in the (MS-)(G)VAR model, $k_{exog} \in \{1, 2, \ldots, K_{exog}\}$
$\tilde{k}_{exog}$	Additional index of the exogenous variables in the (MS-)(G)VAR model, $\tilde{k}_{exog} \in \{1, 2, \ldots, K_{exog}\}$
K'	Number of the exogenous variables (including weakly exogenous variables) for the model selection for the MS-VAR models
K_i	Number of the commodity-specific price determinants in the logistic regression model
k_i	Index for the commodity-specific price determinants in the logistic regression model, $\mathsf{k}_i \in \{1, 2, \ldots, \mathsf{K}_i\}$
$\tilde{\mathsf{k}}$	Additional index for the commodity-specific price determinants in the logistic regression model, $\tilde{\mathsf{k}} \in \{1, 2, \ldots, \mathsf{K}_i\}$
$\boldsymbol{\kappa}, \kappa$	Lagrange multiplier
L	Likelihood function
L^*	Constrained likelihood function
LGD	Loss given default of a loan
LGS	Loss given scarcity in the risk assessment framework, representing the inverse of the substitutability of commodities
ℓ	Log-likelihood function in the EM algorithm for MS-VAR models
l	Index of the states in the MS-VAR model, $l \in \{1, 2, \ldots, M\}$
$\boldsymbol{\Lambda}$	Matrix of parameters of lagged coefficients for external variables in the (MS-)(G)VAR model
$\boldsymbol{\lambda}$	Vector of parameters to be estimated for the MS-VAR model, $\boldsymbol{\lambda} = (\boldsymbol{\theta}', \boldsymbol{\rho}', \boldsymbol{\xi}')'$
$\hat{\boldsymbol{\lambda}}$	Maximum likelihood estimator
M	Number of states in the MS-VAR model
$\hat{M}$	Optimal number of states in the MS-VAR model according to the model selection
M_{exog}	Number of states in the MS-VAR model of the exogenous variables
M_{max}	Maximum number of states in the model selection for MS-VAR models
m	Index of the states in the MS-VAR models, $m \in \{1, 2, \ldots, M\}$
$MSPE$	Mean squared prediction error
μ	Mean of a variable
N	Number of commodities
N_{FEVD}	Horizon of the calculation of the GFEVDs
N_{IRF}	Horizon of the calculation of the GIRFs
n	Index of the horizon in the GIRFs and GFEVDs, $n \in \{0, 1, \ldots, N_{IRF}\}$ and $n \in \{0, 1, \ldots, N_{FEVD}\}$, respectively
$\tilde{n}$	Index used in the calculation for the GFEVD of the GVAR model
N_{pred}	Forecast horizon in the prediction of the (MS-)GVAR model
N_{boot}	Number of bootstrap samples
n_{boot}	Index of bootstrap samples, $n_{boot} \in \{1, 2, \ldots, N_{boot}\}$
N_{hist}	Number of draws of the history for the Monte Carlo integration for the calculation of the GIRFs for the MS-(G)VAR model
n_{hist}	Index of draws of the history for the Monte Carlo integration for the calculation of the GIRFs for the MS-(G)VAR model, $n_{hist} \in \{1, 2, \ldots, N_{hist}\}$
N_{shock}	Number of draws of the shocks for the Monte Carlo integration for the calculation of the GIRFs for the MS-(G)VAR model

n_{shock}	Index of draws of the shocks for the Monte Carlo integration for the calculation of the GIRFs for the MS-(G)VAR model, $n_{shock} \in \{1, 2, \ldots, N_{shock}\}$
$\boldsymbol{\nu}$	Intercept parameter vector in the regression equation of the measurement equation of the MS-VAR model
O	Number of the non-negative parameters to calculate the convergence criteria of the EM algorithm
o	Index for the non-negative parameters to calculate the convergence criteria of the EM algorithm, $o \in \{1, 2, \ldots, O\}$
$\boldsymbol{\Omega}$	Covariance matrix of the error term in the regression equation of the measurement equation of the state space representation of the MS-VAR model
Ω	Information set used for the calculation of GIRFs
ω	Drawn history in the Monte Carlo integration for the calculation of GIRFs for MS-(G)VAR models
$\mathbf{P}$	Transition probability matrix
$p_{i,lm}$	Transition probabilities
$\hat{p}_{i,lm}$	Estimator for the transition probabilities
P	Lag length of the endogenous variables in the (MS-)(G)VAR model
p	Index of the lag length, $p \in \{1, 2, \ldots, P\}$
$\hat{P}$	Optimal lag length of the endogenous variables in the (MS-)(G)VAR model according to the model selection
P'	Lag length of exogenous variables (including weakly exogenous variables) for the model selection for MS-VAR models
P_{exog}	Lag length of the exogenous variables in the (MS-)(G)VAR model
$\tilde{P}_{exog}$	Lag length of the exogenous variables in the (MS-)VAR model of the exogenous variables
$\tilde{p}_{exog}$	Index of the lag length for the exogenous variables in the (MS-)VAR model of the exogenous variables, $\tilde{p}_{exog} \in \{1, 2, \ldots, \tilde{P}_{exog}\}$
P_{max}	Maximal lag length in the model selection for MS-VAR models
PD	Probability of default of a loan
PS	Probability of scarcity of a commodity in the risk assessment framework
pf	Loan portfolio
$\mathbf{price}$	Price of a commodity
$price^*_{i,t}$	Price of external commodities
$proj$	Commodity demanding project
$prod$	Production of a commodity
$\boldsymbol{\Phi}$	Matrix of parameters of lagged coefficients for endogenous variables in the (MS-)(G)VAR model
$\boldsymbol{\Phi}_{exog}$	Matrix of parameters of lagged coefficients for exogenous variables in the (MS-)VAR model of the exogenous variables
$\boldsymbol{\Psi}$	Matrix of parameters of lagged coefficients for exogenous variables in the (MS-)(G)VAR model
$\tilde{\boldsymbol{\Psi}}$	Regime-dependent matrix of parameters of lagged coefficients for exogenous variables in the (MS-)(G)VAR model
$\boldsymbol{\psi}$	Parameter vector for the exogenous variables in the regression equation of the measurement equation of the MS-VAR model
q	q-th quantile
$quant$	Quantity of a commodity required in a project $proj$
R	Number of production countries used in the definition of the Herfindal-Hirschman Index (HHI)

r	Index of production countries, $r \in \{1, 2, \ldots, R\}$
$\boldsymbol{\rho}_i$	Vector of the transition probabilities, $\boldsymbol{\rho}_i = vec\,(\mathbf{P_i})$
$\mathbf{S}$	Supply weight matrix
S	Regime-constellation in the MS-GVAR model
s	Regime, prevailing in a certain market at a certain point in time, in the commodity-specific MS-VAR model, $s_{i,t} \in \{1, 2, \ldots, M\}$
s_{exog}	Regime, prevailing at a certain point in time, in the MS-VAR model of the exogenous variables, $s_{exog,t} \in \{1, 2, \ldots, M_{exog}\}$
SR	Substitutability rate in the risk assessment framework, representing the substitutability of commodities
SIC^{MS}	Criterion of Schwarz (1978) in the model selection of the MS-VAR model
$Spec$	List of specifications of MS-VAR models, used for the model selection
$spec$	Index of specifications of MS-VAR models, $spec \in Spec$
$\hat{spec}$	Optimal specification of the MS-VAR model, according to the model selection
$\mathfrak{s}$	Selection vector
$scarce$	Binary variable, indicating whether a commodity is scarce
sgn	Signum function
$\mathbf{supply}$	Commodity-specific supply variable
$supply^*_{i,t}$	Commodity-specific, external supply variable
$\overline{supply}$	Average world production
$\boldsymbol{\Sigma}$	Covariance matrix in the (MS-)(G)VAR model
$\hat{\boldsymbol{\Sigma}}$	Estimator of the covariance matrix in the (MS-)GVAR model
$\boldsymbol{\Sigma}_{exog}$	Covariance matrix in the (MS-)VAR model of the exogenous variables
$\boldsymbol{\sigma}_i$	Covariances in the maximum likelihood estimation of the MS-VAR models
σ	Element of covariance matrix $\boldsymbol{\Sigma}$
σ_{exog}	Element of covariance matrix $\boldsymbol{\Sigma}_{exog}$
$\tilde{\sigma}$	Standard deviation of a variable
$\mathbf{T}$	Trading volume weight matrix
T	End of sample period
t	Index of time, $t \in \{1, 2, \ldots, T\}$
τ	Index of time $\tau \in \{1, 2, \ldots, T\}$
t_{pred}	Index of time in the out-of-sample period, $t_{pred} \in \{1, 2, \ldots, J\}$
T_{boot}	Number of residuals per bootstrap sample drawn in the bootstrapping procedure
$\hat{T}_{i,m}$	Trace of smoothed probabilities in the MS-VAR model
$\boldsymbol{\theta}$	Parameter in the vector of densities of the vector of endogenous variables conditional on the regime probabilities
θ	Price threshold to classify the commodities into scarce or non-scarce states
$\mathbf{u}$	Error term in the measurement equation of the MS-VAR model
$\mathbf{u}_{i,m,t}$	Regime-dependent error term in the measurement equation of the MS-VAR model
$\hat{\mathbf{u}}_{i,m}$	Estimator for the regime-dependent error term in the measurement equation of the MS-VAR model
$\mathbf{u}^*_i$	Regime-dependent error term of the MS-VAR model
$\hat{\mathbf{u}}^*_i$	Estimator of the regime-dependent error term of the MS-VAR model
VaR	Value at risk
$\mathbf{v}$	General variable for the definition of quantiles
v_q	$q\%$ quantile of a variable
$\boldsymbol{v}$	Innovation process in the transition equation of the MS-VAR model

$\mathbf{W}$	Diagonal matrix, including information about smoothed probabilities as well as the covariance matrix used for the calculation of the likelihood function, in the MS-VAR model
$\mathbf{W}^*$	Diagonal matrix, including information about the covariance matrix used for the calculation of the likelihood function, in the MS-VAR model
$(w_{i,\tilde{\iota}})_{i,\tilde{\iota}}$	Weight matrix for the (MS-)GVAR model
w	Weights between two commodities for the definition of the external variables in the (MS-)GVAR model
$\hat{w}$	Weight between two commodities for the calculation of the supply- and demand-sided information matrix
$\mathbf{X}_{i,t}$	Information set consisting of observed values of $\mathbf{x}_{i,t}, \mathbf{e}_{i,t}, \mathbf{x}^*_{i,t}$
$\mathbf{X}_i$	Matrix consisting of the vectors $\mathbf{X}_{i,m}$
$\mathbf{X}_{i,m}$	Matrix of observed values
$\bar{\mathbf{X}}_{i,m}$	Matrix of endogenous as well as exogenous variables, separately defined for each specification of the MS-VAR models
$\mathbb{X}$	Regressor matrix in the state space representation of the MS-VAR model
$\bar{\mathbf{x}}$	Vector of endogenous as well as exogenous variables in the state space representation of the MS-VAR model
$\mathbf{x}$	Global vector of all commodity-specific variables in the (MS-)(G)VAR model, $\mathbf{x}_t = \left(\mathbf{x}'_{1,t}, \mathbf{x}'_{2,t}, \ldots, \mathbf{x}'_{N,t}\right)'$
$\hat{\mathbf{x}}$	Point forecast of the global vector of all commodity-specific variables in the (MS-)(G)VAR model
$\tilde{\mathbf{x}}$	Average of realizations for the Monte Carlo integration to calculate generalized impulse response functions
$\mathbf{x}_i$	Vector of endogenous variables in the (MS-)(G)VAR model
$\hat{\mathbf{x}}_{i,m,t}$	Predicted values of endogenous variables in the MS-VAR model obtained by the parameters given by the previous iteration in the expectation-maximization algorithm
$\mathbf{x}^*_i$	Vector of external variables in the (MS-)VAR model
$\mathbf{x}^{n_{boot}}$	Bootstrap sample of variables
$\mathbf{x}^{*,n_{boot}}_i$	Bootstrap sample of the vector of external variables in the (MS-)VAR model
X	Dependent variables in the logistic regression model
x	Commodity-specific determinants in the logistic regression model
$\boldsymbol{\Xi}$	Diagonal matrix of regime probabilities in the MS-VAR model
$\hat{\boldsymbol{\Xi}}$	Diagonal matrix of estimated regime probabilities in the MS-VAR model
$\tilde{\boldsymbol{\Xi}}$	Matrix of regimes of N commodities in the MS-GVAR model
$\boldsymbol{\xi}$	Unobservable state of the system and regime probabilities in the MS-VAR model, respectively
$\hat{\boldsymbol{\xi}}$	Estimator of the regime probabilities in the MS-VAR model
$\boldsymbol{\Upsilon}$	Parameter matrix of exogenous variables in the reduced form of the (MS-)GVAR model
$\boldsymbol{\upsilon}$	Error term in the reduced form of the (MS-)GVAR model
$\mathbf{Z}$	Link matrix used for the transformation of (MS-)VAR models to the (MS-)GVAR model
$\mathbf{z}$	Temporary vector used for the transformation of (MS-)VAR models to the (MS-)GVAR model, $\mathbf{z}_{i,t} = (\mathbf{x}'_{i,t}, \mathbf{x}^{*\prime}_{i,t})'$
z	Logit of the logistic regression model
$\mathcal{Z}$	Number of the scenarios of the input variables by the calculation of the probability of scarcity
ζ	Index of the scenarios of the input variables, $\zeta \in \{1, 2, \ldots, \mathcal{Z}\}$

List of Co-authored Papers

Some parts of this dissertation date back to scientific papers in collaboration with co-authors. These papers, as listed below, serve as a basis for some chapters. However, these sections were revised and extended for this dissertation in terms of formulation and content. The author had the decisive share in each of the scientific papers reported in this dissertation.

- Schischke, A., P. Papenfuß, M. Brem, P. Kurz, and A. Rathgeber (2023). Sustainable energy transition and its demand for scarce resources: Insights into the German Energiewende through a new risk assessment framework. Renewable and Sustainable Energy Reviews 176.
  Accepted for presentation at:
  - ProMETS Workshop at DLR-Institute for Networked Energy Systems, 25. - 26. February 2021, Oldenburg, Germany (online)
  - The Global Interdisciplinary Green Cities Conference, 22. - 26. June 2021, Augsburg, Germany (online)
- Schischke, A., P. Papenfuß, and A. Rathgeber (2023). Three Co's to Jointly Model Commodity Markets: Co-Production, Co-Consumption and Co-Trading. Accepted for publication in Empirical Economics
  Accepted for presentation at:
  - Association of Environmental and Resource Economists Annual Sommer Conference (AERE), 2. - 4. June 2021, Miami, USA (online)
  - 5th Commodity Markets Winter Workshop (CMWW), 27. - 29. January 2022, St. Johann, Austria
  - 2022 Annual Meeting of the Commodity & Energy Markets Association (CEMA), 21. - 22. June 2022, Chicago, USA
  - 6th J.P. Morgan Center Commodities Symposium, 14. - 15. August 2023, Denver, USA
- Schischke, A. and A. Rathgeber (2022). Time-varying spillover effects within and between industrial metal markets. Working Paper
  Accepted for presentation at:
  - 6th Commodity Markets Winter Workshop (CMWW), 8. - 9. March 2023, Skeikampen, Norway
  - NCCC-134 Conference on Applied Commodity Price Analysis, Forecasting & Market (NCCC), 24. - 25. April 2023, St. Louis, USA

Further scientific papers of the author:

- Schischke, A., P. Papenfuß, and H. Mihai (2022). Wie nachhaltig ist die deutsche Energiewende wirklich? Eine Analyse der sozialen Bedingungen des Rohstoffabbaus. Die Unternehmung 76 (2), 164 – 191.
- Schischke, A. and A. Rathgeber (2023). Commodities and monetary policy - the role of interest rates revisited. (Working Paper)
  Accepted for presentation at:
  - 2023 Annual Meeting of the Commodity & Energy Markets Association (CEMA), 20. - 21. June 2023, Budapest, Hungary
  - International Conference in Finance, Banking and Accounting (ICFBA), 8. - 9. September 2023, Montpellier, France
- Papenfuß, P., A. Schischke and A. Rathgeber (2021). Factors of Predictive Power for Metal Commodities. (Working Paper)
  Accepted for presentation at:
  - 41st International Symposium on Forecasting (ISF), 27. - 30. June 2021, Oxford, United Kingdom (online)
  - European Conference on Data Analysis (ECDA), 7. - 9. July 2021, Rotterdam, Netherlands (online)

1 Introduction

Sea levels are rising and oceans are becoming warmer. Longer, more intense droughts threaten crops, wildlife and freshwater supplies. From polar bears in the Arctic to marine turtles off the coast of Africa, our planet's diversity of life is at risk from the changing climate. Climate change poses a fundamental threat to the places, species and people's livelihoods[...]. To adequately address this crisis we must urgently reduce carbon pollution and prepare for the consequences of global warming.
World Wildlife Fund (2023)

In December 2015, 196 parties signed a legally binding international agreement on climate change, the Paris Agreement (2015), in which they committed to limit global warming to below 2, preferably 1.5 degrees Celsius compared to pre-industrial levels. In this context, the European Union has implemented the so-called "Green Deal" to achieve climate neutrality by 2050 according to the Paris Agreement (2015), see European Commission and Directorate-General for Communication (2021). A key component of the Green Deal is the decarbonization of the energy sector, since the production and use of energy is responsible for around 75% of the EU's greenhouse gas emissions, according to European Commission and Directorate-General for Communication (2021). In addition, the renovation of buildings towards energy efficient buildings with the use of renewable energy is promoted, as 40% of energy consumption is used for heating buildings. On top of that, Germany committed itself to the nuclear phase-out, as stated by Federal Ministry for Economic Affairs and Climate Action (2011), which constitutes a further constraint of the German energy transition ("German Energiewende"). Hereby, the "transition to climate neutrality could replace today's reliance on fossil fuels with one on raw materials", according to European Commission (2020).

On the way to CO_2 neutrality, green technologies such as wind power and photovoltaic systems, associated storage technologies, and building renovations are key elements. While around 40% of electricity consumption in Germany comes from renewable energies in 2021, their share should already be 80% by 2030, according to Federal Ministry for Economic Affairs and Climate Action (2022b). This transition process will require large amounts of raw materials, see Valero et al. (2018). For example, Marscheider-Weidemann et al. (2021) examine the resource requirements of future technologies based on the global socio-economic scenarios of Kriegler et al. (2012). Hereby, they detect the demand for cobalt and lithium in 2040 will be nearly four and six times, respectively, as high as their global production volume in 2018 in the sustainability scenario, due to the intensified use of the metals in (super-)alloys and storage technologies. Therefore, the question arises whether sufficient resources are available. While sufficient mineral raw materials seem to be available from a geological perspective, rising demand for raw materials can lead to price peaks and delivery bottlenecks in their supply as production capacities are limited in the short-term, see Federal Ministry for Economic Affairs and Climate Action (2022a). In addition, Valero et al. (2018) state the cumulative annual global demand for several metals related to energy technologies is likely to exceed their reserves, indicating potential future shortages. Moreover, the access to raw materials on global commodity markets is becoming more difficult due to an amplified market concentration, according to Federal Ministry for Economic Affairs and Climate Action (2022a). In this context, this thesis aims to analyze the resource scarcity risks of the German Energiewende.

As early as 1979, Skinner (1979) pointed out the "greatest challenge facing the U.S. Geological Survey in its second century will be the problem of resource limitations", as industrial development depends on reliable supplies of metals. Hereby, restrictions on trade relations between individual countries can significantly affect the supply of raw materials in the United States and Europe, since only a small portion is mined in these regions. In recent years, the delivery difficulties during the onset of the Covid-19 pandemic as well as the Ukraine war have further highlighted Europe's, and especially Germany's, dependence on importing raw materials and underline the relevance of material supply security. Since Skinner (1979), various studies analyzed the supply risks, the vulnerability of a system to a potential supply disruption as well as environmental and social impacts of materials, see Arendt et al. (2020), Graedel et al. (2012), and Kolotzek et al. (2018) among others. However, most of these studies are "snapshots in time", according to Graedel et al. (2012), reflecting the supply situation at a certain point in time. Besides the extensive literature on the criticality of materials in general, only few studies address the criticality of materials used in specific technologies. Hereby, this thesis contributes to the literature, as the objective is the scarcity risk of materials in the context of the energy transition, whereby we focus on the scarcity risk rather than specific security of supply issues.

The review study of Liang et al. (2022) analyzes studies in the context of material requirements of renewable energy technologies. Hereby, they reveal most of the previous studies focus on photovoltaic systems and wind power on a global scale. Among the few studies at the national level, the article of Viebahn et al. (2015) analyzes relevant green technologies for the German Energiewende with respect to the geological availability and supply of mineral resources. However, the material requirements in their study are based on a meta-analysis. In contrast, this thesis is a primary study, whereby we compare the scarcity risk of the actual resource requirements for four potential transformation pathways of the German energy system. These pathways are generated in order to optimally reduce Germany's CO_2 emissions by 95% in 2050 compared to 1990 under different assumptions about the acceptance of the energy transition in the German population. Thereby, we examine the annual material requirements from 2020 to 2050 of 28 representative technologies of renewable energy technologies, storage capacities, electricity transports as well as building renovation.

From a geological perspective, sufficient mineral raw materials seem to be available for the German Energiewende, see Federal Ministry for Economic Affairs and Climate Action (2022a). In this context, most previous studies focus only on the required amounts of the commodities and compare them with the available reserves, see for example the study of Valero et al. (2018) on a global scale. However, commodities are industrial goods in real economies and their prices are the result of the supply and demand equilibrium according to the classical fundamental theory. Hereby, an additional demand for commodities causes supply and price reactions that are often disregarded in the previous literature. On the one hand, the supply of resources only reacts slowly to changes in demand, probably leading to (short-term) shortages. On the other hand, the economic dimension is an important but mostly neglected aspect, since higher prices in response to the increased demand can lead to substitution effects, as an alternative commodity or even an alternative technology becomes more cost effective, which finally leads to reductions in the global demand and therefore to a lower scarcity risk. Overall, a good's price reflects all available information, according to Tilton et al. (2018), therefore, we interpret the commodity price as scarcity indicator in this thesis, which can be affected by supply and demand shocks. Based on this assumption, we propose a new framework to assess the scarcity risk of resource-demanding projects, taking into account the substitutability of commodities, the future resource amounts required by the project as well as the commodity market structure. Subsequently, we apply the proposed methodology to investigate and compare the scarcity risks of the material requirements of several transformation pathways of the German Energiewende.

To assess the impact of the German Energiewende on commodity markets, in particular on the demand and price of commodities, we initially examine the structure of commodity markets. In general, a commodity's price is the equilibrium price of its supply and demand and therefore naturally includes all available information, see Tilton et al. (2018). For this reason, we interpret the price as scarcity indicator in this thesis. Hence, a comprehensive understanding of commodity markets is essential. In general, the literature distinguishes two perspectives on commodity markets: On the one hand, the classical fundamental theory states a good's price is the result of its supply and demand equilibrium, see Hotelling (1931) and Deaton and Laroque (2003). In particular, commodity prices, especially industrial metals as industrial goods, are influenced by their fundamentals supply and demand, see Cuddington and Zellou (2013) as well as Stuermer (2018) and Chen et al. (2019). On the other hand, several empirical studies detect similar patterns in commodity prices, characterized first as (excess) co-movement by Pindyck and Rotemberg (1990).

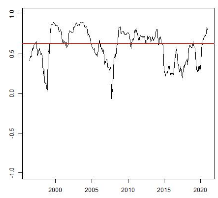
This thesis aims to incorporate both perspectives, since the energy transition requires several commodities, and therefore, the impact of the German Energiewende on a portfolio of commodities is investigated. In addition, co-production and co-consumption relations as well as substitutions of commodities may affect the scarcity risk. Hereby, we aim to reflect the impact of supply and demand, but also of global macroeconomic variables on commodity prices simultaneously. Moreover, spillover effects between commodity markets should be also represented, to account for the co-movement between prices. In particular, we attempt to reflect the relationships between the commodities by their common supply, demand or trading activity. Therefore, a new framework of commodity markets is necessary, as an extension of single commodity market models for the interdependencies between commodities is not feasible due to data limitations. In this context, we propose the global vector autoregression (GVAR) model for commodity markets, originally developed by Pesaran et al. (2004) to analyze the world economy from an individual country level, under the limitation of small sample data sets.

One drawback of the global vector autoregression model is its time-invariance. However, the co-movement between commodity prices varied over time, as major changes like the financialization and the growth in emerging markets caused significant shifts in the commodity markets, see for example Helbling et al. (2008), Le Pen and Sévi (2017), and Ohashi and Okimoto (2016). To illustrate the time-dependent behavior between commodities, we provide an initial bi-variate, time-varying correlation analysis, displayed in Figure 1.1, based on the rolling 18-months[1] correlations of industrial metal prices. Hereby, the correlations between the metal prices fluctuate around the time-invariant correlation based on the entire sample period from 1995 to 2020, indicating the relations between commodities vary over time. While the correlation between copper and zinc prices increases over time, the relations between most of the metal prices do not follow a trend, but instead exhibit simultaneous periods of decreasing followed by increasing correlations. In general, the correlations are higher around the years 2004 and 2009, reflecting the stronger observed co-movement of metal prices due to the financialization as well as the financial crisis, respectively. However, the sharp oil price drop in 2014, combined with a slowdown in Chinese demand, caused decreasing prices of copper, nickel, lead, and tin, whereas supply concerns lead to increasing prices of aluminum, and zinc. Ultimately, the commodity-specific supply and demand shocks in 2014 lead to a reduction in the metals' correlations. Based on this brief, bi-variate co-movement analysis, we claim a time-independent analysis of the relationship between commodity prices, or even a trend analysis, can not fully capture the interaction between commodity markets, especially, as the rolling 18-months variances of the prices observe

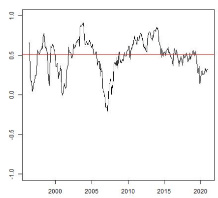
[1]To reduce the impact of potential influential points, we only include data of the previous 18 months, however, the rolling window correlations based on 24 or 36 months reveal similar patterns. In particular, the time-varying correlations fluctuate and do not follow a trend.

similar patterns of increasing followed by decreasing volatility. Moreover, the substantial increase in the demand of resources caused by the energy transition may also affect the structure of commodity markets.

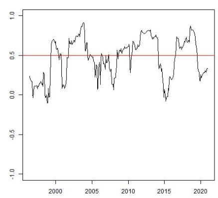
Figure 1.1: Correlation plot of industrial metal prices



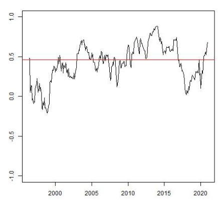
(a) Al P - Cu P



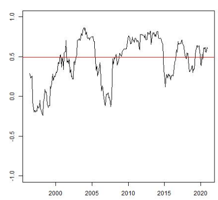
(b) Al P - Ni P



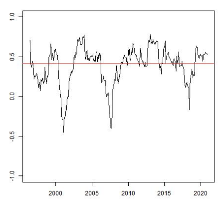
(c) Cu P - Ni P



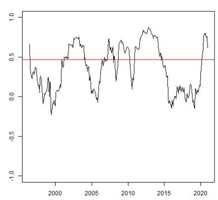
(d) Al P - Pb P



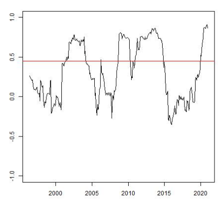
(e) Cu P - Pb P



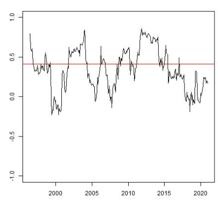
(f) Ni P - Pb P



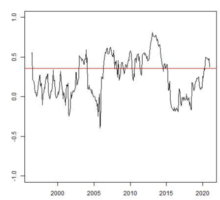
(g) Al P - Sn P



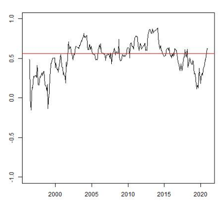
(h) Cu P - Sn P



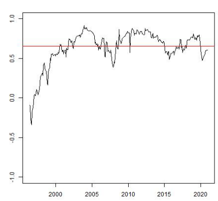
(i) Ni P - Sn P



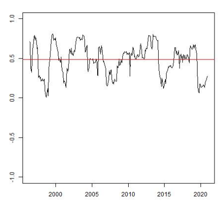
(j) Pb P - Sn P



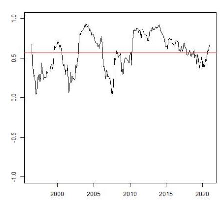
(k) Al P - Zn P



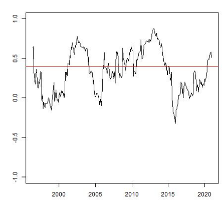
(l) Cu P - Zn P



(m) Ni P - Zn P



(n) Pb P - Zn P



(o) Sn P - Zn P

These figures display the correlations of aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) price to each other, whereby the Pearson correlation is calculated based on a rolling window capturing 18 months and the red line indicates the corresponding correlation derived over the entire sample period from January 1995 to December 2020.

Therefore, the question arises how the constitution of commodity markets, especially the impact of supply and demand on prices as well as the co-movement between prices, changes over time. For this reason, we extend the global vector autoregression (GVAR) framework, which reflects the impact of supply and demand on commodity prices as well as the co-movement between commodity prices, by a Markov-switching component, resulting in a Markov-switching global vector autoregression (MS-GVAR) model, similar to Binder and Gross (2013), which allows for regime-switches, enabling time-varying relations in commodity markets.

Hereby, we incorporate the commodity market model in the scarcity risk assessment framework to analyze and compare the resource requirements of several transformation pathways for the German Energiewende in regard to their availability, respectively their scarcity. In particular, we generate the individual probability of scarcity for each commodity of the project using the modeled prices from the (MS-)GVAR model. Subsequently, we combine the individual probability of scarcity with a substitutability score and the required material amount for a specific project to obtain our commodity-specific risk indicator. Finally, we aggregate the commodity-specific risk scores on project level, to compare the scarcity risk of several project alternatives.

Empirically, we apply the framework to assess the resource scarcity risk of four transformation pathways of the German energy system that differ in the acceptance of German societies for the required actions. On commodity level, cobalt, indium and nickel, mainly allocated to energy storage and solar PV technologies, bear the highest scarcity risks, while on path level the transformation path, which models the transition of the German energy system with full support by the society, exhibits the lowest scarcity risks. Overall, this thesis reveals the economic scarcity risk of commodities strongly depends on the required amounts. Hereby, we detect an active support of the German population for the energy transition significantly reduces the required amounts of raw materials, resulting in a reduced scarcity risk.

1.1 Objectives and Structure of the Thesis

The objective of this thesis is the analysis and comparison of the resource requirements of four transformation pathways for the German Energiewende in terms of their availability, respectively their scarcity, within a commodity market framework. Hereby, we propose and apply a new scarcity risk assessment framework based on commodity market models on the resource requirements of four transformation pathways for the German Energiewende.

This thesis is divided into six chapters. Since a comprehensive understanding of commodity markets is essential for the proposed risk assessment framework, we provide in Chapter 2 an overview of the determinants of commodity prices and of studies that examine common patterns in commodity prices. Hereby, we emphasize previous studies mostly focus either on the impact of supply and demand on prices or on the co-movement between commodities. Moreover, we reveal the literature on criticality in general regards indicators of supply risks, of the vulnerability of a system to potential supply disruptions, as well as of economic, environmental and social impacts, see for example Graedel et al. (2012), Kolotzek et al. (2018), as well as the review study of Erdmann and Graedel (2011), neglecting the impact of demand on the criticality as well as the time aspect. However, the built-up of renewable energies will increase the demand for commodities, which is why the question arises whether the resources are available to achieve the climate goals. Hereby, the studies in the context of the energy transition mostly consider solar PV and wind power on technology level, whereas storage capacities are oftentimes omitted, see Liang et al. (2022).

In Chapter 3, we propose the scarcity risk assessment framework by incorporating new commodity market models, a substitutability score as well as the required resource amounts. Hereby, each commodity market is modeled separately using (Markov-switching) vector autoregression ((MS-)VAR) models with the commodity-specific microeconomic variables supply, demand and price, as well as exogenous, macroeconomic attributes. Subsequently, the individual markets are linked to the (Markov-switching) global vector autoregression ((MS-)GVAR) model via appropriate weight matrices and finally included in the risk assessment framework.

For the scarcity risk assessment of the German Energiewende, we first present the transformation pathways of the German energy system with the associated material requirements in Chapter 4. Besides the actual resource demands, the commodity markets influence the scarcity risk. Therefore, we illustrate the commodity-specific data as well as the price determinants. Finally, possible interdependencies between the commodities are considered in order to combine the individual commodity markets into a holistic commodity market model.

In the empirical part of this thesis in Chapter 5, we exemplary apply the (MS-)GVAR model on the industrial metal markets and investigate the spillover effects between commodity markets. Subsequently, we apply the (MS-)GVAR model on the key resources of the German Energiewende and incorporate the results in the risk assessment framework. In this context, we evaluate the resource scarcity risks of the four transformation pathways of the German Energiewende at both the commodity and path levels and reveal the resource requirements strongly influence the economic scarcity risk, whereby the actual commodity market model plays only a minor role.

Finally, Chapter 6 summarizes the key findings and discusses the limitations of the thesis.

2 Review of Literature

One important part on the way to climate neutrality, in accordance with the international treaty on climate change, the Paris Agreement (2015), is the decarbonization of the energy sector. However, this transition process will require large amounts of raw materials, according to Valero et al. (2018), and "could replace today's reliance on fossil fuels with one on raw materials", see European Commission (2020), possibly leading to price peaks and delivery bottlenecks in the supply of raw materials, according to Federal Ministry for Economic Affairs and Climate Action (2022a). Therefore, the question arises whether enough resources are accessible.

In general, Skinner (1979) already pointed out the "greatest challenge facing the U.S. Geological Survey in its second century will be the problem of resource limitations", as industrial development depend on reliable supplies of metals. In this context, several studies assess the criticality of various commodities, whereby they focus on the supply risks, the vulnerability of a system to a potential supply disruption as well as environmental and social impacts of materials, see Section 2.1. However, the limiting factor for the availability of a commodity is the extraction, see Tilton et al. (2018). While from a geological point of view sufficient resources are available, according to Federal Ministry for Economic Affairs and Climate Action (2022a), there may not be enough reserves to meet the increased demand for raw materials. The resources may be recoverable in the future, but the development of new deposits is tedious, therefore, the production volume of commodities only reacts slowly to changes in demand. In particular, the increased resource requirements for the energy transition can lead to shortages, especially in the short-term, resulting in price peaks due to the supply and demand equilibrium, according to Federal Ministry for Economic Affairs and Climate Action (2022a). For this reason, commodity prices may be interpreted as scarcity indicators. Hereby, a comprehensive understanding of commodity markets and, in particular, of price determinants as well as their common pattern, the so-called co-movement, in commodity prices, is essential, see Section 2.2 and Section 2.3.

2.1 Criticality of Commodities

Since Skinner (1979), several studies proposed frameworks for the assessment and investigation of the criticality of various commodities. In general, the term criticality includes several aspects. While some studies consider the sustainability of the commodities, most criticality assessments examine the vulnerability to supply restrictions as well as supply risks. Hereby, supply risk denotes the specific risk of disruptions in supply of a given commodity, see European Commission (2020). As these analyses neglect the potential risk of (future) demand increases, we focus in this thesis on the scarcity risk of metals which denotes the risk that the commodities are not available in the required quantity at the required time. Tilton et al. (2018) even state commodities become scarce if their price sharply jumps in the short-term or persistently increases over the long-term, regardless of whether supply restrictions or demand increases cause these price changes. In the following, we provide an overview of the literature in the area of criticality. We first regard few important studies, analyzing the criticality of commodities in general, followed by studies, focusing on the criticality of commodities in the context of the energy transition.

The overall review study of Schrijvers et al. (2020) on general criticality assessment frameworks on a product, technology, company, country, region and a global level reveals the main objectives of the reviewed studies are to raise the government's and industry's awareness of supply issues, to provide information to policy and consumers for the mitigation of criticality and to generate a broad information basis for further, in-depth studies.

Moreover, Erdmann and Graedel (2011) highlight in their literature review most of the criticality assessment frameworks identify critical commodities via an analysis of supply risks as well as the vulnerability of a system to potential supply disruptions. However, by the definition of the initial scope, the choice of metrics, the aggregation level of indicators as well as the material group size, the results in the literature differ. For instance, Kolotzek et al. (2018) introduce a new sustainability-oriented raw material assessment and decision support model, based on a literature analysis, best practice in companies, expert questionnaires and interdisciplinary workshops. Due to increasing pressure from customers and competitors, manufacturers have to pay more attention to their responsibility of the sustainability aspects in the selection of the materials used in their products. Therefore, Kolotzek et al. (2018) consider relevant and applicable (semi-)quantitative indicators for the three sustainability dimensions, economic, environmental as well as social, in their raw material assessment and decision support model. In a case study of capacity selection, investigating the criticality of aluminum, niobium and tantalum, they conclude tantalum exhibits the highest total risk. A further general criticality study on 42 materials for Europe is provided by Arendt et al. (2020), who apply the methodology SCARCE, originally developed by Bach et al. (2017) for Germany using the bottom-up and top-down approach of Bach et al. (2016) for identifying the relevant factors, to assess the criticality with regard to the social as well as environmental impacts, considering various social risk and vulnerability indicators. Hereby, Arendt et al. (2020) confirm the results of Kolotzek et al. (2018) that tantalum bears high social risks. However, they identify also cobalt and tin as critical resources from the social perspective, while gold, platinum and niobium perform worst in the environmental perspective, indicating different perspectives draw different conclusions about the criticality. In particular, the review study of Erdmann and Graedel (2011) identify only the platinum group metals and the rare earth elements are found to be critical in various studies.

The more recent review study of Hayes and McCullough (2018) underlines the platinum group metals, the rare earth elements as well as indium are most commonly regarded as critical. In addition, they reveal wolfram, germanium, cobalt, niobium, tantalum, gallium and antimony are determined as critical in several studies, whereby in general by-products are more often identified as critical. While most of the studies analyze a broad set of commodities, the study of Nassar et al. (2015) focuses on the supply risk of by-product metals. In general, by-products are increasingly used in various applications and especially for renewable technologies. In particular, by-products are important for electronic and solar energy applications (gallium, germanium, selenium, indium, and tellurium), offshore wind, lighting, and medical imaging technologies (several rare earth elements), or employed as alloying elements in high-temperature applications (cobalt, hafnium, and rhenium). Hereby, the revenue contribution from a by-product is not sufficient to cover the entire cost of sales from the mine and subsequent beneficiation and refining processes. Therefore, the supply of a by-product is unable to respond to rapid changes in demand, as the production of companion metals depends on their (few) hosts' metals supplies. For that reason, their prices can fluctuate widely, whereby Redlinger and Eggert (2016) confirm a higher volatility in the annual average prices of by-products compared to main products. In addition, the review study of Jordan (2017) reveals the assessment of certain important by-products, such as lithium, is underrepresented in the literature so far. Overall, Nassar et al. (2015) propose to increase recycling, as a possible reduction of the criticality, but Graedel et al. (2015) state various metals see little or no recycling yet, which is why they investigate the substitutability, a further possibility to circumvent availability constraints. Nevertheless, the analysis of Graedel et al. (2015) of 62 metals and metalloids in their major uses reveals several metals have no substitute or the product performance will suffer from substitution.

While Hayes and McCullough (2018) aggregate the literature regarding the critical commodities aside the applied methodology, the review study of Graedel and Reck (2016) highlights the importance of a uniform criticality methodology. They reveal the criticality of elements vary between the investigated studies, caused by different target groups, considered time horizons, included factors and the aggregation of these factors. In this line, Gleich et al. (2013) focus on the aggregation of the factors to a criticality measure and aim to provide accurate weights for further studies. Interpreting the price as scarcity indicator, following Tilton (2010), they analyze the impact of several indicators, inter alia primary as well as secondary production, stocks, U.S. consumption, Herfindal-Hirschman indexes for country- and producer-concentration, U.S. gross domestic product, world population, inflation and interest rates, on the criticality of 42 different raw materials. Their results indicate each commodity is influenced by different factors, highlighting the importance of a commodity-specific indicator selection as well as commodity-specific weights in the assessment of criticality.

Moreover, Graedel and Reck (2016) provide different aspects which further assessment models should include, emphasizing the need for a uniform criticality framework, however, the models proposed in the literature differ. Hereby, the authors emphasize to periodically update the criticality assessment, since criticality is a dynamic state. In this context, Graedel et al. (2012) highlight the importance of re-assessing the criticality with new data. Specifically, they suggest a new robust, reliable methodology to quantify the degree of criticality of metals, whereby they regard the three dimensions supply risk, environmental implications and vulnerability to supply restrictions. First, they focus on the manufacturing and consider geological, technological, economic, social, regulatory as well as geopolitical indicators to measure the supply risk of commodities. Second, they quantify the environmental implications, using information about the toxicity, the energy and water consumption in processing, as well as the emissions to air, water and land. Finally, they consider the vulnerability to supply restrictions under the differentiation on corporate, national and global level. Measuring the distance between the three dimensions, Graedel et al. (2012) get an indicator for the overall criticality of the metals. However, they claim their results are a snapshot in time and the criticality should be re-computed with new data. In this line, Ioannidou et al. (2019) emphasize the importance of dynamic parameters in the criticality assessment. Hereby, the authors propose dynamic indicators, derived from studies outside of the field of criticality, and give specific examples how to include these indicators in the criticality assessment.

To incorporate the time aspect, Rosenau-Tornow et al. (2009) investigate past as well as future supply and demand trends in their proposed methodology for identifying and assessing long-term supply risks for mineral raw materials, using time series data. Finally, they apply their method in a case study on copper and underline the importance of monitoring the exploration of copper as well as mining projects.

Overall, several studies in the literature examine the criticality of commodities. Hereby, the sustainability, the vulnerability to supply disruptions as well as the supply risk of materials are analyzed. As the investigated risk assessment frameworks, indicators and commodities differ across the studies, the identified critical commodities differ, see for example the review study of Erdmann and Graedel (2011). However, several studies emphasize the importance of a time-varying analysis, see Graedel et al. (2012), Ioannidou et al. (2019), and Rosenau-Tornow et al. (2009) among others.

While the previously mentioned studies examine the criticality of commodities in general, without distinction of the various sectors in which the commodities are used, few studies focus on the commodities' availability in the context of the energy transition. However, the built-up of renewable energies will increase the demand for commodities. As the "access to resources is a strategic security question for Europe's ambition to deliver the Green Deal", see European Commission (2020), the resource availability is a barrier to achieve the climate goals. In this context, the European Commission (2020) identifies the most critical raw materials at EU level, whereby the economic importance and supply risk determine the criticality. In particular, they define 30 out of 83 materials as critical in 2020. In addition, Arrobas et al. (2017) examine the future worldwide needs for wind, solar, and energy storage batteries to limit global warming under the target of avoiding global temperature increases of two, four, and six degrees. Hereby, they expect growing demands for aluminum, cobalt, copper, iron ore, lead, lithium, nickel, manganese, the platinum group of metals, silver, steel, titanium and zinc as well as for the rare earth metals including cadmium, molybdenum, neodymium, and indium. However, the actual requirements will depend on the climate goal as well as the intra-technology choices. Moreover, Valero et al. (2018) state the deployment of "green technologies" requires huge amounts of raw materials and compare the future required resource amounts with the global reserves. In particular, they identify the 13 metals, cadmium, chromium, cobalt, copper, gallium, indium, lithium, manganese, nickel, silver, tellurium, tin, and zinc, as critical. Further, Song et al. (2022) detect the clean energy market affects the connectedness between main and by-products in their analysis of the dynamic dependencies between metal markets. In addition, Marscheider-Weidemann et al. (2021) examine the global future resource requirements of 33 technologies within the clusters "mobility and aerospace", "digitization and Industry 4.0", "energy technologies and decarbonization", "recycling and water management", and "power and data networks", based on the global socio-economic scenarios of Kriegler et al. (2012). Hereby, they detect an elevated demand for scandium, lithium, heavy and light rare earth metals, iridium, and cobalt in the sustainability scenario, due to the intensified use of hydrogen technologies, lithium-ion high-performance storage, solid-state batteries, electric traction motors, wind turbines and high-performance permanent magnets.

The review study of Liang et al. (2022), investigating the material requirements of different energy technologies, reveals most of the previous studies focus on global level and only on solar PV and wind power on technology level. One of the few studies on national level is the article of Viebahn et al. (2015), which analyzes the German Energiewende with respect to the geological availability and supply of mineral resources. In this context, Viebahn et al. (2015) assess the cumulative material demand of the German energy transition by 2050 for relevant technologies, requiring at least one critical commodity. Therefore, they combine the market shares of the relevant technologies with their future needs derived from a meta-analysis of nine different long-term energy scenarios for the energy supply system in Germany. In line with the results of the review study of Lee et al. (2020), Viebahn et al. (2015) detect the shift towards an energy system based on renewable sources is feasible. In addition, Roelich et al. (2014) provides a new dynamic methodology based on the supply disruption potential and exposure to supply disruption, to investigate material's criticality during the energy transition and to allow for a comparison between several future scenarios. Hereby, the authors apply their framework on two potential pathways of the energy transition in the United Kingdom and exemplary examine the criticality of neodymium in wind turbines from 2012 to 2050, whereby a step-change in the criticality occurs in 2030.

In line with the findings of Liang et al. (2022), the literature review of Watari et al. (2020) of 88 studies, examining the long-term demand of various critical materials in the context of the energy transition, underlines the emphasis on solar PV, wind power and electric vehicles, as well as the global scale. Hereby, the authors state previous studies mostly neglect the linkages between host and by-products. Further, they reveal the need for a criticality assessment framework which integrates the long-term outlook information.

Overall, several studies investigate the criticality of various metals since Skinner (1979), analyzing inter alia the supply risk, the vulnerability to supply restrictions, the environmental and social impacts, in general or in the context of the energy transition. However, most of the reported results are snapshots in time, see Graedel et al. (2012), neglecting future (expected) supply and demand trends. In addition, most of the criticality studies in the context of the energy transition focus only on solar PV and wind power, neglecting further green technologies, according to Liang et al. (2022). Moreover, the availability of each commodity is analyzed separately, mostly by comparing the future needs with the global reserves or investigating the supply risks and supply disruption potential, and in particular, the joint material risk on path level is neglected, see Watari et al. (2020). Due to the focus on the supply risk, the general scarcity risk assessment is so far underrepresented in the literature. However, commodities become scarce either due to supply restrictions or demand increases, causing price changes, see Tilton et al. (2018), which is why a comprehensive understanding of commodity prices and their determinants is essential.

2.2 Determinants of Commodity Prices

While an imbalance of supply and demand can lead to shortages in the short term, new technologies can prevent long-term shortages due to mineral depletion, see Tilton et al. (2018). In this context, prices may be interpreted as scarcity indicator, where high prices reflect situations of low supply and/or high demand, due to the market equilibrium model. Using this interpretation, Gleich et al. (2013) underline the importance of a commodity-specific analysis of criticality, in particular, a commodity-specific selection of different price influencing indicators, whereas most of the studies apply the same indicators and aggregation for all materials. Hereby, a comprehensive understanding of commodity markets and, in particular, of price determinants as well as the co-movement in commodity prices is essential.

In this section, we provide an overview of potential commodity price influential factors. Since the classical fundamental theory states a price is the result of the supply and demand equilibrium, we first analyze literature regarding the impact of supply and demand on commodity prices. Subsequently, an overview of further determinants investigated in the literature is provided. In particular, we demonstrate how macroeconomic indicators may influence commodity prices. For a comprehensive understanding of commodity markets, important studies analyzing the co-movement between commodity prices are presented in Section 2.3.

2.2.1 Impact of Supply and Demand on Commodity Prices

In general, commodities, especially industrial metals, are industrial goods in real economies and, in particular, key elements of industrial and technological development. Therefore, an increase in the demand should lead to elevated prices due to the classical fundamental theory, see Hotelling (1931), Deaton and Laroque (2003) as well as Frankel and Rose (2010). Ultimately, the higher demand might lead to scarcity (in the short-run), according to Tilton et al. (2018). In the following, we briefly review studies that either focus on the supply and demand equilibrium or empirically examine the effects of supply and demand on prices or the drivers of supply and demand of commodities.

First of all, several studies on commodity markets base their analysis on the supply and demand equilibrium. While Cuddington and Zellou (2013) use a supply and demand equilibrium model to simulate super cycles under the assumption of inelastic supply in the short-run, Deaton and Laroque (1992) extend the supply and demand equilibrium model for the behavior of competitive speculators who hold inventories of commodities. They further introduce a time-series version of the Lewis[1] model in their more recent study, see Deaton and Laroque (2003), whereby supply is assumed to be infinitely elastic in the long-run, whereas demand is related to the level of world income as well as to the price of the commodity. Overall, they detect an imbalance between lagged demand and lagged supply leads to an increase in price, while prices are generally driven by demand fluctuations in the short-run. Moreover, the extension of the "traditional structural approach", including a supply as well as a demand proxy, leads to signficantly improved commodity price forecasts in the study of Borensztein and Reinhart (1994).

While the previously mentioned studies focus on the equilibrium model, several studies investigate the impact of supply and demand on prices empirically. Hereby, Ahumada and Cornejo (2014) detect long-run price effects of supply fluctuations as well as a negative elasticity of real commodity prices with respect to their production in their cointegrated time series-cross section model of eight commodities in the period from 1960 to 2010, whereas Chen et al. (2019) underline the importance of both, supply and demand, for modeling copper price fluctuations in their Markov-switching model based on monthly data from 2004 to 2016. Further, specific determinants still play an important role in metal markets, besides financial characteristics, according to Lutzenberger et al. (2017), analyzing the prices of 30 commodities in the period from 1990 to 2013.

With regard to the energy markets, Thomas et al. (2010) support these findings, as demand as well as supply are significant long-run determinants of the oil price. In contrast, Kilian (2009) reveal the historically observed fluctuations in the oil price are mainly associated with global aggregate demand shocks as well as precautionary demand shocks, but they can not be attributed to oil supply shocks, emphasizing the importance of the demand on commodity prices. Hereby, the different conclusions might be caused by the sample period under consideration, as Thomas et al. (2010) focus on more recent times from 1996 to 2009, whereas Kilian (2009) investigates a longer period from 1968 to 2007, including the 1970s energy crisis. However, Nick and Thoenes (2013) underline the stronger impact of the demand side compared to the supply side in recent times, in their analysis of the German gas market in the period from 2008 to 2012.

Stuermer (2018) reveals similar results for metal markets. While supply factors only affect copper and tin significantly, a long-run demand increase, caused by the technological progress, triggers major increases in real metal prices. In contrast, demand influences in the short-term and supply affects in the long-term commodity prices in the empirical analysis of Guzmán and Silva (2018). These differences in the findings of Stuermer (2018) and Guzmán and Silva (2018) are probably caused by the inhomogeneous definition of short- and long-term horizons. While Guzmán and Silva (2018) refer to short-term as weekly up to monthly and long-term as "several years", Stuermer (2018) evaluates a time span of 150 years, therefore, he defines short-term as up to five years and long-term as up to fifteen years.

Besides the impact of supply and demand on prices, several studies examine the drivers of supply and demand. Hereby, Carter et al. (2011) as well as Helbling et al. (2008) investigate the reasons causing commodity price booms. Both studies highlight the importance of supply and demand on commodity prices, since surprises to the supply and, especially, to the demand of commodities, combined with slow supply responses, historically led to high price levels. In this context, Helbling et al. (2008) mention, besides the increased demand of emerging economies, the biofuels trend and the financialization of commodity markets as key drivers for the demand growth, whereas Kilian and Zhou (2018) focus on the demand side and attribute the commodity price cycles to unexpected fluctuations in the global real economic activity, a proxy for the global demand of commodities. In addition, the stock-to-use-ratio, indirectly reflecting the increased

[1] In general, the article of Lewis (1954) investigates the growth under unlimited labor supplies. In particular, he aimed for reasons explaining the declining West Indian sugar prices relative to the prices of manufactured goods. As long as there was an infinitely elastic supply of labor at the subsistence wage, he argued the world sugar prices could not increase, see Deaton and Laroque (2003).

demand for biofuels, the income effects, as well as weather conditions, affecting the supply of food commodities, is one of the most important drivers of commodity prices, according to Baffes and Dennis (2013).

Overall, supply and demand are important determinants of commodity prices, and therefore also for the scarcity risk, at least in the long-run, as production volumes are inelastic in the short-run. However, due to slow supply responses, fluctuations in the demand influence commodity prices. Hereby, increases in the demand caused by technological developments or growth in emerging countries, among others, have historically led to high price levels.

2.2.2 Impact of Macroeconomic Variables on Commodity Prices

As the increased demand of emerging economies for commodities was one of the key drivers for the demand growth and the corresponding commodity price boom at the beginning of the 2000s, see Helbling et al. (2008), the economic state influences at least the demand of commodities, and ultimately their prices. Therefore, we investigate how several macroeconomic variables affect commodity markets, in the following.

2.2.2.1 Economic Activity

The commodity price boom shows the commodity-specific demand is affected by the global demand. Since the availability of individual demand factors is also limited for many commodities, empirical studies frequently use economic activity, measured by the industrial production or gross domestic product, as a proxy to determine the impact of demand on commodity prices. Overall, several studies underline the global demand is an important determinant of commodity prices and also exhibits predictive power in forecasting commodity prices.

In this context, Issler et al. (2014) show, based on a derived-demand model for cost-minimizing firms, metal price variation and industrial production variation should theoretically be positively correlated. Hereby, they assume fixed supply, which is feasible in the short-run, and optimally chosen demand, taking into account the optimal production for the industrial sector. Besides the theoretical model, Issler et al. (2014) provide empirical evidence of their theory in a long-term analysis from 1900 to 2012 and detect cycles in metal prices are synchronized to those in industrial production. Further, Akram (2009) as well as Smiech et al. (2015) find a significant, positive impact of economic activity on commodity prices in the period from 1989 to 2007 and 1997 to 2013, whereby they measure the global demand by the OECD and euro area industrial production, respectively. Moreover, industrial production helps in explaining the common factors of various commodities, which explain at least partly the common pattern in (non-)energy commodity prices, in the studies of Kagraoka (2016) and Lombardi et al. (2012), investigating the period from 1995 to 2015 and 1975 to 2008, respectively. In addition, Alquist and Coibion (2013) underline the findings of Kagraoka (2016) and Lombardi et al. (2012) for the sample period from 1957 to 2013, as they detect the indirect aggregate common factor, accounting for up to 70% of the variance in forty commodity prices, is highly correlated with their used measure of global industrial production, provided by Baumeister and Peersman (2013).

Regarding the predictive power of the industrial production, Buncic and Moretto (2015) forecast copper price returns with this economic activity proxy, whereas Gargano and Timmermann (2014) find evidence for predictive power in annual forecasts of commodity indices and Borensztein and Reinhart (1994) improve their forecasts of the IMF non-oil all-commodity index by including an aggregated world industrial production index.

While the industrial production covers the output of the industrial sector of the economy and therefore reflects the economic sector which demands especially energy and metal commodities, the gross domestic product is a more general proxy of economic activity, as it includes the output of all final goods and also the agricultural sector. Hence, the studies of Ahumada and Cornejo (2014), Camacho and Perez-Quiros (2014), Dimitropoulos and Yatchew (2018), Frankel and Rose (2010), Klotz et al. (2014) as well as Robinson (2019) consider the gross domestic product as proxy for economic activity. Hereby, Dimitropoulos and Yatchew (2018) obtain a modest better model fit including the growth rate of the gross domestic product of 20 OECD countries in their long-term analysis of eleven energy and metal commodities from 1901 to 2014, whereas Robinson (2019) detects a shock to the OECD's gross domestic product increases the price of gold in the period from 1980 to 2014. In addition, Baffes et al. (2020) reveal an increase in per capita income leads to increasing demand for commodities in their analysis of energy and metal markets in the period from 1965 to 2017.

In general, the results in the literature indicate the economic activity is an important determinant of commodity prices, whereby the results are robust regarding the proxy, commodities and time period considered. However, Kilian (2009) propose an alternative index of the world economic activity, based on dry cargo single voyage ocean freight rates, since the trade with commodities rely on a steady transport. Hence, the freight rates might reflect the actual demand for commodities which is why Kilian and Zhou (2018) state the index of Kilian (2009) is an advantageous proxy for economic activity in modeling industrial commodity markets compared with alternative indicators of global real economic activity.

While most studies consider the economic activity of developed countries, the global demand of emerging markets also affect commodity prices, especially in more recent times. Hereby, Camacho and Perez-Quiros (2014) analyze the dynamic interactions between commodity prices and output growth in Latin America in the period from 1971 to 2009, whereas Ahumada and Cornejo (2014) and Klotz et al. (2014) detect a positive, significant impact of China's gross domestic product on commodity prices in the period from 1960 to 2010 and 1998 to 2012, respectively. Chen (2010) even states the high prices of 21 international traded metals are partially driven by the strong demand from emerging economies, e.g. China and India. In particular, China was responsible for about 90% of the increase in the world consumption of copper from 2000 to 2006, see Helbling et al. (2008).

Overall, the literature reveals economic activity is an important determinant of commodity prices, and thus the associated scarcity risk, although the studies differ in terms of the focus of the study, the methodology used, the commodities considered as well as the proxy included for the economic activity.

2.2.2.2 Exchange Rate

Besides the economic activity, further macroeconomic variables like exchange rates and interest rates affect all commodity prices simultaneously. While most commodities are traded in U.S. dollars, only a small portion is mined in the United States. Hereby, Akram (2009) names the law of one price as the reason for a negative relationship between the dollar and the dollar price of commodities. A decline in the value of the dollar compared to other currencies leads to either an increase in the dollar price of the commodities or to a decrease in the foreign currency price to avoid arbitrage possibilities. Therefore, the returns of foreign commodity suppliers will decrease, which may result in a reduced commodity supply, and ultimately higher prices. In addition, a weaker dollar may raise the purchasing power and, consequently, will increase the commodity demand by foreign consumers, whereas the demand of consumers holding the U.S. dollar decreases.

Empirically, Akram (2009) detects a negative relation between the real dollar exchange rate and commodity prices as well as a significant impact of a shock to the dollar exchange rate to the movement in commodity prices in the period from 1989 to 2007. This finding is also supported by Ahumada and Cornejo (2014) for the period from 1960 to 2010, as they show a dollar depreciation against other major currencies leads to a significant, long-run increase in the dollar price of commodities. Further, Baffes and Dennis (2013), Baffes and Savescu (2014), Gilbert (1989) as well as Zhu et al. (2015) detect a strong and highly significant impact of exchange rates on food and metal prices as well as on commodity indices for the sample periods 1960 to 2012, 1991 to 2012, 1965 to 1986 and 2006 to 2013, respectively. However, Sari et al. (2010), investigating the link between precious metals, oil and the U.S. dollar/Euro exchange rate, do not find a long-run relation in the period from 1999 to 2007, whereas the commodity prices significantly decrease in the short-run to positive exchange rate shocks. Furthermore, the factor models of Lombardi et al. (2012) as well as Vansteenkiste (2009) reveal, for the sample periods from 1975 to 2008 and 1957 to 2008, the exchange rate even partly explains the common factor underlying in non-fuel commodity prices, underlining the relation between exchange rates and prices.

While exchange rates are strongly forward-looking and already include market expectations regarding future price dynamics of the country's commodity exports, commodity price fluctuations are typically more sensitive to short-term demand imbalances. In this context, Chen et al. (2010) theoretically explain the structural link between exchange rates and future commodity prices. They further find in an empirical analysis commodity currency exchange rates, reflecting the exchange rates of the commodity-exporting economies, have surprisingly robust power in predicting global commodity prices, in-sample as well as out-of-sample. However, the reverse relationship, commodity prices forecasting exchange rates, is less robust, highlighting the forward-looking property of exchange rates.

The studies of Buncic and Moretto (2015), Chen et al. (2014) and Pincheira-Brown and Hardy (2019) underline the predictive power of exchange rates, as their prediction models for commodity prices, including an exchange rate, outperform the random walk benchmark in an out-of-sample analysis. Hereby, Chen et al. (2014) use a trade-weighted U.S. exchange rate index against a subset of major currencies to forecast 51 commodity prices, whereas Pincheira-Brown and Hardy (2019) (Buncic and Moretto (2015)) include the Chilean peso (as well as the Australian Dollar) in the prediction of industrial metal prices (copper prices). In contrast, the results in the study of Pierdzioch et al. (2016), focusing on one of the major gold-producing countries, namely Australia, are mixed. In particular, they reveal the out-of-sample predictive power of real interest rates and exchange rates, specifically the Australian dollar/U.S. dollar exchange rate, depends on the shape of a forecaster's loss function.

Overall, exchange rates are an important determinant of commodity prices and also significantly improve commodity price forecasts, due to their forward looking properties. Hereby, the results in the literature emphasize the importance of this factor, as the studies draw the same conclusions regardless of the sample period, the methodology applied, as well as the commodities and/or exchange rates considered.

2.2.2.3 Monetary Policy

A further important determinant of commodity prices is monetary policy. Under the assumption of commodity prices behaving like regular, flexible asset prices traded in efficient markets, expected returns from investing in commodities should equal returns on investing in financial assets, to avoid arbitrage opportunities. Hereby, Akram (2009) and Frankel (2008) list several possible supply- and demand-sided related reasons for an inverse relationship between interest rates and commodity prices. First, investors will invest less in bonds and more in commodities in case of lower nominal interest rates, which leads to a rise in demand and, ultimately, to price increases. Second, lower interest rates reduce carrying costs, while simultaneously increasing inventory demand, which leads, again, to a rise in the overall demand and, ultimately, to price increases. Third, the extraction of exhaustible commodities, such as oil and minerals, is less profitable in low interest rate environments, which leads to a decrease in supply and, ultimately, to price increases. In addition, Frankel (2014) introduces a new carry trade model, whereby, he models the negative effect of interest rates on the demand for inventories and hence on commodity prices, as well as the positive effects of expected future price gains on inventory demand and hence on today's commodity prices. Further, Frankel and Rose (2010) state the inflation rate may approximate the impact of monetary policy on commodity prices, as storable commodities may serve as a hedge against inflation.[2]

In contrast, Svensson (2008) doubts that the theoretical negative correlation between interest rates and commodity prices can also be observed empirically, since the relationship depends on the shock hitting the economy and its effect on all variables, as the interest rate and the economic activity are interdependent themselves. Moreover, Baffes and Dennis (2013) takes up the arguments of Newbery and Stiglitz (1981) that a contrarian monetary policy increases the required rate of return on storage and therefore may increase the commodity prices. In addition, Hammoudeh et al. (2015) argue central banks rise interest rates in response to a "heating" economy, leading to synchronous patterns in prices and interest rates, see for example Gubler and Hertweck (2013) for the empirical evidence of increasing rates in response to a shock in the commodity markets.[3]

Empirically, the evidence in the literature is mixed. While Frankel (2008) confirms his theory of an inverse effect in the analysis of commodity prices in the period from 1950 to 1979 and from 1950 to 2005, the observed relation between interest rates and commodity prices is positive in the period from 1976 to 2005 and from 1980 to 2005. However, several studies support the inverse relation. In particular, Akram (2009) as well as Anzuini et al. (2013) observe shocks to the interest rate significantly affect commodity prices, reflected by commodity indices, in the period from 1989 to 2007 and 1970 to 2008, but Akram (2009) also confirms the shock dependence of Svensson (2008). In particular, a positive shock in the economic activity leads to synchronous responses in the real interest rate, the oil price as well as the commodity price. Moreover, Smiech et al. (2015) detect significant inverse responses of (non-)energy commodity prices to shocks in the interest rates of the euro area economy in the period from 1997 to 2013. Further, interest rates affect precious metal prices and non-fuel commodity prices significantly with a negative sign in the factor models of Apergis et al. (2014) and Byrne et al. (2013), investigating the sample periods from 1981 to 2010 and 1900 to 2008. In addition, interest rates explain, at least partly, the common factor underlying in non-fuel commodity prices in the period from 1957 to 2008 in the study of Vansteenkiste (2009).

While Baffes and Savescu (2014) show a strong negative link between metal prices and interest rates in their bi-variate regressions, the results of their multivariate model are mixed for the sample period from 1991 to 2012. Further, Siami-Namini (2021) investigates the impact of monetary policy, measured by short- and long-term interest rates as well as the M2 money stock, an estimate for the total money supply of the U.S. Federal Reserve, on an aggregate commodity index in the period from 1992 to 2017 and only detects long-term effects, but not in the short-term, whereby the findings are independent of the monetary policy measure under consideration. Moreover, Schischke and Rathgeber (2023) confirm the positive relation between commodity indices and interest rates of Hammoudeh et al. (2015) at least for the period prior to the financial crisis from 1995 to 2008. Hereby, shocks to the interest rate (commodity prices) lead

[2]Empirically, the factor models of Apergis et al. (2014) as well as Kagraoka (2016) reveal the inflation rate helps explaining the common factors underlying in commodity prices.

[3]Gubler and Hertweck (2013) investigate the impact of commodity prices on the economy. Hereby, they reveal a shock to the commodity prices cause a higher inflation rate, whereupon the Federal Reserve reacts with higher interest rates.

to significantly rising prices (interest rates) in the short-(long-)term, emphasizing the argument of Hammoudeh et al. (2015) that central banks react to increasing prices. In addition, Frankel and Rose (2010) find little support for an impact of easy monetary policy and low real interest rates on individual commodity prices, besides any effect monetary policy actions might have via real economic activity and inflation, in their analysis of the period from 1960 to 2008. While the factor models of Lombardi et al. (2012) and Nicola et al. (2016) reveal interest rates do not affect individual commodity prices in the period from 1975 to 2008 and 1970 to 2013, respectively, interest rates are not even selected in the models of Ahumada and Cornejo (2014) and Kagraoka (2016), investigating the sample period from 1960 to 2010 and 1995 to 2015.

However, the model of Ahumada and Cornejo (2014) includes the U.S. Monetary Base as monetary policy variable. Moreover, Apergis et al. (2014) detect the monetary supply positively affects the prices of precious metals, simultaneously to the negative impact of interest rates. Further, commodity prices significantly react to unconventional monetary policy actions in the period following the financial crisis, see Schischke and Rathgeber (2023). Hereby, Apergis et al. (2020) reveal the impact of unconventional monetary policy on commodity prices as well as their volatility is even more pronounced than of interest rates, in the period from 1990 to 2016.

One possible reason for the mixed empirical evidence of an inverse relation between interest rates and commodity prices might be the zero interest rate environment as consequence of the financial crisis. To further strengthen the U.S. economy, the Federal Reserve System (FED) increased its unconventional monetary policy actions, see for example Keating et al. (2019), Eksi and Tas (2017) as well as Peersman et al. (2021), which is why the effect of interest rates on commodity prices might not fully display the impact of the entire monetary policy on commodity prices.

Overall, the evidence on the impact of interest rates on commodity markets is ambiguous in the literature. Hereby, several studies detect an inverse relation of commodity prices to interest rates, whereas especially more recent studies observe synchronous patterns or interest rates are even unrelated to commodities, probably caused by the zero interest rate environment. Hereby, the impact of unconventional monetary policy actions on commodity prices is more pronounced than the effect of interest rates, see Apergis et al. (2020).

2.2.2.4 Further Determinants

Beyond the classical supply and demand, the global demand, the exchange rate and the monetary policy, further determinants of commodity prices are examined in the literature. In particular, the impact of the oil price, the stock market, as well as demographic factors on commodity prices are analyzed.

For instance, the oil price directly influence the costs of the production process. Hereby, Hammoudeh et al. (2004) underline the price of crude oil influences national economies in general, whereby the impact is more pronounced on the highly oil intensive manufacturing industries, e.g. aluminum and steel. Moreover, the oil price also affects the transportation costs, overall leading to changes in metal prices, see Zhang and Tu (2016).

However, the empirical evidence is heterogeneous. Although Baffes and Savescu (2014) investigate the impact of low interest rates on commodity prices, they account for energy costs, by including the oil price as a proxy, but do not detect any significant effect of oil on the analyzed industrial metals prices. In contrast, Dutta (2018), Liberda (2017), Robinson (2019) and Zhu et al. (2015) highlight the importance of oil prices on precious metal prices. Moreover, Baffes and Dennis (2013) detect the food price increase post-2004 was mainly driven by crude oil prices, probably due to the biofuels trend. While in the factor models of Kagraoka (2016) and Vansteenkiste (2009) the oil price represents one of the identified common factors, Lombardi et al. (2012) do not find strong spillovers from oil to non-energy prices, indicating mixed evidence for the effect of the oil price on commodity prices. In particular, the results in the literature depend on the examined commodities, the considered sample, as well as on the applied methodology.

Due to the financialization of commodity markets, financial institutions such as hedge funds and commodity index funds play an important role in commodity futures markets, as commodities represent an alternative asset class for investors, see Büyüksahin and Robe (2014).[4] Therefore, Liberda (2017) expects and confirms spillover effects from the stock market to commodity markets in his comprehensive analysis of the determinants of precious metal prices. In addition, one of the four identified common factors in the study of Kagraoka (2016) is described by the MSCI world stock index (MSCI), which also may be interpreted as a demand indicator.

Further, the demographic structure influences the economy and therefore also commodity prices. Hereby, Aksoy et al. (2019) detect the age profile of the population significantly affects output growth, investment, savings, hours worked per capita, real interest rates, and inflation. Moreover, the expected increase in the share of the population aged 60 and over will lower labor force participation as well as savings rates. Overall, a growing population and a higher employment will lead to a rising demand for commodities. In addition, Apergis et al. (2014) identify the unemployment rate as one common factor in their analysis of precious metal prices, while the model fit is increased in the study of Dimitropoulos and Yatchew (2018), once the unemployment rate is included.

To summarize, several commodity-specific, macroeconomic and demographic conditions might affect commodity prices. First and foremost, commodity prices are determined by their supply and demand, according to the classical fundamental theory, see Deaton and Laroque (2003), whereby many studies approximate the commodity demand by the global economic activity, see Ahumada and Cornejo (2014). Second, macroeconomic variables like the economic activity, exchange rates, and interest rates affect all commodity prices simultaneously, see Akram (2009). In this context, growing economies cause an increased demand, while a decline in the dollar price would lead to an increase in the dollar price of commodities due to the law of one price, since most commodities are traded in U.S. dollars but only a small portion is mined in the United States. Moreover, monetary policy and commodity prices are strongly linked. While the effects of interest rates on prices are ambiguous, several studies emphasize the importance of monetary policy on commodity markets. Overall, the literature reveals supply and demand, economic activity and exchange rates are further important determinants of commodity prices, whereas the evidence of the oil price, stock market indices or demographic variables are mixed. Hereby, the results depend on the considered commodities, included determinants as well as on the applied methodology and sample period.

2.3 Co-movement and Financialization

This thesis aims to analyze the resource requirements of several transformation pathways for the German Energiewende in regard to their availability, respectively their scarcity. Hereby, we examine the impact of the German Energiewende on a portfolio of commodities. However, since the macroeconomic determinants described above affect all commodities simultaneously, commodity prices tend to move synchronously, which is referred to as co-movement. While some studies analyze the common cycles in commodity prices, cointegration as well as correlation analyses aim to empirically proof the observable common pattern in prices and attribute the co-movement, at least partly, to a common factor. Hereby, various studies investigate the extent to which a simple static factor model captures co-movements of commodity prices, as well as

[4]Please refer to Greer (2000) and Gorton and Rouwenhorst (2006) for stylized facts on commodities as an asset class.

the determinants and the time-varying properties of this common factor.

First, the studies of Cashin et al. (2002), Cuddington and Jerrett (2008), and Rossen (2015), analyze the common cycles in commodity prices. Hereby, Cuddington and Jerrett (2008) identify three super cycles, which are highly correlated, in the six London Metal Exchange (LME) industrial metal prices within the past 150 years. However, Rossen (2015) confirms four super cycles in 20 metal commodity prices during the last century on a monthly basis. Moreover, she observes the short-run price cycles differ between metals. In addition, Cashin et al. (2002) examine the properties of the price cycles of 36 commodities and detect an asymmetric behavior of cycles, in particular, the duration of slumps is longer.

Second, Issler et al. (2014) underline the LME industrial metal prices share a common cycle with the industrial production, using cointegration tests. Moreover, their bivariate cointegration tests between the metals suggest common trends. Further, the cointegration results of Ding and Zhang (2020) are commodity-specific. Oil and copper exhibit a long-run price equilibrium with other individual commodity markets as well as the Refinitiv/CoreCommodity CRB index (CRB), whereas the agricultural and the gold market are only cointegrated when controlling for liquidity.

Third, several studies aim to explain the underlying determinants of the co-movement between commodity prices. Hereby, the common factor analyses draw different conclusions, dependent on the commodities and period under consideration. In this context, Zhang et al. (2019), who find evidence for co-movement in prices of crude oil, corn, gold, live cattle and silver in the period from 2005 to 2013, are able to attribute the co-movement to a common liquidity factor. Moreover, the empirical study of West and Wong (2014), analyzing the extent to which a simple static factor model captures co-movements of 22 commodity prices in the period from 1986 to 2012, detects commodity prices tend to revert (slowly) towards the factor. However, Byrne et al. (2013) identify one common factor which is related to interest rates and risk in their long-term analysis of 24 commodities in the period from 1900 to 2008, whereas Chen et al. (2014) detect two underlying common factors in 51 commodity prices in the period from 1980 to 2009, one stationary and one non-stationary, captured by the exchange rate. Further, Lombardi et al. (2012) observe the common trends in 15 individual commodity prices in the period from 1975 to 2008 can be explained by a food or metals factor. Moreover, Yin and Han (2015) decompose 24 commodity prices into global, sectoral and commodity-specific components over the period from 1991 to 2014 and highlight the impact of the global as well as the sectoral component increases significantly around 2004, when commodity index investment started.

Besides the simultaneous influence of the economy on commodity markets, the commodities are interrelated themselves via co-production and co-consumption links. On the one hand, several metals are mined together. While some metals are co-produced, for example 70% of the lead production is derived from mixed Lead-Zinc ores, others are only extracted as companion metals and therefore, their supply depends on the production volume of their hosts' metals, e.g. cobalt and indium, according to Nassar et al. (2015). On the other hand, metals are consumed together. For instance, aluminum and copper are both important metals for various automotive applications, see Zapp et al. (2002).

However, the empirically observed co-movement of prices on exchanges is larger than what would be explainable, see Pindyck and Rotemberg (1990), either by the common consumption and production of commodities, see Shammugam et al. (2019), or by the common factor, reflecting the common effects of macroeconomic determinants, see Section 2.2.2. Hereby, Pindyck and Rotemberg (1990) were the first introducing the concept of excess co-movement, as they detect even unrelated commodities co-move, but the evidence in the literature is ambiguous, depending on the methodology applied, the included control variables as well as the considered time period.

Reconsidering the original study of Pindyck and Rotemberg (1990) using a new methodology, accounting for heteroscedasticity and non-normality, Deb et al. (1996) only find weak to no evidence for the excess co-movement hypothesis. Moreover, while Lescaroux (2009) confirms common patterns in 51 commodities, he rejects the excess co-movement hypothesis, as the observed co-movement is mostly caused by common macroeconomic shocks on the supply and demand of the commodities. In contrast, the results of Fernandez (2015a), testing for the excess co-movement in twelve commodities using data from 1900 to 2013, confirm the excess co-movement, but Fernandez (2015a) states the extent of the excess co-movement varies over time. Moreover, Nicola et al. (2016) conclude energy and agricultural commodities are highly correlated with an increasing co-movement in their analysis of the extent of co-movement in eleven energy, agricultural, and food commodities. Using a new measure of global co-movement, representing the average influence of commodities based on nominal prices and cyclical components of real prices, Fernandez (2015b) detects a strong co-movement between industrial and precious metal prices since 2003. However, besides an increasing long-run trend, starting at the beginning of the 21st century, Ohashi and Okimoto (2016) only find little short-run fluctuations. Further, the reinvestigation study of Le Pen and Sévi (2017) reveals the excess co-movement is time-varying and larger in magnitude after 2007. Hereby, a measure of speculative intensity mainly determines the excess co-movement which is why Le Pen and Sévi (2017) attribute their results to the financialization in commodity prices.

In general, the financialization describes the rapidly growing index investment in commodity markets, starting in the early 2000s, when non-commercials enter commodity futures markets. Hereby, Büyüksahin and Robe (2014) investigate the effects of the financialization by an analysis of the cross-market co-movement between commodity and equity futures markets using unique daily Commodity Futures Trading Commission (CFTC) data from 2000 to 2010. In addition, the correlation analysis between stocks, bonds and commodity futures returns of Silvennoinen and Thorp (2013) indicates a closer integration of markets in the recent years.

Moreover, Basak and Pavlova (2016) investigate, theoretically, how the financialization affects commodity prices. While all commodity futures prices, their volatilities as well as correlations increase with the financialization, this effect is stronger for index futures than for non-indexed commodities. In this context, Tang and Xiong (2012) detect the correlations of oil prices with non-energy commodity futures prices, especially with indexed commodities, grew due to the financialization starting 2004, which is why they conclude the individual commodity prices are not only determined solely by their supply and demand, but also by the investment behavior. Additionally, Pradhananga (2016) state the financialization partly caused the increase in the co-movement between 40 commodity prices. Hereby, Poncela et al. (2014) aim to determine whether the increase in co-movement is long-lasting. Therefore, they consider the extent of co-movement in 44 monthly non-energy commodity prices and detect an increased synchronization among raw materials since December 2003, indicating the financialization changed the relationship between commodity markets.

In this context, Bilgin et al. (2018) and Song et al. (2022) detect the connectedness between commodity markets is time-varying, while the studies of Aepli et al. (2017), Irwin and Sanders (2012), and Zhang and Broadstock (2020) even reveal structural changes in commodity markets. In particular, Zhang and Broadstock (2020) suggest the commodity market structure has changed fundamentally, as the co-dependence among seven major commodity classes tripled after the global financial crisis. Lübbers and Posch (2016) confirm the increased co-movement in 31 commodity futures returns in their generalized dynamic factor model. While Zhang and Broadstock (2020) observe one structural break in the connectedness of commodity markets, Irwin and Sanders (2012) even state three structural changes occurred in agricultural futures markets. By accounting for the time-varying properties in commodity markets, Aepli et al. (2017) also observe a change in the correlation structure in times of the financial crisis, compared to normal regimes. Moreover, the cointegration analysis of Bilgin et al. (2018) indicates most cointegration relations are time-varying and Song et al. (2022) detect time-varying spillovers between main and by-products.

In addition to the demand increase through the financialization, Peersman et al. (2021) name, inter alia, the demand increase from emerging countries, as well as spillovers due to substitutions as possible reasons for the stronger co-movement in commodity prices. In particular, they underline the importance of time-varying analyses of commodity markets. This is further emphasized by Zaremba et al. (2021). They doubt the financialization caused an unprecedented surge in the co-movement, as they identify similar peaks in earlier periods in their long-term analysis of the past 150 years, suggesting the commodity prices observe periods of strong similar patterns, followed by periods of more divergent price behavior. In this line, Ciner et al. (2020) also detect time periods of increasing and decreasing co-movement between the LME non-ferrous industrial metal prices during 1994 to 2016, whereby the spillover effects were stronger after the financial crisis. Moreover, Byrne et al. (2020) investigate responses of commodity markets to demand and real interest shocks, where they reveal heterogeneous results between commodity groups, as well as between different time periods. Hereby, they attribute the time-varying properties to the variation in commodity market participants. In particular, the speculation and therefore the inflow of capital in commodity markets increased in the 2000s, with classical commercial hedgers and non-commercial traders, such as financial institutions, being active in markets simultaneously.

Overall, various studies detect commodity prices tend to move in a synchronized way. While several studies aim to explain the co-movement, at least partly, by a common factor, which is attributed to economic variables like global demand, exchange rates and interest rates, Pindyck and Rotemberg (1990) reveal the empirically observed co-movement of prices on exchanges is larger than what would be explainable by the joint consumption, production as well as by the common factor. Therefore, shocks in one commodity market would probably transmit to the other markets, underlining the importance of jointly modeling commodity markets, especially in risk assessment frameworks. Moreover, due to the rapidly growing index investments in commodity markets, starting in the early 2000s, as well as the demand growth in emerging markets, the co-movement in commodity prices as well as the connectedness with the stock markets increased, see Büyüksahin and Robe (2014) and Tang and Xiong (2012). In particular, the financialization significantly changed the commodity market structure, highlighting the importance of time-varying analyses of commodity markets.

2.4 Summary

Various studies investigate the criticality of commodities, as well as the determinants of and the co-movement between commodity prices, see Table 2.1 (Table A.1) for a (detailed) overview. In particular, several studies investigate the criticality of various metals, analyzing inter alia the supply risk, the vulnerability to supply restrictions, the environmental and social impacts, in general or in the context of the energy transition. However, the increase in the demand for commodities, caused by the energy transition, can lead to price peaks and delivery bottlenecks in their supply, which is oftentimes neglected in previous studies. Therefore, a comprehensive understanding of commodity markets is essential. On the one hand, several studies examine the determinants of commodity prices, starting with the commodity-specific factors, since the classical fundamental theory states that supply and demand determine prices, to macroeconomic factors. On the other hand, the empirical observation of common patterns, the so-called co-movement, in commodity prices is analyzed. Hereby, several studies examine the extent and the determinants of the co-movement. Moreover, the time-varying properties indicate the common pattern in prices changed over time, and especially increased due to the financialization as well as the growth in emerging markets and the corresponding increased interest in commodity markets.

Table 2.1: Literature overview

Subject	Studies
Criticality	Arendt et al. (2020), Bach et al. (2016), Bach et al. (2017), Erdmann and Graedel (2011), Gleich et al. (2013), Graedel et al. (2012), Graedel et al. (2015), Graedel and Reck (2016), Hayes and McCullough (2018), Ioannidou et al. (2019), Jordan (2017), Kolotzek et al. (2018), Nassar et al. (2015), Redlinger and Eggert (2016), Rosenau-Tornow et al. (2009), Schrijvers et al. (2020), Skinner (1979), Tilton (2010), Tilton et al. (2018)
Criticality in the context of the energy transition	Arrobas et al. (2017), European Commission (2020), Lee et al. (2020), Liang et al. (2022), Marscheider-Weidemann et al. (2021), Roelich et al. (2014), Song et al. (2022), Valero et al. (2018), Viebahn et al. (2015), Watari et al. (2020)
Supply and demand	Ahumada and Cornejo (2014), Baffes and Dennis (2013), Borensztein and Reinhart (1994), Carter et al. (2011), Chen et al. (2019), Cuddington and Zellou (2013), Deaton and Laroque (1992), Deaton and Laroque (2003), Frankel and Rose (2010), Guzmán and Silva (2018), Helbling et al. (2008), Hotelling (1931), Kilian (2009), Kilian and Zhou (2018), Lutzenberger et al. (2017), Nick and Thoenes (2013), Stuermer (2018), Thomas et al. (2010)
Economic activity	Ahumada and Cornejo (2014), Akram (2009), Alquist and Coibion (2013), Baffes et al. (2020), Borensztein and Reinhart (1994), Buncic and Moretto (2015), Camacho and Perez-Quiros (2014), Chen (2010), Dimitropoulos and Yatchew (2018), Gargano and Timmermann (2014), Issler et al. (2014), Kagraoka (2016), Kilian (2009), Kilian and Zhou (2018), Klotz et al. (2014), Lombardi et al. (2012), Robinson (2019), Smiech et al. (2015)
Exchange rate	Ahumada and Cornejo (2014), Akram (2009), Baffes and Dennis (2013), Baffes and Savescu (2014), Buncic and Moretto (2015), Chen et al. (2010), Chen et al. (2014), Gilbert (1989), Lombardi et al. (2012), Pierdzioch et al. (2016), Pincheira-Brown and Hardy (2019), Sari et al. (2010), Vansteenkiste (2009), Zhu et al. (2015)
Monetary policy	Ahumada and Cornejo (2014), Akram (2009), Anzuini et al. (2013), Apergis et al. (2014), Apergis et al. (2020), Baffes and Dennis (2013), Baffes and Savescu (2014), Byrne et al. (2013), Eksi and Tas (2017), Frankel (1986), Frankel (2008), Frankel (2014), Frankel and Rose (2010), Gubler and Hertweck (2013), Guzmán and Silva (2018), Hammoudeh et al. (2015), Kagraoka (2016), Keating et al. (2019), Lombardi et al. (2012), Nicola et al. (2016), Peersman et al. (2021), Schischke and Rathgeber (2023), Siami-Namini (2021), Smiech et al. (2015), Svensson (2008), Vansteenkiste (2009)
Inflation rate	Apergis et al. (2014), Frankel and Rose (2010), Kagraoka (2016)
Oil	Baffes and Dennis (2013), Baffes and Savescu (2014), Dutta (2018), Hammoudeh et al. (2004), Kagraoka (2016), Liberda (2017), Lombardi et al. (2012), Robinson (2019), Vansteenkiste (2009), Zhang and Tu (2016), Zhu et al. (2015)
Stock indices	Büyüksahin and Robe (2014), Kagraoka (2016), Liberda (2017)
Demographic attributes	Aksoy et al. (2019), Apergis et al. (2014), Dimitropoulos and Yatchew (2018)
Super-cycles & Cointegration	Cashin et al. (2002), Cuddington and Jerrett (2008), Ding and Zhang (2020), Issler et al. (2014), Rossen (2015)
Common Factor	Byrne et al. (2013), Chen et al. (2014), Lombardi et al. (2012), West and Wong (2014), Yin and Han (2015), Zhang et al. (2019)
Financialization	Basak and Pavlova (2016), Büyüksahin and Robe (2014), Poncela et al. (2014), Pradhananga (2016), Silvennoinen and Thorp (2013), Tang and Xiong (2012)
(Excess) Co-Movement	Deb et al. (1996), Fernandez (2015a), Fernandez (2015b), Le Pen and Sévi (2017), Lescaroux (2009), Nassar et al. (2015), Nicola et al. (2016), Ohashi and Okimoto (2016), Pindyck and Rotemberg (1990), Shammugam et al. (2019), Zapp et al. (2002)
Time-varying co-Movement	Aepli et al. (2017), Bilgin et al. (2018), Byrne et al. (2020), Ciner et al. (2020), Fernandez (2015a), Irwin and Sanders (2012), Le Pen and Sévi (2017), Lübbers and Posch (2016), Ohashi and Okimoto (2016), Peersman et al. (2021), Poncela et al. (2014), Song et al. (2022), Zaremba et al. (2021), Zhang and Broadstock (2020)

This table provides an overview over the above mentioned studies investigating the criticality of commodities (in the context of the energy transition), analyzing potential, commodity price influential factors and focusing on the common behavior in commodity prices.

3 Methodology

The objective of this study is the analysis and comparison of resource-demanding projects, such as the transformation of the energy system towards renewable technologies for the German Energiewende, in terms of their resource scarcity risk. Therefore, we develop a new risk assessment framework in which we explicitly account for the additional resource demand caused by the investigated project. In this context, we initially propose a new model for commodity markets incorporating the impact of fundamentals on - as well as the co-movement between - commodity prices. Subsequently, we extend it to account for time-varying spillover effects. By interpreting the price of a commodity as a scarcity indicator, we are able to quantify the resource scarcity risk, while taking into account the entire commodity market behavior, in particular, the interrelationship between the commodity-specific supply, demand and price, the co-movement between multiple commodity prices as well as the impact of macroeconomic circumstances.

First, we propose a new framework for commodity markets in Section 3.1, combining two perspectives in the commodity market literature: the classical fundamental theory, which examines the micro- and macroeconomic drivers of commodity prices, and the empirical market perspective, which observes common movements of commodity prices on exchanges. While this framework is based on the global vector autoregression (GVAR) model, originally developed by Pesaran et al. (2004) to analyze the world economy from an individual country level, under the limitation of small sample data sets, we are the first adopting this idea to commodity markets. Hereby, we model each commodity market separately using vector autoregression (VAR) models with the commodity-specific, microeconomic variables supply, demand and price as well as exogenous, macroeconomic attributes. Finally, the individual VAR models are linked by appropriate weight matrices based on information on co-production, co-consumption, and co-trading of the commodities to form a global commodity market model, which allows the analysis of spillover effects within and between the commodity markets via generalized impulse response functions (GIRFs).

Second, we extend the global vector autoregression (GVAR) model to a Markov-switching global vector autoregression (MS-GVAR) model in Section 3.2, based on the idea of Binder and Gross (2013) for economies, to account for time-varying relations in commodity markets and to disentangle the differences in the spillover effects at different points in time, since the relation between commodity markets, especially the co-movement between commodity prices, increased during the financialization starting in 2004, see Tang and Xiong (2012). While Binder and Gross (2013) propose their MS-GVAR model with time-varying intercepts, we adopt the idea for commodity markets, but generalize their framework using various Markov-switching specifications. Hereby, we explicitly consider time-varying intercepts, time-varying autoregressive parameters, time-varying parameters associated with the exogenous variables as well as time-varying covariance matrices. Subsequently, we analyze the dynamic properties of the commodity markets via regime-dependent GIRFs.

Third, we propose a framework to analyze resource-demanding projects in terms of their resource scarcity risk by interpreting a commodity's price as a scarcity indicator. In this scarcity framework, we explicitly account for the additional resource demand from large-scale projects such as the German Energiewende, which leads to significant demand increases, due to the build-up of renewable energy technologies. Using the prices derived from the commodity market models, see Section 3.1 and the extended version in Section 3.2, we obtain the individual probability of scarcity (PS) for each commodity of the project. Subsequently, we get our commodity-specific risk indicator by combining the individual probability of scarcity with a substitutability score and the scaled demand of the commodity for a specific project. Finally, we aggregate the commodity risk scores at the project level to compare the scarcity risk of several project alternatives.

3.1 Global Vector Autoregression (GVAR) Model[1]

In the following, we introduce the global vector autoregression (GVAR) model for commodity markets which combines the classical fundamental theory with the empirical observed comovement between prices. Hereby, we first focus on the fundamental theory and model each individual commodity market separately, reflecting the impact of supply and demand on prices. Second, we aggregate these individual models to the global commodity market model, which allows for interdependencies between the commodities. Subsequently, we propose generalized impulse response functions (GIRFs) to investigate the spillover effects within and between commodity markets as well as generalized forecast error variance decompositions (GFEVDs) to provide further insights into the markets.

3.1.1 The Initial GVAR Model

According to the classical fundamental theory, the price of a commodity is the result of its supply and demand equilibrium and hence, determined by its fundamentals, supply and demand. Therefore, we apply individual vector autoregression (VAR) models on all commodities $i = 1, 2, \ldots, N$ of the analysis, to simultaneously model the dependencies between the commodity-specific supply ($\mathbf{supply}_i$), demand ($\mathbf{demand}_i$) and price ($\mathbf{price}_i$)[2] variables, which form the vector $\mathbf{x}_{i,t} = (supply_{i,t}, demand_{i,t}, price_{i,t})'$, for all time periods $t = 1, 2, \ldots, T$:

$$\mathbf{x}_{i,t} = \mathbf{a}_{i,0} + \sum_{p=1}^{P} \mathbf{\Phi}_{i,p}\mathbf{x}_{i,t-p} + \sum_{p=0}^{P_{exog}} \mathbf{\Psi}_{i,p}\mathbf{e}_{t-p} + \boldsymbol{\varepsilon}_{i,t}, \tag{3.1}$$

where $\mathbf{a}_{i,0}$ denotes the $K_i \times 1$ intercept vector and $\mathbf{\Phi}_{i,p}$ are the $K_i \times K_i$ matrices of lagged coefficients for lag $p = 1, 2, \ldots, P$, with $K_{endog} = K_i = 3$, for $i = 1, 2, \ldots, N$, denoting the number of endogenous variables[3] summarized in the vector $\mathbf{x}_{i,t}$ and the maximum lag length P. As the economy affects all commodity markets, we aim to represent the common impact by including K_{exog} macroeconomic factors as exogenous variables[4] in the vector $\mathbf{e}_t$ with $\mathbf{\Psi}_{i,p}$ as $K_i \times K_{exog}$ matrices of the corresponding coefficients for lags $p = 1, 2, \ldots, P_{exog}$. Further, we assume that the $K_i \times 1$ vectors of idiosyncratic commodity-specific shocks $\boldsymbol{\varepsilon}_{i,t}$ are serially

[1]Parts of this section are included in the paper "Three Co's to Jointly Model Commodity Markets: Co-Production, Co-Consumption and Co-Trading", accepted for publication in Empirical Economics, 2023, co-authored by Patric Papenfuß, and Andreas Rathgeber.

[2]The original price variables are non-stationary, according to the augmented Dickey-Fuller (ADF) test, initially proposed in Dickey and Fuller (1979), see Section 4.2.1. Moreover, the Johansen test reveals the variables are not cointegrated, which is why vector error correction (VECM) models would not be feasible from the statistical point of view. Therefore, we estimate VAR models on the logarithmic returns of the variables.

[3]In general, the number of endogenous variables K_i can differ across the commodity markets, see Pesaran et al. (2004). In our case, each commodity market contains the same attributes, commodity-specific supply, demand and price, therefore, the number of endogenous variables equals, $K_{endog} = K_i = 3$.

[4]Since metal markets are comparably small, we assume exogeneity of all macroeconomic fundamentals and include them as exogenous variables in our models.

uncorrelated, independent and identically distributed, with mean zero and covariance matrix $\mathbf{\Sigma}_{ii}$. Therefore, $\boldsymbol{\varepsilon}_{i,t} \sim iid\,(\mathbf{0}, \mathbf{\Sigma}_{ii})$.

While these individual VARs simultaneously model the interdependencies between commodity-specific supply, demand and price, taking into account the common effect of macroeconomic factors, they are unable to holistically represent the relationships between commodity markets, inter alia the entire co-movement observed on exchanges, as the only connection considered between the commodities is via macroeconomic information. However, beyond the macroeconomic factors, the commodity markets are interconnected through co-production, co-consumption, as well as co-trading, which is why individual VAR models are insufficient to represent the complexity of commodity markets.

One solution would be the estimation of a single VAR model, including all commodity-specific variables as well as the macroeconomic determinants. In general, each equation of such a model consists of $P \cdot (K_{endog} \cdot N) + (P_{exog} + 1) \cdot K_{exog}$ parameters,[5] where P denotes the order of the VAR model, N the number of commodities in the analysis, K_{endog} the number of variables per commodity, P_{exog} the number of lags of the exogenous variables and K_{exog} the number of macroeconomic determinants, see Pesaran et al. (2004). In case of three commodities ($N = 3$), three variables per commodity ($K_{endog} = 3$), one lag for the endogenous as well as exogenous variables ($P = 1$ and $P_{exog} = 1$) and three exogenous variables ($K_{exog} = 3$), this leads to 15 parameters per equation. However, once we analyze 10 or 20 commodities, the number of parameters already increases to 36 or 66 respectively.

For a long-term analysis of the relation between commodity markets, commodity-specific supply and demand variables are only available at annual frequency which is why the estimation of these models is infeasible from a statistical point of view. In general, low data frequency in conjunction with many potentially influential variables is a key problem in econometrics. Pesaran et al. (2004) propose a way to overcome these data limitation issues by combining several individual vector autoregression (VAR) models into one global vector autoregression (GVAR) model.

While the GVAR model was initially constructed for the world economy, we are the first adopting the idea to commodity markets.[6] Therefore, we simultaneously estimate several individual, commodity-specific VAR models consisting of commodity-specific supply, demand and price variables, as well as the macroeconomic determinants. To connect these individual models, we present the methodology of the GVAR model based on the original paper of Pesaran et al. (2004) as well as on Dées, di Mauro, Pesaran, and Smith (2007) and Dées, Holly, Pesaran, and Smith (2007) in the following.

So far, the individual commodity market models only account for the impact of the commodity-specific variables as well as the common impact of the economy on commodity markets. To reflect the impact of the other commodity markets on the market of a specific commodity i, we extend the commodity-specific VAR models from Equation 3.1 with the $K_i^* \times 1$ vector $\mathbf{x}_{i,t}^* = \left(supply_{i,t}^*, demand_{i,t}^*, price_{i,t}^*\right)'$ of weighted external variables specific to commodity i:

$$\mathbf{x}_{i,t} = \mathbf{a}_{i,0} + \sum_{p=1}^{P} \mathbf{\Phi}_{i,p}\mathbf{x}_{i,t-p} + \sum_{p=0}^{P} \mathbf{\Lambda}_{i,p}\mathbf{x}_{i,t-p}^* + \sum_{p=0}^{P_{exog}} \mathbf{\Psi}_{i,p}\mathbf{e}_{t-p} + \boldsymbol{\varepsilon}_{i,t}, \tag{3.2}$$

where $\mathbf{\Lambda}_{i,p}$ are $K_i \times K_i^*$ matrices of coefficients associated with the weakly exogenous, external specific variables, for $p = 0, 1, \ldots, P$,[7] and in our case $K_i^* = K_i = 3$. These external commodity

[5] In line with Pesaran et al. (2004), we neglect the parameters of the intercept vector.

[6] The idea to adopt the GVAR model from economies to commodity markets has culminated in the paper "Three Co's to Jointly Model Commodity Markets: Co-Production, Co-Consumption and Co-Trading", accepted for publication in Empirical Economics, 2023, co-authored by Patric Papenfuß, and Andreas Rathgeber.

[7] For a simpler notation, we assume that the lag length of the external variables equals to the lag length of the

variables represent the weighted, aggregated information of the other commodity markets and are defined as:

$$\begin{aligned} supply^*_{i,t} &= \sum_{\tilde{\imath}=1}^{N} w_{i,\tilde{\imath}} \cdot supply_{\tilde{\imath},t}, \\ demand^*_{i,t} &= \sum_{\tilde{\imath}=1}^{N} w_{i,\tilde{\imath}} \cdot demand_{\tilde{\imath},t}, \\ price^*_{i,t} &= \sum_{\tilde{\imath}=1}^{N} w_{i,\tilde{\imath}} \cdot price_{\tilde{\imath},t}, \end{aligned} \tag{3.3}$$

with weights $w_{i,i} = 0$ and $\sum_{\tilde{\imath}=1}^{N} w_{i,\tilde{\imath}} = 1$, for $i = 1, 2, \ldots, N$. The corresponding individual weights $w_{i,\tilde{\imath}}$ may be combined to a weight matrix $(w_{i,\tilde{\imath}})_{i,\tilde{\imath}=1,2,\ldots,N}$. While the initial GVAR model of Pesaran et al. (2004) uses import and export data, the so called trade weights, to link the individual economies into one model, our framework incorporates information from common supply, demand and trading activity to link the individual commodity markets, see Section 4.4.

In order to set up the final global commodity market model, we define the $(K_i + K_i^*) \times 1$ vector $\mathbf{z}_{i,t} = \left(\mathbf{x}'_{i,t}, \mathbf{x}^{*\prime}_{i,t}\right)'$ and rewrite Equation 3.2 for $i = 1, 2, \ldots, N$:

$$\mathbf{A}_{i,0}\mathbf{z}_{i,t} = \mathbf{a}_{i,0} + \sum_{p=1}^{P} \mathbf{A}_{i,p}\mathbf{z}_{i,t-p} + \sum_{p=0}^{P_{exog}} \mathbf{\Psi}_{i,p}\mathbf{e}_{t-p} + \boldsymbol{\varepsilon}_{i,t}, \tag{3.4}$$

where $\mathbf{A}_{i,0} = (\mathbf{I}_{K_i}, -\mathbf{\Lambda}_{i,0})$ is a $K_i \times (K_i + K_i^*)$ dimensional matrix, with $\mathbf{I}_{K_i}$ denoting the $K_i \times K_i$ dimensional identity matrix, and $\mathbf{A}_{i,p} = (\mathbf{\Phi}_{i,p}, \mathbf{\Lambda}_{i,p})$ are also $K_i \times (K_i + K_i^*)$ dimensional matrices, for $p = 1, 2, \ldots, P$. Moreover, we require $\mathbf{A}_{i,0}$ to have full row rank for $i = 1, 2, \ldots, N$.

Further, we denote by $\mathbf{x}_t = \left(\mathbf{x}'_{1,t}, \mathbf{x}'_{2,t}, \ldots, \mathbf{x}'_{N,t}\right)'$ the $K \times 1$ global vector of all commodity-specific variables, where $K = \sum_{i=1}^{N} K_i$. With the $(K_i + K_i^*) \times K$ dimensional link matrices $\mathbf{Z}_i$ of fixed constants defined in terms of the commodity-specific weights $w_{i,\tilde{\imath}}$, we can write $\mathbf{z}_{i,t} = \mathbf{Z}_i\mathbf{x}_t$. Hence, we use this in Equation 3.4:

$$\mathbf{A}_{i,0}\mathbf{Z}_i\mathbf{x}_t = \mathbf{a}_{i,0} + \sum_{p=1}^{P} \mathbf{A}_{i,p}\mathbf{Z}_i\mathbf{x}_{t-p} + \sum_{p=0}^{P_{exog}} \mathbf{\Psi}_{i,p}\mathbf{e}_{t-p} + \boldsymbol{\varepsilon}_{i,t}. \tag{3.5}$$

Stacking Equation 3.5 together for $i = 1, 2, \ldots, N$, we obtain:

$$\mathbf{G}_0\mathbf{x}_t = \mathbf{a}_0 + \sum_{p=1}^{P} \mathbf{G}_p\mathbf{x}_{t-p} + \sum_{p=0}^{P_{exog}} \mathbf{\Psi}_p\mathbf{e}_{t-p} + \boldsymbol{\varepsilon}_t, \tag{3.6}$$

with the $K \times 1$ intercept vector $\mathbf{a}_0 = \left(\mathbf{a}'_{1,0}, \mathbf{a}'_{2,0}, \ldots, \mathbf{a}'_{N,0}\right)'$, the $K \times K$ dimensional matrices $\mathbf{G}_0 = \left((\mathbf{A}_{1,0}\mathbf{Z}_1)', (\mathbf{A}_{2,0}\mathbf{Z}_2)', \ldots, (\mathbf{A}_{N,0}\mathbf{Z}_N)'\right)'$, and $\mathbf{G}_p = \left((\mathbf{A}_{1,p}\mathbf{Z}_1)', (\mathbf{A}_{2,p}\mathbf{Z}_2)', \ldots, (\mathbf{A}_{N,p}\mathbf{Z}_N)'\right)'$, for $p = 1, \ldots, P$, the $K \times K_{exog}$ dimensional matrices $\mathbf{\Psi}_p = \left(\mathbf{\Psi}'_{1,p}, \mathbf{\Psi}'_{2,p}, \ldots, \mathbf{\Psi}'_{N,p}\right)'$, for $p = 0, 1, \ldots, P_{exog}$ and the $K \times 1$ vector $\boldsymbol{\varepsilon}_t = \left(\boldsymbol{\varepsilon}'_{1,t}, \boldsymbol{\varepsilon}'_{2,t}, \ldots, \boldsymbol{\varepsilon}'_{N,t}\right)'$. In case of a non-singular matrix $\mathbf{G}_0$, we define the $K \times 1$ vector $\mathbf{b} = \mathbf{G}_0^{-1}\mathbf{a}_0$, the $K \times K$ dimensional matrices $\mathbf{H}_p = \mathbf{G}_0^{-1}\mathbf{G}_p$ for $p = 1, 2, \ldots, P$, the $K \times K_{exog}$ dimensional matrices $\mathbf{\Upsilon}_p = \mathbf{G}_0^{-1}\mathbf{\Psi}_p$, for $p = 0, 1, \ldots, P_{exog}$, and

commodity-specific variables and coincides for all commodities. However, an extension to differences in the lag length of commodity-specific and external variables as well as between commodities would be possible.

the $K \times 1$ dimensional vector $\boldsymbol{\upsilon}_t = \mathbf{G}_0^{-1}\boldsymbol{\varepsilon}_t$. Rewriting Equation 3.6, we get the GVAR model in its final form:

$$\begin{aligned}\mathbf{x}_t &= \mathbf{G}_0^{-1}\mathbf{a}_0 + \sum_{p=1}^{P} \mathbf{G}_0^{-1}\mathbf{G}_p\mathbf{x}_{t-p} + \sum_{p=0}^{P_{exog}} \mathbf{G}_0^{-1}\boldsymbol{\Psi}_p\mathbf{e}_{t-p} + \mathbf{G}_0^{-1}\boldsymbol{\varepsilon}_t \\ &= \mathbf{b} + \sum_{p=1}^{P} \mathbf{H}_p\mathbf{x}_{t-p} + \sum_{p=0}^{P_{exog}} \boldsymbol{\Upsilon}_p\mathbf{e}_{t-p} + \boldsymbol{\upsilon}_t.\end{aligned} \tag{3.7}$$

Hereby, we model the vector $\mathbf{x}_t$, including all commodity-specific microeconomic variables, while accounting for the dependencies in the cross-commodity dimension as well as for the influence of exogenous macroeconomic variables. This enables us to model all commodity markets simultaneously, under the consideration of cross-commodity dependencies within the markets, as well as the impact of common macroeconomic factors.

In this context, we can predict the commodity markets ahead. For given observations of the commodity markets over the period $t = 1, 2, \ldots, T$, as well as for known exogenous variables $\mathbf{e}_t$,[8] for $t = 1, 2, \ldots, T+1$, the one-step ahead predictor for $\mathbf{x}_{T+1}$ is given by:

$$\mathbf{x}_{T+1} = \mathbf{b} + \sum_{p=1}^{P} \mathbf{H}_p\mathbf{x}_{T+1-p} + \sum_{p=0}^{P_{exog}} \boldsymbol{\Upsilon}_p\mathbf{e}_{T+1-p} + \boldsymbol{\upsilon}_{T+1}. \tag{3.8}$$

While the first component measures the impact of the intercept as well as of the lagged endogenous variables, the second component measures the impact of the exogenous variables, $\mathbf{e}_t$ and the last component reflects the stochastic part. Subsequently, we derive the point forecasts of the endogenous variables:

$$\hat{\mathbf{x}}_{T+1} = \mathbf{b} + \sum_{p=1}^{P} \mathbf{H}_p\mathbf{x}_{T+1-p} + \sum_{p=0}^{P_{exog}} \boldsymbol{\Upsilon}_p\mathbf{e}_{T+1-p}. \tag{3.9}$$

In general, the dynamic properties of the commodity market model depend on the eigenvalues of the matrix $\mathbf{H}_1$. If the roots of $\mathbf{H}_1$ lie inside the unit circle, representing the stationary case, $\mathbf{x}_{T+1}$ will have a stable distribution. Further, the point forecasts, $\hat{\mathbf{x}}_{T+1}$, will have the same linear properties as in the underlying individual VAR models, see Pesaran et al. (2004) for more detailed information.

3.1.2 Analysis of GVAR Models

Similar to VAR models, impulse response functions as well as forecast error variance decompositions are useful tools for a detailed analysis of global VAR models. Frequently, "orthogonalized" impulse responses are calculated, where shocks to the VAR model are orthogonalized using the Cholesky decomposition, see Pesaran and Shin (1998). However, this approach is not invariant to the ordering of the variables in the model. As it is difficult to order the commodity-specific variables supply, demand, and price, as well as the commodities in a meaningful way, we follow Pesaran et al. (2004) and Dées, Holly, Pesaran, and Smith (2007) and apply generalized impulse response functions (GIRFs) as well as generalized forecast error variance decompositions (GFEVDs) for the analysis of the GVAR model, which are robust to different variable orderings. Hereby, the GIRF methodology allows to examine spillover effects within and between commodity markets as well as to investigate the impact of shocks to the exogenous variables, e.g. a global demand shock, on commodity markets. Moreover, the GFEVD analysis aims to explain the variance of the forecast errors by shocks in the commodity variables.

[8] Following Pesaran et al. (2004), we assume known exogenous variables for a simpler notation. However, it is possible to extend the analysis for uncertainty. Please refer to Pesaran et al. (2004) for more details.

3.1.2.1 Generalized Impulse Response Functions

We propose to analyze the dynamic properties of the GVAR model by generalized impulse response functions (GIRFs), first proposed by Koop et al. (1996) and further developed in Pesaran and Shin (1998), since the GIRF analysis needs no ordering of the commodities, due to its invariance property, see Dées, di Mauro, Pesaran, and Smith (2007). Hereby, we first focus on shocks in the commodity markets, subsequently, we investigate how shocks in the global economy transmit to the commodity markets.

3.1.2.1.1 Impulse Response Analysis of Shocks to endogenous, commodity-specific Variables In general, the transmitted effects of a shock to an endogenous variable to the other variables can be reflected by a generalized impulse response function, for $n = 0, 1, \ldots, N_{IRF}$ periods ahead, defined as:

$$\mathbf{GI}\left(n, \boldsymbol{\delta}, \Omega_{t-1}\right) = \mathbb{E}\left[\mathbf{x}_{t+n} | \boldsymbol{\varepsilon}_t = \boldsymbol{\delta}, \Omega_{t-1}\right] - \mathbb{E}\left[\mathbf{x}_{t+n} | \Omega_{t-1}\right], \tag{3.10}$$

where $\boldsymbol{\delta}$ denotes the shock hitting the system at time t and $\Omega_{t-1} = \{\mathbf{x}_{t-1}, \mathbf{x}_{t-2}, \ldots\}$ is the information set at time $t-1$, including all necessary information about the exogenous variables. In our study, we consider the generalized impulse response functions measuring the effect of a shock to the element k of $\mathbf{x}_t$, corresponding to the k_i-th variable in the i-th commodity, by one standard deviation, $\sqrt{\sigma_{kk}}$, therefore, $\boldsymbol{\delta} = (0, \ldots, 0, \sqrt{\sigma_{kk}}, 0, \ldots, 0)'$:

$$\mathbf{GI}_{\mathbf{x}:\mathbf{x}_k}\left(n, \sqrt{\sigma_{kk}}, \Omega_{t-1}\right) = \mathbb{E}\left[\mathbf{x}_{t+n} | \varepsilon_{k,t} = \sqrt{\sigma_{kk}}, \Omega_{t-1}\right] - \mathbb{E}\left[\mathbf{x}_{t+n} | \Omega_{t-1}\right]. \tag{3.11}$$

Under the assumption of multivariate normal distributed residuals $\boldsymbol{\varepsilon}_t$, we can calculate the GIRFs for $n = 0$ as follows:

$$\mathbf{GI}_{\mathbf{x}:\mathbf{x}_k}\left(0, \sqrt{\sigma_{kk}}, \Omega_{t-1}\right) = \frac{1}{\sqrt{\sigma_{kk}}} \mathbf{G}_0^{-1} \boldsymbol{\Sigma} \mathfrak{s}_k, \tag{3.12}$$

where $\boldsymbol{\Sigma}$ is the $K \times K$ variance-covariance matrix of shocks $\boldsymbol{\varepsilon}_t$, σ_{kk} represents the kk-th element of $\boldsymbol{\Sigma}$, $\mathfrak{s}_k$ denotes the $K \times 1$ selection vector, with $\mathfrak{s}_k = 1$ for the k-th element and $\mathfrak{s}_k = 0$ else. Using Equation 3.7 and Equation 3.11, we derive:

$$\mathbf{GI}_{\mathbf{x}:\mathbf{x}_k}\left(n, \sqrt{\sigma_{kk}}, \Omega_{t-1}\right) = \sum_{p=1}^{P} \mathbf{H}_p \mathbf{GI}_{\mathbf{x}:\mathbf{x}_k}\left(n-p, \sqrt{\sigma_{kk}}, \Omega_{t-1}\right), \tag{3.13}$$

where $\mathbf{GI}_{\mathbf{x}:\mathbf{x}_k}\left(n, \sqrt{\sigma_{kk}}, \Omega_{t-1}\right) = \mathbf{0}$ for $n < 0$. Subsequently, we calculate the GIRFs recursively for $n = 1, 2, \ldots, N_{IRF}$. This measures the effect of a one standard error shock to the k-th equation (corresponding to the k_i-th variable in the i-th commodity) at time t on expected values of $\mathbf{x}$ at time $t+n$. In addition to analyzing commodity-specific effects within a market, the GIRF analysis also reveals spillover effects of a shock on the variables of the other commodity markets.

In order to analyze the significance of the GIRFs, we employ the sieve bootstrap technique proposed in Dées, di Mauro, Pesaran, and Smith (2007), which, in general, can be used to derive measures of accuracy, such as confidence intervals, without knowing the underlying distribution,[9] using random sampling with replacement. Our analysis is based on the 68% confidence bounds obtained by a sieve bootstrap procedure with $N_{boot} = 1000$ replications, which we briefly explain in the following.

[9] While we assume the residuals $\boldsymbol{\varepsilon}_t$ of the GVAR model to be multivariate normal distributed, see Section 5, and calculate the GIRFs recursively with help of Equation 3.12, the parameters of the multivariate normal distribution are not known in advance and have to be estimated. Therefore, we propose to use the bootstrap technique, also applicable if the underlying distribution is unknown, instead of Monte Carlo approaches, based on estimated parameters.

1. Calculate the residuals $\hat{\varepsilon}_t, t = 1, 2, \ldots, T$, of the fitted GVAR model, using the estimated parameters of Equation 3.6.
2. Randomly draw N_{boot} times with replacement $T_{boot} \leq T$ residuals to get N_{boot} sets of residuals $\varepsilon^{n_{boot}} = \left(\varepsilon^{n_{boot}}_{T-T_{boot}}, \varepsilon^{n_{boot}}_{T-T_{boot}+1}, \ldots, \varepsilon^{n_{boot}}_{T}\right)$, for $n_{boot} = 1, 2, \ldots, N_{boot}$.
3. Generate N_{boot} bootstrap samples $\mathbf{x}^{n_{boot}}$, according to Equation 3.7, using the resampled, recentered[10] residuals $\varepsilon^{n_{boot}}$ as well as the estimated parameters of the fitted GVAR model and calculate for each commodity $i = 1, 2, \ldots, N$ the corresponding external variables $\mathbf{x}_i^{*,n_{boot}}$ according to Equation 3.3, using the weight matrix $(w_{i,\tilde{\iota}})_{i,\tilde{\iota}=1,2,\ldots,N}$.
4. Estimate the individual, commodity-specific VAR models in Equation 3.2, using the bootstrap sample $\mathbf{x}^{n_{boot}}$ as endogenous variables with corresponding external variables $\mathbf{x}_i^{*,n_{boot}}$ to get new parameters of the VAR models and solve for the GVAR model.[11]
5. Calculate the GIRFs recursively, according to Equation 3.12 and Equation 3.13, based on the new estimated parameters corresponding to the bootstrap sample.
6. Repeat steps 3 to 5 N_{boot} times.
7. Sort the GIRFs into an ascending order for all time periods $n = 0, 1, \ldots, N_{IRF}$, and calculate the $(1 - 0.32) \cdot 100\% = 68\%$ confidence interval by using the $0.32/2$ and $(1 - 0.32/2)$ quantiles of the bootstrap distribution of the GIRFs, in line with Anzuini et al. (2013) and Hammoudeh et al. (2015) among others.

3.1.2.1.2 Impulse Response Analysis of Shocks to exogenous, global Variables Besides the spillover effects within and between commodity markets, we examine how shocks to the global economy, reflected by shocks to the exogenous variables, affect the commodity markets. Therefore, we analyze the effects of a shock to the k_{exog}-th exogenous variable $\mathbf{e}_{k_{exog},t}$, for $k_{exog} = 1, 2, \ldots, K_{exog}$, on the commodity markets via GIRFs.

Initially, a dynamic process for the exogenous variables has to be specified, see Pesaran et al. (2004). Therefore, we assume the vector of exogenous variables follows a vector autoregression process with $\tilde{P}_{exog}$ lags:

$$\mathbf{e}_t = \mathbf{a}_{exog} + \sum_{\tilde{p}_{exog}=1}^{\tilde{P}_{exog}} \boldsymbol{\Phi}_{exog,\tilde{p}_{exog}} \mathbf{e}_{t-\tilde{p}_{exog}} + \varepsilon_{exog,t}, \tag{3.14}$$

where $\mathbf{a}_{exog}$ is the $K_{exog} \times 1$ intercept vector, $\boldsymbol{\Phi}_{exog,\tilde{p}_{exog}}$ is the $K_{exog} \times K_{exog}$ matrix of coefficients for lag $\tilde{p}_{exog} = 1, 2, \ldots, \tilde{P}_{exog}$, and $\varepsilon_{exog,t}$ is the $K_{exog} \times 1$ vector of shocks. Hereby, we assume $\varepsilon_{exog,t}$ to be serially uncorrelated, independent and identically distributed, with mean zero and covariance matrix $\boldsymbol{\Sigma}_{exog}$, therefore, $\varepsilon_{exog,t} \sim iid(\mathbf{0}, \boldsymbol{\Sigma}_{exog})$.

Similar to the GIRFs of a shock to a commodity-specific variable, the generalized impulse response function of the effect of a shock $\boldsymbol{\delta}_{exog}$ to the exogenous variables $\mathbf{e}_t$ on the vector of endogenous variables $\mathbf{x}_t$ is defined for $n = 0, 1, \ldots, N_{IRF}$ periods ahead by:

$$\mathbf{GI}_{\mathbf{x}:\mathbf{e}_{k_{exog}}}(n, \boldsymbol{\delta}_{exog}, \Omega_{t-1}) = \mathbb{E}\left[\mathbf{x}_{t+n} | \varepsilon_{exog,t} = \boldsymbol{\delta}_{exog}, \Omega_{t-1}\right] - \mathbb{E}\left[\mathbf{x}_{t+n} | \Omega_{t-1}\right]. \tag{3.15}$$

[10] To ensure the bootstrap population mean is zero, we follow Dées, di Mauro, Pesaran, and Smith (2007) and recenter the residuals.

[11] As our original GVAR model is stable, we exclude all bootstrap samples where the corresponding GVAR model does not exhibit the stability property, to guarantee representative bootstrap models.

In particular, we investigate the GIRFs of a shock by one standard deviation to the k_{exog}-th exogenous variable $\mathbf{e}_{k_{exog},t}$, therefore, $\boldsymbol{\delta}_{exog} = (0, \ldots, 0, \sqrt{\sigma_{exog,k_{exog}k_{exog}}}, 0, \ldots, 0)'$, on the vector of endogenous variables $\mathbf{x}_t$, which is defined as:

$$\begin{aligned}\mathbf{GI}_{\mathbf{x}:\mathbf{e}_{k_{exog}}}\left(n, \sqrt{\sigma_{exog,k_{exog}k_{exog}}}, \Omega_{t-1}\right) = \mathbb{E}\left[\mathbf{x}_{t+n}|\boldsymbol{\varepsilon}_{exog,k_{exog},t} = \sqrt{\sigma_{exog,k_{exog}k_{exog}}}, \Omega_{t-1}\right] \\ - \mathbb{E}\left[\mathbf{x}_{t+n}|\Omega_{t-1}\right].\end{aligned} \tag{3.16}$$

Using the GVAR model in its final form in Equation 3.7, we derive:

$$\begin{aligned}&\mathbf{GI}_{\mathbf{x}:\mathbf{e}_{k_{exog}}}\left(n, \sqrt{\sigma_{exog,k_{exog}k_{exog}}}, \Omega_{t-1}\right) = \\ &= \sum_{p=1}^{P}\mathbf{H}_p\mathbf{GI}_{\mathbf{x}:\mathbf{e}_{k_{exog}}}\left(n-p, \sqrt{\sigma_{exog,k_{exog}k_{exog}}}, \Omega_{t-1}\right) \\ &+ \sum_{p=0}^{P_{exog}}\boldsymbol{\Upsilon}_p\mathbf{GI}_{\mathbf{e}:\mathbf{e}_{k_{exog}}}\left(n-p, \sqrt{\sigma_{exog,k_{exog}k_{exog}}}, \Omega_{t-1}\right),\end{aligned} \tag{3.17}$$

where $\mathbf{GI}_{\mathbf{e}:\mathbf{e}_{k_{exog}}}\left(n, \sqrt{\sigma_{exog,k_{exog}k_{exog}}}, \Omega_{t-1}\right)$ denotes the impulse response of the exogenous variables to a shock in the k_{exog}-th exogenous variable $\mathbf{e}_{k_{exog}}$:

$$\begin{aligned}\mathbf{GI}_{\mathbf{e}:\mathbf{e}_{k_{exog}}}\left(n, \sqrt{\sigma_{exog,k_{exog}k_{exog}}}, \Omega_{t-1}\right) = \mathbb{E}\left[\mathbf{e}_{t+n}|\boldsymbol{\varepsilon}_{exog,k_{exog},t} = \sqrt{\sigma_{exog,k_{exog}k_{exog}}}, \Omega_{t-1}\right] \\ - \mathbb{E}\left[\mathbf{e}_{t+n}|\Omega_{t-1}\right].\end{aligned} \tag{3.18}$$

For $n < 1$, the impulse responses vanish, therefore, $\mathbf{GI}_{\mathbf{x}:\mathbf{e}_{k_{exog}}}\left(n, \sqrt{\sigma_{exog,k_{exog}k_{exog}}}, \Omega_{t-1}\right) = \mathbf{GI}_{\mathbf{e}:\mathbf{e}_{k_{exog}}}\left(n, \sqrt{\sigma_{exog,k_{exog}k_{exog}}}, \Omega_{t-1}\right) = \mathbf{0}$. Using this fact and Equation 3.17, we derive for $n = 0$:

$$\mathbf{GI}_{\mathbf{x}:\mathbf{e}_{k_{exog}}}\left(0, \sqrt{\sigma_{exog,k_{exog}k_{exog}}}, \Omega_{t-1}\right) = \boldsymbol{\Upsilon}_0\mathbf{GI}_{\mathbf{e}:\mathbf{e}_{k_{exog}}}\left(0, \sqrt{\sigma_{exog,k_{exog}k_{exog}}}, \Omega_{t-1}\right). \tag{3.19}$$

Under the assumption of multivariate normal distributed errors $\boldsymbol{\varepsilon}_{exog}$, we can calculate the impulse responses for the exogenous variables for $n = 0$:

$$\mathbf{GI}_{\mathbf{e}:\mathbf{e}_{k_{exog}}}\left(0, \sqrt{\sigma_{exog,k_{exog}k_{exog}}}, \Omega_{t-1}\right) = \frac{1}{\sqrt{\sigma_{exog,k_{exog}k_{exog}}}}\boldsymbol{\Sigma}_{exog}\mathfrak{s}_{k_{exog}}, \tag{3.20}$$

where $\boldsymbol{\Sigma}_{exog}$ denotes the $K_{exog} \times K_{exog}$ variance-covariance matrix of shocks $\boldsymbol{\varepsilon}_{exog,t}$. Moreover, $\sigma_{exog,k_{exog}k_{exog}}$ is the $k_{exog}k_{exog}$-th element of $\boldsymbol{\Sigma}_{exog}$ and $\mathfrak{s}_{k_{exog}}$ denotes the $K_{exog} \times 1$ selection vector, with $\mathfrak{s}_{k_{exog}} = 1$ for the k_{exog}-th element and $\mathfrak{s}_{k_{exog}} = 0$ else. Overall, the GIRFs of the commodity-specific variables to a shock in the exogenous variable $\mathbf{e}_{k_{exog}}$ are then calculated recursively, using Equation 3.17, Equation 3.19, and Equation 3.20.

Similar to the GIRF analysis of shocks to commodity-specific variables, we analyze the significance of the GIRFs of shocks to the global exogenous variables, by the sieve bootstrap technique, with $N_{boot} = 1000$ replications. Hereby, we adjust the first five steps of the bootstrap technique described in Section 3.1.2.1 as follows:

1. Calculate the residuals $\hat{\boldsymbol{\varepsilon}}_{exog,t}, t = 1, 2, \ldots, T$, of the fitted VAR model of the exogenous variables, using the estimated parameters of Equation 3.14.

2. Randomly draw N_{boot} times with replacement $T_{boot} \leq T$ residuals to get N_{boot} sets of residuals $\boldsymbol{\varepsilon}_{exog}^{n_{boot}} = \left(\boldsymbol{\varepsilon}_{exog,T-T_{boot}}^{n_{boot}}, \boldsymbol{\varepsilon}_{exog,T-T_{boot}+1}^{n_{boot}}, \ldots, \boldsymbol{\varepsilon}_{exog,T}^{n_{boot}}\right)$, for $n_{boot} = 1, 2, \ldots, N_{boot}$.

3. Generate N_{boot} bootstrap samples $\mathbf{e}^{n_{boot}}$, according to Equation 3.14, using the resampled, recentered[12] residuals $\boldsymbol{\varepsilon}_{exog}^{n_{boot}}$ as well as the estimated parameters of the fitted VAR model.
4. Estimate the VAR model in Equation 3.14, using the bootstrap sample $\mathbf{e}^{n_{boot}}$ to get new parameters of the VAR model.
5. Calculate the GIRFs of shocks to the exogenous variables recursively, using Equation 3.17, Equation 3.19, and Equation 3.20, based on the new estimated parameters corresponding to the bootstrap sample.

3.1.2.2 Generalized Forecast Error Variance Decomposition

Forecast error variance decomposition is a further analysis tool to provide insights into general VAR models. Due to the correlated shocks between commodities in our GVAR model, we apply the alternative approach of generalized forecast error variance decomposition (GFEVD), proposed in Dées, Holly, Pesaran, and Smith (2007), which does not depend on the ordering of the variables. Hereby, the GFEVD represents the proportion of the variance of the n-step ahead forecast errors explained by the shocks of variable $k = 1, 2, \ldots, K$. In particular, the GFEVD represents the proportion of the n-step ahead forecast error variance of the $\tilde{k}$-th element of $\mathbf{x}_t$, accounted for by the innovations in the k-th element of $\mathbf{x}_t$, for $n = 1, 2, \ldots, N_{FEVD}$. In case of one lag, hence $P = 1$, the closed form of the GFEVD is defined as:

$$\mathbf{GFEVD}_{\mathbf{x}_{\tilde{k},t},\varepsilon_{k,t}}(n) = \frac{\sigma_{kk}^{-1} \sum_{\tilde{n}=0}^{n} \left(\mathfrak{s}_{\tilde{k}}' \mathbf{H}_1^{\tilde{n}} \mathbf{G}_0^{-1} \boldsymbol{\Sigma} \mathfrak{s}_k \right)^2}{\sum_{\tilde{n}=0}^{n} \mathfrak{s}_{\tilde{k}}' \mathbf{H}_1^{\tilde{n}} \mathbf{G}_0^{-1} \boldsymbol{\Sigma} \mathbf{G}_0^{-1\prime} \mathbf{H}_1^{\tilde{n}\prime} \mathfrak{s}_{\tilde{k}}}, \tag{3.21}$$

where $\boldsymbol{\Sigma}$ is the $K \times K$ variance-covariance matrix of shocks $\boldsymbol{\varepsilon}_t$, σ_{kk} represents the kk-th element of $\boldsymbol{\Sigma}$, $\mathfrak{s}_k$ denotes the $K \times 1$ selection vector, with $\mathfrak{s}_k = 1$ for the k-th element and $\mathfrak{s}_k = 0$ else, and $\mathbf{H}_1$ is defined in Equation 3.7. Since the shocks are correlated, leading to a non-diagonal variance-covariance matrix $\boldsymbol{\Sigma}$, the elements of $\mathbf{GFEVD}_{\mathbf{x}_{\tilde{k},t},\varepsilon_{k,t}}(n)$ across k do not sum to unity, which is why we scale them to guarantee comparability.

3.2 Markov-switching Global Vector Autoregression (MS(M) - GVAR(P)) Model

While the GVAR methodology allows the consideration of cross-commodity dependencies within markets, incorporating commodity-specific microeconomic and exogenous macroeconomic variables, the dependencies between commodities are, so far, time-invariant and do not account for changes in the market over time. However, we aim to examine how the dependencies within and between commodity markets change over time. Therefore, we extend the GVAR model to a Markov-switching global vector autoregression (MS-GVAR) model, based on the idea of Binder and Gross (2013), enabling for time-varying relations.

Similar to the structure in Section 3.1, we first introduce the individual, commodity-specific Markov-switching vector autoregression (MS-VAR) models. Therefore, we briefly discuss the state space representation of MS-VAR models, the filtering algorithm and the likelihood function to derive the expectation-maximization (EM) algorithm, used for the estimation of the models, as well as the model selection procedure. Subsequently, we solve the MS-GVAR model and describe the prediction as well as appropriate analysis methods.

[12]To ensure the bootstrap population mean is zero, we follow Dées, di Mauro, Pesaran, and Smith (2007) and recenter the residuals.

3.2.1 Commodity-specific MS(M)-VAR(P) Models

3.2.1.1 Individual Market Models

Instead of individual VAR models, we apply individual M-state Markov-switching vector autoregression (MS-VAR) models of order P (MS(M[13])-VAR[14](P)) on all commodities $i = 1, 2, \dots, N$ of the analysis. While Binder and Gross (2013) propose the MS-GVAR model for economies based on MS-VAR models with time-varying intercept, we generally introduce the MS-GVAR model based on several specifications of the MS-VAR model, explicitly allowing for time-varying autoregressive parameters, time-varying parameters corresponding to the exogenous variables or a time-varying covariance matrix, see Table 3.1 for an overview over the specifications.

First of all, we follow the idea proposed in Section 3.1 as well as in Schischke et al. (2023) and model the dependencies between the commodity-specific supply ($\mathbf{supply_i}$), demand ($\mathbf{demand_i}$) and price ($\mathbf{price_i}$) variables, which form the $K_i \times 1$ vector $\mathbf{x}_{i,t} = (supply_{i,t}, demand_{i,t}, price_{i,t})'$, extended by the $K_i^* \times 1$ vector $\mathbf{x}_{i,t}^* = \left(supply_{i,t}^*, demand_{i,t}^*, price_{i,t}^*\right)'$ of weighted external variables specific to commodity i, for all time periods $t = 1, 2, \dots, T$:

$$\mathbf{x}_{i,t} = \mathbf{a}_{i,0,s_{i,t}} + \sum_{p=1}^{P} \mathbf{\Phi}_{i,p,s_{i,t}} \mathbf{x}_{i,t-p} + \sum_{p=0}^{P} \mathbf{\Lambda}_{i,p,s_{i,t}} \mathbf{x}_{i,t-p}^* + \sum_{p=0}^{P_{exog}} \mathbf{\Psi}_{i,p,s_{i,t}} \mathbf{e}_{t-p} + \boldsymbol{\varepsilon}_{i,t}, \tag{3.22}$$

where $\mathbf{a}_{i,0,s_{i,t}}$ denotes the regime-dependent intercept vector and $\mathbf{\Phi}_{i,p,s_{i,t}}$ are the regime-dependent $K_i \times K_i$ matrices of lagged coefficients for lag $p = 1, 2, \dots, P$, with $K_{endog} = K_i = 3, i = 1, 2, \dots, N$, denoting the length of vector $\mathbf{x}_{i,t}$. Further, $\mathbf{\Lambda}_{i,p,s_{i,t}}$ are regime-dependent $K_i \times K_i^*$ matrices of coefficients associated with the lagged exogenous, external specific variables for $p = 0, 1, \dots, P$,[15] and in our case $K_i = K_i^* = 3$ for all $i = 1, 2, \dots, N$.

In accordance to Equation 3.3 for the GVAR model, the external commodity variables $\mathbf{x}_{i,t}^*$ represent the information of the other commodity markets and are defined as:

$$\begin{aligned} supply_{i,t}^* &= \sum_{\tilde{\imath}=1}^{N} w_{i,\tilde{\imath}} \cdot supply_{\tilde{\imath},t}, \\ demand_{i,t}^* &= \sum_{\tilde{\imath}=1}^{N} w_{i,\tilde{\imath}} \cdot demand_{\tilde{\imath},t}, \\ price_{i,t}^* &= \sum_{\tilde{\imath}=1}^{N} w_{i,\tilde{\imath}} \cdot price_{\tilde{\imath},t}, \end{aligned} \tag{3.23}$$

with weights $w_{i,i} = 0$ and $\sum_{\tilde{\imath}=1}^{N} w_{i,\tilde{\imath}} = 1$, for $i = 1, 2, \dots, N$. The corresponding individual weights $w_{i,\tilde{\imath}}$ may be aggregated to a weight matrix $(w_{i,\tilde{\imath}})_{i,\tilde{\imath}=1,2,\dots,N}$.

To represent the common factor in our framework, we include K_{exog} macroeconomic factors as exogenous variables in the vector $\mathbf{e}_t$ with $\mathbf{\Psi}_{i,p,s_{i,t}}$ as regime-dependent $K_i \times K_{exog}$ matrices of the corresponding coefficients for lags $p = 0, 1, \dots, P_{exog}$. Further, we assume that the $K_i \times 1$ vectors of idiosyncratic commodity-specific shocks $\boldsymbol{\varepsilon}_{i,t}$ are, in general, serially uncorrelated, independent and identically distributed, with mean zero and regime-dependent covariance matrix $\mathbf{\Sigma}_{ii,s_{i,t}}$. Therefore, $\boldsymbol{\varepsilon}_{i,t} \sim iid\left(\mathbf{0}, \mathbf{\Sigma}_{ii,s_{i,t}}\right)$.

[13] We assume M different states in each commodity market, however, an extension to individual number of states M_i would be possible. In this thesis , $M_i = M$ holds, for $i = 1, 2, \dots, N$.

[14] In the terminology of Krolzig (1997), we focus on MS-VAR models with switching intercept and not on mean adjusted VAR (MSM-VAR) models. Please refer to Krolzig (1997) for more details on MSM-VAR models.

[15] For a simpler notation, we assume that the lag length of the external variables equals to the lag length of the commodity-specific variable and coincides for all commodities. However, an extension to differences in the lag length of commodity-specific and external variables as well as between commodities is possible.

The difference between the commodity-specific VAR models of the original GVAR model displayed in Equation 3.2 in Section 3.1 and the MS-VAR models described in Equation 3.22 lies in the index $s_{i,t}$ attached to the parameters denoting that the parameters depend on the regime $s_{i,t} = 1, 2, \ldots, M$, prevailing in commodity i at time t and, in particular, the parameters are not constant. We assume that this unobserved regime, also called state, is generated by a discrete time, irreducible, ergodic, discrete M-state Markov chain with transition matrix $\mathbf{P}_i$. The corresponding transition probabilities $p_{i,lm}$, denoting the probability of a commodity to switch from regime l to regime m from time t to $t+1$, are defined as:

$$p_{i,lm} = Pr\left(s_{i,t+1} = m | s_{i,t} = l\right), \ \sum_{m=1}^{M} p_{i,lm} = 1, \text{ for } l, m = 1, 2, \ldots, M,^{16} \tag{3.24}$$

and summarized in the transition matrix

$$\mathbf{P}_i = \begin{pmatrix} p_{i,11} & p_{i,12} & \cdots & p_{i,1M} \\ p_{i,21} & p_{i,22} & \cdots & p_{i,2M} \\ \vdots & \vdots & \ddots & \vdots \\ p_{i,M1} & p_{i,M2} & \cdots & p_{i,MM} \end{pmatrix}. \tag{3.25}$$

Further, we can define $\boldsymbol{\xi}_{i,t}$ denoting the unobserved state of the system:

$$\boldsymbol{\xi}_{i,t} = \begin{pmatrix} \mathbb{I}\left(s_{i,t} = 1\right) \\ \mathbb{I}\left(s_{i,t} = 2\right) \\ \vdots \\ \mathbb{I}\left(s_{i,t} = M\right) \end{pmatrix}, \tag{3.26}$$

where

$$\mathbb{I}\left(s_{i,t} = m\right) = \begin{cases} 1, & \text{if } s_{i,t} = m \\ 0, & \text{else} \end{cases} \tag{3.27}$$

is the indicator function for $m = 1, 2, \ldots, M$. As $\boldsymbol{\xi}_{i,t}$ consists only of binary variables, the expected value also represents the probability distribution of $s_{i,t}$:

$$\mathbb{E}\left[\boldsymbol{\xi}_{i,t}\right] = \begin{pmatrix} Pr\left(s_{i,t} = 1\right) \\ Pr\left(s_{i,t} = 2\right) \\ \vdots \\ Pr\left(s_{i,t} = M\right) \end{pmatrix} = \begin{pmatrix} Pr\left(\boldsymbol{\xi}_{i,t} = \boldsymbol{\iota}_1\right) \\ Pr\left(\boldsymbol{\xi}_{i,t} = \boldsymbol{\iota}_2\right) \\ \vdots \\ Pr\left(\boldsymbol{\xi}_{i,t} = \boldsymbol{\iota}_M\right) \end{pmatrix}, \tag{3.28}$$

where $\boldsymbol{\iota}_M$ represents the m-th column of the identity matrix.

3.2.1.2 State Space Representation

For further analysis, it is useful to rewrite the MS(M)-VAR(P) model in Equation 3.22 in its state space representation, consisting of a measurement and a transition equation. Hereby, the measurement equation is generally given by:

$$\mathbf{x}_{i,t} = \mathbb{X}_{i,t}\mathbf{B}_i\boldsymbol{\xi}_{i,t} + \mathbf{u}_{i,t}, \tag{3.29}$$

with system input matrix $\mathbb{X}_{i,t} = \bar{\mathbf{x}}'_{i,t} \otimes \mathbf{I}_{K_i}$, where

$$\bar{\mathbf{x}}'_{i,t} = \left(1, \mathbf{e}'_t, \mathbf{e}'_{t-1}, \ldots, \mathbf{e}'_{t-P_{exog}}, \mathbf{x}^{*\prime}_{i,t-1}, \mathbf{x}^{*\prime}_{i,t-2}, \ldots, \mathbf{x}^{*\prime}_{i,t-P}, \mathbf{x}'_{i,t-1}, \mathbf{x}'_{i,t-2}, \ldots, \mathbf{x}'_{i,t-P}\right),$$

[16] In line with Krolzig (1997), we denote by $Pr(\cdot)$ a discrete probability measure and by $p(\cdot)$ a probability density function.

and $\mathbf{I}_{K_i}$ is the $K_i \times K_i$ dimensional identity matrix, $\otimes$ denotes the Kronecker product, and $\mathbf{B}_i$ includes the regression parameters.[17] The corresponding state or transition equation can be written as:

$$\boldsymbol{\xi}_{i,t+1} = \mathbf{F}_i \boldsymbol{\xi}_{i,t} + \boldsymbol{v}_{i,t+1}, \tag{3.30}$$

where $\mathbf{F}_i = \mathbf{P}_i'$ represents the transpose of the transition probability matrix. We define the mean innovation process $\{\boldsymbol{v}_{i,t}\}$ by $\boldsymbol{v}_{i,t+1} = \boldsymbol{\xi}_{i,t+1} - E\left[\boldsymbol{\xi}_{i,t+1}|\boldsymbol{\xi}_{i,t}\right]$ which is uncorrelated with $\mathbf{u}_{i,t}$ as well as past values of $\boldsymbol{\xi}_i, \mathbf{u}_i, \mathbf{x}_i, \mathbb{X}_i$, due to the Markov property. Additionally, we assume the innovation process $\{\mathbf{u}_{i,t}\}$ follows a normal distribution:

$$\mathbf{u}_{i,t} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Sigma}_{ii}\left(\boldsymbol{\xi}_{i,t} \otimes \mathbf{I}_{K_i}\right)\right). \tag{3.31}$$

In general, there are many specifications of MS-VAR models. In each specification, some or all parameters are regime-dependent, while the remaining parameters are regime-invariant. Krolzig (1997) distinguishes between seven different MS-VAR models corresponding to the intercept form.[18] As Krolzig (1997) does not consider exogenous variables explicitly, we extend the specifications by allowing the parameters corresponding to the exogenous variables to be either regime-dependent or regime-invariant, resulting in 16 different specifications, see Table 3.1.[19]

Table 3.1: Specifications of MS-VAR models

		$\boldsymbol{\psi}$ dependent		$\boldsymbol{\psi}$ invariant	
		$\boldsymbol{\nu}$ dependent	$\boldsymbol{\nu}$ invariant	$\boldsymbol{\nu}$ dependent	$\boldsymbol{\nu}$ invariant
$\boldsymbol{\alpha}$ invariant	$\boldsymbol{\Sigma}$ invariant	MSIX VAR	MSX VAR	MSI VAR	linear VAR
	$\boldsymbol{\Sigma}$ dependent	MSIHX VAR	MSHX VAR	MSIH VAR	MSH VAR
$\boldsymbol{\alpha}$ dependent	$\boldsymbol{\Sigma}$ invariant	MSIAX VAR	MSAX VAR	MSIA VAR	MSA VAR
	$\boldsymbol{\Sigma}$ dependent	MSIAHX VAR	MSAHX VAR	MSIAH VAR	MSAH VAR

This table shows the specifications of general MS-VAR models. The intercept ($\boldsymbol{\nu}$), the autoregressive parameters ($\boldsymbol{\alpha}$), the parameters corresponding to the exogenous variables ($\boldsymbol{\psi}$) as well as the covariance matrix ($\boldsymbol{\Sigma}$) can be either regime-dependent or regime-invariant. In case of the MS-VAR model corresponding to commodity $i = 1, 2, \ldots, N$ in state $m = 1, 2, \ldots, M$, it holds: $\boldsymbol{\nu}_{i,m} = \mathbf{a}_{i,0,m}, \boldsymbol{\alpha}_{i,m} = \left(\boldsymbol{\Phi}_{i,1,m}, \boldsymbol{\Phi}_{i,2,m}, \ldots, \boldsymbol{\Phi}_{i,P,m}\right)'$ and $\boldsymbol{\psi}_{i,m} = \left(\boldsymbol{\Psi}_{i,0,m}, \boldsymbol{\Psi}_{i,1,m}, \ldots, \boldsymbol{\Psi}_{i,P_{exog},m}, \boldsymbol{\Lambda}_{i,0,m}, \boldsymbol{\Lambda}_{i,1,m}, \ldots, \boldsymbol{\Lambda}_{i,P,m}\right)'$, where $\mathbf{a}_{i,0,m}$, $\boldsymbol{\Phi}_{i,1,m}, \boldsymbol{\Phi}_{i,2,m}, \ldots, \boldsymbol{\Phi}_{i,P,m}$ and $\boldsymbol{\Psi}_{i,0,m}, \boldsymbol{\Psi}_{i,1,m}, \ldots, \boldsymbol{\Psi}_{i,P_{exog},m}, \boldsymbol{\Lambda}_{i,0,m}, \boldsymbol{\Lambda}_{i,1,m}, \ldots, \boldsymbol{\Lambda}_{i,P,m}$ are the parameters of Equation 3.22. If the parameter is regime-invariant, it holds for all $m = 1, 2, \ldots, M$, $\boldsymbol{\nu}_{i,m} = \boldsymbol{\nu}_i, \boldsymbol{\alpha}_{i,m} = \boldsymbol{\alpha}_i, \boldsymbol{\psi}_{i,m} = \boldsymbol{\psi}_i$ or $\boldsymbol{\Sigma}_{ii,m} = \boldsymbol{\Sigma}_{ii}$.

In particular, the intercept ($\boldsymbol{\nu}$), the autoregressive parameters ($\boldsymbol{\alpha}$), the parameters corresponding to the exogenous variables ($\boldsymbol{\psi}$) as well as the covariance matrix ($\boldsymbol{\Sigma}$) can be regime-dependent or regime-invariant, resulting in 16 different MS-VAR models. Hereby, the linear VAR model represents the special case when all parameters are regime-invariant. In case of the MS-VAR model corresponding to commodity $i = 1, 2, \ldots, N$ in state $m = 1, 2, \ldots, M$, the intercept is $\boldsymbol{\nu}_{i,m} = \mathbf{a}_{i,0,m}$, the autoregressive parameters are represented by $\boldsymbol{\alpha}_{i,m} = \left(\boldsymbol{\Phi}_{i,1,m}, \boldsymbol{\Phi}_{i,2,m}, \ldots, \boldsymbol{\Phi}_{i,P,m}\right)'$ and $\boldsymbol{\psi}_{i,m} = \left(\boldsymbol{\Psi}_{i,0,m}, \boldsymbol{\Psi}_{i,1,m}, \ldots, \boldsymbol{\Psi}_{i,P_{exog},m}, \boldsymbol{\Lambda}_{i,0,m}, \boldsymbol{\Lambda}_{i,1,m}, \ldots, \boldsymbol{\Lambda}_{i,P,m}\right)'$ denote the parameters corresponding to the exogenous variables, where $\mathbf{a}_{i,0,m}$, $\boldsymbol{\Phi}_{i,1,m}, \boldsymbol{\Phi}_{i,2,m}, \ldots, \boldsymbol{\Phi}_{i,P,m}$ and $\boldsymbol{\Psi}_{i,0,m}, \boldsymbol{\Psi}_{i,1,m}$, $\ldots, \boldsymbol{\Psi}_{i,P_{exog},m}, \boldsymbol{\Lambda}_{i,0,m}, \boldsymbol{\Lambda}_{i,1,m}, \ldots, \boldsymbol{\Lambda}_{i,P,m}$ are the parameters of Equation 3.22. If the parameter is regime-invariant, it holds for all $m = 1, 2, \ldots, M$, $\boldsymbol{\nu}_{i,m} = \boldsymbol{\nu}_i, \boldsymbol{\alpha}_{i,m} = \boldsymbol{\alpha}_i, \boldsymbol{\psi}_{i,m} = \boldsymbol{\psi}_i$ or $\boldsymbol{\Sigma}_{ii,m} = \boldsymbol{\Sigma}_{ii}$.

For each specification, a regression equation can be derived from the measurement equation,[20] using the definition of $\boldsymbol{\Xi}_{i,m} = diag\left(\boldsymbol{\xi}_{i,m}\right)$ as the $T \times T$ diagonal matrix including the regime

[17] Please refer to Appendix B.1.1 for more details, in particular, the definition of matrix $\mathbf{B}_i$, including the regression parameters.

[18] There are also several specifications in the mean-adjusted form, which are not considered within this study. For further information, please refer to Krolzig (1997).

[19] In this study, we are interested in disentangling the change in the behavior of commodity markets between low and high price environments, which is why we do not consider the case, where only the exogenous variables are allowed to switch.

[20] For further information, please refer to the Appendix B.1.1.

probabilities $\boldsymbol{\xi}_{i,m} = \left(\boldsymbol{\xi}'_{i,m,1}, \boldsymbol{\xi}'_{i,m,2}, \dots, \boldsymbol{\xi}'_{i,m,T}\right)$ as diagonal elements and

$$\begin{aligned}\mathbb{X}_{i,t} &= \bar{\mathbf{x}}'_{i,t} \otimes \mathbf{I}_{K_i} = \left(1, \mathbf{e}'_t, \mathbf{e}'_{t-1}, \dots, \mathbf{e}'_{t-P_{exog}}, \mathbf{x}^{*\prime}_{i,t}, \mathbf{x}^{*\prime}_{i,t-1}, \dots, \mathbf{x}^{*\prime}_{i,t-P}, \mathbf{x}'_{i,t-1}, \mathbf{x}'_{i,t-2}, \dots, \mathbf{x}'_{i,t-P}\right) \otimes \mathbf{I}_{K_i} \\ &\coloneqq \left(1, \mathbf{ex}'_{i,t}, \mathbf{en}'_{i,t-1}\right),\end{aligned}$$

with $\mathbf{ex}'_{i,t} \coloneqq \left(\mathbf{e}'_t, \mathbf{e}'_{t-1}, \dots, \mathbf{e}'_{t-P_{exog}}, \mathbf{x}^{*\prime}_{i,t}, \mathbf{x}^{*\prime}_{i,t-1}, \dots, \mathbf{x}^{*\prime}_{i,t-P}\right)$ denoting the exogenous variables, $\mathbf{en}'_{i,t-1} \coloneqq \left(\mathbf{x}'_{i,t-1}, \mathbf{x}'_{i,t-2}, \dots, \mathbf{x}'_{i,t-P}\right)$ the endogenous variables of the model and $\mathbf{I}_{K_i}$ a $K_i \times K_i$ identity matrix. Further, we define $\mathbf{Ex}_i = \left(\mathbf{ex}'_{i,-0}, \mathbf{ex}'_{i,-1}, \dots, \mathbf{ex}'_{i,-P_{exog}}\right)$ with $\mathbf{ex}_{i,-p} = (\mathbf{ex}_{i,0-p}, \mathbf{ex}_{i,1-p}, \dots, \mathbf{ex}_{i,T-p})$, for $p = 0, 1, \dots, P_{exog}$, and $\mathbf{En}_i = \left(\mathbf{en}'_{i,-1}, \mathbf{en}'_{i,-2}, \dots, \mathbf{en}'_{i,-P}\right)$ with $\mathbf{en}_{i,-p} = (\mathbf{en}_{i,1-p}, \mathbf{en}_{i,2-p}, \dots, \mathbf{en}_{i,T-p})$, for $p = 1, 2, \dots, P$ and $\mathbf{x}_i = \left(\mathbf{x}'_{i,1}, \mathbf{x}'_{i,2}, \dots, \mathbf{x}'_{i,T}\right)'$. In the following, we explicitly describe the regression equations as well as the corresponding innovation process $\mathbf{u}_i = \left(\mathbf{u}'_{i,1}, \mathbf{u}'_{i,2}, \dots, \mathbf{u}'_{i,T}\right)'$ for each specification separately, whereby $\mathbf{1}_T$ denotes a $T \times 1$ vector of ones.

- MSI-VAR Model with regime-dependent intercept:
  - Regression equation in case of regime-dependent parameters of exogenous variables:

$$\begin{aligned}\mathbf{x}_i = &\sum_{m=1}^{M} \left(\boldsymbol{\Xi}_{i,m}\mathbf{1}_T \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\nu}_{i,m} + \sum_{m=1}^{M} \left(\boldsymbol{\Xi}_{i,m}\mathbf{Ex}'_i \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\psi}_{i,m} \\ &+ \left(\mathbf{En}'_i \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\alpha}_i + \mathbf{u}_i.\end{aligned} \tag{3.32}$$

  - Regression equation in case of regime-invariant parameters of exogenous variables:

$$\mathbf{x}_i = \sum_{m=1}^{M} \left(\boldsymbol{\Xi}_{i,m}\mathbf{1}_T \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\nu}_{i,m} + \left(\mathbf{Ex}'_i \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\psi}_i + \left(\mathbf{En}'_i \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\alpha}_i + \mathbf{u}_i. \tag{3.33}$$

  - Innovation process:

$$\mathbf{u}_i \sim N\left(\mathbf{0}, \boldsymbol{\Omega}_i\right) \text{ with } \boldsymbol{\Omega}_i = \mathbf{I}_T \otimes \boldsymbol{\Sigma}_{ii}. \tag{3.34}$$

- MSIH-VAR Model with regime-dependent intercept and covariance matrix:
  - Regression equation in case of regime-dependent parameters of exogenous variables:

$$\begin{aligned}\mathbf{x}_i = &\sum_{m=1}^{M} \left(\boldsymbol{\Xi}_{i,m}\mathbf{1}_T \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\nu}_{i,m} + \sum_{m=1}^{M} \left(\boldsymbol{\Xi}_{i,m}\mathbf{Ex}'_i \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\psi}_{i,m} \\ &+ \left(\mathbf{En}'_i \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\alpha}_i + \mathbf{u}_i.\end{aligned} \tag{3.35}$$

  - Regression equation in case of regime-invariant parameters of exogenous variables:

$$\mathbf{x}_i = \sum_{m=1}^{M} \left(\boldsymbol{\Xi}_{i,m}\mathbf{1}_T \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\nu}_{i,m} + \left(\mathbf{Ex}'_i \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\psi}_i + \left(\mathbf{En}'_i \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\alpha}_i + \mathbf{u}_i. \tag{3.36}$$

  - Innovation process:

$$\mathbf{u}_i \sim N\left(\mathbf{0}, \boldsymbol{\Omega}_i\right) \text{ with } \boldsymbol{\Omega}_i = \sum_{m=1}^{M} \boldsymbol{\Xi}_{i,m} \otimes \boldsymbol{\Sigma}_{ii,m}. \tag{3.37}$$

- MSIA-VAR Model with regime-dependent intercept, autoregressive parameters and regime-invariant covariance matrix:

– Regression equation in case of regime-dependent parameters of exogenous variables:

$$\begin{aligned}\mathbf{x}_i = &\sum_{m=1}^{M} \left(\boldsymbol{\Xi}_{i,m}\mathbf{1}_T \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\nu}_{i,m} + \sum_{m=1}^{M} \left(\boldsymbol{\Xi}_{i,m}\mathbf{E}\mathbf{x}_i' \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\psi}_{i,m} \\ &+ \sum_{m=1}^{M} \left(\boldsymbol{\Xi}_{i,m}\mathbf{E}\mathbf{n}_i' \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\alpha}_{i,m} + \mathbf{u}_i.\end{aligned} \tag{3.38}$$

– Regression equation in case of regime-invariant parameters of exogenous variables:

$$\begin{aligned}\mathbf{x}_i = &\sum_{m=1}^{M} \left(\boldsymbol{\Xi}_{i,m}\mathbf{1}_T \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\nu}_{i,m} + \left(\mathbf{E}\mathbf{x}_i' \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\psi}_{i} \\ &+ \sum_{m=1}^{M} \left(\boldsymbol{\Xi}_{i,m}\mathbf{E}\mathbf{n}_i' \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\alpha}_{i,m} + \mathbf{u}_i.\end{aligned} \tag{3.39}$$

– Innovation process:

$$\mathbf{u}_i \sim N\left(\mathbf{0}, \boldsymbol{\Omega}_i\right) \text{ with } \boldsymbol{\Omega}_i = \mathbf{I}_T \otimes \boldsymbol{\Sigma}_{ii}. \tag{3.40}$$

- MSIAH-VAR Model with regime-dependent intercept, autoregressive parameters, as well as covariance matrix:

  – Regression equation in case of regime-dependent parameters of exogenous variables:

$$\begin{aligned}\mathbf{x}_i = &\sum_{m=1}^{M} \left(\boldsymbol{\Xi}_{i,m}\mathbf{1}_T \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\nu}_{i,m} + \sum_{m=1}^{M} \left(\boldsymbol{\Xi}_{i,m}\mathbf{E}\mathbf{x}_i' \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\psi}_{i,m} \\ &+ \sum_{m=1}^{M} \left(\boldsymbol{\Xi}_{i,m}\mathbf{E}\mathbf{n}_i' \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\alpha}_{i,m} + \mathbf{u}_i.\end{aligned} \tag{3.41}$$

  – Regression equation in case of regime-invariant parameters of exogenous variables:

$$\begin{aligned}\mathbf{x}_i = &\sum_{m=1}^{M} \left(\boldsymbol{\Xi}_{i,m}\mathbf{1}_T \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\nu}_{i,m} + \left(\mathbf{E}\mathbf{x}_i' \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\psi}_{i} \\ &+ \sum_{m=1}^{M} \left(\boldsymbol{\Xi}_{i,m}\mathbf{E}\mathbf{n}_i' \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\alpha}_{i,m} + \mathbf{u}_i.\end{aligned} \tag{3.42}$$

  – Innovation process:

$$\mathbf{u}_i \sim N\left(\mathbf{0}, \boldsymbol{\Omega}_i\right) \text{ with } \boldsymbol{\Omega}_i = \sum_{m=1}^{M} \boldsymbol{\Xi}_{i,m} \otimes \boldsymbol{\Sigma}_{ii,m}. \tag{3.43}$$

- MSH-VAR Model with regime-dependent covariance matrix:

  – Regression equation in case of regime-dependent parameters of exogenous variables:

$$\mathbf{x}_i = \left(\mathbf{1}_T \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\nu}_i + \sum_{m=1}^{M} \left(\boldsymbol{\Xi}_{i,m}\mathbf{E}\mathbf{x}_i' \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\psi}_{i,m} + \left(\mathbf{E}\mathbf{n}_i' \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\alpha}_i + \mathbf{u}_i. \tag{3.44}$$

  – Regression equation in case of regime-invariant parameters of exogenous variables:

$$\mathbf{x}_i = \left(\mathbf{1}_T \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\nu}_i + \left(\mathbf{E}\mathbf{x}_i' \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\psi}_i + \left(\mathbf{E}\mathbf{n}_i' \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\alpha}_i + \mathbf{u}_i. \tag{3.45}$$

– Innovation process:

$$\mathbf{u}_i \sim N\left(\mathbf{0}, \boldsymbol{\Omega}_i\right) \text{ with } \boldsymbol{\Omega}_i = \sum_{m=1}^{M} \boldsymbol{\Xi}_{i,m} \otimes \boldsymbol{\Sigma}_{ii,m}. \tag{3.46}$$

- MSA-VAR Model with regime-dependent autoregressive parameters:
  – Regression equation in case of regime-dependent parameters of exogenous variables:

$$\begin{aligned}\mathbf{x}_i = \left(\mathbf{1}_T \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\nu}_i &+ \sum_{m=1}^{M}\left(\boldsymbol{\Xi}_{i,m}\mathbf{E}\mathbf{x}_i' \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\psi}_{i,m} \\ &+ \sum_{m=1}^{M}\left(\boldsymbol{\Xi}_{i,m}\mathbf{E}\mathbf{n}_i' \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\alpha}_{i,m} + \mathbf{u}_i.\end{aligned} \tag{3.47}$$

  – Regression equation in case of regime-invariant parameters of exogenous variables:

$$\mathbf{x}_i = \left(\mathbf{1}_T \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\nu}_i + \left(\mathbf{E}\mathbf{x}_i' \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\psi}_i + \sum_{m=1}^{M}\left(\boldsymbol{\Xi}_{i,m}\mathbf{E}\mathbf{n}_i' \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\alpha}_{i,m} + \mathbf{u}_i. \tag{3.48}$$

  – Innovation process:

$$\mathbf{u}_i \sim N\left(\mathbf{0}, \boldsymbol{\Omega}_i\right) \text{ with } \boldsymbol{\Omega}_i = \mathbf{I}_T \otimes \boldsymbol{\Sigma}_{ii}. \tag{3.49}$$

- MSAH-VAR Model with regime-dependent autoregressive parameters and covariance matrix:
  – Regression equation in case of regime-dependent parameters of exogenous variables:

$$\begin{aligned}\mathbf{x}_i = \left(\mathbf{1}_T \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\nu}_i &+ \sum_{m=1}^{M}\left(\boldsymbol{\Xi}_{i,m}\mathbf{E}\mathbf{x}_i' \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\psi}_{i,m} \\ &+ \sum_{m=1}^{M}\left(\boldsymbol{\Xi}_{i,m}\mathbf{E}\mathbf{n}_i' \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\alpha}_{i,m} + \mathbf{u}_i.\end{aligned} \tag{3.50}$$

  – Regression equation in case of regime-invariant parameters of exogenous variables:

$$\mathbf{x}_i = \left(\mathbf{1}_T \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\nu}_i + \left(\mathbf{E}\mathbf{x}_i' \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\psi}_i + \sum_{m=1}^{M}\left(\boldsymbol{\Xi}_{i,m}\mathbf{E}\mathbf{n}_i' \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\alpha}_{i,m} + \mathbf{u}_i. \tag{3.51}$$

  – Innovation process:

$$\mathbf{u}_i \sim N\left(\mathbf{0}, \boldsymbol{\Omega}_i\right) \text{ with } \boldsymbol{\Omega}_i = \sum_{m=1}^{M} \boldsymbol{\Xi}_{i,m} \otimes \boldsymbol{\Sigma}_{ii,m}. \tag{3.52}$$

3.2.1.3 Filtering Algorithm

In the following, we present the filter algorithm of Hamilton (1989), to calculate the optimal inference of the unobservable regime probabilities $\boldsymbol{\xi}_{i,t+1}$, which provide insights in the state of commodity markets and which are used for the calculation of the maximum likelihood function. In the case of Markov-switching models with exogenous variables, we compute the inference of $\boldsymbol{\xi}_{i,t+1}$ on basis of the information set $\mathbf{X}_{i,t}$, consisting of the observed values of $\mathbf{x}_{i,t}$ and also of the exogenous information:

$$\mathbf{X}_{i,t} = \left(\mathbf{x}_{i,t}', \mathbf{x}_{i,t-1}', \ldots, \mathbf{x}_{i,1-P}', \mathbf{e}_{i,t}', \mathbf{e}_{i,t-1}', \ldots, \mathbf{e}_{i,1-P_{exog}}', \mathbf{x}_{i,t}^{*\prime}, \mathbf{x}_{i,t-1}^{*\prime}, \ldots, \mathbf{x}_{i,1-P}^{*\prime}\right)'.$$

The iterative algorithm described in Krolzig (1997) is a discrete version of the Kalman filter for the state space model in Equation 3.29 and Equation 3.30. First, we define the probability distribution of the state vector $\boldsymbol{\xi}_{i,t}$ conditional on the information available at time τ as:

$$\boldsymbol{\xi}_{i,t|\tau} = \mathbb{E}\left[\boldsymbol{\xi}_{i,t}|\mathbf{X}_{i,\tau}\right] = \begin{pmatrix} Pr\left(\boldsymbol{\xi}_{i,t} = \boldsymbol{\iota}_1|\mathbf{X}_{i,\tau}\right) \\ Pr\left(\boldsymbol{\xi}_{i,t} = \boldsymbol{\iota}_2|\mathbf{X}_{i,\tau}\right) \\ \vdots \\ Pr\left(\boldsymbol{\xi}_{i,t} = \boldsymbol{\iota}_M|\mathbf{X}_{i,\tau}\right) \end{pmatrix}. \tag{3.53}$$

For this study, we differentiate between the filtered, the predicted as well as the smoothed probabilities. While the filtered (predicted) probabilities depend on information available at time t $(t-1)$, the smoothed probabilities are computed conditional on the full-sample information until time T. Starting with the filtered probabilities $\boldsymbol{\xi}_{i,t|t}$, we use the law of Bayes to get the posterior probabilities $Pr\left(\boldsymbol{\xi}_{i,t}|\mathbf{x}_{i,t}, \mathbf{X}_{i,t-1}\right)$:

$$Pr\left(\boldsymbol{\xi}_{i,t}|\mathbf{X}_{i,t}\right) \equiv Pr\left(\boldsymbol{\xi}_{i,t}|\mathbf{x}_{i,t}, \mathbf{X}_{i,t-1}\right) = \frac{p\left(\mathbf{x}_{i,t}|\boldsymbol{\xi}_{i,t}, \mathbf{X}_{i,t-1}\right) Pr\left(\boldsymbol{\xi}_{i,t}|\mathbf{X}_{i,t-1}\right)}{p\left(\mathbf{x}_{i,t}|\mathbf{X}_{i,t-1}\right)}, \tag{3.54}$$

with the prior probability:

$$Pr\left(\boldsymbol{\xi}_{i,t}|\mathbf{X}_{i,t-1}\right) = \sum_{\boldsymbol{\xi}_{i,t-1}} Pr\left(\boldsymbol{\xi}_{i,t}|\boldsymbol{\xi}_{i,t-1}\right) Pr\left(\boldsymbol{\xi}_{i,t-1}|\mathbf{X}_{i,t-1}\right) \tag{3.55}$$

and density

$$p\left(\mathbf{x}_{i,t}|\mathbf{X}_{i,t-1}\right) = \sum_{\boldsymbol{\xi}_{i,t}} p\left(\mathbf{x}_{i,t}, \boldsymbol{\xi}_{i,t}|\mathbf{X}_{i,t-1}\right) = \sum_{\boldsymbol{\xi}_{i,t}} Pr\left(\boldsymbol{\xi}_{i,t}|\mathbf{X}_{i,t-1}\right) p\left(\mathbf{x}_{i,t}|\boldsymbol{\xi}_{i,t}, \mathbf{X}_{i,t-1}\right), \tag{3.56}$$

where the summation is over all possible values of $\boldsymbol{\xi}_{i,t}$ and $\boldsymbol{\xi}_{i,t-1}$, respectively.

Further, we define the vector of densities of $\mathbf{x}_{i,t}$, conditional on $\boldsymbol{\xi}_{i,t}$ and $\mathbf{X}_{i,t-1}$, as $\boldsymbol{\eta}_{i,t}$:

$$\boldsymbol{\eta}_{i,t} = \begin{pmatrix} p\left(\mathbf{x}_{i,t}|\boldsymbol{\theta}_{i,1}, \mathbf{X}_{i,t-1}\right) \\ p\left(\mathbf{x}_{i,t}|\boldsymbol{\theta}_{i,2}, \mathbf{X}_{i,t-1}\right) \\ \vdots \\ p\left(\mathbf{x}_{i,t}|\boldsymbol{\theta}_{i,M}, \mathbf{X}_{i,t-1}\right) \end{pmatrix} = \begin{pmatrix} p\left(\mathbf{x}_{i,t}|\boldsymbol{\xi}_{i,t} = \boldsymbol{\iota}_1, \mathbf{X}_{i,t-1}\right) \\ p\left(\mathbf{x}_{i,t}|\boldsymbol{\xi}_{i,t} = \boldsymbol{\iota}_2, \mathbf{X}_{i,t-1}\right) \\ \vdots \\ p\left(\mathbf{x}_{i,t}|\boldsymbol{\xi}_{i,t} = \boldsymbol{\iota}_M, \mathbf{X}_{i,t-1}\right) \end{pmatrix}, \tag{3.57}$$

where we neglect the parameter vector $\boldsymbol{\theta}_{i,m}$, including the intercept ($\boldsymbol{\nu}_{i,m}$), the autoregressive parameters ($\boldsymbol{\alpha}_{i,m}$,) the parameters corresponding to the exogenous variables ($\boldsymbol{\psi}_{i,m}$) as well as the covariance matrix ($\boldsymbol{\Sigma}_{i,m}$), at the right hand side for a more convenient notation. Using this definition, it follows:

$$p\left(\mathbf{x}_{i,t}|\mathbf{X}_{i,t-1}\right) = \boldsymbol{\eta}_{i,t}'\hat{\boldsymbol{\xi}}_{i,t|t-1} = \mathbf{1}_M'\left(\boldsymbol{\eta}_{i,t} \odot \hat{\boldsymbol{\xi}}_{i,t|t-1}\right), \tag{3.58}$$

where $\hat{\boldsymbol{\xi}}_{i,t|t-1}$ denotes the estimate of $\boldsymbol{\xi}_{i,t|t-1}$, $\mathbf{1}_M$ is the $M \times 1$ vector of ones and $\odot$ denotes the element-wise matrix multiplication. Finally, we calculate the filtered and predicted probabilities recursively for $t = 1, 2, \ldots, T$ based on the initial values $\boldsymbol{\xi}_{i,0}$. Hereby, we estimate the filtered probabilities:

$$\hat{\boldsymbol{\xi}}_{i,t|t} = \frac{\boldsymbol{\eta}_{i,t} \odot \hat{\boldsymbol{\xi}}_{i,t|t-1}}{\mathbf{1}_M'\left(\boldsymbol{\eta}_{i,t} \odot \hat{\boldsymbol{\xi}}_{i,t|t-1}\right)}, \tag{3.59}$$

as well as the predicted probabilities, using the transition equation in Equation 3.30:

$$\hat{\boldsymbol{\xi}}_{i,t+1|t} = \mathbf{F}_i\hat{\boldsymbol{\xi}}_{i,t|t} = \frac{\mathbf{F}_i\left(\boldsymbol{\eta}_{i,t} \odot \hat{\boldsymbol{\xi}}_{i,t|t-1}\right)}{\mathbf{1}_M'\left(\boldsymbol{\eta}_{i,t} \odot \hat{\boldsymbol{\xi}}_{i,t|t-1}\right)}. \tag{3.60}$$

While the estimate of $\boldsymbol{\xi}_{i,t}$ is based on information up to time t so far, we now use the full-sample information until time T for the inference about the unobserved regime, by incorporating the previously neglected sample information $\mathbf{X}_{i,t+1.T}$. The derived smoothing gives therefore the best estimate of the unobservable state at any point in time within the sample.

To obtain the smoothed probabilities, we use the smoothing algorithm, which is the backward filter of Kim (1994), starting at time $t = T$ with the filtered probability $\hat{\boldsymbol{\xi}}_{i,T|T}$. Then, the full-sample smoothed inferences $\hat{\boldsymbol{\xi}}_{i,t|T}$ can be derived by iterating backwards, from $t = T-1, T-2, \ldots, 0$, using the following identity:

$$Pr\left(\boldsymbol{\xi}_{i,t}|\mathbf{X}_{i,T}\right) = \sum_{\boldsymbol{\xi}_{i,t+1}} Pr\left(\boldsymbol{\xi}_{i,t}, \boldsymbol{\xi}_{i,t+1}|\mathbf{X}_{i,T}\right) = \sum_{\boldsymbol{\xi}_{i,t+1}} Pr\left(\boldsymbol{\xi}_{i,t}|\boldsymbol{\xi}_{i,t+1}, \mathbf{X}_{i,T}\right) Pr\left(\boldsymbol{\xi}_{i,t+1}|\mathbf{X}_{i,T}\right). \quad (3.61)$$

For pure VAR models with Markovian parameter shifts, the probability laws for $\mathbf{x}_{i,t}$ and $\boldsymbol{\xi}_{i,t+1}$ depend only on the current state $\boldsymbol{\xi}_{i,t}$ and not on the former history of states. Therefore, using the law of Bayes twice as well as the independence to derive the equality $p\left(\mathbf{X}_{i,t+1.T}|\boldsymbol{\xi}_{i,t}, \boldsymbol{\xi}_{i,t+1}, \mathbf{X}_{i,t}\right) = p\left(\mathbf{X}_{i,t+1.T}|\boldsymbol{\xi}_{i,t}, \mathbf{X}_{i,t}\right)$, we get:

$$\begin{aligned} Pr\left(\boldsymbol{\xi}_{i,t}|\boldsymbol{\xi}_{i,t+1}, \mathbf{X}_{i,T}\right) &\equiv Pr\left(\boldsymbol{\xi}_{i,t}|\boldsymbol{\xi}_{i,t+1}, \mathbf{X}_{i,t}, \mathbf{X}_{i,t+1.T}\right) \\ &\overset{Bayes}{=} \frac{p\left(\mathbf{X}_{i,t+1.T}|\boldsymbol{\xi}_{i,t}, \boldsymbol{\xi}_{i,t+1}, \mathbf{X}_{i,t}\right) Pr\left(\boldsymbol{\xi}_{i,t}|\boldsymbol{\xi}_{i,t+1}, \mathbf{X}_{i,t}\right)}{p\left(\mathbf{X}_{i,t+1.T}|\boldsymbol{\xi}_{i,t+1}, \mathbf{X}_{i,t}\right)} \\ &= Pr\left(\boldsymbol{\xi}_{i,t}|\boldsymbol{\xi}_{i,t+1}, \mathbf{X}_{i,t}\right) \\ &\overset{Bayes}{=} \frac{Pr\left(\boldsymbol{\xi}_{i,t+1}|\boldsymbol{\xi}_{i,t}, \mathbf{X}_{i,t}\right) Pr\left(\boldsymbol{\xi}_{i,t}|\mathbf{X}_{i,t}\right)}{Pr\left(\boldsymbol{\xi}_{i,t+1}|\mathbf{X}_{i,t}\right)} \\ &= \frac{Pr\left(\boldsymbol{\xi}_{i,t+1}|\boldsymbol{\xi}_{i,t}\right) Pr\left(\boldsymbol{\xi}_{i,t}|\mathbf{X}_{i,t}\right)}{Pr\left(\boldsymbol{\xi}_{i,t+1}|\mathbf{X}_{i,t}\right)}, \end{aligned} \quad (3.62)$$

where the last equality follows from the independence of the former history of states. Finally, we calculate the smoothed probabilities, with $\oslash$ denoting the element-wise matrix division:

$$\begin{aligned} \hat{\boldsymbol{\xi}}_{i,t|T} &= \sum_{\boldsymbol{\xi}_{i,t+1}} Pr\left(\boldsymbol{\xi}_{i,t}|\boldsymbol{\xi}_{i,t+1}, \mathbf{X}_{i,T}\right) Pr\left(\boldsymbol{\xi}_{i,t+1}|\mathbf{X}_{i,T}\right) \\ &= \sum_{\boldsymbol{\xi}_{i,t+1}} \frac{Pr\left(\boldsymbol{\xi}_{i,t+1}|\boldsymbol{\xi}_{i,t}\right) Pr\left(\boldsymbol{\xi}_{i,t}|\mathbf{X}_{i,t}\right)}{Pr\left(\boldsymbol{\xi}_{i,t+1}|\mathbf{X}_{i,t}\right)} \hat{\boldsymbol{\xi}}_{i,t+1|T} \\ &= \left(\mathbf{F}_i' \left(\hat{\boldsymbol{\xi}}_{i,t+1|T} \oslash \hat{\boldsymbol{\xi}}_{i,t+1|t}\right)\right) \odot \hat{\boldsymbol{\xi}}_{i,t|t}. \end{aligned} \quad (3.63)$$

3.2.1.4 Maximum Likelihood (ML) Estimation

So far, we determine the state vector $\boldsymbol{\xi}_{i,t}$, given observations of $\mathbf{X}_i$,[21] and known parameters $\boldsymbol{\lambda}_i = \left(\boldsymbol{\theta}_i', \boldsymbol{\rho}_i', \boldsymbol{\xi}_{i,0}'\right)'$, including the parameter vector $\boldsymbol{\theta}_i$ of the VAR model, the transition probabilities $\boldsymbol{\rho}_i = vec\left(\mathbf{P_i}\right)$ as well as the initial values $\boldsymbol{\xi}_{i,0}$ for the filtered probabilities. In this subsection, we aim to obtain the maximum likelihood estimator for the parameter vector $\boldsymbol{\lambda}_i$. First, we observe

[21]In the case of Markov-switching models with exogenous variables, this information set also contains the exogenous information, as stated in the previous subsection.

the likelihood function as a byproduct of the filter:

$$
\begin{aligned}
L\left(\boldsymbol{\lambda}_i|\mathbf{X}_i\right) :=& p\left(\mathbf{X}_{i,T}|\mathbf{X}_{i,0};\boldsymbol{\lambda}_i\right) = \prod_{t=1}^{T} p\left(\mathbf{X}_{i,t}|\mathbf{X}_{i,t-1};\boldsymbol{\lambda}_i\right) \\
=& \prod_{t=1}^{T}\sum_{\boldsymbol{\xi}_{i,t}} p\left(\mathbf{x}_{i,t}|\boldsymbol{\xi}_{i,t},\mathbf{X}_{i,t-1};\boldsymbol{\theta}_i\right) Pr\left(\boldsymbol{\xi}_{i,t}|\mathbf{X}_{i,t-1};\boldsymbol{\lambda}_i\right) \\
=& \prod_{t=1}^{T} \boldsymbol{\eta}'_{i,t}\hat{\boldsymbol{\xi}}_{i,t|t-1} \overset{\text{Transition Equation 3.30}}{=} \prod_{t=1}^{T} \boldsymbol{\eta}'_{i,t}\mathbf{F}_i\hat{\boldsymbol{\xi}}_{i,t-1|t-1}.
\end{aligned}
\tag{3.64}
$$

Since the conditional densities $p\left(\mathbf{x}_{i,t}|\boldsymbol{\xi}_{i,t-1}=\boldsymbol{\iota}_m,\mathbf{X}_{i,t-1}\right)$ are mixtures of normals, the likelihood function is non-normal:

$$
\begin{aligned}
L\left(\boldsymbol{\lambda}_i|\mathbf{X}_i\right) =& \prod_{t=1}^{T}\sum_{l=1}^{M}\sum_{m=1}^{M} p_{i,lm} Pr\left(\boldsymbol{\xi}_{i,t-1}=\boldsymbol{\iota}_l|\mathbf{X}_{i,t-1},\boldsymbol{\lambda}_i\right) p\left(\mathbf{x}_{i,t}|\boldsymbol{\xi}_{i,t}=\boldsymbol{\iota}_m,\mathbf{X}_{i,t-1},\boldsymbol{\theta}_i\right) \\
=& \prod_{t=1}^{T}\sum_{l=1}^{M}\sum_{m=1}^{M} p_{i,lm}\hat{\boldsymbol{\xi}}_{i,l.t-1|t-1}\left\{(2\pi)^{-\frac{K_i}{2}}\left|\boldsymbol{\Sigma}_{ii,m}\right|^{-\frac{1}{2}}\exp\left(-\frac{1}{2}\mathbf{u}'_{i,m,t}\boldsymbol{\Sigma}^{-1}_{ii,m}\mathbf{u}_{i,m,t}\right)\right\},
\end{aligned}
\tag{3.65}
$$

where we assume the conditional densities $p\left(\mathbf{x}_{i,t}|\boldsymbol{\xi}_{i,t}=\boldsymbol{\iota}_m,\mathbf{X}_{i,t-1},\boldsymbol{\theta}_i\right)$ to be normal densities and $\mathbf{u}_{i,m,t} = \mathbf{x}_{i,t} - E\left[\mathbf{x}_{i,t}|\boldsymbol{\xi}_{i,t}=\boldsymbol{\iota}_m,\mathbf{X}_{i,t-1}\right]$. Then, the maximum likelihood (ML) estimates can be derived by maximization of the likelihood function $L\left(\boldsymbol{\lambda}_i|\mathbf{X}_i\right)$, subject to the adding-up restrictions on the matrix of transition probabilities $\boldsymbol{\rho}_i = vec\left(\mathbf{P_i}\right)$ and the initial state $\boldsymbol{\xi}_{i,0}$:

$$
\begin{aligned}
\mathbf{P}_i\mathbf{1}_M &= \mathbf{1}_M \\
\mathbf{1}'_M\boldsymbol{\xi}_{i,0} &= 1,
\end{aligned}
\tag{3.66}
$$

as well as the non-negativity restrictions for the transition probabilities $\boldsymbol{\rho}_i$, the covariances $\boldsymbol{\sigma}_i$ as well as the initial values $\boldsymbol{\xi}_{i,0}$ for the filtered probabilities:

$$
\boldsymbol{\rho}_i \geq \mathbf{0}, \boldsymbol{\sigma}_i \geq \mathbf{0}, \boldsymbol{\xi}_{i,0} \geq \mathbf{0}. \tag{3.67}
$$

If the non-negativity can be ensured, the maximum likelihood estimator $\hat{\boldsymbol{\lambda}}_i$ can be derived by the first order conditions of the constrained log-likelihood function:

$$
\ln L^*\left(\boldsymbol{\lambda}_i\right) = \ln L\left(\boldsymbol{\lambda}_i|\mathbf{X}_{i,T}\right) - \boldsymbol{\kappa}'_{i,1}\left(\mathbf{P}_i\mathbf{1}_M - \mathbf{1}_M\right) - \kappa_{i,2}\left(\mathbf{1}'_M\boldsymbol{\xi}_{i,0} - 1\right), \tag{3.68}
$$

with $\boldsymbol{\kappa}_{i,1}, \kappa_{i,2}$ denoting the Lagrange multipliers associated with the adding-up restrictions on the matrix of transition probabilities $\boldsymbol{\rho}_i$ and the initial state $\boldsymbol{\xi}_{i,0}$. Then, the first order conditions are given by a set of simultaneous equations:

$$
\begin{aligned}
\frac{\partial \ln L\left(\boldsymbol{\lambda}_i|\mathbf{X}_i\right)}{\partial \boldsymbol{\theta}'_i} &= \mathbf{0}, \\
\frac{\partial \ln L\left(\boldsymbol{\lambda}_i|\mathbf{X}_i\right)}{\partial \boldsymbol{\rho}'_i} - \boldsymbol{\kappa}'_{i,1}\left(\mathbf{1}'_M \otimes \mathbf{I}_M\right) &= \mathbf{0}, \\
\frac{\partial \ln L\left(\boldsymbol{\lambda}_i|\mathbf{X}_i\right)}{\partial \boldsymbol{\xi}'_{i,0}} - \kappa_{i,2}\mathbf{1}'_M &= \mathbf{0},
\end{aligned}
\tag{3.69}
$$

where it is assumed that the interior solution of these conditions exists and is well-behaved, such that the non-negativity restrictions are not binding.

3.2.1.5 Expected Maximum Likelihood Estimation

As shown in Hamilton (1990), the expectation-maximization (EM) algorithm introduced by Dempster et al. (1977) can be used in conjunction with the filter to obtain the maximum likelihood estimates of the model's parameters. The EM algorithm is an iterative ML estimation technique designed for a general class of models where the observed time series depends on unobservable stochastic variables. Each iteration of the EM algorithm consists of two steps, the expectation step (E-Step) and the maximization step (M-Step), see Table 3.2.

First, the filtered as well as the smoothed probabilities are calculated within the expectation step, to determine the expectations (of the states). Therefore, the residuals $\mathbf{u}_{i,m,t}$ for each commodity i, state m and time t of the MS-VAR models are calculated using the predicted values $\hat{\mathbf{x}}_{i,m,t}$ gained by the parameters given by the previous iteration: $\mathbf{u}_{i,m,t} = \mathbf{x}_{i,t} - \hat{\mathbf{x}}_{i,m,t}$. Therewith, we calculate $\boldsymbol{\eta}_{i,t}$ under the assumption of normally distributed residuals as well as the estimated parameters from the previous iteration, using Equation 3.57.

Subsequently, starting with the initial value $\hat{\boldsymbol{\xi}}_{i,0|0}$, the filtered $\hat{\boldsymbol{\xi}}_{i,t|t}$ as well as predicted probabilities $\hat{\boldsymbol{\xi}}_{i,t+1|t}$ are estimated iteratively given the parameters of the previous maximization step, according to Equation 3.59 and Equation 3.60 respectively. Within a backward iteration for $t = T-1, T-2, \ldots, 0$, the smoothed probabilities $\hat{\boldsymbol{\xi}}_{i,t|T}$ are derived, according to Equation 3.63, conditional on the previous calculated parameters.

Second, in the maximization step, the parameters $\boldsymbol{\lambda}_i = \left(\boldsymbol{\theta}_i', \boldsymbol{\rho}_i', \boldsymbol{\xi}_{i,0}'\right)'$, including the parameter vector $\boldsymbol{\theta}_i$ of the VAR model, the transition probabilities $\boldsymbol{\rho}_i = vec\left(\mathbf{P_i}\right)$ as well as the initial values $\boldsymbol{\xi}_{i,0}$ for the filtered probabilities, are estimated given the predetermined regimes. In the following, we describe the estimation of the transition probabilities $\boldsymbol{\rho}_i$. Subsequently, we derive the estimator of the parameter vector $\boldsymbol{\theta}_i$ of the VAR model and finally, we explain, how the initial values $\boldsymbol{\xi}_{i,0}$ for the filtered probabilities are calculated.

Starting with the hidden Markov chain step, the transition probabilities $\boldsymbol{\rho}_i = vec\left(\mathbf{P_i}\right)$ are calculated. According to Krolzig (2002), the transition probabilities are derived under given data $\mathbf{X}_{i,T}$ and parameters $\boldsymbol{\lambda}_i$ as follows:

$$\hat{p}_{i,lm} = \frac{\sum_{t=1}^{T} Pr\left(s_{i,t} = m, s_{i,t-1} = l | \mathbf{X}_{i,T}, \boldsymbol{\lambda}_i\right)}{\sum_{t=1}^{T} Pr\left(s_{i,t-1} = l | \mathbf{X}_{i,T}, \boldsymbol{\lambda}_i\right)}. \tag{3.70}$$

Using the definitions of the filtered, predicted as well as smoothed probabilities and the previous transition probabilities, we get:

$$\begin{aligned} & Pr\left(s_{i,t} = m, s_{i,t-1} = l | \mathbf{X}_{i,T}, \boldsymbol{\lambda}_i\right) \\ &= \frac{Pr\left(s_{i,t} = m | s_{i,t-1} = l\right) Pr\left(s_{i,t} = m | \mathbf{X}_{i,T}, \boldsymbol{\lambda}_i\right) Pr\left(s_{i,t-1} = l | \mathbf{X}_{i,t-1}, \boldsymbol{\lambda}_i\right)}{Pr\left(s_{i,t} = m | \mathbf{X}_{i,t-1}, \boldsymbol{\lambda}_i\right)} \\ &= \frac{\hat{p}_{i,lm} \hat{\boldsymbol{\xi}}_{i,m,t|T} \hat{\boldsymbol{\xi}}_{i,l,t-1|t-1}}{\hat{\boldsymbol{\xi}}_{i,m,t|t-1}}, \end{aligned} \tag{3.71}$$

Therewith, we derive the transition probabilities by inserting Equation 3.71 in Equation 3.70:

$$\begin{aligned} \hat{p}_{i,lm} &= \frac{\sum_{t=1}^{T} Pr\left(s_{i,t} = m, s_{i,t-1} = l | \mathbf{X}_{i,T}, \boldsymbol{\lambda}_i\right)}{\sum_{t=1}^{T} Pr\left(s_{i,t-1} = l | \mathbf{X}_{i,T}, \boldsymbol{\lambda}_i\right)} \\ &= \frac{\sum_{t=1}^{T} \frac{Pr\left(s_{i,t}=m | s_{i,t-1}=l\right) Pr\left(s_{i,t}=m | \mathbf{X}_{i,T}\right) Pr\left(s_{i,t-1}=l | \mathbf{X}_{i,t-1}\right)}{Pr\left(s_{i,t}=m | \mathbf{X}_{i,t-1}\right)}}{\sum_{t=1}^{T} Pr\left(s_{i,t-1} = l | \mathbf{X}_{i,T}, \boldsymbol{\lambda}_i\right)} \\ &= \frac{\sum_{t=1}^{T} \frac{\hat{p}_{i,lm} \hat{\boldsymbol{\xi}}_{i,m,t|T} \hat{\boldsymbol{\xi}}_{i,l,t-1|t-1}}{\hat{\boldsymbol{\xi}}_{i,m,t|t-1}}}{\sum_{t=1}^{T} \hat{\boldsymbol{\xi}}_{i,l,t-1|T}}. \end{aligned} \tag{3.72}$$

Table 3.2: Overview of the Expectation-Maximization algorithm

1. Initialization
   (a) Initialize the parameters, for $i = 1, 2, \dots, N$:
   $$\boldsymbol{\theta}_i^{(0)}, \boldsymbol{\rho}_i^0, \boldsymbol{\xi}_{i,1|0}.$$
2. Expectation step
   (a) Calculate the residuals $\mathbf{u}_{i,m,t}$, for $i = 1, 2, \dots, N, m = 1, 2, \dots, M, t = 1, 2, \dots, T$:
   $$\mathbf{u}_{i,m,t} = \mathbf{x}_{i,t} - \hat{\mathbf{x}}_{i,m,t},$$
   using the predicted values $\hat{\mathbf{x}}_{i,m,t}$ gained by the parameters given by the previous iteration.
   (b) Calculate $\boldsymbol{\eta}_{i,t}$, for $i = 1, 2, \dots, N, t = 1, 2, \dots, T$, under the assumption of normally distributed residuals, according to Equation 3.57:
   $$\boldsymbol{\eta}_{i,t} = \begin{pmatrix} p\left(\mathbf{x}_{i,t}|\boldsymbol{\theta}_{i,1}, \mathbf{X}_{i,t-1}\right) \\ p\left(\mathbf{x}_{i,t}|\boldsymbol{\theta}_{i,2}, \mathbf{X}_{i,t-1}\right) \\ \vdots \\ p\left(\mathbf{x}_{i,t}|\boldsymbol{\theta}_{i,M}, \mathbf{X}_{i,t-1}\right) \end{pmatrix}.$$
   (c) Filtering (forward recursion $t = 1, 2, \dots, T$): Calculate iteratively the filtered $\hat{\boldsymbol{\xi}}_{i,t|t}$ and predicted probabilities $\hat{\boldsymbol{\xi}}_{i,t+1|t}$, for $i = 1, 2, \dots, N$, according to Equation 3.59 and Equation 3.60 respectively:
   $$\hat{\boldsymbol{\xi}}_{i,t|t} = \frac{\boldsymbol{\eta}_{i,t} \odot \hat{\boldsymbol{\xi}}_{i,t|t-1}}{\mathbf{1}_M'\left(\boldsymbol{\eta}_{i,t} \odot \hat{\boldsymbol{\xi}}_{i,t|t-1}\right)},$$
   $$\hat{\boldsymbol{\xi}}_{i,t+1|t} = \mathbf{F}_i\hat{\boldsymbol{\xi}}_{i,t|t} = \frac{\mathbf{F}_i\left(\boldsymbol{\eta}_{i,t} \odot \hat{\boldsymbol{\xi}}_{i,t|t-1}\right)}{\mathbf{1}_M'\left(\boldsymbol{\eta}_{i,t} \odot \hat{\boldsymbol{\xi}}_{i,t|t-1}\right)}.$$
   (d) Smoothing (backward recursion $t = T-1, T-2, \dots, 1$): Calculate the smoothed probabilities $\hat{\boldsymbol{\xi}}_{i,t|T}$, for $i = 1, 2, \dots, N$, according to Equation 3.63:
   $$\hat{\boldsymbol{\xi}}_{i,t|T} = \left(\mathbf{F}_i'\left(\hat{\boldsymbol{\xi}}_{i,t+1|T} \oslash \hat{\boldsymbol{\xi}}_{i,t+1|t}\right)\right) \odot \hat{\boldsymbol{\xi}}_{i,t|t}.$$
3. Maximization step
   (a) Hidden Markov Chain Step: Calculate the transition probabilities $\boldsymbol{\rho}_i = vec\left(\mathbf{P_i}\right)$, for $i = 1, 2, \dots, N$, according to Equation 3.72:
   $$\hat{p}_{i,lm} = \frac{\sum_{t=1}^{T} \frac{\hat{p}_{i,lm}\hat{\boldsymbol{\xi}}_{i,m,t|T}\hat{\boldsymbol{\xi}}_{i,l,t-1|t-1}}{\hat{\boldsymbol{\xi}}_{i,m,t|t-1}}}{\sum_{t=1}^{T}\hat{\boldsymbol{\xi}}_{i,l,t-1|T}}.$$
   (b) Regression Step: Estimate the VAR parameter vector $\boldsymbol{\theta}_i$, for $i = 1, 2, \dots, N$:
   - Estimate $\boldsymbol{\gamma}_i$ denoting the structural parameters, including the intercept ($\boldsymbol{\nu}_i$), the autoregressive parameters ($\boldsymbol{\alpha}_i$) as well as the parameters corresponding to the exogenous variables($\boldsymbol{\psi}_i$), according to Equation 3.83:
   $$\hat{\boldsymbol{\gamma}}_i = \left(\sum_{m=1}^{M}\left(\bar{\mathbf{X}}_{i,m}'\hat{\boldsymbol{\Xi}}_{i,m}\bar{\mathbf{X}}_{i,m}\right) \otimes \boldsymbol{\Sigma}_{ii,m}^{-1}\right)^{-1}\left(\sum_{m=1}^{M}\left(\bar{\mathbf{X}}_{i,m}'\hat{\boldsymbol{\Xi}}_{i,m}\right) \otimes \boldsymbol{\Sigma}_{ii,m}^{-1}\right)\mathbf{x}_i.$$
   - Estimate the covariance matrix, according to Equation 3.109 under homoscedasticity and Equation 3.112 under heteroscedasticity:
   $$\hat{\boldsymbol{\Sigma}}_i = \frac{1}{T}\hat{\mathbf{u}}_i^*\left(\boldsymbol{\lambda}_i\right)'\hat{\mathbf{u}}_i^*\left(\boldsymbol{\lambda}_i\right) = \frac{1}{T}\sum_{m=1}^{M}\hat{\mathbf{u}}_{i,m}\left(\boldsymbol{\lambda}_i\right)'\hat{\boldsymbol{\Xi}}_{i,m}\hat{\mathbf{u}}_{i,m}\left(\boldsymbol{\lambda}_i\right),$$
   $$\hat{\boldsymbol{\Sigma}}_{i,m} = \frac{1}{\hat{T}_{i,m}}\hat{\mathbf{u}}_{i,m}^*\left(\boldsymbol{\lambda}_i\right)'\hat{\mathbf{u}}_{i,m}^*\left(\boldsymbol{\lambda}_i\right) = \frac{1}{\hat{T}_{i,m}}\hat{\mathbf{u}}_{i,m}\left(\boldsymbol{\lambda}_i\right)'\hat{\boldsymbol{\Xi}}_{i,m}\hat{\mathbf{u}}_{i,m}\left(\boldsymbol{\lambda}_i\right).$$
   (c) Initial state: Update the initial state $\hat{\boldsymbol{\xi}}_{i,0}$, for $i = 1, 2, \dots, N$, according to Equation 3.113:
   $$\hat{\boldsymbol{\xi}}_{i,0} = \hat{\boldsymbol{\xi}}_{i,0|T}.$$
4. Iterate the expectation step and maximization step until convergence.

This table provides an overview of the expectation-maximization (EM) algorithm for MS-VAR models.

Besides the calculation of the transition probabilities, the parameters of the VAR models have to be estimated within the M-Step of the EM algorithm. In the following, we focus on the estimation of the VAR parameter vector $\boldsymbol{\theta}_i$. Therefore, we first rewrite the objective function, according to Hamilton (1990), to simplify the expected log-likelihood function. Using the regression equation, we derive the maximum likelihood estimator for the MS-VAR parameters as generalized least squares estimator. Subsequently, we derive the estimator for the covariance matrix under homoscedasticity as well as under heteroscedasticity.

Within the M-Step, the estimation of the parameter vector $\boldsymbol{\lambda}_i$ is not simultaneous with the estimation of the smoothed regime probabilities $Pr\left(\boldsymbol{\xi}_i|\mathbf{X}_i, \boldsymbol{\lambda}_i\right)$, which is already estimated within the E-Step, using the predetermined parameter vector $\boldsymbol{\lambda}_i^{(j-1)}$ of the previous iteration $j-1$. Hamilton (1990) shows that, under these circumstances, the first order conditions are equivalent to the following objective function:

$$\begin{aligned} \ell\left(\boldsymbol{\lambda}_i|\mathbf{X}_{i,T}, \boldsymbol{\lambda}_i^{(j-1)}\right) &= \int \ln p\left(\mathbf{X}_i, \boldsymbol{\xi}_i|\boldsymbol{\lambda}_i\right) p\left(\mathbf{X}_i, \boldsymbol{\xi}_i|\boldsymbol{\lambda}_i^{(j-1)}\right) d\boldsymbol{\xi}_i \\ &= \int \ln\left(p\left(\mathbf{X}_i|\boldsymbol{\xi}_i, \boldsymbol{\lambda}_i\right) Pr\left(\boldsymbol{\xi}_i|\boldsymbol{\lambda}_i\right)\right) Pr\left(\boldsymbol{\xi}_i|\mathbf{X}_i, \boldsymbol{\lambda}_i^{(j-1)}\right) p\left(\mathbf{X}_i|\boldsymbol{\lambda}_i^{(j-1)}\right) d\boldsymbol{\xi}_i \\ &= p\left(\mathbf{X}_i|\boldsymbol{\lambda}_i^{(j-1)}\right) \int \ln p\left(\mathbf{X}_i|\boldsymbol{\xi}_i, \boldsymbol{\lambda}_i\right) Pr\left(\boldsymbol{\xi}_i|\mathbf{X}_i, \boldsymbol{\lambda}_i^{(j-1)}\right) d\boldsymbol{\xi}_i \\ &+ p\left(\mathbf{X}_i|\boldsymbol{\lambda}_i^{(j-1)}\right) \int \ln Pr\left(\boldsymbol{\xi}_i|\boldsymbol{\lambda}_i\right) Pr\left(\boldsymbol{\xi}_i|\mathbf{X}_i, \boldsymbol{\lambda}_i^{(j-1)}\right) d\boldsymbol{\xi}_i. \end{aligned} \tag{3.73}$$

Hence, we derive from Equation 3.73:

$$\begin{aligned} \ell\left(\boldsymbol{\lambda}_i|\mathbf{X}_{i,T}, \boldsymbol{\lambda}_i^{(j-1)}\right) \propto &\sum_{t=1}^{T} \sum_{\boldsymbol{\xi}_{i,t}} \ln p\left(\mathbf{x}_{i,t}|\boldsymbol{\xi}_{i,t}, \mathbf{X}_{i,t-1}, \boldsymbol{\theta}_i\right) Pr\left(\boldsymbol{\xi}_{i,t}|\mathbf{X}_{i,T}, \boldsymbol{\lambda}_i^{(j-1)}\right) \\ &+ \sum_{t=1}^{T} \sum_{\boldsymbol{\xi}_{i,t-1}} \ln Pr\left(\boldsymbol{\xi}_{i,t}|\boldsymbol{\xi}_{i,t-1}, \boldsymbol{\rho}_i\right) Pr\left(\boldsymbol{\xi}_{i,t}, \boldsymbol{\xi}_{i,t-1}|\mathbf{X}_{i,T}, \boldsymbol{\lambda}_i^{(j-1)}\right). \end{aligned} \tag{3.74}$$

Within each maximization step, the objective function in Equation 3.73 is maximized. Once the algorithm has converged, we get the ML estimator $\hat{\boldsymbol{\lambda}}_i$:

$$\hat{\boldsymbol{\lambda}}_i \coloneqq \boldsymbol{\lambda}_i^{(j)} = \boldsymbol{\lambda}_i^{(j-1)}. \tag{3.75}$$

For a simpler notation, we drop $\boldsymbol{\lambda}_i^{(j-1)}$ in the following.

At this point, we focus on the estimation of the VAR parameter vector $\boldsymbol{\theta}_i$. Therefore, we consider the first part of Equation 3.74. With the normality of the conditional densities and $\boldsymbol{\gamma}_i$ denoting the structural parameters, including the intercept ($\boldsymbol{\nu}_i$), the autoregressive parameters ($\boldsymbol{\alpha}_i$) as well as the parameters corresponding to the exogenous variables($\boldsymbol{\psi}_i$), as well as the following equality,

$$p\left(\mathbf{x}_{i,t}|s_{i,t}=m, \mathbf{X}_{i,t-1}, \boldsymbol{\theta}_i\right) = (2\pi)^{-\frac{-K_i}{2}} \left|\boldsymbol{\Sigma}_{ii,m}\right|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\mathbf{u}_{i,m,t}\left(\boldsymbol{\gamma}_i\right)' \boldsymbol{\Sigma}_{ii,m}^{-1} \mathbf{u}_{i,m,t}\left(\boldsymbol{\gamma}_i\right)\right\}, \tag{3.76}$$

the expected log-likelihood function simplifies to:

$$\begin{aligned} &\ell\left(\boldsymbol{\theta}_i|\mathbf{X}_{i,T}\right) \propto \\ &\propto \text{const.} - \frac{1}{2}\sum_{t=1}^{T}\sum_{m=1}^{M} \hat{\boldsymbol{\xi}}_{i,m,t|T}\left\{K_i \ln(2\pi) + \ln\left|\boldsymbol{\Sigma}_{ii,m}\right| + \mathbf{u}_{i,m,t}\left(\boldsymbol{\gamma}_i\right)' \boldsymbol{\Sigma}_{ii,m}^{-1}\mathbf{u}_{i,m,t}\left(\boldsymbol{\gamma}_i\right)\right\} \\ &\propto \text{const.} - \frac{1}{2}\sum_{m=1}^{M}\left\{\hat{T}_{i,m}\ln\left|\boldsymbol{\Sigma}_{ii,m}\right| + \sum_{t=1}^{T}\mathbf{u}_{i,m,t}\left(\boldsymbol{\gamma}_i\right)'\left(\hat{\boldsymbol{\xi}}_{i,m,t|T}\boldsymbol{\Sigma}_{ii,m}^{-1}\right)\mathbf{u}_{i,m,t}\left(\boldsymbol{\gamma}_i\right)\right\}, \end{aligned} \tag{3.77}$$

with $\hat{T}_{i,m} = \sum_{t=1}^{T} \hat{\boldsymbol{\xi}}_{i,m,t|T}$, where the term $\hat{T}_{i,m} K_i \ln(2\pi)$ is added to the constant (*const.*) term. This log-likelihood function can be rewritten in matrix notation:

$$\begin{aligned}\ell(\boldsymbol{\theta}_i|\mathbf{X}_{i,T}) &\propto \text{const.} - \frac{1}{2}\sum_{m=1}^{M}\left\{\hat{T}_{i,m}\ln|\boldsymbol{\Sigma}_{ii,m}| + \mathbf{u}_{i,m}(\boldsymbol{\gamma}_i)'\left(\hat{\boldsymbol{\Xi}}_{i,m}\otimes\boldsymbol{\Sigma}_{ii,m}^{-1}\right)\mathbf{u}_{i,m}(\boldsymbol{\gamma}_i)\right\} \\ &\propto \text{const.} - \frac{1}{2}\sum_{m=1}^{M}\hat{T}_{i,m}\ln|\boldsymbol{\Sigma}_{ii,m}| - \frac{1}{2}\mathbf{u}_i(\boldsymbol{\gamma}_i)'\mathbf{W}_i^{-1}\mathbf{u}_i(\boldsymbol{\gamma}_i),\end{aligned} \tag{3.78}$$

where $\mathbf{W}_i^{-1} = \begin{pmatrix} \hat{\boldsymbol{\Xi}}_{i,1}\otimes\boldsymbol{\Sigma}_{ii,1}^{-1} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \hat{\boldsymbol{\Xi}}_{i,M}\otimes\boldsymbol{\Sigma}_{ii,M}^{-1}\end{pmatrix}$,

$\hat{\boldsymbol{\Xi}}_{i,m} = diag\left(\hat{\boldsymbol{\xi}}_{i,m}\right) = \begin{pmatrix}\hat{\boldsymbol{\xi}}_{i,m,1|T} & & \\ & \ddots & \\ & & \hat{\boldsymbol{\xi}}_{i,m,T|T}\end{pmatrix}$, $\mathbf{u}_i(\boldsymbol{\gamma}_i) = \begin{pmatrix}\mathbf{u}_{i,1}(\boldsymbol{\gamma}_i) \\ \mathbf{u}_{i,2}(\boldsymbol{\gamma}_i) \\ \vdots \\ \mathbf{u}_{i,M}(\boldsymbol{\gamma}_i)\end{pmatrix} = \mathbf{1}_M \otimes \mathbf{x}_i - \mathbf{X}_i\boldsymbol{\gamma}_i$, with

$\mathbf{u}_{i,m}(\boldsymbol{\gamma}_i) = \mathbf{x}_i - \mathbf{X}_{i,m}\boldsymbol{\gamma}_i$, and $\mathbf{X}_i = \begin{pmatrix}\mathbf{X}_{i,1} \\ \mathbf{X}_{i,2} \\ \vdots \\ \mathbf{X}_{i,M}\end{pmatrix}$, with $\mathbf{X}_{i,m} = \begin{pmatrix}\mathbf{X}_{i,m,1} \\ \mathbf{X}_{i,m,2} \\ \vdots \\ \mathbf{X}_{i,m,T}\end{pmatrix}$.

The considered MS-VAR models in the intercept form are linear in the vector $\boldsymbol{\gamma}_i$ of structural parameters, i.e. the regression equation can be written as:

$$\mathbf{x}_{i,t} = \sum_{m=1}^{M}\boldsymbol{\xi}_{i,m,t}\mathbf{X}_{i,m,t}\boldsymbol{\gamma}_i + \mathbf{u}_{i,t}. \tag{3.79}$$

Therefore, the residuals associated with regime m at time t are given by:

$$\mathbf{u}_{i,m,t}(\boldsymbol{\gamma}_i) = \mathbf{x}_{i,t} - \mathbf{X}_{i,m,t}\boldsymbol{\gamma}_i. \tag{3.80}$$

We can show that the maximum likelihood estimator equals to the generalized least squares estimator of the corresponding linear regression model, with many observations per cell, where the pseudo observations $(\mathbf{x}_{i,t}, \mathbf{X}_{i,m,t}, \boldsymbol{\xi}_{i,t} = \boldsymbol{\iota}_m)$ are weighted with the smoothed probabilities $\hat{\boldsymbol{\xi}}_{i,m,t|T}\left(\boldsymbol{\lambda}_i^{j-1}\right) = Pr\left(\boldsymbol{\xi}_{i,t} = \boldsymbol{\iota}_m|\mathbf{X}_{i,T}, \boldsymbol{\lambda}_i^{j-1}\right)$:

$$\frac{\partial\ell(\boldsymbol{\theta}_i|\mathbf{X}_{i,T})}{\partial\boldsymbol{\gamma}_i} = -\frac{1}{2}\frac{\partial\mathbf{u}_i(\boldsymbol{\gamma}_i)'\mathbf{W}_i^{-1}\mathbf{u}_i(\boldsymbol{\gamma}_i)}{\partial\boldsymbol{\gamma}_i} = -\mathbf{u}_i(\boldsymbol{\gamma}_i)'\mathbf{W}_i^{-1}\frac{\partial\mathbf{u}_i(\boldsymbol{\gamma}_i)}{\partial\boldsymbol{\gamma}_i}. \tag{3.81}$$

Using the definition of the residuals $\mathbf{u}_i$ and setting the Equation 3.81 equal to zero, see Appendix B.1.2, we get the estimator:

$$\hat{\boldsymbol{\gamma}}_i = \left(\mathbf{X}_i'\mathbf{W}_i^{-1}\mathbf{X}_i\right)^{-1}\mathbf{X}_i'\mathbf{W}_i^{-1}(\mathbf{1}_M\otimes\mathbf{x}_i). \tag{3.82}$$

To reduce the computational effort, we rewrite[22] the maximum likelihood estimator for identical regressors in each equation $\mathbf{x}_{i,k_i}$, $k_i = 1, 2, \ldots, K_i$, where it holds $\mathbf{X}_{i,m} = \bar{\mathbf{X}}_{i,m}\otimes\mathbf{I}_{K_i}$ to:

$$\hat{\boldsymbol{\gamma}}_i = \left(\sum_{m=1}^{M}\left(\bar{\mathbf{X}}_{i,m}'\hat{\boldsymbol{\Xi}}_{i,m}\bar{\mathbf{X}}_{i,m}\right)\otimes\boldsymbol{\Sigma}_{ii,m}^{-1}\right)^{-1}\left(\sum_{m=1}^{M}\left(\bar{\mathbf{X}}_{i,m}'\hat{\boldsymbol{\Xi}}_{i,m}\right)\otimes\boldsymbol{\Sigma}_{ii,m}^{-1}\right)\mathbf{x}_i. \tag{3.83}$$

[22]For more information, please refer to the Appendix B.1.2, Equation B.9.

Hence, we derive the particular estimator for the different MS-VAR model specifications, using the definition of $\mathbf{Ex}_i$ and $\mathbf{En}_i$ in Section 3.2.1.2. Note the estimator for the MSIAX- as well as MSIAHX-VAR models can be calculated for each regime separately. As each regime separately is homogeneous, the estimators of both models coincide. The estimators for the homogeneous models can be simplified using the calculations in Equation B.6.

- MSI-VAR model
  - ML estimator of the maximization step:

$$\hat{\gamma}_i = \left(\left(\sum_{m=1}^{M}\left(\bar{\mathbf{X}}'_{i,m}\hat{\mathbf{\Xi}}_{i,m}\bar{\mathbf{X}}_{i,m}\right)\right)^{-1}\sum_{m=1}^{M}\left(\bar{\mathbf{X}}'_{i,m}\hat{\mathbf{\Xi}}_{i,m}\right)\otimes \mathbf{I}_{K_i}\right)\mathbf{x}_i \tag{3.84}$$

  - Definition of $\bar{\mathbf{X}}_{i,m}$ in case of regime-dependent exogenous variables:

$$\bar{\mathbf{X}}_{i,m} = \left(\mathbf{1}_T \otimes \boldsymbol{\iota}'_m, \boldsymbol{\iota}'_m \otimes \mathbf{Ex}_i, \mathbf{En}_i\right) \tag{3.85}$$

  - Definition of $\bar{\mathbf{X}}_{i,m}$ in case of regime-invariant exogenous variables:

$$\bar{\mathbf{X}}_{i,m} = \left(\mathbf{1}_T \otimes \boldsymbol{\iota}'_m, \mathbf{Ex}_i, \mathbf{En}_i\right) \tag{3.86}$$

- MSIH-VAR model
  - ML estimator of the maximization step:

$$\hat{\gamma}_i = \left(\sum_{m=1}^{M}\left(\bar{\mathbf{X}}'_{i,m}\hat{\mathbf{\Xi}}_{i,m}\bar{\mathbf{X}}_{i,m}\right)\otimes \mathbf{\Sigma}^{-1}_{ii,m}\right)^{-1}\left(\sum_{m=1}^{M}\left(\bar{\mathbf{X}}'_{i,m}\hat{\mathbf{\Xi}}_{i,m}\right)\otimes \mathbf{\Sigma}^{-1}_{ii,m}\right)\mathbf{x}_i \tag{3.87}$$

  - Definition of $\bar{\mathbf{X}}_{i,m}$ in case of regime-dependent exogenous variables:

$$\bar{\mathbf{X}}_{i,m} = \left(\mathbf{1}_T \otimes \boldsymbol{\iota}'_m, \boldsymbol{\iota}'_m \otimes \mathbf{Ex}_i, \mathbf{En}_i\right) \tag{3.88}$$

  - Definition of $\bar{\mathbf{X}}_{i,m}$ in case of regime-invariant exogenous variables:

$$\bar{\mathbf{X}}_{i,m} = \left(\mathbf{1}_T \otimes \boldsymbol{\iota}'_m, \mathbf{Ex}_i, \mathbf{En}_i\right) \tag{3.89}$$

- MSIA-VAR model
  - ML estimator of the maximization step in case of regime-dependent exogenous variables:

$$\hat{\gamma}_{i,m} = \left(\left(\left(\bar{\mathbf{X}}'_{i,m}\hat{\mathbf{\Xi}}_{i,m}\bar{\mathbf{X}}_{i,m}\right)^{-1}\left(\bar{\mathbf{X}}'_{i,m}\hat{\mathbf{\Xi}}_{i,m}\right)\right)\otimes \mathbf{I}_{K_i}\right)\mathbf{x}_i \tag{3.90}$$

  - Definition of $\bar{\mathbf{X}}_{i,m}$ in case of regime-dependent exogenous variables:

$$\bar{\mathbf{X}}_{i,m} = \left(\mathbf{1}_T, \mathbf{Ex}_i, \mathbf{En}_i\right) \tag{3.91}$$

  - ML estimator of the maximization step in case of regime-invariant exogenous variables:

$$\hat{\gamma}_i = \left(\left(\sum_{m=1}^{M}\left(\bar{\mathbf{X}}'_{i,m}\hat{\mathbf{\Xi}}_{i,m}\bar{\mathbf{X}}_{i,m}\right)\right)^{-1}\sum_{m=1}^{M}\left(\bar{\mathbf{X}}'_{i,m}\hat{\mathbf{\Xi}}_{i,m}\right)\otimes \mathbf{I}_{K_i}\right)\mathbf{x}_i \tag{3.92}$$

  - Definition of $\bar{\mathbf{X}}_{i,m}$ in case of regime-invariant exogenous variables:

$$\bar{\mathbf{X}}_{i,m} = \left(\mathbf{1}_T \otimes \boldsymbol{\iota}'_m, \mathbf{Ex}_i, \boldsymbol{\iota}'_m \otimes \mathbf{En}_i\right) \tag{3.93}$$

- MSIAH-VAR model
  - ML estimator of the maximization step in case of regime-dependent exogenous variables:
$$\hat{\gamma}_{i,m} = \left(\left(\left(\bar{\mathbf{X}}'_{i,m} \hat{\mathbf{\Xi}}_{i,m} \bar{\mathbf{X}}_{i,m} \right)^{-1} \left(\bar{\mathbf{X}}'_{i,m} \hat{\mathbf{\Xi}}_{i,m} \right) \right) \otimes \mathbf{I}_{K_i} \right) \mathbf{x}_i \tag{3.94}$$
  - Definition of $\bar{\mathbf{X}}_{i,m}$ in case of regime-dependent exogenous variables:
$$\bar{\mathbf{X}}_{i,m} = (\mathbf{1}_T, \mathbf{Ex}_i, \mathbf{En}_i) \tag{3.95}$$
  - ML estimator of the maximization step in case of regime-invariant exogenous variables:
$$\hat{\gamma}_i = \left(\sum_{m=1}^{M} \left(\bar{\mathbf{X}}'_{i,m} \hat{\mathbf{\Xi}}_{i,m} \bar{\mathbf{X}}_{i,m} \right) \otimes \mathbf{\Sigma}_{ii,m}^{-1} \right)^{-1} \left(\sum_{m=1}^{M} \left(\bar{\mathbf{X}}'_{i,m} \hat{\mathbf{\Xi}}_{i,m} \right) \otimes \mathbf{\Sigma}_{ii,m}^{-1} \right) \mathbf{x}_i \tag{3.96}$$
  - Definition of $\bar{\mathbf{X}}_{i,m}$ in case of regime-invariant exogenous variables:
$$\bar{\mathbf{X}}_{i,m} = (\mathbf{1}_T \otimes \boldsymbol{\iota}'_m, \mathbf{Ex}_i, \boldsymbol{\iota}'_m \otimes \mathbf{En}_i) \tag{3.97}$$
- MSH-VAR model
  - ML estimator of the maximization step:
$$\hat{\gamma}_i = \left(\sum_{m=1}^{M} \left(\bar{\mathbf{X}}'_{i,m} \hat{\mathbf{\Xi}}_{i,m} \bar{\mathbf{X}}_{i,m} \right) \otimes \mathbf{\Sigma}_{ii,m}^{-1} \right)^{-1} \left(\sum_{m=1}^{M} \left(\bar{\mathbf{X}}'_{i,m} \hat{\mathbf{\Xi}}_{i,m} \right) \otimes \mathbf{\Sigma}_{ii,m}^{-1} \right) \mathbf{x}_i \tag{3.98}$$
  - Definition of $\bar{\mathbf{X}}_{i,m}$ in case of regime-dependent exogenous variables:
$$\bar{\mathbf{X}}_{i,m} = (\mathbf{1}_T, \boldsymbol{\iota}'_m \otimes \mathbf{Ex}_i, \mathbf{En}_i) \tag{3.99}$$
  - Definition of $\bar{\mathbf{X}}_{i,m}$ in case of regime-invariant exogenous variables:
$$\bar{\mathbf{X}}_{i,m} = (\mathbf{1}_T, \mathbf{Ex}_i, \mathbf{En}_i) \tag{3.100}$$
- MSA-VAR model
  - ML estimator of the maximization step:
$$\hat{\gamma}_i = \left(\left(\sum_{m=1}^{M} \left(\bar{\mathbf{X}}'_{i,m} \hat{\mathbf{\Xi}}_{i,m} \bar{\mathbf{X}}_{i,m} \right) \right)^{-1} \sum_{m=1}^{M} \left(\bar{\mathbf{X}}'_{i,m} \hat{\mathbf{\Xi}}_{i,m} \right) \otimes \mathbf{I}_{K_i} \right) \mathbf{x}_i \tag{3.101}$$
  - Definition of $\bar{\mathbf{X}}_{i,m}$ in case of regime-dependent exogenous variables:
$$\bar{\mathbf{X}}_{i,m} = (\mathbf{1}_T, \boldsymbol{\iota}'_m \otimes \mathbf{Ex}_i, \boldsymbol{\iota}'_m \otimes \mathbf{En}_i) \tag{3.102}$$
  - Definition of $\bar{\mathbf{X}}_{i,m}$ in case of regime-invariant exogenous variables:
$$\bar{\mathbf{X}}_{i,m} = (\mathbf{1}_T, \mathbf{Ex}_i, \boldsymbol{\iota}'_m \otimes \mathbf{En}_i) \tag{3.103}$$
- MSAH-VAR model
  - ML estimator of the maximization step:
$$\hat{\gamma}_i = \left(\sum_{m=1}^{M} \left(\bar{\mathbf{X}}'_{i,m} \hat{\mathbf{\Xi}}_{i,m} \bar{\mathbf{X}}_{i,m} \right) \otimes \mathbf{\Sigma}_{ii,m}^{-1} \right)^{-1} \left(\sum_{m=1}^{M} \left(\bar{\mathbf{X}}'_{i,m} \hat{\mathbf{\Xi}}_{i,m} \right) \otimes \mathbf{\Sigma}_{ii,m}^{-1} \right) \mathbf{x}_i \tag{3.104}$$

– Definition of $\bar{\mathbf{X}}_{i,m}$ in case of regime-dependent exogenous variables:

$$\bar{\mathbf{X}}_{i,m} = \left(\mathbf{1}_T, \boldsymbol{\iota}'_m \otimes \mathbf{Ex}_i, \boldsymbol{\iota}'_m \otimes \mathbf{En}_i\right) \tag{3.105}$$

– Definition of $\bar{\mathbf{X}}_{i,m}$ in case of regime-invariant exogenous variables:

$$\bar{\mathbf{X}}_{i,m} = \left(\mathbf{1}_T, \mathbf{Ex}_i, \boldsymbol{\iota}'_m \otimes \mathbf{En}_i\right) \tag{3.106}$$

Besides the VAR parameters, the covariance matrix $\boldsymbol{\Sigma}_{ii,m}$ has to be estimated within the M-Step of the algorithm. Therefore, we differentiate between the homoscedastic and heteroscedastic case. Under homoscedasticity, it holds: $\boldsymbol{\Sigma}_{ii,m} = \boldsymbol{\Sigma}_{ii}$, for all states $m = 1, 2, \ldots, M$. Hence, we rewrite the expected log-likelihood function with $\mathbf{W}_i^{-1} = diag\left(\hat{\boldsymbol{\xi}}'_{i,1|t}, \hat{\boldsymbol{\xi}}'_{i,2|t}, \ldots, \hat{\boldsymbol{\xi}}'_{i,T|t}\right) \otimes \boldsymbol{\Sigma}_{ii}^{-1}$:

$$\ell\left(\boldsymbol{\lambda}_i|\mathbf{X}_{i,T}\right) \propto \text{const.} - \frac{K_i T}{2}\ln\left(2\pi\right) - \frac{T}{2}\ln\left|\boldsymbol{\Sigma}_{ii}\right| - \frac{1}{2}\mathbf{u}_i^*\left(\boldsymbol{\lambda}_i\right)'\mathbf{W}_i^{*-1}\mathbf{u}_i^*\left(\boldsymbol{\lambda}_i\right), \tag{3.107}$$

where $\mathbf{u}_i^*\left(\boldsymbol{\lambda}_i\right) = (diag(\sqrt{\boldsymbol{\xi}_{i,11|t}}, \sqrt{\boldsymbol{\xi}_{i,21|t}}, \ldots, \sqrt{\boldsymbol{\xi}_{i,M1|t}}, \sqrt{\boldsymbol{\xi}_{i,12|t}}, \sqrt{\boldsymbol{\xi}_{i,22|t}}, \ldots, \sqrt{\boldsymbol{\xi}_{i,M2|t}}, \ldots, \sqrt{\boldsymbol{\xi}_{i,1T|t}}, \sqrt{\boldsymbol{\xi}_{i,2T|t}}, \ldots, \sqrt{\boldsymbol{\xi}_{i,MT|t}}) \otimes \mathbf{I}_{K_i})\mathbf{u}_i\left(\boldsymbol{\lambda}_i\right)$ and $\mathbf{W}_i^{*-1} = \left(\mathbf{I}_T \otimes \boldsymbol{\Sigma}_{ii}^{-1}\right)$. Then, the partial derivatives of the expected log-likelihood function, with respect to $\boldsymbol{\Sigma}_{ii}$, are given by:

$$\frac{\partial\ell\left(\boldsymbol{\lambda}_i|\mathbf{X}_{i,T}\right)}{\partial\boldsymbol{\Sigma}_{ii}} = -\frac{T}{2}\boldsymbol{\Sigma}_{ii}^{-1} - \frac{1}{2}\boldsymbol{\Sigma}_{ii}^{-1}\mathbf{u}_i^*\left(\boldsymbol{\lambda}_i\right)'\mathbf{u}_i^*\left(\boldsymbol{\lambda}_i\right)\boldsymbol{\Sigma}_{ii}^{-1}. \tag{3.108}$$

If we set the partial derivatives to zero and solve Equation 3.108 for the covariance matrix $\boldsymbol{\Sigma}_{ii}$, using the estimator $\hat{\mathbf{u}}_i^*\left(\boldsymbol{\lambda}_i\right)$ for $\mathbf{u}_i^*\left(\boldsymbol{\lambda}_i\right)$, we get:

$$\hat{\boldsymbol{\Sigma}}_{ii} = \frac{1}{T}\hat{\mathbf{u}}_i^*\left(\boldsymbol{\lambda}_i\right)'\hat{\mathbf{u}}_i^*\left(\boldsymbol{\lambda}_i\right) = \frac{1}{T}\sum_{m=1}^{M}\hat{\mathbf{u}}_{i,m}\left(\boldsymbol{\lambda}_i\right)'\hat{\boldsymbol{\Xi}}_{i,m}\hat{\mathbf{u}}_{i,m}\left(\boldsymbol{\lambda}_i\right). \tag{3.109}$$

Under heteroscedasticity, we rewrite the expected log-likelihood function:

$$\ell\left(\boldsymbol{\lambda}_i|\mathbf{X}_{i,T}\right) \propto \text{const.} - \sum_{m=1}^{M}\left\{+\frac{\hat{T}_{i,m}}{2}\ln\left|\boldsymbol{\Sigma}_{ii,m}\right| + \frac{1}{2}\mathbf{u}_{i,m}^*\left(\boldsymbol{\lambda}_i\right)'\mathbf{W}_{i,m}^{*-1}\mathbf{u}_{i,m}^*\left(\boldsymbol{\lambda}_i\right)\right\}, \tag{3.110}$$

where $\mathbf{u}_{i,m}^*\left(\boldsymbol{\lambda}_i\right) = \left(diag\left(\sqrt{\hat{\boldsymbol{\xi}}_{i,m,1|T}}, \sqrt{\hat{\boldsymbol{\xi}}_{i,m,2|T}}, \ldots, \sqrt{\hat{\boldsymbol{\xi}}_{i,m,T|T}}\right) \otimes \mathbf{I}_{K_i}\right)\left(\mathbf{x}_i - \mathbf{X}_{i,m}\boldsymbol{\lambda}_i\right)$, and $\hat{T}_{i,m} = \sum_{t=1}^{T}\hat{\boldsymbol{\xi}}_{i,m,t|T}$, as well as $\mathbf{W}_{i,m}^{*-1} = \left(\mathbf{I}_T \otimes \boldsymbol{\Sigma}_{ii,m}\right)^{-1}$. Then, the partial derivatives of the expected log-likelihood function, with respect to $\boldsymbol{\Sigma}_{ii,m}$, are given by:

$$\frac{\partial\ell\left(\boldsymbol{\lambda}_i|\mathbf{X}_{i,T}\right)}{\partial\boldsymbol{\Sigma}_{ii,m}} = -\frac{\hat{T}_{i,m}}{2}\boldsymbol{\Sigma}_{ii,m}^{-1} - \frac{1}{2}\boldsymbol{\Sigma}_{ii,m}^{-1}\mathbf{u}_{i,m}^*\left(\boldsymbol{\lambda}_i\right)'\mathbf{u}_{i,m}^*\left(\boldsymbol{\lambda}_i\right)\boldsymbol{\Sigma}_{ii,m}^{-1}. \tag{3.111}$$

If we set the partial derivatives to zero and solve Equation 3.111 for the covariance matrix $\boldsymbol{\Sigma}_{ii,m}$, we get:

$$\hat{\boldsymbol{\Sigma}}_{ii,m} = \frac{1}{\hat{T}_{i,m}}\hat{\mathbf{u}}_{i,m}^*\left(\boldsymbol{\lambda}_i\right)'\hat{\mathbf{u}}_{i,m}^*\left(\boldsymbol{\lambda}_i\right) = \frac{1}{\hat{T}_{i,m}}\hat{\mathbf{u}}_{i,m}\left(\boldsymbol{\lambda}_i\right)'\hat{\boldsymbol{\Xi}}_{i,m}\hat{\mathbf{u}}_{i,m}\left(\boldsymbol{\lambda}_i\right). \tag{3.112}$$

Although the estimates for $\boldsymbol{\lambda}_i$ and $\boldsymbol{\Sigma}_{ii}$ are interdependent, Krolzig (1997) states it is sufficient to calculate one single generalized least squares estimation within each maximization step, to ensure the convergence of the EM algorithm.

As the last step of the algorithm, the initial state is updated by the smoothed probability:

$$\hat{\boldsymbol{\xi}}_{i,0} = \hat{\boldsymbol{\xi}}_{i,0|T}. \tag{3.113}$$

Given initial values and applying recursively the EM algorithm, we derive estimates for the regime probabilities as well as the parameters. As the EM algorithm is repeated iteratively until convergence is guaranteed, several convergence criteria are used. First, the relative change of the log-likelihood:

$$\Delta_1 = \frac{\ln L\left(\boldsymbol{\lambda}_i^{(j+1)}|\mathbf{X}_{i,T}\right) - \ln L\left(\boldsymbol{\lambda}_i^{(j)}|\mathbf{X}_{i,T}\right)}{\ln L\left(\boldsymbol{\lambda}_i^{(j)}|\mathbf{X}_{i,T}\right)} \tag{3.114}$$

has to be smaller than a predefined value, 0.0001 in our case.

Second, we control for the parameter variation, using the maximum norm,

$$\Delta_2 = \|\boldsymbol{\lambda}_i^{(j+1)} - \boldsymbol{\lambda}_i^{(j)}\| = \max_{o\in\{1,2,\ldots,O\}}\{|\boldsymbol{\lambda}_{i,o}^{(j+1)} - \boldsymbol{\lambda}_{i,o}^{(j)}|\}, \tag{3.115}$$

where the changes in the parameters have to be smaller than the predefined value,[23] again 0.0001 in our case, with O denoting the number of parameters.

3.2.1.6 Model Specification

To determine the optimal number of states, lag length as well as the specification of the MS-VAR models, we extend the model selection process based on the Hannan and Quinn (1979) (HQC^{MS}) and Schwarz (1978) (SIC^{MS}) criteria, presented in Li and Kwok (2021). While Li and Kwok (2021) propose to estimate the number of states as well as the lag length of MSI-VAR models with the information criteria, hence, only for models where the intercept is regime-dependent, we generalize their idea for all of our considered specifications of the MS-VAR models. Subsequently, we use the criteria to simultaneously decide for the optimal number of states, the lag length as well as the specification of the MS-VAR models. As we require our individual, commodity-specific models to have the same model characteristics, we further adjust the procedure of Li and Kwok (2021) slightly. In this study, we follow mostly the notation of Li and Kwok (2021), but add the index *spec* to highlight the dependency on the specification. Hereby, *spec* can attain all possible considered specifications *Spec* presented in Table 3.1.

In general, we estimate the optimal number of states, up to the predefined maximum number of states M_{max}, the optimal lag length, up to the predefined number of lags P_{max}, as well as the specification out of all considered specifications $spec \in Spec$, by minimizing a criterion function of the form:

$$IC\left(M, P, spec\right) = -2\ell\left(\hat{\boldsymbol{\lambda}}_{M,P,spec}|\mathbf{X}_{i,T}\right) + c_{T,M,P,spec}, \tag{3.116}$$

where the second term $c_{T,M,P,spec}$ is a sequence indexed by the sample size and penalizes the inclusion of redundant states and lags. As we require our individual, commodity-specific models to have the same model characteristics, we adjust the procedure of Li and Kwok (2021) for our final MS-GVAR model slightly and denote by $\ell\left(\hat{\boldsymbol{\lambda}}_{M,P,spec}|\mathbf{X}_{i,T}\right)$ the mean of all commodity-specific maximized log-likelihood values, corresponding to the MS-VAR models with M states, P lags and specification *spec*. The selection strategy proposed by Li and Kwok (2021) jointly selects the optimal model parameters $\left(\hat{M}, \hat{P}, \hat{spec}\right)$ by satisfying:

$$\left(\hat{M}, \hat{P}, \hat{spec}\right) = \arg\min_{0\leq P\leq P_{max}, 1\leq M\leq M_{max}, spec\in Spec} IC\left(M, P, spec\right). \tag{3.117}$$

We consider the two criteria HQC^{MS}, according to Hannan and Quinn (1979), or SIC^{MS}, according to Schwarz (1978), for the penalty term $c_{T,M,P,spec}$, defined as:

$$\begin{aligned} c_{T,M,P,spec} &= 2\cdot d\cdot dim\left(\boldsymbol{\lambda}_{M,P,spec}\right)\log\log T \\ c_{T,M,P,spec} &= dim\left(\boldsymbol{\lambda}_{M,P,spec}\right)\log T, \end{aligned} \tag{3.118}$$

[23]We do not consider parameter-specific values, as all parameters are located in a similar range.

where we use $d = 1$ for the HQC^{MS} criterion, according to Li and Kwok (2021). Hereby, the term $dim\left(\boldsymbol{\lambda}_{M,P,spec}\right)$[24] denotes the number of parameters estimated in the MS-VAR models and is defined conditional on the number of states M, on the number of endogenous variables K, on the number of lags P, associated with the endogenous variables, as well as on the number of exogenous variables K' and the number of lags P', associated with the exogenous variables. In our case, the external variables $\mathbf{x}_i^*$, included in the model with P lags, as well as the macroeconomic variables $\mathbf{e}$, included in the model with P_{exog} lags, form the vector of exogenous variables, therefore, $K' = K^* + K_{exog}$. For simplicity, we only consider the case where the number of lags P of the external variables coincides with the number of lags P_{exog} of the macroeconomic variables, so $P' = P = P_{exog}$. However, an extension for different number of lags is possible.

The overall number of the estimated parameters is hereby dependent on the specification of the MS-VAR model. It is composed by the number of parameters estimated for the transition probability matrix, the intercept vector and the covariance matrix. Further, it relates to the parameters associated with the exogenous as well as to the endogenous variables. While the number of parameters for the transition probability matrix equals $M\left(M-1\right)$ for all specifications of the MS-VAR models, the number of parameters of the MS-VAR model depends on whether the intercept, the autoregressive parameters as well as the parameters associated with the exogenous variables and the covariance matrix are regime-invariant or regime-dependent. In case of regime-dependent parameters, there are M times as many parameters to estimate.

In particular, the number of parameters for the intercept vector equals either K in the regime-invariant case, or MK, if the intercept switches between the regimes. Further, either K^2P or MK^2P parameters associated with the endogenous variables and $KK'P'$ or $MKK'P'$ parameters associated with the exogenous variables have to be estimated for the regime-invariant or regime-dependent case, respectively. In addition, the number of parameters estimated for the covariance matrix is either $\frac{1}{2}K\left(K+1\right)$ in the regime-invariant case or $\frac{1}{2}MK\left(K+1\right)$ in the regime-dependent case. In the following, we provide the resulting number of parameters estimated for the different specifications of the MS-VAR models:

- MSI-VAR model:
  - Dimension in case of regime-dependent parameters of exogenous variables:
$$dim\left(\boldsymbol{\lambda}_{M,P,spec}\right) = M\left(M-1\right) + MK + MKK'P' + K^2P + \frac{1}{2}K\left(K+1\right) \tag{3.119}$$
  - Dimension in case of regime-invariant parameters of exogenous variables:
$$dim\left(\boldsymbol{\lambda}_{M,P,spec}\right) = M\left(M-1\right) + MK + KK'P' + K^2P + \frac{1}{2}K\left(K+1\right) \tag{3.120}$$
- MSIH-VAR model:
  - Dimension in case of regime-dependent parameters of exogenous variables:
$$dim\left(\boldsymbol{\lambda}_{M,P,spec}\right) = M\left(M-1\right) + MK + MKK'P' + K^2P + \frac{1}{2}MK\left(K+1\right) \tag{3.121}$$
  - Dimension in case of regime-invariant parameters of exogenous variables:
$$dim\left(\boldsymbol{\lambda}_{M,P,spec}\right) = M\left(M-1\right) + MK + KK'P' + K^2P + \frac{1}{2}MK\left(K+1\right) \tag{3.122}$$

[24]The term $dim\left(\boldsymbol{\lambda}_{M,P,spec}\right)$ represents the dimension of estimated parameters in the MS-VAR model, and therefore, it is not interpreted as a matrix function. In our case, the dimension of estimated parameters in the MS-VAR model equals for all commodity-specific models which is why we can neglect the commodity-specific index.

- MSIA-VAR model:
  - Dimension in case of regime-dependent parameters of exogenous variables:

$$dim\left(\boldsymbol{\lambda}_{M,P,spec}\right) = M\left(M-1\right) + MK + MKK'P' + MK^2P + \frac{1}{2}K\left(K+1\right) \quad (3.123)$$

  - Dimension in case of regime-invariant parameters of exogenous variables:

$$dim\left(\boldsymbol{\lambda}_{M,P,spec}\right) = M\left(M-1\right) + MK + KK'P' + MK^2P + \frac{1}{2}K\left(K+1\right) \quad (3.124)$$

- MSIAH-VAR model:
  - Dimension in case of regime-dependent parameters of exogenous variables:

$$dim\left(\boldsymbol{\lambda}_{M,P,spec}\right) = M\left(M-1\right) + MK + MKK'P' + MK^2P + \frac{1}{2}MK\left(K+1\right) \quad (3.125)$$

  - Dimension in case of regime-invariant parameters of exogenous variables:

$$dim\left(\boldsymbol{\lambda}_{M,P,spec}\right) = M\left(M-1\right) + MK + KK'P' + MK^2P + \frac{1}{2}MK\left(K+1\right) \quad (3.126)$$

- MSH-VAR model:
  - Dimension in case of regime-dependent parameters of exogenous variables:

$$dim\left(\boldsymbol{\lambda}_{M,P,spec}\right) = M\left(M-1\right) + K + MKK'P' + K^2P + \frac{1}{2}MK\left(K+1\right) \quad (3.127)$$

  - Dimension in case of regime-invariant parameters of exogenous variables:

$$dim\left(\boldsymbol{\lambda}_{M,P,spec}\right) = M\left(M-1\right) + K + KK'P' + K^2P + \frac{1}{2}MK\left(K+1\right) \quad (3.128)$$

- MSA-VAR model:
  - Dimension in case of regime-dependent parameters of exogenous variables:

$$dim\left(\boldsymbol{\lambda}_{M,P,spec}\right) = M\left(M-1\right) + K + MKK'P' + MK^2P + \frac{1}{2}K\left(K+1\right) \quad (3.129)$$

  - Dimension in case of regime-invariant parameters of exogenous variables:

$$dim\left(\boldsymbol{\lambda}_{M,P,spec}\right) = M\left(M-1\right) + K + KK'P' + MK^2P + \frac{1}{2}K\left(K+1\right) \quad (3.130)$$

- MSAH-VAR model:
  - Dimension in case of regime-dependent parameters of exogenous variables:

$$dim\left(\boldsymbol{\lambda}_{M,P,spec}\right) = M\left(M-1\right) + K + MKK'P' + MK^2P + \frac{1}{2}MK\left(K+1\right) \quad (3.131)$$

  - Dimension in case of regime-invariant parameters of exogenous variables:

$$dim\left(\boldsymbol{\lambda}_{M,P,spec}\right) = M\left(M-1\right) + K + KK'P' + MK^2P + \frac{1}{2}MK\left(K+1\right) \quad (3.132)$$

3.2.2 Solution of the MS-GVAR Model

To derive the final MS-GVAR model, allowing for interdependencies between the commodities, we link the commodity-specific MS-VAR models, described above, in a similar manner to the individual VAR models in Section 3.1.[25] Therefore, we define the $(K_i + K_i^*) \times 1$ vector $\mathbf{z}_{i,t} = \left(\mathbf{x}'_{i,t}, \mathbf{x}^{*\prime}_{i,t}\right)'$ and rewrite Equation 3.22 for $i = 1, 2, \ldots, N$:

$$\mathbf{A}_{i,0,s_{i,t}}\mathbf{z}_{i,t} = \mathbf{a}_{i,0,s_{i,t}} + \sum_{p=1}^{P} \mathbf{A}_{i,p,s_{i,t}}\mathbf{z}_{i,t-p} + \sum_{p=0}^{P_{exog}} \mathbf{\Psi}_{i,p,s_{i,t}}\mathbf{e}_{t-p} + \boldsymbol{\varepsilon}_{i,t}, \tag{3.133}$$

where

$$\begin{aligned} \mathbf{A}_{i,0,s_{i,t}} &= \left(\mathbf{I}_{K_i}, -\mathbf{\Lambda}_{i,0,s_{i,t}}\right) \\ \mathbf{A}_{i,p,s_{i,t}} &= \left(\mathbf{\Phi}_{i,p,s_{i,t}}, \mathbf{\Lambda}_{i,p,s_{i,t}}\right), \text{ for } p = 1, 2, \ldots, P \end{aligned} \tag{3.134}$$

are $K_i \times (K_i + K_i^*)$ dimensional matrices, with $\mathbf{I}_{K_i}$ denoting the $K_i \times K_i$ dimensional unit matrix. Moreover, we require $\mathbf{A}_{i,0,s_{i,t}}$ to have full row rank for $i = 1, 2, \ldots, N$.

We denote by $\mathbf{x}_t = \left(\mathbf{x}'_{1,t}, \mathbf{x}'_{2,t}, \ldots, \mathbf{x}'_{N,t}\right)'$ the $K \times 1$ global vector of all commodity-specific variables, where $K = \sum_{i=1}^{N} K_i$. With the link matrices $\mathbf{Z}_i$ of fixed constants defined in terms of the commodity-specific weights $w_{i,\tilde{\imath}}$, we can write $\mathbf{z}_{i,t} = \mathbf{Z}_i\mathbf{x}_t$. Using this in Equation 3.133, we obtain:

$$\mathbf{A}_{i,0,s_{i,t}}\mathbf{Z}_i\mathbf{x}_t = \mathbf{a}_{i,0,s_{i,t}} + \sum_{p=1}^{P} \mathbf{A}_{i,p,s_{i,t}}\mathbf{Z}_i\mathbf{x}_{t-p} + \sum_{p=0}^{P_{exog}} \mathbf{\Psi}_{i,p,s_{i,t}}\mathbf{e}_{t-p} + \boldsymbol{\varepsilon}_{i,t}. \tag{3.135}$$

Stacking Equation 3.135 together for $i = 1, 2, \ldots, N$, we get:

$$\mathbf{G}_{0,S_t}\mathbf{x}_t = \mathbf{a}_{0,S_t} + \sum_{p=1}^{P} \mathbf{G}_{p,S_t}\mathbf{x}_{t-p} + \sum_{p=0}^{P_{exog}} \mathbf{\Psi}_{p,S_t}\mathbf{e}_{t-p} + \boldsymbol{\varepsilon}_t, \tag{3.136}$$

where $S_t = \{s_{1,t}, s_{2,t}, \ldots, s_{N,t}\}$ denotes the regime-constellation across all commodities that is assumed while forming the matrices $\mathbf{G}_{0,S_t}$, and with the $K \times 1$ intercept vector $\mathbf{a}_{0,S_t} = \left(\mathbf{a}'_{1,0,s_{1,t}}, \mathbf{a}'_{2,0,s_{2,t}}, \ldots, \mathbf{a}'_{N,0,s_{N,t}}\right)'$, the $K \times K$ dimensional matrices $\mathbf{G}_{p,S_t} = \left(\left(\mathbf{A}_{1,p,s_{1,t}}\mathbf{Z}_1\right)', \left(\mathbf{A}_{2,p,s_{2,t}}\mathbf{Z}_2\right)', \ldots, \left(\mathbf{A}_{N,p,s_{N,t}}\mathbf{Z}_N\right)'\right)'$, for $p = 0, 1, \ldots, P$, the $K \times K_{exog}$ dimensional matrices $\mathbf{\Psi}_{p,S_t} = \left(\mathbf{\Psi}'_{1,p,s_{1,t}}, \mathbf{\Psi}'_{2,p,s_{2,t}}, \ldots, \mathbf{\Psi}'_{N,p,s_{N,t}}\right)'$, for $p = 0, 1, \ldots, P$, and the $K \times 1$ vector $\boldsymbol{\varepsilon}_t = \left(\boldsymbol{\varepsilon}'_{1,t}, \boldsymbol{\varepsilon}'_{2,t}, \ldots, \boldsymbol{\varepsilon}'_{N,t}\right)'$.

In case of a non-singular matrix $\mathbf{G}_{0,S_t}$ for a given regime-constellation S_t, we get the reduced form:

$$\begin{aligned} \mathbf{x}_t &= \mathbf{G}_{0,S_t}^{-1}\mathbf{a}_{0,S_t} + \sum_{p=1}^{P} \mathbf{G}_{0,S_t}^{-1}\mathbf{G}_{p,S_t}\mathbf{x}_{t-p} + \sum_{p=0}^{P_{exog}} \mathbf{G}_{0,S_t}^{-1}\mathbf{\Psi}_{p,S_t}\mathbf{e}_{t-p} + \mathbf{G}_{0,S_t}^{-1}\boldsymbol{\varepsilon}_t \\ &= \mathbf{b}_{S_t} + \sum_{p=1}^{P} \mathbf{H}_{p,S_t}\mathbf{x}_{t-p} + \sum_{p=0}^{P_{exog}} \mathbf{\Upsilon}_{p,S_t}\mathbf{e}_{t-p} + \boldsymbol{\upsilon}_{t,S_t}, \end{aligned} \tag{3.137}$$

[25] Please note, the only difference between the aggregation of the individual VAR models to the GVAR model and the aggregation of the individual MS-VAR models to the MS-GVAR model lies in the regime-dependence of the parameters.

with $\mathbf{b}_{S_t} = \mathbf{G}_{0,S_t}^{-1}\mathbf{a}_{0,S_t}$, $\mathbf{H}_{p,S_t} = \mathbf{G}_{0,S_t}^{-1}\mathbf{G}_{p,S_t}$, for $p = 1, 2, \ldots, P$, $\boldsymbol{\Upsilon}_{p,S_t} = \mathbf{G}_{0,S_t}^{-1}\boldsymbol{\Psi}_{p,S_t}$, for $p = 0, 1, \ldots, P$ and $\boldsymbol{\upsilon}_{t,S_t} = \mathbf{G}_{0,S_t}^{-1}\boldsymbol{\varepsilon}_t$. This enables us to model markets of all commodities simultaneously, while accounting for the (time-varying) dependencies in the cross-commodity dimension.

3.2.2.1 Regime-constellation-dependent Solution

The global model can only be solved by assuming a regime-constellation S, as inferred throughout the sample period. We define a $M \times N$ matrix $\tilde{\boldsymbol{\Xi}}$ indicating the desired regimes for the N commodities. There are two possible ways for setting the desired regimes to solve the global model:

1. Choose an arbitrary regime-constellation, such that each commodity i is set to the desired regime m_i with weight 1.
2. Set the regimes according to the estimated constellation at a selected point in time τ. Then, $\tilde{\boldsymbol{\Xi}}_i = \hat{\boldsymbol{\xi}}_{i,\tau|T}$. Hereby, the columns, denoted by $\tilde{\boldsymbol{\Xi}}_i$, each sum to one.

With the definition of $\tilde{\boldsymbol{\Xi}}$, the weighted average of the local models' parameter space can be calculated for $p = 0, 1, \ldots, P$:

$$\tilde{\mathbf{A}}_{i,p} = \sum_{m=1}^{M} \mathbf{A}_{i,p,s_i=m} \cdot \tilde{\boldsymbol{\Xi}}_{m,i}, \tag{3.138}$$

where $\tilde{\boldsymbol{\Xi}}_{m,i}$ denotes the m-th row of $\tilde{\boldsymbol{\Xi}}_i$. This weighting is also used to get the intercept:

$$\tilde{\mathbf{a}}_{i,0} = \sum_{m=1}^{M} \mathbf{a}_{i,0,s_i=m} \cdot \tilde{\boldsymbol{\Xi}}_{m,i}, \tag{3.139}$$

as well as the parameter matrices for the exogenous variables, for $p = 0, 1, \ldots, P_{exog}$:

$$\tilde{\boldsymbol{\Psi}}_{i,p} = \sum_{m=1}^{M} \boldsymbol{\Psi}_{i,p,s_i=m} \cdot \tilde{\boldsymbol{\Xi}}_{m,i}. \tag{3.140}$$

Subsequently, the solution of the global model is calculated according to Equation 3.136, using the parameters $\tilde{\mathbf{A}}_{i,p}, \tilde{\mathbf{a}}_{i,0}, \tilde{\boldsymbol{\Psi}}_{i,p}$, corresponding to the predefined regime-constellation S.

3.2.3 Prediction of the MS-GVAR Model

According to Krolzig (1997), an advantage of the MS-VAR models is the simplicity in forecasting if the optimal predictor is given by the conditional mean for a given information set Ω_T, including the observations of commodity markets over the period $t = 1, 2, \ldots, T$, as well as the exogenous variables $\mathbf{e}_t$ for $t = T+1, T+2, \ldots, T+N_{pred}$:

$$\hat{\mathbf{x}}_{T+N_{pred}|T} = \mathbb{E}\left[\mathbf{x}_{T+N_{pred}|T}|\Omega_T\right]. \tag{3.141}$$

In case of regime-dependent autoregressive parameters, the linearity property of the multi-step predictor does not hold. Therefore, Krolzig (1997) proposes to approximate the N_{pred}-step ahead predictor by iterative pseudo one-step ahead predictions, which is why we focus on them in the following. Given the regime probabilities $\hat{\boldsymbol{\xi}}_{T|T}$ at time T, the predicted probabilities for $T+1$ can be derived by Equation 3.60. Subsequently, using the inferred predicted probabilities for the regime-constellation S_{T+1} and the MS-GVAR model in its reduced form, presented in Equation 3.137, we get the one-step ahead predictor $\hat{\mathbf{x}}_{T+1|T}$:

$$\hat{\mathbf{x}}_{T+1|T} = \mathbf{b}_{S_{T+1}} + \sum_{p=1}^{P} \mathbf{H}_{p,S_{T+1}} \mathbf{x}_{T-p} + \sum_{p=0}^{P_{exog}} \mathbf{\Upsilon}_{p,S_{T+1}} \mathbf{e}_{T-p}. \tag{3.142}$$

3.2.3.1 Evaluation of the Predictability

To underline the importance of a time-varying analysis, we compare the predictability of the MS-GVAR model with the time-invariant GVAR model. Therefore, we simultaneously forecast all commodity-specific variables included in the global commodity vector $\mathbf{x}_t$ one-step ahead, using a rolling window approach, and assess the goodness of fit of the predictions via the test of Clark and West (2007). Hereby we base our evaluation only on out-of-sample data, since in-sample predictability may occur spuriously.

Consequently, we split the data set with observations $t = 1, 2, \ldots, T$ in an in-sample set with observations $t = 1, 2, \ldots, I$ and an out-of-sample set with observations $t = I+1, I+2, \ldots, I+J = I+1, I+2, \ldots, T$. Subsequently, we forecast the variables one-step ahead, by a rolling window procedure. In particular, for each time in the out-of-sample set $t_{pred} = 1, 2, \ldots, J$, we estimate the (MS-)GVAR model based on observations in the period $\{t_{pred}, 1 + t_{pred}, \ldots, I + t_{pred} - 1\}$ and predict the commodity markets one-step ahead, $\hat{\mathbf{x}}_{I+t_{pred}}$, using Equation 3.9 or Equation 3.142, respectively.

Subsequently, we examine whether the time-varying MS-GVAR model outperforms the time-invariant GVAR model in terms of out-of-sample predictability. Therefore, we compare the performance of the models via the test of Clark and West (2007) for nested models, as the GVAR model equals the MS-GVAR model with one state. The null hypothesis of Clark and West (2007) assumes the unrestricted model, in our case the MS-GVAR model, includes excess parameters, whereas in the alternative hypothesis the restricted model, the GVAR model, underperforms in terms of the mean squared prediction error (MSPE). In general, the mean squared prediction error measures the average squared deviation of the predicted ($\hat{\mathbf{x}}_{k,t_{pred}}$) to the true ($\mathbf{x}_{k,t_{pred}}$) value of variable $\mathbf{x}_{k,t_{pred}}$, for $k = 1, 2, \ldots, K$:

$$MSPE_k = \frac{1}{J} \sum_{t_{pred}=1}^{J} \left(\mathbf{x}_{k,t_{pred}} - \hat{\mathbf{x}}_{k,t_{pred}} \right)^2. \tag{3.143}$$

The test of Clark and West (2007) corrects for the excess noise in the mean squared prediction error of the unrestricted model, caused by the additional parameters, whose population values are zero, under the null hypothesis, which is why the MSPE of the restricted model is expected to be smaller. Therefore, the MSPE of the restricted GVAR model ($MSPE_k^{\text{GVAR}}$) is compared for each variable k to the adjusted MSPE value for the unrestricted MS-GVAR model ($MSPE_{k,adj}^{\text{MS-GVAR}}$):

$$\begin{aligned} MSPE_{k,adj}^{\text{MS-GVAR}} &= MSPE_k^{\text{MS-GVAR}} - adj_k^{\text{MS-GVAR}} \\ &= \frac{1}{J} \sum_{t_{pred}=1}^{J} \left(\mathbf{x}_{k,t_{pred}} - \hat{\mathbf{x}}_{k,t_{pred}}^{\text{MS-GVAR}} \right)^2 \\ &- \frac{1}{J} \sum_{t_{pred}=1}^{J} \left(\hat{\mathbf{x}}_{k,t_{pred}}^{\text{GVAR}} - \hat{\mathbf{x}}_{k,t_{pred}}^{\text{MS-GVAR}} \right)^2, \end{aligned} \tag{3.144}$$

where $adj_k^{\text{MS-GVAR}}$ denotes the adjustment term, and $\hat{\mathbf{x}}_{k,t_{pred}}^{\text{GVAR}}$ ($\hat{\mathbf{x}}_{k,t_{pred}}^{\text{MS-GVAR}}$) denotes the predicted value of the GVAR (MS-GVAR) model. Hereby, Clark and West (2007) state the difference between the MSPE values of the restricted and unrestricted model, whereby the adjusted version is used in the latter case, $MSPE_k^{\text{GVAR}} - MSPE_{k,adj}^{\text{MS-GVAR}}$, equals zero under the null hypothesis. In particular, Clark and West (2007) propose to test the null with a t-test using 1.282 as critical value for the 10% significance level, since they show standard normal critical values are close to the actual values.

3.2.4 Analysis of the MS-GVAR Model via Generalized Impulse Response Functions

Similar to the GVAR framework, we analyze the dynamic properties of the MS-GVAR model by generalized impulse response functions (GIRFs). However, in case of the MS-GVAR model, we require an estimate for the global covariance matrix to calculate appropriate GIRFs, which we describe first, before the calculation of GIRFs is presented.

3.2.4.1 Estimation of the Global Covariance Matrix

To estimate the global covariance matrix $\mathbf{\Sigma}_S$ for a given regime-constellation $S = \{s_1, s_2, \ldots, s_N\}$ with corresponding matrix $\tilde{\mathbf{\Xi}}$, required in the GIRF analysis, we partition the matrix into blocks, according to Binder and Gross (2013) and Andersson (2014):

$$\begin{aligned}\mathbf{\Sigma}_S &= \begin{bmatrix} \mathbf{\Sigma}_{11,s_1,s_1} & \mathbf{\Sigma}_{12,s_1,s_2} & \cdots & \mathbf{\Sigma}_{1N,s_1,s_N} \\ \mathbf{\Sigma}_{21,s_2,s_1} & \mathbf{\Sigma}_{22,s_2,s_2} & \cdots & \mathbf{\Sigma}_{2N,s_2,s_N} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{\Sigma}_{N1,s_N,s_1} & \mathbf{\Sigma}_{N2,s_N,s_2} & \cdots & \mathbf{\Sigma}_{NN,s_N,s_N} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{\Sigma}_{11,\tilde{\mathbf{\Xi}}_1\tilde{\mathbf{\Xi}}_1} & \mathbf{\Sigma}_{12,\tilde{\mathbf{\Xi}}_1,\tilde{\mathbf{\Xi}}_2} & \cdots & \mathbf{\Sigma}_{1N,\tilde{\mathbf{\Xi}}_1,\tilde{\mathbf{\Xi}}_N} \\ \mathbf{\Sigma}_{21,\tilde{\mathbf{\Xi}}_2,\tilde{\mathbf{\Xi}}_1} & \mathbf{\Sigma}_{22,\tilde{\mathbf{\Xi}}_2\tilde{\mathbf{\Xi}}_2} & \cdots & \mathbf{\Sigma}_{2N,\tilde{\mathbf{\Xi}}_2,\tilde{\mathbf{\Xi}}_N} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{\Sigma}_{N1,\tilde{\mathbf{\Xi}}_N,\tilde{\mathbf{\Xi}}_1} & \mathbf{\Sigma}_{N2,\tilde{\mathbf{\Xi}}_N,\tilde{\mathbf{\Xi}}_2} & \cdots & \mathbf{\Sigma}_{NN,\tilde{\mathbf{\Xi}}_N\tilde{\mathbf{\Xi}}_N} \end{bmatrix}.\end{aligned} \tag{3.145}$$

Each diagonal block is calculated as:

$$\hat{\mathbf{\Sigma}}_{ii,\tilde{\mathbf{\Xi}}_i} = \frac{\sum_{t=1}^{T} \boldsymbol{\varepsilon}_{it,\tilde{\mathbf{\Xi}}_i} \boldsymbol{\varepsilon}'_{it,\tilde{\mathbf{\Xi}}_i} \left(\hat{\boldsymbol{\xi}}'_{i,t|T}\tilde{\mathbf{\Xi}}_i\right)}{\sum_{t=1}^{T} \hat{\boldsymbol{\xi}}'_{i,t|T}\tilde{\mathbf{\Xi}}_i}. \tag{3.146}$$

The off-diagonal blocks $\mathbf{\Sigma}_{i\tilde{\imath}}$ of the global matrix $\mathbf{\Sigma}$, denoting the covariances between the commodities i and $\tilde{\imath}$ can be estimated analogously:

$$\hat{\mathbf{\Sigma}}_{i\tilde{\imath},\tilde{\mathbf{\Xi}}_i\tilde{\mathbf{\Xi}}_{\tilde{\imath}}} = \frac{\sum_{t=1}^{T} \boldsymbol{\varepsilon}_{it,\tilde{\mathbf{\Xi}}_i} \boldsymbol{\varepsilon}'_{\tilde{\imath}t,\tilde{\mathbf{\Xi}}_{\tilde{\imath}}} \sqrt{\left(\hat{\boldsymbol{\xi}}'_{i,t|T}\tilde{\mathbf{\Xi}}_i\right)\left(\hat{\boldsymbol{\xi}}'_{\tilde{\imath},t|T}\tilde{\mathbf{\Xi}}_{\tilde{\imath}}\right)}}{\sum_{t=1}^{T} \sqrt{\left(\hat{\boldsymbol{\xi}}'_{i,t|T}\tilde{\mathbf{\Xi}}_i\right)\left(\hat{\boldsymbol{\xi}}'_{\tilde{\imath},t|T}\tilde{\mathbf{\Xi}}_{\tilde{\imath}}\right)}}. \tag{3.147}$$

The global matrix $\mathbf{\Sigma}_S$ is symmetric by construction, with non-zero variances on the diagonal. However, we can not ensure the positive semi-definiteness. Therefore, we follow Binder and Gross (2013) and implement the iterative algorithm proposed by Higham (2002), such that the global matrix will be positive semi-definite.

3.2.4.2 Generalized Impulse Response Functions

In line with the GVAR model, we analyze the dynamic properties of the MS-GVAR model by generalized impulse response functions (GIRFs), as the GIRF analysis needs no ordering of the commodities, see Dées, di Mauro, Pesaran, and Smith (2007) and Section 3.1.2.1. However, due to the time-varying properties of the MS-GVAR model, we calculate the regime-dependent GIRFs of Markov-switching models proposed by Ehrmann et al. (2001) and Ehrmann et al. (2003). Hereby, we first focus on shocks in the commodity markets, subsequently, we investigate how shocks in the global economy transmit to the commodity markets.

3.2.4.2.1 Impulse Response Analysis of Shocks to endogenous, commodity-specific Variables For a given regime-constellation S_t, the generalized impulse response function, reflecting the impact of a shock to the endogenous variables, for $n = 0, 1, \ldots, N_{IRF}$ periods ahead, is defined as:

$$\mathbf{GI}\left(n, \boldsymbol{\delta}, \Omega_{t-1}\right) = \mathbb{E}\left[\mathbf{x}_{t+n} | \boldsymbol{\varepsilon}_{t,S_t} = \boldsymbol{\delta}, \Omega_{t-1}\right] - \mathbb{E}\left[\mathbf{x}_{t+n} | \Omega_{t-1}\right], \tag{3.148}$$

where $\boldsymbol{\delta}$ denotes the shock and Ω_{t-1} is the history, consisting of all known information until time $t-1$, including all necessary information about the exogenous variables. As we consider regime-dependent GIRFs, the predictions $\mathbf{x}_{t+n}$ are calculated under the assumption the system is in the predefined state S_t. In our study, we consider the GIRFs of the effect of a shock of the k-th variable of $\mathbf{x}_t$, corresponding to the k_i-th variable in the i-th commodity, by one standard deviation, $\sqrt{\sigma_{kk,S_t}}$, therefore, $\boldsymbol{\delta} = \left(0, \ldots, 0, \sqrt{\sigma_{kk,S_t}}, 0, \ldots, 0\right)'$, whereby σ_{kk,S_t} represents the kk-th element of $\boldsymbol{\Sigma}_{S_t}$.

In general, we do not assume the innovations of the MS-GVAR model are multivariate normal distributed for each considered regime-constellation. Therefore, we calculate the GIRFs via Monte Carlo integration, according to the algorithms presented in Gonçalves et al. (2021), Karamé (2012), Koop et al. (1996) as well as Weise (1999). In the following, we describe the algorithm used to compute the GIRFs, hereby ignoring sampling variability.

1. Draw with replacement a block of P consecutive observations from the observed data to get N_{hist} randomly drawn histories $\omega_{t-1}^{n_{hist}}$, $n_{hist} = 1, 2, \ldots, N_{hist}$.

2. Randomly sample with replacement $(N_{IRF} + 1) \times N_{shock}$ values of the K-dimensional estimated residuals of the model to get a sequence $\{\boldsymbol{\varepsilon}_{t+n}^{n_{shock}}\}_{n=0}^{N_{IRF}}$ of K-dimensional shocks $\boldsymbol{\varepsilon}_{t+n}^{n_{shock}}, n = 0, 1, \ldots, N_{IRF}$, $n_{shock} = 1, 2, \ldots, N_{shock}$. Under the assumption of jointly distributed shocks, if date t' s shock is drawn, all K residuals for date t are collected.

3. For a specific n_{shock} as well as n_{hist}, use the $N_{IRF} + 1$ random shocks $\{\boldsymbol{\varepsilon}_{t+n}^{n_{shock}}\}$ to compute the realization $\mathbf{x}_{t+n}^{n_{hist}, n_{shock}}\left(\boldsymbol{\varepsilon}_{t+n}^{n_{shock}}, \omega_{t-1}^{n_{hist}}\right)$ for $n = 0, 1, \ldots, N_{IRF}$, using Equation 3.142 and iterating on the estimated, nonlinear time series model under consideration from the given initial conditions $\boldsymbol{\varepsilon}_{t+n}^{n_{shock}}, \omega_{t-1}^{n_{hist}}$.

4. Use the same draw of $N_{IRF} + 1$ random shocks $\{\boldsymbol{\varepsilon}_{t+n}^{n_{shock}}\}$, but replace the first shock $\boldsymbol{\varepsilon}_{t+0}^{n_{shock}}$ by $\boldsymbol{\varepsilon}_{t+0}^{n_{shock},\boldsymbol{\delta}} = \boldsymbol{\varepsilon}_{t+0}^{n_{shock}} + \boldsymbol{\delta}$ to produce a realization, $\mathbf{x}_{t+n}^{n_{hist}, n_{shock}, \boldsymbol{\delta}}\left(\boldsymbol{\varepsilon}_{t+n}^{n_{shock},\boldsymbol{\delta}}, \omega_{t-1}^{n_{hist}}\right)$, of the time series for $n = 0, 1, \ldots, N_{IRF}$, based on the initial conditions $\boldsymbol{\varepsilon}_{t+n}^{n_{shock},\boldsymbol{\delta}}, \omega_{t-1}^{n_{hist}}$.

5. Repeat steps 3 and 4 N_{shock} times and form the averages for each individual component:

$$\begin{aligned} \tilde{\mathbf{x}}_{t+n}^{n_{hist},\boldsymbol{\delta}}\left(\boldsymbol{\varepsilon}_{t+n}^{\boldsymbol{\delta}}, \omega_{t-1}^{n_{hist}}\right) &= \frac{1}{N_{shock}} \sum_{n_{shock}=1}^{N_{shock}} \mathbf{x}_{t+n}^{n_{hist}, n_{shock}, \boldsymbol{\delta}}\left(\boldsymbol{\varepsilon}_{t+n}^{n_{shock},\boldsymbol{\delta}}, \omega_{t-1}^{n_{hist}}\right), \\ \tilde{\mathbf{x}}_{t+n}^{n_{hist}}\left(\boldsymbol{\varepsilon}_{t+n}, \omega_{t-1}^{n_{hist}}\right) &= \frac{1}{N_{shock}} \sum_{n_{shock}=1}^{N_{shock}} \mathbf{x}_{t+n}^{n_{hist}, n_{shock}}\left(\boldsymbol{\varepsilon}_{t+n}^{n_{shock}}, \omega_{t-1}^{n_{hist}}\right). \end{aligned} \tag{3.149}$$

According to Koop et al. (1996), these averages will converge by the law of large numbers to the conditional expectations $\mathbb{E}\left[\mathbf{x}_{t+n} | \boldsymbol{\varepsilon}_{t,S_t} = \boldsymbol{\delta}, \omega_{t-1}^{n_{hist}}\right]$ and $\mathbb{E}\left[\mathbf{x}_{t+n} | \omega_{t-1}^{n_{hist}}\right]$.

6. The Monte Carlo estimate of the history dependent GIRF is calculated by taking the difference:

$$\mathbf{GI}_{\mathbf{x}:\mathbf{x}_k}\left(\boldsymbol{\varepsilon}_{t+n}, \omega_{t-1}^{n_{hist}}\right) = \tilde{\mathbf{x}}_{t+n}^{n_{hist},\boldsymbol{\delta}}\left(\boldsymbol{\varepsilon}_{t+n}^{\boldsymbol{\delta}}, \omega_{t-1}^{n_{hist}}\right) - \tilde{\mathbf{x}}_{t+n}^{n_{hist}}\left(\boldsymbol{\varepsilon}_{t+n}, \omega_{t-1}^{n_{hist}}\right). \tag{3.150}$$

7. Repeat steps 2 to 6 N_{hist} times and take the average over $\mathbf{GI}_{\mathbf{x}:\mathbf{x}_k}\left(\boldsymbol{\varepsilon}_{t+n}, \omega_{t-1}^{n_{hist}}\right)$ to get the history independent estimate of the GIRF $\mathbf{GI}_{\mathbf{x}:\mathbf{x}_k}\left(\boldsymbol{\varepsilon}_{t+n}, \omega_{t-1}\right)$. With an increasing number of repetitions the pointwise convergence will be guaranteed by the law of large numbers, according to Koop et al. (1996).

3.2.4.2.2 Significance of the Spillover Effects via Bootstrapping In order to analyze the significance of the GIRFs, we employ an adjusted version of the bootstrap techniques proposed in Ehrmann et al. (2001). In general, the bootstrapping, using random sampling with replacement, has the advantage to assign measures of accuracy, such as confidence intervals, without knowing the underlying distribution. However, two major problems arise within Markov-switching models: Due to the presence of the hidden Markov-chain, there are two processes underlying a Markov-switching model, one for the regimes and one for the endogenous variables, which is why standard bootstrapping techniques, as described in Section 3.1.2.1 for the GVAR model, can not be applied straightforward.

In our case, we create a history for the regimes as well as for the endogenous variables within the bootstrapping. As we apply the bootstrapping to generate confidence bounds for our GIRF analysis, the bootstrapping is also regime-dependent, just like the GIRF, which is why the bootstrapping is calculated for one predetermined regime-constellation S.

In the following, we describe the algorithm for the bootstrapping procedure:

1. Create a history for the regimes $S_t, t = 1, 2, \ldots, T$. As the regimes are not observable, but the smoothed probabilities represent their best estimate, we assume the history of regimes to correspond to the estimated smoothed probabilities $\hat{\boldsymbol{\xi}}_{i,t|T}$.

2. Calculate the residuals $\hat{\boldsymbol{\varepsilon}}_{t,S_t}, t = 1, 2, \ldots, T$ of the fitted MS-GVAR model in Equation 3.136, with the estimated parameters for the prevailing regime-constellations.

3. Draw randomly with replacement $T_{boot} \leq T$ residuals to get N_{boot} sets of residuals $\boldsymbol{\varepsilon}^{n_{boot}} = \left(\boldsymbol{\varepsilon}_{T-T_{boot}}^{n_{boot}}, \boldsymbol{\varepsilon}_{T-T_{boot}+1}^{n_{boot}}, \ldots, \boldsymbol{\varepsilon}_{T}^{n_{boot}}\right)$, $n_{boot} = 1, 2, \ldots, N_{boot}$.

4. Generate N_{boot} bootstrap samples $\mathbf{x}^{n_{boot}} = \left(\mathbf{x}_{T-T_{boot}}^{n_{boot}}, \mathbf{x}_{T-T_{boot}+1}^{n_{boot}}, \ldots, \mathbf{x}_{T}^{n_{boot}}\right)$, according to Equation 3.137, using the resampled, recentered[26] residuals $\boldsymbol{\varepsilon}^{n_{boot}}$ as well as with the estimated parameters of the fitted MS-GVAR model for the prevailing regime-constellations and calculate the corresponding external variables $\mathbf{x}_i^{*,n_{boot}}$, according to Equation 3.23, using the weight matrix $\left(w_{i,\tilde{\iota}}\right)_{i,\tilde{\iota}=1,2,\ldots,N}$, for $i = 1, 2, \ldots, N$.

5. Estimate the MS-GVAR model for a specific bootstrap sample, $\mathbf{x}^{n_{boot}}$, with corresponding external variables $\mathbf{x}_i^{*,n_{boot}}$, for $i = 1, 2, \ldots, N$, by estimating the commodity-specific MS-VAR models in Equation 3.22 via the EM algorithm, described in Section 3.2.1.5, and aggregating them to the global model, according to Equation 3.137, using $\mathbf{x}^{n_{boot}}$ as endogenous variables.[27]

[26] In line with the bootstrapping procedure for the GVAR model, described in Section 3.1.2.1, we follow Dées, di Mauro, Pesaran, and Smith (2007) and recenter the residuals to ensure the bootstrap population mean is zero.

[27] In line with the bootstrapping procedure for the GVAR model, described in Section 3.1.2.1, we exclude all bootstrap samples, where the corresponding MS-GVAR model does not exhibit the stability property, to guarantee representative bootstrap models.

6. Calculate the GIRFs for the predefined regime-constellation S for the specific bootstrap sample, $\mathbf{x}^{n_{boot}}$, based on the new estimated parameters corresponding to the bootstrap sample.

7. Repeat steps 4 to 6 N_{boot} times.

8. Sort the GIRFs into an ascending order for all time periods $n = 0, 1, \ldots, N_{IRF}$, and calculate the 68% confidence interval by using the 0.16 and 0.84 quantiles of the bootstrap distribution of the GIRFs, in line with the GVAR model.

We apply the bootstrap technique for $N_{boot} = 500$ runs, which is sufficiently large to be a good numerical approximation of the distribution of the underlying estimates. Subsequently, the confidence intervals of the GIRF analysis are based on this distribution.

3.2.4.2.3 Magnitude of the Spillover Effects The GIRF analysis and the corresponding confidence bounds, obtained via the bootstrapping procedure in Section 3.2.4.2.2, indicates whether a shock leads to significant changes in the commodity markets. In case of the MS-GVAR model, we investigate the spillover effects under different predetermined regime-constellations and compare the results. In particular, we evaluate the differences in the significance of the spillover effects, indicating whether shocks only affect the commodity markets under a certain regime. Moreover, we also compare the magnitude of the GIRFs between the states. Hereby, we apply the one-sided Wilcoxon signed rank test (Wilcoxon)[28] test on the GIRFs to examine whether the response of a shock to a variable is stronger under one regime-constellation compared to another one.

In addition, we examine the implied risks of the spillover effects under the different regime-constellations. Hereby, we compare the conditional value at risk (CoVaR) of the variables under the regime-constellations. Therefore, we first define the value at risk (VaR), which generally measures the risk of a loss. In particular, the value at risk at the $q\%$ level of a variable $\mathbf{v}$ is defined as the maximum value VaR_q which satisfies:

$$P_{\mathbf{v}} \left( ]-\infty, VaR_q] \right) \leq 1 - q, \tag{3.151}$$

representing the maximum loss, excluding the worst $q\%$ possible losses, whereby $P_{\mathbf{v}}$ denotes the probability distribution of $\mathbf{v}$. The corresponding conditional value at risk is then defined as the expected return in the worst $q\%$ cases and thus corresponds to the average loss in a loss event that was triggered by the VaR being exceeded:

$$CoVaR_q = \mathbb{E} \left[\mathbf{v} | \mathbf{v} > VaR_q \right]. \tag{3.152}$$

Subsequently, a comparison of the CoVaR[29] values under different regimes provide more insights into the spillover risks under the different regimes.

3.2.4.2.4 Impulse Response Analysis of Shocks to exogenous, global Variables Similar to the GVAR model, we also examine how shocks to the global economy, reflected by shocks to the exogenous variables, affect the commodity markets. Therefore, we analyze the effects of a shock to the k_{exog}-th exogenous variable $\mathbf{e}_{k_{exog},t}$ on the commodity markets via GIRFs.

[28] In contrast to a t-test, the Wilcoxon signed rank test test do not require the normal distribution.

[29] In particular, we calculate the CoVaR as the average upper $q\%$ spillover effects, whereby we use the bootstraps derived in Section 3.2.4.2.2.

In line with the time-invariant case, a dynamic process for the exogenous variables has to be specified. Hereby, we also allow for regime-switches in the exogenous variables and model them by a Markov-switching vector autoregression (MS-VAR) model with M_{exog} states and $\tilde{P}_{exog}$ lags:

$$\mathbf{e}_t = \mathbf{a}_{exog,s_{exog,t}} + \sum_{\tilde{p}_{exog}=1}^{\tilde{P}_{exog}} \mathbf{\Phi}_{exog,\tilde{p}_{exog},s_{exog,t}} \mathbf{e}_{t-\tilde{p}_{exog}} + \boldsymbol{\varepsilon}_{exog,t}, \tag{3.153}$$

where $\mathbf{a}_{exog,s_{exog,t}}$ denotes the regime-dependent intercept, $\mathbf{\Phi}_{exog,\tilde{p}_{exog},s_{exog,t}}$ is the regime-dependent $K_{exog} \times K_{exog}$ matrix of coefficients for lag $\tilde{p}_{exog} = 1, 2, \ldots, \tilde{P}_{exog}$, and $\boldsymbol{\varepsilon}_{exog,t}$ is the $K_{exog} \times 1$ vector of shocks. Hereby, we assume $\boldsymbol{\varepsilon}_{exog,t}$ to be serially uncorrelated, independent and identically distributed, with mean zero and covariance matrix $\mathbf{\Sigma}_{exog,s_{exog,t}}$, therefore, $\boldsymbol{\varepsilon}_{exog,t} \sim iid(\mathbf{0}, \mathbf{\Sigma}_{exog,s_{exog,t}})$. Moreover, $s_{exog,t} = 1, 2, \ldots, M_{exog}$ denotes the regime prevailing in the system at time t.

Similar to the GIRFs of a shock to a commodity-specific variable, the generalized impulse response function of the effect of a shock to the k_{exog}-th exogenous variable $\mathbf{e}_{k_{exog},t}$ on the vector of endogenous variables $\mathbf{x}_t$ is defined for $n = 0, 1, \ldots, N_{IRF}$ periods ahead by:

$$\mathbf{GI}_{\mathbf{x}:\mathbf{e}_{k_{exog}}}\left(n, \boldsymbol{\delta}_{exog}, \Omega_{t-1}\right) = \mathbb{E}\left[\mathbf{x}_{t+n} | \boldsymbol{\varepsilon}_{exog,t,s_{exog,t}} = \boldsymbol{\delta}_{exog}, \Omega_{t-1}\right] - \mathbb{E}\left[\mathbf{x}_{t+n} | \Omega_{t-1}\right], \tag{3.154}$$

where $\boldsymbol{\delta}_{exog}$ denotes the shock. In line with the regime-dependent GIRFs for shocks in the endogenous variables, the predictions $\mathbf{x}_{t+n}$ are calculated under the assumption the system of the endogenous variables is in a predefined state S_t and the system of the exogenous variables is in a predefined state $s_{exog,t}$. In this study, we consider the GIRFs of the effect of a shock of the k_{exog}-th exogenous variable by one standard deviation, therefore, $\boldsymbol{\delta}_{exog} = \left(0, \ldots, 0, \sqrt{\sigma_{exog,k_{exog}k_{exog},s_{exog,t}}}, 0, \ldots, 0\right)'$, whereby $\sigma_{exog,k_{exog}k_{exog},s_{exog,t}}$ is the $k_{exog}k_{exog}$-th element of $\mathbf{\Sigma}_{exog,s_{exog,t}}$. Similar to the GIRFs of the GVAR model, we use the MS-GVAR model in its final form in Equation 3.137 and derive:

$$\begin{aligned}\mathbf{GI}_{\mathbf{x}:\mathbf{e}_{k_{exog}}}\left(n, \boldsymbol{\delta}_{exog}, \Omega_{t-1}\right) = &\sum_{p=1}^{P} \mathbf{H}_{p,S_t} \mathbf{GI}_{\mathbf{x}:\mathbf{e}_{k_{exog}}}\left(n-p, \boldsymbol{\delta}_{exog}, \Omega_{t-1}\right) \\ &+ \sum_{p=0}^{P_{exog}} \mathbf{\Upsilon}_{p,S_t} \mathbf{GI}_{\mathbf{e}:\mathbf{e}_{k_{exog}}}\left(n-p, \boldsymbol{\delta}_{exog}, \Omega_{t-1}\right),\end{aligned} \tag{3.155}$$

where $\mathbf{GI}_{\mathbf{e}:\mathbf{e}_{k_{exog}}}\left(n, \boldsymbol{\delta}_{exog}, \Omega_{t-1}\right)$ denotes the impulse response of the exogenous variables to a shock in the k_{exog}-th exogenous variable $\mathbf{e}_{k_{exog}}$:

$$\mathbf{GI}_{\mathbf{e}:\mathbf{e}_{k_{exog}}}\left(n, \boldsymbol{\delta}_{exog}, \Omega_{t-1}\right) = \mathbb{E}\left[\mathbf{e}_{t+n} | \boldsymbol{\varepsilon}_{exog,t,s_{exog,t}} = \boldsymbol{\delta}_{exog}, \Omega_{t-1}\right] - \mathbb{E}\left[\mathbf{e}_{t+n} | \Omega_{t-1}\right]. \tag{3.156}$$

Using the fact that generalized impulse responses vanish for $n < 1$, $\mathbf{GI}_{\mathbf{x}:\mathbf{e}_{k_{exog}}}\left(n, \boldsymbol{\delta}_{exog}, \Omega_{t-1}\right) = \mathbf{GI}_{\mathbf{e}:\mathbf{e}_{k_{exog}}}\left(n, \boldsymbol{\delta}_{exog}, \Omega_{t-1}\right) = 0$, and Equation 3.155, we derive for $n = 0$:

$$\mathbf{GI}_{\mathbf{x}:\mathbf{e}_{k_{exog}}}\left(0, \boldsymbol{\delta}_{exog}, \Omega_{t-1}\right) = \mathbf{\Upsilon}_{0,S_t} \mathbf{GI}_{\mathbf{e}:\mathbf{e}_{k_{exog}}}\left(0, \boldsymbol{\delta}_{exog}, \Omega_{t-1}\right). \tag{3.157}$$

In general, we do not assume neither the innovations of the MS-GVAR model nor the innovations of the MS-VAR model of the exogenous variables are multivariate normal distributed for the considered regime-constellation. Therefore, we also calculate the responses of the exogenous variables to shocks to the exogenous variables, $\mathbf{GI}_{\mathbf{e}:\mathbf{e}_{k_{exog}}}\left(n, \boldsymbol{\delta}_{exog}, \Omega_{t-1}\right)$, via a Monte Carlo integration, similar to the procedure described in Section 3.2.4.2.1. However, we slightly adjust the integration, see Appendix B.1.3. Overall, the GIRFs of the commodity-specific variables to a shock in the exogenous variable $\mathbf{e}_{k_{exog}}$ are then derived recursively, using Equation 3.155, Equation 3.157, and $\mathbf{GI}_{\mathbf{e}:\mathbf{e}_{k_{exog}}}\left(n, \boldsymbol{\delta}_{exog}, \Omega_{t-1}\right)$ obtained from the Monte Carlo integration.

Similar to the GIRFs above, which measure the impact of shocks to endogenous variables, we also analyze the significance of the GIRFs for shocks to the exogenous variables, by an adjusted version of the bootstrap techniques proposed in Ehrmann et al. (2001), where we adjust the procedure in Section 3.2.4.2.2 slightly, see Appendix B.1.4.

3.3 Risk Assessment Framework[30]

While the (MS-)GVAR model reflects the impact of fundamentals on - as well as the co-movement between - commodity prices, the objective of this thesis is the scarcity risk assessment of resource-demanding projects, which require large amounts of commodities in their realization, leading to a significant demand increase. Therefore, we develop a general framework, which includes the future required resource amounts of the project, the substitutability of commodities, as well as the commodity market structure, to determine the scarcity risk of the projects.

As the scarcity risk of a commodity is generally not observable, a commodity's price is interpreted as scarcity indicator, in accordance to Gleich et al. (2013) and Tilton (2010), since high prices are the result of high demand and/or low supply situations. With this interpretation, we initially calculate the individual probability of scarcity (PS) for each commodity of the project in Section 3.3.1, using the (MS-)GVAR framework proposed in Section 3.1 and Section 3.2, which considers the impact of fundamentals as well as the spillover effects between commodity markets. Subsequently, we aggregate the individual resource-specific risk measures at the project level to the risk indicator expected loss due to scarcity (ES), taking into account the actual required quantities and the substitutability of the commodities, which allows us to compare multiple project alternatives in terms of their resource scarcity risk. Hereby, a high value indicates a high risk of scarcity for the resource-demanding project.

3.3.1 Probability of Scarcity

The first stage of our framework estimates the probability of scarcity per commodity. Hereby, we use the (MS-)GVAR framework proposed in Section 3.1 as well as Section 3.2, to account for the commodity market structure. In addition, we propose in Section 3.3.1.2 logistic regression models, enabling for commodity-specific price determinants, as an alternative way to obtain the probabilities of scarcity.

In general, extremely high prices indicate either high demand and/or low supply situations, which is why we interpret the commodity's price as scarcity indicator. Therefore, we specify an appropriate price threshold θ_i for each commodity $i = 1, 2, \ldots, N$, which, once exceeded, determines the commodity to be scarce, and consequently, allows for a classification of the commodities into scarce or non-scarce. Hereby, we define the commodity-specific, binary latent variable $scarce_{i,t}$, for all commodities i and times $t = 1, 2, \ldots, T$, with value 1 if the price ($price_{i,t}$) exceeds the threshold (θ_i) at a certain point in time (t), indicating scarcity of the commodity, and value 0 else:

$$scarce_{i,t} = \begin{cases} 1, \ price_{i,t} > \theta_i \\ 0, \ price_{i,t} \leq \theta_i. \end{cases} \tag{3.158}$$

In general, we propose two approaches for setting an appropriate price threshold. First, the threshold can be derived based on expert knowledge of the respective commodity markets. As

[30] Parts of this section are published in the paper "Sustainable energy transition and its demand for scarce resources: Insights into the German Energiewende through a new risk assessment framework", Renewable and Sustainable Energy Reviews 176, 2023, co-authored by Patric Papenfuß, Max Brem, Paul Kurz, and Andreas Rathgeber.

the commodities under consideration are included in the profitability calculation of a project, experts determine the price above which the utilization of a commodity becomes uneconomic, leading to infeasible projects.

Second, the threshold price can be derived statistically based on historical data. In our case, we suggest to use the one-sigma approach, as, for normally distributed random variables, this leads to approximately $100\% - 68\% = 32\%$ observations being classified as scarce, see Foster et al. (1997), which in turn enables a statistically valid analysis:

$$\theta_i = \mu_{price_i} + \tilde{\sigma}_{price_i}, \tag{3.159}$$

with μ_{price_i} denoting the historical mean of the price of commodity $i = 1, 2, \ldots, N$ and $\tilde{\sigma}_{price_i}$ being the corresponding standard deviation.[31]

These threshold prices enable a classification into scarce and non-scarce commodities. However, we are interested in the probability of scarcity, i.e. the probability that the price of a commodity will exceed its threshold, in the context of a resource-demanding project. Therefore, we either use the (MS-)GVAR models, which take into account the commodity market structure, to estimate the probability distribution of the commodity price under several predefined scenarios and indirectly derive the probability of scarcity by determining quantiles, or we use logistic regression models, considering individual price determinants, to derive the probability of scarcity directly.

3.3.1.1 Calculation of the Probability of Scarcity using the (MS-)GVAR Model

For the calculation of the probability of scarcity, taking into account the commodity market structure, we forecast the commodity prices one-step ahead,[32] with help of the estimated (MS-)GVAR model in Equation 3.7 and Equation 3.137, respectively. Subsequently, we are able to derive the probability distribution of the price forecasts by a bootstrapping procedure, in analogy to Section 3.1.2.1 and Section 3.2.4.2.2, respectively. Finally, we use the definition of quantiles and get the probability of a commodity's price exceeding its threshold under the predefined scenarios.

While the (MS-)GVAR model accounts for the interdependencies between - as well as for the impact of the economy on - the commodity markets, the price forecasts are derived under predefined scenarios for the (historical) endogenous as well as exogenous variables. In this thesis, we focus on a mean scenario, a shock scenario, an extreme value scenario, focus and extreme focus scenarios as well as quantile scenarios, for the endogenous as well as exogenous variables, indicated by $\zeta = 1, 2, \ldots, \mathcal{Z}$, which are briefly described in the following. The mean scenario considers the probability of scarcity under normal circumstances, whereas we also analyze how the probability of scarcity increases in stressed periods.[33] In addition, we provide the (extreme) focus scenarios which allow to investigate the sensitivity of the probability of scarcity to one specific variable. Moreover, the quantile scenarios analyze how the probability of scarcity increases if the variables are at different states of their distributions.

- In the mean scenario $\zeta = 1$, the K_{endog} endogenous variables $\mathbf{x}_{k_{endog},i}$ for commodity i as well as the K_{exog} variables $\mathbf{e}_{k_{exog}}$ follow their sample average, $\mu_{k_{endog},i}$, and $\mu_{k_{exog}}$,

[31] Instead of the historical mean and standard deviation over a predefined time period, a rolling window approach can be used to determine a time-varying price threshold. However, this approach is only feasible for the logistic regression model and therefore not considered in this thesis.

[32] Due to data limitations, we only forecast the prices one-step ahead, although we consider the resource requirements of the following decades. However, we also include the accumulated resource amounts in the calculation of the scarcity risk such that the final risk score reflects the risk of the considered time period.

[33] We use the absolute values of the estimated coefficient matrices of the (MS-)GVAR model, to ensure an increase in the input value will lead to a higher probability of scarcity, hereby, penalizing for any disturbances. This approach might overestimate the scarcity risk, however, an overestimation is preferable to an underestimation.

respectively:

$$\begin{aligned} x_{k_{endog},i,1} &= \mu_{k_{endog},i}, k_{endog} = 1,2,\ldots,K_{endog}, \\ e_{k_{exog},1} &= \mu_{k_{exog}}, k_{exog} = 1,2,\ldots,K_{exog}. \end{aligned} \tag{3.160}$$

- In the shock scenario $\zeta = 2$, each variable follows the one-sigma approach:

$$\begin{aligned} x_{k_{endog},i,2} &= \mu_{k_{endog},i} + \tilde{\sigma}_{k_{endog},i}, k_{endog} = 1,2,\ldots,K_{endog}, \\ e_{k_{exog},2} &= \mu_{k_{exog}} + \tilde{\sigma}_{k_{exog}}, k_{exog} = 1,2,\ldots,K_{exog}, \end{aligned} \tag{3.161}$$

  in which $\mu_{k_{endog},i}$, and $\mu_{k_{exog}}$ denote the sample means, and $\tilde{\sigma}_{k_{endog},i}$, and $\tilde{\sigma}_{k_{exog}}$ are the standard deviations of the sample.

- In the extreme scenario $\zeta = 3$, each covariate follows the two-sigma approach:

$$\begin{aligned} x_{k_{endog},i,3} &= \mu_{k_{endog},i} + 2\tilde{\sigma}_{k_{endog},i}, k_{endog} = 1,2,\ldots,K_{endog}, \\ e_{k_{exog},3} &= \mu_{k_{exog}} + 2\tilde{\sigma}_{k_{exog}}, k_{exog} = 1,2,\ldots,K_{exog}. \end{aligned} \tag{3.162}$$

- In the focus scenario $\zeta = 4_{\tilde{k}_{exog}}$, the $\tilde{k}_{exog}$-th exogenous variable follows the shock scenario, whereas the remaining variables follow the mean scenario:[34]

$$\begin{aligned} x_{k_{endog},i,4_{\tilde{k}_{exog}}} &= \mu_{k_{endog},i}, k_{endog} = 1,2,\ldots,K_{endog}, \\ e_{k_{exog},4_{\tilde{k}_{exog}}} &= \mu_{k_{exog}}, k_{exog} = 1,2,\ldots,K_{exog}, \text{ with } k_{exog} \neq \tilde{k}_{exog}, \\ e_{\tilde{k}_{exog},i,4_{\tilde{k}_{exog}}} &= \mu_{\tilde{k}_{exog},i} + \tilde{\sigma}_{\tilde{k}_{exog},i}. \end{aligned} \tag{3.163}$$

- In the extreme focus scenario $\zeta = 5_{\tilde{k}_{exog}}$, the $\tilde{k}_{exog}$-th exogenous variable follows the extreme scenario, instead of the shock scenario, whereas the remaining variables follow the mean scenario:[35]

$$\begin{aligned} x_{k_{endog},i,5_{\tilde{k}_{exog}}} &= \mu_{k_{endog},i}, k_{endog} = 1,2,\ldots,K_{endog}, \\ e_{k_{exog},5_{\tilde{k}_{exog}}} &= \mu_{k_{exog}}, k_{exog} = 1,2,\ldots,K_{exog}, \text{ with } k_{exog} \neq \tilde{k}_{exog}, \\ e_{\tilde{k}_{exog},i,5_{\tilde{k}_{exog}}} &= \mu_{\tilde{k}_{exog},i} + 2\tilde{\sigma}_{\tilde{k}_{exog},i}. \end{aligned} \tag{3.164}$$

- In the quantile scenarios $\zeta = 6_q$, each variable takes on the value of its $q\%$ quantile:

$$\begin{aligned} x_{k_{endog},i,6_q} &= x_{k_{endog},i,q}, k_{endog} = 1,2,\ldots,K_{endog}, \\ e_{k_{exog},6_q} &= e_{k_{exog},q}, k_{exog} = 1,2,\ldots,K_{exog}, \end{aligned} \tag{3.165}$$

  in which $x_{k_{endog},i,q}, e_{k_{exog},q}$ denote the $q\%$ quantiles of the sample.

Overall, these scenarios enable a scenario-based risk assessment. On the one hand, the scenario-specific comparison between multiple commodities may be used to choose between possible substitute materials from a risk perspective, while on the other hand, the commodity-specific comparison between the scenarios may be used as a sensitivity analysis.

[34] For a simpler notation, we introduce the focus scenario only for the exogenous variables. However, instead of an exogenous variable, an endogenous variable can follow the shock scenario, while the remaining variables follow the mean scenario.

[35] In line with the focus scenario, we also introduce the extreme focus scenario only for the exogenous variables, for a simpler notation. However, instead of an exogenous variable, an endogenous variable can follow the extreme scenario, while the remaining variables follow the mean scenario.

Using the estimated (MS-)GVAR model of Equation 3.7 and Equation 3.137, respectively, we forecast the commodity prices one-step ahead, according to Equation 3.9 and 3.142, respectively, whereby we assume the variables as well as their lags are equal to the predefined scenario values $x_{k_{endog},i,\zeta}$, for $k_{endog} = 1, 2, \ldots, K_{endog}$ as well as $e_{k_{exog},\zeta}$, for $k_{exog} = 1, 2, \ldots, K_{exog}$, for scenario $\zeta = 1, 2, \ldots, \mathcal{Z}$.[36] Subsequently, we calculate the probability distribution of the prices per scenario by a bootstrapping procedure.

In this context, we slightly adjust the bootstrapping methodologies of the (MS-)GVAR models for the significance of the generalized impulse response functions, described in Section 3.1.2.1 and Section 3.2.4.2.2, respectively, since we use the bootstrapping procedure to obtain a probability distribution of the commodity prices instead of calculating generalized impulse response functions. In case of the GVAR framework, we adapt step five and seven of the bootstrap technique proposed in Dées, di Mauro, Pesaran, and Smith (2007) and described in Section 3.1.2.1 as follows:

5. For each scenario ζ, we use the scenario values of the variables $\mathbf{x}_{k_{endog},i,\zeta}$, for $k_{endog} = 1, 2, \ldots, K_{endog}$ as well as $\mathbf{e}_{k_{exog},i,\zeta}$, for $k_{exog} = 1, 2, \ldots, K_{exog}$ and the new estimated parameters of the GVAR model corresponding to the bootstrap sample to forecast the commodity prices one-step ahead.

7. Sort the predicted prices for each commodity and each scenario into an ascending order. Comparing the predefined threshold with the quantiles of the bootstrap distribution of the predicted prices, we can derive the probability of scarcity, using the definition of quantiles.[37]

In case of the MS-GVAR framework, we adapt step six and eight of the regime-dependent bootstrap technique proposed in Ehrmann et al. (2001) and described in Section 3.2.4.2.2 as follows:

6. For each scenario ζ, we use the scenario values of the variables $\mathbf{x}_{k_{endog},i,\zeta}$, for $k_{endog} = 1, 2, \ldots, K_{endog}$ as well as $\mathbf{e}_{k_{exog},i,\zeta}$, for $k_{exog} = 1, 2, \ldots, K_{exog}$ and the new estimated parameters of the MS-GVAR model corresponding to the bootstrap sample to forecast the commodity prices one-step ahead.

8. Sort the predicted prices for each commodity and each scenario into an ascending order. Comparing the predefined threshold with the quantiles of the bootstrap distribution of the predicted prices, we can derive the probability of scarcity, using the definition of quantiles.

3.3.1.2 An Alternative Model for the Calculation of the Probability of Scarcity: Logistic Regression Model

The (MS-)GVAR framework jointly models commodity markets, considering the impact of fundamentals on - as well as spillover effects between - prices. Hereby, the prices are influenced by the other commodity markets as well as by common exogenous variables. As Gleich et al. (2013) state the determinants of prices are heterogeneous between different metals, we alternatively propose to estimate the probability of scarcity directly via commodity-specific logistic

[36] In case of the MS-GVAR framework, the forecasts of the commodity prices are regime-dependent, which is why a regime-constellation has to be predefined. As we apply our risk framework on the resource requirements of the German Energiewende in the period 2020 to 2050, we use the estimated regime probabilities at the end of our sample period, December 2019.

[37] The $q\%$ quantile of a variable $\mathbf{v}$ is generally defined as the maximum value v_q which satisfies $P_{\mathbf{v}}\left(]-\infty, v_q]\right) \leq q$, whereby $P_{\mathbf{v}}$ denotes the probability distribution of $\mathbf{v}$.

regression models, allowing for individual selected price influential factors. However, the comovement between prices is only covered by common price determinants, in contrast to the (MS-)GVAR framework, which allows for spillover effects between commodity markets.

Overall, we derive the probability of scarcity directly by estimating logistic regression models on the predefined price threshold. In particular, we model the dependencies between various price determinants on the variable $scarce_{i,t}$, defined via the threshold in Equation 3.158. Hereby, we consider different, possible price influential factors of five dimensions: Macroeconomic, demographic, capital market driven as well as supply and demand variables.

Since the inclusion of all, potential price determinants is unfeasible from the statistical point of view, due to data limitations, we perform a two-step model selection for each commodity to identify the commodity-specific price determining factors, which will be included in the calculation of the probability of scarcity. In the first step, we extract all factors which do have a significant impact on the commodity price from a broad list of variables. Hereby, we implement univariate linear regression models for all potential variables, with the commodity price as dependent variable and preselect the influential variables, using a t-test based on the 5% significance level. In the second step, we determine the final set of covariates per commodity by applying a bi-directional stepwise model selection, using the Bayesian information criterion (BIC) on the preselected set of factors. With help of the variance inflation factor, we ensure none of our final models suffers from multicollinearity, as we exclude one of the highly correlated variables from the analysis, based on an economic justification, in case of an initial model with variance inflation factor above five.

Using the identified price influential factors, we calculate the probability of scarcity per commodity. Therefore, we first estimate a logistic regression model. Hereby, we regress the K_i price determinants $\mathbf{x}_{i,t} = (\mathsf{x}_{1,i,t}, \mathsf{x}_{2,i,t}, \ldots, \mathsf{x}_{\mathsf{K}_i,i,t})$, derived from the previous, commodity-specific model selection, on the dependent variable $scarce_{i,t}$, defined in Equation 3.158 by the threshold θ_i:

$$P\left(scarce_{i,t} = 1 | \mathbf{X} = \mathbf{x}_{i,t}\right) = \frac{1}{1 + \exp\left(-\mathsf{z}_{i,t}\right)}, \tag{3.166}$$

with the logit $\mathsf{z}_{i,t}$:

$$\mathsf{z}_{i,t} = \beta_0 + \beta_1 \mathsf{x}_{1,i,t} + \beta_2 \mathsf{x}_{2,i,t} + \ldots + \beta_{\mathsf{K}_i} \mathsf{x}_{\mathsf{K}_i,i,t} + \varepsilon_{i,t}, \tag{3.167}$$

in which β_0 denotes the intercept, $\beta_1, \beta_2, \ldots, \beta_{\mathsf{K}_i}$ are the coefficients corresponding to the K_i covariates $\mathsf{x}_{1,i,t}, \mathsf{x}_{2,i,t}, \ldots, \mathsf{x}_{\mathsf{K}_i,i,t}$ for commodity $i = 1, 2, \ldots, N$ at time $t = 1, 2, \ldots, T$ and $\varepsilon_{i,t}$ represents the error term. Hereby, we obtain the estimated parameters $\hat{\beta}_0, \hat{\beta}_1, \ldots, \hat{\beta}_{\mathsf{K}_i}$ of Equation 3.166 via the Maximum-Likelihood approach. In particular, the estimated logistic regression model per commodity enables us to directly calculate the probability of scarcity, which is the probability of a commodity becoming scarce.

In this context, we examine several predefined conditions for the covariates to investigate how stressed situations affect the probability of scarcity, similar to the scenarios for the endogenous as well as exogenous variables in the (MS-)GVAR model in Section 3.3.1. Hereby, we consider the scenarios $\zeta = 1, 2, \ldots, \mathcal{Z}$ for the covariates, namely a mean scenario, a shock scenario, an extreme value scenario, focus and extreme focus scenarios as well as quantile scenarios.

- In the mean scenario $\zeta = 1$, the covariates $\mathsf{x}_{\mathsf{k}_i,i,1}$ for commodity i follow their sample average $\mu_{\mathsf{k}_i,i}$:

$$\mathsf{x}_{\mathsf{k}_i,i,1} = \mu_{\mathsf{k}_i,i}, \mathsf{k}_i = 1, 2, \ldots, \mathsf{K}_i. \tag{3.168}$$

- In the shock scenario $\zeta = 2$, each covariate follows the one-sigma approach:

$$\mathsf{x}_{\mathsf{k}_i,i,2} = \mu_{\mathsf{k}_i,i} + sgn\left(\beta_{\mathsf{k}_i}\right) \tilde{\sigma}_{\mathsf{k}_i,i}, \mathsf{k}_i = 1, 2, \ldots, \mathsf{K}_i, \tag{3.169}$$

in which $\mu_{\mathsf{k}_i,i}$ denotes the sample mean, $\tilde{\sigma}_{\mathsf{k}_i,i}$ is the standard deviation of the sample and $sgn\left(\beta_{\mathsf{k}_i}\right)$ is the signum function[38] of the estimated coefficient in Equation 3.167.

- In the extreme scenario $\zeta = 3$, each covariate follows the two-sigma approach:

$$\mathsf{x}_{\mathsf{k}_i,i,3} = \mu_{\mathsf{k}_i,i} + 2sgn\left(\beta_{\mathsf{k}_i}\right)\tilde{\sigma}_{\mathsf{k}_i,i}, \mathsf{k}_i = 1, 2, \ldots, \mathsf{K}_i. \tag{3.170}$$

- In the focus scenario $\zeta = 4_{\tilde{\mathsf{k}}}$, the $\tilde{\mathsf{k}}$-th covariate follows the shock scenario, whereas the remaining variables follow the mean scenario:

$$\begin{aligned} \mathsf{x}_{\mathsf{k}_i,i,4_{\tilde{\mathsf{k}}}} &= \mu_{\mathsf{k}_i,i}, \mathsf{k}_i = 1, 2, \ldots, \mathsf{K}_i, \text{ with } \mathsf{k}_i \neq \tilde{\mathsf{k}}, \\ \mathsf{x}_{\tilde{\mathsf{k}},i,4_{\tilde{\mathsf{k}}}} &= \mu_{\tilde{\mathsf{k}},i} + sgn\left(\beta_{\tilde{\mathsf{k}}}\right)\tilde{\sigma}_{\tilde{\mathsf{k}},i}. \end{aligned} \tag{3.171}$$

- In the extreme focus scenario $\zeta = 5_{\tilde{\mathsf{k}}}$, the $\tilde{\mathsf{k}}$-th covariate follows the extreme scenario, instead of the shock scenario, whereas the remaining variables follow the mean scenario:

$$\begin{aligned} \mathsf{x}_{\mathsf{k}_i,i,5_{\tilde{\mathsf{k}}}} &= \mu_{\mathsf{k}_i,i}, \mathsf{k}_i = 1, 2, \ldots, \mathsf{K}_i, \text{ with } \mathsf{k}_i \neq \tilde{\mathsf{k}}, \\ \mathsf{x}_{\tilde{\mathsf{k}},i,5_{\tilde{\mathsf{k}}}} &= \mu_{\tilde{\mathsf{k}},i} + 2sgn\left(\beta_{\tilde{\mathsf{k}}}\right)\tilde{\sigma}_{\tilde{\mathsf{k}},i}. \end{aligned} \tag{3.172}$$

- In the quantile scenarios $\zeta = 6_q$, each covariate takes on the value of its $q\%$ quantile:

$$\mathsf{x}_{\mathsf{k}_i,i,6_q} = \mathsf{x}_{\mathsf{k}_i,i,q}, \mathsf{k}_i = 1, 2, \ldots, \mathsf{K}_i, \tag{3.173}$$

in which $\mathsf{x}_{\mathsf{k}_i,i,q}$ denotes the $q\%$ quantile of the sample.

Using the scenario values of the covariates $\mathsf{x}_{1,i,\zeta}, \mathsf{x}_{2,i,\zeta}, \ldots, \mathsf{x}_{\mathsf{K}_i,i,\zeta}$ and the corresponding estimated logistic regression of Equation 3.166, we directly calculate the probability of scarcity per commodity i and scenario ζ.

3.3.2 Expected Loss due to Scarcity on Commodity and Project Level

The main objective of the framework is the comparison of resource-demanding projects in respect to the economic scarcity risk of the commodities they require. Hereby, these projects may differ in the selection and quantity of the commodities included. In particular, we know the required amount of each commodity as well as the probability of scarcity for a certain scenario and project, derived from the (MS-)GVAR model, taking into account the commodity market structure, or from the logistic regression model, enabling for commodity-specific price determinants. The general idea of the framework is to aggregate this commodity-specific information to a risk measure on project level, similar to the combination of multiple credit contracts into a portfolio-based risk measure.

Within credit risk modeling, the risk of portfolios is compared by the expected loss (EL), defined as:

$$EL_{pf} = \sum_{cred \in pf} EL_{cred,pf} = \sum_{cred \in pf} EAD_{cred,pf} \cdot LGD_{cred,pf} \cdot PD_{cred,pf}, \tag{3.174}$$

in which $EL_{cred,pf}$ denotes the expected loss of a loan $cred$ in the portfolio pf, $EAD_{cred,pf}$ the corresponding exposure at default, $LGD_{cred,pf}$ the loss given default and $PD_{cred,pf}$ the respective probability of default, see Basel Committee on Banking Supervision (2005). While the probability of default (PD) and the probability of scarcity (PS) may be regarded equivalently,

[38] Similar to the scenarios based on the (MS-)GVAR model, we use the signum function on the estimated coefficients to ensure the addition of the standard deviation will lead to a higher probability of scarcity.

the adoption of the expected loss (EL) to a scarcity risk measure of resource-demanding projects requires adjustments on the loss given default (LGD) and the exposure at default (EAD).

In general, the loss given default represents the loss a bank realizes in case a borrower defaults. In contrast, the respective measure in our framework, the loss given scarcity (LGS), should reflect whether the commodity is substitutable, since the risk of scarcity is negligible if one commodity can be substituted for another. Therefore, the loss given scarcity is linked to the substitutability rate (SR), representing a normalized indicator for the substitutability of commodity i, by:

$$LGS_i = 1 - SR_i \in [0, 1]\,. \tag{3.175}$$

Hereby, a LGS_i of 0 indicates the commodity i is perfectly substitutable by other commodities, hence, its scarcity is irrelevant in a project context, whereas a LGS_i of 1 indicates no substitute for the commodity i is available and the project is infeasible in case of scarcity.

While the exposure at default represents the amount of loss a bank is exposed to in case of a defaulted loan, the respective measure in our framework, the exposure at scarcity (EAS), should reflect the required resource amount of the project. Thereby, we assume, the entire amount of the specific commodity required in project $proj$ is not accessible in case of scarcity, independent of the project's state at which the scarcity occurs. For a comparison of several commodities and projects, we scale the required resource amount of the project ($quant_{proj,i}$) by the average world production ($\overline{supply_i}$) of the respective commodity i, resulting in the project and commodity-specific exposure at scarcity:

$$EAS_{proj,i} = \frac{quant_{proj,i}}{\overline{supply_i}}. \tag{3.176}$$

Hereby, commodities are considered to be more risky if the absolute required quantities are comparatively small, but which are only mined in small quantities, since the world supply is relatively small in contrast to the additional demand required for the project.

Using the adjusted parameters, we are able to calculate the expected loss due to scarcity for project $proj$, commodity i and scenario ζ:

$$ES_{proj,i,\zeta} = 100 \cdot EAS_{proj,i} \cdot LGS_i \cdot PS_{proj,i,\zeta}. \tag{3.177}$$

Subsequently, we aggregate the commodity-specific expected loss due to scarcity values, in analogy to credit risk modeling, on project level:

$$ES_{proj,\zeta} = \sum_{i=1}^{N} ES_{proj,i,\zeta} = \sum_{i=1}^{N} 100 \cdot EAS_{proj,i} \cdot LGS_i \cdot PS_{proj,i,\zeta}. \tag{3.178}$$

Hereby, the assumption of independence between the commodities allows for the additivity of the expected loss due to scarcity values, as potential dependencies between commodities are directly reflected in the probability of scarcity[39] via the (MS-)GVAR framework or indirectly reflected by common macroeconomic determinants in case of the logistic regression model.

The resulting expected loss due to scarcity enables the comparison of various projects from a commodity scarcity risk perspective. Hereby, the framework takes advantage of the commodity market models included, as the impact of supply and demand, macroeconomic conditions as well as spillover effects between commodities are reflected. Further, the determination of the commodity-specific threshold θ_i, which is required for the definition of the binary variable $scarce_i$, is adjustable to the project of interest and economic background. Additionally, depending on the use-case, multiple of the proposed scenarios, as well as combinations and extensions of them, allow for a detailed sensitivity analysis.

[39] In case of the (MS-)GVAR model, we combine the individual commodity market models via information on the transformation paths, see Section 4.4.4, therefore, the probability of scarcity also depends on the considered project. In contrast, the probability of scarcity derived from the logistic regression model is independent of the underlying project.

4 Data

The objective of the empirical part of this thesis is the analysis and comparison of the resource requirements of several transformation pathways for the German Energiewende in regard to their availability, respectively their scarcity. Therefore, we analyze the resource demands of four expansion pathways of the German energy system, see Section 4.1, within a commodity market framework, based on commodity-specific data as well as data from several potential influencing factors on commodity prices, which are described in Section 4.2 and Section 4.3, respectively. Hereby, the individual commodity market models are linked via information on co-production, co-consumption and co-trading, reflecting the relations between commodities, see Section 4.4.

4.1 Transformation Pathways of the German Energiewende

In this thesis, we consider the resource requirements of four expansion pathways of the German energy system, called expansion path of the German energy system ($REMod$) paths, which are all generated under the restriction of a 95% CO_2 reduction in 2050, compared to Germany's emissions in 1990, see Sterchele et al. (2020). These pathways differentiate by the underlying assumptions of the German society's acceptance for actions to fulfill these reduction goals. While the initial energy expansion pathways of the energy system are modeled according to Sterchele et al. (2020), a translation of these pathways into resource demands from 2020 to 2050, on an annual basis, is performed via a life-cycle assessment as well as a system dynamics model,[1] see Betten et al. (2020). A general overview of the pathways is displayed in Table 4.1.

Within the reference path ($REMod - REF$), which marks the baseline scenario, the energy system is calculated at optimal costs, without further boundary conditions that promote or aggravate the achievement of the 95% CO_2 reduction goal. In contrast, the sufficiency path ($REMod - SUF$) models a substantial change in the behavior of the German population towards a reduction in its energy consumption, e.g., by tripling the maximum renovation rate for buildings, resulting in a reduction in heated building area, which overall results in the lowest energy demand of all the $REMod$ pathways. However, mental reservations of the population for new technologies in the private sector could lead to a substantial time delay for the implementation of renewable energy technologies, as well as a continuous high demand of conventional energy technologies, represented in the persistence path ($REMod - PER$). In addition, strong resistance to the expansion of large infrastructures, such as wind energy parks or power grid expansions, are modeled in the unacceptance path ($REMod - UNA$). In order to achieve the German climate targets under these boundary conditions, the demand for photovoltaic and storage technologies will increase drastically, as will the corresponding demand for commodities.

[1] These calculations are generated within the project InteRessE, Grant-Nr: 03ET4065B, supported by the German Federal Ministry for Economic Affairs and Energy, which aims to analyze the resource demand of the German Energiewende from various perspectives. Hereby, the specific data for the technologies considered as well as the resource requirement are not yet published.

Table 4.1: Energy system pathways - demand and installed capacities

Abbreviation Name	$REMod - REF$ Reference	$REMod - SUF$ Sufficiency	$REMod - PER$ Persistence	$REMod - UNA$ Unacceptance
Energy demand in 2050 [in TWh]	1447	1068	1464	1282
Limits for installed capacity in GW:				
Photovoltaic	530	530	530	800
Wind Onshore	230	230	230	80
Wind Offshore	80	80	80	40
Electricity import	40	40	40	20
Consumption development:				
Classic power applications	constant	decreasing	constant	constant
Traffic performance	increasing	decreasing	increasing	increasing
Heated building area	increasing	decreasing	increasing	increasing
Process heat	slightly	decreasing	slightly	slightly

This table displays an overview of central boundary conditions of the transformation pathways, reference path ($REMod - REF$), sufficiency path ($REMod - SUF$), persistence path ($REMod - PER$), and unacceptance path ($REMod - UNA$), according to Sterchele et al. (2020). In particular, the total energy demand in 2050 in terrawatt-hour (TWh), the limits for installed capacity in gigawatt (GW) for photovoltaic, wind on- and offshore, and the electricity import per path as well as the assumed consumption development per path are displayed.

The annual resource requirements of 28 metals are calculated from 2020 to 2050 based on 28 representative technologies[2] and evaluated from an ecological, an economic, and a social perspective for each of these pathways within the project InteRessE, Grant-Nr: 03ET4065B, supported by the German Federal Ministry for Economic Affairs and Energy. In this thesis, we utilize these resource requirements to assess the potential scarcity risk of the transformation pathways. Hereby, we focus on the requirements of the nine commodities silver (Ag), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), and platinum (Pt), which are key resources for the German Energiewende, according to Bastian et al. (2019).

For the analysis of the resource demands within the risk assessment framework, we introduce new models for commodity markets, incorporating the (time-varying) impact of fundamentals, especially of demand, on prices, as well as the (time-varying) co-movement between prices, which we exemplary apply on the industrial metal markets. Subsequently, we use the methodology to assess the resource risk of the German Energiewende. Therefore, we enlarge the set of crucial commodities defined by Bastian et al. (2019) with the remaining industrial metals aluminum (Al), lead (Pb), tin (Sn) as well as zinc (Zn), which significantly contribute to the co-movement in metal markets. While these metals are also part of energy technologies, Bastian et al. (2019) does not classify them as key resources.

The main uses and the largest mining countries of these resources are summarized in Table C.1, while the main uses in the context of the energy transition of the metals are described in Table 4.2, according to Rohstoffagentur (2016). Hereby, silver, indium and platinum are utilized for the installation of photovoltaic systems, while copper, tin as well as the rare earth metals dysprosium and neodymium are used for wind turbines. Moreover, the metals silver, aluminum, cobalt, indium, lithium, nickel, lead, platinum and zinc, are required for energy storage within lithium-ion batteries, redox flow storage, aluminum based electrolytic capacitors or general batteries.

[2] The representative technologies can be classified into the following sectors with associated overarching categories:

- electricity and heat generation: PV, wind on- and offshore, combined heat and power (CHP), fuel cell, heat pump, condensing boiler, solar thermal energy
- storage and mobility: power-to-X, batteries, central thermal energy storage
- electricity transport: transmission network, trade
- reduction of energy demand: building renovation

Table 4.2: Main uses of the metals in the context of the energy transition

	Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
Photovoltaic systems	x					x					x		
Wind turbines				x	x			x				x	
Lithium-ion batteries	x		x				x		x				
Redox flow storage													x
Solar thermal power plant	x												
Aluminum based electrolytic capacitors		x											
Fuel cell		x											
Chassis electric car		x											
Magnets			x		x			x					
Alloys			x										
Catalysts			x								x		
Electric traction motors				x									
Batteries	x		x			x	x			x	x		
Smart Grid (display)						x							
Micro Energy Harvesting					x			x					

This table displays the main uses in the context of energy technologies of the metals silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), according to Bastian et al. (2019).

4.1.1 Path-specific Commodity Requirements

For a better understanding of the resource requirements of the four *REMod* transformation pathways for the German Energiewende in the period from 2020 to 2050, we graphically display them in Figure C.1. The corresponding descriptive statistics, the total amount over time as well as the average annual world production from 2010 to 2019 are given in Table 4.3. Overall, the demand for the commodities will increase over time, except for silver, indicating more renewable energy technologies will be built up at the end of the considered time period. In addition, the total demand of the German Energiewende for cobalt is outstandingly high, compared to the average annual world production of the previous decade, whereas the requirements for platinum are neglectable.

Comparing the commodity demands for the four different expansion pathways, the sufficiency path requires the least metals, except for the rare earth elements dysprosium and neodymium, as well as for lead and zinc. In particular, the least demand for dysprosium and neodymium in the unacceptance path can be attributed to the modeled resistance with respect to large infrastructural projects. In this context, less wind energy parks will be installed, resulting in a reduced demand for the rare earth metals. However, in order to achieve the energy targets despite the few wind parks, more photovoltaic systems and storage technologies need to be set up, which is why the required amounts of silver, cobalt, indium, lithium as well as nickel are comparably high in the unacceptance path.

Table 4.3: Descriptive statistics of the path-specific commodity requirements

	Path	Min.	5% Q.	25% Q.	Med.	Mean	75% Q.	95% Q.	Max.	av. S.	Total	SD	Skew.	Kurt.
Ag [t]	$REMod-REF$	55	62	125	172	171	196	294	348	25440	5302	71	0.54	-0.02
	$REMod-SUF$	37	39	91	131	115	145	149	150	25440	3579	36	-0.94	-0.44
	$REMod-PER$	53	61	101	204	195	267	343	362	25440	6039	97	0.10	-1.37
	$REMod-UNA$	45	55	165	263	273	363	492	550	25440	8453	137	0.21	-0.91
Al [tmt]	$REMod-REF$	47	50	332	374	335	413	431	450	54630	10381	118	-1.45	0.80
	$REMod-SUF$	30	33	218	265	227	285	298	303	54630	7038	88	-1.28	0.10
	$REMod-PER$	46	53	281	302	294	375	405	413	54630	9113	102	-1.17	0.57
	$REMod-UNA$	42	48	340	371	375	494	551	559	54630	11626	147	-0.87	0.07
Co [tmt]	$REMod-REF$	1	1	31	35	31	39	43	43	173	974	13	-1.40	0.71
	$REMod-SUF$	0	1	20	26	21	27	28	29	173	655	10	-1.29	-0.02
	$REMod-PER$	1	1	21	26	23	28	31	31	173	702	9	-1.42	0.80
	$REMod-UNA$	1	2	33	37	32	38	43	45	173	997	12	-1.61	1.32
Cu [tmt]	$REMod-REF$	31	40	218	262	233	294	324	335	18580	7238	91	-1.14	-0.08
	$REMod-SUF$	22	28	112	203	169	229	255	266	18580	5231	79	-0.66	-1.10

Descriptive statistics of the path-specific commodity requirement

	Path	Min.	5% Q.	25% Q.	Med.	Mean	75% Q.	95% Q.	Max.	av. S.	Total	SD	Skew.	Kurt.
	$REMod-PER$	36	50	187	229	198	241	254	260	18580	6147	66	-1.33	0.41
	$REMod-UNA$	26	36	207	255	221	271	286	289	18580	6849	82	-1.35	0.32
Dy [t]	$REMod-REF$	6	8	39	52	59	88	102	108	1280	1814	32	-0.12	-1.33
	$REMod-SUF$	5	7	13	27	42	78	88	89	1280	1297	33	0.33	-1.79
	$REMod-PER$	6	8	40	62	56	78	91	93	1280	1747	28	-0.45	-1.09
	$REMod-UNA$	5	7	17	28	28	41	44	44	1280	865	13	-0.26	-1.43
In [t]	$REMod-REF$	16	19	34	50	55	79	94	106	774	1700	26	0.21	-1.24
	$REMod-SUF$	10	12	20	32	40	65	76	80	774	1245	23	0.34	-1.53
	$REMod-PER$	16	19	46	62	56	70	75	85	774	1742	18	-0.76	-0.52
	$REMod-UNA$	15	16	62	91	84	113	131	147	774	2597	37	-0.43	-0.92
Li [tmt]	$REMod-REF$	1	1	19	23	20	25	27	27	998	614	8	-1.37	0.63
	$REMod-SUF$	0	0	12	16	13	17	18	19	998	413	6	-1.26	-0.07
	$REMod-PER$	1	1	13	16	14	18	20	20	998	443	6	-1.35	0.68
	$REMod-UNA$	1	1	20	23	20	24	27	28	998	629	8	-1.57	1.26
Nd [t]	$REMod-REF$	38	49	273	445	457	722	829	839	31400	14168	278	-0.13	-1.44
	$REMod-SUF$	31	42	81	159	325	642	742	756	31400	10077	287	0.42	-1.71
	$REMod-PER$	38	49	277	441	423	637	712	743	31400	13115	236	-0.37	-1.34
	$REMod-UNA$	29	40	83	191	196	305	340	341	31400	6070	110	-0.07	-1.59
Ni [tmt]	$REMod-REF$	4	5	97	120	106	135	150	152	2330	3283	44	-1.20	0.26
	$REMod-SUF$	2	2	62	87	72	94	101	104	2330	2219	34	-1.15	-0.28
	$REMod-PER$	4	6	72	97	84	105	117	120	2330	2597	34	-1.21	0.22
	$REMod-UNA$	5	6	101	123	106	133	139	142	2330	3286	41	-1.51	0.99
Pb [t]	$REMod-REF$	514	682	2011	2347	2157	2691	2892	2969	4838000	66874	698	-1.02	-0.03
	$REMod-SUF$	408	447	760	1012	1467	2468	2596	2597	4838000	45481	850	0.29	-1.75
	$REMod-PER$	491	736	1998	2389	2121	2478	2559	2621	4838000	65763	599	-1.54	1.15
	$REMod-UNA$	444	653	1120	1276	1213	1372	1400	1573	4838000	37597	249	-1.73	2.87
Pt [kg]	$REMod-REF$	14	16	90	124	114	161	180	183	435500	3525	58	-0.61	-1.08
	$REMod-SUF$	11	13	26	47	75	141	161	172	435500	2329	58	0.42	-1.61
	$REMod-PER$	14	16	90	125	142	148	354	382	435500	4414	108	0.84	-0.39
	$REMod-UNA$	10	12	42	83	81	104	197	236	435500	2523	56	0.93	0.73
Sn [t]	$REMod-REF$	402	422	2058	2503	2307	3152	3320	3381	282500	71531	1000	-0.85	-0.78
	$REMod-SUF$	281	317	750	1119	1572	2714	3008	3035	282500	48732	1023	0.32	-1.66
	$REMod-PER$	380	444	2030	2589	2219	2780	2999	3060	282500	68786	868	-1.11	-0.41
	$REMod-UNA$	313	345	1517	1971	1702	2128	2212	2224	282500	52767	623	-1.20	-0.08
Zn [tmt]	$REMod-REF$	7	43	71	83	78	95	1034	106	12720	2413	22	-1.15	1.21
	$REMod-SUF$	7	30	66	82	79	102	120	122	12720	2454	33	-0.51	-0.88
	$REMod-PER$	40	40	70	71	64	71	72	72	12720	1979	13	-1.24	-0.46
	$REMod-UNA$	12	43	70	93	83	102	104	104	12720	2563	24	-1.18	0.61

This table displays the descriptive statistics (minimum (Min.), 5% quantile (5% Q.), 25% quantile (25% Q.), 75% quantile (75% Q.), 95% quantile (95% Q.), median (Med.), mean (Mean), maximum (Max.), standard deviation (SD), skewness (Skew.) and excess kurtosis (Kurt.)) of the annual requirements in kilogram (kg), metric ton (t), and thousand metric tons (tmt), from 2020 to 2050 for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn) of the $REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$ transformation pathways. Hereby, we also report the average annual world production (av. S.) of 2010 to 2019 as well as the total required amount (Total) of the considered time period 2020 to 2050.

4.2 Commodity Markets

The objective of this thesis is the analysis and comparison of the scarcity risk of the resource requirements of the transformation pathways of the German energy system from 2020 until 2050. In this context, we intend to analyze the commodity scarcity risk under consideration of the long-term relation between commodity markets. Therefore, we base our risk assessment framework as well as the time-invariant commodity market model, the GVAR model, on annual data in the period from 1970 to 2019, excluding the time span of the COVID-19 pandemic to ensure the observed relationships are not biased by this extreme event. Moreover, the additional resource demands for the German Energiewende are calculated from 2020 to 2050, which is why we analyze the commodity markets based on data until 2019 and investigate the scarcity risk of the future additional commodity demand from 2020 until 2050.

However, due to the large number of parameters estimated in the time-varying commodity market model, the MS-GVAR model, implied by the time-varying component, we base this analysis on monthly data. In contrast to the GVAR model, the Markov-switching methodology allows the distinct analysis of volatile as well as calm periods, which is why we explicitly include the start of the COVID-19 pandemic in our dataset for the exemplary application of this framework on commodity markets. Therefore, our data spans from January 1995 to December 2020 in this part, see Section 5.2.

Since the (MS-)GVAR methodology models the individual commodity markets, considering the interactions between supply, demand and price, we consider the worldwide primary production per commodity per metric ton, reported by U.S. Geological Survey (2020b), as our commodity-specific supply variable, instead of a global supply proxy. Following the idea of Fernandez (2015a), we approximate the global commodity-specific demand by the global apparent consumption. Therefore, we adjust the commodity-specific U.S. apparent consumption per metric ton,[3] as provided by U.S. Geological Survey (2020a), by the ratio of reported U.S. gross domestic product (U.S. GDP) and world gross domestic product (GDP),[4] drawn from U.S. Bureau of Economic Analysis (2022) and The World Bank (2022a). In case of the monthly supply and demand data of the industrial metals for the MS-GVAR model, we use the world production and consumption data per metric ton, provided by The World Bureau of Metal Statistics (2021). In addition, the sources of the commodity price[5] data are displayed in Table 4.4. As U.S. Geological Survey (2013a) only provide the averaged annual prices until 2010, we extend these series by the annual mean of daily prices, see Table 4.4 for the corresponding sources. In contrast, we base our monthly analysis on end of month prices, provided by Thomson Reuters Eikon, see Table 4.4. In general, all metal prices are standardized to U.S. dollar per metric ton, following Chen (2010).

Table 4.4: Sources of the commodity prices

	Source until 2010	Source from 2010 to 2020
Ag	U.S. Geological Survey (2013a)	Thomson Reuters Eikon (2022l)
Al	Thomson Reuters Eikon (2022d)	Thomson Reuters Eikon (2022d)
Co	U.S. Geological Survey (2013a)	Thomson Reuters Eikon (2022a)
Cu	Thomson Reuters Eikon (2022e)	Thomson Reuters Eikon (2022e)
Dy		Asian Metal (2022a)
In	U.S. Geological Survey (2013a)	Thomson Reuters Eikon (2022b)
Li	U.S. Geological Survey (2013a)	Thomson Reuters Eikon (2022c)
Nd		Asian Metal (2022b)
Ni	U.S. Geological Survey (2013a)	Thomson Reuters Eikon (2022g)
Pb	U.S. Geological Survey (2013a)	Thomson Reuters Eikon (2022f)
Pt	U.S. Geological Survey (2013a)	Thomson Reuters Eikon (2022k)
Sn	Thomson Reuters Eikon (2022i)	Thomson Reuters Eikon (2022i)
Zn	Thomson Reuters Eikon (2022h)	Thomson Reuters Eikon (2022h)

This table displays the sources of the prices for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn). While U.S. Geological Survey (2013a) only provides prices until 2010, we extend them with prices provided by Thomson Reuters Eikon (2022l), Thomson Reuters Eikon (2022d), Thomson Reuters Eikon (2022a), Thomson Reuters Eikon (2022e), Thomson Reuters Eikon (2022b), Thomson Reuters Eikon (2022c), Thomson Reuters Eikon (2022g), Thomson Reuters Eikon (2022f), Thomson Reuters Eikon (2022k), Thomson Reuters Eikon (2022i) and Thomson Reuters Eikon (2022h) for 2011 to 2020.

[3] In this thesis, we intend to analyze the long-term interdependencies between commodity markets in the period from 1970 to 2019. For this period, only the apparent consumption data of the U.S. is available, provided by U.S. Geological Survey (2020a). Therefore, we approximate the world metal consumption by the U.S. apparent consumption, applying the ratio of reported U.S. GDP and World GDP. However, this approach may underestimate the metal-intense rise of emerging markets in the previous 20 years, as the increased demand for commodities of the emerging markets is hereby not reflected.

[4] Please refer to Section 4.3 for the descriptive statistics of the U.S. GDP and GDP.

[5] In particular, we consider nominal prices in this thesis, as we are interested in the probability distribution of the actual price forecasts to derive the probability of scarcity per commodity.

Within the risk assessment framework, the probability of scarcity, i.e. the probability that a commodity's price exceeds a certain threshold, is estimated. Therefore, we propose, as an alternative to the (MS-)GVAR methodology, the application of logistic regression models, to consider the effect of commodity-specific price determinants, identified by a two-step model selection. Arendt et al. (2020) and Graedel et al. (2012) regard the Herfindal-Hirschman index (HHI), see Rhoades (1993), capturing the global supply concentration of raw materials, as scarcity indicator. Therefore, we consider this index as a further potential, commodity-specific price determinant.[6] It is defined for commodity $i = 1, 2, \ldots, N$ at time $t = 1, 2, \ldots, T$ as:

$$HHI_{i,t} = 10000 \cdot \sum_{r=1}^{R} \left(\frac{prod_{i,t,r}}{\sum_{r=1}^{R} prod_{i,t,r}} \right)^2, \tag{4.1}$$

in which $prod_{i,t} = \sum_{r=1}^{R} prod_{i,t,r}$ represents the production for commodity i at time t, for all production countries $r = 1, 2, \ldots, R$, whereby the production data is the per country breakdown of the commodity-specific worldwide production ($\mathbf{supply}_i$), provided by U.S. Geological Survey (2020b).

To avoid spurious regressions, we apply the augmented Dickey-Fuller (ADF) test, initially proposed in Dickey and Fuller (1979), to each of the commodity-specific variables and calculate logarithmic returns in case of non-stationary data, based on the five percent significance level. All time series were non-stationary at first, which yields in a final dataset of stationary, logarithmic return variables. Moreover, we adjust the monthly variables in the time-varying MS-GVAR framework for seasonality by demeaning.

4.2.1 Descriptive Statistics

Descriptive statistics of the commodity-specific, stationary variables as well as the results of the test statistics of the augmented Dickey-Fuller test for stationarity, according to Dickey and Fuller (1979), and the Shapiro-Wilk (SW) test[7] for normality, according to Shapiro and Wilk (1965), are given in Table 4.5,[8] with corresponding plots displayed in Appendix C.2.3, whereby we also report the results of the monthly data for the industrial metals in the shorter time period of 1995 to 2020. Moreover, we subsequently standardize these logarithmic returns to have mean of zero and standard deviation of one.[9]

Overall, most of the logarithmic return variables are non-normal, caused by leptokurtic or platykurtic data as well as by skewed data. In particular, apart from some supply and demand variables, the annual prices of aluminum, cobalt, copper, dysprosium, neodymium,[10] lead as well as tin are normal. In case of the industrial metals, the monthly prices of aluminum and nickel are also normal, whereas the Shapiro-Wilk test does not confirm normality for the remaining metal prices with data starting in 1995. These mixed findings are in line with the literature. While Chen (2010) detects the copper and lead prices are normally distributed, the commodity indices analyzed in the study of Zhang and Broadstock (2020) as well as most of the considered metal prices in the studies of Aepli et al. (2017), Chen (2010), Le Pen and Sévi

[6]Due to data limitations, we exclude this variable from the analysis of the rare earth metals, dysprosium and neodymium.

[7]We also applied the Jarque-Bera (JB) test for normality, based on the skewness as well as the kurtosis, according to Jarque and Bera (1980), and obtain similar results.

[8]The descriptive statistics of the original, unadjusted variables as well as the corresponding results of the test statistics of the augmented Dickey-Fuller test and the Shapiro-Wilk test are displayed in Table C.2.

[9]For notation reasons, we always refer to the variables with their original names, i.e. supply, demand and price, also for adjusted data.

[10]The prices of the two rare earth metals are available in the period from 2012 to 2019. The Shapiro-Wilk test do not reject the null hypothesis of normality for these prices, however, it is doubtful whether these prices are normally distributed. For an accurate statement, more data for a longer time period have to be analyzed.

(2017), Rossen (2015), Yin and Han (2015), Zhang and Tu (2016) and Zhu et al. (2015) show a non-normal behavior.

Table 4.5: Descriptive statistics of the commodity-specific variables

			Min.	5% Q.	25% Q.	Med.	Mean	75% Q.	95% Q.	Max.	SD	Skew.	Kurt.	ADF	SW
Ag	HHI	a	-0.14	-0.07	-0.02	0.00	0.00	0.02	0.06	0.20	0.05	0.53	4.54	-6.16**	0.89***
	supply	a	-0.07	-0.05	0.00	0.03	0.02	0.04	0.08	0.10	0.04	-0.40	-0.18	-4.31**	0.97
	demand	a	-0.31	-0.25	-0.08	0.02	0.02	0.10	0.31	0.44	0.17	0.31	-0.08	-7.40**	0.98
	price	a	-0.67	-0.28	-0.13	-0.01	0.05	0.15	0.59	0.72	0.27	0.42	0.50	-5.32**	0.96.
Al	HHI	a	-0.20	-0.12	-0.03	-0.00	0.01	0.07	0.18	0.23	0.09	0.20	0.25	-5.37**	0.97
	supply	a	-0.12	-0.05	0.02	0.04	0.04	0.07	0.11	0.12	0.05	-0.79	1.01	-4.43**	0.95*
	supply	m	-0.10	-0.04	-0.01	0.00	0.00	0.01	0.03	0.13	0.02	0.45	4.51	-23.96**	0.94***
	demand	a	-0.31	-0.20	-0.04	0.02	0.02	0.11	0.22	0.27	0.12	-0.29	0.00	-5.93**	0.98
	demand	m	-0.11	-0.06	-0.02	-0.00	-0.00	0.02	0.07	0.13	0.04	0.29	0.51	-25.88**	0.99*
	price	a	-0.43	-0.25	-0.10	0.01	0.02	0.17	0.29	0.52	0.19	0.03	-0.28	-6.24**	0.99
	price	m	-0.18	-0.08	-0.03	-0.00	-0.00	0.03	0.09	0.14	0.05	-0.08	0.35	-17.24**	0.99
Co	HHI	a	-0.42	-0.27	-0.07	0.00	0.01	0.08	0.27	0.57	0.17	0.41	1.81	-8.07**	0.95*
	supply	a	-0.70	-0.25	-0.01	0.05	0.02	0.11	0.22	0.28	0.16	-1.89	6.05	-6.16**	0.85***
	demand	a	-0.50	-0.28	-0.09	0.01	0.02	0.12	0.34	0.75	0.21	0.51	1.99	-7.92**	0.96.
	price	a	-0.78	-0.57	-0.20	0.08	0.04	0.24	0.71	1.47	0.42	0.64	1.24	-5.79**	0.96
Cu	HHI	a	-0.10	-0.06	-0.04	-0.00	0.01	0.04	0.17	0.23	0.07	1.32	1.50	-5.71**	0.87***
	supply	a	-0.05	-0.02	0.00	0.02	0.02	0.05	0.08	0.10	0.03	0.28	-0.09	-4.43**	0.98
	supply	m	-0.09	-0.04	-0.01	-0.00	0.00	0.02	0.04	0.09	0.03	0.17	0.93	-23.11**	0.99*
	demand	a	-0.36	-0.26	-0.03	0.01	0.01	0.08	0.18	0.25	0.12	-0.96	1.81	-7.10**	0.90***
	demand	m	-0.18	-0.07	-0.03	-0.00	-0.00	0.03	0.07	0.16	0.05	-0.02	0.83	-25.01**	0.99
	price	a	-0.53	-0.30	-0.12	-0.01	0.03	0.15	0.44	0.60	0.23	0.27	-0.05	-6.01 **	0.98
	price	m	-0.43	-0.11	-0.04	0.00	-0.00	0.04	0.11	0.25	0.07	-0.68	4.59	-15.59**	0.95***
Dy[11]	**supply**	a	-0.29	-0.15	-0.01	0.05	0.05	0.14	0.28	0.36	0.13	-0.21	0.05	-5.75**	0.98
	demand	a	-0.95	-0.55	-0.22	0.02	0.02	0.16	0.63	0.85	0.36	0.06	0.35	-8.79**	0.98
	price	a	-0.49	-0.41	-0.14	0.07	0.02	0.22	0.36	0.40	0.32	-0.36	-1.54	-6.43**	0.97
In	HHI	a	-0.18	-0.13	-0.07	-0.02	0.02	0.04	0.26	0.59	0.15	2.12	5.23	-5.65**	0.76***
	supply	a	-0.76	-0.18	-0.01	0.04	0.06	0.11	0.44	0.68	0.22	-0.25	3.71	-6.62**	0.89***
	demand	a	-1.43	-0.28	-0.06	0.02	0.05	0.14	0.49	1.27	0.35	-0.41	8.10	-8.23**	0.76***
	price	a	-0.81	-0.55	-0.25	-0.06	0.02	0.23	0.97	1.31	0.45	0.86	0.62	-5.11**	0.94**
Li	HHI	a	-0.25	-0.23	-0.08	0.00	0.01	0.11	0.21	0.36	0.14	0.19	-0.44	-6.54**	0.98
	supply	a	-1.32	-0.21	-0.01	0.05	0.07	0.13	0.34	1.39	0.34	0.29	8.66	-8.49**	0.71***
	demand	a	-0.73	-0.48	-0.06	0.00	0.01	0.09	0.38	0.67	0.25	-0.47	1.64	-6.66**	0.92***
	price	a	-0.24	-0.13	0.00	0.03	0.04	0.07	0.18	0.42	0.10	0.63	3.56	-4.07**	0.89***
Nd	**supply**	a	-0.29	-0.15	-0.01	0.05	0.05	0.14	0.28	0.36	0.13	-0.21	0.05	-5.75**	0.98
	demand	a	-0.95	-0.55	-0.22	0.02	0.02	0.16	0.63	0.85	0.36	0.06	0.35	-8.79**	0.98
	price	a	-0.27	-0.23	-0.09	0.00	0.02	0.09	0.33	0.40	0.22	0.46	-1.08	-3.02**	0.95
Ni	HHI	a	-0.24	-0.16	-0.05	-0.02	-0.01	0.04	0.15	0.32	0.10	0.75	1.68	-5.54**	0.95*
	supply	a	-0.23	-0.15	-0.01	0.04	0.03	0.08	0.14	0.31	0.09	-0.19	1.38	-5.44**	0.96.
	supply	m	-0.24	-0.10	-0.03	0.00	0.00	0.03	0.10	0.19	0.06	-0.33	1.46	-19.94**	0.97***
	demand	a	-0.23	-0.20	-0.06	0.01	0.02	0.10	0.18	0.59	0.14	1.02	3.46	-6.46**	0.92***
	demand	m	-0.25	-0.13	-0.05	0.00	0.00	0.05	0.13	0.27	0.08	0.03	0.43	-22.05**	1.00
	price	a	-0.57	-0.38	-0.15	0.05	0.03	0.18	0.42	1.05	0.28	0.64	1.66	-6.10**	0.96.
	price	m	-0.33	-0.14	-0.07	-0.00	-0.00	0.06	0.15	0.29	0.10	-0.06	0.33	-16.98**	0.99
Pb	HHI	a	-0.40	-0.08	0.00	0.04	0.03	0.06	0.16	0.27	0.11	-1.70	6.56	-5.28**	0.80***
	supply	a	-0.10	-0.09	-0.03	0.00	0.01	0.04	0.10	0.13	0.05	0.22	-0.32	-6.23**	0.98
	supply	m	-0.25	-0.08	-0.03	-0.00	0.00	0.02	0.08	0.32	0.06	0.74	6.55	-23.53**	0.91***
	demand	a	-0.30	-0.10	-0.04	0.02	0.02	0.08	0.16	0.28	0.10	-0.26	0.94	-7.32**	0.98
	demand	m	-0.19	-0.07	-0.02	-0.00	0.00	0.02	0.08	0.19	0.05	0.31	2.05	-26.35**	0.97***
	price	a	-0.36	-0.31	-0.12	0.04	0.04	0.15	0.47	0.70	0.23	0.62	0.39	-6.08**	0.96
	price	m	-0.31	-0.12	-0.05	-0.00	-0.00	0.05	0.14	0.22	0.08	-0.27	1.38	-17.87**	0.98***
Pt	HHI	a	-0.20	-0.07	-0.02	0.01	0.01	0.03	0.13	0.20	0.07	0.21	1.56	-7.91**	0.94*
	supply	a	-0.17	-0.12	-0.01	0.03	0.02	0.05	0.17	0.20	0.08	-0.01	0.90	-7.92**	0.94*
	demand	a	-0.91	-0.31	-0.17	0.06	0.02	0.17	0.37	0.50	0.25	-0.98	2.49	-8.46**	0.92***
	price	a	-0.41	-0.27	-0.07	-0.01	0.04	0.19	0.44	0.53	0.21	0.47	-0.07	-5.55**	0.96.
Sn	HHI	a	-0.18	-0.12	-0.03	0.01	0.00	0.05	0.11	0.15	0.07	-0.36	-0.28	-5.79**	0.98
	supply	a	-0.15	-0.10	-0.05	-0.01	0.00	0.05	0.13	0.15	0.07	0.25	-0.75	-5.52**	0.97
	supply	m	-0.27	-0.12	-0.03	-0.00	0.00	0.04	0.11	0.26	0.07	-0.34	1.97	-22.79**	0.97***
	demand	a	-0.48	-0.22	-0.06	0.01	0.00	0.10	0.22	0.27	0.14	-0.85	1.49	-8.05**	0.95*
	demand	m	-0.27	-0.14	-0.04	-0.00	-0.00	0.05	0.12	0.34	0.08	-0.01	1.72	-27.41**	0.97***
	price	a	-0.57	-0.31	-0.09	0.00	0.03	0.16	0.47	0.57	0.22	0.23	0.61	-6.38**	0.97
	price	m	-0.24	-0.10	-0.04	-0.01	-0.00	0.03	0.11	0.21	0.06	0.05	1.06	-15.85**	0.98***

[11] Due to data limitations, we exclude the HHI from the analysis of the rare earth metals, dysprosium and neodymium.

Descriptive statistics of the commodity-specific variables

			Min.	5% Q.	25% Q.	Med.	Mean	75% Q.	95% Q.	Max.	SD	Skew.	Kurt.	ADF	SW
Zn	HHI	a	-0.12	-0.09	-0.00	0.02	0.03	0.06	0.14	0.20	0.07	0.09	0.24	-6.89**	0.98
	supply	a	-0.06	-0.03	0.00	0.02	0.02	0.04	0.07	0.10	0.03	0.03	0.22	-5.38**	0.99
	supply	m	-0.24	-0.07	-0.02	-0.00	0.00	0.02	0.07	0.17	0.04	-0.43	4.21	-20.83**	0.95***
	demand	a	-0.31	-0.17	-0.05	0.02	0.01	0.09	0.16	0.34	0.11	-0.10	0.65	-5.94**	0.98
	demand	m	-0.27	-0.08	-0.03	0.00	0.00	0.03	0.08	0.23	0.05	-0.28	3.35	-25.53**	0.97***
	price	a	-0.55	-0.29	-0.12	0.03	0.04	0.19	0.40	0.86	0.26	0.77	1.62	-5.46**	0.95*
	price	m	-0.41	-0.11	-0.05	0.00	-0.00	0.05	0.11	0.24	0.07	-0.53	2.83	-17.13**	0.97***

This table displays the descriptive statistics (minimum (Min.), 5% quantile (5% Q.), 25% quantile (25% Q.), 75% quantile (75% Q.), 95% quantile (95% Q.), median (Med.), mean (Mean), maximum (Max.), standard deviation (SD), skewness (Skew.) and excess kurtosis (Kurt.)) of the stationary, commodity-specific variables HHI, supply (**supply**), demand (**demand**) and price (**price**) for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), as well as the results of the test statistics of the augmented Dickey-Fuller (ADF) test and the Shapiro-Wilk (SW) test with corresponding significance level (0.1% (***), 1% (**), 5% (*) and 10% (.)). Hereby, we report the descriptive statistics for the considered time period from 1970 to 2019 for the annual (a) analysis or from 1995 to 2020 in case of the monthly (m) analysis.

The non-normality can be partly explained by the excess kurtosis. In particular, the prices of silver, cobalt, indium, lithium, nickel, zinc as well as the monthly prices of all industrial metals are leptokurtic, indicating fat tails, whereas the excess kurtosis of the annual prices of aluminum, copper, and platinum is slightly negative. Deaton and Laroque (1992), Le Pen and Sévi (2017), Rossen (2015) as well as Yin and Han (2015) confirm the leptokurtic behavior of individual metal prices, inter alia for silver, aluminum, cobalt, copper, nickel, platinum, tin and zinc. In addition, Dutta (2018), Gargano and Timmermann (2014) as well as Zhang and Broadstock (2020) detect fat tails in their considered commodity price indices, indicating commodity prices are generally leptokurtic.

Further, our results regarding the skewness are ambiguous, which is in line with the mixed evidence in the literature. In general, the metal prices are right skewed in our annual dataset from 1970 to 2019. However, the monthly prices of copper, lead and zinc follow a left skewed distribution in the time period from 1995 to 2020. While the long-term analyses of Deaton and Laroque (1992) and Rossen (2015), considering data from the beginning of the 20th century, confirm metal prices are right skewed, inter alia for copper and tin, as well as for silver, aluminum, cobalt, nickel, lead, platinum and zinc, the aluminum, copper and platinum prices follow a negatively skewed distribution in the studies of Le Pen and Sévi (2017), Yin and Han (2015) and Zhang and Tu (2016), investigating monthly data from the end of the 20th century until 2015, emphasizing the price structure changed over time, which underlines our mixed findings.

Moreover, we detect the highest standard deviation for the logarithmic returns of prices, compared to the logarithmic returns of the remaining commodity-specific variables. Hereby, the annual prices show more fluctuations than the monthly prices, similar to Pincheira-Brown and Hardy (2019). In particular, the minor as well as rare earth metals, cobalt, dysprosium and indium show the highest volatility, underlining the evidence of Redlinger and Eggert (2016), who reveal the annual prices of by-products fluctuate more than prices of main products. The few fluctuations in the commodity-specific supply and demand variables are rather intuitive, since demand and, in particular, supply as well as the concentration of producing countries are more inelastic variables in the short-term, even at an annual frequency.

Further, we observe a moderate positive mean and median for most of the logarithmic returns of the commodity prices, indicating the rise in commodity prices,[12] over the entire period starting in 1970, in accordance with the studies of Chen (2010), Gargano and Timmermann (2014), Pincheira-Brown and Hardy (2019), Yin and Han (2015) and Zhang and Tu (2016). In contrast,

[12]Please also refer to the level plots of the commodity prices, displayed in Figures C.2, C.3, C.4, C.5, C.6, C.7, C.8, C.9, C.10, C.11, C.12, C.13 and C.14.

the mean of the logarithmic returns of the monthly prices since 1995 is slightly negative.

In general, the metal prices increased rapidly in the early 2000s, caused by the growing demand of emerging markets, especially of China, as well as the entrance of institutional investors into commodity futures markets. Hereby, Akram (2009), Carter et al. (2011) and Gargano and Timmermann (2014) as well as Frankel and Rose (2010), Liberda (2017) and Lombardi et al. (2012) confirm the commodity price boom around 2008 for commodity indices as well as for individual commodity prices, respectively. In case of copper, Buncic and Moretto (2015) observe the price increased since 2003 with a peak not only in the year 2008 but also in 2011, followed by gradually declining copper prices thereafter, see also Figure C.5. While the copper, lead, platinum and tin (nickel) prices quadruple (increases sevenfold) from the beginning of 2000 to 2010, the aluminum price rose the least, according to Lombardi et al. (2012) as well as Figure C.3, where only a moderate trend can be observed. Despite the increasing trend in the production of commodities, the growth in demand, particularly the demand of emerging economies, outweighs the increase in supply, resulting in increasing prices. Hereby, the remarkable trend in lithium's price can be explained by the increasing demand for lithium-ion batteries, especially in more recent times. However, the slowdown in Chinese demand for commodities, combined with the sharp oil price drop in 2014 caused declining commodity prices in the mid-2010s, see World Bank (2015). While most of the prices decreased, supply concerns as well as continuously falling inventories lead to an increase in aluminum's, lead's, and zinc's price.

Overall, the supply and demand of each commodity increased over the considered time period. In particular, the exceptional growth rates in indium supply and demand, see Figure C.7, originate from the high demand for indium tin oxide, used for electrical conductive purposes in a variety of flat-panel displays as well as in alloys and solders, compounds, electrical components and semiconductors, see U.S. Geological Survey (2019). Moreover, the continuous expansion in lithium supply, see Figure C.8, is in response to the increased lithium demand for battery applications, in portable electronic devices, electric tools, electric vehicles, and grid storage applications, see U.S. Geological Survey (2019). However, due to declining prices, the world production of lithium slightly decreased in 2019, see U.S. Geological Survey (2020c).

While the world production volume increases in response to an increased demand for most of the considered metals, cobalt's supply depends on the production volume of copper and nickel, as it is mostly mined as a by-product. Therefore, the world production of cobalt decreased in 2016, see Figure C.4, due to lower production from nickel operations, despite the high demand for cobalt in rechargeable battery and aerospace industries, see U.S. Geological Survey (2017). Regarding the global supply concentration, measured by the Herfindal-Hirschman index (HHI), cobalt as well as aluminum and lead are outstanding, see Figure C.4, Figure C.3 and Figure C.11, since around half of the global world production of cobalt is mined in Congo, whereas aluminum and lead are mainly mined in China, see U.S. Geological Survey (2020b), indicating an amplified market concentration and associated supply risks.

4.3 Determinants of Commodity Prices

Apart from the commodity-specific supply and demand variables, there are numerous potential influencing factors of commodity prices, see Section 2.2. In case of the risk assessment framework, we also propose a logistic regression model combined with a commodity-specific model selection to calculate the probability of scarcity per commodity, based on the most influencing determinants of the commodity prices. Therefore, we choose the determinants from a wide range of possible factors, which we classify into five dimensions: Macroeconomic, demographic, capital market driven as well as supply- and demand-sided variables. Although this thesis investigates the risk of several German energy transformation pathways, global factors are considered as potential price influential factors, instead of their German counterpart. Hereby, we assume global factors would affect the commodity prices more than changes in the German economy. As some variables are not available on a global scope for the considered time period from 1970 to 2019, the global behavior is then approximated by U.S. based data. However, this approach may underestimate the metal-intense rise of emerging markets in the recent years, as the impact of the emerging markets are hereby not reflected. An overview of the determinants, their data availability and sources is presented in Table 4.6.

Table 4.6: Overview of the price determinants

Abbr.	**Factor**	**Freq.**	**Data per.**	**Data source**
U.S. IP	U.S. industrial production, unadjusted, Index 2017=100	Monthly / Annually	1935 - 2021	Board of Governors of the Federal Reserve System (U.S.) (2022b) and Board of Governors of the Federal Reserve System (U.S.) (2022c)
IP	world industrial production, unadjusted, in U.S. dollar	Monthly	1991 - 2021	The World Bank (2022d)
U.S. GDP	real U.S. gross domestic product, in billions of chained 2012 dollars, seasonally adjusted annual rate	Annually	1947 - 2021	U.S. Bureau of Economic Analysis (2022)
GDP	world gross domestic product, in current U.S. dollar	Annually	1969 - 2021	The World Bank (2022a)
GDPc	world gross domestic product per capita, in current U.S. dollar	Annually	1960 - 2021	The World Bank (2022b)
FX	U.S. dollar index	Monthly / Annually	1967 - 2021	ICE Futures U.S. (2022a) and ICE Futures U.S. (2022b)
FFR	Federal Funds Effective Rate, in %, not seasonally adjusted	Monthly / Annually	1955 - 2021	Board of Governors of the Federal Reserve System (U.S.) (2022a)
SIR	3-month U.S. Treasury rate, in % per annum	Monthly / Annually	1965 - 2021	Organization for Economic Co-operation and Development (OECD) (2022b) and Organization for Economic Co-operation and Development (OECD) (2022c)
LIR	10-year U.S. Treasury rate, in %	Annually	1953 - 2021	Board of Governors of the Federal Reserve System (U.S.) (2022d)
MB	U.S. monetary base, in millions of U.S. dollar	Monthly	1959 - 2020	Federal Reserve Bank of St. Louis (2022a)
CPI	U.S. consumer price index, in %, not seasonally adjusted	Annually	1960 - 2021	The World Bank (2022c)
EMP	U.S. employment, representing the % of working age population	Annually	1955 - 2020	Organization for Economic Co-operation and Development (OECD) (2022a)
POP	world population	Annually	1960 - 2021	The World Bank (2022e)
MSCI	MSCI world stock index, annual index level, closing price in basis points	Daily	1969 - 2021	MSCI (2022)
SPX	Standard & Poor's 500 index	Annually	1963 - 2020	Standard & Poor's (2022)
OIL	West Texas Intermediate spot crude oil price, in U.S. dollar per barrel	Annually	1946 - 2021	Federal Reserve Bank of St. Louis (2022b)
ND	global natural disasters	Annually	1970 - 2021	Guha-Sapir (2021)
KOF	KOF globalization index	Annually	1970 - 2019	Gygli et al. (2021)
HHI	Herfindal-Hirschman index (mining countries)	Annually	1970 - 2019	U.S. Geological Survey (2020b)
Supply	world production of each commodity	Annually	1969 - 2021	The World Bureau of Metal Statistics (2021) and U.S. Geological Survey (2020b)
Demand	estimated world apparent consumption per commodity, based on data of the U.S. apparent consumption as well as the ratio of reported U.S. GDP and World GDP	Annually	1969 - 2021	The World Bureau of Metal Statistics (2021) and U.S. Geological Survey (2020a)
Price	price per commodity in U.S. dollar per ton	Monthly / Annually	1970 - 2021	see Table 4.4

This table displays the variable names of the input factors U.S. industrial production (U.S. IP), world industrial production (IP), U.S. gross domestic product (U.S. GDP), world gross domestic product (GDP), world gross domestic product per capita (GDPc), U.S. dollar index (FX), Federal Funds Effective Rate (FFR), 3-month U.S. Treasury rate (SIR), 10-year U.S. Treasury rate (LIR), U.S. monetary base (MB), U.S. consumer price index (CPI), U.S. employment (EMP), world population (POP), MSCI world stock index (MSCI), Standard & Poor's 500 index (SPX), West Texas Intermediate spot crude oil price (OIL), global natural disasters (ND) and KOF globalization index (KOF), and the commodity-specific variables, the corresponding data frequency (Freq.), data period (Data per.) as well as data source.

In general, the economic state affects the commodity markets. Therefore, we include the U.S.

industrial production (U.S. IP),[13] drawn from Board of Governors of the Federal Reserve System (U.S.) (2022c), as well as the world gross domestic product (GDP), drawn from The World Bank (2022a), as proxies for the economic activity, see Kagraoka (2016) and Robinson (2019). Moreover, we add the world gross domestic product per capita (GDPc), drawn from The World Bank (2022b), which is expected to shift the demand curve over time, see Cuddington and Zellou (2013) and Helbling et al. (2008), and the U.S. dollar index (FX), drawn from ICE Futures U.S. (2022b), as proxy for the exchange rate, according to Baffes and Savescu (2014). Further, we consider as monetary policy variables the Federal Funds Effective Rate (FFR), drawn from Board of Governors of the Federal Reserve System (U.S.) (2022a), the 3-month U.S. Treasury rate (SIR), drawn from Organization for Economic Co-operation and Development (OECD) (2022c) as short-term interest rates as well as the 10-year U.S. Treasury rate (LIR), drawn from Board of Governors of the Federal Reserve System (U.S.) (2022d), as long-term interest rate, see Siami-Namini (2021). In addition, we take into account the U.S. monetary base (MB), drawn from Federal Reserve Bank of St. Louis (2022a), which is also interpreted as a liquidity measure, according to Guzmán and Silva (2018), and the U.S. consumer price index (CPI), drawn from The World Bank (2022c), to account for the inflation rate, see Akram (2009).

Furthermore, demographic factors, covering indicators of societal evolution including socio-economic progress and population growth, also influence commodity prices. Following Apergis et al. (2014), we take into consideration the U.S. employment (EMP), drawn from Organization for Economic Co-operation and Development (OECD) (2022a), as well as the world population (POP), drawn from The World Bank (2022e). Additionally, we account for spillover effects from the stock market to commodity markets and reflect the capital market driven determinants by the MSCI world stock index (MSCI), drawn from MSCI (2022), as well as the Standard & Poor's 500 index (SPX), drawn from Standard & Poor's (2022), according to Kagraoka (2016).

In addition to the actual world production as well as the indicator of global supply concentration (HHI), we add the West Texas Intermediate spot crude oil price (OIL), drawn from Federal Reserve Bank of St. Louis (2022b), which is a proxy for energy costs, according to Baffes and Savescu (2014). Moreover, we include the global natural disasters (ND), considering the occurrence and effects of disasters, drawn from Guha-Sapir (2021), and the KOF globalization index (KOF), measuring the economic, social and political dimensions of globalization, drawn from Gygli et al. (2021), as further determinants of the supply side of markets.

In case of the (MS-)GVAR model, we add exogenous variables to account for the common effects of macroeconomic factors on commodity markets. Due to data limitations, we only consider proxies for the global demand, the exchange rate[14] and the monetary policy, which affect commodity prices, see for example Akram (2009).[15] In particular, we include the world gross domestic product (GDP) or world industrial production (IP), drawn from The World Bank (2022a) or The World Bank (2022d) for monthly values, as proxy for economic activity,[16] the U.S. dollar index (FX), drawn from ICE Futures U.S. (2022b) or ICE Futures U.S. (2022a) for monthly values, as proxy for the exchange rate, as well as the Federal Funds Effective Rate (FFR), drawn from Board of Governors of the Federal Reserve System (U.S.) (2022a), as

[13] As data for the world industrial production (IP) is only available since 1990, we use the U.S. industrial production (U.S. IP) as proxy for the global behavior.

[14] The findings in the literature indicate the exchange rate is a determinant of commodity prices, however, several studies emphasize its predictive power, see for example Chen et al. (2010).

[15] The oil price, reflecting the energy costs of the production of the commodities, might also influence the prices, however, due to data limitations we exclude this variable from the (MS-)GVAR models, as the supply side is already represented by the commodity-specific production volume and the evidence of the impact of oil prices on commodity markets is mixed in the literature, see Section 2.2.2.4. Actually, we exemplary included the price of oil as further exogenous variable in the GVAR model, but this did not provide added value, therefore, we excluded this variable for our analysis.

[16] As the world gross domestic product is only reported quarterly, we include the world industrial production, in the monthly analysis of the MS-GVAR model.

interest rate.

For a long-term analysis of the scarcity risk as well as of the commodity market structure, we base the first part of this thesis on annual data in the period from 1970 to 2019. In case of the time-varying MS-GVAR model, we use monthly data from 1995 to 2020, due to the large number of parameters. In line with the commodity-specific variables, we take the respective annual average or the end of month value in case of higher frequency determinants. Further, we apply the augmented Dickey-Fuller test for stationarity and calculate logarithmic returns[17] in case of non-stationary variables. Moreover, we adjust the monthly variables in the time-varying MS-GVAR framework for seasonality by demeaning.

4.3.1 Descriptive Statistics

The descriptive statistics of the price influencing variables as well as the test statistics of the augmented Dickey-Fuller test and the Shapiro-Wilk test, based on data in the considered time period, are given in Table 4.7,[18] with corresponding plots displayed in Appendix C.3. Finally, we standardize the data for the (MS-)GVAR model to guarantee interpretability and comparability.

Overall, several determinants are non-normal regarding the Shapiro-Wilk test, only the logarithmic returns of the world gross domestic product as well as the U.S. gross domestic product, the world gross domestic product per capita, the U.S. dollar index, the long-term interest rate (10-year U.S. Treasury rate), the U.S. employment, and the global natural disasters show a normal behavior. While the Federal Funds Effective Rate, the annual U.S. short-term interest rate (3-month U.S. Treasury rate), the U.S. consumer price index, the world population, the KOF globalization index as well as the West Texas Intermediate spot crude oil price show lots of variation, the standard deviation of the remaining variables is comparably small. Hereby, the price of oil fluctuates more than most of the metal prices, displayed in Section 4.2, in accordance to Fernandez (2015a) and Sari et al. (2010), as well as more than the stock market indices MSCI world stock index and Standard & Poor's 500 index, in accordance to Buncic and Moretto (2015), Liberda (2017) as well as Pierdzioch et al. (2016).

Regarding the reasons for the observed non-normality, the U.S. industrial production, the monthly world industrial production, the U.S. gross domestic product, the annual U.S. dollar index, the (monthly) Federal Funds Effective Rate, the U.S. short- and long-term interest rate (3-month U.S. Treasury rate, 10-year U.S. Treasury rate), the U.S. employment as well as the two stock indices MSCI world stock index and Standard & Poor's 500 index follow a negatively skewed distribution, in line with Buncic and Moretto (2015), whereas the U.S. consumer price index, the U.S. monetary base, the world population, the West Texas Intermediate spot crude oil price, global natural disasters, and KOF globalization index are right skewed. Moreover, the U.S. industrial production, the world industrial production, the Federal Funds Effective Rate, the U.S. short-term interest rate (3-month U.S. Treasury rate), the U.S. monetary base, the U.S. consumer price index, the world population, as well as the KOF globalization index are leptokurtic, implying fat tails.

Table 4.7: Descriptive statistics of the price determinants

		Min.	5% Q.	25% Q.	Med.	Mean	75% Q.	95% Q.	Max.	SD	Skew.	Kurt.	ADF	SW
U.S. IP	a	-0.12	-0.05	0.00	0.03	0.02	0.05	0.08	0.09	0.04	-1.05	1.86	-4.85**	0.93**
IP	m	-0.13	-0.03	-0.01	-0.00	-0.00	0.01	0.03	0.07	0.02	-0.61	4.87	-25.66**	0.96***
U.S. GDP	a	-0.03	-0.01	0.02	0.03	0.03	0.04	0.06	0.08	0.02	-0.25	0.24	-2.69**	0.98

[17]We have to calculate returns twice for CPI, KOF as well as POP to obtain stationary variables.

[18]The descriptive statistics of the original, unadjusted variables as well as the corresponding results of the test statistics of the augmented Dickey-Fuller test and the Shapiro-Wilk test are displayed in Table C.3.

Descriptive statistics of the price determinants

		Min.	5% Q.	25% Q.	Med.	Mean	75% Q.	95% Q.	Max.	SD	Skew.	Kurt.	ADF	SW
GDP	a	-0.06	-0.01	0.02	0.06	0.07	0.11	0.16	0.20	0.06	0.03	-0.69	-2.37**	0.98
GDPc	a	-0.07	-0.02	0.01	0.05	0.05	0.10	0.14	0.18	0.06	0.00	-0.70	-2.76**	0.98
FX	a	-0.24	-0.13	-0.05	-0.00	-0.00	0.05	0.11	0.17	0.08	-0.35	0.56	-4.94**	0.99
FX	m	-0.07	-0.04	-0.01	0.00	-0.00	0.01	0.04	0.07	0.02	0.02	0.48	-16.63**	0.99
FFR	a	-2.49	-0.73	-0.26	0.01	-0.02	0.22	0.78	1.09	0.55	-1.59	6.40	-4.61**	0.87***
FFR	m	-2.48	-0.18	-0.03	0.01	0.00	0.06	0.19	0.69	0.20	-6.68	75.99	-12.10**	0.54***
SIR	a	-1.67	-0.58	-0.29	-0.01	-0.02	0.20	0.62	1.04	0.45	-0.60	2.12	-3.90**	0.96.
LIR	a	-0.44	-0.27	-0.14	-0.02	-0.02	0.08	0.21	0.27	0.15	-0.28	-0.26	-6.51**	0.98
MB	a	-0.10	-0.02	0.05	0.06	0.07	0.09	0.20	0.59	0.09	3.30	16.17	-3.34**	0.66***
CPI	a	-5.61	-0.72	-0.25	0.00	0.10	0.25	0.91	9.63	1.64	3.04	22.48	-7.01**	0.47***
EMP	a	-0.03	-0.01	0.00	0.01	0.01	0.01	0.02	0.03	0.01	-0.40	0.38	-3.21**	0.98
POP	a	-2.94	-1.89	-0.44	-0.06	-0.01	0.32	1.80	5.72	1.34	1.57	5.72	-7.49**	0.85***
MSCI	a	-0.33	-0.21	0.00	0.09	0.06	0.15	0.25	0.39	0.14	-0.64	0.56	-4.86**	0.94*
SPX	a	-0.27	-0.19	0.00	0.09	0.07	0.16	0.23	0.29	0.13	-0.86	0.28	-4.45**	0.93**
OIL	a	-0.65	-0.42	-0.07	0.05	0.06	0.20	0.43	0.99	0.27	0.15	2.25	-6.34**	0.94**
ND	a	-0.34	-0.18	-0.09	0.03	0.03	0.13	0.29	0.48	0.17	0.34	-0.14	-8.76**	0.99
KOF[19]	a	-15.03	-7.20	-0.82	-0.26	-0.44	0.15	4.40	16.71	4.05	0.26	7.94	-6.69**	0.69***

This table displays the descriptive statistics (minimum (Min.), 5% quantile (5% Q.), 25% quantile (25% Q.), 75% quantile (75% Q.), 95% quantile (95% Q.), median (Med.), mean (Mean), maximum (Max.), standard deviation (SD), skewness (Skew.) and excess kurtosis (Kurt.)) of the stationary, determinants U.S. industrial production (U.S. IP), world industrial production (IP), U.S. gross domestic product (U.S. GDP), world gross domestic product (GDP), world gross domestic product per capita (GDPc), U.S. dollar index (FX), Federal Funds Effective Rate (FFR), 3-month U.S. Treasury rate (SIR), 10-year U.S. Treasury rate (LIR), U.S. monetary base (MB), U.S. consumer price index (CPI), U.S. employment (EMP), world population (POP), MSCI world stock index (MSCI), Standard & Poor's 500 index (SPX), West Texas Intermediate spot crude oil price (OIL), global natural disasters (ND) and KOF globalization index (KOF), as well as the results of the test statistics of the augmented Dickey-Fuller (ADF) test and the Shapiro-Wilk (SW) test with corresponding significance level (0.1% (***), 1% (**), 5% (*) and 10% (.)). Hereby, we report the descriptive statistics for the considered time period from 1970 to 2019 for the annual (a) analysis or from 1995 to 2020 in case of the monthly (m) analysis.

4.4 Dependencies between Commodity Markets

In the scarcity risk assessment of four transformation pathways of the German Energiewende, we take into account the commodity market structure. Therefore, we propose and apply the (MS-)GVAR framework, reflecting the impact of fundamentals on - as well as the spillover effects between - commodity prices, while controlling for the effect of the economy on commodity markets. In contrast to Pesaran et al. (2004), who introduce the GVAR model to link individual economies via trade weights, representing the shares of exports and imports between countries, we propose several weight matrices to connect the individual commodity markets to the global model. As the simultaneous impact of the economy on commodity markets is represented via the exogenous variables, we aim to reflect the individual relationships between the commodities. In particular, the metals are mined or used together, so their production or consumption, and hence their prices, are related. Moreover, the increased interest in commodities as financial assets and the accompanying increase in index investment in commodity markets since 2004 provide another source of common patterns in prices. Therefore, the weight matrices in this thesis include information on the co-production, co-consumption and co-trading relationships among commodities, reflecting possible linkages in commodity markets.

[19] As noted above, we calculate returns twice instead of logarithmic returns in case of the CPI to ensure stationarity. Further, we calculate returns twice for KOF as well as three times POP to obtain stationary variables.

4.4.1 Overview of Possible Linkages between Commodity Markets

First of all, metals are produced together. For instance, 70% of the lead production is derived from mixed Lead-Zinc ores, according to Nassar et al. (2015) and Shammugam et al. (2019). Further, several metals, like cobalt as well as indium, are extracted as companion metals. In particular, Nassar et al. (2015) state the production of by-products is unable to respond to demand increases in the short-term, as these metals are financially dependent on other metals for recovery. Hereby, the production volume of cobalt, which is mostly mined as a by-product of copper and nickel, declined in 2016 caused by a reduced nickel supply, despite the high demand for cobalt in rechargeable battery and aerospace industries, see U.S. Geological Survey (2017). Moreover, supply constraints due to e.g. natural disasters, political instability, or trade restrictions affect the global supply of all metals extracted in the same mining project or region.

Dependencies between metals also occur on the demand side of markets. In particular, there are substitution, but also co-consumption links. Baffes et al. (2020) state copper demand is highly correlated to aluminum prices, which originates from the substitutability of copper by aluminum in certain industries, electricity for example, whereas the use statistics of aluminum and copper show their co-consumption inter alia in AlSi9Cu3 alloys, used for various automotive applications, see Zapp et al. (2002). Additionally, renewable energy technologies, which are the key element of the energy transition, require different raw materials which is why the demand for several metals will significantly increase over the next decades, according to Valero et al. (2018).

However, the common trend in prices, observed during the last decades, can no longer be explained via their co-production and co-consumption links only, as the co-movement in commodity prices has significantly increased since the financialization of commodities and the associated increase in index investments. In general, commodity prices tend to move in a synchronized way, as they are simultaneously influenced by macroeconomic determinants. However, Basak and Pavlova (2016) reveal the increasing investments of index funds in commodity markets should further elevate the co-movement. Although Hamilton and Wu (2015) find no direct effect of futures traders positions on prices, the study of Tang and Xiong (2012) empirically detects an increase in the co-movement of commodities, starting in the year 2004, due to the financialization, which is stronger for indexed commodities.

In our framework, we aim to account for the possible linkages between the commodity markets. Therefore, we connect the individual commodity markets via matrices including information on the co-production, co-consumption and co-trading relations between commodities in the exemplary application of the (MS-)GVAR framework on the industrial metals. Moreover, we use the dependencies inferred from the transformation paths to directly reflect the relations in the context of the energy transition in case of the risk assessment framework.

4.4.2 Co-Production

First, we use information on the common supply concentration of the commodities.[20] Hereby, we aggregate the common production per country as a measure of co-production via:

$$\hat{w}_{i,\tilde{\imath}} = \sum_{r=1}^{R} prod_{r,i} \cdot prod_{r,\tilde{\imath}} \tag{4.2}$$

[20]Actually, we consider the common mining countries instead of actual co-production data, due to data limitations, where we do not differentiate whether the metals originate from the same mine. However, potential supply risks, geological risks as well as political risks are similar in the same region, therefore, the consideration of the common mining countries approximates the common supply relation between commodities. However, we use the umbrella term co-production for notation reasons.

with $\hat{w}_{i,\tilde{\iota}}$ denoting the relation between commodity i and commodity $\tilde{\iota}$, for $i, \tilde{\iota} = 1, 2, \ldots, N, i \neq \tilde{\iota}$ and $\hat{w}_{i,i} = 0$. Moreover, $prod_{i,r}$ denotes the country-specific share of annual world production of country $r = 1, 2, \ldots, R$ for commodity i, whereby the production data is the averaged, per-country breakdown of the commodity-specific worldwide production ($\mathbf{supply}_i$), in the period from 2010 to 2019, provided by U.S. Geological Survey (2020b). Overall, this dimension shows a comparably strong link between aluminum, lead, tin, and zinc, while copper and nickel are connected less, see Table 4.8.

Table 4.8: Co-production of the commodities

	Al	Cu	Ni	Pb	Sn	Zn
Al	0.00	0.05	0.04	0.24	0.17	0.19
Cu	0.05	0.00	0.02	0.06	0.05	0.06
Ni	0.04	0.02	0.00	0.04	0.06	0.03
Pb	0.24	0.06	0.04	0.00	0.16	0.19
Sn	0.17	0.05	0.06	0.16	0.00	0.13
Zn	0.19	0.06	0.03	0.19	0.13	0.00

This table displays the co-production, based on the common supply concentration, of aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn).

4.4.3 Co-Consumption

Besides the co-production, we analyze the sectors in which industrial metals are co-consumed, approximating the economy by the five industries automotive/ transportation, chemistry/ pharmaceutics, electrics, construction and mechanical engineering, which in summary account for up to 90% of the worldwide demand of the considered commodities. Therefore, we use the worldwide consumption data from Brandtzæg (2018) and Leder (2020), hereby implying that the proportion between the industries remains unchanged over the investigated time period.[21]

The consumption of aluminum per industry, provided by Brandtzæg (2018), is given in Table 4.9. In order to join the different usages of the commodities, we assume the consumption of aluminum in the categories foil, packaging, consumer goods and other is negligible. The corresponding consumption per industry, displayed in Table 4.16, is used as input data for the calculation of the weights. We note aluminum is mainly used in the automotive sector as well as for construction, whereas there is no consumption in the chemical industry.

The worldwide consumption per industry of copper, according to Leder (2020), is displayed in Table 4.10. We assign these branches as shown in Table 4.15 to the five industries and obtain the corresponding weights. As the entry trade and other can not be matched to the five considered industries in a reasonable way, we assume their proportion is zero. Similar to aluminum, copper is not used in the sector chemistry/ pharmaceutics, while its majority is utilized for electrics purposes.

As displayed in Table 4.11, more than half of the worldwide amount of nickel is used for stainless steel (automotive, construction, engineering), see Leder (2020), which we equally attribute to the industries automotive/ transportation, construction and mechanical engineering, see Table 4.15. We further assume the consumption of nickel in the industries nickel alloys, plating, steel refiner, foundries and other is zero to join the different usages of the commodities.

[21] Reliable data on the consumption per industry of the commodities is rare and only reflects a snapshot in time. However, for the exemplary introduction of the (MS-)GVAR framework on commodity markets as well as the data limitations, we assume the proportion between the industries remains unchanged over the investigated time period. Within the risk assessment framework, we are able to reflect the co-consumption of the commodities via the correlation of the annual resource requirement per possible transformation path, which reflects the relation between the metals in the period from 2020 to 2050 in the context of the German Energiewende, see Section 4.4.4.

Table 4.9: Aluminum consumption

Industry	%
Automotive/ transportation	0.26
Construction industry	0.24
Mechanical and plant engineering	0.11
Electrical engineering	0.11
Foil	0.08
Packaging	0.08
Consumer goods	0.06
Other	0.06

This table displays the proportion of aluminum (Al) consumption per industry.

Table 4.10: Copper consumption

Industry	%
Cables and electrics	0.57
Construction industry	0.15
Automotive	0.09
Mechanical engineering	0.08
Trade	0.05
Other	0.06

This table displays the proportion of copper (Cu) consumption per industry.

Table 4.11: Nickel consumption

Industry	%
Stainless steel (automotive, construction, mechanical engineering)	0.57
Electrical engineering	0.03
Nickel alloys	0.13
Plating	0.11
Steel refiner	0.09
Foundries	0.06
Other	0.01

This table displays the proportion of nickel (Ni) consumption per industry.

Table 4.12: Lead consumption

Industry	%
Electrical engineering (lead-acid batteries)	0.74
Construction (roof, facade)	0.06
Plant construction (radiation protection, anodes)	0.06
Chemistry (pigments)	0.05
Other (alloys, cable sheath, glass)	0.09

This table displays the proportion of lead (Pb) consumption per industry.

Table 4.13: Tin consumption

Industry	%
Electronics industry (solder)	0.52
Chemical industry	0.15
Brass bronze	0.06
Float glass	0.02
Packaging (tinplate)	0.16
Other	0.09

This table displays the proportion of tin (Sn) consumption per industry.

Table 4.14: Zinc consumption

Industry	%
Automotive engineering (galvanizing)	0.50
Construction (zinc, brass products)	0.23
Chemistry / pharmaceutics	0.06
Other (zinc casting alloys)	0.21

This table displays the proportion of zinc (Zn) consumption per industry.

Table 4.15: Commodity - industry mapping

	Automotive / transportation	Chemistry/ pharmaceutics	Electrics	Construction	Mechanical engineering
Al	automotive / transportation	-	electrical engineering	building industry	mechanical and plant engineering
Cu	automotive	-	cables and electrics	building industry	mechanical engineering
Ni	stainless steel (automotive, construction, mechanical engineering)	-	electrical engineering	stainless steel (automotive, construction, mechanical engineering)	stainless steel (automotive, construction, mechanical engineering)
Pb	-	chemistry	electrical engineering & other	construction	plant construction
Sn	-	chemical industry	electronics industry (solder)	-	-
Zn	automotive engineering	chemistry / pharmaceutics	-	construction	

This table displays the mapping of the industry data for aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) to the five industry sectors automotive/ transportation, chemistry/ pharmaceutics, electrics, construction and mechanical engineering.

Table 4.12 displays the consumption of lead per industry, according to Leder (2020). We are able to assign all of the consuming branches to the five industries considered, as displayed in Table

4.15. The majority of lead is used in the electrics sector,[22] as well as in construction, mechanical engineering and chemistry/ pharmaceutics, but none in the automotive/ transportation industry.

In Table 4.13, we display the demand of tin per industry, where we neglect the consumption for brass bronze, float glass, packaging (tinplate) as well as other, since there is no direct assignment to the five considered industries. Hereby, the main use of tin is in electrics, followed by the chemistry/ pharmaceutics sector.

Finally, we consider the consumption of zinc by the industries, displayed in Table 4.14. Since the information about other (zinc casting alloys) is rather unspecific, we neglect it for further calculations. The assigned branches are displayed in Table 4.15. Hereby, zinc is mainly used in the automotive/ transportation sector, whereas there is no significant application in electrics and mechanical engineering, according to Leder (2020). However, there is little consumption in the industries chemistry/ pharmaceutics and construction.

Table 4.16: Consumption of the commodities

Industry	Al	Cu	Ni	Pb	Sn	Zn
Automotive/ transportation	0.36	0.10	0.32	0.00	0.00	0.63
Chemistry/ pharmaceutics	0.00	0.00	0.00	0.05	0.22	0.08
Electrics	0.15	0.64	0.05	0.83	0.78	0.00
Construction	0.33	0.17	0.32	0.06	0.00	0.29
Mechanical engineering	0.15	0.09	0.32	0.06	0.00	0.00

This table displays the aggregated consumption of aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) in the industry sectors automotive/ transportation, chemistry/ pharmaceutics, electrics, construction and mechanical engineering.

Table 4.16 displays the proportion of consumption per commodity and industry. We aggregate these industry-specific values to a demand-sided information matrix, see Table 4.17, via the following formula:

$$\hat{w}_{i,\tilde{\imath}} = \sum_h ind_{h,i} \cdot ind_{h,\tilde{\imath}}, \tag{4.3}$$

where $\hat{w}_{i,\tilde{\imath}}$ denotes the estimated weight between commodity i and commodity $\tilde{\imath}$, for $i, \tilde{\imath} = 1, 2, \ldots, N$, $i \neq \tilde{\imath}$ and $\hat{w}_{i,i} = 0$. Further, $ind_{h,i}$ denotes the proportion of consumption of commodity i in industry $h \in \{$automotive/ transportation, chemistry/ pharmaceutics, electrics, construction, mechanical engineering$\}$. Overall, the weights indicate a strong link between copper, lead and tin due to their consumption in the electrics sector.

Table 4.17: Demand-sided information matrix

	Al	Cu	Ni	Pb	Sn	Zn
Al	0.00	0.20	0.27	0.15	0.12	0.33
Cu	0.20	0.00	0.15	0.55	0.50	0.11
Ni	0.28	0.15	0.00	0.08	0.04	0.29
Pb	0.16	0.55	0.08	0.00	0.66	0.02
Sn	0.12	0.50	0.04	0.66	0.00	0.02
Zn	0.33	0.11	0.29	0.02	0.02	0.00

This table displays the demand-sided information matrix, based on the common proportion the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), belonging to the considered industry sectors automotive/ transportation, chemistry/ pharmaceutics, electrics, construction and mechanical engineering, according to Equation 4.3.

[22] Actually, lead is used in the electrics sector to construct inter alia lead acid batteries, which are used for the automotive industry. Due to data limitations, we decided to assign the entire consumption of lead in the topic electrical engineering to the initial electrics sector and do not distinguish whether the products are processed further.

4.4.4 Co-Consumption in the context of the German Energiewende

Within our risk assessment framework, we compare the scarcity risk of the resource demands of several transformation pathways of the German energy system from 2020 to 2050 for the key resources silver (Ag), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), and platinum (Pt) as well as the remaining industrial metals aluminum (Al), lead (Pb), tin (Sn), and zinc (Zn).[23] As the clean energy market affects the connectedness of metal markets, according to Song et al. (2022), the co-consumption of the commodities in the context of the energy transition can directly be reflected via the correlation of the annual resource requirements per transformation path.[24]

Focusing on the $REMod-REF$ path, the commodities are highly correlated, except silver, which even has a negative relation to indium, see Table 4.18. While the dependence with copper is highest for most of the commodities, indium and zinc are only moderately related to the other metals. Moreover, cobalt and lithium are almost perfectly correlated, which can be explained by their common usage in storage technologies, like (lithium-ion) batteries.

In general, the relationship between the metals is comparable between the pathways. In case of the $REMod-SUF$ as well as $REMod-UNA$ paths, displayed in Table 4.19 and Table 4.21 respectively, silver is more connected, while the dependencies of the metals to lead and platinum are smaller. In addition, the link between indium and the remaining commodities is highest in the $REMod-PER$ path, see Table 4.20.

Table 4.18: Demand-sided information matrix based on the $REMod-REF$ path

	Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
Ag	1.00	0.60	0.37	0.38	-0.06	0.36	0.28	0.25	0.16	0.28	0.66
Al	0.60	1.00	0.95	0.91	0.58	0.94	0.92	0.79	0.77	0.84	0.83
Co	0.37	0.95	1.00	0.96	0.74	1.00	0.99	0.88	0.89	0.92	0.77
Cu	0.38	0.91	0.96	1.00	0.82	0.97	0.97	0.94	0.94	0.98	0.79
In	-0.06	0.58	0.74	0.82	1.00	0.74	0.80	0.85	0.86	0.87	0.49
Li	0.36	0.94	1.00	0.97	0.74	1.00	0.99	0.89	0.90	0.93	0.76
Ni	0.28	0.92	0.99	0.97	0.80	0.99	1.00	0.90	0.92	0.94	0.74
Pb	0.25	0.79	0.88	0.94	0.85	0.89	0.90	1.00	0.93	0.95	0.67
Pt	0.16	0.77	0.89	0.94	0.86	0.90	0.92	0.93	1.00	0.98	0.67
Sn	0.28	0.84	0.92	0.98	0.87	0.93	0.94	0.95	0.98	1.00	0.73
Zn	0.66	0.83	0.77	0.79	0.49	0.76	0.74	0.67	0.67	0.73	1.00

This table displays the demand-sided information matrix in the context of the German Energiewende based on the correlation of the annual resource demands of the $REMod-REF$ path for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn).

Table 4.19: Demand-sided information matrix based on the $REMod-SUF$ path

	Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
Ag	1.00	0.71	0.68	0.48	0.20	0.69	0.59	0.20	0.33	0.36	0.54
Al	0.71	1.00	0.99	0.88	0.56	0.99	0.97	0.40	0.54	0.59	0.82
Co	0.68	0.99	1.00	0.92	0.63	1.00	0.99	0.48	0.62	0.65	0.86
Cu	0.48	0.88	0.92	1.00	0.85	0.93	0.96	0.71	0.82	0.84	0.93
In	0.20	0.56	0.63	0.85	1.00	0.65	0.73	0.88	0.91	0.93	0.84
Li	0.69	0.99	1.00	0.93	0.65	1.00	0.99	0.52	0.65	0.69	0.88

[23] In case of dysprosium and neodymium, we approximate their supply and demand data by the relative supply and demand of rare earths metals provided by U.S. Geological Survey (2020b) and U.S. Geological Survey (2020a), which is why we have to exclude these two metals for the risk assessment via the (MS-)GVAR framework to avoid multicollinearity.

[24] In contrast to the observed data of co-production and (approximated) co-consumption, the co-consumption information matrix in the context of the German Energiewende is based on estimated correlations of the future resource requirements. While the basic idea of Pesaran et al. (2004) is to reduce the number of parameters using the observed trade weights to link the individual country-specific models, Gross (2013) propose a procedure to estimate the weights jointly with the GVAR's parameters. In this line, the estimated correlations of the co-consumption matrix for the German Energiewende will also lead to a feasible weight matrix.

Demand-sided information matrix based on the $REMod - SUF$ path

	Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
Ni	0.59	0.97	0.99	0.96	0.73	0.99	1.00	0.57	0.70	0.73	0.90
Pb	0.20	0.40	0.48	0.71	0.88	0.52	0.57	1.00	0.95	0.95	0.70
Pt	0.33	0.54	0.62	0.82	0.91	0.65	0.70	0.95	1.00	0.99	0.83
Sn	0.36	0.59	0.65	0.84	0.93	0.69	0.73	0.95	0.99	1.00	0.86
Zn	0.54	0.82	0.86	0.93	0.84	0.88	0.90	0.70	0.83	0.86	1.00

This table displays the demand-sided information matrix in the context of the German Energiewende based on the correlation of the annual resource demands of the $REMod - SUF$ path for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn).

Table 4.20: Demand-sided information matrix based on the $REMod - PER$ path

	Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
Ag	1.00	0.68	0.28	0.23	0.07	0.26	0.24	0.35	0.21	0.34	0.28
Al	0.68	1.00	0.86	0.83	0.68	0.84	0.85	0.81	0.54	0.84	0.80
Co	0.28	0.86	1.00	0.99	0.91	1.00	0.99	0.89	0.64	0.94	0.87
Cu	0.23	0.83	0.99	1.00	0.95	0.98	0.98	0.91	0.63	0.96	0.92
In	0.07	0.68	0.91	0.95	1.00	0.92	0.91	0.85	0.61	0.91	0.86
Li	0.26	0.84	1.00	0.98	0.92	1.00	0.98	0.89	0.65	0.94	0.87
Ni	0.24	0.85	0.99	0.98	0.91	0.98	1.00	0.85	0.66	0.93	0.89
Pb	0.35	0.81	0.89	0.91	0.85	0.89	0.85	1.00	0.60	0.93	0.89
Pt	0.21	0.54	0.64	0.63	0.61	0.65	0.66	0.60	1.00	0.70	0.60
Sn	0.34	0.84	0.94	0.96	0.91	0.94	0.93	0.93	0.70	1.00	0.95
Zn	0.28	0.80	0.87	0.92	0.86	0.87	0.89	0.89	0.60	0.95	1.00

This table displays the demand-sided information matrix in the context of the German Energiewende based on the correlation of the annual resource demands of the $REMod - PER$ path for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn).

Table 4.21: Demand-sided information matrix based on the $REMod - UNA$ path

	Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
Ag	1.00	0.84	0.56	0.55	0.11	0.54	0.45	0.20	0.10	0.49	0.67
Al	0.84	1.00	0.88	0.86	0.51	0.85	0.81	0.37	0.41	0.78	0.86
Co	0.56	0.88	1.00	0.96	0.79	1.00	0.99	0.57	0.62	0.94	0.85
Cu	0.55	0.86	0.96	1.00	0.84	0.96	0.96	0.55	0.61	0.96	0.88
In	0.11	0.51	0.79	0.84	1.00	0.79	0.86	0.49	0.70	0.86	0.65
Li	0.54	0.85	1.00	0.96	0.79	1.00	0.99	0.59	0.63	0.95	0.84
Ni	0.45	0.81	0.99	0.96	0.86	0.99	1.00	0.58	0.65	0.95	0.83
Pb	0.20	0.37	0.57	0.55	0.49	0.59	0.58	1.00	0.37	0.56	0.32
Pt	0.10	0.41	0.62	0.61	0.70	0.63	0.65	0.37	1.00	0.69	0.46
Sn	0.49	0.78	0.94	0.96	0.86	0.95	0.95	0.56	0.69	1.00	0.84
Zn	0.67	0.86	0.85	0.88	0.65	0.84	0.83	0.32	0.46	0.84	1.00

This table displays the demand-sided information matrix in the context of the German Energiewende based on the correlation of the annual resource demands of the $REMod - UNA$ path for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn).

4.4.5 Co-Trading

Finally, we represent the co-trading of investors in commodity markets by calculating the Pearson correlation coefficient between the daily total volume of traded contracts from the London Metal Exchange (LME), provided by Thomson Reuters Eikon (2022j), in the period from 2010 to 2019, see Table 4.22.[25] Hereby, nickel, lead and zinc are strongly connected, whereby aluminum has a negative correlation with nickel, and zinc.

[25] In contrast to the observed data of co-production and (approximated) co-consumption, the co-trading information matrix is based on estimated correlations. In line with the co-consumption information matrices in the context of the German Energiewende as well as Gross (2013), the estimated correlations of the trading volume matrix will also lead to a feasible weight matrix.

Table 4.22: Correlation matrix of Futures trading volumes

	Al	Cu	Ni	Pb	Sn	Zn
Al	1.00	0.05	-0.15	0.04	0.15	-0.01
Cu	0.05	1.00	0.35	0.37	0.20	0.42
Ni	-0.15	0.35	1.00	0.35	-0.08	0.57
Pb	0.04	0.37	0.35	1.00	0.28	0.66
Sn	0.15	0.20	-0.08	0.28	1.00	0.14
Zn	-0.01	0.42	0.57	0.66	0.14	1.00

This table displays the Pearson correlation matrix between the daily total volume of traded contracts of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) from the London Metal Exchange (LME), calculated over the period from 2010 to 2019.

4.4.6 Final Weight Matrices

To convert the presented information matrices into weight matrices, connecting the individual commodity markets within the GVAR framework, we scale the values to row sums of one. The resulting weight matrices for the dimensions supply (**S**), demand (**D**) and trading (**T**) for the industrial metals are displayed in Table 4.23, Table 4.24, and Table 4.25.[26] As co-production, co-consumption and co-trading occur simultaneously in practice, we estimate a fourth, common weight matrix (**C**), which aggregates all three dimensions, see Table 4.26. Hereby, we construct the common weight matrix by equally weighting the previously calculated, individual weight matrices.

Table 4.23: Supply weight matrix (**S**)

	Al	Cu	Ni	Pb	Sn	Zn
Al	0.00	0.08	0.06	0.34	0.25	0.27
Cu	0.21	0.00	0.10	0.26	0.19	0.24
Ni	0.19	0.12	0.00	0.21	0.32	0.16
Pb	0.34	0.09	0.06	0.00	0.23	0.28
Sn	0.30	0.08	0.11	0.28	0.00	0.23
Zn	0.32	0.10	0.05	0.32	0.22	0.00

This table displays the supply-sided weight matrix of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn).

Table 4.24: Demand weight matrix (**D**)

	Al	Cu	Ni	Pb	Sn	Zn
Al	0.00	0.19	0.26	0.14	0.11	0.30
Cu	0.14	0.00	0.10	0.36	0.33	0.08
Ni	0.33	0.18	0.00	0.10	0.05	0.35
Pb	0.11	0.37	0.05	0.00	0.45	0.01
Sn	0.09	0.37	0.03	0.49	0.00	0.01
Zn	0.42	0.15	0.38	0.03	0.03	0.00

This table displays the demand-sided weight matrix of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn).

Table 4.25: Trading weight matrix (**T**)

	Al	Cu	Ni	Pb	Sn	Zn
Al	0.00	0.12	0.37	0.11	0.39	0.02
Cu	0.03	0.00	0.25	0.27	0.15	0.30
Ni	0.10	0.23	0.00	0.23	0.05	0.38
Pb	0.03	0.22	0.21	0.00	0.16	0.39
Sn	0.18	0.23	0.10	0.32	0.00	0.17
Zn	0.00	0.23	0.32	0.37	0.08	0.00

This table displays the trading weight matrix of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn).

Table 4.26: Common weight matrix (**C**)

	Al	Cu	Ni	Pb	Sn	Zn
Al	0.00	0.13	0.23	0.20	0.25	0.20
Cu	0.13	0.00	0.15	0.30	0.22	0.20
Ni	0.21	0.18	0.00	0.18	0.14	0.30
Pb	0.16	0.23	0.11	0.00	0.28	0.23
Sn	0.19	0.23	0.08	0.36	0.00	0.14
Zn	0.25	0.16	0.25	0.24	0.11	0.00

This table displays the common weight matrix of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn).

Overall, aluminum, lead, and zinc show the strongest interdependencies on the supply side,[27] whereas aluminum, nickel, and zinc as well as copper, lead, and tin are highly connected on the demand side. Moreover, nickel and lead depend mostly on zinc in case of the trading weight

[26]Table 4.23, Table 4.24, Table 4.25, and Table 4.26 show rounded values, whereas further calculations in the model use the true values without rounding.

[27]While the dependency between lead and zinc originates from their co-production of lead-zinc ores, the strong relation to aluminum is mainly caused by their common production countries, particularly China, see Table C.1 for the largest mining countries per commodity.

matrix. In contrast, the weights of the common matrix are more balanced, as the individual effects compensate each other to a certain extent.

The weight matrices in the context of the risk assessment framework for the German Energiewende, connecting the commodities via their common resource requirements for the four different transformation pathways, are displayed in Table 4.27, Table 4.28, Table 4.29, and Table 4.30. Due to data limitations, the MS-GVAR model is based on the six industrial metals aluminum, copper, nickel, lead, tin and zinc. Hereby, the corresponding weight matrices for the German Energiewende, connecting the commodities via their common resource requirements for the four different transformation pathways, are displayed in Table 4.31, Table 4.32, Table 4.33, and Table 4.34.

Table 4.27: Demand-sided weight matrix based on the $REMod - REF$ path

	Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
Ag	0.00	0.18	0.11	0.11	0.02	0.11	0.08	0.07	0.05	0.08	0.19
Al	0.07	0.00	0.12	0.11	0.07	0.12	0.11	0.10	0.10	0.10	0.10
Co	0.04	0.11	0.00	0.11	0.09	0.12	0.12	0.10	0.10	0.11	0.09
Cu	0.04	0.11	0.11	0.00	0.09	0.11	0.11	0.11	0.11	0.11	0.09
In	0.01	0.09	0.11	0.12	0.00	0.11	0.12	0.12	0.13	0.13	0.07
Li	0.04	0.11	0.12	0.11	0.09	0.00	0.12	0.10	0.11	0.11	0.09
Ni	0.03	0.11	0.12	0.11	0.10	0.12	0.00	0.11	0.11	0.11	0.09
Pb	0.03	0.10	0.11	0.12	0.11	0.11	0.11	0.00	0.12	0.12	0.08
Pt	0.02	0.10	0.11	0.12	0.11	0.11	0.11	0.12	0.00	0.12	0.08
Sn	0.03	0.10	0.11	0.12	0.10	0.11	0.11	0.11	0.12	0.00	0.09
Zn	0.09	0.12	0.11	0.11	0.07	0.11	0.10	0.09	0.09	0.10	0.00

This table displays the demand-sided weight matrix in the context of the German Energiewende based on the annual resource demands of the $REMod - REF$ path for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn).

Overall, the weights for the German Energiewende almost equal, whereas the correlation between the metals differ. However, the scaling of the correlations with the corresponding row sums leads to the balanced weight matrices. Hereby, the matrices regarding the four transformation pathways barely differ, due to the similar initial correlation matrices.

Table 4.28: Demand-sided weight matrix based on the $REMod - SUF$ path

	Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
Ag	0.00	0.15	0.14	0.10	0.04	0.14	0.12	0.04	0.07	0.08	0.11
Al	0.09	0.00	0.13	0.12	0.08	0.13	0.13	0.05	0.07	0.08	0.11
Co	0.09	0.13	0.00	0.12	0.08	0.13	0.13	0.06	0.08	0.08	0.11
Cu	0.06	0.11	0.11	0.00	0.10	0.11	0.12	0.09	0.10	0.10	0.11
In	0.03	0.08	0.09	0.12	0.00	0.09	0.10	0.12	0.13	0.13	0.12
Li	0.09	0.12	0.13	0.12	0.08	0.00	0.12	0.06	0.08	0.09	0.11
Ni	0.07	0.12	0.12	0.12	0.09	0.12	0.00	0.07	0.09	0.09	0.11
Pb	0.03	0.06	0.08	0.11	0.14	0.08	0.09	0.00	0.15	0.15	0.11
Pt	0.04	0.07	0.08	0.11	0.12	0.09	0.09	0.13	0.00	0.14	0.11
Sn	0.05	0.08	0.09	0.11	0.12	0.09	0.10	0.13	0.13	0.00	0.11
Zn	0.07	0.10	0.11	0.11	0.10	0.11	0.11	0.09	0.10	0.11	0.00

This table displays the demand-sided weight matrix in the context of the German Energiewende based on the annual resource demands of the $REMod - SUF$ path for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn).

Table 4.29: Demand-sided weight matrix based on the $REMod - PER$ path

	Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
Ag	0.00	0.23	0.09	0.08	0.02	0.09	0.08	0.12	0.07	0.11	0.10
Al	0.09	0.00	0.11	0.11	0.09	0.11	0.11	0.10	0.07	0.11	0.10
Co	0.03	0.10	0.00	0.12	0.11	0.12	0.12	0.11	0.08	0.11	0.10
Cu	0.03	0.10	0.12	0.00	0.11	0.12	0.12	0.11	0.07	0.11	0.11
In	0.01	0.09	0.12	0.12	0.00	0.12	0.12	0.11	0.08	0.12	0.11
Li	0.03	0.10	0.12	0.12	0.11	0.00	0.12	0.11	0.08	0.11	0.10

Demand-sided weight matrix based on the $REMod - PER$ path

	Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
Ni	0.03	0.10	0.12	0.12	0.11	0.12	0.00	0.10	0.08	0.11	0.11
Pb	0.04	0.10	0.11	0.11	0.11	0.11	0.11	0.00	0.08	0.12	0.11
Pt	0.04	0.09	0.11	0.11	0.10	0.11	0.11	0.10	0.00	0.12	0.10
Sn	0.04	0.10	0.11	0.11	0.11	0.11	0.11	0.11	0.08	0.00	0.11
Zn	0.04	0.10	0.11	0.12	0.11	0.11	0.11	0.11	0.08	0.12	0.00

This table displays the demand-sided weight matrix in the context of the German Energiewende based on the annual resource demands of the $REMod - PER$ path for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn).

Table 4.30: Demand-sided weight matrix based on the $REMod - UNA$ path

	Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
Ag	0.00	0.19	0.12	0.12	0.02	0.12	0.10	0.04	0.02	0.11	0.15
Al	0.12	0.00	0.12	0.12	0.07	0.12	0.11	0.05	0.06	0.11	0.12
Co	0.07	0.11	0.00	0.12	0.10	0.12	0.12	0.07	0.08	0.12	0.10
Cu	0.07	0.11	0.12	0.00	0.10	0.12	0.12	0.07	0.07	0.12	0.11
In	0.02	0.08	0.12	0.13	0.00	0.12	0.13	0.07	0.11	0.13	0.10
Li	0.07	0.11	0.12	0.12	0.10	0.00	0.12	0.07	0.08	0.12	0.10
Ni	0.06	0.10	0.12	0.12	0.11	0.12	0.00	0.07	0.08	0.12	0.10
Pb	0.04	0.08	0.12	0.12	0.11	0.13	0.13	0.00	0.08	0.12	0.07
Pt	0.02	0.08	0.12	0.12	0.13	0.12	0.12	0.07	0.00	0.13	0.09
Sn	0.06	0.10	0.12	0.12	0.11	0.12	0.12	0.07	0.09	0.00	0.10
Zn	0.09	0.12	0.12	0.12	0.09	0.12	0.11	0.04	0.06	0.12	0.00

This table displays the demand-sided weight matrix in the context of the German Energiewende based on the annual resource demands of the $REMod - UNA$ path for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn).

Table 4.31: Demand-sided weight matrix based on the $REMod - REF$ path for the industrial metals

	Al	Cu	Ni	Pb	Sn	Zn
Al	0.00	0.21	0.21	0.18	0.20	0.19
Cu	0.20	0.00	0.21	0.21	0.21	0.17
Ni	0.20	0.22	0.00	0.20	0.21	0.17
Pb	0.19	0.22	0.21	0.00	0.22	0.16
Sn	0.19	0.22	0.21	0.21	0.00	0.17
Zn	0.22	0.21	0.20	0.18	0.19	0.00

This table displays the demand-sided weight matrix in the context of the German Energiewende based on the annual resource demands of the $REMod - REF$ path for the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn).

Table 4.32: Demand-sided weight matrix based on the $REMod - SUF$ path for the industrial metals

	Al	Cu	Ni	Pb	Sn	Zn
Al	0.00	0.24	0.26	0.11	0.16	0.22
Cu	0.20	0.00	0.22	0.16	0.19	0.22
Ni	0.23	0.23	0.00	0.14	0.18	0.22
Pb	0.12	0.21	0.17	0.00	0.29	0.21
Sn	0.15	0.21	0.18	0.24	0.00	0.22
Zn	0.19	0.22	0.21	0.17	0.20	0.00

This table displays the demand-sided weight matrix in the context of the German Energiewende based on the annual resource demands of the $REMod - SUF$ path for the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn).

Table 4.33: Demand-sided weight matrix based on the $REMod - PER$ path for the industrial metals

	Al	Cu	Ni	Pb	Sn	Zn
Al	0.00	0.20	0.21	0.20	0.20	0.19
Cu	0.18	0.00	0.21	0.20	0.21	0.20
Ni	0.19	0.22	0.00	0.19	0.21	0.20
Pb	0.18	0.21	0.19	0.00	0.21	0.20
Sn	0.18	0.21	0.20	0.20	0.00	0.21
Zn	0.18	0.21	0.20	0.20	0.21	0.00

This table displays the demand-sided weight matrix in the context of the German Energiewende based on the annual resource demands of the $REMod - PER$ path for the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn).

Table 4.34: Demand-sided weight matrix based on the $REMod - UNA$ path for the industrial metals

	Al	Cu	Ni	Pb	Sn	Zn
Al	0.00	0.23	0.22	0.10	0.21	0.23
Cu	0.20	0.00	0.23	0.13	0.23	0.21
Ni	0.20	0.23	0.00	0.14	0.23	0.20
Pb	0.16	0.23	0.24	0.00	0.23	0.13
Sn	0.19	0.24	0.23	0.14	0.00	0.21
Zn	0.23	0.24	0.22	0.09	0.23	0.00

This table displays the demand-sided weight matrix in the context of the German Energiewende based on the annual resource demands of the $REMod - UNA$ path for the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn).

5 Empirical Results

This thesis aims to compare the scarcity risk of the annual material requirements of four potential transformation pathways of the German energy system. Therefore, we apply the proposed framework in Section 3, which assesses the scarcity risk of resource-demanding projects under the consideration of the substitutability of commodities, the future resource amounts required by the project, as well as the historical information available, in the context of the German Energiewende. By interpreting the price of a commodity as scarcity indicator, we are able to quantify the resource scarcity risk of each transformation path, while taking into account the entire commodity market behavior. Thus, a comprehensive understanding of commodity markets is essential, which is why we base our framework on the (Markov-switching) global vector autoregression ((MS-)GVAR) model to reflect commodity markets holistically, especially, the interdependencies between supply, demand and price, as well as the co-movement between commodity prices. Hereby, we exemplary introduce the (MS-)GVAR model on the industrial metal markets. First, we investigate the dynamic properties of the GVAR model under several weight matrices, reflecting the co-production, co-consumption and co-trading relationship between commodities, see Section 5.1. Second, we disentangle the differences in the spillover effects between the industrial metal markets at different points in time within the MS-GVAR model, explicitly allowing for time-varying relations in commodity markets, see Section 5.2. Finally, we apply the (MS-)GVAR model in the context of the German Energiewende to compare the resource requirements of four transformation pathways with regard to their scarcity risk, see Section 5.3 and conclude the empirical part of this study with a brief discussion in Section 5.4.

5.1 A Joint Model for Industrial Metal Markets[1]

In this section, we exemplary model the industrial metal markets jointly, using the global vector autoregression (GVAR) model to analyze the interrelations within and between the commodities. In contrast to Pesaran et al. (2004), who use the GVAR model to link individual economies via trade weights, we propose to link the commodity markets via information on co-production, co-consumption and co-trading relationships among commodities. Therefore, we apply the GVAR model several times using the different weight matrices supply ($\mathbf{S}$), demand ($\mathbf{D}$), trading ($\mathbf{T}$) and common ($\mathbf{C}$), see Section 4.4. To analyze the spillover effects in the metal markets, we calculate generalized impulse response functions (GIRFs) recursively, according to Equation 3.13, with 68% confidence bounds, in line with Anzuini et al. (2013) and Hammoudeh et al. (2015) among others, obtained by a sieve bootstrap procedure[2] and the recent observations as input variables, as proposed in Dées, di Mauro, Pesaran, and Smith (2007). This methodology investigates direct as well as indirect effects on the attributes to an innovation of one standard deviation in

[1]Parts of this section are included in the paper "Three Co's to Jointly Model Commodity Markets: Co-Production, Co-Consumption and Co-Trading", accepted for publication in Empirical Economics, 2023, co-authored by Patric Papenfuß, and Andreas Rathgeber.

[2]In particular, we draw $N_{boot} = 1000$ times $T_{boot} = 40$ residuals with replacement to generate the bootstrap sample.

a certain variable. Thereby, we analyze the impact of supply and demand on commodity prices, spillover effects between the commodity markets as well as the effects of shocks to the global economy on commodity markets. Hereby, we first estimate individual, commodity-specific VAR models and investigate the impact of the fundamentals on the price via GIRFs. Subsequently, we analyze the spillover effects between the commodity markets of the GVAR models, based on the different weight matrices. Finally, we underline our findings through a generalized forecast error variance decomposition (GFEVD) analysis, as well as a correlation analysis.

5.1.1 Individual Commodity Market Models

First, we examine the spillover effects within the individual industrial metal markets, by analyzing the generalized impulse response functions of the commodity-specific VAR models. Therefore, we estimate each industrial metal market separately, with the commodity-specific variables supply, demand, and price, via individual VAR models with one lag and without intercept, according to Equation 3.1, based on annual data from 1970 to 2019.[3] Since various studies emphasize the common effects of the economy on commodity markets, especially the impact of economic activity, exchange rates, and monetary policy, see Section 2.2, we include the exogenous variables world gross domestic product (GDP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR) with one lag to account for macroeconomic factors.

The results of the Durbin-Watson (DW) test, the multivariate ARCH Lagrange multiplier (ARCH-LM) test and the OLS-cumulative sums of standardized residuals (OLS-CUSUM) test to each commodity-specific VAR model indicate neither model suffers from autocorrelation, heteroscedasticity nor structural breaks at the 5% significance level, except the aluminum market, which exhibits heteroscedasticity, see Table D.1. The Henze-Zirkler (HZ) test for normality shows the VAR models of aluminum, copper, nickel, lead, and tin have multivariate normal distributed residuals, whereas the Henze-Zirkler test rejects the null hypothesis of multivariate normality for the zinc market at the 5% level. While the multivariate normal distributed residuals are generally not required for the estimation of VAR models via ordinary least squares, the GIRFs are calculated recursively under the assumption of normal errors, based on Equation 3.13. Since we only provide the GIRFs of the individual VAR models as preliminary analysis, to allow for a comparison of the GVAR model with individual commodity market models, we do not adjust the specifications of the VAR model of the zinc market, although the corresponding residuals show a non-normal behavior. Hereby, we keep in mind the true generalized impulse response functions may deviate from the presented ones.

To aggregate the GIRF results of each individual commodity market and to facilitate the comparison with the corresponding results of the GVAR model, we present the results of the GIRF analysis in Table 5.1, where we indicate significant positive, or negative, responses of the column variables to a shock in the row variables by a (+) or (-) respectively.

While we detect no interdependencies in the copper and zinc market, indicating their prices are not affected by their individual supply and demand, spillover effects within the aluminum, nickel, lead, and tin markets imply the fundamentals and prices are correlated. Hereby, demand and price negatively interact to each other for aluminum and nickel, indicating a higher price (demand) implies less demand (price). Moreover, nickel's and tin's supply and price positively affect each other. In addition, we observe interactions between the supply and demand of tin and lead. Hereby, the positive responses in the lead market are reasonably, whereas the negative relation in the tin market is counterintuitive. However, as the GIRF methodology investigates direct as well as indirect effects on the attributes to an innovation of one standard deviation in

[3] In particular, we estimate the VAR model without intercept to guarantee comparability to the GVAR models in Section 5.1.2.

a certain variable, the observed reactions may be caused by unobservable, indirect effects.

Table 5.1: GIRF results of the individual, commodity-specific VAR models

	Al			Cu			Ni			Pb			Sn			Zn		
	supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
supply	+			+			+		+	+	+		+	-	+	+		
demand		+	-		+			+	-	+	+		-	+			+	
price		-	+			+	+	-	+			+	+		+			+

This table displays the results of the GIRF analysis of the individual, commodity-specific VAR models, showing the response of the column variables to a shock of the row variables supply (**supply**), demand (**demand**) and price (**price**) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), where significant positive (+) or negative (-) effects are displayed, based on the 68%- level.

Besides the spillover effects within the individual commodity markets, we also investigate the impact of shocks to the exogenous variables on the commodity markets. In particular, we examine how the commodity-specific variables respond to innovations in the economic activity, the exchange rate or the interest rate. Therefore, we first model the exogenous variables, world gross domestic product (GDP), U.S. dollar index (FX) and Federal Funds Effective Rate (FFR), via a VAR model with two lags to avoid autocorrelated residuals, but without intercept vector, in line with the commodity market models.[4] Subsequently, we derive the impact of a shock to each exogenous variable on the individual commodity markets recursively, using Equation 3.17, Equation 3.19, and Equation 3.20, and display the responses in analogy to the GIRF analysis of shocks within the commodity markets in Table 5.2.

Overall, the GIRF analysis reveals the shocks to the macroeconomic variables affect each commodity market to a similar extent. In particular, an increase in the global demand, associated with an expansion of the economic activity and thus indicated by a positive shock in the world gross domestic product, leads to a significant increase in commodity markets, as production, consumption, and prices rise simultaneously, which supports the synchronized pattern of commodity markets and economic activity, see Issler et al. (2014) among others. However, the production of copper declines in response to a global demand increase. Since copper is regarded as a leading indicator of the global economic situation, see Bauer (2023), copper producers may reduce their supply in times of economic booms, to prevent their losses due to the subsequent recession phases.

In addition, a positive shock to the exchange rate, U.S. dollar index, representing an appreciation of the U.S. dollar, leads to decreasing commodity markets. In particular, since the metals are quoted in U.S. dollars, a stronger U.S. dollar implies the metals become more expensive for consumers holding other currencies, see Vansteenkiste (2009), and therefore, the demand and ultimately, the price of the commodities decrease. Moreover, as the profits of the producers raise, see Vansteenkiste (2009), the copper supply increases, and therefore, the copper price declines. However, the production volumes of the other metals reduce, probably caused by indirect effects of the reduction in demand and price, which are also represented in the GIRF analysis at annual frequency.

Further, the reactions of the supply variables to a contrarian monetary policy, i.e. represented by a positive shock to the interest rate, are mixed. While we observe no reaction in the supply of aluminum, lead, and zinc, a positive shock in the interest rate leads to an increase in the

[4] The results of the Durbin-Watson (DW) test, the multivariate multivariate ARCH Lagrange multiplier (ARCH-LM) test and the OLS-cumulative sums of standardized residuals (OLS-CUSUM) test to the VAR model indicate the model does not suffer from autocorrelation, heteroscedasticity or structural breaks at the 5% significance level, see Table D.1. Moreover, the Henze-Zirkler (HZ) test for normality shows the VAR model has multivariate normal distributed residuals.

production volume of copper, nickel, and tin, probably due to the more profitable extraction of commodities in high interest rates environments, see Akram (2009) and Frankel (2008). In contrast, a contrarian monetary policy shock implies increasing commodity-specific demand and price. This contradicts the hypothesis of an inverse relationship of Frankel (2008), who argues the cost of capital for holding a commodity should decrease during periods of expansionary monetary policy, while at the same time the demand for commodities acting as an alternative asset class should rise. However, Frankel (2008) only confirms his theory in the empirical analysis of commodity prices in the period from 1950 to 1979 (2005), whereas the observed relation between interest rates and commodity prices is positive in the period from 1980 (1976) to 2005, underlining the direction of relation between monetary policy and commodity prices changed over time. Moreover, in times of increasing commodity prices, and therefore, a period of increasing inflation, the central banks raise the interest rates. This increase in the interest rates dampens the boom in commodity prices, but prices continue to rise in the short-term, which could explain the positive reaction in the metals' markets, see Schischke and Rathgeber (2023).

Table 5.2: GIRF results of the individual VAR models for shocks to the exogenous variables

	Al			Cu			Ni			Pb			Sn			Zn		
	supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
GDP	+	+	+	-	+	+	+	+	+	+	+	+	+	+	+	+	+	+
FX	-	-	-	+	-	-	-	-	-	-	-	-		-	-	-	-	
FFR		+	+	+	+	+	+	+	+		+	+	+	+				+

This table displays the results of the GIRF analysis of the individual, commodity-specific VAR models, showing the response of the column variables supply (**supply**), demand (**demand**) and price (**price**) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), to a shock of the row variables world gross domestic product (GDP), U.S. dollar index (FX), and the Federal Funds Effective Rate (FFR), where significant positive (+) or negative (-) effects are displayed, based on the 68%-level.

Overall, we detect the commodity markets react to shocks in the global economy as well as in the individual commodity markets. However, as the copper and zinc markets do not exhibit any spillover effects, one may conclude microeconomic information of supply and demand is already included in the price of copper and zinc, inducing their consideration is irrelevant and negligible in modern commodity markets. However, the individual industrial metal market models do not reflect spillover effects between the commodities yet, which is why we turn our attention to the results of the GVAR models.

5.1.2 Global Commodity Market Model

To account for the impact of supply and demand on commodity prices as well as for spillover effects between commodity markets, we apply the GVAR model, according to Equation 3.7, on the industrial metal markets, whereby we include the impact of macroeconomic factors on the commodity markets via the exogenous variables world gross domestic product (GDP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR). Hereby, we calculate the GVAR model several times, using the different weight matrices supply (**S**), demand (**D**), trading (**T**) and common (**C**), as outlined in Section 4.4, for a comparison between the possible linkages of commodity markets. In particular, we estimate the GVAR models on annual data from 1970 to 2019, with one lag for the endogenous as well as exogenous variables, and without intercept, due to data limitations. While the results of the Durbin-Watson (DW) test, the ARCH-LM test and the OLS-cumulative sums of standardized residuals (OLS-CUSUM) test indicate neither model suffers from autocorrelation, heteroscedasticity nor structural breaks at the 5% significance level, the Henze-Zirkler (HZ) test implies the residuals of the GVAR model, based on the different weight matrices, are multivariate normal distributed, see Table D.1.

In line with the individual VAR models, we analyze the dynamic properties of the global vector autoregression (GVAR) models, via generalized impulse response functions (GIRFs), according to Equation 3.13. Hereby, direct as well as indirect effects on the attributes to an innovation of one standard deviation in a certain variable are investigated. Our analysis is based on the 68% confidence bounds obtained by a sieve bootstrap procedure with 1000 replications and the recent observations as input variables, as proposed in Dées, di Mauro, Pesaran, and Smith (2007). Runkle (1987) and Lütkepohl (1990) both point out impulse response functions can inflate false negatives, a problem also Galesi and Lombardi (2009) suffer from, in their analysis of unrestricted GVAR models. Although data limitations, as caused by a small sample size, could further harm the results, our analysis detects numerous significant responses.

To aggregate the GIRF analysis and to provide an holistic overview of the results, we indicate significant positive, or negative, responses of the column variables to a shock in the row variables by a (+) or (-) in Table 5.3. Hereby, we differentiate between the models with weight matrices supply (**S**), demand (**D**), trading (**T**) and common (**C**), respectively.

The diagonal of Table 5.3 shows significant results for all variables and weight matrix combinations, which is rather unsurprising, as it captures the effect of a shock to the response variable itself. Despite the concerns of false negatives within GIRF analyses, we obtain numerous significant spillover effects in the cross-commodity dimension, underlining the importance of jointly modeling commodity markets and making the findings of the framework even more pronounced.

Regarding the differences between the weight matrices, the GVAR model using the demand weight matrix detects slightly more spillover effects, compared to the other models, indicating the relations between commodities may be best modeled by their co-consumption. Since the GVAR models based on the supply and trading weight matrices represent less spillover effects, the relations between the commodities may be best reflected by their co-consumption or the aggregated information including the co-consumption, indicating the importance of the demand side in the (relation between) commodity markets. As the common weight matrix simultaneously represents information on co-production, co-consumption and co-trading with equal weights, the individual effects of the consumption behavior are potentially diminished to a certain extent.

Table 5.3: GIRF results of the GVAR models based on the supply, demand, trading and common weight matrices

			Al			Cu			Ni			Pb			Sn			Zn		
		w	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
Al	**supply**	**S**	+						+											
		D	+				+		+				-	+				-		
		T	+												-					
		C	+						+											
	demand	**S**		+	-		+												+	
		D		+			+									-				
		T		+			+						+			-			+	
		C		+			+									-			+	
	price	**S**			+	-		+			+									
		D			+				+		+									-
		T			+	-										+				
		C			+	-					+									
Cu	**supply**	**S**			-	+														
		D			-	+														
		T			-	+														
		C			-	+														
	demand	**S**	+				+	-						+				-		-
		D	+	+			+	-			-							-	+	-
		T	+	+			+	-						+						
		C	+	+			+	-						+				-		-
		S					-	+						-						+

GIRF results of the GVAR models based on the supply, demand, trading and common weight matrices

			Al			Cu			Ni			Pb			Sn			Zn		
		w	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
	price	**D**		-	+		-	+	+		+									+
		T		-			-	+						-	-					+
		C		-			-	+			+			-						+
Ni	**supply**	**S**	+						+			+			-					
		D	+			-			+						-					
		T	+						+			+				+				
		C	+						+											
	demand	**S**								+				-						
		D								+										
		T								+				-						
		C								+				-						
	price	**S**			+						+				+					
		D									+				+	-				
		T			+						+				+				+	
		C			+						+				+				+	
Pb	**supply**	**S**	-									+	+	-				+		
		D							+			+	+					+		
		T				-						+	+					+		
		C		+		-			+			+	+					+		
	demand	**S**			-			+		+		+	+						+	
		D		+		+	+	+		+		+	+			-			+	
		T						+				+	+			-			+	
		C		+				+		+		+	+			-			+	
	price	**S**												+			+			
		D												+						
		T					+							+			+			
		C					+							+			+			
Sn	**supply**	**S**									+	+			+					
		D							-		+				+					
		T									+				+					
		C							-		+				+					
	demand	**S**					+		+		-				-	+				
		D	+				+		+							+				
		T							+							+				
		C					+		+							+				
	price	**S**							-					+			+			
		D															+			
		T															+			
		C															+			
Zn	**supply**	**S**	-									+			-			+		
		D					-					+						+		
		T										+						+		
		C	-				-					+						+		
	demand	**S**				-					+								+	
		D		+			+			-			+						+	
		T		+			+				+		+						+	
		C							+	-			+						+	
	price	**S**					-	+			+									+
		D						+			+									+
		T					-	+		+										+
		C					-	+			+									+

This table displays the results of the GIRF analysis of the GVAR model based on the different weight matrices (w) supply (**S**), demand (**D**), trading (**T**) and common (**C**). We analyze the response of the column variables to a shock of the row variables supply (**supply**), demand (**demand**) and price (**price**) of aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn). Significant positive (+) or negative (-) effects on the 68%- level are displayed.

To start, we compare the results of the individual VAR models to the commodity-specific results of the GVAR model, before we analyze the spillover effects in the cross-commodity dimension in detail. Overall, the spillover effects in the individual commodity markets change, as the GVAR model connects the commodity markets and therefore accounts for unobserved, indirect effects between the commodities, which are not represented in the individual VAR models. Hereby, the significant impact between supply (demand) and price in the nickel and tin markets as well as the response of aluminum demand to price shocks vanish once the interdependence between the commodities is included, whereas copper demand and price significantly influence each other in the GVAR models. However, in line with the findings of the individual VAR models in Table 5.1, the GVAR analysis also detects no significant responses of zinc's supply, demand and price to shocks to its own variables. Moreover, the interdependencies in the lead market remain valid, indicating the GVAR models also reflect the spillover effects in the individual commodity markets.

Besides these responses in the individual markets, we observe various spillover effects in the cross-commodity dimension, underlining the importance of jointly modeling commodity markets. While tin, the smallest metal market in terms of the trading volume, is least connected to the other markets, which is reasonable, as it is not co-mined with any of the remaining metals,[5] nor is there a specific, common use case, the nickel market is highly affected by the other metals. Since nickel is mostly used in alloys, e.g. nickel-aluminum- or copper-nickel-(zinc)-alloys, the demand for the other metals determines the demand, and ultimately, the price and supply of nickel. Hereby, nickel depends on the other markets, whereas shocks to the lead and zinc market cause various spillover effects, probably due to their co-production and co-consumption relation to the other metal markets. However, the majority of spillover effects is from, and to, the aluminum and copper markets, the largest metal markets in terms of the production, consumption, and trading volume, and therefore, the most influential and connected metals. Hereby, shocks in the two metal markets cause changes in all other markets, emphasizing the impact of both metals.

Overall, aluminum and copper also exhibit the strongest interrelation, even though their link reflected in the weight matrices is not overwhelmingly large, see Section 4.4.6. The observed strong spillover effects between these two metals are most likely due to their common applications in electrical conduction, automotive and aerospace industries. Therefore, an increase in the demand for one of these two metals leads to higher demand for the other metal, as there is probably an increase in the demand for a common application, which increases the demand for both metals. In addition, higher copper prices lead to a lower demand for aluminum, since an increase in the price of copper reduces the demand for copper and the common applications and increases the price of aluminum, which is why the demand for aluminum decreases. In addition, an increase in the price of aluminum leads to lower production volume of copper, as the supply is likely to react to the associated decrease in demand in response to the higher price. However, a shock to the supply of one metal does not lead to any significant reaction in the supply of the other metal, as aluminum and copper are not co-mined together.

In contrast, 70% of the lead production is derived from mixed Lead-Zinc ores, see Nassar et al. (2015).[6] Due to this strong co-production relationship between lead and zinc, a positive shock to the supply of one of both metals cause an increase in the supply of the other metal. Moreover, a shock to the demand of lead (or zinc) probably leads to an increase in the demand of zinc (or lead). This observed interrelation between the demand might originate from indirect effects of the co-production relation and the supply and demand equilibrium, combined with their common use in the automotive industry. Hereby, zinc is mainly used for rust protection, whereas the

[5] Actually, tin is not co-mined with the other industrial metals. However, as we reflect the supply weight matrix on the common supply concentration, based on the country-specific production volumes, the weight matrix displays a co-production relation.

[6] As the supply weight matrix is based on the common supply concentration, based on the country-specific production volumes, the relation between lead and zinc is probably underestimated, as both metals are actually co-produced.

most important application of lead is in car batteries.

Besides the interdependencies between lead and zinc, we observe strong spillover effects between copper and zinc. Similar to the relation between aluminum and copper, the interactions in the copper and zinc market probably originate from their common applications, in particular, in the copper-zinc alloy brass, which is used due to its good electrical conductivity and mechanical stability. While the demand variables interact, an increase in zinc's price lead to a reduction in copper's demand, probably caused by the associated reduction in zinc's demand or the corresponding increase in copper's price.

Overall, regarding the spillover effects between supply, demand and price in the cross-commodity dimension, the strongest effects are the positive interactions between supply and supply, demand and demand, as well as price and price, with remarkable interdependencies between the demand variables. Since the global demand is reflected by the exogenous variable world gross domestic product (GDP), the common applications of the metals are an important determinant of the observed strong relationship between the individual demand variables, as common consumption leads to a concurrent behavior in the demand. In this context, the influence of the common uses of the metals underlines the importance of the demand weight matrix, as the corresponding model detects most of the interdependencies.

The demand-sided relationships are also reflected in the interdependencies between the prices. Hereby, the various spillover effects between the prices are the direct result of the co-trading as well as the indirect result of joint applications, as an increase in the demand leads to higher prices. Overall, an increase in one commodity's price generally leads to rising prices of the other commodities, indicating a common behavior in the metals' prices. In particular, aluminum, copper, nickel and zinc prices influence each other. Hereby, shocks to the copper price affect the other commodities, while the copper price reacts only to changes in the zinc price, indicating a strong impact of copper on the other commodity markets. However, there are no spillover effects from lead and tin prices, probably because these metals are the smallest in terms of their trading volume. In addition, tin is only processed for specific applications with few common uses with the other commodities, while lead has been substituted in several original use cases in more recent years.

Besides the interdependencies between the demand and price variables, we also observe various spillover effects between the supply variables, but to a lesser extent. On the one hand, the co-production of the metals leads to the increase in supply in response to a positive shock to another supply variable, for example for lead and zinc, as 70% of the lead production is derived from mixed Lead-Zinc ores. On the other hand, the observed positive reactions may be caused by indirect effects from the co-consumption relation, as an increase in the demand implies rising prices and ultimately an increase in the production volume. In particular, the spillover effects between aluminum and nickel can be explained by their common use in aircraft and vehicle construction as well as in nickel-aluminum alloys.

Moreover, there are few spillover effects between supply and demand variables, whereby the demand for one commodity more likely results in changes in the supply for another metal than vice versa, underlining the (indirect) effect of the co-consumption on the supply side of the markets. In addition, the fundamentals, especially the demand, significantly influence prices. More importantly, shocks to the prices affect the supply and demand variables. While the supply tendentially increases in response to a positive shock to the prices, higher prices cause a reduction in the demand.

Similar to the GIRF analysis of the individual commodity markets, we also examine the effects of global shocks to the commodity markets modeled by the GVAR framework, based on the different weight matrices supply ($\mathbf{S}$), demand ($\mathbf{D}$), trading ($\mathbf{T}$) and common ($\mathbf{C}$). In particular, we examine how the commodity-specific variables respond to innovations in the global economic activity, reflecting the global demand, the exchange rate or the interest rate. Therefore, we use the VAR model of the exogenous variables, world gross domestic product (GDP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR), specified in Section 5.1.1, and analyze the impacts of a shock to each exogenous variable on the commodity markets, using the GIRFs derived recursively, using Equation 3.17, Equation 3.19, and Equation 3.20. In analogy to the GIRF analysis of shocks within and between the commodity markets, we display the responses of the commodity markets to innovations in the exogenous variables in Table 5.4.

Overall, similar to the analysis of the individual commodity markets, we observe the shocks to the macroeconomic variables affect each commodity market to a similar extent, across all models. In particular, an increase in the global demand cause increasing supply, demand and prices for all commodities, similar to the results of the individual commodity market models in Section 5.1.1. The only exception is the copper production which reduces in response to the demand increase, underlining the special role of copper. Moreover, a positive shock to the exchange rate, reflecting an appreciation of the U.S. dollar, leads to a reduction in the metal markets, except for copper supply, in line with Vansteenkiste (2009) as well as the results of the individual VAR models.

Further, a contrarian monetary policy, reflected via a positive interest rate shock, leads to an increase in the production volume of aluminum, copper and tin (zinc), underlining the arguments of Akram (2009) and Frankel (2008), whereas the production volumes of lead (zinc) decrease. Moreover, both, demand and price, increase in response to rising interest rates, contrary to Frankel (2008), but confirming the reactions of the commodity markets in the individual VAR models. Hereby, the central banks probably raise the interest rates in response to higher inflation, caused by higher prices, leading to a concurrent behavior between metal markets and interest rates, see Schischke and Rathgeber (2023).

Table 5.4: GIRF results of the GVAR models based on the supply, demand, trading and common weight matrices for shocks to the exogenous variables

		Al			Cu			Ni			Pb			Sn			Zn		
	w	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
GDP	**S**	+	+	+	-	+	+	+	+	+	+	+	+	+	+	+	+	+	+
	D	+	+	+	-	+	+	+	+	+	+	+	+	+	+	+	+	+	+
	T	+	+	+	-	+	+	+	+	+	+	+	+	+	+	+	+	+	+
	C	+	+	+	-	+	+	+	+	+	+	+	+	+	+	+	+	+	+
FX	**S**	-	-	-	+	-	-	-	-	-	-	-	-		-	-	-	-	
	D	-	-	-	+	-	-	-	-	-	-	-	-		-	-	-	-	
	T	-	-	-	+	-	-	-	-	-	-	-	-		-	-	-	-	
	C	-	-	-	+	-	-	-	-	-	-	-	-		-	-	-	-	
FFR	**S**	+	+	+	+	+	+		+		-	+	+	+	+		-	+	+
	D	+	+	+	+	+	+		+		-	+	+	+	+		+	+	+
	T		+		+	+	+		+		-	+	+	+	+		-		+
	C		+		+	+	+		+		-	+	+	+	+			+	+

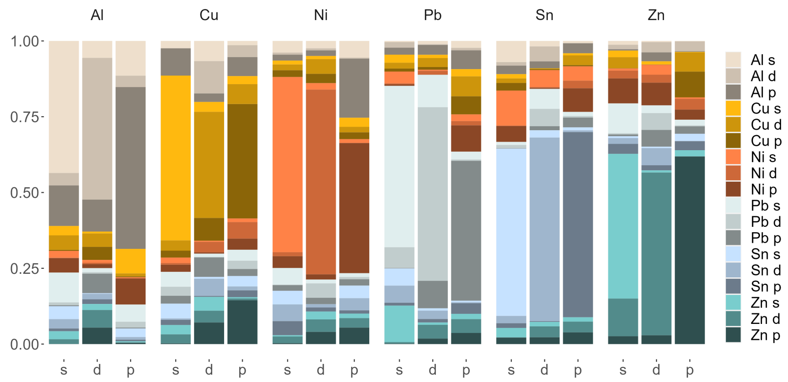
This table displays the results of GIRF analysis of the GVAR model based on the different weight matrices (w) supply (**S**), demand (**D**), trading (**T**) and common (**C**). We analyze the response of the column variables, supply (**supply**), demand (**demand**) and price (**price**) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), to a shock of the row variables world gross domestic product (GDP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR), where significant positive (+) or negative (-) effects are displayed, based on the 68%- level.

Overall, the GIRF analysis reveals the industrial metal markets are strongly interrelated. While the fundamentals still affect the commodity prices, various spillover effects underline innovations in the supply, demand or price of one commodity lead to changes in the other metal markets, highlighting the importance of jointly modeling commodity markets. Moreover, shocks in the global economy affect all commodity markets simultaneously, indicating global shocks cause similar patterns in the metals markets, supporting the impact of a common factor in commodity prices.

5.1.2.1 Forecast Error Variance Analysis

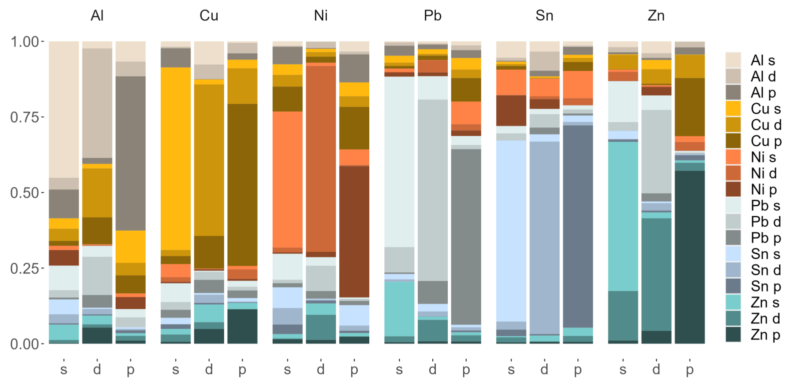
We further investigate the interdependencies between the commodity markets through a generalized forecast error variance decomposition (GFEVD) analysis, based on the GVAR models with the weight matrices supply (**S**), demand (**D**), trading (**T**) or common (**C**). Hereby, we calculate the generalized forecast error variance decompositions for each variable for one to ten steps ahead, according to Equation 3.21, and display the attributes' forecast errors variances, which are decomposed by aggregated shocks of each endogenous variable as their mean of 1 to 10 years ahead, see Figure 5.1, Figure 5.2, Figure 5.3, and Figure 5.4.

Figure 5.1: Generalized forecast error variance decomposition for the GVAR model based on the weight matrix supply (**S**)



This figure displays the scaled and aggregated generalized forecast error variance decomposition (GFEVD) of the GVAR model based on the weight matrix supply (**S**) by the mean of 1 to 10 steps ahead per attribute, decomposed by the shocks to each endogenous variable.

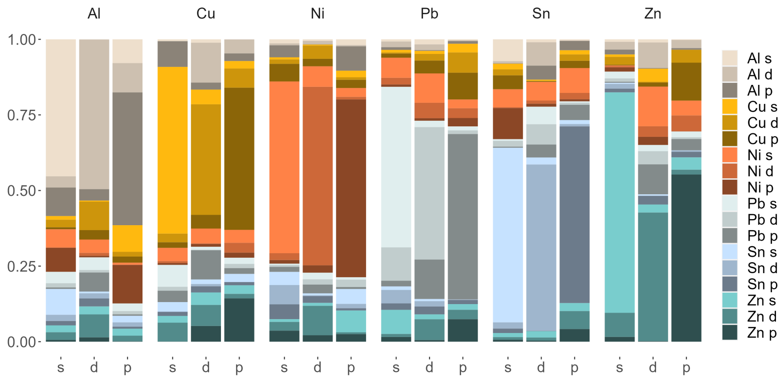
Figure 5.2: Generalized forecast error variance decomposition for the GVAR model based on the weight matrix demand (**D**)



This figure displays the scaled and aggregated generalized forecast error variance decomposition (GFEVD) of the GVAR model based on the weight matrix demand (**D**) by the mean of 1 to 10 steps ahead per attribute, decomposed by the shocks to each endogenous variable.

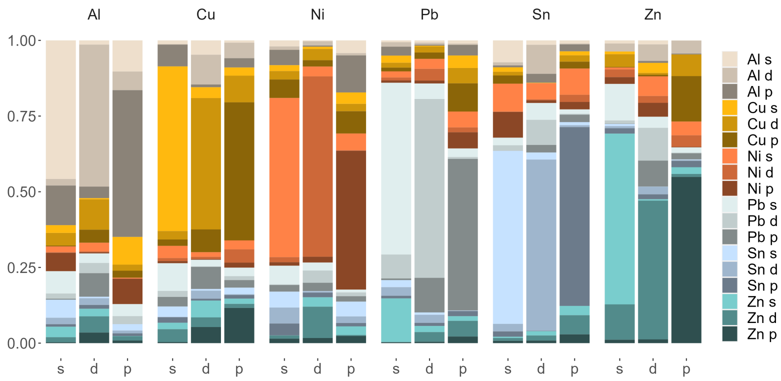
In general, the forecast error variances are mainly influenced by each variable itself. Hereby, the nickel supply as well as aluminum and zinc demand are outstanding, as over 50% of their forecast error variances are explained by other commodities' variables, indicating the high impact of the other metals. Moreover, the GVAR model based on the demand weight matrix indicates the forecast error variances are slightly more affected by the other commodities, however, the results are similar across the models.

Figure 5.3: Generalized forecast error variance decomposition for the GVAR model based on the weight matrix trading (**T**)



This figure displays the scaled and aggregated generalized forecast error variance decomposition (GFEVD) of the GVAR model based on the weight matrix trading (**T**) by the mean of 1 to 10 steps ahead per attribute, decomposed by the shocks to each endogenous variable.

Figure 5.4: Generalized forecast error variance decomposition for the GVAR model based on the weight matrix common (**C**)



This figure displays the scaled and aggregated generalized forecast error variance decomposition (GFEVD) of the GVAR model based on the weight matrix common (**C**) by the mean of 1 to 10 steps ahead per attribute, decomposed by the shocks to each endogenous variable.

Turning to a commodity-specific perspective, we observe the forecast error variance of aluminum's supply is particularly influenced by the copper, lead, and tin markets. While the GIRF analysis also detects spillover effects from copper and lead demand to the aluminum supply, shocks to the tin market do not significantly affect the aluminum supply, however, tin's variables contribute to the forecast error variance. Further, copper demand and price, followed by lead and zinc demand, describe the forecast error variance of the aluminum demand, underlining the findings of the GIRF analysis, where copper demand and price, as well as lead and zinc demand significantly affect the aluminum demand. Besides aluminum's fundamentals, the copper market mostly affects the forecast error variance of aluminum's price, further highlighting the strong impact of copper on the aluminum market, as already outlined in Section 5.1.2. However, the strong interdependencies between copper and aluminum are most pronounced in case of the GVAR model based on the demand-sided weight matrix, as both metals are jointly consumed in the field of electrical conduction, automotive and aerospace industries.

For the copper market, the forecast error variances are mainly influenced by its own, commodity-specific attributes, further indicating copper affects the other metal markets, but it reacts to a smaller extent to changes in the other commodities. While copper supply is equally described by aluminum, nickel and lead, a relative large proportion of the forecast error variance of copper's demand is determined by aluminum's and zinc's attributes. Hereby, the strong impact of aluminum on the copper demand underlines the observed, significant spillover effects in Section 5.1.2, again highlighting the strong interrelation between these two metals, probably caused by their co-consumption. Moreover, the GFEVD analysis also reveals the special position of copper, as only the zinc price has a pronounced influence on the copper's price forecast error variance, in line with the GIRF analysis, which reveals the copper price affects the other commodity prices, but only reacts itself to changes in the zinc price.

Nickel's supply and price forecast error variances are mostly described by aluminum and copper, underlining the results of the GIRF analysis, since a shock to the aluminum price leads to significant responses in the nickel supply and price. Moreover, similar to the spillover effects, lead and tin explain parts of the forecast error variance of nickel's supply, while lead and zinc demand determine the demand's forecast error variance.

For lead, the forecast error variances of supply and demand are mainly explained by the variables of zinc, apart from the contribution of lead's own variables, underlining the findings of the GIRF analysis, which detects lead's fundamentals respond significantly to shocks in the zinc market, probably caused by their strong co-production relation. Moreover, the variance of lead's price is affected by the copper and nickel market to a larger extent, in line with the spillover effects, indicating the price of lead significantly responses to shocks to copper demand and price as well as nickel demand.

Further, the strong interdependencies between lead and zinc are underlined by the forecast error variance analysis of zinc. Hereby, lead determines the majority of the forecast error variances of zinc's supply and demand, caused by their co-production relation. In contrast, copper contributes to a smaller extent to the forecast error variance, which is contrary to the significant reactions of the zinc market to shocks in the copper demand. However, the forecast error variance of zinc's price is mostly determined by copper, in particular, by the copper price, followed by copper demand, underlining the co-movement between copper and zinc prices.

In case of tin, aluminum mostly explains the forecast error variance of tin's demand, while the forecast error variances of supply and price are mostly affected by nickel, underlining the significant impact of shocks to the nickel market on tin. In general, the GFEVD analysis reveals tin is explained by the other commodities, but rarely contributes to the forecast error variances of the other metals itself, underlining tin is least connected to the other markets, as it is not co-mined with any of the remaining metals, nor is there a specific common use case, in line with the results of the GIRF analysis.

Overall, the copper price helps in explaining the metal's price variances, further underlining the strong impact of copper on commodity markets, in line with the results of the GIRF analysis. Moreover, the GFEVD analysis emphasizes the interdependencies between aluminum and copper, copper and zinc as well as lead and zinc, caused by their co-consumption and co-production links.

5.1.2.2 Correlation Analysis

The GVAR model considers the impact of commodity-specific supply and demand on prices, but also allows for spillover effects between the commodity markets. To highlight the framework's ability to represent the co-movement in commodity prices, we compare the price correlations induced by the GVAR model with the market observed correlations. Therefore, we split our dataset into an expanding in-sample window with data from 1971 to 2009 and an out-of-sample window covering the years 2010 to 2019. For each time step in the out-of-sample window, we estimate the GVAR model with the weight matrices supply (**S**), demand (**D**), trading (**T**) or common (**C**) based on the in-sample data and forecast all commodities' annual prices[7] one-step ahead, using Equation 3.9. Subsequently, we calculate Pearson correlation matrices, using the predicted and observed prices, respectively.

The comparison of the correlation matrices, presented in Table 5.5, Table 5.6, Table 5.7, Table 5.8, and Table 5.9, highlights the dependencies between the commodity markets are well modeled by our framework, except for zinc.[8] The focus of this brief correlation analysis is on the replication's accuracy of the observed co-movement, where the predictive power of the models is not evaluated in further detail. While the correlations of copper (tin) with the other commodities observed from the GVAR models based on the different weight matrices supply (**S**), demand (**D**), trading (**T**), or common (**C**), are similar, the negative relation observed between zinc and the other metals is best reflected by the model based on the demand weight matrix, whereas the other models overestimate the correlations, indicating the GVAR model based on the weight matrix demand (**D**) performs best.

Apart from the negative relation observed between zinc and the other metals, which is not reflected, the GVAR framework based on the weight matrix demand (**D**) performs exceptionally well, with differences in the correlations for the predicted and real prices being smaller than 10%, except for the links between aluminum and lead. While the model based on the common weight matrix also reflect the dependencies between aluminum, copper, nickel, lead and tin, the negative correlation to zinc is missed. In contrast, the correlations derived from the GVAR models based on the supply and trading weight matrix deviate more, underlining the relations between the commodities may be best reflected by their co-consumption or the aggregated information including the co-consumption.

Since the GIRF analysis reveals strong spillover effects between the commodity markets, especially between the commodity-specific demand variables, indicating the strong impact of demand and the common applications of the metals, we focus in the following on the weight matrix demand (**D**), based on information about the co-consumption between commodities. Overall, the correlation analysis underlines the importance of jointly modeling commodity markets, confirming the results of the GIRF and GFEVD analysis. However, the GVAR model is a time-invariant framework, which is why changes in the correlation structure of the commodities are not reflected.

[7]As all original variables were non-stationary, we base the entire analysis on the logarithmic return data and hence, also the price forecasts are forecasts of logarithmic returns, although we use the term price for notation reasons.

[8]Throughout the sample period from 1970 to 2019, all metal prices are positively related, especially in the early 2000s. However, the price of zinc gradually increased over the period from 2010 to 2019, whereas the metal prices generally declined around 2015, caused by the slowdown in Chinese demand and the oil price drop. As our analysis is based on annual data, the sharp decline in the prices, contrary to the increase in zinc's price, skews the calculation of the correlations, leading to the observed negative relationship between the metals. The depletion of aluminum stocks in LME-linked warehouses and a sustained demand growth around 2015 prevented the price of aluminum from declining further, which is why the two metals show a slightly positive relation. Moreover, the price of lead, while also declining around 2015, shows similar patterns throughout the sample period, likely due to its co-production with zinc, which is why zinc's correlation with lead is the highest. Overall, the observed correlations in the period from 2010 to 2019 differ from those over the entire sample period or the rolling 18-months correlations, displayed in Figure 1.1, emphasizing the importance of a time-varying analysis which allows for changing dependencies between the metal prices.

Table 5.5: Correlation matrix of the observed spot prices

	Al	Cu	Ni	Pb	Sn	Zn
Al	1.00	0.84	0.76	0.88	0.83	0.29
Cu	0.84	1.00	0.92	0.68	0.90	-0.18
Ni	0.76	0.92	1.00	0.51	0.85	-0.30
Pb	0.88	0.68	0.51	1.00	0.81	0.50
Sn	0.83	0.90	0.85	0.81	1.00	-0.05
Zn	0.29	-0.18	-0.30	0.50	-0.05	1.00

This table displays the correlation matrix of the observed spot prices of aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), between 2010 and 2019.

Table 5.6: Correlation matrix of predicted prices based on the GVAR model with supply weight matrix

	Al	Cu	Ni	Pb	Sn	Zn
Al	1.00	0.76	0.72	0.77	0.76	0.45
Cu	0.76	1.00	0.86	0.60	0.84	0.18
Ni	0.72	0.86	1.00	0.55	0.87	-0.01
Pb	0.77	0.60	0.55	1.00	0.84	0.73
Sn	0.76	0.84	0.87	0.84	1.00	0.30
Zn	0.45	0.18	-0.01	0.73	0.30	1.00

This table displays the correlation matrix of the predicted spot prices of aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), using the global vector autoregression (GVAR) framework with weight matrix supply **S**, in an out-of-sample rolling window forecast from 2010 to 2019.

Table 5.7: Correlation matrix of predicted prices based on the GVAR model with demand weight matrix

	Al	Cu	Ni	Pb	Sn	Zn
Al	1.00	0.84	0.81	0.70	0.76	0.34
Cu	0.84	1.00	0.84	0.66	0.84	0.04
Ni	0.81	0.84	1.00	0.65	0.86	-0.09
Pb	0.70	0.66	0.65	1.00	0.88	0.54
Sn	0.76	0.84	0.86	0.88	1.00	0.18
Zn	0.34	0.04	-0.09	0.54	0.18	1.00

This table displays the correlation matrix of the predicted spot prices of aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), using the global vector autoregression (GVAR) framework with weight matrix demand **D**, in an out-of-sample rolling window forecast from 2010 to 2019.

Table 5.8: Correlation matrix of predicted prices based on the GVAR model with trading weight matrix

	Al	Cu	Ni	Pb	Sn	Zn
Al	1.00	0.75	0.57	0.63	0.68	0.38
Cu	0.75	1.00	0.87	0.67	0.89	0.32
Ni	0.57	0.87	1.00	0.49	0.84	-0.01
Pb	0.63	0.67	0.49	1.00	0.86	0.78
Sn	0.68	0.89	0.84	0.86	1.00	0.41
Zn	0.38	0.32	-0.01	0.78	0.41	1.00

This table displays the correlation matrix of the predicted spot prices of aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), using the global vector autoregression (GVAR) framework with weight matrix trading **T**, in an out-of-sample rolling window forecast from 2010 to 2019.

Table 5.9: Correlation matrix of predicted prices based on the GVAR model with common weight matrix

	Al	Cu	Ni	Pb	Sn	Zn
Al	1.00	0.91	0.82	0.76	0.81	0.58
Cu	0.91	1.00	0.90	0.61	0.80	0.35
Ni	0.82	0.90	1.00	0.65	0.87	0.23
Pb	0.76	0.61	0.65	1.00	0.88	0.75
Sn	0.81	0.80	0.87	0.88	1.00	0.41
Zn	0.58	0.35	0.23	0.75	0.41	1.00

This table displays the correlation matrix of the predicted spot prices of aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), using the global vector autoregression (GVAR) framework with weight matrix common **C**, in an out-of-sample rolling window forecast from 2010 to 2019.

5.2 Time-varying Spillover Effects within and between Industrial Metal Markets

The financialization of commodity markets, i.e. the entry of institutional investors into commodity futures markets, corresponding with an increase in index investment in commodity markets, has significantly changed the structure of commodity markets. In particular, the co-movement between commodity prices increased during the financialization starting in 2004, see Le Pen and Sévi (2017), Ohashi and Okimoto (2016), Peersman et al. (2021), Tang and Xiong (2012) and Yin and Han (2015). Moreover, Peersman et al. (2021) name also the demand increase from emerging countries, as well as spillover effects due to substitutions as possible reasons for the stronger co-movement in commodity prices observed at the beginning of the 2000s. This raises the question whether a time-independent analysis of the relationship between commodity prices, or even a trend analysis, can fully represent the interdependencies in commodity markets.

The initial, bi-variate, and time-varying correlation analysis, displayed in Figure 1.1, indicates the rolling 18-months correlations between the industrial metal prices fluctuate around the time-invariant correlation based on the entire sample period, underlining the relation between prices is time-varying. Therefore, the question arises whether and how the constitution of commodity markets, especially the impact of fundamentals on prices as well as the co-movement between prices, changed over time.

For this reason, we extend the global vector autoregression (GVAR) framework, which reflects the impact of supply and demand on commodity prices as well as the co-movement between commodity prices, by a Markov-switching component, and exemplary apply the time-dependent Markov-switching global vector autoregression (MS-GVAR) model on the six industrial metals, using monthly data from 1995 to 2020.[9] In particular, we reveal how the interdependencies between markets change over time. Hereby, we simultaneously estimate individual, commodity-specific Markov-switching vector autoregression (MS-VAR) models, accounting for the aggregated impact of the other commodity markets as well as for the macroeconomic factors world industrial production (IP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR),[10] and investigate the regime inference of each commodity market. Subsequently, we aggregate the industrial metal markets to the overall commodity market model, the Markov-switching global vector autoregression (MS-GVAR) model, via the weight matrix based on the co-consumption of the commodities, see Table 4.24, as the information about the common use of the metals reflects their dependencies best, see Section 5.1.

Moreover, we analyze the dynamic properties of the model via regime-dependent GIRFs, whereby we investigate the spillover effects within and between the commodity markets. To disentangle whether the impacts of shocks to the commodity markets differ in time, we artificially attribute all industrial metal markets to calm or volatile periods and examine how shocks to the commodity variables, or even to exogenous variables, transmit to the commodity markets. Finally, we underline the importance of a time-varying analysis by assessing the out-of-sample forecast performance of the MS-GVAR model and compare it to the performance of the time-invariant GVAR model.

5.2.1 Model Specification

To determine the optimal number of states, the lag length as well as the specification of the commodity-specific MS-VAR models, we apply the slightly adjusted model selection procedure of Li and Kwok (2021), described in Section 3.2.1.6, using the information criterion of Hannan and Quinn (1979).[11] Overall, we estimate the optimal number of states, up to the predefined maximum number of states, $M_{max} = 3$, the optimal lag length, up to the predefined number of lags, $P_{max} = 2$, as well as the specification out of all considered specifications $spec \in \{MSI, MSIH, MSH, MSA, MSAH\}$, see Table 5.10. Hereby, the models with regime-dependent exogenous variables are excluded from the selection process, as the inclusion of time-

[9] While the exemplary application of the GVAR model on the industrial metal markets is based on annual data from 1970 to 2019, the MS-GVAR model is applied to monthly data from 1995 to 2020, due to data limitations. Within the comparison of the results of the time-invariant GVAR model with those of the time-dependent MS-GVAR model, the different data period as well as data frequency have to be taken into account.

[10] Similar to the GVAR model, we include exogenous variables to take into account the common effects of the economy on commodity markets. However, the GDP is only reported on quarterly frequency, therefore, we consider the world industrial production as proxy for the economic activity.

[11] We also apply the model selection based on the information criterion of Schwarz (1978) and obtain similar results.

varying parameters for the exogenous variables leads to unstable MS-GVAR models in our case. However, the results of the model selection are not affected.

Table 5.10: Results of the model selection procedure for the MS-GVAR model based on the demand weight matrix

Nr. States	Nr. Lags	MSI	MSIH	MSH	MSA	MSAH
2	1	2523.44	2455.25	2447.64	2633.16	2599.09
	2	2545.49	2481.92	2480.27	2785.01	2716.14
3	1	2515.10	2457.54	2446.58	2774.68	2717.13
	2	2543.76	2444.17	2465.09	3033.36	2950.37

This table displays the model selection results for the MS-GVAR model based on the demand weight matrix, using the information criterion of Hannan and Quinn (1979), proposed in Section 3.2.1.6. Hereby, the MS-GVAR model is estimated using different number of states (Nr. States), lag lengths (Nr. lags), as well as specifications (MSI, MSIH, MSH, MSA, MSAH).

In particular, the MSIH(3)-VAR(2) model performs best, followed by the MSH(3)-VAR(1) and MSH(2)-VAR(1) models. Since the models based on three states exhibit unstable regime inferences, we base our analysis on the MSH(2)-VAR(1) model with regime-dependent covariance matrix, but regime-invariant intercept as well as regime-invariant parameters for the endogenous and exogenous variables, indicating the correlation between the variables differ, whereas the magnitude of the impact of the variables do not change between the regimes. The two regimes enable to capture calm as well as volatile periods, while they lead to stable regime inferences, see Section 5.2.2. The results of the Durbin-Watson (DW) test indicate neither of the commodity-specific MSH(2)-VAR(1) model suffers from autocorrelation, see Table D.2, indicating the lag length of one chosen by the model selection procedure is feasible from a statistical point of view.

5.2.2 Regime Inferences

Since the MS-GVAR model aggregates the individual, commodity-specific MS-VAR models to one, global commodity market model, the transition probabilities, i.e. the probabilities to switch from state one to state two and vice versa, displayed in Table 5.11, as well as the smoothed probabilities, indicating in which state a commodity is located in at a specific point in time, differ across the markets. Therefore, we briefly discuss the transition and regime probabilities of each industrial metal market in the following.

Table 5.11: Transition probability matrices for the individual, commodity-specific MS-VAR models

Al		Cu		Ni		Pb		Sn		Zn	
0.92	0.08	0.82	0.18	0.50	0.50	0.84	0.16	0.90	0.10	0.82	0.18
0.16	0.84	0.50	0.50	0.56	0.44	0.08	0.92	0.12	0.88	0.23	0.77

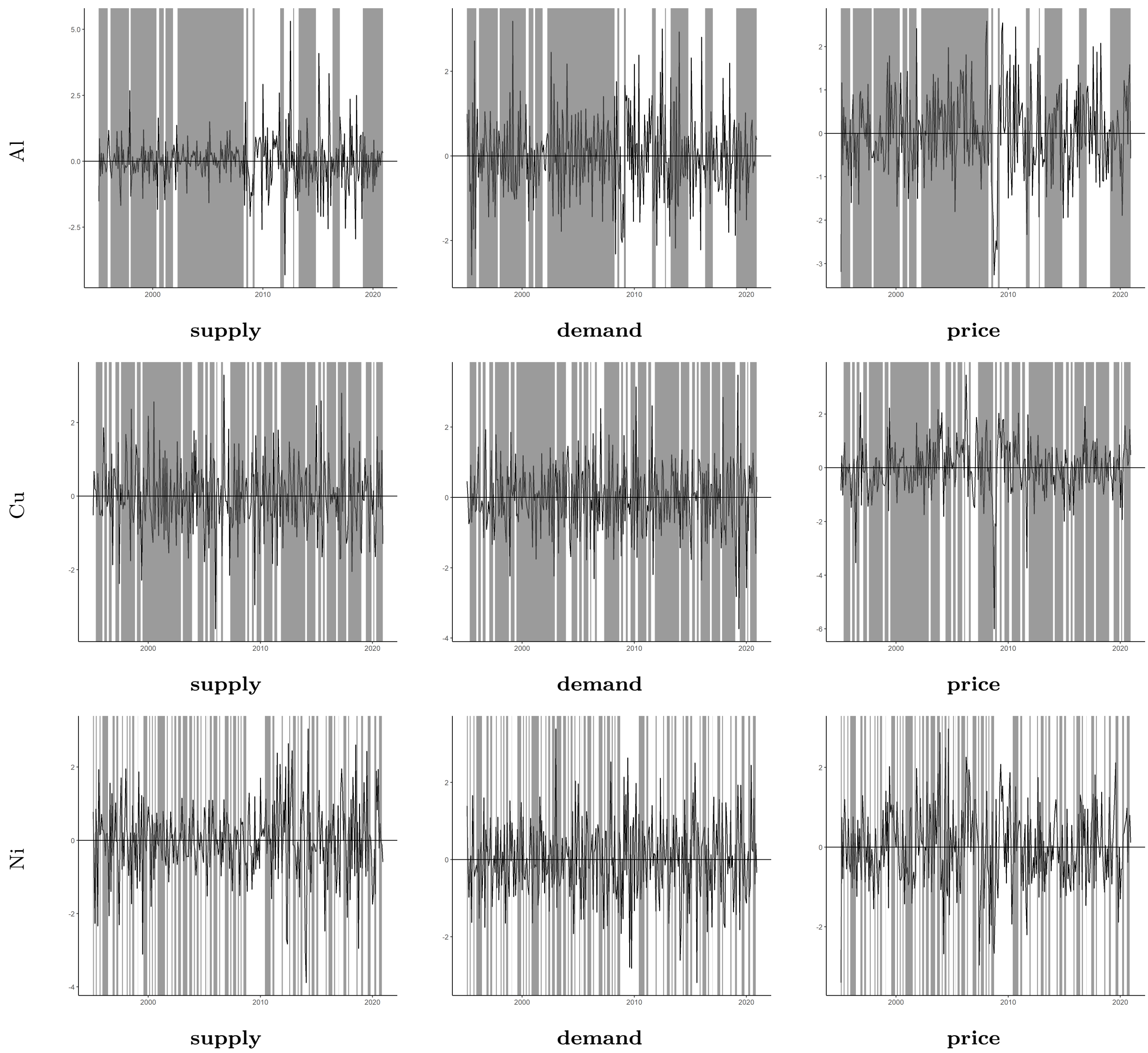
This table displays the transition probability matrices for the individual, commodity-specific MS-VAR models of aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn).

The transition probabilities indicate it is more likely to stay in the present state for aluminum, lead, tin and zinc, whereas the copper market exhibits a strong tendency to remain in, or move to, state one. Moreover, the probability to switch between the regimes is almost equal to the probability to stay, in case of the nickel market. Therefore, the nickel market switches its states numerous times, whereas the other metal markets show only few periods in regime two, see Figure 5.5,[12] indicating the industrial metal markets are stable, except the nickel market.

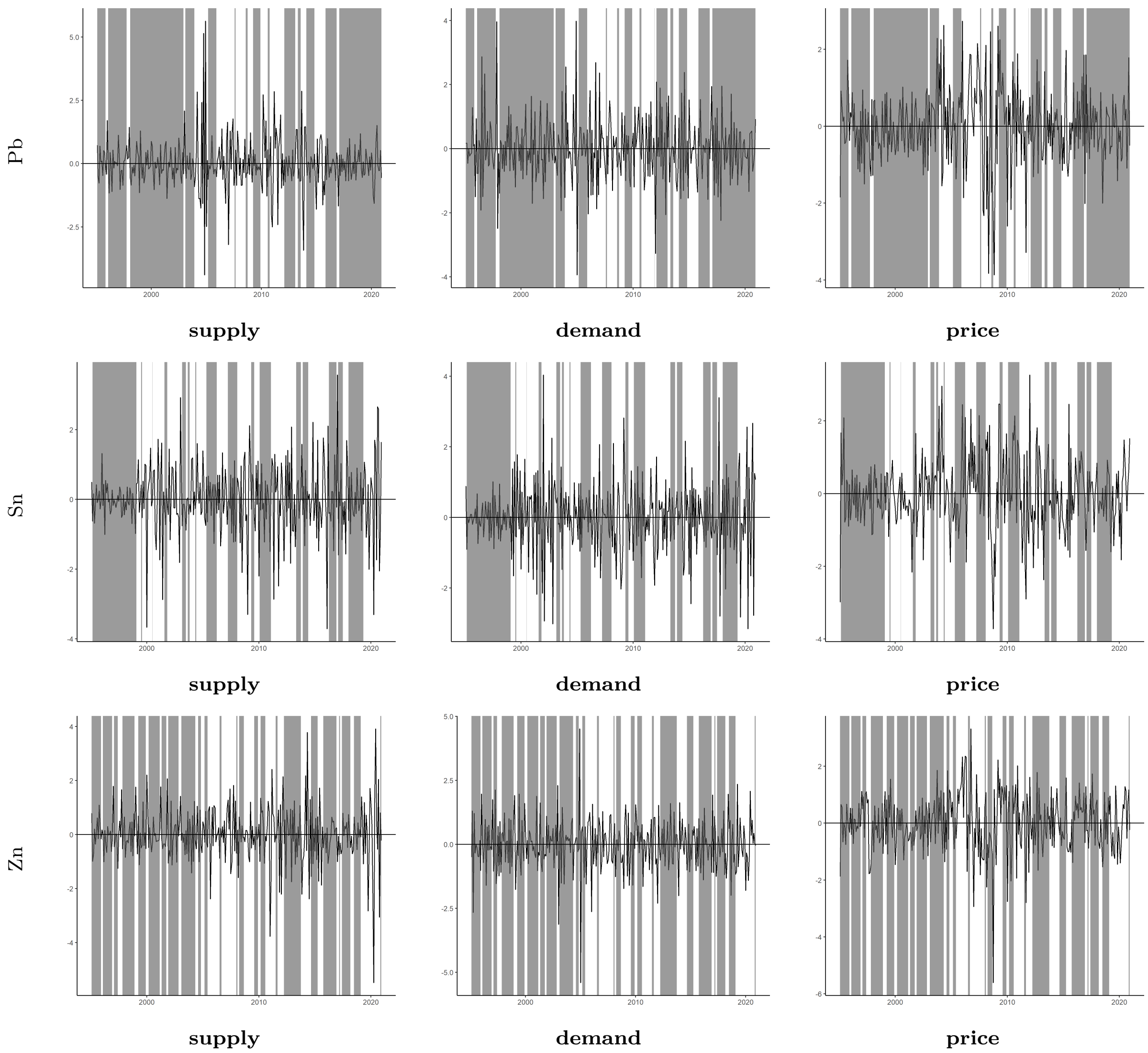
[12]These figures show the returns of each individual supply, demand and price variable over the entire sample period. Shaded areas indicate the smoothed probability to be in state two exceeds 50%, hence, it is more likely for the commodity market to be in state two at these points in time.

Overall, the metal markets are mainly located in regime one until 2004, the beginning of the financialization of commodity markets, see Tang and Xiong (2012), with only few and brief periods in regime two, indicating the increase in index investment changed the industrial metal markets. Thereafter, the markets show longer periods in regime two, especially in the years 2006/07, 2009, 2011 and 2015, corresponding to the boom in commodity prices, the financial crisis, the European debt crisis, and the sharp drop in the oil price. In the following, we turn to a commodity-specific perspective and examine each commodity market and its regimes separately, whereby we exemplary focus on the aluminum market. However, the regime switches are similar in the copper, lead, tin and zinc markets, whereas the regimes in the nickel market fluctuate more.

Figure 5.5: Regime inferences of the commodity markets, derived from the MS-GVAR model based on the demand weight matrix



Regime inferences of the commodity markets, derived from the MS-GVAR model based on the demand weight matrix



These figures show the logarithmic returns of each individual supply (**supply**), demand (**demand**) and price (**price**) variable of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), over the entire sample period from January 1995 to December 2020. Shaded areas indicate the smoothed probability to be in state one exceeds 50%, hence, it is more likely for the individual commodity market to be in state one at these points in time.

While at the beginning of the considered time period the aluminum market shows longer periods in state one, with only brief periods in state two, the market switches to state two at the end of 2008, during the financial crisis, and remains almost four years in this regime. Moreover, the aluminum market is located in state two in 2015 as well as 2017 for almost two years each. Hereby, aluminum's variables, especially the aluminum price, fluctuate widely within these periods.

After the significant price increase from 2004 to mid-2008, caused by a strong demand growth partly driven by the emerging countries, the physical as well as speculative demand dropped at the end of the year 2008, due to the financial crisis, resulting in rising inventories and declining prices, probably provoking the observed switch in aluminum's regimes. While the prices were recovered in 2010, due to the increase in Chinese demand, see U.S. Geological Survey (2013a), the supply side of the aluminum market remained turbulent. The production from U.S. primary aluminum smelters increased during the first half of 2011, whereas the leading U.S. aluminum producer announced to close several smelters permanently at the beginning of 2012, see U.S. Geological Survey (2012). However, caused by new capacity in China, India, and Qatar, the world production increased in 2012, see U.S. Geological Survey (2013b).

While the fluctuations in aluminum's supply and price probably caused the market to be located in regime two over the period 2009 to 2012, the decrease in aluminum inventories held by LME-bonded warehouses, combined with an ongoing demand growth, might explain the market is situated in state two in the year 2015, see U.S. Geological Survey (2016). In contrast, the volatile price in the years 2017 and 2018, caused by supply concerns, inter alia due to regulation problems of the world's largest alumina refinery, Alunorte, sanctions on the Russian aluminum producer Rusal as well as a labour strike at Alcoa, might be the reasons for the market being in state two during the years 2017 and 2018, see U.S. Geological Survey (2020d).

The copper market rarely changes its regimes and exhibits only brief periods in regime two, indicating its stability. While it is in state two for a longer period in 2006, just before the onset of the financial crisis, and corresponding to periods of a more volatile copper price, the periods in state one reflect calmer markets. In general, U.S. Geological Survey (2013a) state copper prices react to disturbances to production at any given large mine, due to copper's close balance between production and consumption. For example, the pit wall failure in Indonesia and a mine strike in Mexico in the year 2006, combined with the growth in global consumption during this period, resulted in a price increase, which might explain the longer lasting period of the copper market in state two.

As indicated by the transition probabilities, the nickel market exhibits the most regime switches, with numerous but brief periods in state two. Hereby, the slightly longer lasting period in regime two around 2009 is probably caused by the financial crisis, whereas the period in state two at the end of 2013 to the beginning of 2014 corresponds to a period of weak prices, associated with an oversupply of nickel, see U.S. Geological Survey (2015).

The lead market is situated in state two in periods of high price volatility, in particular, during the beginning of the financialization of commodity markets in 2004, the commodity price boom and the financial crisis. Hereby, the price increase between 2004 and 2007 is mainly driven by the strong demand for lead in China, see U.S. Geological Survey (2013a), while the surplus in the lead market might explain why the market is situated in state two in 2011, see U.S. Geological Survey (2012). In addition, the decline in the average London Metal Exchange (LME) cash lead price by about 13% in 2015 from that in 2014 caused the switch to regime two in 2015, see U.S. Geological Survey (2016).

In contrast to the other metal markets, tin is in both regimes for almost the same length of time, which can be explained by the corresponding transition probabilities, in particular, tin is equally likely to remain in one of the two states, see Table 5.11. While the tin market is only situated in state one until 1999, tin remains in regime two for longer periods at the beginning of the recent century, during the financial crisis, between 2011 to 2013, 2014 to 2016 and 2019 to 2020. Hereby, the increase in the tin price around 2008, leading to a more volatile period, was caused by supply disruptions in China as well as Indonesia, a demand growth of emerging economies, the decrease in U.S. inventories as well as the increasing role of investment funds, see U.S. Geological Survey (2013a). Additionally, the tin price increased around 2012, as major tin-consuming countries aim to switch to lead-free solders that usually contain larger amounts of tin, probably causing the switch of the market to regime two, see U.S. Geological Survey (2012). In contrast, the volatile price, due to the price decline in response to the increased tin production in Burma in 2014, as well as the reduced consumption of tin in the United States, probably caused the tin markets staying in regime two in 2015, see U.S. Geological Survey (2016).

Similar to the copper market, the zinc market shows longer periods in state one, with only brief periods in state two. However, the zinc market switches to - and remains in - state two for several years during the commodity price boom from 2005 to 2008, and the financial crisis starting in 2009. Moreover, the production-to-consumption surplus in 2015, see U.S. Geological Survey

(2016), and the increase in global zinc mine production, see U.S. Geological Survey (2020d), both periods of higher price volatility, probably caused the switch of the zinc market to regime two.

For an additional in-depth analysis of the commodity markets, we report distinct descriptive statistics for each regime in Table 5.12, where we assume the commodities are either located in state one or state two, depending on their respective smoothed probability exceeding 50%. While copper is rarely located in state two, the tin market is situated in both states each half of the time. Overall, the minimum, quantiles as well as maximum values show the markets take on more extreme values in the second state. Moreover, the commodity variables exhibit a higher volatility in state two, indicating periods of more fluctuations. In particular, the markets remain in regime two during periods with high fluctuations in supply, demand and price, which is why we refer to state two as *volatile* and state one as *calm* period in the following.

Table 5.12: Descriptive statistics of the commodity-specific variables based on the regime inferences of the MS-GVAR model

		State	Min.	5% Q.	25% Q.	Med.	Mean	75% Q.	95% Q.	Max.	SD	Nr. Obs.
Al	**supply**	1	-0.04	-0.03	-0.01	0.00	-0.00	0.01	0.02	0.04	0.01	215
		2	-0.10	-0.05	-0.02	0.00	0.00	0.02	0.06	0.13	0.04	97
	demand	1	-0.11	-0.06	-0.02	0.00	0.00	0.02	0.06	0.13	0.04	215
		2	-0.09	-0.08	-0.02	-0.00	-0.00	0.02	0.09	0.12	0.05	97
	price	1	-0.18	-0.07	-0.03	-0.00	0.00	0.04	0.08	0.14	0.05	215
		2	-0.18	-0.11	-0.04	-0.00	-0.00	0.03	0.11	0.14	0.07	97
Cu	**supply**	1	-0.06	-0.04	-0.01	-0.00	0.00	0.01	0.04	0.07	0.02	260
		2	-0.09	-0.06	-0.02	-0.00	0.00	0.02	0.05	0.09	0.04	52
	demand	1	-0.11	-0.06	-0.02	0.00	0.00	0.03	0.06	0.13	0.04	260
		2	-0.18	-0.11	-0.05	-0.01	-0.01	0.04	0.12	0.16	0.07	52
	price	1	-0.16	-0.08	-0.03	0.00	0.00	0.03	0.09	0.16	0.05	260
		2	-0.43	-0.19	-0.09	-0.00	-0.00	0.10	0.16	0.25	0.13	52
Ni	**supply**	1	-0.09	-0.05	-0.01	0.00	0.01	0.03	0.05	0.12	0.03	198
		2	-0.24	-0.16	-0.08	-0.03	-0.01	0.07	0.13	0.19	0.09	114
	demand	1	-0.14	-0.11	-0.04	0.00	0.00	0.05	0.09	0.16	0.06	198
		2	-0.25	-0.15	-0.08	-0.00	-0.00	0.06	0.17	0.27	0.10	114
	price	1	-0.33	-0.15	-0.07	0.00	0.00	0.07	0.14	0.29	0.10	198
		2	-0.26	-0.14	-0.07	-0.01	-0.00	0.06	0.18	0.22	0.10	114
Pb	**supply**	1	-0.09	-0.05	-0.02	-0.00	-0.00	0.02	0.05	0.09	0.03	217
		2	-0.25	-0.14	-0.05	0.01	0.01	0.07	0.16	0.32	0.09	95
	demand	1	-0.11	-0.07	-0.02	-0.00	-0.00	0.02	0.07	0.14	0.04	217
		2	-0.19	-0.08	-0.04	-0.00	0.00	0.04	0.10	0.19	0.06	95
	price	1	-0.19	-0.09	-0.04	-0.00	0.00	0.04	0.09	0.18	0.06	217
		2	-0.31	-0.18	-0.08	0.00	-0.00	0.06	0.17	0.22	0.11	95
Sn	**supply**	1	-0.11	-0.09	-0.03	-0.01	-0.01	0.02	0.06	0.09	0.04	150
		2	-0.27	-0.15	-0.04	0.01	0.01	0.07	0.13	0.26	0.09	162
	demand	1	-0.17	-0.08	-0.03	-0.00	-0.00	0.03	0.08	0.13	0.05	150
		2	-0.27	-0.18	-0.06	-0.00	0.00	0.07	0.17	0.34	0.11	162
	price	1	-0.10	-0.05	-0.03	0.00	0.01	0.04	0.11	0.16	0.05	150
		2	-0.24	-0.12	-0.05	-0.01	-0.01	0.03	0.11	0.21	0.07	162
Zn	**supply**	1	-0.08	-0.05	-0.02	-0.00	-0.00	0.01	0.04	0.06	0.03	196
		2	-0.24	-0.10	-0.03	0.01	0.00	0.05	0.09	0.17	0.06	116
	demand	1	-0.13	-0.06	-0.02	0.00	0.00	0.03	0.06	0.11	0.04	196
		2	-0.27	-0.09	-0.04	0.00	-0.00	0.04	0.10	0.23	0.06	116
	price	1	-0.21	-0.09	-0.04	-0.00	0.00	0.04	0.09	0.14	0.06	196
		2	-0.41	-0.13	-0.06	0.01	-0.00	0.07	0.13	0.24	0.09	116

This table displays the descriptive statistics (minimum (Min.), 5%, 25%, 75%, 95% quantile (Q.), median (Med.), mean (Mean), maximum (Max.), standard deviation (SD), and number ob observations (Nr. Obs.)) of the stationary, commodity-specific variables supply, demand and price for the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), being either in state one or in state two, when assuming the markets are in state two if the smoothed probability to be in state two exceeds 50%.

5.2.3 Time-varying Spillover Effects in Commodity Markets

The analysis of the regime inferences in the industrial metal markets indicate the commodity markets switch their regimes from state one to state two in more volatile periods. Hereby, the metal markets exhibit calm periods until the financialization, while inter alia the boom in commodity prices, and the financial crisis lead to more fluctuations in the metals' prices and therefore, to a regime switch. However, the regimes between the markets differ, as for example nickel changes its regimes more often than aluminum. To disentangle whether the interdependencies between the commodity markets change between the regimes, we artificially attribute all industrial metal markets to either the calm or volatile regime and examine how shocks to the commodity variables, or even to the exogenous variables, transmit to the commodity markets under the calm or volatile regime.

In particular, we assume the commodity markets are all simultaneously in their calm (volatile) state, and we calculate the corresponding regime-dependent generalized impulse response functions, according to Ehrmann et al. (2003),[13] based on the 68% confidence bounds, obtained by the adjusted bootstrap procedure of Ehrmann et al. (2001),[14] see Section 3.2.4.2.2. Hereby, we shock each commodity-specific variable by one standard deviation to analyze the direct as well as the indirect effects of this shock on the remaining variables, in the individual, commodity-specific markets, as well as in the cross-commodity dimension. In addition, we also investigate how shocks to the exogenous variables affect the commodity markets.

5.2.3.1 Spillover Effects within and between Commodity Markets under the Calm Regime

First of all, we examine the spillover effects within and between the industrial metal markets under the calm regime. To condense the GIRF results and to facilitate the comparison to the GIRFs under the volatile regime as well as to the GIRFs of the GVAR model in Section 5.1, we provide an overview of the results in Table 5.13. Hereby, we indicate significant positive, or negative, responses of the column variables to a shock in the row variables by a (+), or (-).

Table 5.13: GIRF results of the MS-GVAR model based on the demand weight matrix under the calm regime

		Al			Cu			Ni			Pb			Sn			Zn		
		supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
	supply	+			-		+	-								-	+		
Al	**demand**		+		+					-		+				-			
	price		+	+	+	+			+	+			+	+	+		+	+	-
	supply		-		+	-		-								-	+		
Cu	**demand**	+	+		+	+				-		+	+			-			
	price		+	+	+		+		+	+				+			+	+	-
	supply						+	+						+		-	+		
Ni	**demand**	+	+		+				+			+			+	-			
	price		+	+	+	+				+			+	+	+		+	+	-
	supply						+	-			+				+	-			
Pb	**demand**	+	+		+					-		+				-			
	price		+	+	+	+			+				+	+			+	+	
	supply				-		+	-						+			+		
Sn	**demand**	+	+		+					-		+	+		+	-	+		
	price		+	+	+	+			+							+	+	+	
	supply						+	-						+			+		
Zn	**demand**	+	+		+					-		+			+	-		+	
	price		+	+	+	+			+	+		+	+	+	+		+	+	+

This table displays the results of the GIRF analysis of the MS-GVAR model based on the weight matrix demand (**D**), assuming each commodity market is situated in its calm regime. We analyze the response of the column variables to a shock of the row variables supply (**supply**), demand (**demand**) and price (**price**) of aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn). Significant positive (+) or negative (-) effects on the 68%- level are displayed.

[13]We calculate the generalized impulse response functions via a Monte Carlo integration, described in Section 3.2.4.2. Hereby, we draw $N_{hist} = 500$ histories, and $N_{shock} = 500$ shocks.

[14]In particular, we draw $N_{boot} = 500$ times $T_{boot} = 250$ residuals with replacement to generate the bootstrap sample.

In analogy to Table 5.3 displaying the GIRF results of the GVAR model in Section 5.1, the diagonal of Table 5.13 shows significant positive responses for all variables, which is rather unsurprising, as it captures the effect of a shock to the response variable itself. Despite the concerns of false negatives within GIRF analyses in general, see Runkle (1987), Lütkepohl (1990) and Galesi and Lombardi (2009), we obtain numerous significant responses in the cross-commodity dimension, underlining the importance of jointly modeling commodity markets, as discussed in Section 5.1.

To start, we examine the spillover effects within the individual commodity markets, before we focus on the effects in the cross-commodity dimension. In particular, the copper market is affected most by its own shocks. Hereby, an increase in the copper demand or price leads to a significantly rising supply, indicating the production volume of copper increases in response to a higher demand and price. Moreover, the demand of copper (price of tin) reduces in case of increasing supply (demand), which is rather counterintuitive, since we would expect a synchronous reaction. However, the GIRF methodology investigates direct as well as indirect effects on the attributes to an innovation of one standard deviation in a certain variable, therefore, the observed responses may be caused by unobservable, indirect effects. In addition, the aluminum price positively affects its demand, whereas zinc's supply and demand significantly react to a shock in the price of zinc. While these results indicate the MS-GVAR model reflects the spillover effects in the individual markets of aluminum, copper, tin, and zinc, the nickel and lead market do not exhibit any significant responses to shocks in their own supply, demand and price, however, we observe various spillover effects in the cross-commodity dimension.

Similar to the results of the time-invariant GVAR model on annual data, presented in Section 5.1, the majority of the spillover effects is between the supply and supply, demand and demand, as well as price and price variables, indicating a concurrent behavior between the metal markets. While the impacts between the supply variables are less pronounced, the prices are the most important driver, underlining the common pattern in commodity prices.

Overall, we observe strong interdependencies between the prices themselves, except for tin, the smallest metal in terms of its trading volume, as its price does not influence and is not influenced by the other commodity prices. However, an increase in the price of aluminum, copper, nickel, lead, or zinc leads to rising prices of the other commodities, indicating a common behavior in the metals' prices. The only exception is zinc, as its price reduces in response to a positive shock in the aluminum, copper, and nickel price. This decrease in the zinc price might be caused by a disproportionaly increase in its supply, compared to the demand increase, also caused by rising prices. Moreover, shocks to the copper price affect the other commodities, while the copper price itself does not react to changes in the other prices, indicating a strong impact of copper on the other commodity markets, which is in line with the findings of the time-invariant GVAR model.

While the prices influence each other, supply and demand are also highly affected by rising prices. Hereby, the positive reactions of the supply in response to price shocks can be explained by indirect effects due to the co-production relation. A higher price of one commodity cause an increase in its production volume, and therefore also in the production volume of a co-produced metal. Thereafter, the demand might adjust in response to the increase in the overall supply, leading to a demand increase. Further, the inverse relation between zinc's and the other metals' prices cause the positive response of demand to price shocks. In particular, a higher metal's price lead to a reduction in another price, and ultimately, to a higher demand.

Turning to a commodity perspective, we observe the majority of the spillover effects between the aluminum, copper, nickel, tin, and zinc markets, whereby especially copper and tin are affected by the other markets. While the GIRF results of the time-invariant GVAR model, based on annual data from 1970 to 2019, indicate tin is least connected to the other markets, the time-varying MS-GVAR model detects various spillover effects from, and to, the tin market, underlining the markets are more connected in recent times. Hereby, an increase in the demand for copper-tin alloys, used in machine- and tool-construction, and in electrical engineering and electronics, might explain the increased connectedness of tin with the other markets. Moreover, since the late 2000s, there is an increased interest in copper-zinc-tin-sulfides, a quaternary semiconducting compound, for applications in thin film solar cells. In contrast, lead is least connected to the other markets. While the GVAR model reflects the time-invariant dependencies between the metal markets in the period from 1970 to 2019, the MS-GVAR model considers the time-varying interdependencies in the period from 1995 to 2020, especially, a period where lead was largely banned from many applications. Moreover, the dependencies are reflected by the co-consumption relation between the metals, which probably explains that the co-production relation between lead and zinc is not represented.

In contrast, the co-consumption of aluminum with the other metals explains the various spillover effects from, and to the aluminum market. In particular, demand shocks lead to a significant increase in the supply and demand of aluminum, indicating aluminum's production and consumption are highly affected by the demand of the other commodity markets, probably caused by their common applications. Moreover, aluminum's demand and price significantly increase in response to price shocks, further underlining the sensitivity of the aluminum market to changes in the other commodities.

Similar to the time-invariant analysis in Section 5.1, aluminum and copper are related most. In particular, the increase in aluminum demand (price) in response to a positive shock to the copper demand (price) can be explained by their common applications in electrical conduction, automotive and aerospace industries. Further, an increase in the demand of aluminum (copper) leads to a rising production volume, as the copper (aluminum) supply is likely to respond to the associated increase in the demand of copper (aluminum). Moreover, higher prices, and the associated higher production volumes, indirectly lead to an increase in the demand variables. However, a positive shock to the aluminum supply leads to a reduction in copper supply, which might be explained by the indirect effect of substitution of copper by aluminum, e.g. in electrics, as the oversupply in aluminum might be used for copper's applications and therefore, reduces the production volume of copper.

In addition, the price of copper reacts significantly to shocks in the production volume of the other commodities, whereas changes in the metals' demand lead to a higher interest in copper due to common applications, and therefore to an increase in copper supply. Moreover, a positive shock to prices leads to an increase in copper's supply and demand, suggesting the production volume as well as the consumption of copper increases in periods of rising prices. However, the copper price does not react to price changes, whereas it affects the other commodity prices, indicating a strong impact of copper on the other commodity markets, since copper is one of the largest metal in terms of the trading volume. Moreover, indirect effects in the GIRF analysis and the balance between its supply and demand increase might compensate the reaction of copper's price.

While the lead demand increases in response to demand shocks, supply shocks negatively affect the supply of nickel, whereas rising prices lead to increasing demand (and price) of nickel. Moreover, we observe various negative spillover effects from demand variables to the prices of nickel and tin, which are rather counterintuitive, since we would expect a synchronous reaction of prices on demand shocks. However, unobservable, indirect effects, which are also reflected in the GIRF analysis, might cause these results. Further, a positive supply shock reduces tin's price, while the supply of tin increases in response to rising prices, indicating the production volume increases, probably due to higher demand and price expectations for tin. Moreover, supply shocks positively affect zinc's supply, whereas rising prices increase the supply and demand of zinc simultaneously.

Comparing the spillover effects under the calm regime in the monthly analysis with the time-invariant responses in the annual analysis of the GVAR model, displayed in Table 5.3, we observe different spillover effects within the individual commodity markets. Hereby, the time-invariant model detects more interactions between supply and demand, whereas the time-varying model indicates more spillover effects from prices to the fundamentals. The differences in the spillover effects within the markets might be explained by the different time period under consideration, as the GVAR model examines a longer period, in particular, before the financialization, and the resulting stronger connection between the fundamentals. Furthermore, there are various spillover effects between the commodity markets under the calm regime, which are not significant in the annual, time-invariant model, possibly caused by the considered horizon, as in the annual analysis, short-term fluctuations may be aggregated out. In particular, aluminum reacts significantly to shocks, while changes in the nickel, lead and zinc price affect the other commodities. Moreover, the tin market, which is least connected in the GVAR model, affects and is affected by the other commodities in the MS-GVAR model, underlining an increase in the interdependencies between the markets. However, the numerous significant responses in the cross-commodity dimension, observed in both models, underline the importance of jointly modeling commodity markets. Hereby, the fundamentals, especially demand, also affect, and are affected by, commodity prices. Moreover, we observe spillover effects between the commodity prices, which are even more pronounced in the time-varying MS-GVAR model. The strong connections between aluminum and copper in the time-invariant analysis can also be confirmed in the time-varying analysis, however, the connection between these two markets with the remaining commodities is stronger in the MS-GVAR model, probably due to the stronger influence of the post-financialization period.

5.2.3.2 Differences in the Spillover Effects between Calm and Volatile Regimes

The presented generalized impulse response functions in Table 5.13 indicate the spillover effects within and between the commodity markets under the assumption each market is situated in its calm regime. However, we are interested in whether and how the impact of shocks changes over time. Therefore, we estimate the spillover effects under the volatile regime and examine the differences in the corresponding results, see Table 5.13 and Table 5.14 for the effects under the calm and volatile regime, respectively. In general, the shock size is larger under the volatile regime due to the higher variances in the variables, which might lead to stronger responses. However, the bootstrap bands are wider at the same time, which is why the results do not differ across the states in terms of significance. In particular, we do not observe any differences in the significant responses between the two regimes, indicating a shock to one variable either leads to a significant response or not, independent of the underlying regime.

However, the magnitude of the spillover effects differs between the states. Hereby, we apply the one-sided Wilcoxon-test[15] on the absolute value of the median GIRFs, to examine whether the responses of a shock to a variable is stronger under the volatile regime, compared to the calm regime in absolute terms, see Table 5.15 for the test statistics. In particular, apart from shocks to the aluminum and nickel price as well as to the lead and tin market, the responses are significantly stronger under the volatile regime, at least at the 10% level, indicating the spillover effects are stronger in more volatile periods. Thereby, shocks to the aluminum, copper, nickel, and zinc market cause significantly stronger reactions in the commodity markets, especially the prices are affected to a larger extent.

[15]Since the GIRFs do not follow a normal distribution, we apply the non-parametric Wilcoxon signed rank test (Wilcoxon) test instead of the t-test. However, the results of the t-test are similar.

Table 5.14: GIRF results of the MS-GVAR model based on the demand weight matrix under the volatile regime

		Al			Cu			Ni			Pb			Sn			Zn		
		supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
Al	**supply**	+			-		+	-								-	+		
	demand		+		+					-		+				-			
	price		+	+	+	+			+	+			+	+	+		+	+	-
Cu	**supply**		-		+	-		-								-	+		
	demand	+	+		+	+				-		+	+			-			
	price		+	+	+		+		+	+				+			+	+	-
Ni	**supply**						+	+						+		-	+		
	demand	+	+		+				+			+			+	-			
	price		+	+	+	+				+			+	+	+		+	+	-
Pb	**supply**						+	-			+				+	-			
	demand	+	+		+					-		+				-			
	price		+	+	+	+			+				+	+			+	+	
Sn	**supply**				-		+	-						+			+		
	demand	+	+		+					-		+	+		+	-	+		
	price		+	+	+	+			+							+	+	+	
Zn	**supply**						+	-						+			+		
	demand	+	+		+					-		+			+	-		+	
	price		+	+	+	+			+	+		+	+	+	+		+	+	+

This table displays the results of the GIRF analysis of the MS-GVAR model based on the weight matrix demand (**D**), assuming each commodity market is situated in its volatile regime. We analyze the response of the column variables to a shock of the row variables supply (**supply**), demand (**demand**) and price (**price**) of aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn). Significant positive (+) or negative (-) effects on the 68%- level are displayed.

Table 5.15: Results of the Wilcoxon-test for the assessment of differences in the magnitude of spillover effects under the calm and volatile regime

		Al			Cu			Ni			Pb			Sn			Zn		
		supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
Al	**supply**	81**	91***	91***	89**	86**	91***	74*	91***	91***	85**	88**	81**	91***	87**	91***	91***	87**	89**
	demand	91***	82**	81**	91***	85**	70*	91***	77*	86**	89**	90***	80**	91***	71*	87**	88**	88**	91***
	price	25	15	15	13	3	23	18	20	15	28	4	42	13	5	7	24	9	20
Cu	**supply**	38	53	52	71*	62	73*	41	66.	64.	58	67.	36	51	34	66.	64.	66.	55
	demand	74*	70*	66.	71*	49	59	65.	62	52	69.	86**	51	69.	71*	45	59	47	51
	price	90***	87**	91***	91***	88**	91***	91***	91***	91***	82**	90***	91***	89**	79**	72*	73*	85**	84**
Ni	**supply**	89**	88**	91***	62	83**	73*	91***	89**	87**	78*	78*	57	91***	45	91***	84**	87**	84**
	demand	59	61	61	71*	72*	59	64.	65.	70*	67.	80**	41	91***	68.	53	70*	72*	73*
	price	86**	68.	63	84**	42	45	84**	60	74*	53	51	79**	52	45	64.	48	71*	79**
Pb	**supply**	1	1	2	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0
	demand	10	7	0	0	0	0	0	0	0	0	0	0	1	1	0	7	0	7
	price	0	1	18	3	0	2	4	0	0	1	2	0	0	0	0	0	0	11
Sn	**supply**	29	19	31	64.	0	4	36	5	14	18	22	41	13	5	20	46	12	30
	demand	28	11	39	55	46	52	42	51	29	58	20	24	26	29	31	33	34	65.
	price	29	37	24	34	12	38	17	31	2	18	29	31	36	25	28	33	28	47
Zn	**supply**	91***	91***	81**	81**	91***	80**	87**	85**	74*	84**	86**	91***	79**	73*	91***	82**	86**	83**
	demand	91***	84**	91***	82**	91***	91***	89**	89**	74*	89**	88**	81**	67.	83**	82**	86**	91***	81**
	price	78*	66.	78*	80**	79**	69.	85**	77*	89**	63	72*	76*	88**	72*	60	83**	84**	83**

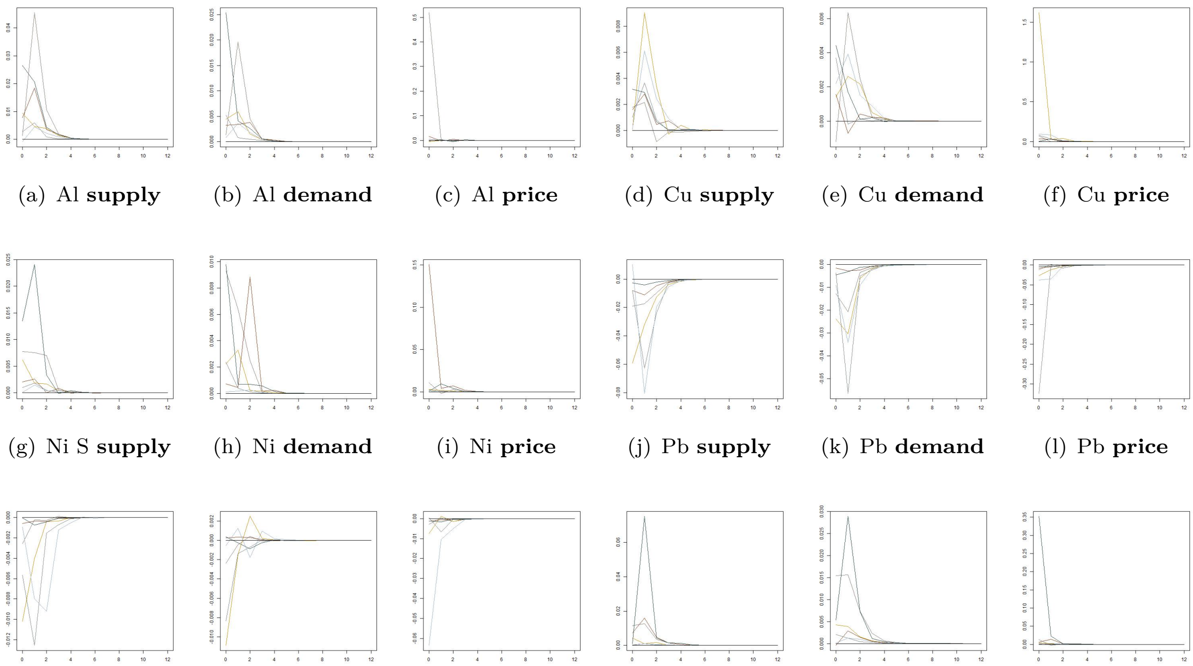
This table displays the statistics of the Wilcoxon-test with corresponding significance level (0.1% (***), 1% (**), 5% (*) and 10% (.)), to assess whether the magnitude of the spillover effects are stronger in absolute terms, under the volatile regime than under the calm regime. Hereby, we investigate the differences in the spillover effects of the column variables to shocks in the row variables (**supply**), demand (**demand**) and price (**price**) of aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn).

Due to the stronger reactions in the commodity markets, the spillover effects may lead to higher risks in volatile periods, compared to calm periods. For that reason, we further investigate the conditional value at risk (CoVaR)[16] of the variables, a risk measure which represents the

[16] As all original variables were non-stationary, we base the entire analysis on the logarithmic return data and hence, also the conditional value at risk values are based on logarithmic returns.

expected return in the worst 10% of cases. In particular, the conditional value at risk indicates the average maximum increase in the variables and therefore, provides an indication of the increases in the commodity markets, especially the price, in response to shocks. In this context, we focus on the price risk and graphically display the difference between the conditional value at risk values under the volatile and calm regime of the prices for shocks to each variable in Figure 5.6. The corresponding differences of the conditional value at risk of the supply and demand variables are presented in Figure D.1 and Figure D.2.

Figure 5.6: Differences in the conditional value at risk of the spillover effects on the **price** variables in the MS-GVAR model under the volatile vs. the calm regime



(a) Al **supply** (b) Al **demand** (c) Al **price** (d) Cu **supply** (e) Cu **demand** (f) Cu **price**

(g) Ni S **supply** (h) Ni **demand** (i) Ni **price** (j) Pb **supply** (k) Pb **demand** (l) Pb **price**

(m) Sn **supply** (n) Sn **demand** (o) Sn **price** (p) Zn **supply** (q) Zn **demand** (r) Zn **price**

These figures indicate the differences in the conditional value at risk (CoVaR) of the spillover effects from the individual supply (**supply**), demand (**demand**) and price (**price**) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) to the commodity prices of aluminum (Al), copper (Cu), Nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) under the volatile vs. the calm regime.

Overall, we observe shocks to the aluminum, copper, nickel and zinc market cause an increased price risk under the volatile regime, indicating the prices react stronger to changes in the other markets in periods of more fluctuations. Moreover, the supply and demand variables exhibit higher risks in response to these spillover effects. However, the conditional value at risk is higher under the calm period in response to changes in the lead and tin market, confirming the reactions and the corresponding risks are not stronger in the volatile periods, in line with the Wilcoxon-test results.

Although the comparison of the significance of the GIRFs under the calm and volatile regime indicate the spillover effects equal under the calm and volatile regime, the impact of fundamentals as well as the effects between prices are stronger in the volatile regime, as the magnitude is generally larger, implying stronger spillover effects. These larger effects in terms of the magnitude cause higher risks of the spillover effects for shocks in the aluminum, copper, nickel and zinc markets under the volatile regime.

Therefore, the commodity markets react stronger to any changes in the market in periods of high fluctuations, whereas under the calm period they only slightly respond to shocks. This finding underlines the stability of the markets under the calm regime, whereas the responses are more pronounced in the volatile regime, suggesting the risk is even higher under the volatile regime, due to possible reactions in response to shocks in other commodity markets. However, while the magnitude of the responses is higher for almost all shocks under the volatile regime, the significance does not change, indicating neither the impact of fundamentals nor spillover effects between prices affect commodity markets more in calm or volatile regime. In contrast, the markets react more sensitive to both, shocks in the supply and demand as well as to prices, under the volatile regime.

5.2.3.3 Global Spillover Effects to Commodity Markets

Besides the spillover effects within and between the commodity markets, we also examine how global shocks affect the commodity markets under the calm and volatile regime in the MS-GVAR model. In particular, the responses of the commodity variables to innovations in the global economic activity, the exchange rate and the interest rate can be analyzed, similar to Section 5.1.2. However, due to the observed heteroscedasticity in the monthly exogenous variables and in line with the MS-GVAR model, we model the exogenous variables, world industrial production (IP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR), via a MSH(2)-VAR(1) model, enabling for a time-varying covariance matrix, in contrast to the time-invariant VAR model in Section 5.1.[17]

Hereby, the transition probabilities in Table 5.16 indicate a high probability to remain in the current state, especially, in the first state. Moreover, similar to the commodity-specific regime inferences in Figure 5.5, the regime inference presented in Figure 5.7 suggests the exogenous variables switch their regime to state two during the financial crisis, the European debt crisis as well as the onset of the Covid-19 pandemic,[18] each times of higher fluctuations, which is why we refer to state two as the volatile regime, whereas state one represents the calm periods. The distinct descriptive statistics for each regime in Table 5.17, where we assume the economy to be located either in state one or state two, underlines the variables, especially the Federal Funds Effective Rate, exhibit a higher volatility in state two.

Table 5.16: Transition probability matrices for the individual MS-VAR model of the exogenous variables

Exogenous variables	
0.97	0.03
0.30	0.70

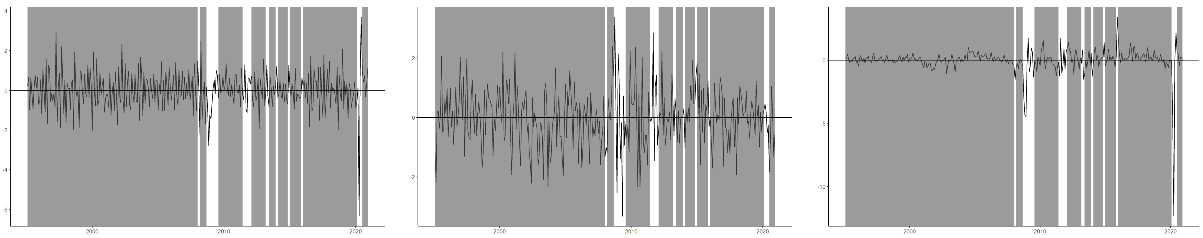
This table displays the transition probability matrix of the individual MS-VAR model of the exogenous variables world industrial production (IP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR).

Since the MS-VAR model of the exogenous variables also classifies the economy into calm and volatile periods, we examine in the following the impacts of global shocks on the commodity markets under the assumption the economy, as well as the commodity markets, are either under the calm or volatile regime and display the corresponding GIRF results in Table 5.18, whereby we first focus on the results derived under the calm regime.

[17] The results of the Durbin-Watson (DW) test indicate the MSH(2)-VAR(1) model of the exogenous variables does not suffer from autocorrelation, see Table D.2, therefore, the lag length of one is feasible from a statistical point of view.

[18] The extreme drop in the world industrial production and Federal Funds Effective Rate in response to the onset of the Covid-19 pandemic might affect the results of the MS-GVAR model and lead to biased impacts. Therefore, we examine whether the MS-GVAR model and the corresponding MS-VAR model of the exogenous variables change if these extreme values are replaced by a zero return. However, the results are comparable which is why we base our analysis on the original data.

Figure 5.7: Regime inferences of the exogenous variables



(a) IP (b) FX (c) FFR

These figures show the logarithmic returns of the exogenous variables world industrial production (IP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR), over the entire sample period from January 1995 to December 2020. Shaded areas indicate the smoothed probability to be in state one exceeds 50%, hence, it is more likely for the exogenous variables to be in state one at these points in time.

Table 5.17: Descriptive statistics of the exogenous variables based on the regime inferences of the MS-VAR model

	State	Min.	5% Q.	25% Q.	Med.	Mean	75% Q.	95% Q.	Max.	SD	Nr. Obs.
IP	1	-0.04	-0.03	-0.01	-0.00	0.00	0.01	0.03	0.06	0.02	285.00
	2	-0.13	-0.05	-0.02	0.00	-0.00	0.01	0.03	0.07	0.04	27.00
FX	1	-0.05	-0.04	-0.01	0.00	-0.00	0.01	0.03	0.05	0.02	285.00
	2	-0.07	-0.05	-0.02	0.00	0.00	0.02	0.06	0.07	0.03	27.00
FFR	1	-0.25	-0.12	-0.02	0.01	0.02	0.05	0.16	0.35	0.08	285.00
	2	-2.48	-0.89	-0.28	-0.09	-0.18	0.17	0.41	0.69	0.60	27.00

This table displays the descriptive statistics (minimum (Min.), 5%, 25%, 75%, 95% quantile (Q.), median (Med.), mean (Mean), maximum (Max.), standard deviation (SD), and number ob observations (Nr. Obs.)) of the stationary, exogenous variables world industrial production (IP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR), being either in state one or in state two, when assuming the markets are in state two if the smoothed probability to be in state two exceeds 50%.

Overall, similar to the analysis of the GVAR model, we observe the shocks to the macroeconomic variables affect each commodity market to a similar extent, however, the impact differ, indicating the relation between the economy and the commodity markets changed between the different considered time periods. First of all, a global demand shock, represented by a positive shock to the world industrial production cause decreasing commodity markets, the only exceptions are tin and zinc supply as well as copper and tin demand which increase. The observed decline in the markets is rather counterintuitive, since we would expect a synchronous reaction, similar to the positive responses to the world gross domestic product shock in the GVAR model. While the world industrial production and the commodity markets are positively correlated, the lagged world industrial production exhibits a negative correlation with the markets, probably caused by its negative autocorrelation, as the world industrial production fluctuates around its mean, see Figure C.15. Due to the recursive calculation of the GIRFs, the lagged exogenous variables also affect the commodity markets, and probably their effects predominate, compared to the positive contemporaneous relation. In addition, indirect effects, also reflected in the GIRF analysis, may intensify the negative reactions of the commodity markets, as an increase in the global demand cause rising interest rates, which lead to decreasing demand and ultimately declining commodity prices.

Moreover, an appreciation of the U.S. dollar, represented by a positive shock to the U.S. dollar index, positively affects the commodity markets, contrary to the negative reaction observed in the annual long-term analysis of the GVAR model. In line with the results of the GVAR model, nickel's, lead's, and zinc's demand as well as nickel's price decline probably caused by the predominate reduction in the demand for consumers holding other currencies, for which a stronger U.S. dollar implies the metals, quoted in U.S. dollars, become more expensive. In contrast, aluminum, copper, and tin markets increase in response to an appreciation of the U.S. dollar. Hereby, the higher demand for commodities of consumers holding the U.S. dollar probably predominates the reduction in demand for consumers holding other currencies, finally, causing rising markets. The mixed results between the time-invariant and time-varying analysis might be explained by the different considered time horizon, indicating the relation between the economy and the commodity markets changed over time. Since the MS-GVAR model only reflects the more recent times, with more financialized commodity markets, the demand for consumers holding the U.S. dollar probably predominates the foreign demand, at least for aluminum and copper, the largest metal markets in terms of the trading volume.

Besides the effects of global demand and exchange rate, we also investigate how changes in the monetary policy affect commodity markets. Hereby, a contrarian monetary policy, reflected via a positive interest rate shock, cause increasing supply for copper and nickel, decreasing demand for all commodities, as well as decreasing prices for aluminum, copper and nickel. These inverse spillover effects underline the theory of Frankel (2008), who argues the cost of capital for holding a commodity should decrease and the demand for commodities as an alternative asset class should increase in response to an expansionary monetary policy. In contrast, the positive reaction of the lead, tin and zinc price are in line with the findings of the GVAR model as well as the studies of Hammoudeh et al. (2015), and Schischke and Rathgeber (2023). These heterogeneous results further underline the commodity markets changed over time, as the GVAR model, reflecting the annual relations in the period from 1970 to 2019, observes rising commodity markets, whereas the MS-GVAR model, representing the monthly relations in the period from 1995 to 2020, detects a synchronous behavior in the lead, tin, and zinc price, probably caused by a disproportionate reduction in the supply, and an inverse reaction in the aluminum, copper and nickel market.

Table 5.18: GIRF results of the MS-GVAR model for shocks to the exogenous variables

		Al			Cu			Ni			Pb			Sn			Zn		
		supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
calm	IP	-	-	-	-	+	-	-	-	-	-	-	-	+	+	-	+	-	-
	FX	+	+	+	+	+	+	+	-	-	+	-	+	+	+	+	+	-	+
	FFR	-	-	-	+	-	-	+	-	-	-	-	+	-	-	+	-	-	+
volatile	IP	-	-	-	-	+	-	-	-	-	-	-	-	+	+	-	+	-	-
	FX	+	+	+	+	+	+	+	-	-	+	-	+	+	+	+	+	-	+
	FFR	-	-	-	+	-	-	+	-	-	-	-	+	-	-	+	-	-	+

This table displays the results of the GIRF analysis of the MS-GVAR model based on the weight matrix demand (**D**), assuming each commodity market as well as the exogenous variables are situated in their calm or volatile regime. We analyze the response of the column variables, supply (**supply**), demand (**demand**) and price (**price**) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), to a shock of the row variables world industrial production (IP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR), where significant positive (+) or negative (-) effects are displayed, based on the 68%- level.

Comparing the impacts of the global shocks under the calm and volatile regime in Table 5.18, we do not detect any differences in the significance of the effects, similar to the results of the endogenous shocks presented in Section 5.2.3.2. While the Wilcoxon test above underline the spillover effects within and between commodity markets are generally more pronounced in volatile periods, the one-sided Wilcoxon test applied on the GIRFs of the exogenous shocks indicates the absolute magnitude of the spillover effects is comparable across the regimes, see Table 5.19. In particular, the global shocks highly affect commodity markets in calm as well as volatile periods.

Moreover, we examine how the risk of increasing prices differs between calm and volatile periods. Therefore, we calculate the conditional value at risk (CoVaR) under both regimes, which indicates the expected increase in the markets in the worst 10% of cases in response to a global shock. The differences in the conditional value at risk values under the volatile and calm regime are displayed in Figure D.3. Hereby, we observe mixed results. While the risk of increasing variables is in the short-term particularly stronger under the volatile regime, the overall risk of increases in the commodity markets is comparable under both regimes, underlining the economy affect the commodity markets to a similar extent, in calm as well as volatile periods.

Overall, although the comparison of the significant GIRFs of the endogenous as well as exogenous variables under the calm and volatile regime indicate the spillover effects coincide in both regimes, the impact of fundamentals, as well as the effects between the markets, especially between prices, are stronger in the volatile regime, as the magnitude is generally larger, implying stronger spillover effects and ultimately higher risks.

Table 5.19: Results of the Wilcoxon-test for the assessment of differences in the magnitude of spillover effects from the exogenous variables to the commodity markets of the MS-GVAR model under the calm and volatile regime

	Al			Cu			Ni			Pb			Sn			Zn		
	supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
IP	27	18	78**	78**	23	68*	54	20	36	23	9	0	47	34	0	9	9	0
FX	32	21	59.	55	26	36	38	12	43	32	19	5	43	33	32	29	28	0
FFR	21	22	25	54	31	32	34	36	41	12	29	7	33	46	37	28	22	8

This table displays the statistics of the Wilcoxon-test with corresponding significance level (0.1% (***), 1% (**), 5% (*) and 10% (.)), to assess whether the magnitude of the spillover effects are stronger in absolute terms, under the volatile regime than under the calm regime. Hereby, we investigate the differences in the spillover effects of the column variables supply (**supply**), demand (**demand**) and price (**price**) of aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) to shocks in the row variables world industrial production (IP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR).

5.2.4 Out-of-sample Forecast Performance of the (time-varying) Commodity Market Models

To underline the importance of a time-varying analysis, we assess the out-of-sample forecast performance of the MS-GVAR model and compare our results against the GVAR model as benchmark model. Therefore, we forecast the commodity-specific variables with a rolling window approach and compare the predictability using the test of Clark and West (2007) for nested models.[19] Hereby, we split our dataset into a rolling in-sample window covering $J/(I+J) \approx 3/4$ observations of the entire sample, starting with data from January 1995 to December 2013, and an out-of-sample window covering the period from January 2014 to December 2020.[20] The model selection of the MS-GVAR model, based on the in-sample data, indicates the MSH(2)-VAR(1) and MSH(3)-VAR(1) model, which allows for regime-switches in the covariance matrix, perform best. For simplicity and for consistence with the MS-GVAR model in Section 3.2.1.6, we apply the MSH(2)-VAR(1) model throughout the entire analysis.

For each step in time of the out-of-sample window, we estimate the (MS-)GVAR model based on the in-sample data and forecast all commodities' monthly variables one-step ahead.[21] Thus, we estimate and solve the (MS-)GVAR model recursively, whereby we re-evaluate the regime-probabilities in each step in the out-of-sample period.

[19]The MS-GVAR model nests the GVAR model in a natural way, as the only difference is in the time-varying parameters. For a better comparability of the models, we include an intercept in the GVAR model for the predictions, contrary to the time-invariant models in Section 5.1, and Section 5.3.

[20]Please refer to Appendix D.2.3 for more detailed information on the models.

[21]Hereby, we include the observed exogenous variables, instead of predicting the corresponding values, such that the predictions are not biased by the forecasts of the exogenous variables.

Table 5.20: Results of the model selection procedure for the MS-GVAR model in the in-sample period

Nr. States	Nr. lags	MSI	MSIH	MSH	MSA	MSAH
2	1	1857.72	1819.43	1811.27	1975.10	1957.00
	2	1880.21	1847.98	1828.52	2110.88	2041.71
3	1	1859.84	1831.94	1811.11	2104.18	2078.70
	2	1888.51	1821.18	1813.98	2353.61	2253.04

This table displays the model selection results for the MS-GVAR model in the in-sample period, based on the information criterion of Hannan and Quinn (1979), proposed in Section 3.2.1.6. Hereby, the MS-GVAR model is estimated using different number of states (Nr. States), lag lengths (Nr. lags), as well as specifications (MSI, MSIH, MSH, MSA, MSAH).

The results of the Clark and West (2007) test are displayed in Table 5.21. Hereby, we compare the predictability of each commodity-specific variable. Overall, the MS-GVAR model significantly outperforms the time-invariant GVAR model in 6 of the 18 variables at the 10% significance level. In particular, the MS-GVAR model outperforms the predictions of the aluminum, copper, nickel, and zinc price, whereas the time-varying analysis barely improves the predictions of the supply and demand variables.

Table 5.21: Results of the out-of-sample Clark and West (2007) test

Al			Cu			Ni			Pb			Sn			Zn		
supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
0.12	-0.81	**1.38.**	0.72	1.27	**1.43.**	0.56	**1.58.**	**1.30.**	**2.18***	0.70	0.75	0.06	-0.01	-0.01	0.57	1.22	**2.31***

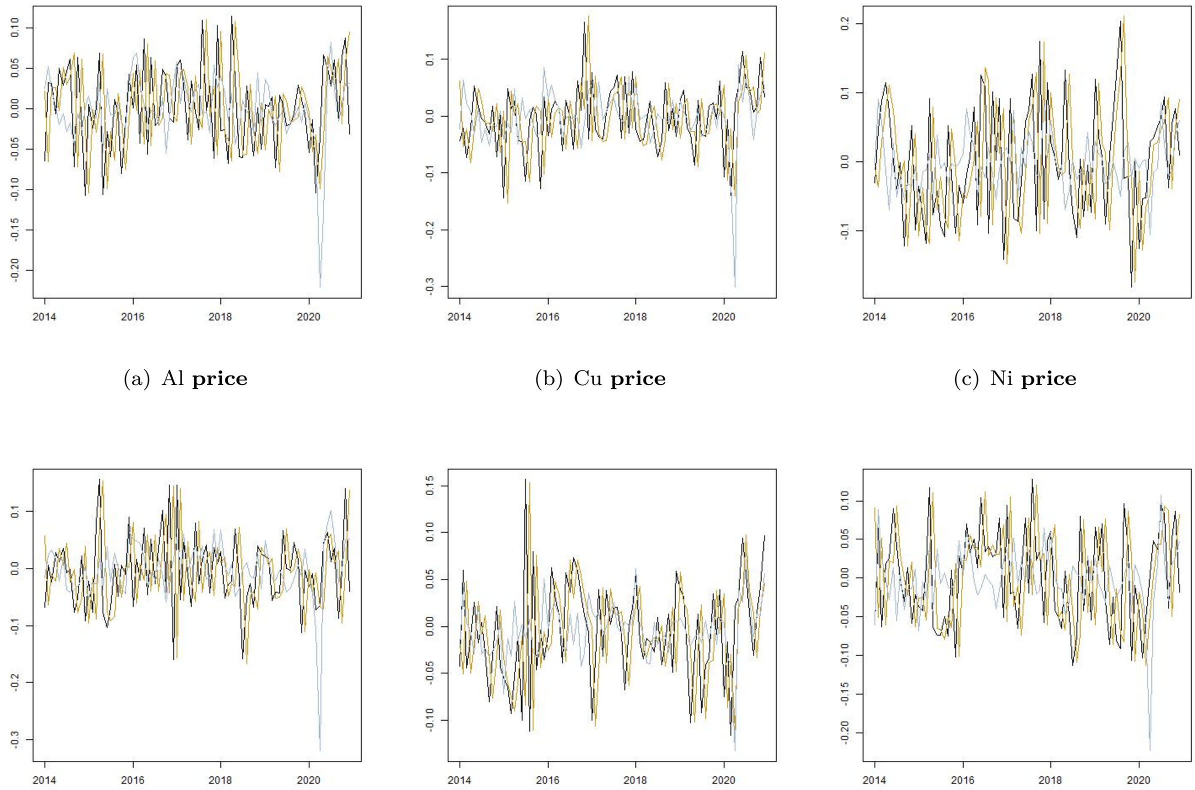
This table displays the out-of-sample forecast performance of the MS-GVAR model compared to the GVAR model, based on the statistic of the Clark and West (2007) test with corresponding significance level (0.1% (***), 1% (**), 5% (*) and 10% (.)). Hereby, the forecasts are estimated for the out-of-sample window from January 2014 to December 2020 for each individual variable supply (**supply**), demand (**demand**) and price (**price**) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn).

Figure 5.8 shows a comparison between the observed prices in the out-of-sample period as well as the estimated prices from the (MS-)GVAR model. The corresponding plots for the supply and demand variables are displayed in Figure D.12 and Figure D.13. While the predicted prices of the MS-GVAR model follow the direction and magnitude of the fluctuations in the actual prices, with a brief delay, the GVAR model is not able to reflect the price movements. However, the GVAR model slightly underestimates the fluctuations in the supply and demand variables, but reflects the movements better compared to its price forecasts. Overall, the results indicate the time-varying analysis reproduces the fluctuations in the variables, especially for the commodity prices, leading to an improved predictability.

To highlight the MS-GVAR model is able to forecast commodity prices, but also represents the co-movement between the prices, we compare the predicted price correlations of the (MS-)GVAR model with the market observed dependencies, similar to the correlation analysis in Section 5.1. Therefore, we use the predicted prices of the (MS-)GVAR model, displayed in Figure 5.8, calculate the corresponding Pearson correlation matrices and compare them with the observed correlations in the period from January 2014 to December 2020, see Table 5.22, Table 5.23, and Table 5.24.[22] Hereby, we recognize the GVAR model overestimates the true correlations. This

[22]In contrast to the correlation analysis of the GVAR model in Section 5.1.2.2, we focus on the monthly correlations in the period from 2014 to 2020, instead of the annual correlations in the period from 2010 to 2019. Overall, the observed correlations differ, as the dependencies between the metal prices change over time, further highlighting the importance of a time-varying analysis. In general, we notice the metal prices are less dependent on each other in the more recent period, indicating the common pattern decreased at the end of the 2010s. However, the declining prices in 2015 in contrast to the rising zinc price caused by supply problems no longer distort the results of the monthly analysis as much, which is why zinc observes a positive correlation with all other prices.

Figure 5.8: Observed and predicted prices in the out-of-sample period



(a) Al **price** (b) Cu **price** (c) Ni **price**

(d) Pb **price** (e) Sn **price** (f) Zn **price**

These figures compare the observed prices (**price**) of aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) in the out-of-sample period from January 2014 to December 2020, indicated by the black line, with the estimated prices of the GVAR model and the MS-GVAR model.

might be caused by the high correlations in the period until 2013, whereas the decrease in the dependencies around 2015, see Figure 1.1, is not reflected by the time-invariant analysis. While the slowdown in Chinese demand and the oil price drop cause generally declining metal prices, aluminum's and especially zinc's price increased due to demand and supply concerns, resulting in divergent price behavior. In contrast, the MS-GVAR model is able to reflect the dependencies between the prices exceptionally well, with differences in the correlations for the predicted and observed prices being smaller than 5%, indicating the time-varying analysis takes into account the change in the correlation structure. Hereby, the predictions of the MS-GVAR model follow the direction and magnitude of the fluctuations in the observed prices with a delay, see Figure 5.8, but represent the correlation structure.

Overall, the results indicate the MS-GVAR model reproduces the fluctuations as well as the relations between the metal prices. Moreover, the time-varying properties allow for an in-depth analysis of the commodity market structure. In particular, the spillover effects and the associated risks can be analyzed in more detail. Hereby, the application of the MS-GVAR model on the industrial metal markets reveals the spillover effects are stronger during volatile periods, implying higher risks. Our results emphasize the importance of time-varying analyses in the context of risk assessments. Especially the transformation of the energy system, to achieve carbon neutrality, will affect commodity markets, which is why fluctuations should be taken into account in the risk analysis.

Table 5.22: Correlation matrix of the observed spot prices

	Al	Cu	Ni	Pb	Sn	Zn
Al	1.00	0.48	0.46	0.46	0.21	0.51
Cu	0.48	1.00	0.47	0.50	0.25	0.63
Ni	0.46	0.47	1.00	0.41	0.23	0.43
Pb	0.46	0.50	0.41	1.00	0.13	0.60
Sn	0.21	0.25	0.23	0.13	1.00	0.24
Zn	0.51	0.63	0.43	0.60	0.24	1.00

This table displays the correlation matrix of the observed spot prices, between 2014 and 2020.

Table 5.23: Correlation matrix of predicted prices based on the GVAR model

	Al	Cu	Ni	Pb	Sn	Zn
Al	1.00	0.77	0.70	0.81	0.63	0.83
Cu	0.77	1.00	0.71	0.95	0.85	0.89
Ni	0.70	0.71	1.00	0.70	0.67	0.81
Pb	0.81	0.95	0.70	1.00	0.82	0.87
Sn	0.63	0.85	0.67	0.82	1.00	0.83
Zn	0.83	0.89	0.81	0.87	0.83	1.00

This table displays the correlation matrix of the predicted prices, using the GVAR framework, in an out-of-sample rolling window forecast from 2014 to 2020.

Table 5.24: Correlation matrix of predicted prices based on the MS-GVAR model

	Al	Cu	Ni	Pb	Sn	Zn
Al	1.00	0.48	0.45	0.45	0.22	0.49
Cu	0.48	1.00	0.44	0.49	0.24	0.61
Ni	0.45	0.44	1.00	0.39	0.23	0.43
Pb	0.45	0.49	0.39	1.00	0.14	0.59
Sn	0.22	0.24	0.23	0.14	1.00	0.23
Zn	0.49	0.61	0.43	0.59	0.23	1.00

This table displays the correlation matrix of the predicted prices, using the MS-GVAR framework, in an out-of-sample rolling window forecast from 2014 to 2020.

5.3 Scarcity Risk of the German Energiewende[23]

The objective of this thesis is the risk analysis and comparison of the resource requirements for the German energy transition in regard to their availability, respectively their scarcity. Therefore, we investigate four potential transformation paths, generated to optimally reduce Germany's CO_2 emissions by 95% in 2050 compared to 1990, with regards to their resource scarcity risk. Hereby, each path represents a resource-demanding project, requiring an enormous amount of metals in its realization. In particular, we apply the proposed risk assessment framework, introduced in Section 3.3, taking into account the substitutability of commodities, the future required resource amounts of the project as well as the commodity market structure. The general idea of the developed framework is based on the supply and demand equilibrium, which is why we interpret a commodity's price as scarcity indicator. First, we calculate the risk of each considered commodity to become scarce. Subsequently, we aggregate the individual, commodity-specific risk measures on path level, enabling the comparison of the alternative transformation pathways of the German Energiewende regarding their resource scarcity risk.

Within the first step, we calculate the individual probability of scarcity (PS) per commodity. Hereby, we either use the (MS-)GVAR framework, generally introduced in Section 3.1 (Section 3.2), or simple logistic regression models. While the (MS-)GVAR framework models the commodity markets and their interrelations in a holistic way under consideration of their prospective relations induced by the resource requirements of the transformation paths of the German Energiewende, the logistic regression model enables for commodity-specific price influencing factors. Subsequently, we determine the commodity-specific risk measures, combining

[23]Parts of this section are published in the paper "Sustainable energy transition and its demand for scarce resources: Insights into the German Energiewende through a new risk assessment framework", Renewable and Sustainable Energy Reviews 176, 2023, co-authored by Patric Papenfuß, Max Brem, Paul Kurz, and Andreas Rathgeber.

the individual probability of scarcity with an appropriate substitutability score as well as the required resource demands. Finally, we aggregate the individual risk measures to the resulting expected loss due to scarcity (ES) on project level and compare the potential transformation paths of the German Energiewende with regards to their resource scarcity risk.

5.3.1 Commodity-specific Probability of Scarcity

In order to calculate the individual probability of scarcity per commodity, we define an appropriate price threshold, which, once exceeded, determines the scarcity of the commodity and, therefore, classifies the commodities into scarce and non-scarce states. In our case, we set this threshold statistically, based on annual price data from 2010 to 2019, representing the commodity prices of the previous decade, via the one-sigma approach displayed in Equation 3.159.[24]

Table 5.25: Commodity price threshold

	Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
$\$/t$	901638	2187	53318	7951	608299	647207	111979	81920	19813	2294	50831971	23155	2655

This table displays the price threshold for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn) in $U.S.\$/t$, derived from the one-sigma approach in Equation 3.159, based on annual price data in the period from 2010 to 2019.

With help of the resulting threshold values given in Table 5.25, we calculate the commodity-specific probability of scarcity, using either the (MS-)GVAR framework or the logistic regression model. Therefore, we first present the results of the probability of scarcity derived from the estimated (MS-)GVAR framework, subsequently, we display the two-step model selection for the logistic regression models as well as the corresponding probability of scarcity values.

5.3.1.1 Probability of Scarcity derived from the (MS-)GVAR Model

While we introduce the (MS-)GVAR methodology exemplary in Section 5.1 (Section 5.2), investigating the (time-varying) spillover effects between the industrial metal markets, we predict the commodity prices under consideration of their prospective relations induced by the German Energiewende to get the individual probabilities of scarcity per commodity. Hereby, we apply the GVAR[25] framework on annual data from 1970 to 2019 and focus on the industrial metals, as well as on the crucial commodities defined by Bastian et al. (2019)[26], whereas we estimate the MS-GVAR[27] model only for the industrial metals, using monthly data from 1995 to 2019, due to data limitations. Subsequently, we forecast the commodity prices under several, predefined scenarios of the endogenous as well as exogenous variables to derive individual probabilities of scarcity per commodity. Thereby, we get the probabilities of scarcity using a bootstrapping procedure, to obtain the probability distribution of the prices, as well as the definition of quantiles, see Section 3.3.1.

[24] In general, the supply of commodities is rather inelastic in the short-term, since the launch of a new mining project takes on average one decade. Therefore, we use data of the previous ten years to approximate the actual market situation. However, we examine to what extent the results remain valid if the sample period is reduced or enlarged, see the robustness analyses in Section 5.3.4.

[25] Please refer to Appendix D.3.1.1 for detailed information about the models. In particular, we include the exogenous variables world gross domestic product (GDP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR) to account for the common impact of the economy on prices.

[26] In case of dysprosium and neodymium, we approximate their supply and demand data by the relative supply and demand of rare earths metals, provided by U.S. Geological Survey (2018), which is why we have to exclude these two metals for the risk assessment via the (MS-)GVAR framework to avoid multicollinearity.

[27] Please refer to Appendix D.3.1.2 for detailed information about the models. In particular, we include the exogenous variables world industrial production (IP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR) to account for the common impact of the economy on prices.

The exemplary application of the (MS-)GVAR framework in Section 5.1 (Section 5.2) reveals strong spillover effects between the commodity-specific demand variables, indicating the strong impact of demand and the common applications of the metals, as the co-consumption leads to a concurrent behavior in the demand and ultimately in the commodity markets. As we compare the actual resource demands of several transformation pathways, the correlation of the future annual resource requirements per path might reflect best the corresponding co-consumption relation in the context of the German Energiewende, see Section 4.4.4. Therefore, we estimate the (MS-)GVAR model four times, using the weight matrices displayed in Table 4.27, Table 4.28, Table 4.29 and Table 4.30 (Table 4.31, Table 4.32, Table 4.33 and Table 4.34), representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$ paths, respectively. Detailed information on the models as well as the spillover effects can be found in Appendix D.3.1.

Due to data limitations, we estimate each GVAR model with one lag for the endogenous, external as well as exogenous variables, and without intercept. Hereby, the Durbin-Watson (DW) test implies all variables in the GVAR models, except lithium supply, do not exhibit any autocorrelation at the 5% significance level. Moreover, neither model suffers from heteroscedasticity nor structural breaks at the 5% significance level, except the indium market, indicating the time-invariant model is not able to fully display the time-varying relationship between indium's variables, demanding for a time-dependent model, see Table D.9.[28]

In case of the MS-GVAR model, we apply the model selection, see Section D.3.1.2.1, for each model using the different weight matrices, representing the four energy transition paths, separately. Hereby, we conclude, similar to the exemplary application of the MS-GVAR model based on the demand weight matrix in Section 5.2, the MSH(2)-VAR(1) models perform best, see Section D.3.1.2.1. Therefore, we estimate the MS-GVAR models with two states, one lag, a regime-dependent covariance matrix, but a regime-invariant intercept as well as regime-invariant parameters corresponding to the endogenous and exogenous variables. In particular, the two states enable to capture calm as well as volatile periods, while the time-varying covariance matrix solves the problem of heteroscedasticity, observed in the time-invariant models.

As the (MS-)GVAR models are based on stationary variables, we actually forecast the logarithmic returns of prices and calculate the prediction of the actual price under the assumption of an initial price level. In particular, since we specifically consider the resource requirements of the period from 2020 to 2050, we use the average price level of 2019 and the logarithmic returns forecasts[29] to predict an artificial price under several scenarios that we compare with the threshold price, see Table 5.26. For a sensitivity analysis, we also examine the average price of the previous decade as initial basis level, which coincides to the considered data used to calculate the threshold price.

While the prices of lithium and zinc are higher in the year 2019 compared to the average price level of the previous decade, the average prices of the other commodities are higher than the price observed in 2019, indicating the commodity prices declined at the end of the decade, see Appendix C.2.3. Hereby, the average indium and platinum prices are a multiple of their price in 2019, due to the decrease in their prices in mid 2010s, probably caused by the slowdown in Chinese demand and the sharp oil price drop. Overall, the initial price levels do not exceed their threshold prices, based on the one-sigma approach, using price data from 2010 to 2019.

[28]Overall, autocorrelated (or heteroscedastic) residuals might lead to too narrow confidence intervals, obtained by the bootstrap procedure, and therefore, the GVAR model might underestimate the probability of scarcity for lithium (indium), as the predictions are based on the bootstrap procedure. However, a modification of the lag length is not feasible due to data limitations.

[29]Actually, we base the (MS-)GVAR model on standardized logarithmic return data. Therefore, we first recalculate the logarithmic returns with help of the corresponding estimated mean and standard deviation used for the standardization. Subsequently, we compute the predicted level values.

However, due to the high initial price levels in 2019 (in the last decade) of zinc and lithium (lead), we expect these three commodities exhibit a higher probability exceeding their threshold prices. While the extreme rise in the price of zinc is caused by ongoing supply concerns, see Figure C.14, the increased interest in lithium for batteries leads to the positive trend in its price, see Figure C.8. Due to the additional demand caused by the energy transition, we expect the prices will continue to rise, especially for lithium, probably leading to (short-term) shortages. Therefore, we do not adjust the threshold price or the initial price level to reflect their higher risks of scarcity.

Table 5.26: Initial commodity price levels

	Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
2019	521485	1794	33289	6008	187818	103128	13917	1997	27770710	18634	2549
Mean	679740	1945	38077	6761	443541	86304	15304	2105	40119566	20452	2276

This table displays the initial price level for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn) to calculate price forecasts based on the return forecasts derived from the (MS-)GVAR models. Hereby, we use either the price observed in 2019, or the average price of the previous decade, from 2010 to 2019.

Using the (MS-)GVAR models with weight matrices representing the resource requirements of the $REMod - REF$, $REMod - SUF$, $REMod - PER$, and $REMod - UNA$ paths, respectively, as well as the initial price level, we estimate the price probability distribution via the bootstrapping procedure and compare the derived prices with the threshold prices to extract the path-specific probabilities of scarcity per commodity. In this context, the price distributions are derived by forecasting the commodity prices one-step ahead, under predefined conditions for the (historical) endogenous as well as exogenous variables, see Section 3.3.1, using data from 2010 to 2019 in line with the period of the threshold price.[30] Hereby, we consider a mean scenario, examining the probability of scarcity under normal circumstances, while we investigate in the shock and extreme scenario how the risk increases in periods when the variables take on extremer values. Within the (extreme) focus scenarios, we analyze the sensitivity of the probability of scarcity to an extreme value in an exogenous variable, while all other variables behave normal. Further, we examine in the quantile scenarios how the probability of scarcity increases if the variables are at different states of their distributions. The resulting values of the endogenous and exogenous variables are displayed in Table 5.27.

In general, the scenario values differ across the commodities and scenarios. While we observe the annual demand (and supply) generally increased over the previous decade, leading to positive mean scenario values, most of the prices decreased, resulting in negative values, as displayed in their time series plots in Section C.2.3. Overall, the values under the mean scenario are comparably high in absolute terms, especially for aluminum, lithium, nickel, lead, and platinum, indicating volatile markets over the previous decade, since we standardized the data over the entire sample period from 1970 to 2019. In contrast, the monthly variables exhibit more moderate values, as they reflect the monthly returns. However, the values under the median scenario differ from the values under the mean scenario for the annual as well as monthly variables, indicating the underlying variables are skewed. As the input variables are all standardized over the entire sample period from 1970 (1995) to 2019 to have mean zero and standard deviation one, the input values under the shock, extreme and (extreme) focus scenarios are relatively high, whereby the values of cobalt's, lead's, and platinum's supply and demand, nickel's supply, lithium's price as well as the Federal Funds Effective Rate (FFR) are outstanding. Moreover, the 25% and 40% scenarios mostly indicate negative returns, except for the world gross domestic product (GDP) and world industrial production (IP), whereas the input values naturally increase to higher values under the 60% and 75% quantile scenarios, which are, however, smaller

[30]For a robustness analysis, we examine to what extent the results remain valid if the scenario values are derived based on a reduced or an enlarged sample period.

than the values under the shock and extreme scenario. Overall, we would expect the highest probability of scarcity under the shock and extreme scenarios, while the negative input values under the 25% and 40% scenario probably lead to a reduced scarcity risk.

Table 5.27: Scenario values for the input variables

			Mean	Shock	Extr.	Foc. EA	Foc. FX	Foc. FFR	Foc. Extr. EA	Foc. Extr. FX	Foc. Extr. FFR	Q. 25%	Q. 40%	Q. 50%	Q. 60%	Q. 75%
GVAR	Ag	**supply**	-0.10	0.87	1.85	-0.10	-0.10	-0.10	-0.10	-0.10	-0.10	-0.53	-0.15	0.30	0.54	0.54
		demand	0.03	1.19	2.36	0.03	0.03	0.03	0.03	0.03	0.03	-0.65	-0.21	0.17	0.44	0.56
		price	-0.13	0.83	1.79	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.83	-0.54	-0.33	-0.13	0.11
	Al	**supply**	0.29	1.09	1.90	0.29	0.29	0.29	0.29	0.29	0.29	-0.21	0.15	0.27	0.38	0.57
		demand	0.36	1.31	2.26	0.36	0.36	0.36	0.36	0.36	0.36	-0.23	0.53	0.57	0.60	0.80
		price	-0.08	0.71	1.49	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.69	-0.42	-0.18	0.06	0.36
	Co	**supply**	-0.14	1.49	3.12	-0.14	-0.14	-0.14	-0.14	-0.14	-0.14	-0.24	0.05	0.18	0.30	0.40
		demand	0.25	1.61	2.98	0.25	0.25	0.25	0.25	0.25	0.25	-0.53	-0.21	-0.12	0.04	0.32
		price	-0.13	0.80	1.74	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.49	-0.31	-0.26	-0.11	0.18
	Cu	**supply**	-0.05	0.89	1.83	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.83	-0.72	-0.41	0.05	0.67
		demand	0.22	1.01	1.80	0.22	0.22	0.22	0.22	0.22	0.22	-0.18	-0.11	-0.07	0.00	0.25
		price	-0.06	0.76	1.57	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06	-0.55	-0.48	-0.45	-0.20	0.44
	In	**supply**	-0.08	0.45	0.98	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.24	-0.06	-0.04	0.02	0.12
		demand	-0.17	0.20	0.56	-0.17	-0.17	-0.17	-0.17	-0.17	-0.17	-0.37	-0.24	-0.18	-0.16	-0.12
		price	-0.19	0.65	1.49	-0.19	-0.19	-0.19	-0.19	-0.19	-0.19	-0.89	-0.33	0.01	0.27	0.41
	Li	**supply**	0.29	1.27	2.25	0.29	0.29	0.29	0.29	0.29	0.29	-0.19	-0.13	-0.04	0.00	0.38
		demand	0.25	1.31	2.36	0.25	0.25	0.25	0.25	0.25	0.25	-0.46	-0.22	-0.05	0.02	0.26
		price	-0.05	1.73	3.52	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.70	-0.48	-0.23	-0.11	0.27
	Ni	**supply**	0.34	1.80	3.26	0.34	0.34	0.34	0.34	0.34	0.34	-0.46	0.35	0.57	0.62	0.78
		demand	0.19	1.15	2.10	0.19	0.19	0.19	0.19	0.19	0.19	-0.43	0.16	0.50	0.70	0.97
		price	-0.13	0.70	1.52	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.80	-0.23	0.07	0.13	0.27
	Pb	**supply**	0.29	1.61	2.92	0.29	0.29	0.29	0.29	0.29	0.29	-0.54	-0.14	0.21	0.76	1.13
		demand	0.25	1.44	2.63	0.25	0.25	0.25	0.25	0.25	0.25	-0.82	-0.16	0.36	0.67	0.94
		price	-0.09	0.51	1.12	-0.09	-0.09	-0.09	-0.09	-0.09	-0.09	-0.57	-0.27	-0.12	0.02	0.25
	Pt	**supply**	-0.42	1.09	2.59	-0.42	-0.42	-0.42	-0.42	-0.42	-0.42	-1.86	-0.42	0.00	0.26	0.40
		demand	-0.23	1.29	2.81	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23	-0.84	-0.18	0.11	0.21	0.44
		price	-0.34	0.34	1.02	-0.34	-0.34	-0.34	-0.34	-0.34	-0.34	-0.54	-0.50	-0.44	-0.38	-0.30
	Sn	**supply**	0.23	1.19	2.14	0.23	0.23	0.23	0.23	0.23	0.23	-0.42	-0.06	0.09	0.17	0.83
		demand	0.15	0.81	1.46	0.15	0.15	0.15	0.15	0.15	0.15	-0.16	0.09	0.16	0.18	0.19
		price	-0.01	0.93	1.87	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.43	-0.17	-0.02	0.20	0.36
	Zn	**supply**	-0.26	0.59	1.44	-0.26	-0.26	-0.26	-0.26	-0.26	-0.26	-0.74	-0.26	-0.06	-0.05	-0.04
		demand	0.18	1.42	2.65	0.18	0.18	0.18	0.18	0.18	0.18	-0.49	0.11	0.25	0.40	0.61
		price	-0.00	0.61	1.22	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.52	-0.18	-0.12	-0.01	0.27
	exog	GDP	-0.56	0.24	1.03	-0.56	-0.56	-0.56	-0.56	-0.56	-0.56	-0.88	-0.76	-0.72	-0.49	-0.12
		FX	0.29	1.01	1.73	0.29	1.01	0.29	0.29	1.73	0.29	0.04	0.14	0.17	0.21	0.48
		FFR	0.52	1.46	2.40	0.52	0.52	1.46	0.52	0.52	2.40	-0.17	0.29	0.48	0.68	1.04
log. Reg.		U.S. IP	-0.09	0.48	1.06	-0.09	-0.09	-0.09	-0.09	-0.09	-0.09	-0.53	-0.08	0.10	0.22	0.25
		GDPc	-0.51	0.30	1.11	-0.51	-0.51	-0.51	-0.51	-0.51	-0.51	-0.84	-0.72	-0.68	-0.44	-0.05
		LIR	-0.11	1.45	3.02	-0.11	-0.11	-0.11	-0.11	-0.11	-0.11	-0.92	-0.78	-0.34	0.31	1.37
		CPI	0.18	2.43	4.67	0.18	0.18	0.18	0.18	0.18	0.18	-0.26	-0.23	-0.11	0.01	0.27
		MSCI	0.10	0.59	1.08	0.10	0.10	0.10	0.10	0.10	0.10	-0.33	-0.05	0.12	0.25	0.54
		OIL	-0.24	0.73	1.70	-0.24	-0.24	-0.24	-0.24	-0.24	-0.24	-0.59	-0.30	-0.16	0.11	0.43
		ND	-0.16	0.71	1.58	-0.16	-0.16	-0.16	-0.16	-0.16	-0.16	-0.95	-0.42	-0.23	-0.03	0.46
MS-GVAR	Al	**supply**	0.01	1.34	2.67	0.01	0.01	0.01	0.01	0.01	0.01	-0.66	-0.27	-0.08	0.30	0.53
		demand	0.02	1.08	2.14	0.02	0.02	0.02	0.02	0.02	0.02	-0.63	-0.31	-0.06	0.28	0.59
		price	-0.04	0.89	1.81	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.69	-0.30	-0.05	0.19	0.57
	Cu	**supply**	-0.03	0.94	1.91	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.58	-0.29	-0.01	0.18	0.55
		demand	0.00	1.08	2.17	0.00	0.00	0.00	0.00	0.00	0.00	-0.68	-0.18	0.06	0.24	0.73
		price	-0.09	0.71	1.51	-0.09	-0.09	-0.09	-0.09	-0.09	-0.09	-0.54	-0.33	-0.02	0.10	0.36
	Ni	**supply**	0.02	1.30	2.58	0.02	0.02	0.02	0.02	0.02	0.02	-0.74	-0.18	0.03	0.31	0.80
		demand	0.01	1.06	2.10	0.01	0.01	0.01	0.01	0.01	0.01	-0.71	-0.11	0.09	0.32	0.63
		price	-0.07	0.79	1.64	-0.07	-0.07	-0.07	-0.07	-0.07	-0.07	-0.77	-0.29	-0.08	0.05	0.58
	Pb	**supply**	-0.03	0.89	1.81	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.54	-0.21	-0.03	0.09	0.38
		demand	-0.03	0.96	1.96	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.68	-0.28	-0.02	0.11	0.53
		price	-0.08	0.76	1.59	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.56	-0.33	-0.09	0.17	0.40
	Sn	**supply**	-0.02	1.13	2.28	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.59	-0.24	-0.07	0.16	0.62
		demand	-0.02	1.02	2.06	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.60	-0.23	0.05	0.20	0.58
		price	-0.12	0.87	1.86	-0.12	-0.12	-0.12	-0.12	-0.12	-0.12	-0.68	-0.42	-0.19	0.10	0.54
	Zn	**supply**	-0.07	1.17	2.41	-0.07	-0.07	-0.07	-0.07	-0.07	-0.07	-0.74	-0.23	-0.02	0.24	0.62
		demand	-0.02	0.87	1.76	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.60	-0.25	-0.09	0.20	0.64

Scenario values for the input variables

		Mean	Shock	Extr.	Foc. EA	Foc. FX	Foc. FFR	Foc. Extr. EA	Foc. Extr. FX	Foc. Extr. FFR	Q. 25%	Q. 40%	Q. 50%	Q. 60%	Q. 75%
	price	-0.04	0.82	1.68	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.72	-0.34	0.09	0.26	0.52
exog	IP	0.18	1.22	2.26	0.18	0.18	1.22	0.18	0.18	2.26	-0.23	0.06	0.11	0.34	0.61
	FX	0.07	0.94	1.81	0.07	0.94	0.07	0.07	1.81	0.07	-0.50	-0.10	0.03	0.21	0.60
	FFR	-0.01	0.90	1.81	0.90	-0.01	-0.01	1.81	-0.01	-0.01	-0.52	-0.28	-0.16	0.15	0.53

This table displays the scenario values of the (potential) input variables under the different scenarios Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%). Hereby, the endogenous as well as exogenous variables of the annual (monthly) (MS-)GVAR model as well as the commodity-specific determinants of the logistic regression (log. Reg.) model are displayed. In particular, we report the scenario values of supply (**supply**), demand (**demand**), and price (**price**) of the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), as well as U.S. industrial production (U.S. IP), world industrial production (IP), world gross domestic product (GDP), world gross domestic product per capita (GDPc), U.S. dollar index (FX), Federal Funds Effective Rate (FFR), 10-year U.S. Treasury rate (LIR), U.S. consumer price index (CPI), MSCI world stock index (MSCI), West Texas Intermediate spot crude oil price (OIL), and global natural disasters (ND). Hereby, the values are derived statistically, using data in the period from 2010 to 2019.

In the following, we present the individual probability of scarcity per commodity derived from the (MS-)GVAR model, using the threshold prices, the initial price levels as well as the scenario values, presented in Table 5.25, Table 5.26, and Table 5.27.

5.3.1.1.1 Probability of Scarcity derived from the time-invariant GVAR Model First, we focus on the commodity-specific probability of scarcity derived from the time-invariant GVAR models, see Table 5.28, based on weights representing the dependencies of the commodities in the reference path ($REMod - REF$), as well as under the assumption of the initial price level being equal to the prices observed in 2019. Overall, the probability of scarcity under the mean, 25%, 40% and 50% quantile scenario (focus scenario) is moderate (medium), whereas the commodities exhibit high scarcity risks under the shock, extreme as well as 60% and 75% quantile scenarios, indicating an increased risk in more stressed periods.

In particular, the mean scenario, proposed in Equation 3.160, which assumes all endogenous and exogenous variables behave normal, leads to moderate results for silver, aluminum, cobalt, copper, indium, nickel, platinum as well as tin. In contrast, lithium, lead and zinc exhibit a higher risk of scarcity which is partly explained by their high price level in 2019, used as initial basis level, see Table 5.26, the historical rise in prices, as well as the relatively high input values, see Table 5.37. Except from indium and platinum which exhibit comparable low initial price levels, the commodities will exceed their predefined thresholds almost surely under the shock, and extreme scenarios, as in the shock and especially in the extreme scenario, all variables simultaneously exhibit an extreme value. Hereby, the interdependencies between the commodity markets, reflected by the GVAR model, cause these high risks, since the variables affect each other.

Moreover, as shocks to the global economy affect all commodities simultaneously, see Section D.3.1.1.2, which in turn influence each other, the probability of scarcity under the (extreme) focus scenarios, reflecting a shock to one global variable, are higher than under the mean scenario. In general, the industrial metals are more affected by the global economy as they are generally more connected with the macroeconomy. However, the effects of the economy on the commodity markets are relatively strong. While the shock and extreme scenarios assume the markets are stressed overall, the focus scenarios only consider a shock to one global variable. The resulting high probabilities of scarcity indicate the risks of a shortage in response to a stressed economy are comparatively high, underlining the strong influence of macroeconomics on commodity markets, consistent with the results of the GIRF analysis.

Hereby, the commodities are affected most by changes in the global demand, underlining the strong impact of demand in commodity markets. In particular, the six industrial metals, as well as lithium almost surely exceed their thresholds if the world gross domestic product attains extreme values. While niche metals react more to specific sector shocks, shocks to the global economic activity influence the industrial metals through many applications, leading to these strong reactions. In addition, extreme values in the exchange rate lead to increased price risks, especially for the industrial metals, while most of the commodities bear only moderate risks in a stressed interest rate environment, indicating the commodities react less to monetary policy changes.

Table 5.28: Probability of scarcity per commodity derived from the GVAR models

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
REMod − REF	2019	Mean	0.00	0.00	0.00	0.00	0.00	0.15	0.00	0.06	0.00	0.00	0.31
		Shock	1.00	1.00	1.00	1.00	0.96	1.00	1.00	1.00	0.85	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.01	0.58	0.24	0.57	0.00	0.97	0.79	0.96	0.00	0.88	1.00
		Foc. FX	0.00	0.10	0.05	0.12	0.00	0.73	0.37	0.48	0.00	0.29	0.99
		Foc. FFR	0.00	0.06	0.06	0.02	0.00	0.78	0.06	0.27	0.00	0.05	0.69
		Foc. Extr. EA	0.32	0.98	0.89	0.98	0.03	1.00	0.98	1.00	0.08	1.00	1.00
		Foc. Extr. FX	0.10	0.53	0.36	0.77	0.01	0.91	0.86	0.84	0.00	0.81	1.00
		Foc. Extr. FFR	0.06	0.25	0.21	0.11	0.01	0.93	0.31	0.52	0.00	0.27	0.87
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.02	0.00	0.00	0.11
		Q. 60%	0.00	0.14	0.07	0.03	0.00	0.89	0.12	0.45	0.00	0.09	0.81
		Q. 75%	0.64	1.00	0.94	1.00	0.18	1.00	1.00	1.00	0.04	1.00	1.00
	Mean	Mean	0.00	0.03	0.00	0.01	0.00	0.00	0.00	0.17	0.00	0.01	0.05
		Shock	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.35	0.96	0.61	0.99	0.30	0.01	0.94	1.00	0.11	1.00	0.99
		Foc. FX	0.18	0.38	0.16	0.53	0.35	0.01	0.57	0.72	0.04	0.62	0.87
		Foc. FFR	0.12	0.23	0.12	0.10	0.20	0.01	0.14	0.45	0.03	0.18	0.31
		Foc. Extr. EA	0.86	1.00	0.97	1.00	0.83	0.21	1.00	1.00	0.85	1.00	1.00
		Foc. Extr. FX	0.54	0.80	0.56	0.94	0.79	0.12	0.92	0.92	0.16	0.93	0.98
		Foc. Extr. FFR	0.38	0.46	0.38	0.31	0.47	0.12	0.45	0.68	0.11	0.48	0.62
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.06	0.00	0.00	0.02
		Q. 60%	0.24	0.41	0.18	0.12	0.27	0.02	0.27	0.70	0.03	0.28	0.38
		Q. 75%	0.99	1.00	0.99	1.00	0.99	0.78	1.00	1.00	0.80	1.00	1.00
REMod − SUF	2019	Mean	0.00	0.00	0.00	0.00	0.00	0.13	0.00	0.04	0.00	0.00	0.28
		Shock	1.00	1.00	1.00	1.00	0.96	1.00	1.00	1.00	0.85	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.01	0.56	0.23	0.54	0.00	0.96	0.77	0.94	0.00	0.86	1.00
		Foc. FX	0.00	0.07	0.05	0.10	0.00	0.71	0.32	0.44	0.00	0.27	0.99
		Foc. FFR	0.00	0.05	0.04	0.02	0.00	0.81	0.04	0.25	0.00	0.04	0.67
		Foc. Extr. EA	0.28	0.97	0.87	0.98	0.03	1.00	0.98	1.00	0.08	1.00	1.00
		Foc. Extr. FX	0.09	0.50	0.33	0.77	0.01	0.90	0.87	0.83	0.00	0.81	1.00
		Foc. Extr. FFR	0.04	0.22	0.18	0.11	0.00	0.95	0.27	0.49	0.00	0.25	0.86
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.01	0.00	0.00	0.11
		Q. 60%	0.00	0.09	0.03	0.02	0.00	0.84	0.07	0.40	0.00	0.07	0.77
		Q. 75%	0.56	0.99	0.92	0.99	0.12	1.00	1.00	1.00	0.04	1.00	1.00
	Mean	Mean	0.00	0.02	0.01	0.01	0.00	0.00	0.00	0.14	0.00	0.00	0.04
		Shock	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.31	0.96	0.59	0.98	0.25	0.00	0.93	1.00	0.10	1.00	0.99
		Foc. FX	0.15	0.35	0.12	0.52	0.33	0.00	0.56	0.70	0.03	0.62	0.86
		Foc. FFR	0.10	0.20	0.10	0.08	0.16	0.00	0.10	0.41	0.02	0.16	0.30
		Foc. Extr. EA	0.85	1.00	0.98	1.00	0.80	0.15	1.00	1.00	0.87	1.00	1.00
		Foc. Extr. FX	0.51	0.80	0.52	0.94	0.78	0.08	0.93	0.92	0.16	0.93	0.98
		Foc. Extr. FFR	0.36	0.44	0.32	0.28	0.44	0.09	0.42	0.65	0.10	0.46	0.60
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Probability of scarcity per commodity derived from the GVAR models

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
		Q. 50%	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.05	0.00	0.00	0.01
		Q. 60%	0.19	0.32	0.11	0.10	0.23	0.01	0.18	0.63	0.03	0.21	0.34
		Q. 75%	0.98	1.00	0.99	1.00	0.99	0.61	1.00	1.00	0.82	1.00	1.00
$REMod - PER$	2019	Mean	0.00	0.00	0.00	0.00	0.00	0.18	0.00	0.08	0.00	0.00	0.30
		Shock	1.00	1.00	1.00	1.00	0.95	1.00	1.00	1.00	0.87	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.02	0.61	0.26	0.61	0.00	0.97	0.81	0.97	0.00	0.90	1.00
		Foc. FX	0.01	0.11	0.06	0.14	0.00	0.76	0.38	0.56	0.00	0.32	0.99
		Foc. FFR	0.00	0.07	0.06	0.02	0.00	0.83	0.05	0.31	0.00	0.06	0.69
		Foc. Extr. EA	0.36	0.98	0.89	0.98	0.03	1.00	0.99	1.00	0.08	1.00	1.00
		Foc. Extr. FX	0.14	0.54	0.36	0.78	0.01	0.92	0.88	0.87	0.01	0.83	1.00
		Foc. Extr. FFR	0.09	0.28	0.24	0.14	0.01	0.95	0.31	0.56	0.00	0.27	0.87
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.02	0.00	0.00	0.11
		Q. 60%	0.02	0.15	0.07	0.04	0.00	0.90	0.10	0.49	0.00	0.10	0.78
		Q. 75%	0.74	1.00	0.95	0.99	0.15	1.00	1.00	1.00	0.05	1.00	1.00
	Mean	Mean	0.01	0.03	0.01	0.02	0.00	0.00	0.01	0.20	0.00	0.02	0.05
		Shock	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.42	0.97	0.63	0.99	0.29	0.01	0.95	1.00	0.11	1.00	1.00
		Foc. FX	0.23	0.43	0.17	0.56	0.37	0.01	0.62	0.76	0.04	0.65	0.87
		Foc. FFR	0.18	0.26	0.13	0.11	0.20	0.01	0.14	0.51	0.03	0.20	0.32
		Foc. Extr. EA	0.90	1.00	0.97	1.00	0.84	0.22	1.00	1.00	0.86	1.00	1.00
		Foc. Extr. FX	0.58	0.82	0.58	0.95	0.81	0.12	0.94	0.94	0.16	0.95	0.99
		Foc. Extr. FFR	0.45	0.51	0.40	0.33	0.47	0.13	0.46	0.72	0.11	0.51	0.61
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.07	0.00	0.00	0.02
		Q. 60%	0.34	0.43	0.18	0.15	0.26	0.02	0.24	0.73	0.04	0.29	0.37
		Q. 75%	1.00	1.00	0.99	1.00	0.99	0.74	1.00	1.00	0.82	1.00	1.00
$REMod - UNA$	2019	Mean	0.00	0.00	0.00	0.00	0.00	0.16	0.00	0.06	0.00	0.00	0.28
		Shock	1.00	1.00	1.00	1.00	0.95	1.00	1.00	1.00	0.83	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.01	0.58	0.25	0.56	0.00	0.96	0.80	0.97	0.00	0.88	1.00
		Foc. FX	0.00	0.09	0.05	0.12	0.00	0.73	0.36	0.51	0.00	0.30	0.99
		Foc. FFR	0.01	0.06	0.06	0.02	0.00	0.82	0.05	0.28	0.00	0.04	0.67
		Foc. Extr. EA	0.33	0.98	0.89	0.99	0.03	1.00	0.99	1.00	0.08	1.00	1.00
		Foc. Extr. FX	0.11	0.50	0.35	0.77	0.01	0.90	0.88	0.85	0.00	0.81	1.00
		Foc. Extr. FFR	0.06	0.23	0.21	0.11	0.01	0.95	0.30	0.53	0.00	0.25	0.85
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.01	0.00	0.00	0.09
		Q. 60%	0.00	0.10	0.05	0.02	0.00	0.85	0.07	0.43	0.00	0.06	0.73
		Q. 75%	0.61	1.00	0.92	0.99	0.11	1.00	1.00	1.00	0.04	1.00	1.00
	Mean	Mean	0.00	0.03	0.01	0.02	0.00	0.00	0.00	0.17	0.00	0.01	0.04
		Shock	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.36	0.98	0.61	0.99	0.27	0.01	0.94	1.00	0.10	1.00	0.99
		Foc. FX	0.18	0.37	0.15	0.53	0.35	0.00	0.59	0.73	0.03	0.63	0.87
		Foc. FFR	0.12	0.21	0.12	0.10	0.18	0.00	0.12	0.46	0.03	0.17	0.30
		Foc. Extr. EA	0.87	1.00	0.98	1.00	0.83	0.16	1.00	1.00	0.87	1.00	1.00
		Foc. Extr. FX	0.56	0.81	0.56	0.94	0.81	0.09	0.94	0.94	0.14	0.94	0.98
		Foc. Extr. FFR	0.38	0.46	0.38	0.30	0.45	0.12	0.44	0.70	0.11	0.49	0.59
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.06	0.00	0.00	0.01
		Q. 60%	0.20	0.33	0.13	0.11	0.22	0.01	0.19	0.67	0.02	0.22	0.32
		Q. 75%	0.99	1.00	0.99	1.00	0.99	0.63	1.00	1.00	0.78	1.00	1.00

This table displays the probability of scarcity (PS) of the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), derived from the GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod - REF$, $REMod - SUF$, $REMod - PER$, and $REMod - UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). Hereby, the probability of scarcity is calculated for the scenarios Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%) of the input variables.

The sensitivities of the commodities to the quantile scenarios are heterogeneous. Due to the negative scenario values, the commodities are available almost surely under the 25% and 40% quantile scenarios. Moreover, the probabilities of scarcity of indium and platinum are comparably low, even under the assumption all variables attain their 75% quantiles, probably caused by the comparable low initial price level in 2019. In contrast, lithium's, lead's and zinc's risks increase to a relative high value already in the 50% quantile scenario, indicating these commodities bear higher risks in the model setup, when all variables take on their median values.

For a sensitivity analysis, we use the average price level of the period 2010 to 2019, instead of the price level of the year 2019, to reflect the market situation of the previous decade. In this case, the probabilities of scarcity of aluminum, copper, lead, and tin are higher under the mean scenario than using 2019 as basis level, whereas lithium and zinc exhibit a lower risk, due to the comparably lower initial price level, see Table 5.26. However, the prices of all commodities will exceed their predefined threshold almost surely under the shock, and extreme scenarios. Moreover, even the probability of scarcity values under the (extreme) focus scenarios are higher, except for lithium, and zinc, shocks to the U.S. dollar index mostly affect the commodity markets, whereas the risk of price increases in response to shocks in the Federal Funds Effective Rate and world gross domestic product is less pronounced. Further, lithium's and platinum's probability of scarcity values are relatively small, even under the 60% scenario, whereas the risk of lead is comparably high even under the 50% scenario. In the quantile scenarios, zinc also bear a relatively high risk, however, the risk is higher in case of the price level of 2019. Moreover, in contrast to the results under the initial price level of 2019, the probability of scarcity of indium increases significantly in the quantile scenarios, underlining the sensitivity of the results on the initial price level.

In general, the initial price level highly determines the final probability of scarcity values, indicating the scarcity risk depends on the price level, whereby the risk increases in response to a currently high price. While the actual probabilities of scarcity differ, the impact of the scenarios on the commodity markets equals, indicating the effects of stressed circumstances coincide, regardless of the assumed price level.

Comparing the probabilities of scarcity derived from the GVAR model based on weights representing the dependencies of the commodities in the $REMod - REF$ path, with those of the GVAR models under the $REMod - SUF$, $REMod - PER$, and $REMod - UNA$ path, also displayed in Table 5.28, we observe similar results, in particular the probabilities of scarcity of the $REMod - REF$ and $REMod - UNA$ path almost equal. Overall, regardless of the considered scenario and assumed price level, the probabilities of scarcity are slightly higher in the GVAR model based on the $REMod - PER$ path, probably originating from the highest dependency between indium and the remaining commodities. Due to the stronger link of silver combined with weaker dependencies of the metals to lead and platinum within the $REMod - SUF$ path, the corresponding probabilities of scarcity are tendentially smaller compared to the risks of the $REMod - REF$ path.

5.3.1.1.2 Probability of Scarcity derived from the time-varying MS-GVAR Model
Second, we focus on the probability of scarcity derived from the time-varying MS-GVAR model, see Table 5.29, based on weights representing the dependencies of the commodities in the reference path ($REMod - REF$), as well as under the assumption of the initial price level being equal to the prices observed in 2019. Overall, the MS-GVAR models generally predict higher probabilities of scarcity than the time-invariant GVAR model. While the MS-GVAR model indicates the probability of scarcity under the mean, 25%, 40% and 50% quantile scenario (focus scenario) is moderate (medium), especially for aluminum, copper, nickel, and tin, all commodities exhibit high scarcity risks under the shock, extreme as well as 60% and 75% quantile scenarios, indicating an increased risk in more stressed periods.

In particular, the mean scenario, proposed in Equation 3.160, indicates the probability of scarcity is moderate for aluminum, copper, nickel and tin, whereas lead and zinc bear high risks of scarcity, although the variables only attain their average value, probably caused by their high initial prices, in line with the results of the GVAR model. However, the probability of scarcity increases to a high risk under the shock and extreme scenarios for all commodities, implying the commodities will almost surely exceed their predefined thresholds if all variables simultaneously exhibit an extreme value.

Moreover, shocks to the global economy lead to higher scarcity risks due to simultaneous spillover effects from the economy to the commodity markets. In particular, the commodity markets and the associated scarcity risks are more affected by a stressed economy in the case of the MS-GVAR model compared to the GVAR model, due to the different considered period, indicating the markets are more connected in recent times. Hereby, the risk of scarcity of all commodities (except tin) increases to a high value if a shock hits the Federal Funds Effective Rate, whereas the commodities react more moderately to shocks in the exchange rate. However, the global demand affects the commodities the least, contrary to the finding of the GVAR model, observing the most sensitive reactions in response to extreme values in the world gross domestic product. These differences might be explained by the different considered time period, indicating the impact of the Federal Funds Effective Rate (FFR) and the U.S. dollar index (FX) on commodity markets increased in more recent times. Moreover, parts of this heterogeneous results might also be explained by the different proxy for the economic activity, included in our analysis. While we use the world gross domestic product (GDP), representing the change in the value of all goods and services, in the annual GVAR model due to data limitations, we consider the world industrial production (IP), measuring the change in physical volume of the industrial output, in the monthly MS-GVAR model. However, if the exogenous variables attain extreme values, the commodities will almost surely exceed their threshold price, underlining the commodity prices are highly affected by the economy, in line with the significant spillover effects from the economy to the commodity markets, see Section 5.2.3.3.

Similar to the results derived from the GVAR model, the sensitivities of the commodities to the quantile scenarios are heterogeneous. While zinc almost surely exceeds its predefined threshold already under the median scenario, the risk of scarcity is neglectable for aluminum, copper, nickel and tin. This is rather counterintuitive, especially for aluminum, as the probability of scarcity under the mean scenario indicates a higher risk. As the variables generally attain smaller values in absolute terms under the median scenario, compared to the mean scenario, the corresponding risk is higher in the latter case. However, all commodities bear high scarcity risks under the 60% and 75% quantile scenarios, indicating stressed markets lead to an increased scarcity risk.

Table 5.29: Probability of scarcity per commodity derived from the MS-GVAR models

			Al	Cu	Ni	Pb	Sn	Zn
REMod – REF	2019	Mean	0.25	0.11	0.11	0.91	0.05	1.00
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.96	0.61	0.78	1.00	0.51	1.00
		Foc. FX	1.00	0.92	0.93	1.00	0.58	1.00
		Foc. FFR	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. Extr. EA	0.99	0.88	0.93	1.00	0.84	1.00
		Foc. Extr. FX	1.00	0.98	0.99	1.00	0.85	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.01
		Q. 50%	0.00	0.00	0.00	0.25	0.01	0.97
		Q. 60%	1.00	1.00	1.00	1.00	0.96	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
	Mean	Mean	1.00	0.99	0.53	1.00	0.43	0.61
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	0.97	1.00	0.93	0.98

Probability of scarcity per commodity derived from the MS-GVAR models

			Al	Cu	Ni	Pb	Sn	Zn
REMod – REF	Mean	Foc. FX	1.00	1.00	1.00	1.00	0.94	0.97
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	1.00	1.00	0.99	1.00	0.97	1.00
		Foc. Extr. FX	1.00	1.00	1.00	1.00	0.98	0.99
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.01	0.00	0.00
		Q. 50%	0.20	0.23	0.07	0.72	0.13	0.12
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
REMod – SUF	2019	Mean	0.20	0.11	0.09	0.91	0.04	1.00
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.97	0.58	0.80	1.00	0.50	1.00
		Foc. FX	1.00	0.91	0.88	1.00	0.59	1.00
		Foc. FFR	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. Extr. EA	0.99	0.86	0.95	1.00	0.82	1.00
		Foc. Extr. FX	1.00	0.98	0.97	1.00	0.83	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.01
		Q. 50%	0.00	0.01	0.00	0.27	0.01	0.96
		Q. 60%	1.00	1.00	1.00	1.00	0.95	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
	Mean	Mean	0.98	0.98	0.49	1.00	0.41	0.68
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	0.98	1.00	0.93	0.99
		Foc. FX	1.00	1.00	0.98	1.00	0.92	0.99
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	1.00	1.00	1.00	1.00	0.97	1.00
		Foc. Extr. FX	1.00	1.00	1.00	1.00	0.98	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.01	0.00	0.00
		Q. 50%	0.18	0.23	0.05	0.74	0.14	0.13
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
REMod – PER	2019	Mean	0.25	0.12	0.12	0.92	0.06	1.00
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.96	0.61	0.81	1.00	0.50	1.00
		Foc. FX	1.00	0.92	0.94	1.00	0.60	1.00
		Foc. FFR	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. Extr. EA	1.00	0.87	0.95	1.00	0.82	1.00
		Foc. Extr. FX	1.00	0.98	0.99	1.00	0.85	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.02
		Q. 50%	0.00	0.01	0.01	0.27	0.01	0.97
		Q. 60%	1.00	1.00	1.00	1.00	0.96	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
	Mean	Mean	1.00	0.99	0.57	1.00	0.41	0.64
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	0.98	1.00	0.93	0.99
		Foc. FX	1.00	1.00	1.00	1.00	0.94	0.99
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	1.00	1.00	0.99	1.00	0.98	1.00
		Foc. Extr. FX	1.00	1.00	1.00	1.00	0.98	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.01	0.00	0.00
		Q. 50%	0.19	0.25	0.08	0.73	0.14	0.12
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
	2019	Mean	0.24	0.12	0.12	0.90	0.05	1.00
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.96	0.58	0.80	1.00	0.50	1.00
		Foc. FX	1.00	0.91	0.95	1.00	0.57	1.00

Probability of scarcity per commodity derived from the MS-GVAR models

			Al	Cu	Ni	Pb	Sn	Zn
$REMod-UNA$	2019	Foc. FFR	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. Extr. EA	1.00	0.84	0.93	1.00	0.79	1.00
		Foc. Extr. FX	1.00	0.97	0.99	1.00	0.84	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.02
		Q. 50%	0.00	0.01	0.01	0.25	0.01	0.97
		Q. 60%	1.00	1.00	1.00	1.00	0.95	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
	Mean	Mean	1.00	0.98	0.54	1.00	0.45	0.70
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	0.98	1.00	0.91	1.00
		Foc. FX	1.00	1.00	1.00	1.00	0.94	0.99
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	1.00	1.00	1.00	1.00	0.98	1.00
		Foc. Extr. FX	1.00	1.00	1.00	1.00	0.97	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.01	0.00	0.00
		Q. 50%	0.19	0.24	0.07	0.70	0.13	0.16
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00

This table displays the probability of scarcity (PS) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), derived from the MS-GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). Hereby, the probability of scarcity is calculated for the scenarios Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%) of the input variables.

The GIRF analysis in Section D.3.1.2.3 reveals the MS-GVAR models detect various spillover effects, which might cause the high risk values, as the commodity markets influence each other. Hereby, due to data limitations, the applied MS-GVAR model only reflects the relations in industrial metal markets. In contrast, the GVAR model includes the industrial metal markets as well as further key commodities of the German Energiewende, which might compensate the strong relations between the industrial metals. Moreover, the GVAR model is based on an annual dataset in the period from 1970 to 2019, including a longer time period prior to the financialization and the corresponding increase in the relation between commodity markets. In contrast, the MS-GVAR model is based on monthly data from 1995 to 2019. Hereby, we annualize the predicted logarithmic returns of the prices, assuming the return would not change during the year. This approach slightly overestimates the scarcity risks, as the returns obviously change during one year and the annualization approach does not account for the mean reverting behavior in commodity prices. However, the estimation of the annual return via 12-step ahead predictions would not be feasible, since this would require more assumptions for the exogenous variables. Moreover, an overestimation of the risk is preferable to an underestimation, probably obtained by the 12-step ahead forecasts. In addition, this thesis focuses on the comparison of the resource risks of the four transformation paths, which is possible, even if the probabilities of scarcity obtained slightly overestimate the risk.

In line with the GVAR model, we use the average price level of the previous decade, instead of the price level of the year 2019, to aggregate out extreme underlying price levels, for a sensitivity analysis. Overall, the derived probabilities of scarcity are higher, using the average price level, caused by the corresponding higher initial price levels, see Table 5.26. Hereby, aluminum, copper and lead almost surely exceed their thresholds even under the mean scenario. Additionally, the probabilities of scarcity of nickel and tin are higher compared to the results based on the price level of the year 2019, whereas zinc's risk reduces. However, the prices of (almost) all commodities will exceed their predefined threshold almost surely under the shock, extreme, ((extreme) focus) as well as 60% and 75% quantile scenarios. Moreover, lead bear even under the median scenario high risks, due to its higher initial price level, whereas nickel, tin and zinc (aluminum and copper) show a moderate (medium) risk.

Comparing the probabilities of scarcity derived from the MS-GVAR model based on weights representing the dependencies of the commodities in the $REMod - REF$ path, with those of the MS-GVAR models under the $REMod - SUF$, $REMod - PER$ as well as $REMod - UNA$ paths, also displayed in Table 5.29, we observe similar results with only small deviations, due to the similar relations, reflected in the induced weight matrices, see Section 4.4.6. Hereby, the probabilities of scarcity indicate a higher risk of the MS-GVAR model based on the $REMod - PER$ path, whereas the $REMod - SUF$ path exhibits slightly smaller risk values, in line with the results of the GVAR model.

Overall, the MS-GVAR model indicates higher scarcity risks compared to the GVAR model. However, both frameworks attribute higher scarcity risks to zinc and lead. Moreover, the models using the different weight matrices derive similar risk values, due to the comparable induced weight matrices. In addition, the $REMod - PER$ path exhibits higher risks, whereas the risk is lowest in the $REMod - SUF$ path, regardless of the considered framework, price level or scenario.

5.3.1.2 Probability of Scarcity derived from the alternative Logistic Regression Model

As the determinants of commodity prices are heterogeneous between the different metals, see Gleich et al. (2013), we alternatively propose to estimate the path-independent probability of scarcity directly via commodity-specific logistic regression models, allowing for individual selected price influential factors. However, the co-movement between prices is only covered by common price determinants, in contrast to the (MS-)GVAR framework, which allows for spillover effects between commodity markets.

To start, we apply a two-step model selection to identify the price influential factors for each commodity separately. Subsequently, we estimate individual logistic regression models and calculate the probability of scarcity per commodity, considering different scenarios for the commodity-specific price determinants.

In particular, the two-step model selection procedure is applied for each commodity on the 18 potential price determinants, described in Section 4.3, capturing the dimensions macroeconomic, demographic, capital market driven as well as supply and demand factors. Hereby, the short-term interest rate 3-month U.S. Treasury rate (SIR), the monetary policy proxy U.S. monetary base (MB), the demographic factors U.S. employment (EMP) and world population (POP), the capital market related variable Standard & Poor's 500 index (SPX), as well as the supply-sided variables Herfindal-Hirschman index (HHI) and KOF globalization index (KOF) are not selected for any commodity. However, the economic activity proxies, U.S. industrial production (U.S. IP), world gross domestic product (GDP), and world gross domestic product per capita (GDPc), the exchange rate U.S. dollar index (FX), the monetary policy variables, Federal Funds Effective Rate (FFR) and 10-year U.S. Treasury rate (LIR), the inflation measure U.S. consumer price index (CPI), the capital market related variable MSCI world stock index (MSCI), the supply-sided variables commodity-specific supply, West Texas Intermediate spot crude oil price (OIL) and global natural disasters (ND), as well as the commodity-specific demand are identified as influential attributes for at least one commodity. The resulting coefficients of the logistic regression model per commodity are displayed in Table 5.30.

In general, the heterogeneity in the variable selection is remarkable, as the determinants identified by the individual model selection differ, indicating the importance of a commodity-specific analysis. Hereby, the exchange rate, measured by the U.S. dollar index, and the West Texas Intermediate spot crude oil price are outstanding as they are price determining factors for up to six of the thirteen commodities.[31] Moreover, the global demand, reflected by the U.S. industrial production (U.S. IP), world gross domestic product (GDP) and world gross domestic product per capita (GDPc), as well as the monetary policy, measured by the Federal Funds Effective Rate (FFR) and the 10-year U.S. Treasury rate (LIR), affect several commodities, indicating the economic activity, the exchange rate, the monetary policy as well as the oilprice influence the commodity markets, in line with Guzmán and Silva (2018), while most of the remaining factors are only included in up to three models.

Table 5.30: Estimated coefficients of the logistic regression models

	Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
U.S. IP			0.05										
GDP					0.90			0.90					
GDPc	-0.16	0.75	0.65										
FX				0.03	0.14	-0.31		0.14		-0.58	-0.33		
FFR				-0.20	-0.93			-0.93					1.73
LIR			0.73						-0.30	0.39			1.22
CPI							0.16						
MSCI		-0.21											
supply									1.42			0.43	
OIL	0.18			0.17		0.30					0.57	-0.05	
ND			-0.80										
demand													0.91

This table displays the estimated coefficients of the individual logistic regression models of the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), based on the identified independent variables from the two stage model selection. Hereby, the independent variables are U.S. industrial production (U.S. IP), world gross domestic product (GDP), world gross domestic product per capita (GDPc), U.S. dollar index (FX), Federal Funds Effective Rate (FFR), 10-year U.S. Treasury rate (LIR), U.S. consumer price index (CPI), MSCI world stock index (MSCI), commodity-specific supply (**supply**), West Texas Intermediate spot crude oil price (OIL), global natural disasters (ND), and commodity-specific demand (**demand**).

Overall, the economic activity determines the commodity prices. In particular, the proxies U.S. industrial production and world gross domestic product positively influence the probability of scarcity of cobalt, dysprosium and neodymium, which is in line with the literature, see for example Akram (2009), Baffes and Savescu (2014), and Issler et al. (2014), also detecting a positive relation between economic activity and commodity prices, as well as with the positive spillover effects from globald demand shocks to commodity markets in Section 5.1. Moreover, the determinant world gross domestic product per capita positively influences the probability of scarcity of aluminum and cobalt, whereas it slightly affects silver in the inverse direction.

In line with the results of Ahumada and Cornejo (2014), we find a negative relationship between the prices of indium, lead and platinum, with the U.S. dollar index, indicating a lower exchange rate leads to a higher probability of scarcity, confirming the negative spillover effects in Section 5.1. This is reasonable, as a decline in the dollar will cause an increase in the dollar price of the commodity or a fall in the foreign currency price, which, finally, leads to a higher commodity price, see Akram (2009). In contrast, dysprosium's and neodymium's (and copper's) price

[31] Due to data limitation issues, the model selection could not be performed for dysprosium and neodymium. Instead, we manually select the world gross domestic product (GDP), U.S. dollar index (FX), and the Federal Funds Effective Rate (FFR), in line with the exogenous variables of the (MS-)GVAR model, for their logistic regression models. Since the models of dysprosium and neodymium also show identical times, in which they are classified as scarce, their results in the framework coincide.

increase in response to an appreciation of the dollar, implying the stronger U.S. dollar leads to an increased demand for these, mostly in China (Chile) produced, metals of consumers holding the U.S. dollar, whereby the findings of copper underline the results of the global spillover effects in the time-varying MS-GVAR model, see Section 5.2.3.3.

Further, the short-term (long-term) interest rate, measured by the Federal Funds Effective Rate (10-year U.S. Treasury rate), negatively affects the probability of scarcity of copper, nickel and the rare earth metals, indicating a lower interest rate will lead to a higher risk of scarcity. However, the positive coefficients in the logistic regression models of cobalt, lead as well as zinc suggest a high interest rate is associated with a high probability of scarcity, in line with the positive spillover effects from a contrarian monetary policy to commodity markets in Section 5.1. Further, a higher inflation rate implies higher prices, which is why the scarcity risk of lithium positively reacts to changes in the U.S. consumer price index.

In line with Kagraoka (2016), who detects the MSCI world stock index is one of four identified factors explaining commodity prices, the capital market related determinant MSCI world stock index affects the risk of aluminum, indicating rising markets lead to a reduced scarcity risk, probably due to more liquidity in the markets or less demand of speculators in commodities. In contrast, the Standard & Poor's 500 index is not part of any commodity-specific model in our study. This might be explained by the long time period analyzed beginning in 1970, whereas the effect of the capital market on commodities raises during the financialization beginning around 2004, see Tang and Xiong (2012). Moreover, the demographic variables U.S. employment and world population are not selected by our two-stage model selection, implying the demographic situation is not the main determinant of commodity prices and their scarcity risk.

The results of the supply-sided variables are counter-intuitive. Hereby, a higher supply leads to a higher probability of scarcity of nickel, whereas a higher global natural disasters value leads to a lower probability of scarcity of cobalt, probably caused by a smaller economic activity in response to the natural disaster. Moreover, the positive impact of the West Texas Intermediate spot crude oil price, a proxy for energy costs, on the probability of scarcity of silver, copper, indium as well as platinum, indicates a higher risk in case of higher oil prices, in line with the literature, see for example Liberda (2017) and Vansteenkiste (2009). However, the scarcity risk of tin slightly decreases in case of rising oil prices, further underlining the heterogeneous reactions of the commodities to the price determinants. Moreover, the spillover effects in the industrial metal markets in Section 5.1.2 reveal tin is least connected to the other metals, and therefore, tin probably shows a less cyclical behavior to the price of oil. In addition, the demand variable has a positive impact on the probability of scarcity of zinc, indicating a higher consumption of zinc leads to a higher scarcity risk.

Overall, the heterogeneity of the selected variables as well as the (opposite) signs of the coefficients reveal the probability of scarcity is commodity-specific and the reaction to changes in determinants may influence different commodities in a different way. This finding suggests the importance of modeling commodity markets individually, however, in contrast to the (MS-)GVAR frameworks, the dependencies between commodities and, especially, spillover effects between the markets, are not reflected within this analysis.

Using the logistic regression models, the individual probability of scarcity can be calculated, taking into account the commodity-specific determinants, see Table 5.31. Hereby, we investigate different scenarios of the input variables, displayed in Table 5.27, similar to the (MS-)GVAR models. As interdependencies are not reflected within the logistic regression model, the scarcity risk is generally lower.

Table 5.31: Probability of scarcity per commodity derived from the logistic regression models

	Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
Mean	0.04	0.06	0.07	0.04	0.01	0.10	0.04	0.01	0.09	0.06	0.03	0.02	0.04
Shock	0.05	0.12	0.46	0.06	0.05	0.16	0.06	0.05	0.56	0.14	0.07	0.03	0.80
Extr.	0.06	0.21	0.90	0.09	0.24	0.24	0.08	0.24	0.94	0.32	0.14	0.05	1.00
Foc. EA	0.04	0.06	0.07	0.04	0.02	0.10	0.04	0.02	0.09	0.06	0.03	0.02	0.04
Foc. FX	0.04	0.06	0.07	0.04	0.01	0.12	0.04	0.01	0.09	0.08	0.04	0.02	0.04
Foc. FFR	0.04	0.06	0.07	0.05	0.02	0.10	0.04	0.02	0.09	0.06	0.03	0.02	0.17
Foc. Extr. EA	0.04	0.06	0.07	0.04	0.04	0.10	0.04	0.04	0.09	0.06	0.03	0.02	0.04
Foc. Extr. FX	0.04	0.06	0.07	0.04	0.01	0.15	0.04	0.01	0.09	0.12	0.05	0.02	0.04
Foc. Extr. FFR	0.04	0.06	0.07	0.06	0.06	0.10	0.04	0.06	0.09	0.06	0.03	0.02	0.51
Q. 25%	0.03	0.04	0.02	0.04	0.00	0.09	0.04	0.00	0.02	0.04	0.02	0.02	0.00
Q. 40%	0.03	0.05	0.03	0.04	0.01	0.10	0.04	0.01	0.08	0.04	0.03	0.02	0.01
Q. 50%	0.04	0.05	0.05	0.04	0.01	0.10	0.04	0.01	0.11	0.05	0.03	0.02	0.03
Q. 60%	0.04	0.07	0.11	0.05	0.01	0.11	0.04	0.01	0.14	0.06	0.04	0.02	0.10
Q. 75%	0.04	0.09	0.34	0.05	0.03	0.13	0.04	0.03	0.22	0.11	0.05	0.03	0.47

This table displays the probability of scarcity (PS) of the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), derived from the logistic regression models based on pre-selected determinants. Hereby, the probability of scarcity is calculated for the scenarios Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%) of the input variables.

While the initial mean scenario, proposed in Equation 3.168, leads to moderate results, with the highest probability of scarcity observed for cobalt, indium and nickel with approximately 10%, the sensitivities of the commodities to the scenarios are heterogeneous. Cobalt's, nickel's as well as zinc's risks increase to a high and extremely high value for the shock and extreme scenario, whereas the probability of scarcity of silver, copper, lithium, platinum and tin are comparably low, even under the extreme scenario. In addition, the rare earth metals show remarkably moderate risks in all scenarios. Overall, the probabilities of scarcity are lower, especially under the shock and extreme scenario, compared to the probabilities of scarcity derived from the (MS-)GVAR models, as spillover effects between the markets are not reflected within the logistic regression models and therefore, the scarcity risk is probably underestimated.

In line with the (MS-)GVAR models, we also consider shocks to the exogenous variables, the world gross domestic product, the U.S. dollar index as well as the Federal Funds Effective Rate. Hereby, the commodity markets remain rather unaffected by shocks in the three considered macroeconomic variables, except zinc, which is highly affected by extreme values in the Federal Funds Effective Rate, due to its comparably high corresponding coefficient. This is rather unsurprising for silver, aluminum, cobalt, indium, lithium, nickel, lead, platinum, and tin, as the macroeconomic variables are not included in the corresponding models due to the two-stage model selection. However, the probabilities of scarcity of copper as well as the rare earth metals barely react to the shocks, although the Federal Funds Effective Rate is included, implying the logistic regression model underestimates the scarcity risk inferred from the economy.

In case of the scenarios based on different quantiles, we recognize the shock and extreme scenarios lead to slightly higher probabilities of scarcity than even the 75% quantile scenario. The scarcity risk of cobalt, nickel and zinc at least doubles in the 75% quantile scenario compared to the 25% quantile scenario, while the quantile scenarios barely differ in the other commodity markets, suggesting the logistic regression model is not as sensitive to the quantiles of the input variables as the (MS-)GVAR model. Moreover, the logistic regression models underestimate the scarcity risk, since spillover effects are not reflected at all, leading to the comparable moderate probabilities of scarcity.

Overall, the MS-GVAR model indicates the highest risks for all commodities, whereas the logistic regression model underestimates the scarcity, as spillover effects between the commodity markets are not reflected. While the probability of scarcity, derived from the (MS-)GVAR models under the mean, 25%, 40% and 50% quantile scenario (focus scenario) is moderate (medium), the commodities exhibit high scarcity risks under the shock, extreme as well as 60% and 75% quantile scenarios, indicating an increased risk in more stressed periods. As the MS-GVAR model focuses on the industrial metal markets, which are highly connected, the resulting scarcity risks are highest due to interdependencies between the markets, whereas the GVAR model includes more metals, but slightly underestimates the risks induced from the spillover effects. Moreover, shocks to the global economy lead to higher scarcity risks for the (MS-)GVAR models due to simultaneous spillover effects from the economy to the commodity markets, whereas the logistic regression model clearly underestimates these spillover risks. On commodity level, (lithium) lead and zinc are outstanding for the commodity market models, whereas the logistic regression model attributes cobalt, nickel and zinc a high probability of scarcity. The comparison of the scarcity risk derived from the (MS-)GVAR model based on weights representing the dependencies of the commodities in the $REMod - REF$ path, with those of the (MS-)GVAR models under the $REMod - SUF$, $REMod - PER$ as well as $REMod - UNA$ paths reveals similar results, whereby the $REMod - SUF$ ($REMod - PER$) path exhibits slightly smaller (higher) risk values.

5.3.2 Loss Given Scarcity and Exposure at Scarcity

For a holistic assessment of the scarcity risk of the resource demands of the German Energiewende, the derived commodity-specific probabilities of scarcity are aggregated with an appropriate substitutability score as well as the required resource demands to the final scarcity risk measure.

To reflect the substitutability, the loss given scarcity (LGS), of the commodities, we use the information about the major metals applications with primary substitutes and substitute performance from Figure 5 of Graedel et al. (2015). For other applications of the framework, where a metal is only used in a specific application, we propose to apply technology-specific parameters for the substitutability of the commodities.

Table 5.32: Loss given scarcity per commodity

	Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
LGS	0.44	0.44	0.54	0.70	1.00	0.60	0.41	0.41	0.62	1.00	0.66	0.36	0.38

This table displays the loss given scarcity (LGS) of the commodities aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn).

Table 5.32 displays the final loss given scarcity values per commodity, highlighting the inability to substitute dysprosium and lead, indicated by a loss given scarcity of 1. In contrast, silver, aluminum, cobalt, lithium, neodymium, tin as well as zinc, have adequate substitutes, resulting in a score of approximately 50%. Thereby, the comparably low loss given scarcity of neodymium is caused by the possibility to substitute it by dysprosium within its major application, neodymium magnets. However, as dysprosium bears a high scarcity risk itself, this example shows the substitutability score neglects the scarcity risks of the substitutes. Moreover, the study of Graedel et al. (2015) examines the general substitutability of 62 metals and metalloids in their major uses, and do not focus on the application of the metals in the context of renewable energies. Further, they reveal several metals have no substitute or the product performance will suffer from substitution. Therefore, we provide a robustness analysis in Section 5.3.4.3, investigating how the results change if neither commodity is regarded as substitutable.

In addition to the commodity-specific probability of scarcity and substitutability, reflected by the loss given scarcity, the required amount per commodity, represented by the exposure at scarcity (EAS), determines the expected loss due to scarcity. As described in Equation 3.176, the required quantity per commodity and pathway is scaled by the mean of the previous ten years' annual world production.[32] The resulting exposure at scarcity values per path and commodity are displayed in Table 5.33.

Overall the scaled resource requirements of lead and platinum are relatively low in each path, as platinum is barely needed, whereas the required demand for lead is relatively low compared to the average world production, see Table 4.3. In contrast, the scaled demand for cobalt is exceptionally high, which is due to the high requirements of cobalt for storage capacities, combined with a comparatively low average production volume, as cobalt is mostly mined as a by-product of copper and nickel and thus depends on the demand and price for the other metals. Moreover, the resource requirements of indium and dysprosium are relatively high, compared to their average world production, indicating the demand for these metals will increase due to their use in photovoltaic systems, and wind power plants. Hereby, the required amounts for dysprosium are alarming, as, so far, this rare earth metal is not substitutable, see Table 5.32, and there is an amplified market concentration, since dysprosium is mainly mined in China. In addition, the exposure at scarcity of nickel is remarkable, as its total amounts, required for the built-up of storage capacities, indicate an increased interest in this industrial metal in the following decades.

Table 5.33: Exposure at scarcity per commodity

	Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
$REMod-REF$	0.21	0.19	5.64	0.39	1.42	2.20	0.62	0.45	1.41	0.01	0.01	0.25	0.19
$REMod-SUF$	0.14	0.13	3.79	0.28	1.01	1.61	0.41	0.32	0.95	0.01	0.01	0.17	0.19
$REMod-PER$	0.24	0.17	4.06	0.33	1.36	2.25	0.44	0.42	1.11	0.01	0.01	0.24	0.16
$REMod-UNA$	0.33	0.21	5.77	0.37	0.68	3.36	0.63	0.19	1.41	0.01	0.01	0.19	0.20

This table displays the exposure at scarcity (EAS) of the commodities aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn) per transformation path ($REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$). In particular, the exposure at scarcity is derived as the total required amount per commodity and path scaled by the average world production in the period from 2010 to 2019, according to Equation 3.176.

In general, we clearly notice an elevated exposure at scarcity for almost all commodities of the $REMod-UNA$ path, caused by the assumed protests against big infrastructural projects, leading to a compensation through additional solar parks and energy storage build-ups, resulting in the highest demand in cobalt and indium. Hereby, the required quantities for cobalt in the $REMod-UNA$ path are one and a half times the amounts in the $REMod-SUF$ path, and as high as six times of the average annual world production, allocated only for the German Energiewende. Due to less installed wind energy parks, the $REMod-UNA$ path requires the least amounts of the rare earth metals, in particular, the $REMod-REF$ path requires more than twice the amount compared to the $REMod-UNA$ path. In this context, the $REMod-REF$ path is notable, as the energy system is hereby modeled without further boundary conditions, but results into comparable high resource amounts, indicating the behavior of the population can significantly reduce the required amounts of commodities. Further, the comparably low exposure at scarcity values for the $REMod-PER$ path are remarkable, as the persistence on - and usage of - conventional technologies in the German population, for the transportation and housing sector, leads to reduced commodity demands, especially in cobalt and zinc, because of a reduced demand for storage capacities. Moreover, the $REMod-SUF$ path shows the lowest exposure at scarcity values for most of the commodities, indicating the acceptance of the society significantly reduces the resource requirements, except for the rare earth metals.

Overall, the $REMod-SUF$ path exhibits the least average resource requirements for almost all commodities, followed by the $REMod-PER$ path, whereas the $REMod-REF$ path, and especially the $REMod-UNA$ path, require the most resource amounts. The only exceptions

[32] For a robustness analysis, we examine to what extent the results remain valid if the considered period of the world production is reduced or enlarged, see Section 5.3.4.4.

are zinc, which is required least for the $REMod - PER$ path, and the rare earth metals, which are barely needed for the $REMod - UNA$ path, since only few wind turbines will be installed.

All in all, lead and zinc are outstanding in terms of their high probability of scarcity, whereas we clearly identify cobalt as a key commodity, followed by indium, nickel as well as the rare earth metal dysprosium, due to their high resource requirements. While cobalt, indium and nickel are, at least partly, substitutable, dysprosium can not be replaced by any other metal, indicating the severity of its high demand. Moreover, the risk indicators probability of scarcity and the exposure at scarcity show the commodities exhibit the least risks under the $REMod - SUF$ path, whereas the $REMod - UNA$ path bears the highest risks.

5.3.3 Expected Loss due to Scarcity

The objective of this thesis is the risk assessment of the four transformation paths of the German Energiewende. Therefore, the probability of scarcity, the loss given scarcity as well as the exposure at scarcity are aggregated following Equation 3.177, to calculate the scarcity risk, the expected loss due to scarcity (ES), per commodity, scenario and path. Hereby, we use either the path-specific probabilities of scarcity obtained via the (MS-)GVAR framework, by reflecting the relationship between the commodities over their common requirements within the energy transition pathway, or the path-independent probabilities of scarcity from the logistic regression model.

5.3.3.1 Expected Loss due to Scarcity per Commodity

The commodity- and path-specific expected loss due to scarcity (ES) values for the different scenarios, are displayed in Table 5.34 (Table 5.35) in case of the (MS-)GVAR model, with weight matrices corresponding to the paths considered, and Table 5.36 for the results of the logistic regression model.

Overall, the GVAR model indicates cobalt bears by far the greatest risk, followed by indium and nickel. In addition, the risk from copper and lithium is noticeable, whereas the risk of lead and platinum is negligible. While lithium and zinc (cobalt and indium) bear higher scarcity risks, especially under the mean scenario, using the price level of 2019 (the average price level), caused by their comparable high prices in 2019 (the previous decade), the risks of the other commodities are comparable, which is why we focus on the GVAR model based on the price level of the basis year 2019 in the following.

Table 5.34: Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the GVAR models

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
Mean	2019	$REMod - REF$	0.00	0.03	0.30	0.00	0.00	3.84	0.17	0.08	0.00	0.03	2.26
		$REMod - SUF$	0.00	0.02	0.20	0.00	0.00	2.58	0.12	0.05	0.00	0.02	2.30
		$REMod - PER$	0.00	0.02	0.22	0.00	0.00	2.77	0.14	0.07	0.00	0.03	1.86
		$REMod - UNA$	0.00	0.03	0.31	0.00	0.00	3.92	0.17	0.04	0.00	0.02	2.40
	Mean	$REMod - REF$	0.04	0.23	1.52	0.35	0.40	0.00	0.35	0.24	0.00	0.12	0.35
		$REMod - SUF$	0.02	0.16	1.02	0.26	0.29	0.00	0.24	0.16	0.00	0.08	0.35
		$REMod - PER$	0.04	0.21	1.10	0.30	0.41	0.00	0.28	0.24	0.00	0.11	0.28
		$REMod - UNA$	0.06	0.26	1.56	0.34	0.60	0.00	0.35	0.13	0.00	0.09	0.37
Shock	2019	$REMod - REF$	9.17	8.36	304.32	27.27	126.63	25.23	87.36	1.38	0.45	9.12	7.21
		$REMod - SUF$	6.19	5.67	204.60	19.71	92.73	16.96	59.05	0.94	0.30	6.21	7.33
		$REMod - PER$	10.44	7.34	219.38	23.16	129.75	18.20	69.11	1.36	0.57	8.77	5.91
		$REMod - UNA$	14.62	9.36	311.47	25.80	193.47	25.82	87.45	0.78	0.32	6.72	7.66
	Mean	$REMod - REF$	9.17	8.36	304.32	27.27	131.77	25.23	87.36	1.38	0.53	9.12	7.21
		$REMod - SUF$	6.19	5.67	204.60	19.71	96.49	16.96	59.05	0.94	0.35	6.21	7.33
		$REMod - PER$	10.44	7.34	219.38	23.16	135.01	18.20	69.11	1.36	0.67	8.77	5.91
		$REMod - UNA$	14.62	9.36	311.47	25.80	201.32	25.82	87.45	0.78	0.38	6.72	7.66
		$REMod - REF$	9.17	8.36	304.32	27.27	131.77	25.23	87.36	1.38	0.53	9.12	7.21

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the GVAR models

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
Extr.	2019	$REMod-SUF$	6.19	5.67	204.60	19.71	96.49	16.96	59.05	0.94	0.35	6.21	7.33
		$REMod-PER$	10.44	7.34	219.38	23.16	135.01	18.20	69.11	1.36	0.67	8.77	5.91
		$REMod-UNA$	14.62	9.36	311.47	25.80	201.32	25.82	87.45	0.78	0.38	6.72	7.66
	Mean	$REMod-REF$	9.17	8.36	304.32	27.27	131.77	25.23	87.36	1.38	0.53	9.12	7.21
		$REMod-SUF$	6.19	5.67	204.60	19.71	96.49	16.96	59.05	0.94	0.35	6.21	7.33
		$REMod-PER$	10.44	7.34	219.38	23.16	135.01	18.20	69.11	1.36	0.67	8.77	5.91
		$REMod-UNA$	14.62	9.36	311.47	25.80	201.32	25.82	87.45	0.78	0.38	6.72	7.66
Foc. EA	2019	$REMod-REF$	0.09	4.87	73.34	15.57	0.00	24.40	68.84	1.33	0.00	7.99	7.21
		$REMod-SUF$	0.06	3.30	49.31	11.25	0.00	16.40	46.53	0.90	0.00	5.45	7.33
		$REMod-PER$	0.10	4.27	52.87	13.22	0.00	17.60	54.46	1.30	0.00	7.69	5.91
		$REMod-UNA$	0.15	5.45	75.06	14.73	0.00	24.96	68.91	0.75	0.00	5.90	7.66
	Mean	$REMod-REF$	3.24	8.07	184.72	26.89	39.79	0.35	81.86	1.38	0.06	9.09	7.16
		$REMod-SUF$	2.18	5.47	124.19	19.43	29.14	0.24	55.33	0.94	0.04	6.19	7.28
		$REMod-PER$	3.69	7.08	133.17	22.84	40.77	0.25	64.75	1.36	0.07	8.74	5.87
		$REMod-UNA$	5.16	9.04	189.06	25.44	60.80	0.36	81.94	0.78	0.04	6.70	7.60
Foc. FX	2019	$REMod-REF$	0.04	0.83	16.43	3.16	0.00	18.44	32.15	0.67	0.00	2.64	7.12
		$REMod-SUF$	0.02	0.56	11.05	2.29	0.00	12.40	21.73	0.46	0.00	1.80	7.24
		$REMod-PER$	0.04	0.73	11.85	2.69	0.00	13.30	25.43	0.66	0.00	2.54	5.83
		$REMod-UNA$	0.06	0.93	16.82	2.99	0.00	18.87	32.18	0.38	0.00	1.95	7.56
	Mean	$REMod-REF$	1.61	3.14	47.17	14.59	46.78	0.23	49.97	1.00	0.02	5.66	6.26
		$REMod-SUF$	1.09	2.13	31.71	10.54	34.25	0.15	33.78	0.68	0.01	3.86	6.36
		$REMod-PER$	1.84	2.75	34.00	12.39	47.93	0.16	39.53	0.98	0.02	5.44	5.13
		$REMod-UNA$	2.57	3.51	48.28	13.80	71.47	0.23	50.02	0.56	0.01	4.18	6.65
Foc. FFR	2019	$REMod-REF$	0.05	0.49	17.04	0.65	0.00	19.61	5.59	0.38	0.00	0.44	5.00
		$REMod-SUF$	0.03	0.33	11.46	0.47	0.00	13.18	3.78	0.26	0.00	0.30	5.09
		$REMod-PER$	0.05	0.43	12.29	0.56	0.00	14.14	4.42	0.37	0.00	0.42	4.10
		$REMod-UNA$	0.07	0.55	17.44	0.62	0.00	20.06	5.60	0.21	0.00	0.32	5.31
	Mean	$REMod-REF$	1.08	1.89	36.82	2.62	26.49	0.18	11.88	0.62	0.02	1.67	2.26
		$REMod-SUF$	0.73	1.28	24.76	1.89	19.39	0.12	8.03	0.42	0.01	1.14	2.29
		$REMod-PER$	1.23	1.66	26.55	2.22	27.14	0.13	9.40	0.61	0.02	1.60	1.85
		$REMod-UNA$	1.73	2.12	37.69	2.48	40.46	0.18	11.89	0.35	0.01	1.23	2.40
Foc. Extr. EA	2019	$REMod-REF$	2.90	8.19	271.46	26.78	4.08	25.16	86.05	1.38	0.04	9.10	7.21
		$REMod-SUF$	1.96	5.55	182.50	19.35	2.99	16.91	58.16	0.94	0.03	6.20	7.33
		$REMod-PER$	3.30	7.19	195.69	22.74	4.19	18.14	68.07	1.36	0.05	8.75	5.91
		$REMod-UNA$	4.62	9.18	277.83	25.34	6.24	25.74	86.14	0.78	0.03	6.71	7.66
	Mean	$REMod-REF$	7.87	8.36	295.50	27.24	109.37	5.32	87.27	1.38	0.46	9.12	7.20
		$REMod-SUF$	5.31	5.67	198.66	19.69	80.09	3.58	58.99	0.94	0.30	6.21	7.32
		$REMod-PER$	8.96	7.34	213.02	23.14	112.06	3.84	69.04	1.36	0.57	8.77	5.90
		$REMod-UNA$	12.54	9.36	302.44	25.78	167.09	5.45	87.36	0.78	0.33	6.72	7.65
Foc. Extr. FX	2019	$REMod-REF$	0.94	4.46	108.64	21.02	1.58	22.99	75.39	1.16	0.00	7.37	7.20
		$REMod-SUF$	0.64	3.02	73.04	15.20	1.16	15.45	50.96	0.79	0.00	5.02	7.32
		$REMod-PER$	1.08	3.91	78.32	17.86	1.62	16.58	59.64	1.14	0.00	7.09	5.90
		$REMod-UNA$	1.51	4.99	111.20	19.89	2.42	23.52	75.47	0.65	0.00	5.44	7.65
	Mean	$REMod-REF$	4.95	6.65	169.51	25.74	104.36	3.08	80.55	1.28	0.09	8.44	7.09
		$REMod-SUF$	3.34	4.51	113.96	18.61	76.42	2.07	54.44	0.87	0.06	5.75	7.21
		$REMod-PER$	5.64	5.84	122.20	21.86	106.93	2.22	63.72	1.26	0.11	8.12	5.82
		$REMod-UNA$	7.89	7.44	173.49	24.36	159.44	3.15	80.63	0.72	0.06	6.23	7.53
Foc. Extr. FFR	2019	$REMod-REF$	0.55	2.08	62.69	3.00	1.45	23.47	27.26	0.72	0.00	2.43	6.24
		$REMod-SUF$	0.37	1.41	42.15	2.17	1.06	15.78	18.42	0.49	0.00	1.66	6.35
		$REMod-PER$	0.63	1.83	45.19	2.55	1.49	16.92	21.56	0.70	0.00	2.34	5.12
		$REMod-UNA$	0.88	2.33	64.16	2.84	2.21	24.01	27.28	0.40	0.00	1.80	6.63
	Mean	$REMod-REF$	3.46	3.87	114.73	8.37	61.80	3.00	39.75	0.94	0.06	4.42	4.44
		$REMod-SUF$	2.33	2.62	77.13	6.05	45.25	2.02	26.87	0.64	0.04	3.01	4.52
		$REMod-PER$	3.94	3.40	82.71	7.11	63.32	2.17	31.44	0.93	0.07	4.25	3.64
		$REMod-UNA$	5.51	4.34	117.42	7.92	94.42	3.07	39.79	0.53	0.04	3.26	4.72
Q. 25%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Q. 40%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the GVAR models

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Q. 50%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.61	0.09	0.02	0.00	0.00	0.77
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.41	0.06	0.02	0.00	0.00	0.78
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.44	0.07	0.02	0.00	0.00	0.63
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.62	0.09	0.01	0.00	0.00	0.82
	Mean	$REMod-REF$	0.02	0.13	0.30	0.03	0.00	0.00	0.26	0.09	0.00	0.01	0.14
		$REMod-SUF$	0.01	0.09	0.20	0.02	0.00	0.00	0.18	0.06	0.00	0.01	0.14
		$REMod-PER$	0.02	0.11	0.22	0.02	0.00	0.00	0.21	0.09	0.00	0.01	0.11
		$REMod-UNA$	0.03	0.14	0.31	0.03	0.00	0.00	0.26	0.05	0.00	0.01	0.15
Q. 60%	2019	$REMod-REF$	0.04	1.15	21.61	0.85	0.00	22.51	10.31	0.62	0.00	0.83	5.82
		$REMod-SUF$	0.02	0.78	14.53	0.61	0.00	15.13	6.97	0.42	0.00	0.57	5.92
		$REMod-PER$	0.04	1.01	15.58	0.72	0.00	16.23	8.15	0.61	0.00	0.80	4.77
		$REMod-UNA$	0.06	1.28	22.11	0.80	0.00	23.03	10.32	0.35	0.00	0.61	6.18
	Mean	$REMod-REF$	2.22	3.44	54.17	3.35	35.31	0.56	23.76	0.97	0.02	2.55	2.75
		$REMod-SUF$	1.50	2.33	36.42	2.42	25.86	0.37	16.06	0.66	0.01	1.74	2.80
		$REMod-PER$	2.53	3.02	39.05	2.85	36.18	0.40	18.80	0.96	0.02	2.45	2.26
		$REMod-UNA$	3.54	3.85	55.44	3.17	53.95	0.57	23.79	0.55	0.01	1.88	2.92
Q. 75%	2019	$REMod-REF$	5.90	8.33	286.67	27.16	23.98	25.23	87.36	1.38	0.02	9.11	7.21
		$REMod-SUF$	3.98	5.65	192.73	19.63	17.56	16.96	59.05	0.94	0.01	6.20	7.33
		$REMod-PER$	6.72	7.31	206.66	23.07	24.57	18.20	69.11	1.36	0.03	8.76	5.91
		$REMod-UNA$	9.40	9.33	293.41	25.70	36.64	25.82	87.45	0.78	0.02	6.72	7.66
	Mean	$REMod-REF$	9.12	8.36	301.58	27.27	130.98	19.61	87.36	1.38	0.43	9.12	7.21
		$REMod-SUF$	6.15	5.67	202.76	19.71	95.91	13.18	59.05	0.94	0.28	6.21	7.33
		$REMod-PER$	10.38	7.34	217.41	23.16	134.20	14.14	69.11	1.36	0.54	8.77	5.91
		$REMod-UNA$	14.53	9.36	308.67	25.80	200.11	20.06	87.45	0.78	0.31	6.72	7.66

This table displays the expected loss due to scarcity for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), per path ($REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$), as well as per scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) for the input variables. Hereby, the values are derived from the GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean).

In general, the results are driven by the exposure at scarcity, as higher resource requirements cause higher scarcity risks. Hereby, the resource demands from 2020 to 2050 for the energy expansion pathways of the energy system derived from the project InteRessE,[33] combined with the probability of scarcity based on the different commodity frameworks highly determine the risk of scarcity. Especially in case of the shock, extreme, as well as 60% and 75% quantile scenarios, for which the GVAR model indicates for most of the commodities to exceed their threshold almost surely, the corresponding commodity-specific expected loss due to scarcity values equal to the product of substitutability and exposure at scarcity. However, with a few exceptions, the order of the most risky commodities remains the same across all scenarios. Hereby, cobalt, indium and nickel are outstanding, but also the risk of lithium and copper is remarkable, whereas the low demand for lead and platinum indicate their scarcity risks are neglectable. While the probability of scarcity of lead indicates a general high scarcity risk, the low scaled demand cause the negligible overall risk for lead.

Under the mean scenario, lithium and zinc are outstanding, caused by their high probabilities of scarcity, while the other expected loss due to scarcity values are near zero, as their probabilities of scarcity indicate the commodities almost never exceed their thresholds. Hereby, the risks of cobalt and nickel are relatively high, although their probabilities of scarcity are comparable to those of the other commodities, however, their high required resource amounts cause their expected loss due to scarcity values, underlining the impact of the resource requirements on the risk results. In contrast, the probability of scarcity of copper determines its low scarcity risk, despite its comparable high resource requirements.

[33] The specific data for the technologies considered as well as the resource requirement are not yet published.

Comparing the expected loss due to scarcity values between the scenarios, we detect the results highly depend on the probability of scarcity and the exposure at scarcity. Since most of the commodities exceed their threshold almost surely under the shock, extreme, as well as 60% and 75% quantile scenarios, the expected loss due to scarcity is determined by their substitutability as well as their resource requirements. In particular, cobalt and indium, followed by nickel, bear the highest risks, caused by their high scaled demand, reflected in their exposure at scarcity, while the expected loss due to scarcity of lead and platinum is almost neglectable, caused by their lower requirements. Since extreme situations in the global demand affects the probability of scarcity of the commodities most, the scarcity risk under the (extreme) focus scenario with stressed world gross domestic product is higher than the respective scenarios with stressed U.S. dollar index or Federal Funds Effective Rate. Hereby, the risks of the industrial metals, especially copper and nickel, increases most, indicating they are impacted most by the economy. In contrast, the probability of scarcity vanishes in case of the 25% and 40% quantile scenarios, resulting in expected loss due to scarcity values of zero, indicating the risk of commodity scarcity is almost neglectable under these circumstances.

Overall, the results of the GVAR model reveal cobalt, indium and nickel bear the highest scarcity risks, caused by their resource requirements, indicating the demand for the metals is the most important risk driver.

The MS-GVAR model reflects the time-varying spillover effects in the industrial metal markets to assess the scarcity risks. Hereby, the commodities bear higher risks if the average price level is used to derive the probability of scarcity, since the average prices of the previous decade are higher than the prices in 2019, except for zinc. However, regardless of the initial price level, copper and nickel are outstanding, while the scarcity risk of lead is neglectable, in line with the results of the GVAR model, see Table 5.35.

Table 5.35: Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the MS-GVAR models

			Al	Cu	Ni	Pb	Sn	Zn
Mean	2019	$REMod-REF$	2.11	3.05	9.26	1.26	0.47	7.21
		$REMod-SUF$	1.43	2.21	6.26	0.86	0.32	7.33
		$REMod-PER$	1.85	2.59	7.33	1.24	0.46	5.91
		$REMod-UNA$	2.36	2.89	9.27	0.71	0.35	7.66
	Mean	$REMod-REF$	8.33	26.89	46.30	1.38	3.92	4.38
		$REMod-SUF$	5.65	19.43	31.30	0.94	2.67	4.46
		$REMod-PER$	7.31	22.84	36.63	1.36	3.77	3.59
		$REMod-UNA$	9.33	25.44	46.35	0.78	2.89	4.66
Shock	2019	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
Extr.	2019	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
Foc. EA	2019	$REMod-REF$	8.04	16.58	68.32	1.38	4.65	7.21
		$REMod-SUF$	5.45	11.98	46.18	0.94	3.17	7.33
		$REMod-PER$	7.06	14.08	54.04	1.36	4.47	5.91
		$REMod-UNA$	9.01	15.69	68.38	0.78	3.43	7.66
	Mean	$REMod-REF$	8.36	27.27	84.57	1.38	8.51	7.08
		$REMod-SUF$	5.67	19.71	57.16	0.94	5.80	7.20
		$REMod-PER$	7.34	23.16	66.90	1.36	8.19	5.80
		$REMod-UNA$	9.36	25.80	84.65	0.78	6.28	7.52
		$REMod-REF$	8.34	25.20	81.07	1.38	5.32	7.21

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the MS-GVAR models

			Al	Cu	Ni	Pb	Sn	Zn
Foc. FX	2019	$REMod-SUF$	5.66	18.21	54.80	0.94	3.63	7.33
Foc. FX	2019	$REMod-PER$	7.33	21.40	64.13	1.36	5.12	5.91
Foc. FX	2019	$REMod-UNA$	9.34	23.84	81.15	0.78	3.93	7.66
Foc. FX	Mean	$REMod-REF$	8.36	27.27	87.19	1.38	8.53	7.02
Foc. FX	Mean	$REMod-SUF$	5.67	19.71	58.93	0.94	5.81	7.14
Foc. FX	Mean	$REMod-PER$	7.34	23.16	68.97	1.36	8.20	5.76
Foc. FX	Mean	$REMod-UNA$	9.36	25.80	87.27	0.78	6.29	7.46
Foc. FFR	2019	$REMod-REF$	8.36	27.27	87.36	1.38	9.02	7.21
Foc. FFR	2019	$REMod-SUF$	5.67	19.71	59.05	0.94	6.15	7.33
Foc. FFR	2019	$REMod-PER$	7.34	23.16	69.11	1.36	8.68	5.91
Foc. FFR	2019	$REMod-UNA$	9.36	25.80	87.45	0.78	6.66	7.66
Foc. FFR	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.08	7.21
Foc. FFR	Mean	$REMod-SUF$	5.67	19.71	59.05	0.94	6.19	7.33
Foc. FFR	Mean	$REMod-PER$	7.34	23.16	69.11	1.36	8.73	5.91
Foc. FFR	Mean	$REMod-UNA$	9.36	25.80	87.45	0.78	6.70	7.66
Foc. Extr. EA	2019	$REMod-REF$	8.31	24.00	81.25	1.38	7.66	7.21
Foc. Extr. EA	2019	$REMod-SUF$	5.63	17.34	54.92	0.94	5.22	7.33
Foc. Extr. EA	2019	$REMod-PER$	7.30	20.38	64.27	1.36	7.36	5.91
Foc. Extr. EA	2019	$REMod-UNA$	9.31	22.71	81.33	0.78	5.65	7.66
Foc. Extr. EA	Mean	$REMod-REF$	8.36	27.27	86.49	1.38	8.86	7.18
Foc. Extr. EA	Mean	$REMod-SUF$	5.67	19.71	58.46	0.94	6.04	7.30
Foc. Extr. EA	Mean	$REMod-PER$	7.34	23.16	68.42	1.36	8.52	5.89
Foc. Extr. EA	Mean	$REMod-UNA$	9.36	25.80	86.57	0.78	6.54	7.63
Foc. Extr. FX	2019	$REMod-REF$	8.36	26.78	86.31	1.38	7.73	7.21
Foc. Extr. FX	2019	$REMod-SUF$	5.67	19.35	58.34	0.94	5.27	7.33
Foc. Extr. FX	2019	$REMod-PER$	7.34	22.74	68.28	1.36	7.43	5.91
Foc. Extr. FX	2019	$REMod-UNA$	9.36	25.34	86.40	0.78	5.70	7.66
Foc. Extr. FX	Mean	$REMod-REF$	8.36	27.27	87.19	1.38	8.97	7.17
Foc. Extr. FX	Mean	$REMod-SUF$	5.67	19.71	58.93	0.94	6.11	7.29
Foc. Extr. FX	Mean	$REMod-PER$	7.34	23.16	68.97	1.36	8.63	5.88
Foc. Extr. FX	Mean	$REMod-UNA$	9.36	25.80	87.27	0.78	6.62	7.61
Foc. Extr. FFR	2019	$REMod-REF$	8.36	27.27	87.36	1.38	9.08	7.21
Foc. Extr. FFR	2019	$REMod-SUF$	5.67	19.71	59.05	0.94	6.19	7.33
Foc. Extr. FFR	2019	$REMod-PER$	7.34	23.16	69.11	1.36	8.73	5.91
Foc. Extr. FFR	2019	$REMod-UNA$	9.36	25.80	87.45	0.78	6.70	7.66
Foc. Extr. FFR	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
Foc. Extr. FFR	Mean	$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
Foc. Extr. FFR	Mean	$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
Foc. Extr. FFR	Mean	$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
Q. 25%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00
Q. 25%	2019	$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00
Q. 25%	2019	$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00
Q. 25%	2019	$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00
Q. 25%	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00
Q. 25%	Mean	$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00
Q. 25%	Mean	$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00
Q. 25%	Mean	$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00
Q. 40%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.10
Q. 40%	2019	$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.10
Q. 40%	2019	$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.08
Q. 40%	2019	$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.11
Q. 40%	Mean	$REMod-REF$	0.00	0.00	0.00	0.01	0.00	0.00
Q. 40%	Mean	$REMod-SUF$	0.00	0.00	0.00	0.01	0.00	0.00
Q. 40%	Mean	$REMod-PER$	0.00	0.00	0.00	0.01	0.00	0.00
Q. 40%	Mean	$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00
Q. 50%	2019	$REMod-REF$	0.00	0.11	0.35	0.34	0.09	7.02
Q. 50%	2019	$REMod-SUF$	0.00	0.08	0.24	0.23	0.06	7.14
Q. 50%	2019	$REMod-PER$	0.00	0.09	0.28	0.34	0.09	5.76
Q. 50%	2019	$REMod-UNA$	0.00	0.10	0.35	0.19	0.07	7.46
Q. 50%	Mean	$REMod-REF$	1.69	6.16	6.12	0.99	1.22	0.87
Q. 50%	Mean	$REMod-SUF$	1.14	4.45	4.13	0.67	0.83	0.88
Q. 50%	Mean	$REMod-PER$	1.48	5.23	4.84	0.98	1.17	0.71
Q. 50%	Mean	$REMod-UNA$	1.89	5.83	6.12	0.56	0.90	0.92
Q. 60%	2019	$REMod-REF$	8.36	27.27	87.36	1.38	8.75	7.21
Q. 60%	2019	$REMod-SUF$	5.67	19.71	59.05	0.94	5.96	7.33
Q. 60%	2019	$REMod-PER$	7.34	23.16	69.11	1.36	8.42	5.91
Q. 60%	2019	$REMod-UNA$	9.36	25.80	87.45	0.78	6.46	7.66
Q. 60%	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
Q. 60%	Mean	$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
Q. 60%	Mean	$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the MS-GVAR models

			Al	Cu	Ni	Pb	Sn	Zn
		$REMod - UNA$	9.36	25.80	87.45	0.78	6.72	7.66
Q. 75%	2019	$REMod - REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod - SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod - PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod - UNA$	9.36	25.80	87.45	0.78	6.72	7.66
	Mean	$REMod - REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod - SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod - PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod - UNA$	9.36	25.80	87.45	0.78	6.72	7.66

This table displays the expected loss due to scarcity for the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), per path ($REMod - REF$, $REMod - SUF$, $REMod - PER$ and $REMod - UNA$), as well as per scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) for the input variables. Hereby, the values are derived from the MS-GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod - REF$, $REMod - SUF$, $REMod - PER$ and $REMod - UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean).

As the MS-GVAR model considers the time period from 1995 to 2019 and only includes the industrial metals, which are most connected to the economy, the scarcity risks are highly affected by shocks to the global variables, compared to the GVAR model. Overall, the stronger interdependencies between the commodity markets as well as between the commodities and the economy lead to higher probabilities of scarcity and ultimately higher expected loss due to scarcity values, compared to the GVAR model. However, the expected loss due to scarcity of the industrial metals derived from the MS-GVAR model coincide with those derived from the GVAR model under the shock, extreme, 60% and 75% quantile scenarios, as both models indicate the commodities almost surely exceed their threshold prices, which is why the expected loss due to scarcity equals to the product of loss given scarcity and exposure at scarcity. Despite the higher risk values, copper and nickel are outstanding, caused by their comparable high probability of scarcity as well as exposure at scarcity, whereas the risk of lead is comparable low, in line with the results derived from the GVAR model.

Since spillover effects between the commodity markets are not reflected within the logistic regression models to derive the probability of scarcity, leading to smaller probability of scarcity values, the risk of scarcity is underestimated, compared to the results of the (MS-)GVAR model, see Table 5.36. However, the results derived from the logistic regression model also reveal cobalt, followed by indium and nickel are the most risky commodities, similar to the GVAR model. In contrast, the risk of copper and lithium is by far lower, due to their lower probability of scarcity. Moreover, contrary to the results of the (MS-)GVAR model, the expected loss due to scarcity of the rare earth metals, dysprosium as well as neodymium, can be derived under the logistic regression model. Hereby, dysprosium bears a comparable high risk, caused by its subsitutability score as well as its required resource amount, however, its expected loss due to scarcity value is not outstanding, due to the low probability of scarcity. In addition, the scaled resource requirement of neodymium is comparable low, resulting in moderate expected loss due to scarcity values. While lithium exhibits comparable high risks under the mean scenario, zinc's risks increases in case of the shock scenario, caused by its higher probability of scarcity. Moreover, the risks of the commodities barely react to shocks to the global economy, as spillover effects are not reflected in the logistic regression model. Only copper and indium are affected by shocks to the Federal Funds Effective Rate and U.S. dollar index, respectively. In general, the risks of the precious metals, the industrial metals, except nickel, as well as neodymium are comparably low, even under the extreme scenario, while cobalt, indium, and nickel are outstanding, exhibiting comparably high risks even under the mean scenario.

Table 5.36: Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the logistic regression models

		Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
Mean	$REMod - REF$	0.33	0.50	22.05	1.16	1.48	13.36	1.05	0.19	7.88	0.08	0.02	0.19	0.27
	$REMod - SUF$	0.22	0.34	14.82	0.84	1.06	9.78	0.70	0.14	5.33	0.05	0.01	0.13	0.28
	$REMod - PER$	0.37	0.44	15.89	0.99	1.42	13.69	0.75	0.18	6.23	0.08	0.02	0.18	0.22
	$REMod - UNA$	0.52	0.56	22.57	1.10	0.70	20.41	1.07	0.08	7.89	0.04	0.01	0.14	0.29
Shock	$REMod - REF$	0.44	0.96	140.56	1.66	7.70	21.03	1.46	1.01	48.70	0.20	0.04	0.29	5.80
	$REMod - SUF$	0.30	0.65	94.50	1.20	5.51	15.40	0.98	0.72	32.92	0.14	0.02	0.20	5.90
	$REMod - PER$	0.50	0.84	101.33	1.41	7.42	21.54	1.05	0.93	38.53	0.20	0.05	0.28	4.76
	$REMod - UNA$	0.70	1.08	143.86	1.57	3.67	32.12	1.49	0.43	48.75	0.11	0.03	0.22	6.16
Extr.	$REMod - REF$	0.59	1.75	275.15	2.34	33.85	31.90	2.01	4.42	82.22	0.44	0.08	0.46	7.19
	$REMod - SUF$	0.40	1.18	184.98	1.69	24.19	23.36	1.35	3.14	55.58	0.30	0.05	0.31	7.31
	$REMod - PER$	0.67	1.53	198.35	1.99	32.60	32.69	1.45	4.09	65.04	0.43	0.09	0.44	5.90
	$REMod - UNA$	0.94	1.96	281.61	2.22	16.14	48.74	2.06	1.89	82.30	0.25	0.05	0.34	7.64
Foc. EA	$REMod - REF$	0.33	0.50	22.05	1.16	2.99	13.36	1.05	0.39	7.88	0.08	0.02	0.19	0.27
	$REMod - SUF$	0.22	0.34	14.82	0.84	2.14	9.78	0.70	0.28	5.33	0.05	0.01	0.13	0.28
	$REMod - PER$	0.37	0.44	15.89	0.99	2.88	13.69	0.75	0.36	6.23	0.08	0.02	0.18	0.22
	$REMod - UNA$	0.52	0.56	22.57	1.10	1.43	20.41	1.07	0.17	7.89	0.04	0.01	0.14	0.29
Foc. FX	$REMod - REF$	0.33	0.50	22.05	1.19	1.63	16.34	1.05	0.21	7.88	0.12	0.02	0.19	0.27
	$REMod - SUF$	0.22	0.34	14.82	0.86	1.17	11.96	0.70	0.15	5.33	0.08	0.01	0.13	0.28
	$REMod - PER$	0.37	0.44	15.89	1.01	1.57	16.74	0.75	0.20	6.23	0.11	0.03	0.18	0.22
	$REMod - UNA$	0.52	0.56	22.57	1.12	0.78	24.96	1.07	0.09	7.89	0.07	0.02	0.14	0.29
Foc. FFR	$REMod - REF$	0.33	0.50	22.05	1.40	3.51	13.36	1.05	0.46	7.88	0.08	0.02	0.19	1.20
	$REMod - SUF$	0.22	0.34	14.82	1.01	2.51	9.78	0.70	0.33	5.33	0.05	0.01	0.13	1.22
	$REMod - PER$	0.37	0.44	15.89	1.19	3.38	13.69	0.75	0.42	6.23	0.08	0.02	0.18	0.99
	$REMod - UNA$	0.52	0.56	22.57	1.32	1.67	20.41	1.07	0.20	7.89	0.04	0.01	0.14	1.28
Foc. Extr. EA	$REMod - REF$	0.33	0.50	22.05	1.16	6.00	13.36	1.05	0.78	7.88	0.08	0.02	0.19	0.27
	$REMod - SUF$	0.22	0.34	14.82	0.84	4.29	9.78	0.70	0.56	5.33	0.05	0.01	0.13	0.28
	$REMod - PER$	0.37	0.44	15.89	0.99	5.78	13.69	0.75	0.73	6.23	0.08	0.02	0.18	0.22
	$REMod - UNA$	0.52	0.56	22.57	1.10	2.86	20.41	1.07	0.34	7.89	0.04	0.01	0.14	0.29
Foc. Extr. FX	$REMod - REF$	0.33	0.50	22.05	1.21	1.80	19.86	1.05	0.23	7.88	0.17	0.03	0.19	0.27
	$REMod - SUF$	0.22	0.34	14.82	0.87	1.29	14.54	0.70	0.17	5.33	0.12	0.02	0.13	0.28
	$REMod - PER$	0.37	0.44	15.89	1.02	1.73	20.35	0.75	0.22	6.23	0.17	0.03	0.18	0.22
	$REMod - UNA$	0.52	0.56	22.57	1.14	0.86	30.34	1.07	0.10	7.89	0.10	0.02	0.14	0.29
Foc. Extr. FFR	$REMod - REF$	0.33	0.50	22.05	1.67	8.16	13.36	1.05	1.07	7.88	0.08	0.02	0.19	3.64
	$REMod - SUF$	0.22	0.34	14.82	1.21	5.84	9.78	0.70	0.76	5.33	0.05	0.01	0.13	3.70
	$REMod - PER$	0.37	0.44	15.89	1.42	7.86	13.69	0.75	0.99	6.23	0.08	0.02	0.18	2.99
	$REMod - UNA$	0.52	0.56	22.57	1.58	3.89	20.41	1.07	0.46	7.89	0.04	0.01	0.14	3.87
Q. 25%	$REMod - REF$	0.29	0.37	5.39	0.96	0.57	11.32	0.98	0.07	2.11	0.05	0.01	0.14	0.02
	$REMod - SUF$	0.20	0.25	3.62	0.69	0.41	8.29	0.66	0.05	1.42	0.03	0.01	0.09	0.02
	$REMod - PER$	0.33	0.32	3.89	0.81	0.55	11.59	0.71	0.07	1.67	0.05	0.02	0.13	0.01
	$REMod - UNA$	0.47	0.41	5.52	0.91	0.27	17.29	1.00	0.03	2.11	0.03	0.01	0.10	0.02
Q. 40%	$REMod - REF$	0.31	0.42	9.96	1.10	0.98	12.58	0.98	0.13	6.63	0.06	0.02	0.16	0.08
	$REMod - SUF$	0.21	0.29	6.70	0.79	0.70	9.21	0.66	0.09	4.48	0.04	0.01	0.11	0.08
	$REMod - PER$	0.36	0.37	7.18	0.93	0.94	12.89	0.71	0.12	5.25	0.06	0.02	0.16	0.06
	$REMod - UNA$	0.50	0.47	10.20	1.04	0.47	19.22	1.01	0.05	6.64	0.03	0.01	0.12	0.08
Q. 50%	$REMod - REF$	0.32	0.45	16.10	1.17	1.22	13.20	1.00	0.16	9.89	0.07	0.02	0.18	0.21
	$REMod - SUF$	0.22	0.30	10.82	0.85	0.87	9.66	0.67	0.11	6.68	0.05	0.01	0.12	0.21
	$REMod - PER$	0.37	0.39	11.60	0.99	1.17	13.52	0.72	0.15	7.82	0.07	0.02	0.17	0.17

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the logistic regression models

		Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
	$REMod-UNA$	0.52	0.50	16.48	1.11	0.58	20.16	1.02	0.07	9.90	0.04	0.01	0.13	0.22
Q. 60%	$REMod-REF$	0.35	0.55	34.00	1.27	1.81	14.35	1.02	0.24	12.51	0.09	0.02	0.18	0.70
	$REMod-SUF$	0.24	0.37	22.86	0.92	1.29	10.51	0.69	0.17	8.46	0.06	0.01	0.13	0.71
	$REMod-PER$	0.40	0.48	24.51	1.08	1.74	14.70	0.74	0.22	9.90	0.09	0.03	0.18	0.57
	$REMod-UNA$	0.56	0.61	34.80	1.20	0.86	21.92	1.04	0.10	12.53	0.05	0.01	0.14	0.74
Q. 75%	$REMod-REF$	0.40	0.75	104.03	1.44	3.63	16.83	1.06	0.47	19.44	0.15	0.03	0.25	3.37
	$REMod-SUF$	0.27	0.51	69.94	1.04	2.59	12.32	0.71	0.34	13.14	0.10	0.02	0.17	3.42
	$REMod-PER$	0.45	0.66	74.99	1.23	3.50	17.24	0.76	0.44	15.38	0.15	0.03	0.24	2.76
	$REMod-UNA$	0.63	0.84	106.47	1.37	1.73	25.71	1.08	0.20	19.46	0.08	0.02	0.18	3.58

This table displays the expected loss due to scarcity for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), per path ($REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$), as well as per scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) for the input variables, derived from the logistic regression model.

Overall, independent of the model used to derive the probability of scarcity, the $REMod-SUF$ path mostly exhibits the lowest commodity-specific expected loss due to scarcity values, while the $REMod-REF$, $REMod-PER$ and $REMod-UNA$ paths bear higher risks, caused by their higher probabilities of scarcity and, especially, their higher resource demands, reflected in the exposure at scarcity (EAS). Hereby, the $REMod-UNA$ path bears the highest risks for almost all commodities caused by its high resource demands, whereas the risk of copper, lead and tin (platinum) is slightly higher under the $REMod-REF$ ($REMod-PER$) path. However, the probability of scarcity vanishes in case of the 25% and 40% quantile scenarios for all paths, resulting in expected loss due to scarcity values of zero, indicating the risk of commodity scarcity is almost neglectable under these circumstances.

5.3.3.2 Expected Loss due to Scarcity per Path

Aggregating the commodity-specific expected loss due to scarcity values on path level, we obtain our final measure of scarcity, the expected loss due to scarcity $ES_{p,\zeta}$ per path and scenario displayed in Table 5.37. Hereby, the expected loss due to scarcity values differ in the underlying commodities. The results of the logistic regression model incorporates the risks of the industrial metals as well as the key elements of the German Energiewende, defined by Bastian et al. (2019). However, the GVAR model excludes the rare earth metals, dysprosium and neodymium, whereas the MS-GVAR model only reflects the risks of the industrial metals markets, due to data limitations. In Section 5.3.4.5, we restrict the commodity set to the industrial metals to allow a comparison between the models.

Table 5.37: Path-specific expected loss due to scarcity

		Mean	Shock	Extr.	Foc. EA	Foc. FX	Foc. FFR	Foc. Extr. EA	Foc. Extr. FX	Foc. Extr. FFR	Q. 25%	Q. 40%	Q. 50%	Q. 60%	Q. 75%
GVAR 2019	$REMod-REF$	6.71	606.51	611.73	203.64	81.48	49.25	442.34	250.77	129.89	0.00	0.00	1.49	63.72	482.35
	$REMod-SUF$	4.46	419.38	423.50	136.75	52.02	31.04	295.84	166.61	81.00	0.00	0.00	1.22	32.60	318.89
	$REMod-PER$	5.79	492.78	499.35	163.65	66.28	37.55	335.94	195.08	106.35	0.00	0.00	1.18	46.81	369.25
	$REMod-UNA$	6.74	680.83	691.38	207.29	81.09	50.42	449.17	251.12	131.82	0.00	0.00	1.32	52.18	482.21

Path-specific expected loss due to scarcity

Model	Path	Mean	Shock	Extr.	Foc. EA	Foc. FX	Foc. FFR	Foc. Extr. EA	Foc. Extr. FX	Foc. Extr. FFR	Q. 25%	Q. 40%	Q. 50%	Q. 60%	Q. 75%
GVAR Mean	$REMod-REF$	3.60	611.73	611.73	362.61	176.42	85.51	559.09	411.73	244.85	0.00	0.00	0.97	129.11	602.41
	$REMod-SUF$	2.07	423.48	423.50	241.14	114.03	48.49	383.79	279.13	152.78	0.00	0.00	0.23	65.87	414.73
	$REMod-PER$	3.67	499.35	499.35	293.24	159.69	76.89	456.97	354.01	210.74	0.00	0.00	0.81	107.32	492.34
	$REMod-UNA$	5.22	691.38	691.38	382.96	198.92	94.83	626.61	474.60	277.47	0.00	0.00	0.62	114.91	677.09
MS-GVAR 2019	$REMod-REF$	23.37	140.70	140.70	106.18	128.52	140.60	129.80	137.77	140.66	0.00	0.10	7.91	140.33	140.70
	$REMod-SUF$	17.35	98.91	98.91	75.27	87.27	98.86	91.89	95.74	98.89	0.00	0.06	7.76	98.62	98.91
	$REMod-PER$	20.48	115.64	115.64	88.74	106.11	115.54	107.25	113.56	115.62	0.00	0.09	7.07	115.26	115.64
	$REMod-UNA$	24.25	137.77	137.77	105.80	127.90	137.73	126.18	135.42	137.76	0.00	0.14	8.41	137.45	137.77
MS-GVAR Mean	$REMod-REF$	91.20	140.70	140.70	137.17	139.75	140.66	139.54	140.33	140.70	0.00	0.01	17.05	140.70	140.70
	$REMod-SUF$	62.63	98.91	98.91	97.34	97.40	98.89	98.48	98.65	98.89	0.00	0.01	11.11	98.91	98.91
	$REMod-PER$	78.46	115.64	115.64	113.35	115.03	115.62	115.03	115.42	115.62	0.00	0.01	15.48	115.64	115.64
	$REMod-UNA$	90.88	137.77	137.77	135.43	136.97	137.76	137.26	137.57	137.76	0.00	0.02	16.47	137.76	137.77
log. Reg.	$REMod-REF$	48.56	229.84	442.40	50.27	51.77	52.01	53.68	55.56	59.99	22.27	33.42	43.97	67.08	151.84
	$REMod-SUF$	33.70	158.42	303.86	34.93	36.05	36.46	37.36	38.82	42.90	15.75	23.38	30.59	46.40	104.58
	$REMod-PER$	40.48	178.83	345.28	42.12	43.75	43.64	45.39	47.63	50.92	20.15	29.05	37.18	54.62	117.83
	$REMod-UNA$	55.40	240.19	446.14	56.20	60.07	57.69	57.81	65.60	63.02	28.16	39.84	50.74	74.57	161.36

This table displays the expected loss due to scarcity per path ($REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$) and scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) of the input variables. Hereby, the values are derived from the (MS-)GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean) as well as from the logistic regression (log. Reg.) model.

While the expected loss due to scarcity values of the 25% and 40% quantile scenarios of the (MS-)GVAR model are equal to zero due to the vanishing probabilities of scarcity, the risks under the shock, extreme, focus and 75% quantile scenarios are exceptionally high, mainly caused by the high expected loss due to scarcity of cobalt, indium and nickel (copper and nickel). As the path-specific expected loss due to scarcity in the mean scenario is comparably low, caused by the almost neglectable commodity-specific probabilities of scarcity under the GVAR model, the MS-GVAR model exhibits the highest values, since the commodities almost surely exceed their threshold price, although only the industrial metals are considered. However, except for the mean scenario, the expected loss due to scarcity values are significantly smaller for the MS-GVAR models, due to the reduced commodity set. Moreover, the (MS-)GVAR models indicate stressed values in the global demand, reflected by the world gross domestic product (world industrial production) cause the highest (smallest) risks, whereas shocks to the Federal Funds Effective Rate incrases the scarcity risk the least (most) in the time-invariant (time-varying) analysis. However, the relative high expected loss due to scarcity values indicate the global economy affect the commodity markets and their associated scarcity risks, in line with the spillover effects from the economy to the commodities in Section 5.1.2 and Section 5.2.3.3. In case of the logistic regression models, the expected loss due to scarcity values of the shock, extreme and 75% scenario are outstanding, while the remaining scenario specific values are comparable, indicating the scarcity risk barely differ across the scenarios. In particular, a stressed economy leads only to moderate risks, as the commodities barely react to shocks to the global economy, especially, only few commodities are determined by the macroeconomic variables at all. However, the commodity-specific analysis neglects the interdependencies between the commodities and therefore, underestimates the scarcity risk, especially in the shock, extreme, (extreme) focus as well as 60% and 75% quantile scenarios.

Overall, the sufficiency path ($REMod - SUF$) bears the lowest risk, due to the least required amounts for the commodities. In contrast, the unacceptance path ($REMod - UNA$), closely followed by the reference path ($REMod - REF$), shows the highest expected loss due to scarcity values of all pathways for the GVAR model and the logistic regression model, in almost all scenarios considered, inter alia due to the high demand in cobalt, caused by the large amount of battery storage required in this path. However, in case of the 60% (40% and 50%) quantile scenario using the prices of 2019 (average prices) as basis level, the reference path ($REMod - REF$) path bears higher risks in the mean scenario, caused by the higher expected loss due to scarcity values of copper, lead, and tin. The higher expected loss due to scarcity values of copper, lead, and tin also lead to higher overall risks of the $REMod - REF$ path in the MS-GVAR model. Overall, despite the different frameworks applied, despite the different time periods used for the initial price level, and despite the different metals considered, our analysis suggests, the $REMod - SUF$ path should be accomplished, as an higher acceptance of the society reduces the commodity requirements, and therefore the scarcity risks, substantially.

5.3.4 Robustness Analyses

The derived scarcity risk of the transformation pathways is based on assumptions of a price threshold, the underlying scenario values, the substitutability, as well as a scaling factor for the commodity requirements, which we choose due to the close connection with our setup, as well as for its simplicity and statistical validity. In this section, we examine to what extent our results remain valid by using different assumptions for the price threshold, the definition of the scenario values, the substitutability, as well as for the scaling factor, whereby we focus on one assumption each, see Section 5.3.4.1, Section 5.3.4.2, Section 5.3.4.3, and Section 5.3.4.4.[34] Moreover, due to data limitations, the results presented above correspond to different commodities analyzed. While the MS-GVAR model is only based on the industrial metal markets, the logistic regression models incorporates all key resources for the German Energiewende, according to Bastian et al. (2019), including the rare earth metals, which are omitted for the GVAR model. For the comparability of the results derived from the different models, we restrict the commodities to the industrial metals, for which the data is available in each case. Subsequently, we are able to compare the corresponding expected loss due to scarcity per path, scenario and model, see Section 5.3.4.5.

5.3.4.1 Robustness Analysis for the Threshold Price

First, we consider the sensitivity of the proposed risk assessment framework to the threshold. As we set the price threshold statistically with the one-sigma approach, its calculation depends on the considered time period. Instead of approximating the historical mean and standard deviation on data over the previous decade, we reduce and enlarge the considered time period to five and 25 years, respectively, to analyze the sensitivity of the risk assessment framework to changes in the price threshold.[35]

Table 5.38 displays the resulting thresholds based on the reduced as well as enlarged sample period, which are generally lower than the original ones, presented in Table 5.25. As the commodity prices stayed at a high level for several years after the price boom in the beginning of 2000s, the thresholds based on the previous decade includes a period of high prices, while the thresholds based on the last 25 years also include a time span of a calmer period, resulting in

[34]Adjusting two or more assumptions at the same time lead to different expected loss due to scarcity values, however, the main findings, especially the hierarchy between the paths, remain valid.

[35]Due to data availability issues, the prices of the rare earth metals dysprosium and neodymium are only available within the last decade, thus, the considered time period can not be enlarged.

lower values, especially for the precious metal prices. Moreover, the decline in the prices around 2015, caused by the slowdown in Chinese demand as well as the oil price drop, lead to lower threshold prices if only the recent five years are considered. The exceptions are cobalt, lithium, and zinc, as their prices increased over the previous years. While supply concerns cause the ongoing growth in zinc's price, the increased interest in cobalt and lithium for batteries lead to the observed rising prices.

Table 5.38: Commodity price threshold of the robustness analysis

			Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
2015	2019	$\$/t$	548814	2037	63650	6461	317529	373960	129735	70520	13576	2270	32914653	20258	2929
1995	2019	$\$/t$	692596	2229	59546	7165	608299	659014	101435	81920	21167	2194	44638013	20183	2549

This table displays the price threshold for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn) in $U.S.\$/t$, derived from the one-sigma approach in Equation 3.159, based on annual price data in the period from 1995 to 2019 or from 2015 to 2019.

Table D.28, Table D.29 and Table D.31 (Table D.35, Table D.36 and Table D.38) display the resulting probabilities of scarcity derived under the thresholds considering five years (25 years) for the (MS-)GVAR model and the logistic regression model, respectively. In general, the results suggest a higher threshold price leads to a reduced scarcity risk for the (MS-)GVAR models with different weight matrices as well as the logistic regression model.

Regarding the GVAR model and the mean scenario as well as the scenarios, representing a stressed economy, the risk of lithium, lead, tin as well as zinc are significantly higher under the thresholds based on the previous 25 years for most of the model specifications, whereas silver, aluminum, copper, indium, nickel and platinum observe the highest risk under the thresholds based on the previous five years.

In case of the MS-GVAR model, the probabilities of scarcity under the thresholds based on the previous 25 years are slightly lower for aluminum and nickel. However, the results of the adapted thresholds lead generally to higher risk values, especially, the risks observed under the mean scenario are exceptionally high if the threshold based on the previous five years is used, except for zinc.

In case of the logistic regression model, the model selection identifies the same determinants under the new threshold, except for lithium, which is now driven by the 10-year U.S. Treasury rate as well as the U.S. consumer price index, see Table D.30 and Table D.37, which is why lithium's risks differ across the scenarios. Hereby, the mean scenario indicates a neglectable, risk under the threshold based on the previous five years, whereas lithium's risk is exceptionally high under the shock and extreme scenarios. In general, the commodity-specific probabilities of scarcity are slightly higher under the adapted thresholds, except for cobalt, caused by its smaller threshold value. Overall, the (MS-)GVAR models as well as the logistic regression model indicate higher scarcity risks in case of smaller threshold prices.

Due to the different probabilities of scarcity, the expected loss due to scarcity values differ across the analyses. However, a higher threshold price cause a smaller probability of scarcity and therefore a reduced expected loss due to scarcity value, see Table D.39, Table D.40 and Table D.41 for the (MS-)GVAR and the logistic regression model, respectively. While the GVAR model exhibits an increased risk of silver and nickel (lithium) under the threshold based on the previous five years (25 years), aluminum, copper and nickel (tin) show high expected loss due to scarcity values in case of the MS-GVAR model, using the threshold prices, based on the previous five years (25 years). Besides nickel, the logistic regression model also indicates higher risks for the rare earth metals.

Overall, the models based on the threshold price of the previous five years exhibits the highest path-specific expected loss due to scarcity, see Table 5.39. However, although the commodity-and path-specific expected loss due to scarcity values differ, the ordering between the paths remains the same for all considered models as well as threshold prices. While the $REMod - SUF$ path bears the lowest commodity-specific risks, the $REMod - UNA$ path exhibits the highest values across all scenarios, except for the MS-GVAR model, similar to the results in Table 5.35.

Table 5.39: Path-specific expected loss due to scarcity of the robustness analysis for the threshold price

			Mean	Shock	Extr.	Foc. EA	Foc. FX	Foc. FFR	Foc. Extr. EA	Foc. Extr. FX	Foc. Extr. FFR	Q. 25%	Q. 40%	Q. 50%	Q. 60%	Q. 75%
2015 to 2019	GVAR, 2019	$REMod - REF$	31.92	611.73	611.73	171.07	138.67	94.74	405.98	244.87	173.36	0.00	0.09	11.54	120.36	506.24
		$REMod - SUF$	20.50	423.50	423.50	116.77	96.46	62.48	275.05	167.52	113.06	0.00	0.06	6.79	76.87	332.55
		$REMod - PER$	31.12	499.35	499.35	143.88	118.95	83.13	326.02	214.22	151.21	0.00	0.07	10.10	99.59	411.93
		$REMod - UNA$	33.88	691.38	691.38	174.45	144.65	98.74	431.52	275.04	184.06	0.00	0.09	10.25	116.56	546.66
	GVAR, Mean	$REMod - REF$	207.68	611.02	611.73	335.79	287.66	267.67	539.25	374.10	320.32	0.02	18.21	131.15	281.59	565.34
		$REMod - SUF$	146.15	422.42	423.50	234.49	201.90	186.72	372.19	257.57	220.67	0.01	13.20	93.25	193.46	381.29
		$REMod - PER$	204.63	498.73	499.35	302.40	265.09	250.13	448.69	329.49	292.77	0.01	22.07	134.09	259.19	463.86
		$REMod - UNA$	280.23	689.88	691.38	413.03	358.72	341.07	615.67	446.09	394.69	0.05	31.69	188.24	349.68	630.30
	MS-GVAR	$REMod - REF$	136.53	140.70	140.70	140.54	140.49	140.70	140.66	140.66	140.70	0.00	11.53	120.38	140.70	140.70
		$REMod - SUF$	96.00	98.91	98.91	98.83	98.82	98.89	98.89	98.88	98.91	0.00	7.42	83.27	98.91	98.91
		$REMod - PER$	112.04	115.64	115.64	115.52	115.46	115.62	115.61	115.59	115.62	0.00	9.85	97.52	115.64	115.64
		$REMod - UNA$	134.58	137.77	137.77	137.68	137.65	137.76	137.72	137.72	137.77	0.00	13.25	118.56	137.77	137.77
	MS-GVAR, Mean	$REMod - REF$	133.89	140.70	140.70	137.78	138.43	140.70	139.76	139.99	140.70	0.00	65.20	133.04	140.70	140.70
		$REMod - SUF$	92.14	98.91	98.91	96.81	97.37	98.91	98.26	98.41	98.91	0.00	47.79	91.37	98.91	98.91
		$REMod - PER$	110.10	115.64	115.64	113.58	114.13	115.64	115.00	115.20	115.64	0.00	53.97	109.38	115.64	115.64
		$REMod - UNA$	130.56	137.77	137.77	135.29	135.52	137.77	137.11	137.20	137.77	0.00	65.73	129.91	137.77	137.77
	log. Reg.	$REMod - REF$	126.06	294.99	435.87	143.31	149.96	187.31	162.06	176.02	224.74	81.38	95.92	108.85	152.70	238.66
		$REMod - SUF$	89.66	208.57	306.35	101.98	106.76	133.47	115.39	125.40	160.28	58.04	68.32	77.48	108.46	168.85
		$REMod - PER$	117.33	265.95	379.16	133.90	140.27	176.08	151.93	165.26	211.86	76.49	89.70	101.53	141.47	217.56
		$REMod - UNA$	123.84	252.08	389.54	132.02	136.53	153.20	140.91	150.25	171.20	95.04	104.39	112.38	143.26	207.25
1995 to 2019	GVAR, 2019	$REMod - REF$	29.37	606.04	611.73	165.21	80.10	46.50	407.32	219.44	111.50	0.00	1.35	22.53	54.99	452.67
		$REMod - SUF$	20.42	419.25	423.50	113.31	52.87	31.82	275.66	146.11	70.44	0.00	1.66	16.02	34.27	293.14
		$REMod - PER$	21.76	491.90	499.35	137.23	65.99	39.65	315.27	176.47	91.02	0.00	1.18	17.00	43.89	347.49
		$REMod - UNA$	29.64	680.50	691.38	170.12	78.45	47.51	418.20	224.74	110.66	0.00	2.73	23.80	49.93	451.15
	GVAR, Mean	$REMod - REF$	11.81	611.73	611.73	262.85	154.38	83.22	549.05	368.39	219.81	0.00	0.00	4.41	113.33	598.32
		$REMod - SUF$	7.25	423.50	423.50	175.78	101.71	50.26	372.85	249.97	140.67	0.00	0.00	2.87	60.40	409.39
		$REMod - PER$	13.44	499.35	499.35	219.27	142.05	75.93	449.24	321.28	195.32	0.00	0.01	5.30	99.02	488.54
		$REMod - UNA$	14.02	691.38	691.38	283.79	177.61	95.00	615.67	428.78	248.62	0.00	0.00	5.37	104.59	670.36
	MS-GVAR	$REMod - REF$	43.78	140.70	140.70	98.97	124.32	140.70	127.75	137.67	140.70	0.00	0.46	15.40	140.52	140.70
		$REMod - SUF$	33.28	98.91	98.91	71.95	83.67	98.89	92.09	95.45	98.91	0.00	0.44	12.78	98.91	98.91
		$REMod - PER$	38.64	115.64	115.64	83.71	104.29	115.62	106.23	113.53	115.62	0.00	0.37	13.55	115.50	115.64
		$REMod - UNA$	40.81	137.77	137.77	96.80	123.86	137.76	125.55	135.09	137.77	0.00	0.73	15.12	137.60	137.77
	MS-GVAR, Mean	$REMod - REF$	70.20	140.70	140.70	128.99	137.38	140.70	137.38	140.35	140.70	0.00	1.79	42.40	140.70	140.70
		$REMod - SUF$	49.08	98.91	98.91	91.81	95.25	98.91	97.25	97.84	98.91	0.00	1.21	30.80	98.91	98.91
		$REMod - PER$	60.96	115.64	115.64	108.73	113.57	115.64	113.57	115.64	115.64	0.00	1.63	37.02	115.64	115.64
		$REMod - UNA$	66.44	137.77	137.77	127.10	134.97	137.77	134.80	137.25	137.77	0.00	1.65	38.18	137.77	137.77
	log. Reg.	$REMod - REF$	34.39	150.39	295.42	36.11	35.85	39.18	39.51	37.46	48.34	20.32	28.75	35.04	44.53	69.32
		$REMod - SUF$	24.10	105.04	204.71	25.33	25.17	28.13	27.76	26.33	35.67	14.33	20.17	24.54	31.23	48.84
		$REMod - PER$	29.36	121.65	237.99	31.00	30.80	33.62	34.27	32.38	41.88	18.13	24.94	29.97	37.52	57.09
		$REMod - UNA$	39.21	155.18	290.00	40.01	41.07	42.88	41.62	43.10	49.43	24.79	33.55	40.00	49.81	74.76

This table displays the expected loss due to scarcity per path ($REMod - REF$, $REMod - SUF$, $REMod - PER$, and $REMod - UNA$) and scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) of the input variables. Hereby, the values are derived from the (MS-)GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod - REF$, $REMod - SUF$, $REMod - PER$, and $REMod - UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean) as well as from the logistic regression model (Log Reg). The displayed expected loss due to scarcity values represent the results of the robustness analysis, in which the threshold is derived based on data from 1995 to 2019 or 2015 to 2019.

While the aggregated expected loss due to scarcity values on path level, displayed in Table 5.39, are higher than the original ones, displayed in Table 5.37, in case of the (MS-)GVAR model, the risk on path level, derived from the logistic regression model, is smaller. The higher expected loss due to scarcity values are caused by the higher probability of scarcity for almost all commodities, whereas the effect of the smaller probability of scarcity of cobalt in combination with its exposure at scarcity on the expected loss due to scarcity outweighs the higher probabilities of the remaining commodities in case of the logistic regression model, resulting in smaller expected loss due to scarcity values. However, the ordering between the paths remains the same, similar to the commodity-specific results. This finding holds for the (MS-)GVAR models based on different initial price levels and for the logistic regression models as well as for the various scenarios. In particular, the $REMod - UNA$ (and $REMod - REF$) path, requiring the highest demand in cobalt for battery storage, bears the highest risk derived from the GVAR model and the logistic regression model (MS-GVAR model), while the acceptance of the society reduces the risk, reflected by the smallest expected loss due to scarcity values for the $REMod - SUF$ path.

5.3.4.2 Robustness Analysis for the Scenario Values

Besides the threshold price, the definition of the scenario values also depends on the considered time periods, as the scenario values reflect the (adjusted) historical mean or quantiles of the variables. Similar to the above robustness analysis, we reduce and enlarge the period from the previous decade to five and 25 years and display the resulting scenario values in Table D.42 and Table D.49, respectively.

Overall, the comparison of the scenario values based on the previous decade or on the period from 2015 (1995) to 2019 provide mixed results. In general, the values are highest based on the previous 25 years, followed by the values based on the previous decade. However, several demand variables, especially for cobalt, and the monthly prices are higher if only data of the previous five years is considered, indicating the recent increase in demand. In contrast, the scenario values of the supply and price variables are higher in the enlarged sample, probably caused by the commodity price boom at the beginning of the 2000s. Moreover, the annual global demand proxies (monetary policy variables) obtain higher values in the period from 1995 (2015) to 2019, whereas their monthly counterparts exhibit higher values in the period from 2015 (1995) to 2019. In addition, the comparison of the mean, and (extreme) focus scenario values differ from the comparison of the shock, extreme and quantile scenarios, further underlining the heterogeneity of the results.

Using the scenario values based on the previous five years (25 years), the probability of scarcity of the (MS-)GVAR model as well as the logistic regression model can be derived, see Section D.3.2.2. The mixed evidence of the comparison of the scenario values is also reflected in the probability of scarcity values. While the GVAR model exhibits the highest (smallest) values for the quantile scenarios under the values based on the previous 25 years (five years), the risks of the focus scenarios are mostly higher in case of the scenario values based on the previous decade, whereas the risks under the mean, shock, and extreme scenarios are comparable, except for lithium, which is highest in case of the values of the previous five years. In contrast, the MS-GVAR model obtains the highest risks for all commodities and scenarios under the scenario values based on the previous five years, confirming the heterogeneous results. However, the probabilities of scarcity derived from the logistic regression model barely differ, only the results of nickel and zinc are outstanding, as their probabilities of scarcity are highest under the values based on the previous ten or five years, respectively. As the logistic regression model enables for individual selected price influential factors, the results of the commodities differ, whereas the (MS-)GVAR model reflects spillover effects between the metals, which is why higher (smaller) scenario values affect all markets.

These heterogeneous results of the probability of scarcity are also reflected in the final risk measure expected loss due to scarcity (ES), on commodity as well as path level, see Section D.3.2.2. While the risks from the MS-GVAR model is higher (smaller) across almost all scenarios, using the scenario values based on the previous five years (25 years), the results of the GVAR model differ across the scenarios. Hereby, the mean (quantile) scenario indicates higher expected loss due to scarcity using the scenario values based on the previous five years (25 years). In case of the logistic regression model, the expected loss due to scarcity values are comparable, whereby the results differ more under the quantile scenarios.

However, even if the expected loss due to scarcity values differ, the comparison of the resource scarcity risk of the paths is mostly preserved. In particular, the $REMod-SUF$ path carries the lowest risks across most of the scenarios and models, the only exceptions are under the mean scenario, whereby the $REMod-PER$ and $REMod-UNA$ path exhibits the smallest risk once. In general, the heterogeneous scenario values cause mixed evidence for the different models and scenarios, however, the ordering between the paths generally remains the same, underlining the $REMod-SUF$ path exhibits the lowest scarcity risks.

Table 5.40: Path-specific expected loss due to scarcity of the robustness analysis for the scenario values

				Mean	Shock	Extr.	Foc. EA	Foc. FX	Foc. FFR	Foc. Extr. EA	Foc. Extr. FX	Foc. Extr. FFR	Q. 25%	Q. 40%	Q. 50%	Q. 60%	Q. 75%
2015 to 2019	GVAR	2019	$REMod-REF$	7.96	606.98	611.73	125.20	75.04	29.76	404.07	245.79	60.90	0.00	0.00	0.72	49.06	469.20
			$REMod-SUF$	6.00	419.82	423.50	83.61	51.46	20.12	276.54	165.52	39.62	0.00	0.02	0.83	29.15	307.00
			$REMod-PER$	6.61	493.53	499.35	98.89	60.24	22.79	311.05	191.86	47.10	0.00	0.00	0.81	34.19	355.65
			$REMod-UNA$	9.23	682.16	691.38	131.98	80.00	32.13	413.52	250.11	64.00	0.00	0.03	1.52	46.64	470.04
		Mean	$REMod-REF$	10.26	611.73	611.73	218.52	151.64	43.86	527.49	379.10	110.54	0.00	0.00	0.86	79.44	599.69
			$REMod-SUF$	6.80	423.48	423.50	152.45	103.55	27.17	362.32	262.39	69.84	0.00	0.01	0.61	46.05	411.59
			$REMod-PER$	7.75	499.33	499.35	181.85	139.67	40.09	425.72	325.85	98.00	0.00	0.00	0.46	64.79	488.66
			$REMod-UNA$	14.18	691.38	691.38	250.10	188.08	56.74	590.50	445.39	131.30	0.00	0.00	0.75	86.95	672.74
	MS-GVAR	2019	$REMod-REF$	116.54	140.70	140.70	139.25	138.95	140.55	140.07	140.37	140.64	0.00	0.46	22.49	140.40	140.70
			$REMod-SUF$	81.81	98.91	98.91	97.88	97.66	98.82	98.61	98.51	98.88	0.00	0.68	17.07	98.67	98.91
			$REMod-PER$	99.30	115.64	115.64	114.12	114.13	115.48	115.00	115.26	115.59	0.00	0.45	19.48	115.34	115.64
			$REMod-UNA$	116.61	137.77	137.77	136.51	136.68	137.69	137.34	137.43	137.76	0.00	0.92	22.54	137.54	137.77
		Mean	$REMod-REF$	139.19	140.70	140.70	140.41	140.60	140.66	140.64	140.64	140.70	0.00	0.32	89.53	140.70	140.70
			$REMod-SUF$	97.78	98.91	98.91	98.83	98.83	98.88	98.88	98.87	98.89	0.00	0.25	63.73	98.91	98.91
			$REMod-PER$	114.21	115.64	115.64	115.55	115.52	115.61	115.59	115.57	115.62	0.00	0.42	79.20	115.64	115.64
			$REMod-UNA$	136.85	137.77	137.77	137.68	137.69	137.76	137.72	137.73	137.76	0.00	0.42	92.48	137.76	137.77
	log. Reg.		$REMod-REF$	44.39	219.87	432.75	47.12	48.52	48.05	52.69	53.61	54.21	21.33	27.05	33.00	65.19	186.40
			$REMod-SUF$	31.10	152.16	297.76	33.05	34.11	34.13	37.03	37.83	39.15	15.13	19.06	23.18	45.45	128.72
			$REMod-PER$	37.30	170.79	338.81	39.92	41.51	40.61	45.26	46.70	46.20	19.52	23.94	28.50	53.07	143.62
			$REMod-UNA$	50.50	230.50	439.22	51.79	56.40	53.11	54.41	63.69	57.32	26.56	32.30	38.28	71.44	195.10
1995 to 2019	GVAR	2019	$REMod-REF$	0.19	606.14	611.73	179.58	43.53	32.38	420.74	235.27	150.20	0.00	0.00	0.09	117.71	532.57
			$REMod-SUF$	0.19	419.40	423.50	122.22	26.65	21.57	287.79	159.69	91.77	0.00	0.00	0.13	58.04	356.96
			$REMod-PER$	0.16	492.94	499.35	138.58	33.67	23.75	319.81	183.80	118.96	0.00	0.00	0.08	81.05	409.54
			$REMod-UNA$	0.19	681.05	691.38	177.94	38.67	32.96	430.17	232.47	151.54	0.00	0.00	0.07	91.77	553.22
		Mean	$REMod-REF$	0.00	611.73	611.73	306.89	104.85	51.55	526.52	392.08	269.14	0.00	0.00	0.00	273.37	609.08
			$REMod-SUF$	0.00	423.48	423.50	209.52	69.17	29.27	362.86	272.13	176.96	0.00	0.00	0.00	155.97	420.12
			$REMod-PER$	0.00	499.33	499.35	236.69	90.65	42.69	423.76	333.39	226.61	0.00	0.00	0.00	213.97	497.16
			$REMod-UNA$	0.00	691.35	691.38	314.45	114.18	53.51	583.14	454.33	301.42	0.00	0.00	0.00	255.41	686.63
	MS-GVAR	2019	$REMod-REF$	3.36	140.70	140.70	23.91	65.78	140.57	80.71	120.37	140.66	0.00	0.00	6.10	114.23	140.70
			$REMod-SUF$	3.51	98.91	98.91	20.47	42.01	98.82	61.70	81.79	98.88	0.00	0.00	6.60	77.48	98.91
			$REMod-PER$	2.68	115.64	115.64	19.42	55.76	115.50	66.07	99.88	115.61	0.00	0.00	5.22	95.11	115.64
			$REMod-UNA$	3.42	137.77	137.77	24.01	66.27	137.72	80.75	119.70	137.76	0.00	0.00	6.96	112.79	137.77
		Mean	$REMod-REF$	1.03	140.70	140.70	57.10	108.58	140.66	112.21	133.67	140.70	0.00	0.00	4.68	140.49	140.70
			$REMod-SUF$	0.70	98.91	98.91	44.79	72.40	98.89	80.76	91.90	98.89	0.00	0.00	3.28	98.91	98.91
			$REMod-PER$	1.10	115.64	115.64	47.27	90.85	115.61	92.19	110.40	115.62	0.00	0.00	4.19	115.49	115.64
			$REMod-UNA$	0.90	137.77	137.77	58.47	106.92	137.76	109.55	131.15	137.77	0.00	0.00	4.11	137.74	137.77
	log. Reg.		$REMod-REF$	48.29	190.97	424.10	49.67	51.94	51.95	52.64	56.38	63.82	22.91	34.29	48.10	68.79	139.40
			$REMod-SUF$	33.42	131.57	292.24	34.41	36.10	36.25	36.53	39.34	45.65	16.10	23.94	33.37	47.46	95.50
			$REMod-PER$	39.99	149.41	335.17	41.32	43.73	43.38	44.17	48.26	54.30	20.27	29.52	40.15	56.06	108.90
			$REMod-UNA$	55.45	200.81	419.75	56.10	60.83	57.72	57.50	67.36	65.30	28.41	41.05	55.67	76.98	149.28

This table displays the expected loss due to scarcity per path ($REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$) and scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) of the input variables. Hereby, the values are derived from the (MS-)GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean) as well as from the logistic regression model (Log Reg). The displayed expected loss due to scarcity values represent the results of the robustness analysis, in which the scenario values are derived using data from 1995 to 2019 or from 2015 to 2019.

5.3.4.3 Robustness Analysis for the Loss given Scarcity

In our main analysis, we use information about the metals' applications with primary substitutes and substitute performance from Graedel et al. (2015) to reflect the substitutability of the considered commodities. However, Graedel et al. (2015) do not only focus on technologies for the energy transition, therefore, we might assume a metal can be replaced, although this is not the case for the German Energiewende. Moreover, the substitute materials may be equally or even more scarce than the current material. Consequently, we investigate whether the resource risks change if we assume there is no possibility to substitute any commodity within the pathways, resulting in a loss given scarcity of one for each commodity.

In general, the expected loss due to scarcity values on commodity as well as on path level are higher under the assumption the metals are not substitutable, see Appendix D.3.2.3 and Table 5.41 for the commodity-specific and aggregated expected loss due to scarcity values, respectively. However, the ordering between the paths remains the same for each model considered as well as for the different scenarios. While the $REMod - UNA$ path bears the highest risks in case of the GVAR and logistic regression model, the $REMod - REF$ path has the highest values in case of the MS-GVAR model. In contrast, the $REMod - SUF$ path exhibits the lowest expected loss due to scarcity overall, further underlining the path with the lowest resource requirements bears the smallest scarcity risk.

Table 5.41: Path-specific expected loss due to scarcity of the robustness analysis for the loss given scarcity

			Mean	Shock	Extr.	Foc. EA	Foc. FX	Foc. FFR	Foc. Extr. EA	Foc. Extr. FX	Foc. Extr. FFR	Q. 25%	Q. 40%	Q. 50%	Q. 60%	Q. 75%
GVAR	2019	$REMod - REF$	16.37	1102.22	1110.90	382.38	160.49	105.32	818.79	464.40	253.88	0.00	0.00	3.67	133.66	890.10
		$REMod - SUF$	11.10	762.57	769.44	258.62	105.01	68.80	548.93	309.83	161.24	0.00	0.00	3.12	72.26	590.52
		$REMod - PER$	13.91	891.97	902.91	306.92	130.18	80.72	622.12	361.21	207.24	0.00	0.00	2.98	98.50	682.37
		$REMod - UNA$	16.54	1230.31	1247.89	388.74	160.18	108.88	831.69	464.56	258.35	0.00	0.00	3.35	112.54	891.50
	Mean	$REMod - REF$	6.64	1110.91	1110.91	650.95	311.32	153.63	1005.39	731.92	440.51	0.00	0.00	1.83	231.83	1090.51
		$REMod - SUF$	3.99	769.40	769.44	434.45	202.27	87.88	690.72	496.20	275.32	0.00	0.00	0.50	118.49	749.43
		$REMod - PER$	6.69	902.91	902.91	526.89	281.57	138.09	819.62	627.79	378.71	0.00	0.00	1.56	192.99	887.42
		$REMod - UNA$	9.63	1247.89	1247.89	686.30	348.58	169.32	1120.75	838.07	496.57	0.00	0.00	1.21	205.08	1216.16
MS-GVAR	2019	$REMod - REF$	45.64	244.53	244.53	185.42	220.86	244.28	225.83	238.29	244.43	0.00	0.27	19.79	243.52	244.53
		$REMod - SUF$	35.52	173.76	173.76	133.40	152.33	173.62	161.63	167.46	173.73	0.00	0.16	19.58	172.97	173.76
		$REMod - PER$	39.58	202.49	202.49	155.52	183.39	202.20	187.66	197.69	202.44	0.00	0.25	17.08	201.42	202.49
		$REMod - UNA$	47.81	238.79	238.79	185.04	219.95	238.68	219.18	233.84	238.75	0.00	0.36	21.11	237.89	238.79
	Mean	$REMod - REF$	155.82	244.53	244.53	238.01	242.14	244.43	242.34	243.73	244.53	0.00	0.01	29.17	244.53	244.53
		$REMod - SUF$	108.61	173.76	173.76	170.65	170.70	173.73	172.86	173.19	173.73	0.00	0.01	19.30	173.76	173.76
		$REMod - PER$	134.30	202.49	202.49	198.07	200.81	202.44	201.29	201.88	202.44	0.00	0.01	26.38	202.49	202.49
		$REMod - UNA$	156.52	238.79	238.79	234.33	236.98	238.75	237.78	238.22	238.75	0.00	0.04	28.47	238.75	238.79
log. Reg.		$REMod - REF$	85.20	409.49	774.27	87.20	90.43	90.66	91.17	96.63	103.61	38.75	57.79	76.36	118.25	273.83
		$REMod - SUF$	59.09	283.51	532.80	60.52	62.91	63.73	63.35	67.44	74.92	27.32	40.36	53.07	81.84	189.27
		$REMod - PER$	70.46	316.57	597.99	72.37	75.81	75.31	76.16	82.13	86.76	34.78	49.88	64.11	95.58	211.25
		$REMod - UNA$	97.30	430.19	792.38	98.23	105.02	101.46	100.08	114.17	111.50	48.89	68.98	88.22	131.61	291.61

This table displays the expected loss due to scarcity per path ($REMod - REF$, $REMod - SUF$, $REMod - PER$, and $REMod - UNA$) and scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) of the input variables. Hereby, the values are derived from the (MS-)GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod - REF$, $REMod - SUF$, $REMod - PER$, and $REMod - UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean) as well as from the logistic regression model (Log Reg). The displayed expected loss due to scarcity values represent the results of the robustness analysis, in which the loss given scarcity is set equal to one.

5.3.4.4 Robustness Analysis for the Exposure at Scarcity

Besides the threshold price and the scenario values, the scaling factor for the exposure at scarcity also depends on the considered time period. Similar to those robustness analyses, we reduce and enlarge the period from the previous ten years to five and 25 years and display the resulting exposure at scarcity in Table 5.42.[36] In general, the rise in commodities over the last decades lead to an increase in the production volume of the commodities, which is why the exposure at scarcity is highest for the enlarged sample period, whereas the scaled demand based on the scaling factor of the previous five and ten years are comparable. Hereby, cobalt is outstanding, as albeit its high demand for storage capacities, the production volume decreased in the previous years, as cobalt's supply depends on the supply of its host metals, resulting in a higher exposure at scarcity.

Table 5.42: Exposure at scarcity values of the robustness analysis

		Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
2015 to 2019	$REMod-REF$	0.20	0.17	6.52	0.36	1.42	2.20	0.44	0.45	1.42	0.01	0.01	0.24	0.19
	$REMod-SUF$	0.14	0.12	4.39	0.26	1.01	1.61	0.29	0.32	0.96	0.01	0.01	0.16	0.19
	$REMod-PER$	0.23	0.15	4.70	0.31	1.36	2.25	0.31	0.42	1.13	0.01	0.01	0.23	0.16
	$REMod-UNA$	0.32	0.19	6.68	0.34	0.68	3.35	0.45	0.19	1.42	0.01	0.01	0.18	0.20
1995 to 2019	$REMod-REF$	0.25	0.27	7.78	0.46	1.42	3.04	1.09	0.45	1.89	0.02	0.01	0.27	0.23
	$REMod-SUF$	0.17	0.18	5.23	0.34	1.01	2.22	0.74	0.32	1.28	0.01	0.01	0.18	0.23
	$REMod-PER$	0.28	0.24	5.61	0.39	1.36	3.11	0.79	0.42	1.50	0.02	0.01	0.26	0.19
	$REMod-UNA$	0.39	0.30	7.96	0.44	0.68	4.64	1.12	0.19	1.89	0.01	0.01	0.20	0.24

This table displays the exposure at scarcity (EAS) of the commodities aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn) per transformation path ($REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$). In particular, the exposure at scarcity is derived as the total required amount per commodity and path scaled by the average world production in the period from 2015 to 2019 or 1995 to 2019, according to Equation 3.176.

The corresponding expected loss due to scarcity values on commodity as well as path level for the considered scenarios are displayed in Section D.3.2.4, as well as Table 5.43. In general, the expected loss due to scarcity values per commodity and per path are higher if the resource demands are scaled by the average production volume of the previous 25 years, whereas the results of the other scaling factors are similar.

While the absolute risk values differ across the scenarios and scaling factors, similar to the other robustness analyses, the ordering between the paths does not change by varying the assumptions on the scaling factor for the exposure at scarcity. In line with the results above, the $REMod-SUF$ path, reflecting a high acceptance of the society for the German Energiewende, bears the lowest risks, while the $REMod-UNA$ ($REMod-REF$) path, modeling strong resistances to actions for the energy transition, shows the highest risk values in case of the GVAR and logistic regression model (MS-GVAR model).

Table 5.43: Path-specific expected loss due to scarcity of the robustness analysis for the exposure at scarcity

				Mean	Shock	Extr.	Foc. EA	Foc. FX	Foc. FFR	Foc. Extr. EA	Foc. Extr. FX	Foc. Extr. FFR	Q. 25%	Q. 40%	Q. 50%	Q. 60%	Q. 75%
2015 to 2019	GVAR	2019	$REMod-REF$	5.65	644.33	649.55	206.72	78.59	46.18	475.28	259.54	132.64	0.00	0.00	1.32	60.47	517.54
			$REMod-SUF$	3.88	444.72	448.84	138.60	49.97	28.45	317.08	171.65	81.85	0.00	0.00	1.11	29.44	341.81
			$REMod-PER$	4.93	519.48	526.05	165.95	64.01	35.12	359.09	200.99	109.20	0.00	0.00	1.04	44.28	394.25
			$REMod-UNA$	5.60	719.55	730.10	211.10	78.17	47.06	482.78	260.05	134.70	0.00	0.00	1.15	48.24	517.46
		Mean	$REMod-REF$	3.79	649.55	649.55	389.09	182.57	90.89	601.39	435.18	261.09	0.00	0.00	1.01	136.93	641.44
			$REMod-SUF$	2.25	448.83	448.84	258.34	117.15	51.41	412.63	293.98	162.09	0.00	0.00	0.22	69.23	441.81
			$REMod-PER$	3.86	526.05	526.05	312.57	164.18	81.06	487.01	371.18	222.96	0.00	0.00	0.83	112.78	520.24
			$REMod-UNA$	5.66	730.10	730.10	410.40	204.96	100.26	670.61	498.83	294.52	0.00	0.00	0.65	120.86	718.21
	MS-GVAR		$REMod-REF$	23.10	138.31	138.31	104.67	126.46	138.22	127.68	135.50	138.28	0.00	0.10	7.97	137.97	138.31
			$REMod-SUF$	17.20	97.22	97.22	74.27	85.80	97.17	90.44	94.12	97.21	0.00	0.06	7.81	96.95	97.22
			$REMod-PER$	20.24	113.48	113.48	87.40	104.25	113.38	105.37	111.50	113.47	0.00	0.10	7.11	113.12	113.48
			$REMod-UNA$	23.98	135.53	135.53	104.42	125.97	135.49	124.27	133.29	135.51	0.00	0.14	8.46	135.22	135.53
		Mean	$REMod-REF$	88.75	138.31	138.31	134.79	137.40	138.28	137.16	137.95	138.31	0.00	0.01	16.46	138.31	138.31
			$REMod-SUF$	60.89	97.22	97.22	95.67	95.74	97.21	96.80	96.97	97.21	0.00	0.01	10.68	97.22	97.22

[36]Due to data availability issues, the production data of the rare earth metals dysprosium and neodymium, the scaling factors and therefore the exposure at scarcity coincide.

Path-specific expected loss due to scarcity of the robustness analysis for the exposure at scarcity

			Mean	Shock	Extr.	Foc. EA	Foc. FX	Foc. FFR	Foc. Extr. EA	Foc. Extr. FX	Foc. Extr. FFR	Q. 25%	Q. 40%	Q. 50%	Q. 60%	Q. 75%
		$REMod-PER$	76.34	113.48	113.48	111.21	112.91	113.47	112.88	113.27	113.47	0.00	0.01	14.94	113.48	113.48
		$REMod-UNA$	88.50	135.53	135.53	133.21	134.75	135.51	135.02	135.33	135.51	0.00	0.02	15.87	135.51	135.53
log. Reg.		$REMod-REF$	47.14	243.64	477.07	49.87	51.27	50.80	55.44	56.36	56.96	21.60	27.88	34.43	70.40	207.44
log. Reg.		$REMod-SUF$	32.94	168.15	327.57	34.89	35.96	35.97	38.87	39.67	41.00	15.31	19.62	24.13	48.95	142.87
log. Reg.		$REMod-PER$	39.26	187.91	370.74	41.88	43.47	42.57	47.22	48.65	48.16	19.70	24.52	29.50	56.80	158.77
log. Reg.		$REMod-UNA$	53.31	254.81	484.55	54.60	59.21	55.92	57.22	66.49	60.14	26.83	33.14	39.73	76.76	216.62
1995 to 2019	GVAR 2019	$REMod-REF$	10.35	832.92	840.11	281.33	115.78	74.36	607.57	344.59	184.98	0.00	0.00	2.15	94.99	662.16
1995 to 2019	GVAR 2019	$REMod-SUF$	6.62	575.29	580.96	188.40	73.57	47.15	405.61	228.21	115.51	0.00	0.00	1.69	49.25	437.08
1995 to 2019	GVAR 2019	$REMod-PER$	8.92	674.63	683.68	224.45	93.35	56.63	459.47	266.51	150.32	0.00	0.00	1.73	69.46	504.68
1995 to 2019	GVAR 2019	$REMod-UNA$	10.55	935.94	950.49	287.33	115.59	76.70	617.86	345.82	188.29	0.00	0.00	1.93	79.20	662.70
1995 to 2019	GVAR Mean	$REMod-REF$	4.75	840.11	840.11	487.75	235.72	116.02	759.76	556.87	333.26	0.00	0.00	1.29	175.17	825.05
1995 to 2019	GVAR Mean	$REMod-SUF$	2.75	580.93	580.96	323.54	151.73	65.57	520.58	376.59	207.43	0.00	0.00	0.29	89.09	566.25
1995 to 2019	GVAR Mean	$REMod-PER$	4.83	683.68	683.68	393.39	213.16	104.05	619.71	478.14	286.26	0.00	0.00	1.06	145.12	672.19
1995 to 2019	GVAR Mean	$REMod-UNA$	7.02	950.49	950.49	516.49	267.12	128.97	852.76	643.81	378.53	0.00	0.00	0.83	156.16	927.01
1995 to 2019	MS-GVAR 2019	$REMod-REF$	29.75	181.56	181.56	138.07	166.60	181.46	167.83	178.10	181.52	0.00	0.12	9.53	181.17	181.56
1995 to 2019	MS-GVAR 2019	$REMod-SUF$	21.81	127.27	127.27	97.66	112.65	127.21	118.58	123.37	127.25	0.00	0.07	9.33	126.96	127.27
1995 to 2019	MS-GVAR 2019	$REMod-PER$	26.09	148.77	148.77	115.24	137.22	148.66	138.40	146.39	148.75	0.00	0.11	8.57	148.36	148.77
1995 to 2019	MS-GVAR 2019	$REMod-UNA$	30.98	178.59	178.59	138.25	166.52	178.54	164.06	175.84	178.57	0.00	0.17	10.15	178.25	178.59
1995 to 2019	MS-GVAR Mean	$REMod-REF$	117.07	181.56	181.56	177.01	180.48	181.52	180.08	181.12	181.56	0.00	0.01	21.50	181.56	181.56
1995 to 2019	MS-GVAR Mean	$REMod-SUF$	79.99	127.27	127.27	125.30	125.40	127.25	126.75	126.96	127.25	0.00	0.01	13.83	127.27	127.27
1995 to 2019	MS-GVAR Mean	$REMod-PER$	100.69	148.77	148.77	145.87	148.11	148.75	148.01	148.53	148.75	0.00	0.01	19.46	148.77	148.77
1995 to 2019	MS-GVAR Mean	$REMod-UNA$	117.11	178.59	178.59	175.61	177.63	178.57	177.95	178.37	178.57	0.00	0.02	20.82	178.57	178.59
1995 to 2019	log. Reg.	$REMod-REF$	66.14	311.14	591.78	67.85	70.50	69.82	71.26	75.66	78.33	30.55	45.53	59.84	91.28	206.70
1995 to 2019	log. Reg.	$REMod-SUF$	45.87	214.04	405.64	47.10	49.06	48.84	49.53	52.83	55.81	21.58	31.84	41.59	63.07	142.11
1995 to 2019	log. Reg.	$REMod-PER$	54.99	241.32	459.03	56.64	59.44	58.34	59.90	64.71	66.06	27.56	39.49	50.48	74.15	159.99
1995 to 2019	log. Reg.	$REMod-UNA$	75.93	327.16	604.64	76.74	82.35	78.46	78.34	89.95	84.35	38.82	54.64	69.47	102.03	220.66

This table displays the expected loss due to scarcity per path ($REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$) and scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) of the input variables. Hereby, the values are derived from the (MS-)GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean) as well as from the logistic regression model (Log Reg). The displayed expected loss due to scarcity values represent the results of the robustness analysis, in which the exposure at scarcity is derived by a scaling factor using data from 1995 to 2019 or from 2015 to 2019.

5.3.4.5 Robustness Analysis for the Industrial Metal Markets

Due to data limitations, we are not able to apply the MS-GVAR model to all key resources for the German Energiewende, according to Bastian et al. (2019). Therefore, the expected loss due to scarcity values on path level differ in the underlying commodities. To guarantee comparability between the models, we restrict the commodities to the industrial metals, for which the data is available in each case.

The resulting probabilities of scarcity for the GVAR model, taking into account the spillover effects between the industrial metal markets, are presented in Table D.65. Hereby, the probabilities of scarcity values are higher compared to the original GVAR model, but smaller compared to the MS-GVAR model, indicating the time-invariant model underestimates the scarcity risk. Since the logistic regression model do not reflect the spillover effects between commodity markets, the adapted commodity set do not affect the probability of scarcity. Aggregating the probability of scarcity with the substitutability and the scaled resource amounts to the risk measure expected loss due to scarcity (ES), we observe the highest risks induced by the time-varying MS-GVAR model, whereas the time-invariant GVAR model underestimates the scarcity risk, especially under the mean, focus, and quantile scenarios, see Table 5.44. However, the logistic regression model underestimates the scarcity risk the most, as the interdependencies between the markets are neglected. Despite the differences in the expected loss due to scarcity values, the ordering between the paths equals across all models. Hereby, the $REMod - REF$ path bears the highest risks, whereas a reduction in the resource requirements due to an optimal energy system with full acceptance of the society for the German Energiewende leads to the smallest scarcity risks for the $REMod - SUF$ path.

Table 5.44: Path-specific expected loss due to scarcity of the robustness analysis for the industrial metal markets

			Mean	Shock	Extr.	Foc. EA	Foc. FX	Foc. FFR	Foc. Extr. EA	Foc. Extr. FX	Foc. Extr. FFR	Q. 25%	Q. 40%	Q. 50%	Q. 60%	Q. 75%
Industrial Metals	GVAR 2019	$REMod - REF$	4.41	140.70	140.70	122.51	58.53	13.80	140.23	126.64	42.10	0.00	0.09	2.06	23.14	140.49
		$REMod - SUF$	5.62	98.91	98.91	89.58	52.54	16.81	98.57	89.98	39.06	0.00	0.07	2.77	28.65	98.90
		$REMod - PER$	3.79	115.64	115.64	100.78	48.40	11.20	115.30	104.06	35.02	0.00	0.07	1.69	19.06	115.57
		$REMod - UNA$	4.28	137.77	137.77	119.52	55.82	13.03	137.39	122.97	39.47	0.00	0.05	1.89	20.35	137.63
	GVAR Mean	$REMod - REF$	4.36	140.70	140.70	139.14	102.96	26.10	140.70	135.89	67.42	0.00	0.00	1.77	42.05	140.70
		$REMod - SUF$	6.70	98.91	98.91	98.03	77.62	28.23	98.91	95.51	55.93	0.00	0.01	2.93	44.93	98.91
		$REMod - PER$	3.50	115.64	115.64	114.52	84.90	21.37	115.64	112.00	55.29	0.00	0.00	1.56	35.19	115.64
		$REMod - UNA$	4.02	137.77	137.77	136.41	98.85	23.81	137.77	132.83	63.44	0.00	0.02	1.81	36.27	137.77
	log. Reg.	$REMod - REF$	10.09	57.62	94.40	10.09	10.15	11.25	10.09	10.22	13.96	3.64	8.45	11.96	15.30	25.40
		$REMod - SUF$	6.97	41.01	66.38	6.97	7.01	8.08	6.97	7.06	10.76	2.51	5.79	8.21	10.64	18.39
		$REMod - PER$	8.15	46.02	75.34	8.15	8.20	9.11	8.15	8.27	11.34	3.00	6.83	9.62	12.29	20.41
		$REMod - UNA$	10.03	57.89	94.70	10.03	10.07	11.23	10.03	10.12	14.08	3.58	8.39	11.90	15.26	25.51

This table displays the expected loss due to scarcity per path ($REMod - REF$, $REMod - SUF$, $REMod - PER$, and $REMod - UNA$) and scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) of the input variables. Hereby, the values are derived from the (MS-)GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod - REF$, $REMod - SUF$, $REMod - PER$, and $REMod - UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean) as well as from the logistic regression model (Log Reg). The displayed expected loss due to scarcity values represent the results of the robustness analysis, in which only the industrial metal markets are considered.

Overall, the expected loss due to scarcity values differ between the robustness analyses, investigating different assumptions for the threshold, the substitutability as well as the scaling factor. However, the ordering between the paths remains the same, indicating the risk assessment framework is stable in its assumptions. Moreover, neglecting the time-dependence or the interdependencies between the commodities lead to an underestimation of the risk, as (time-varying) spillover effects are not accounted for. In addition, the robustness analyses underline the importance of the acceptance of the society for the German Energiewende, as the associated resource risks are significantly lower.

5.4 Discussion

The objective of this thesis is the analysis and comparison of the resource requirements of four transformation pathways of the German energy transition in regard to their availability, respectively their scarcity. In this context, we propose and apply a new framework to assess the scarcity risk of resource-demanding projects under the consideration of the substitutability of commodities, the future required resource amounts of the project as well as the commodity market structure, using new commodity market models, reflecting the impact of fundamentals on - as well as the spillover effects between - commodity prices.

In the following, we first discuss the methodological developments of this thesis. Thereafter, we relate our findings of the commodity market models as well as the results of the risk assessment framework to the literature.

5.4.1 Discussion of the methodological extensions

Overall, we propose a new risk assessment framework for resource-demanding projects based on newly introduced commodity market models derived from economics. In particular, Pesaran et al. (2004) initially proposed the global vector autoregression (GVAR) model as a feasible statistical approach to analyze the world economy from an individual country level, under the limitation of small sample data sets. Hereby, they link individual economies into one model via import and export data, the so-called trade weights. In contrast, we adopt the approach to commodity markets. While Basak and Pavlova (2016) are the first combining the classical fundamental theory with the empirical observation of co-movement between commodity prices in a theoretical model, the GVAR framework enables the empirical analysis. Thereby, we link the commodity markets, using information about their common supply, demand and trading activity, instead of the originally proposed trade weights. While previous studies on the co-production, see Nassar et al. (2015) or the co-consumption of metals, see Shammugam et al. (2019), only analyze their effects on prices, we incorporate this information to reflect the interdependencies between commodity markets in a holistic model. In addition, the macroeconomic variables included reflect the simultaneous effects of the global demand, the exchange rate, and the monetary policy on commodity markets, taking into account the interrelationships among commodity markets and not just prices. Moreover, the GVAR model investigates the co-movement between - as well as the impact of commodity-specific supply and demand on - commodity prices, whereas previous models in the literature either examine the common behavior in prices, see Le Pen and Sévi (2017), Nicola et al. (2016), Ohashi and Okimoto (2016) and Pindyck and Rotemberg (1990) among others, or the influence of the individual supply and demand on prices, see Chen et al. (2019), Lutzenberger et al. (2017), Stuermer (2018) and Thomas et al. (2010).

Due to the increase in the co-movement between commodity prices since the financialization and the growth in emerging countries, according to Tang and Xiong (2012) and Helbling et al. (2008), the question arises how the constitution of commodity markets, especially the impact of fundamentals on prices as well as the co-movement between prices, changes over time. While previous studies focus on whether the relation between commodity prices changed over time, see inter alia Aepli et al. (2017), Fernandez (2015a), Le Pen and Sévi (2017), Poncela et al. (2014), we aim to consider the change in the co-movement between prices, but also the change in the impact of supply and demand on prices. Therefore, we extend the GVAR framework, which includes the interrelations between commodity-specific supply, demand and price information, the effects of marcoeconomic variables on - as well as the interdependencies between - the commodity markets, to a Markov-switching global vector autoregression (MS-GVAR) model to account for time-varying relations in commodity markets and to disentangle the differences in the spillover effects at different points in time. Hereby, we extend the initially proposed MS-GVAR model of Binder and Gross (2013) for economies, by allowing inter alia for time-varying interdependencies between supply, demand and prices, as well as for time-varying correlations between prices. Moreover, we extend the model selection procedure for MS-VAR models of Li and Kwok (2021) to handle global MS-VAR models with different specifications.

While the (MS-)GVAR model disentangles (time-varying) single-market effects from (time-varying) inter-market effects, this thesis focuses on the application of these models within the proposed risk assessment framework. Hereby, we aim to analyze and compare the actual resource requirements of four transformation pathways for the German Energiewende in regard to their availability, respectively their scarcity.

In this context, we propose a new framework for the risk assessment of resource-demanding projects. While Rosenau-Tornow et al. (2009) include past and future trends in commodity markets to identify the long-term supply risks of commodities, the results of most of the previous risk assessment frameworks are only snapshots in time and do not account for future developments. Hereby, they examine the general criticality of commodities via several indicators of the dimensions supply risks, vulnerability of a system to potential supply disruptions, as well as economic, environmental and social impacts, see inter alia Bach et al. (2017), Graedel et al. (2012), Kolotzek et al. (2018), as well as the review study of Erdmann and Graedel (2011).

Since the built-up of renewable energies will increase the demand for commodities, the question arises whether the resources are available to achieve the climate goals. In this context, only few studies include the total future resource amounts for the energy transition in their risk assessment or only focus on solar PV and wind power on technology level, see Liang et al. (2022) and Watari et al. (2020). While the study of European Commission (2020) identifies critical commodities in terms of their economic importance and supply risk, Valero et al. (2018) and Viebahn et al. (2015) determine possible bottlenecks of the energy transition by comparing the future required resource amounts with the global reserves, neglecting the economic implications of the increase in demand. Moreover, linkages between the required metals are so far neglected in the criticality assessment in the context of the energy transition, according to the literature review study of Watari et al. (2020).

In this regard, we propose a new risk assessment framework, interpreting the commodity price as scarcity indicator, following Gleich et al. (2013) and Tilton et al. (2018). Hereby, we consider the actual future required resource amounts of the energy transition, the substitutability of commodities, potentially reducing the scarcity risk, as well as the commodity market structure, which accounts for the (time-varying) impact of fundamentals on prices as well as the (time-varying) relations between commodities, since changes in one commodity market also affect the (scarcity of the) other markets and the clean energy market affects the connectedness of metal markets, according to Song et al. (2022). Moreover, the proposed framework allows for an aggregation of the commodity-specific scarcity risks on path level to compare potential transformation pathways of the energy transition in regard to their scarcity risk, enabling policy recommendations.

5.4.2 Discussion of the Major Findings with regard to the Literature

The application of the proposed models provides us new insights into commodity markets and the scarcity risk, induced from the German Energiewende. First of all, our results reveal the global economy affect each metal market to a similar extent, indicating the common behavior in commodity prices is partly caused by the economy. Moreover, commodity-specific supply and demand still influence commodity prices, but we also observe strong spillover effects between the markets, underlining the importance of jointly modeling commodity markets. Hereby, the co-consumption of the metals leads to a concurrent behavior in the markets, especially in the prices. Further, the results highlight shocks cause more pronounced reactions in the commodity markets in periods of high fluctuations. In particular, stressed commodity markets or stressed economic conditions increase the scarcity risk of the resources required for the German Energiewende, due to spillover effects. However, the risk analysis reveals a full support of the society for the energy transition generally reduces the scarcity risk.

5.4.2.1 Impact of the Global Economy on Commodity Markets

The proposed commodity market models disentangle (time-varying) single-market effects from (time-varying) inter-market effects, but also account for the impact of the economy on commodities. Hereby, our analysis underlines a strong impact of the global economy on metal markets, indicating the common pattern between prices can be partly attributed to global effects. In particular, we examine the impact on the individual, commodity-specific supply, demand, and price variables, and reveal each metal market is affected to a similar extent, whereas most studies in the literature only investigate the impact of global shocks on the commodity price, see among others Baffes and Savescu (2014), Issler et al. (2014), Kagraoka (2016), Lombardi et al. (2012), and Vansteenkiste (2009) or even on a commodity index, see among others Akram (2009), Anzuini et al. (2013), Chen et al. (2010), Hammoudeh et al. (2015), Schischke and Rathgeber (2023), and Smiech et al. (2015).

In particular, the results of the time-invariant GVAR model, reflecting the long-term relations within and between commodity markets, suggest an increase in the global demand, represented by the world gross domestic product, causes rising commodity markets, in line with various studies in the literature, for example Issler et al. (2014), Robinson (2019), and Smiech et al. (2015). However, we observe an inverse response of the industrial metal markets to a global demand shock in the time-varying MS-GVAR analysis. Hereby, the counterintuitive results can be partly explained by the different frequency, time period and economic activity proxy considered, as we include the world industrial production in the monthly analysis which only represents the output of the industrial sector of the economy. Moreover, indirect effects from the monetary policy as well as the negative impact of the lagged economic activity may cause the observed inverse reactions.

The different sample period may also explain the mixed evidence in the impact of the exchange rate on commodity markets, indicating the relation between the economy and the commodity markets changed over time, as the time-varying analysis only reflects the more recent times, with more financialized and more connected commodity markets, see inter alia Silvennoinen and Thorp (2013). In general, an appreciation of the U.S. dollar implies the metals become more expensive for consumers holding other currencies, as the metals are quoted in U.S. dollars, and therefore, the demand of foreign consumers decrease, whereas the demand of consumers holding the U.S. dollar increase. While the growth in the domestic demand for aluminum, copper, and tin cause rising prices in the time-varying analysis, the decline in the foreign demand predominates the increase in the demand of consumers holding the U.S. dollar in the time-invariant analysis as well as for nickel, lead, and zinc in the time-varying analysis, leading to decreasing commodity prices, in line with the findings of Akram (2009), Ahumada and Cornejo (2014), and Gilbert (1989) among others.

Moreover, the reaction of the metal markets to a contrarian monetary policy shock are heterogeneous, confirming the mixed evidence in the literature and indicating the impact of interest rates on commodities varies over time. Hereby, the results of the time-varying analysis, based on monthly data in the period from 1995 to 2020, underline the theory of Frankel (2008), who argues the cost of capital for holding a commodity as well as carrying costs should decrease and the demand for commodities as an alternative asset class as well as the demand for inventories should increase in response to an expansionary monetary policy, supporting the empirical results of Akram (2009), Anzuini et al. (2013) and Smiech et al. (2015). In contrast, the time-invariant analysis, based on annual data from 1970 to 2019, detects rising interest rates lead to increasing demand and price of the commodities, in line with Hammoudeh et al. (2015) as well as Schischke and Rathgeber (2023), probably indicating the central banks respond to high commodity prices via the interest rate, and thus prices run ahead of the interest rate.

Overall, the results imply the economy highly affect commodity markets, in accordance with Akram (2009), Byrne et al. (2013), Chen et al. (2014), Kagraoka (2016), Lombardi et al. (2012) among others. Hereby, all commodity markets react to a similar extent, indicating the economy partly causes the common behavior in commodity prices.

5.4.2.2 Evidence on the Structure of Commodity Markets

In addition to the impact of the economy, the commodity market models also reflect the impact of supply and demand on prices as well as the co-movement between them. Hereby, the literature addressing the influence of commodity-specific supply and demand factors on commodity prices is limited probably due to data constraints. In this regard, most of the previous studies approximate the demand of commodities by economic growth indicators, see for example Ahumada and Cornejo (2014), Borensztein and Reinhart (1994), Deaton and Laroque (2003), Helbling et al. (2008), Kilian (2009), and Stuermer (2018). Moreover, some studies use proxies for the supply as well as demand to determine the impact on commodity prices. In particular, Baffes and Dennis (2013) use the combined stock-to-use-ratio for food commodities, which represents the supply side, by indirectly reflecting the weather conditions, as well as the demand side, by indirectly reflecting the increased demand for biofuels and the income effects, whereas Nick and Thoenes (2013) consider unexpected supply shortfalls as well as temperature induced demand spikes in their analysis of the German gas market. In contrast, Ahumada and Cornejo (2014) and Stuermer (2018) examine the impact of the actual world production of the commodities. Hereby, Stuermer (2018) only detects a significant impact for tin and copper, while Ahumada and Cornejo (2014) reveal long-run price effects for a broad spectrum of eight different storable commodities, including metals.

Some of the few studies with commodity-specific supply and demand variables are Chen et al. (2019), Lutzenberger et al. (2017), and Thomas et al. (2010). In particular, Thomas et al. (2010) and Chen et al. (2019) investigate the impact of supply, demand and speculation on the price of oil and copper, respectively, and underline the importance of both fundamentals, while Lutzenberger et al. (2017) reveal metal-specific determinants, reflected by the global mine production, secondary production, reserves, stocks, as well as apparent consumption, still play a relevant role on metal prices. Hereby, these studies only focus on the specific commodity markets and are not able to account for cross-commodity linkages.

In contrast, we reflect the commodity-specific supply and demand by the world production and consumption and examine their impact on the commodity prices (in the cross-commodity dimension). In particular, our time-invariant as well as time-varying analyses reveal commodity-specific supply and demand still have a significant impact on commodity prices in the individual markets, confirming the results of Chen et al. (2019), Lutzenberger et al. (2017), and Thomas et al. (2010). In addition, the results imply the fundamentals also affect the prices in the cross-commodity dimension, suggesting supply and demand contribute to the common pattern in the markets. Moreover, various spillover effects from prices to the supply and demand variables indicate a price shock leads to an increase in the supply and a reduction in the demand, implying the production as well as the consumption respond to changes in the price (in the cross-commodity dimension).

While only few studies consider the impact of the actual supply and demand on prices, the literature focusing on the co-movement between prices is more pronounced. As early as 1990, Pindyck and Rotemberg (1990) investigate the common pattern in commodity prices and detect the co-movement exceeds the effects which can be explained by common macroeconomic impacts. Thereafter, several studies attempted to characterize the determinants and the magnitude of the co-movement between prices, see inter alia Byrne et al. (2013), Chen et al. (2014), Le Pen and Sévi (2017), Nicola et al. (2016), Tang and Xiong (2012), West and Wong (2014), and Zhang et al. (2019). In particular, Tang and Xiong (2012) conclude the individual commodity prices are not only determined solely by their supply and demand, but also by the investment behavior. However, these studies only focus on the correlation between the commodity prices, but do not account for the commodity-specific impact of supply and demand.

In contrast, our (MS-)GVAR analysis models the co-movement between prices as well as the impact of the fundamentals, by connecting the commodity markets via information based on the co-production, co-consumption as well as co-trading. Moreover, the models consider the simultaneous effects from the economy on commodities, partly explaining the common behavior in prices. Hereby, we provide new insights into the relations between the markets, since we observe various spillover effects in the cross-commodity dimension, underlining the importance of jointly modeling commodities.

While supply and supply, as well as demand and demand show a concurrent behavior, indicating the fundamentals are interrelated between the metal markets, we also observe strong interdependencies between the prices themselves, confirming the co-movement between commodity prices observed in several studies, see Ciner et al. (2020), Fernandez (2015a), and Tang and Xiong (2012) among others. In particular, a brief correlation analysis highlights the (MS-)GVAR framework performs exceptionally well in replicating the dependency structure underlying in the prices. In general, our analysis also reveals the degree of connectedness between the markets. Hereby, copper has the strongest impact on the co-movement between prices, as shocks to the copper price affect the other commodities, whereas the copper price itself only responds to changes in the zinc price, underlining the findings of Ciner et al. (2020). Moreover, the aluminum and copper markets, the largest metal markets in terms of the production, consumption, and trading volume, are most connected with the other markets as well as with each other, most likely due to their common applications in electrical conduction, automotive and aerospace industries. Hereby, the substantial dependency between aluminum and copper, due to their co-consumption, is in line with the studies of Ciner et al. (2020) and Shammugam et al. (2019), whereby we can not confirm the strong correlation between copper and tin found by Shammugam et al. (2019), whereas we also detect copper and zinc are related.

In general, our analysis reveals the common applications of the metals are an important determinant of the observed relationships between the commodities, confirming the findings of Shammugam et al. (2019), since the common consumption leads to a concurrent behavior in the demand, and ultimately in the markets. Hereby, the strong interdependencies between the commodity-specific demand variables as well as the impact of the demand on the supply and the price (in the cross-commodity dimension) are remarkable. In particular, the spillover effects are caused by the actual co-consumption relation, as the impact of the global demand on commodity markets is represented by the exogenous variable world Gross Domestic Product (GDP). In contrast, the commodity-specific supply is less important, underlining the findings of Kilian (2009), Nick and Thoenes (2013) for the energy markets, as well as of Shammugam et al. (2019) for metal markets. Moreover, our results indicate information about the co-consumption models best the interdependencies between the commodity markets, whereas information about the co-production or co-trading does not fully reflect the relations, further highlighting the relevance of the demand-side.

Overall, our results reveal the individual commodity-specific supply and demand still affect commodity prices, also in the cross-commodity dimension, indicating the fundamentals contribute to the common patterns in the markets, especially to the co-movement between the prices. Moreover, the impact of the demand is more pronounced, suggesting the co-consumption of the metals is an important determinant of the co-movement.

5.4.2.3 Time-varying Structure in Commodity Markets

While the GVAR framework disentangles the impact of supply, demand and the economy on - as well as the co-movement between - prices, one major limitation is its time-invariance. However, the studies of Ciner et al. (2020), Fernandez (2015a), Le Pen and Sévi (2017), and Ohashi and Okimoto (2016) detect the (excess) co-movement between commodities varies over time. In addition, Aepli et al. (2017), Irwin and Sanders (2012) and Zhang and Broadstock (2020) even observe a change in the connectedness of commodity markets. In particular, Peersman et al. (2021), Poncela et al. (2014) and Tang and Xiong (2012) attribute the closer integration in commodity markets to the financialization starting in 2004, whereas Song et al. (2022) state the clean energy market is a determinant of the dynamic connectedness between metal markets. Moreover, Ciner et al. (2020) and Zaremba et al. (2021) detect periods of increasing as well as decreasing co-movement between prices, indicating the degree of connectedness varies over time. Overall, the evidence in the literature implies the importance of a time-varying commodity market model.

In this regard, the application of the time-varying MS-GVAR framework supports the findings in the literature, as the analysis reveals the commodity markets react stronger to any changes in the market in periods of high fluctuations, in accordance with the findings of Ciner et al. (2020). In addition, the regime inferences indicate the individual commodity markets change between calm and volatile states. In particular, the metal markets are attributed to the volatile regime in the years 2004, 2006/07, 2009, 2011, and 2015, corresponding to the financialization of commodity markets, the boom in commodity prices, the financial crisis, the European debt crisis, and the sharp drop in the oil price, combined with the slowdown of Chinese demand, suggesting the structure of the commodity markets changed during time, in line with Aepli et al. (2017), Irwin and Sanders (2012) and Zhang and Broadstock (2020). Hereby, the increased demand for copper-tin alloys and copper-zinc-tin-sulfides in more recent times, caused the observed stronger interdependencies between tin and the other metal markets. Further, the spillover effects in the commodity markets are more pronounced in the volatile regime, implying higher risks of increasing prices. Moreover, the out-of-sample forecasting analysis shows the time-varying MS-GVAR model significantly outperforms the time-invariant analysis in predicting prices and reproduces the fluctuations as well as the relations between the metal prices exceptionally well.

Overall, the (time-varying) commodity market models reveal various spillover effects between the markets, underlining the importance of jointly modeling commodities. The interdependencies between the prices hereby confirm the co-movement. Moreover, as the models include the impact of the economy on the commodity markets, the observed strong impacts of commodity-specific demand shocks on the markets underline the co-consumption between the commodities highly affects the markets and their common patterns. In addition, the results suggest the structure of the markets changed over time, as the spillover effects are more pronounced in volatile periods.

5.4.2.4 Scarcity Risk of the German Energiewende

The objective of this thesis is to analyze and compare the scarcity risk of the resource requirements of the German Energiewende. Since the results of the commodity markets underline the inclusion of a global demand proxy does not fully reflect the demand-sided effects on prices, commodity-specific demand data should be included in the analysis of scarcity risk of the energy transition.

However, various studies investigate the supply risks, the vulnerability of a system to a potential supply disruption as well as environmental, economic and social impacts of materials to determine the general criticality of commodities, see Arendt et al. (2020), Graedel et al. (2012), and Kolotzek et al. (2018) among others. Hereby, the review study of Erdmann and Graedel (2011) detects the results in the literature vary, due to different definitions and methodologies applied, and only the platinum group metals and the rare earth elements are found to be critical in various studies. These studies only analyze the criticality of materials in general, and do not focus on their risk induced by the energy transition, but the "access to resources is a strategic security question for Europe's ambition to deliver the Green Deal", see European Commission (2020).

Regarding the literature with focus on the material requirements of different renewable energy technologies, the review study of Liang et al. (2022) reveals most of the previous studies focus on a global level on photovoltaic systems and wind power, neglecting further important technological requirements. Hereby, the studies of Valero et al. (2018) and Arrobas et al. (2017) examine the future demand of several renewable technologies on a global scale under a business as usual scenario or under different climate goals, respectively. Moreover, Marscheider-Weidemann et al. (2021) investigate the future resource amounts of various different technologies of global socio-economic scenarios and emphasize the intensified needs of energy technologies in the sustainability scenario. In addition, the article of Viebahn et al. (2015) analyzes relevant green technologies for the German Energiewende with respect to the geological availability and supply of mineral resources, but their considered material requirements are based on a meta-analysis. Moreover, Roelich et al. (2014) examine the criticality risk of two potential scenarios of the energy transition of the United Kingdom from 2012 to 2050, but they focus their case study only on the requirements of neodymium for wind turbines. However, linkages between metals are so far neglected in the criticality assessment in the context of the energy transition, according to the literature review of Watari et al. (2020).

In contrast, this thesis compares the joint resource scarcity risk of the actual future material requirements of 28 representative technologies of renewable energy technologies, storage capacities, electricity transport as well as building renovation, for four potential transformation pathways of the German energy system. Hereby, the transformation paths are all generated under the restriction of 95% CO_2 reduction in 2050, compared to Germany's emissions in 1990, see Sterchele et al. (2020), but differentiate by the underlying assumptions of the German society's acceptance for actions to fulfill these reduction goals.

In this context, our results of the commodity-specific probability of scarcity indicate similar risks across the pathways, whereby the $REMod-SUF$ ($REMod-PER$) path exhibits slightly smaller (higher) risk values. In general, lithium, lead and zinc bear the highest probability of scarcity under normal circumstances, however all commodities exhibit high scarcity risks in more stressed periods. Moreover, shocks to the global economy lead to higher scarcity risks due to simultaneous spillover effects from the economy to the commodity markets. Furthermore, the results underline the importance of jointly modeling commodity markets, as the alternative individual logistic regression model, enabling for commodity-specific price influencing factors, underestimates the scarcity risks.

The final risk measure, the expected loss due to scarcity, derived by aggregating the probability of scarcity with the substitutability as well as the required resource demand, highlights cobalt, indium, and nickel, followed by copper and lithium, which are mainly allocated to energy storage, solar PV technologies, and wind farms, due to their high required quantities combined with higher probabilities of scarcity. Hereby, these results confirm the studies of Arrobas et al. (2017), Valero et al. (2018) and European Commission (2020), which focus on the criticality in the context of the energy transition on a global and European level. However, lead and platinum show a negligible risk in our analysis, in contrast to the findings of Arrobas et al. (2017) and European Commission (2020). These differences might be explained by the choice of representative technologies, as the actual demand is also a question of "which wind, solar technologies, and zero/low emission vehicles" will be deployed, see Arrobas et al. (2017). Moreover, the scope of the study may lead to different results, since this thesis only focuses on the German energy transition, whereas the studies of Arrobas et al. (2017) and European Commission (2020) consider the resources on a global and European level, respectively.

Overall, this thesis reveals the $REMod-SUF$ path, representing a substantial change in the behavior of the German population towards a reduction in the energy consumption, mostly exhibits the lowest commodity-specific expected loss due to scarcity values. Hereby, several robustness analyses reveal this result remains valid, despite the different frameworks applied, despite the different time periods used for the definition of the model setup, and despite the different metals considered, as a higher acceptance of the society reduces the commodity requirements, and therefore the scarcity risks, substantially. In contrast, delays or a resistance of the population for new technologies cause higher risks. Hereby, the probabilities of scarcity increase, but more important the higher resource demands lead to the higher risks, indicating the economic scarcity risk of commodities highly depends on the required amounts. Especially, the $REMod-UNA$ path, representing a strong resistance against infrastructure projects, bear the highest risks, inter alia due to the high demand in cobalt, caused by the large amount of battery storage required in this path. Overall, a reduced energy demand combined with a resource-optimal energy system fully supported by the German population lead to a reduced scarcity risk.

6 Conclusion

> *Humans and wild animals face new challenges for survival because of climate change. More frequent and intense drought, storms, heat waves, rising sea levels, melting glaciers and warming oceans can directly harm animals, destroy the places they live, and wreak havoc on people's livelihoods and communities.*
>
> World Wildlife Fund (2023)

To keep the climate change under control, 196 parties signed the legally binding, international treaty on climate change, the Paris Agreement (2015), in which they committed themselves to limit global warming to below 2, preferably to 1.5 degrees Celsius, compared to pre-industrial levels. Therefore, the decarbonization of the energy sector is an important part on the way to CO_2 neutrality. As a consequence, renewable energy technologies like wind power and photovoltaic systems, associated storage technologies as well as building renovations are key elements. However, the built-up of these technologies requires large amounts of raw materials, see Valero et al. (2018). Hereby, Europe, and in particular Germany, highly depend on importing raw materials, as the delivery difficulties during the beginning of the Covid-19 pandemic as well as the Ukraine war demonstrated. Especially, an increased demand for resources can lead to (short-term) shortages, see Federal Ministry for Economic Affairs and Climate Action (2022a).

In this context, this thesis analyzed the resource scarcity risks in the context of the German Energiewende. Hereby, we compared the risk of the annual material requirements from 2020 to 2050 of four potential transformation pathways of the German energy system, which are generated in order to optimally reduce Germany's CO_2 emissions by 95% in 2050 compared to 1990, under different assumptions about the acceptance of the energy transition in the German population. While from a geological point of view, enough mineral raw materials seem to be available, see Federal Ministry for Economic Affairs and Climate Action (2022a), the increased resource requirements can lead to (short-term) shortages, and therefore to price peaks. Interpreting the commodity price as scarcity indicator, we proposed and applied a new framework to assess the resource scarcity risk of four potential transformation pathways of the German energy system to reach the climate goals. In particular, the framework accounts for the substitutability of commodities, the future required commodity amounts as well as the commodity market structure and enables an aggregation and comparison of the commodity-specific scarcity risks on path level.

Initially, a comprehensive understanding of commodity markets is essential. While the classical fundamental theory states a good's price is the result of its supply and demand equilibrium, see Hotelling (1931) and Deaton and Laroque (2003), several empirical studies detect a common behavior in commodity prices, characterized first as (excess) co-movement by Pindyck and Rotemberg (1990). To account for the impact of fundamentals on prices, due to the increased resource demand of the German Energiewende, as well as for the interdependencies between commodity markets, as the German Energiewende affects various commodities simultaneously, we proposed a new empirical commodity market framework, which overcomes the problem of limited data in large-scale models. In this context, we adopted the global vector autoregression (GVAR) model, which was initially designed by Pesaran et al. (2004) to analyze the world economy from an individual country level, to commodity markets. Hereby, each commodity market is modeled separately using VAR models with the commodity-specific, microeconomic variables supply, demand, and price, as well as exogenous, macroeconomic attributes. Subsequently, these individual models are connected via appropriate weight matrices to a global commodity market model, which allows for spillover effects between the commodities.

The exemplary application of the GVAR model on the industrial metal markets, connecting the commodities via information based on the co-production, co-consumption as well as co-trading relation, disentangled single-market effects from inter-market effects. In particular, the results indicate the framework is able to showcase the strong co-movement in commodity prices as well as the simultaneous impact of the economy on all commodity markets. Moreover, various spillover effects of commodity-specific supply and demand, both within and across commodity markets, as well as their impact on prices, underline the importance to account for fundamentals, but also to jointly model commodity markets. Hereby, the results reveal the commodity demand as well as the common applications of the metals are important determinants of the observed relationships between the commodities. Overall, the analysis provided new insights into the relation between commodity markets, in particular, the impact of fundamentals on prices (in the cross-commodity dimension).

One of the major limitation of the standard GVAR framework is its time-invariance, while the commodity markets changed over time. Especially, the entry of institutional investors in commodity futures markets during the so-called financialization led to a closer integration in commodity markets. Therefore, the question arises how the constitution of commodity markets, especially the impact of fundamentals on prices as well as the co-movement between prices, changes over time. For this reason, we extended the GVAR framework, by a Markov-switching component, to a Markov-switching global vector autoregression (MS-GVAR) model, enabling for time-varying interdependencies in commodity markets. Hereby, regime-switches in the impact of supply and demand on commodity prices as well as the co-movement between commodity prices allow to disentangle the differences in the spillover effects at different points in time.

In particular, we exemplary applied the MS-GVAR model on the industrial metal markets. Hereby, the regime inference of the individual commodity-specific models reveals the framework is able to classify the commodity markets into calm and volatile states. The distinct analysis of calm and volatile regimes underline the importance to account for supply and demand, but also to jointly model commodity markets, suggested by the time-invariant model, as we detected several spillover effects within and between the commodity markets. However, the results imply the commodity variables either react significantly to shocks or not, independent of the underlying regime. In spite of that, the responses of the commodity markets are more pronounced in periods of high fluctuations, implying higher risks of increasing prices. An out-of-sample forecasting analysis underlines the importance of a time-varying analysis, as the MS-GVAR model significantly outperforms the GVAR model in the price predictions. Moreover, the MS-GVAR model reproduces the fluctuations as well as the relations between the metal prices better than its time-invariant counterpart.

While the exemplary application of the (MS-)GVAR model on industrial metal markets revealed the models are able to disentangle (time-varying) single-market effects from (time-varying) inter-market effects, this thesis focuses on the analysis and comparison of the resource scarcity risk of four transformation pathways for the German Energiewende. Therefore, the (time-varying) global commodity market model was incorporated in the proposed scarcity risk assessment framework. In particular, as the commodity price is interpreted as scarcity indicator, the individual probability of scarcity for each commodity of the transformation pathway is generated with help of the modeled prices from the (MS-)GVAR model. Subsequently, the commodity-specific risk indicators per pathway are derived, combining the individual probability of scarcity with a substitutability score and the scaled resource requirements per transformation pathway. Finally, the aggregation of the commodity-specific risk indicators on path level allows for a comparison of the four transformation paths with regard to their resource risks and enables policy suggestions.

The application of the proposed risk assessment framework on four potential transformation pathways of the German Energiewende revealed lithium, lead and zinc bear the highest probability of scarcity. However, when additionally accounting for the substitutability as well as the required resource demand for the German Energiewende, cobalt, indium and nickel, followed by copper and lithium, mainly allocated to energy storage, solar PV technologies and wind parks, bear the highest scarcity risks, and therefore, will be the key commodities for the German Energiewende. The comparison of the four transformation paths, suggests the path which models the transition of the German energy system with full support by the society, shows the lowest scarcity risks, as an active support of the German population for the energy transition significantly reduces the required amounts of raw materials and therefore the scarcity risks.

While the proposed risk assessment framework considers the substitutability, the required resources as well as the commodity market structure, the model also has its weaknesses and limitations that must be thoughtfully considered. First of all, the proposed risk assessment is a statistical approach aiming to examine and compare the scarcity risk of four potential transformation pathways of the German energy system, however, as all statistical analyses it is based on historical data and the corresponding assumption that past data can be extrapolated to the future. However, the climate change and the associated decarbonization of the energy system will change the future energy system and especially the applied energy technologies. Second, although the future resource demands are included, the impacts on the economy can only be roughly estimated. In particular, the resource requirements used reflect the demands of 28 representative technologies for wind, solar, storage, electricity transport and building renovation, however, the actual choice of the deployed technology may change the results. Third, the actual resource demands for the transformation paths are scaled with the historical average of world production to allow a comparison between the commodities. Since not all of a metal's resource consumption is used for energy applications, the scaling used could skew the results. However, the analyses on the robustness of the scaling factor show that the main findings remain valid even if the scaling factor is changed. Fourth, the analysis investigates the commodity markets and the risks induced from the energy transition from a global perspective and does not focus on individual commodity trades. Those might be affected by delivery difficulties inter alia caused by an amplified market concentration, which are not considered within this thesis and which may lead to resource shortages in the short-term. Fifth, this thesis investigates the German energy system and the associated resource demands, however, all countries have to contribute to keep the climate change under control which is why the actual resource requirements for a worldwide energy system based on renewable technologies will be a multiple of the German demand.

Because of these limitations, the final risk scores should be interpreted carefully. However, various robustness analyses reveal the hierarchy between the alternative transformation paths remains the same, indicating the risk assessment framework helps in comparing resource-demanding projects, independent of the assumption under focus. Therefore, the main finding, a reduced energy demand combined with a resource-optimal energy system reduces the resource amounts and ultimately the resource scarcity, keeps valid in either case.

Overall, a new risk assessment framework is developed and applied for the comparison of the resource requirements of four potential transformation pathways of the German Energiewende. Hereby, the framework accounts for the substitutability of commodities, the actual future required resource amounts of the project as well as the commodity market structure, using new commodity market models, reflecting the impact of fundamentals on - as well as the spillover effects between - commodity prices. The results indicate on commodity level cobalt, indium, and nickel are potential bottlenecks of the German Energiewende. In particular, the analysis reveals the fewer resources are required, the better. On the one hand, electricity consumption must be reduced, on the other hand, it is important that infrastructure projects for the construction of wind turbines and electricity lines are made possible, especially been accepted and supported by the German population, since the development of an optimal electricity market reduces the required raw materials and thus reduces the scarcity risk. Therefore, policy should raise the awareness in the German population to save electricity and stand behind the necessary infrastructure projects such that the energy system can be set up optimally.

Bibliography

Aepli, M. D., R. Füss, T. E. S. Henriksen, and F. Paraschiv (2017). Modeling the multivariate dynamic dependence structure of commodity futures portfolios. *Journal of Commodity Markets 6*, 66–87.

Ahumada, H. and M. Cornejo (2014). Explaining commodity prices by a cointegrated time series-cross section model. *Empirical Economics 48*, 1667–1690.

Akram, Q. F. (2009). Commodity prices, interest rates and the dollar. *Energy Economics 31*(6), 838–851.

Aksoy, Y., H. S. Basso, R. P. Smith, and T. Grasl (2019). Demographic structure and macroeconomic trends. *American Economic Journal: Macroeconomics 11*(1), 193–222.

Alquist, R. and O. Coibion (2013). The comovement in commodity prices: Sources and implications. (140), 1–62.

Andersson, S. (2014). On the Specification of Local Models in a Global Vector Autoregression: A Comparison of Markov-Switching Alternatives.

Anzuini, A., M. J. Lombardi, and P. Pagano (2013). The Impact of Monetary Policy Shocks on Commodity Prices. *International Journal of Central Banking 9*(3), 125–150.

Apergis, N., I. Chatziantoniou, and A. Cooray (2020). Monetary policy and commodity markets: Unconventional versus conventional impact and the role of economic uncertainty. *International Review of Financial Analysis 71.*

Apergis, N., C. Christou, and J. E. Payne (2014). Precious metal markets, stock markets and the macroeconomic environment: a FAVAR model approach. *Applied Financial Economics 24*(10), 691–703.

Arendt, R., M. Muhl, V. Bach, and M. Finkbeiner (2020). Criticality assessment of abiotic resource use for Europe - application of the SCARCE method. *Resources Policy 67.*

Arrobas, D. L. P., K. L. Hund, M. S. Mccormick, J. Ningthoujam, and J. R. Drexhage (2017). The Growing Role of Minerals and Metals for a Low Carbon Future.

Asian Metal (2022a). Dysprosium Price 99.5%min FOB China U$/MT. https://www.asianmetal.com/Dysprosium-Price-Index/.

Asian Metal (2022b). Neodymium Price 99.5%min FOB China U$/MT. https://www.asianmetal.com/Neodymium-Price-Index/.

Bach, V., M. Berger, M. Henßler, M. Kirchner, S. Leiser, L. Mohr, E. Rother, K. Ruhland, L. Schneider, L. Tikana, W. Volkhausen, F. Walachowicz, and M. Finkbeiner (2016). Integrated method to assess resource efficiency – ESSENZ. *Journal of Cleaner Production 137*, 118–130.

Bach, V., N. Finogenova, M. Berger, L. Winter, and M. Finkbeiner (2017). Enhancing the assessment of critical resource use at the country level with the SCARCE method – Case study of Germany. *Resources Policy 53*, 283–299.

Baffes, J. and A. Dennis (2013). Long-Term Drivers of Food Prices.

Baffes, J., A. Kabundi, and P. Nagle (2020). The role of income and substitution in commodity demand. *Policy Research Working Paper Series* (9122).

Baffes, J. and C. Savescu (2014). Monetary conditions and metal prices. *Applied Economics Letters 21*(7), 447–452.

Basak, S. and A. Pavlova (2016). A Model of Financialization of Commodities. *The Journal of Finance 71*(4), 1511–1556.

Basel Committee on Banking Supervision (2005). An Explanatory Note on the Basel II IRB Risk Weight Functions. *Bank for International Settlements*, 1–19.

Bastian, D., T. Brandenburg, P. Buchholz, D. Huy, M. Liedtke, M. Schmidt, and H. Sievers (2019). DERA-Rohstoffliste 2019. *DERA-Rohstoffinformationen 40*, 1–116.

Bauer, S. (2023). Copper as an economic indicator.

Baumeister, C. and G. Peersman (2013). The role of time-varying price elasticities in accounting for volatility changes in the crude oil market. *Journal of Applied econometrics 28*(7), 1087–1109.

Betten, T., S. Shammugam, and R. Graf (2020). Adjustment of the Life Cycle Inventory in Life Cycle Assessment for the Flexible Integration into Energy Systems Analysis. *Energies 13*(17).

Bilgin, M. H., F. Gogolin, M. C. K. Lau, and S. A. Vigne (2018). Time-variation in the relationship between white precious metals and inflation: A cross-country analysis. *Journal of International Financial Markets, Institutions and Money 56*, 55–70.

Binder, M. and M. Gross (2013). Regime-switching global vector autoregressive models. Technical report, ECB Working Paper.

Board of Governors of the Federal Reserve System (U.S.) (2022a). Federal Funds Effective Rate (FEDFUNDS). https://fred.stlouisfed.org/series/FEDFUNDS.

Board of Governors of the Federal Reserve System (U.S.) (2022b). Industrial Production: Total Index (IPB50001N). https://fred.stlouisfed.org/series/IPB50001N.

Board of Governors of the Federal Reserve System (U.S.) (2022c). Industrial Production: Total Index (IPB50001N). https://fred.stlouisfed.org/series/IPB50001N.

Board of Governors of the Federal Reserve System (U.S.) (2022d). Market Yield on U.S. Treasury Securities at 10-Year Constant Maturity, Quoted on an Investment Basis (DGS10). https://fred.stlouisfed.org/series/DGS10.

Borensztein, E. and C. Reinhart (1994). The Macroeconomic Determinants of Commodity Prices. *IMF Staff Papers 41*(2), 236–261.

Brandtzæg, S. R. (2018). Hydro annual report 2018.

Buncic, D. and C. Moretto (2015). Forecasting copper prices with dynamic averaging and selection models. *The North American Journal of Economics and Finance 33*, 1–38.

Byrne, J. P., G. Fazio, and N. Fiess (2013). Primary commodity prices: Co-movements, common factors and fundamentals. *Journal of Development Economics 101*, 16–26.

Byrne, J. P., R. Sakemoto, and B. Xu (2020). Commodity price co-movement: heterogeneity and the time-varying impact of fundamentals. *European Review of Agricultural Economics 47*(2), 499–528.

Büyüksahin, B. and M. A. Robe (2014). Speculators, commodities and cross-market linkages. *Journal of International Money and Finance 42*, 38–70.

Camacho, M. and G. Perez-Quiros (2014). Commodity Prices and the Business Cycle in Latin America: Living and Dying by Commodities? *Emerging Markets Finance and Trade 50*(2), 110–137.

Carter, C. A., G. C. Rausser, and A. Smith (2011). Commodity Booms and Busts. *Annual Review of Resource Economics 3*(1), 87–118.

Cashin, P., C. McDermott, and A. Scott (2002). Booms and slumps in world commodity prices. *Journal of Development Economics 69*(1), 277–296.

Chen, J., X. Zhu, and M. Zhong (2019). Nonlinear effects of financial factors on fluctuations in nonferrous metals prices: A Markov-switching VAR analysis. *Resources Policy 61*, 489–500.

Chen, M.-H. (2010). Understanding world metals prices — Returns, volatility and diversification. *Resources Policy 35*(3), 127–140.

Chen, S.-L., J. Jackson, H. Kim, and P. Resiandini (2014). What drives commodity prices? *American Journal of Agricultural Economics 96*, 1455–1468.

Chen, Y.-C., K. S. Rogoff, and B. Rossi (2010). Can Exchange Rates Forecast Commodity Prices? *The Quarterly Journal of Economics 125*(3), 1145–1194.

Ciner, C., B. Lucey, and L. Yarovaya (2020). Spillovers, integration and causality in LME non-ferrous metal markets. *Journal of Commodity Markets 17*.

Clark, T. E. and K. D. West (2007). Approximately normal tests for equal predictive accuracy in nested models. *Journal of Econometrics 138*(1), 291–311.

Cuddington, J. T. and D. Jerrett (2008). Super cycles in real metals prices? *IMF staff Papers 55*(4), 541–565.

Cuddington, J. T. and A. M. Zellou (2013). A simple mineral market model: Can it produce super cycles in prices? *Resources Policy 38*(1), 75–87.

Deaton, A. and G. Laroque (1992). On the Behaviour of Commodity Prices. *The Review of Economic Studies 59*(1), 1–23.

Deaton, A. and G. Laroque (2003). A Model of Commodity Prices After Sir Arthur Lewis. *Journal of Development Economics 71*(2), 289–310.

Deb, P., P. K. Trivedi, and P. Varangis (1996). The excess co-movement of commodity prices reconsidered. *Journal of Applied Econometrics 11*(3), 275–291.

Dempster, A. P., N. M. Laird, and D. B. Rubin (1977). Maximum likelihood from incomplete data via the EM algorithm. *Journal of the Royal Statistical Society: Series B (Methodological) 39*(1), 1–22.

Dickey, D. A. and W. A. Fuller (1979). Distribution of the Estimators for Autoregressive Time Series With a Unit Root. *Journal of the American Statistical Association 74*(366), 427–431.

Dimitropoulos, D. and A. Yatchew (2018). Discerning trends in commodity prices. *Macroeconomic Dynamics 22*(3), 683–701.

Ding, S. and Y. Zhang (2020). Cross market predictions for commodity prices. *Economic Modelling 91*(C), 455–462.

Dutta, A. (2018). Impacts of oil volatility shocks on metal markets: A research note. *Resources Policy 55*, 9–19.

Dées, S., F. di Mauro, M. H. Pesaran, and L. V. Smith (2007). Exploring the international linkages of the euro area: a global VAR analysis. *Journal of Applied Econometrics 22*, 1–38.

Dées, S., S. Holly, M. H. Pesaran, and L. V. Smith (2007). Long Run Macroeconomic Relations in the Global Economy. *CESifo Working Paper Series 1904*.

Ehrmann, M., M. Ellison, and N. Valla (2001). Regime-dependent impulse response functions in a Markov-switching vector autoregression model. *Bank of Finland Discussion Paper* (11).

Ehrmann, M., M. Ellison, and N. Valla (2003). Regime-dependent impulse response functions in a Markov-switching vector autoregression model. *Economics Letters 78*(3), 295–299.

Eksi, O. and B. K. O. Tas (2017). Unconventional monetary policy and the stock market's reaction to federal reserve policy actions. *The North American Journal of Economics and Finance 40*, 136–147.

Erdmann, L. and T. E. Graedel (2011). Criticality of non-fuel minerals: a review of major approaches and analyses. *Environmental science & technology 45*(18), 7620–7630.

European Commission (2020). Communication from the commission to the European parliament, the council, the European economic and social committee and the committee of the regions.

European Commission and Directorate-General for Communication (2021). *Decarbonising our energy system to meet our climate goals*. Publications Office of the European Union.

Federal Ministry for Economic Affairs and Climate Action (2011). Der Weg zur Energie der Zukunft - sicher, bezahlbar und umweltfreundlich -.

Federal Ministry for Economic Affairs and Climate Action (2022a). Keine Zukunftstechnologie ohne mineralische Rohstoffe. *Federal Ministry for Economic Affairs and Climate Action*.

Federal Ministry for Economic Affairs and Climate Action (2022b). Unsere Energiewende: sicher, sauber, bezahlbar. *Federal Ministry for Economic Affairs and Climate Action*.

Federal Reserve Bank of St. Louis (2022a). Monetary Base; Total (BOGMBASE). https://fred.stlouisfed.org/series/BOGMBASE.

Federal Reserve Bank of St. Louis (2022b). Spot Crude Oil Price: West Texas Intermediate (WTI) (WTISPLC). https://fred.stlouisfed.org/series/WTISPLC.

Fernandez, V. (2015a). Commodity price excess co-movement from a historical perspective: 1900–2010. *Energy Economics 49*, 698–710.

Fernandez, V. (2015b). Influence in commodity markets: Measuring co-movement globally. *Resources Policy 45*, 151–164.

Foster, D. P., R. A. Stine, and R. P. Waterman (1997). New York, NY: Springer New York.

Frankel, J. A. (1986). Expectations and Commodity Price Dynamics: The Overshooting Model. *American Journal of Agricultural Economics 68*(2), 344–348.

Frankel, J. A. (2008). The Effect of Monetary Policy on Real Commodity Prices. In *Asset Prices and Monetary Policy*, pp. 291–333. National Bureau of Economic Research, Inc.

Frankel, J. A. (2014). Effects of speculation and interest rates in a "carry trade" model of commodity prices. *Journal of International Money and Finance 42*(C), 88–112.

Frankel, J. A. and A. K. Rose (2010). Determinants of Agricultural and Mineral Commodity Prices. *HKS Faculty Research Working Paper Series RWP10*(38), 1–48.

Galesi, A. and M. J. Lombardi (2009). External shocks and international inflation linkages: a global VAR analysis. *ECB Working Paper Series* (1062).

Gargano, A. and A. Timmermann (2014). Forecasting commodity price indexes using macroeconomic and financial predictors. *International Journal of Forecasting 30*(3), 825–843.

Gilbert, C. L. (1989). The Impact of Exchange Rates and Developing Country Debt on Commodity Prices. *The Economic Journal 99*(397), 773–784.

Gleich, B., B. Achzet, H. Mayer, and A. Rathgeber (2013). An empirical approach to determine specific weights of driving factors for the price of commodities — A contribution to the measurement of the economic scarcity of minerals and metals. *Resources Policy 38*(3), 350–362.

Gonçalves, S., A. M. Herrera, L. Kilian, and E. Pesavento (2021). Impulse response analysis for structural dynamic models with nonlinear regressors. *Journal of Econometrics 225*(1), 107–130.

Gorton, G. and K. G. Rouwenhorst (2006). Facts and Fantasies about Commodity Futures. *Financial Analysts Journal 62*(2), 47–68.

Graedel, T. E., R. Barr, C. Chandler, T. Chase, J. Choi, L. Christoffersen, E. Friedlander, C. Henly, C. Jun, N. T. Nassar, et al. (2012). Methodology of metal criticality determination. *Environmental science & technology 46*(2), 1063–1070.

Graedel, T. E., E. M. Harper, N. T. Nassar, and B. K. Reck (2015). On the materials basis of modern society. *Proceedings of the National Academy of Sciences 112*(20), 6295–6300.

Graedel, T. E. and B. K. Reck (2016). Six years of criticality assessments: What have we learned so far? *Journal of Industrial Ecology 20*(4), 692–699.

Greer, R. J. (2000). The nature of commodity index returns. *The Journal of Alternative Investments 3*(1), 45–52.

Gross, M. (2013). Estimating GVAR weight matrices. *ECB Working Paper Series* (1523).

Gubler, M. and M. S. Hertweck (2013). Commodity price shocks and the business cycle: Structural evidence for the U.S. *Journal of International Money and Finance 37*, 324–352.

Guha-Sapir, D. (2021). EMDAT: OFDA/CRED International Disaster Database. www.emdat.be.

Guzmán, J. I. and E. Silva (2018, October). Copper price determination: fundamentals versus non-fundamentals. *Mineral Economics 31*(3), 283–300.

Gygli, S., F. Haelg, N. Potrafke, and J. Sturm (2021). The KOF globalisation index-revisited. https://kof.ethz.ch/prognosen-indikatoren/indikatoren/kof-globalisierungsindex.html.

Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica: Journal of the econometric society*, 357–384.

Hamilton, J. D. (1990). Analysis of time series subject to changes in regime. *Journal of Econometrics 45*(1), 39–70.

Hamilton, J. D. and J. C. Wu (2015). Effects of Index-Fund Investing on Commodity Futures Prices. *International Economic Review 56*(1), 187–205.

Hammoudeh, S., S. Dibooglu, and E. Aleisa (2004). Relationships among U.S. oil prices and oil industry equity indices. *International Review of Economics & Finance 13*(4), 427–453.

Hammoudeh, S., D. K. Nguyen, and R. M. Sousa (2015). US monetary policy and sectoral commodity prices. *Journal of International Money and Finance 57*, 61–85.

Hannan, E. J. and B. G. Quinn (1979). The Determination of the Order of an Autoregression. *Journal of the Royal Statistical Society. Series B (Methodological) 41*(2), 190–195.

Hayes, S. M. and E. A. McCullough (2018). Critical minerals: A review of elemental trends in comprehensive criticality studies. *Resources Policy 59*, 192–199.

Helbling, T., V. Mercer-Blackman, and K. Cheng (2008). Riding a Wave - Soaring commodity prices may have a lasting impact. *Finance and Development 45*(1), 10–15.

Higham, N. J. (2002). Computing the nearest correlation matrix — a problem from finance. *IMA journal of Numerical Analysis 22*(3), 329–343.

Hotelling, H. (1931). The Economics of Exhaustible Resources. *Journal of Political Economy 39*, 137–175.

ICE Futures U.S. (2022a). US Dollar INDEX DXY - PRICE INDEX (.DXY). data retrieved from Investing.com, https://de.investing.com/indices/usdollar-historical-data.

ICE Futures U.S. (2022b). US Dollar INDEX DXY - PRICE INDEX (.DXY). data retrieved from stooq.com, https://stooq.com/q/d/?s=dx.f.

Ioannidou, D., N. Heeren, G. Sonnemann, and G. Habert (2019). The future in and of criticality assessments. *Journal of Industrial Ecology 23*(4), 751–766.

Irwin, S. H. and D. R. Sanders (2012). Financialization and structural change in commodity futures markets. *Journal of agricultural and applied economics 44*(3), 371–396.

Issler, J., C. Rodrigues, and R. Burjack (2014). Using common features to understand the behavior of metal-commodity prices and forecast them at different horizons. *Journal of International Money and Finance 42*, 310–335.

Jarque, C. M. and A. K. Bera (1980). Efficient tests for normality, homoscedasticity and serial independence of regression residuals. *Economics Letters 6*(3), 255–259.

Jordan, B. W. (2017). Companions and competitors: Joint metal-supply relationships in gold, silver, copper, lead and zinc mines. *Resource and Energy Economics 49*, 233–250.

Kagraoka, Y. (2016). Common dynamic factors in driving commodity prices: Implications of a generalized dynamic factor model. *Economic Modelling 52*, 609–617.

Karamé, F. (2012). An algorithm for generalized impulse-response functions in Markov-switching structural VAR. *Economics Letters 117*(1), 230–234.

Keating, J. W., L. J. Kelly, L. A. Smith, and V. J. Valcarcel (2019). A Model of Monetary Policy Shocks for Financial Crises and Normal Conditions. *Journal of Money, Credit and Banking 51*(1), 227–259.

Kilian, L. (2009). Not All Oil Price Shocks Are Alike: Disentangling Demand and Supply Shocks in the Crude Oil Market. *American Economic Review 99*(3), 1053–1069.

Kilian, L. and X. Zhou (2018). Modeling fluctuations in the global demand for commodities. *Journal of International Money and Finance 88*(C), 54–78.

Kim, C.-J. (1994). Dynamic linear models with Markov-switching. *Journal of Econometrics 60*(1), 1–22.

Klotz, P., T. C. Lin, and S.-H. Hsu (2014). Global commodity prices, economic activity and monetary policy: The relevance of China. *Resources Policy 42*, 1–9.

Kolotzek, C., C. Helbig, A. Thorenz, A. Reller, and A. Tuma (2018). A company-oriented model for the assessment of raw material supply risks, environmental impact and social implications. *Journal of Cleaner Production 176*, 566–580.

Koop, G., M. H. Pesaran, and S. M. Potter (1996). Impulse response analysis in nonlinear multivariate models. *Journal of Econometrics 74*(1), 119–147.

Kriegler, E., B. C. O'Neill, S. Hallegatte, T. Kram, R. J. Lempert, R. H. Moss, and T. Wilbanks (2012). The need for and use of socio-economic scenarios for climate change analysis: A new approach based on shared socio-economic pathways. *Global Environmental Change 22*(4), 807–822.

Krolzig, H.-M. (1997). *Markov-Switching Vector Autoregressions: Modelling, Statistical Inference, and Application to Business Cycle Analysis.* Berlin, Heidelberg: Springer Berlin Heidelberg.

Krolzig, H.-M. (2002). Regime-Switching Models. *Department of Economics and Nuffield College, University of Oxford*.

Le Pen, Y. and B. Sévi (2017, 09). Futures Trading and the Excess Co-movement of Commodity Prices*. *Review of Finance 22*(1), 381–418.

Leder, R. (2020). 19.20 Der Geschäftsbericht der Nichteisen-Metallindustrie.

Lee, J., M. Bazilian, B. Sovacool, K. Hund, S. Jowitt, T. Nguyen, A. Månberger, M. Kah, S. Greene, C. Galeazzi, K. Awuah-Offei, M. Moats, J. Tilton, and S. Kukoda (2020). Reviewing the material and metal security of low-carbon energy transitions. *Renewable and Sustainable Energy Reviews 124*.

Lescaroux, F. (2009). On the excess co-movement of commodity prices—A note about the role of fundamental factors in short-run dynamics. *Energy Policy 37*(10), 3906–3913.

Lewis, W. A. (1954). Economic development with unlimited supplies of labour. *The Manchester School 22*(2), 139–191.

Li, N. and S. S. Kwok (2021). Jointly determining the state dimension and lag order for Markov-switching vector autoregressive models. *Journal of Time Series Analysis 42*(4), 471–491.

Liang, Y., R. Kleijn, A. Tukker, and E. van der Voet (2022). Material requirements for low-carbon energy technologies: A quantitative review. *Renewable and Sustainable Energy Reviews 161*.

Liberda, M. (2017). Mixed-frequency Drivers of Precious Metal Prices. *Acta Universitatis Agriculturae et Silviculturae Mendelianae Brunensis 65*(6), 2007–2015.

Lombardi, M. J., C. Osbat, and B. Schnatz (2012). Global commodity cycles and linkages: a FAVAR approach. *Empirical Economics 43*(2), 651–670.

Lutzenberger, F., B. Gleich, H. G. Mayer, C. Stepanek, and A. W. Rathgeber (2017). Metals: resources or financial assets? A multivariate cross-sectional analysis. *Empirical Economics 53*(3), 927–958.

Lübbers, J. and P. N. Posch (2016). Commodities' common factor: An empirical assessment of the markets' drivers. *Journal of Commodity Markets 4*(1), 28–40.

Lütkepohl, H. (1990). Asymptotic Distributions of Impulse Response Functions and Forecast Error Variance Decompositions of Vector Autoregressive Models. *The Review of Economics and Statistics 72*(1).

Marscheider-Weidemann, F., S. Langkau, E. Eberling, L. Erdmann, M. Haendel, M. Krail, A. Loibl, C. Neef, M. Neuwirth, L. Rostek, et al. (2021). Rohstoffe für Zukunftstechnologien 2021.

MSCI (2022). MSCI World. data retrieved from Wikipedia.com, `https://de.wikipedia.org/wiki/MSCI_World#:~:text=Der%20MSCI%20World%20startete%20am,er%20auf%20134%2C81%20Punkte.`

Nassar, N. T., T. E., Graedel, and E. M. Harper (2015). By-product metals are technologically essential but have problematic supply. *Science Advances 1*(3).

Newbery, D. M. and J. E. Stiglitz (1981). The theory of commodity price stabilization; a study in the economics of risk.

Nick, S. and S. Thoenes (2013). What Drives Natural Gas Prices? – A Structural VAR Approach. *EWI Working Paper 13*(02).

Nicola, F., P. D. Pace, and M. Hernandez (2016). Co-Movement of Major Energy, Agricultural, and Food Commodity Price Returns: A Time-Series Assessment. *Energy Economics 57*.

Ohashi, K. and T. Okimoto (2016). Increasing trends in the excess comovement of commodity prices. *Journal of Commodity Markets 1*(1), 48–64.

Organization for Economic Co-operation and Development (OECD) (2022a). USA Employment Rate. `https://data.oecd.org/emp/employment-rate.htm`.

Organization for Economic Co-operation and Development (OECD) (2022b). USA Short-term interest rates (indicator). `https://data.oecd.org/interest/short-term-interest-rates.htm`.

Organization for Economic Co-operation and Development (OECD) (2022c). USA Short-term interest rates (indicator). `https://data.oecd.org/interest/short-term-interest-rates.htm`.

Paris Agreement (2015). Paris agreement. In *Report of the Conference of the Parties to the United Nations Framework Convention on Climate Change (21st Session, 2015: Paris). Retrived December*, Volume 4, pp. 2017. HeinOnline.

Peersman, G., S. K. Rüth, and W. Van der Veken (2021). The interplay between oil and food commodity prices: Has it changed over time? *Journal of International Economics 133*.

Pesaran, M., T. Schuermann, and S. Weiner (2004). Modeling Regional Interdependencies Using a Global Error-Correcting Macroeconometric Model. *Journal of Business & Economic Statistics 22*(2), 129–162.

Pesaran, M. and Y. Shin (1998). Generalized impulse response analysis in linear multivariate models. *Economics Letters 58*(1), 17–29.

Pierdzioch, C., M. Risse, and S. Rohloff (2016). Fluctuations of the real exchange rate, real interest rates, and the dynamics of the price of gold in a small open economy. *Empirical Economics 51*(4), 1481–1499.

Pincheira-Brown, P. and N. Hardy (2019). Forecasting base metal prices with the Chilean exchange rate. *Resources Policy 62*, 256–281.

Pindyck, R. and J. Rotemberg (1990). The Excess Co-movement of Commodity Prices. *Economic Journal 100*(403), 1173–1189.

Poncela, P., E. Senra, and L. P. Sierra (2014). Common dynamics of nonenergy commodity prices and their relation to uncertainty. *Applied Economics 46*(30), 3724–3735.

Pradhananga, M. (2016). Financialization and the rise in co-movement of commodity prices. *International Review of Applied Economics 30*(5), 547–566.

Redlinger, M. and R. Eggert (2016). Volatility of by-product metal and mineral prices. *Resources Policy 47*, 69–77.

Rhoades, S. A. (1993). The Herfindahl-Hirschman Index. *Fed. Res. Bull. 79*.

Robinson, Z. (2019). Revisiting gold price behaviour: a structural VAR. *Mineral Economics 32*, 365–372.

Roelich, K., D. A. Dawson, P. Purnell, C. Knoeri, R. Revell, J. Busch, and J. K. Steinberger (2014). Assessing the dynamic material criticality of infrastructure transitions: A case of low carbon electricity. *Applied Energy 123*, 378–386.

Rohstoffagentur, D.-D. (2016). Rohstoffe für Zukunftstechnologien 2016. *DERA Rohstoffinformationen 28*.

Rosenau-Tornow, D., P. Buchholz, A. Riemann, and M. Wagner (2009, 12). Assessing the long-term supply risks for mineral raw materials - a combined evaluation of past and future trends. *Resources Policy 34*(4), 161–175.

Rossen, A. (2015). What are metal prices like? Co-movement, price cycles and long-run trends. *Resources Policy 45*, 255–276.

Runkle, D. E. (1987). Vector autoregressions and reality. *Journal of Business & Economic Statistics 5*(4), 437–442.

Sari, R., S. Hammoudeh, and U. Soytas (2010). Dynamics of oil price, precious metal prices, and exchange rate. *Energy Economics 32*(2), 351–362.

Schischke, A., P. Papenfuß, and A. Rathgeber (2023). Three Co's to Jointly Model Commodity Markets: Co-Production, Co-Consumption and Co-Trading. *Empirical economics*.

Schischke, A. and A. Rathgeber (2023). Commodities and monetary policy - the role of interest rates revisited. *Available at SSRN: http://ssrn.com/abstract=4365481*.

Schrijvers, D., A. Hool, G. A. Blengini, W.-Q. Chen, J. Dewulf, R. Eggert, L. van Ellen, R. Gauss, J. Goddin, K. Habib, et al. (2020). A review of methods and data to determine raw material criticality. *Resources, Conservation and Recycling 155*.

Schwarz, G. (1978). Estimating the dimension of a model. *The annals of statistics*, 461–464.

Shammugam, S., A. W. Rathgeber, and T. Schlegl (2019). Causality between metal prices: is joint consumption a more important determinant than joint production of main and by-product metals? *Resources Policy 61*, 49–66.

Shapiro, S. S. and M. B. Wilk (1965, 12). An analysis of variance test for normality (complete samples). *Biometrika 52*(3-4), 591–611.

Siami-Namini, S. (2021). U.S. Monetary Policy and Commodity Prices: A SVECM Approach. *Economic Papers: A journal of applied economics and policy 40*(4), 288–312.

Silvennoinen, A. and S. Thorp (2013). Financialization, crisis and commodity correlation dynamics. *Journal of International Financial Markets, Institutions and Money 24*, 42–65.

Skinner, B. J. (1979). Earth resources. *Proceedings of the National Academy of Sciences 76*(9), 4212–4217.

Smiech, S., M. Papiez, and M. A. Dabrowski (2015). Does the euro area macroeconomy affect global commodity prices? Evidence from a SVAR approach. *International Review of Economics and Finance 39*, 485–503.

Song, H., C. Wang, X. Lei, and H. Zhang (2022). Dynamic dependence between main-byproduct metals and the role of clean energy market. *Energy Economics 108*.

Standard & Poor's (2022). S&P 500 (SPX). data retrieved from Investing.com, https://de.investing.com/indices/us-spx-500-historical-data.

Sterchele, P., J. Brandes, J. Heilig, D. Wrede, C. Kost, T. Schlegl, A. Bett, and H.-M. Henning (2020). Die deutsche Energiewende im Kontext gesellschaftlicher Verhaltensweisen.

Stuermer, M. (2018). 150 Years of Boom and Bust: What Drives Mineral Commodity Prices? *Macroeconomic Dynamics 22*(3), 702–717.

Svensson, L. E. O. (2008). The effect of monetary policy on real commodity prices: comment. *Campbell, John Y. (ed.), Asset Prices and Monetary Policy, University of Chicago Press*.

Tang, K. and W. Xiong (2012). Index Investment and the Financialization of Commodities. *Financial Analysts Journal 68*(6), 54–74.

The World Bank (2022a). GDP (current US$)[WLD]. https://data.worldbank.org/indicator/NY.GDP.MKTP.CD.

The World Bank (2022b). GDP per capita (current US$). https://data.worldbank.org/indicator/NY.GDP.PCAP.CD.

The World Bank (2022c). Inflation, consumer prices for the United States (FPCPITOTLZGUSA). https://fred.stlouisfed.org/series/FPCPITOTLZGUSA.

The World Bank (2022d). World Industrial Production (IPTOTNSKD), seasonnaly unadjusted. https://databank.worldbank.org/source/global-economic-monitor-(gem).

The World Bank (2022e). World Total Population. https://data.worldbank.org/indicator/SP.POP.TOTL.

The World Bureau of Metal Statistics (2021). Bespoke Production and Consumption Report 1995-2021. https://world-bureau.co.uk/.

Thomas, A. H., M. Mühleisen, and M. Pant (2010). Peaks, spikes, and barrels: Modeling sharp movements in oil prices. *IMF Working Papers*, 1–17.

Thomson Reuters Eikon (2022a). Cobalt Cathode 99.8% CIF NWE U$/LB.

Thomson Reuters Eikon (2022b). Indium CIF NWE U$/MT.

Thomson Reuters Eikon (2022c). Lithium Metal =99%, Battery Grade U$/MT.

Thomson Reuters Eikon (2022d). LME-Aluminium 99.7% Cash U$/MT.

Thomson Reuters Eikon (2022e). LME-Copper Grade A Cash U$/MT.

Thomson Reuters Eikon (2022f). LME-Lead Cash U$/MT.

Thomson Reuters Eikon (2022g). LME-Nickel Cash U$/MT.

Thomson Reuters Eikon (2022h). LME-SHG Zinc 99.995% Cash U$/MT.

Thomson Reuters Eikon (2022i). LME-Tin 99.85% Cash U$/MT.

Thomson Reuters Eikon (2022j). LME Total VM and OI - VOLUME TRADED.

Thomson Reuters Eikon (2022k). London Platinum Free Market U$/MT.

Thomson Reuters Eikon (2022l). Silver, Handy&Harman (NY) U$/MT. OZ.

Tilton, J. E. (2010). *On borrowed time: assessing the threat of mineral depletion*. Routledge.

Tilton, J. E., P. C. Crowson, J. H. DeYoung, R. G. Eggert, M. Ericsson, J. I. Guzmán, D. Humphreys, G. Lagos, P. Maxwell, M. Radetzki, D. A. Singer, and F.-W. Wellmer (2018). Public policy and future mineral supplies. *Resources Policy 57*, 55–60.

U.S. Bureau of Economic Analysis (2022). Real Gross Domestic Product (GDPC1). https://fred.stlouisfed.org/series/GDPC1.

U.S. Geological Survey (1970-2018). Metals and minerals: U.S. Geological Survey Minerals Yearbooks [1970-2018]. https://www.usgs.gov/centers/national-minerals-information-center/mineral-industry-surveys/.

U.S. Geological Survey (2012). Mineral Commodity Summaries 2012. http://pubs.er.usgs.gov/publication/mcs2012.

U.S. Geological Survey (2013a). Metal prices in the United States through 2010: U.S. Geological Survey Scientific Investigations Report 2012–5188. https://pubs.usgs.gov/sir/2012/5188/.

U.S. Geological Survey (2013b). Mineral Commodity Summaries 2013. http://pubs.er.usgs.gov/publication/mcs2013.

U.S. Geological Survey (2015). Mineral Commodity Summaries 2015. http://pubs.er.usgs.gov/publication/mcs2015.

U.S. Geological Survey (2016). Mineral Commodity Summaries 2023. http://pubs.er.usgs.gov/publication/mcs2016.

U.S. Geological Survey (2017). Metals and minerals: U.S. Geological Survey Minerals Yearbooks 2017. https://www.usgs.gov/centers/national-minerals-information-center/mineral-industry-surveys/.

U.S. Geological Survey (2019). Metals and minerals: U.S. Geological Survey Minerals Yearbooks 2019. https://www.usgs.gov/centers/national-minerals-information-center/mineral-industry-surveys/.

U.S. Geological Survey (2020a). Historical statistics for mineral and material commodities in the United States: U.S. Geological Survey Data Series 140. https://www.usgs.gov/centers/national-minerals-information-center/historical-statistics-mineral-and-material-commodities.

U.S. Geological Survey (2020b). Metals and minerals: U.S. Geological Survey Minerals Yearbooks [1970-2020]. https://www.usgs.gov/centers/national-minerals-information-center/mineral-industry-surveys/.

U.S. Geological Survey (2020c). Metals and minerals: U.S. Geological Survey Minerals Yearbooks 2020. https://www.usgs.gov/centers/national-minerals-information-center/mineral-industry-surveys/.

U.S. Geological Survey (2020d). Mineral Commodity Summaries 2020. http://pubs.er.usgs.gov/publication/mcs2020.

Valero, A., A. Valero, G. Calvo, and A. Ortego (2018). Material bottlenecks in the future development of green technologies. *Renewable and Sustainable Energy Reviews 93*, 178–200.

Vansteenkiste, I. (2009). How important are common factors in driving non-fuel commodity prices? A dynamic factor analysis. *ECB Working Paper Series* (1072).

Viebahn, P., O. Soukup, S. Samadi, J. Teubler, K. Wiesen, and M. Ritthoff (2015). Assessing the need for critical minerals to shift the german energy system towards a high proportion of renewables. *Renewable and Sustainable Energy Reviews 49*, 655–671.

Watari, T., K. Nansai, and K. Nakajima (2020). Review of critical metal dynamics to 2050 for 48 elements. *Resources, Conservation and Recycling 155*.

Weise, C. L. (1999). The Asymmetric Effects of Monetary Policy: A Nonlinear Vector Autoregression Approach. *Journal of Money, Credit and Banking 31*(1), 85–108.

West, K. and K.-F. Wong (2014). A factor model for co-movements of commodity prices. *Journal of International Money and Finance 42*(C), 289–309.

World Bank (2015). Commodity Markets Outlook.

World Wildlife Fund (2023). Effects of Climate Change.

Yin, L. and L. Han (2015). Co-movements in commodity prices: Global, sectoral and commodity-specific factors. *Economics Letters 126*, 96–100.

Zapp, P., G. Rombach, and W. Kuckshinrichs (2002). *The future of automotive aluminium.*

Zaremba, A., Z. Umar, and M. Mikutowski (2021). Commodity financialisation and price co-movement: Lessons from two centuries of evidence. *Finance Research Letters 38*.

Zhang, C. and X. Tu (2016). The effect of global oil price shocks on China's metal markets. *Energy Policy 90*, 131–139.

Zhang, D. and D. C. Broadstock (2020). Global financial crisis and rising connectedness in the international commodity markets. *International Review of Financial Analysis 68*.

Zhang, Y., S. Ding, and E. M. Scheffel (2019). A key determinant of commodity price Co-movement: The role of daily market liquidity. *Economic Modelling 81*, 170–180.

Zhu, X., J. Chen, and M. Zhong (2015). Dynamic interacting relationships among international oil prices, macroeconomic variables and precious metal prices. *Transactions of Nonferrous Metals Society of China 25*(2), 669–676.

A Literature Overview

The transition process to climate neutrality will require large amounts of raw materials, see Valero et al. (2018). In this context, the demand for raw materials can cause price peaks and delivery bottlenecks in their supply, see Federal Ministry for Economic Affairs and Climate Action (2022a). Since Skinner (1979), who emphasizes the importance of reliable supplies of metals, several studies assessed the criticality of various commodities. However, the limiting factor for the availability of a commodity is the extraction, see Tilton et al. (2018), since the supply of resources only reacts slowly to changes in demand. For this reason, commodity prices may be interpreted as scarcity indicators which is why a comprehensive understanding of commodity markets and, in particular, of price determinants as well as their common pattern is essential. Hereby, previous studies examine various commodity-specific as well as global price determinants as well as the reasons for the observed co-movement.

Table A.1 provides an overview of the studies analyzing the criticality of commodities, the price determinants or the co-movement between prices. Hereby, the table displays the used methodology, the considered time period and frequency of data and whether individual commodities or a commodity index is analyzed. Moreover, the included determinants are tagged as well as whether the study focuses on the criticality of commodities - in general or in the context of the energy transition - or on the (time-varying) (excess) co-movement between prices.

Table A.1: Overview of studies on criticality (in the context of the energy transition), determinants of, and co-movement between commodity prices

Name	Commodity-specific	Metals	Commodity index	Methodology	Startdate	Enddate	Frequency	Criticality	Energy transition	Supply & Demand	Economic activity	Exchange rate	Monetary policy	Inflation	Oil	Financial market index	Demographic	Super-cycles	Common factor	(Excess-)Co-movement	Time-varying co-movement	Financialization
Aepli et al. (2017)	x	Ag, Au, Cu	x	multivariate dynamic copula models	1998	2013	w														x	
Ahumada and Cornejo (2014)	x	Al, Au, Cu		cointegrated time series-cross section model	1960	2010	a			x	x	x	x	x		x						
Akram (2009)			x	structural VAR model	1989	2007	q				x	x	x	x								
Aksoy et al. (2019)				panel VAR model	1970	2014	a										x					
Alquist and Coibion (2013)	x	Al, Cu, Ni, Pb, Sn, Zn		structural factor model	1957	2013	m				x			x								
Anzuini et al. (2013)			x	VAR model	1970	2008	m				x		x	x								
Apergis et al. (2014)	x	Ag, Au		factor-augmented VAR model	1981	2010	m				x	x	x	x	x	x	x					
Apergis et al. (2020)	x	Ag, Au, Al, Cu, Ni, Pt		EGARCH-X model	1990	2016	q						x									
Arendt et al. (2020)	x							x			x	x			x							
Arrobas et al. (2017)	x			criticality methodology					x													
Bach et al. (2016)	x	Ag, Al		ESSENZ				x														
Bach et al. (2017)	x	40 raw materials		SCARCE				x														
Baffes and Dennis (2013)	x			regression analysis	1960	2012	a			x	x	x	x		x							
Baffes and Savescu (2014)	x	Al, Cu, Ni, Pb, Sn, Zn		regression analysis	1991	2012	q				x	x	x		x							
Baffes et al. (2020)	x	Al, Cu, Ni, Pb, Sn, Zn		VAR & ARDL model	1965	2017	a			x	x	x					x					
Basak and Pavlova (2016)				theoretical model												x						
Bilgin et al. (2018)	x	Ag, Pa, Pt		(non-)linear cointegration model	1988	2016	m							x							x	
Borensztein and Reinhart (1994)			x	regression analysis	1971	1992	q			x	x	x										
Büyüksahin and Robe (2014)			x	dynamic conditional correlations	1991	2010	d / w									x						x
Buncic and Moretto (2015)	x	Cu		dynamic model averaging & selection framework	1996	2014	m			x	x	x	x		x	x						
Byrne et al. (2013)	x	Ag, Al, Cu, Pb, Sn, Zn	x	factor-augmented VAR model	1900	2008	a				x		x	x	x	x			x			
Byrne et al. (2020)	x	Ag, Al, Au, Cu, Pb, Sn, Zn	x	time-varying factor-augmented VAR model	1974	2014	q				x		x								x	
Camacho and Perez-Quiros (2014)			x	reduced-form Markov-switching model	1971	2009	m				x											
Carter et al. (2011)			x	data analysis	1957	2010	m			x												
Cashin et al. (2002)	x	Al, Au, Cu, Fe, Pb, Sn, Zn		commodity price cycle-dating algorithm	1957	1999	m											x				
Chen (2010)	x	Ag, Al, Au, Boron, Co, Cr, Cu, Fe, Steel, Ma, Mg, Mo, Ni, Pb, Pt, Si, Sulfur, Sn, V, W, Zn		decomposition analysis	1900	2007	a			x	x											
Chen et al. (2010)			x	regression analysis & Granger causality	1973	2008	q					x										
Chen et al. (2014)	x	Al, Cu, Fe, Ni, Pb, Sn, Zn, Uranium		PANIC framework	1980	2009	m					x							x			
Chen et al. (2019)	x	Cu		Markov-switching VAR model	2004	2016	m			x		x	x		x							
Ciner et al. (2020)	x	Al, Al alloy, Cu, Ni, Pb, Sn, Zn		Diebold-Yilmaz spillover index & wavelet analysis	1994	2016	w														x	
Cuddington and Jerrett (2008)	x	Al, Cu, Ni, Pb, Sn, Zn		bandpass filter	1850	2002	a											x				
Cuddington and Zellou (2013)				supply and demand model & simulation analysis						x												
Deaton and Laroque (1992)	x	Cu, Sn		rational expectations competitive storage model	1900	1987	a			x												
Deaton and Laroque (2003)	x	Cu, Sn		time-series version of the Lewis model	1900	1987	a			x												
Deb et al. (1996)	x	Au, Pb		GARCH	1960	1992	m													x		
Dimitropoulos and Yatchew (2018)	x	Ag, Al, Cu, Fe, Ni, Pb, Sn, Zn		regression analysis	1901	2014	a				x						x					
Ding and Zhang (2020)	x	Au, Cu	x		2005	2018	d															
Dutta (2018)			x	GARCH-jump model	2007	2016	d								x							
Eksi and Tas (2017)				empirical asset pricing model	1979	2015	m						x									
Erdmann and Graedel (2011)				literature review				x														
European Commission (2020)	x			criticality methodology					x													
Fernandez (2015a)	x	Ag, Al, Cu, Pb, Sn, Zn		see Pindyck and Rotemberg (1990)	1900	2010	a			x	x	x	x			x				x	x	
Fernandez (2015b)	x	Ag, Al, Au, Cu, Ni, Pb, Pt, Sn, Zn	x	price decomposition	1968	2013	m													x	x	x
Frankel (1986)				theoretical model									x									
Frankel (2008)	x	Ag, Al, Au, Cu, Ni, Pa, Pb, Pt, Sn, Zn	x	regression analysis	1950	2005	m / a				x	x	x									
Frankel (2014)	x	Ag, Cu, Pt	x	regression analysis	1950	2012	a				x		x	x								
Frankel and Rose (2010)	x	Ag, Au, Cu, Pt	x	regression analysis	1960	2008	a			x	x		x	x								

Overview of studies on criticality (in the context of the energy transition), determinants of, and co-movement between commodity prices

Name	Commodity-specific	Metals	Commodity index	Methodology	Startdate	Enddate	Frequency	Criticality	Energy transition	Supply & Demand	Economic activity	Exchange rate	Monetary policy	Inflation	Oil	Financial market index	Demographic	Super-cycles	Common factor	(Excess-)Co-movement	Time-varying co-movement	Financialization
Gargano and Timmermann (2014)			x	regression analysis	1947	2010	m				x	x	x	x		x	x					
Gilbert (1989)			x	regression analysis	1965	1986	q				x	x	x									
Gleich et al. (2013)	x	42 elements		regression analysis	1984	2009	a	x		x	x	x		x			x					
Graedel et al. (2012)				criticality methodology				x														
Graedel et al. (2015)	x	62 metals						x														
Graedel and Reck (2016)				literature review				x														
Gubler and Hertweck (2013)			x	VAR model	1955	2007	q						x									
Guzmán and Silva (2018)	x	Cu		VAR model	1996	2015	m			x	x	x	x		x	x						
Hammoudeh et al. (2004)	x			VECM & GARCH	1995	2001	d								x							
Hammoudeh et al. (2015)			x	structural VAR	1957	2008	q				x	x	x	x								
Hayes and McCullough (2018)				literature review				x														
Helbling et al. (2008)	x	Al, Cu		data analysis	1985	2007	a			x												
Hotelling (1931)				theoretical model						x			x									
Ioannidou et al. (2019)				literature review				x														
Irwin and Sanders (2012)			x	structural change analysis	1995	2011	m														x	
Issler et al. (2014)	x	Al, Cu, Ni, Pb, Sn, Zn		VECM	1900	2012	m / a				x	x	x			x						
Jordan (2017)								x														
Kagraoka (2016)	x	Al, Cu, Ni, Pb, Sn, Zn	x	dynamic factor model	1995	2015	m				x	x	x	x	x	x						
Keating et al. (2019)			x	VAR model	1967	2015	q				x		x	x								
Kilian (2009)	x			structural VAR model	1968	2007	m			x	x											
Kilian and Zhou (2018)			x	data analysis	1970	2016	q			x	x											
Klotz et al. (2014)			x	Granger causality & GIRF	1998	2012	m				x	x	x									
Kolotzek et al. (2018)	x	Al, Nb, Ta		raw material assessment				x														
Lee et al. (2020)	x			literature review					x													
Le Pen and Sévi (2017)	x	Ag, Cu		large approximate factor models	1993	2013	m				x	x	x	x		x	x			x	x	
Lescaroux (2009)	x	Al, Cu, Fe, Ni, Pb, Sn, U, Zn		price decomposition, correlation analysis	1987	2008	m			x										x		
Liang et al. (2022)				literature review					x													
Liberda (2017)	x	Ag, Au, Pa, Pt		MIDAS regression analysis	1997	2016	d / m				x	x	x	x	x	x						
Lombardi et al. (2012)	x	Al, Cu, Fe, Ni, Pb, Sn, Zn		factor-augmented VAR model	1975	2008	q				x	x	x		x				x			
Lübbers and Posch (2016)	x	31 commodities		generalized dynamic factor model	1996	2015	m									x					x	
Lutzenberger et al. (2017)	x	30 metals		cross-sectional methodology	1990	2013	m			x	x	x	x	x		x						
Marscheider-Weidemann et al. (2021)	x			scenario analysis					x													
Nassar et al. (2015)	x	62 metals			2008	2008		x														
Nick and Thoenes (2013)	x			structural VAR model	2008	2012	w			x												
Nicola et al. (2016)	x			multivariate dynamic conditional correlation models & regression analysis	1970	2013	m					x	x			x				x		
Ohashi and Okimoto (2016)			x	see Pindyck and Rotemberg (1990) & dynamic conditional correlation models	1983	2011	m				x	x	x	x		x				x	x	
Peersman et al. (2021)			x	time-varying-parameter Bayesian VAR model	1990	2015	m						x								x	
Pierdzioch et al. (2016)	x	Au		regression analysis	1977	2014	q				x	x	x		x	x						
Pincheira-Brown and Hardy (2019)	x	Al, Cu, Ni, Pb, Sn, Zn		regression analysis	1999	2017	m / q					x										
Pindyck and Rotemberg (1990)	x	Au, Cu		correlation analysis & regression analysis	1960	1985	m				x	x	x	x		x				x		
Poncela et al. (2014)	x	Al, Cu, Fe, Ni, Pb, Sn, Zn, Uranium		dynamic factor model & factor augmented VAR model	1992	2012	m				x	x	x		x	x					x	
Pradhananga (2016)	x	Al, Cu, Ni, Pb, Sn, Zn		factor-augmented VAR model	1995	2012	m															x
Redlinger and Eggert (2016)	x	36 metals		regression analysis	1960	2013	w / a	x														
Robinson (2019)	x	Au		structural VAR model	1980	2014	a				x	x	x		x							
Roelich et al. (2014)	x	Nd		risk assessment	2012	2050	a		x													
Rosenau-Tornow et al. (2009)	x	Cu		long-term supply risk analysis	1960	2005	a	x														

Overview of studies on criticality (in the context of the energy transition), determinants of, and co-movement between commodity prices

Name	Commodity-specific	Metals	Commodity index	Methodology	Startdate	Enddate	Frequency	Criticality	Energy transition	Supply & Demand	Economic activity	Exchange rate	Monetary policy	Inflation	Oil	Financial market index	Demographic	Super-cycles	Common factor	(Excess-)Co-movement	Time-varying co-movement	Financialization
Rossen (2015)	x	Ag, Al, Au, Bi, Co, Cr, Cu, Fe, Mg, Mo, Ni, Pa, Pb, Pt, Sb, Steel, Sn, W, Zn		cointegration analysis & price cycles	1910	2011	m											x				
Sari et al. (2010)	x	Ag, Au, Pa, Pt		GFEVD & GIRF	1999	2007	d					x			x							
Schischke and Rathgeber (2023)			x	VAR	1996	2019	m				x	x	x	x								
Schrijvers et al. (2020)				literature review & round tables				x														
Shammugam et al. (2019)	x	Ag, Al, As, Au, Bi, Cd, Co, Cr, Cu, Fe, Ga, Ge, In, Li, Mo, Ni, Pb, Pd, Pt, Rh, Sb, Se, Sn, Zn		Toda-Yamamoto approach of Granger causality	1960	2013	m													x		
Siami-Namini (2021)			x	structural VECM	1992	2017	m				x	x	x	x	x							
Silvennoinen and Thorp (2013)	x	Ag, Al, Au, Cu, Ni, Pb, Pt, Sn, Zn		DSTCC–GARCH	1990	2009	w					x	x			x						x
Skinner (1979)								x														
Smiech et al. (2015)			x	structural VAR model	1997	2013	m				x		x									
Song et al. (2022)	x	Al, Cd, Co, Cu, Ga, Ge, In, Ni, Pb, Se, Zn		time-varying parameter VAR	2005	2021	m		x												x	
Stuermer (2018)	x	Cu, Pb, Sn, Zn		VAR	1840	2014	a			x	x											
Svensson (2008)				comment									x									
Tang and Xiong (2012)	x	Ag, Au, Cu, Pa, Pt		regression analysis	1986	2011	d															x
Thomas et al. (2010)	x			regression analysis & structural VAR model	1996	2009	a			x	x		x		x							
Tilton et al. (2018)	x	Al, Cu, Ni, Pb, Sn, Zn		resource availability				x														
Valero et al. (2018)	x	31 raw materials		risk assessment	2016	2050	a		x													
Vansteenkiste (2009)	x	Al, Cu, Ni, Pb, Sn, Zn		dynamic common factor model	1957	2008	m				x	x	x	x		x			x			
Viebahn et al. (2015)				meta analysis					x													
Watari et al. (2020)				literature review					x													
West and Wong (2014)	x	Al, Au, Cu, Ni, Pb, Sn, Zn		factor model	1989	2012	m				x	x							x			
Yin and Han (2015)	x	Ag, Au, Cu, Pa, Pt		Bayesian dynamic latent factor model	1991	2014	m												x			
Zapp et al. (2002)	x	Al		chain modelling						x												
Zaremba et al. (2021)	x			correlation analysis	1850	2019	m														x	
Zhang and Broadstock (2020)			x	VAR model & Diebold-Yilmaz spillover index	1960	2017	m														x	
Zhang et al. (2019)	x	Ag, Au		regression analysis & Granger causality	2005	2013	d												x			
Zhang and Tu (2016)	x	Al, Cu		GARCH	2004	2015	d								x							
Zhu et al. (2015)	x	Ag, Au, Pt		causality test& GIRF	2006	2013	d					x	x		x							

This table provides an overview on studies investigating the criticality of commodities, analyzing potential, commodity price influential factors and focusing on the common behavior in commodity prices. These differentiate by the methodology used, the underlying time span and frequency (daily (d), weekly (w), monthly (m), quarterly (q), annual (a)) of data analyzed, as well as the influential variables considered. Moreover, entries in the columns *Criticality* or *Energy transition* indicate the studies investigate the criticality of commodities in general or in the context of the energy transition. While the column *Supply & Demand* denotes supply- or demand-related variables are included, the columns *Economic activity*, *Exchange rate*, *Monetary policy*, *Inflation*, *Oil*, *Financial market index* and *Demographic* reflects studies which include variables related to economic activity, exchange rate, monetary policy, inflation rate, oil price, financial market indices or demography, whereby the focus of the studies does not necessarily have to be on these variables. In addition, studies with entries in the columns *Super-cycles* or *Financialization* consider historic super-cycles in commodity prices or examine whether the so-called financialization affect commodity markets, respectively. Further, studies investigating common factors underlying in commodity prices or generally the (excess-)co-movement between prices (and the time-varying behavior of the co-movement) are represented in the columns *Common factor*, *(Excess-)Co-movement* (*Time-varying co-movement*).

B Methodology

In this thesis, we propose a new framework to analyze resource-demanding projects in terms of the resource scarcity risk. Hereby, we account for the time-varying commodity market structure, via the Markov-switching global vector autoregression (MS-GVAR) model. In the following, additional information about the state space representation, and the expectation-maximization (EM) algorithm for the commodity-specific MS-VAR models, as well as about the generalized impulse response functions (GIRFs) for the global model, is provided.

B.1 Markov-switching Global Vector Autoregression Model

B.1.1 State Space Representation

In general, the commodity-specific MS(M)-VAR(P) models in Equation 3.22 can be rewritten into their state space representation, consisting of a measurement and a transition equation. In this context, the regression equations for all MS-VAR specifications are derived from the measurement equation in Equation 3.29 of the state space representation of the model. In the following, the regression equation for the most general MSIAHX-VAR model will be derived, where all parameters, the intercept term ($\boldsymbol{\nu}$), the parameters corresponding to the exogenous variables ($\boldsymbol{\psi}$), the lagged coefficient matrices ($\boldsymbol{\alpha}$) as well as the covariance matrix ($\boldsymbol{\Sigma}$), are regime-dependent.

Hereby, we define $\mathbb{X}_{i,t}$ as follows:

$$\begin{aligned}
\mathbb{X}_{i,t} &= \bar{\mathbf{x}}'_{i,t} \otimes \mathbf{I}_{K_i} \\
&= \left(1, \mathbf{e}'_t, \mathbf{e}'_{t-1}, \ldots, \mathbf{e}'_{t-P_{exog}}, \mathbf{x}^{*\prime}_{i,t}, \mathbf{x}^{*\prime}_{i,t-1}, \ldots, \mathbf{x}^{*\prime}_{i,t-P}, \mathbf{x}'_{i,t-1}, \mathbf{x}'_{i,t-2}, \ldots, \mathbf{x}'_{i,t-P}\right) \otimes \mathbf{I}_{K_i} \\
&\coloneqq \left(1, \mathbf{ex}'_{i,t}, \mathbf{en}'_{i,t-1}\right),
\end{aligned} \tag{B.1}$$

with $\mathbf{ex}'_{i,t} \coloneqq \left(\mathbf{e}'_t, \mathbf{e}'_{t-1}, \ldots, \mathbf{e}'_{t-P_{exog}}, \mathbf{x}^{*\prime}_{i,t}, \mathbf{x}^{*\prime}_{i,t-1}, \ldots, \mathbf{x}^{*\prime}_{i,t-P}\right)$ denoting the exogenous variables, $\mathbf{en}'_{i,t-1} \coloneqq \left(\mathbf{x}'_{i,t-1}, \mathbf{x}'_{i,t-2}, \ldots, \mathbf{x}'_{i,t-P}\right)$ the endogenous variables of the model, $\mathbf{I}_{K_i}$ a $K_i \times K_i$ identity matrix and $\otimes$ the Kronecker product.

Then, the measurement equation can be rewritten:

$$
\begin{aligned}
\mathbf{x}_{i,t} &= \mathbb{X}_{i,t}\mathbf{B}_i\boldsymbol{\xi}_{i,t} + \mathbf{u}_{i,t} \\
&= \left(\left(1, \mathbf{ex}'_{i,t-1}, \mathbf{en}'_{i,t-1}\right) \otimes \mathbf{I}_{K_i}\right) \begin{pmatrix} \boldsymbol{\nu}_{i,1} & \boldsymbol{\nu}_{i,2} & \cdots & \boldsymbol{\nu}_{i,M} \\ \boldsymbol{\psi}_{i,1} & \boldsymbol{\psi}_{i,2} & \cdots & \boldsymbol{\psi}_{i,M} \\ \boldsymbol{\alpha}_{i,1} & \boldsymbol{\alpha}_{i,2} & \cdots & \boldsymbol{\alpha}_{i,M} \end{pmatrix} \begin{pmatrix} \boldsymbol{\xi}_{i,1,t} \\ \boldsymbol{\xi}_{i,2,t} \\ \vdots \\ \boldsymbol{\xi}_{i,M,t} \end{pmatrix} + \mathbf{u}_{i,t} \\
&= \left(1 \otimes \mathbf{I}_{K_i}, \mathbf{ex}'_{i,t-1} \otimes \mathbf{I}_{K_i}, \mathbf{en}'_{i,t-1} \otimes \mathbf{I}_{K_i}\right) \begin{pmatrix} \boldsymbol{\nu}_{i,1}\boldsymbol{\xi}_{i,1,t} + \boldsymbol{\nu}_{i,2}\boldsymbol{\xi}_{i,2,t} + \cdots + \boldsymbol{\nu}_{i,M}\boldsymbol{\xi}_{i,M,t} \\ \boldsymbol{\psi}_{i,1}\boldsymbol{\xi}_{i,1,t} + \boldsymbol{\psi}_{i,2}\boldsymbol{\xi}_{i,2,t} + \cdots + \boldsymbol{\psi}_{i,M}\boldsymbol{\xi}_{i,M,t} \\ \boldsymbol{\alpha}_{i,1}\boldsymbol{\xi}_{i,1,t} + \boldsymbol{\alpha}_{i,2}\boldsymbol{\xi}_{i,2,t} + \cdots + \boldsymbol{\alpha}_{i,M}\boldsymbol{\xi}_{i,M,t} \end{pmatrix} \\
&+ \mathbf{u}_{i,t} \\
&= \left(1 \otimes \mathbf{I}_{K_i}\right)\left(\boldsymbol{\nu}_{i,1}\boldsymbol{\xi}_{i,1,t} + \boldsymbol{\nu}_{i,2}\boldsymbol{\xi}_{i,2,t} + \cdots + \boldsymbol{\nu}_{i,M}\boldsymbol{\xi}_{i,M,t}\right) \\
&+ \left(\mathbf{ex}'_{i,t-1} \otimes \mathbf{I}_{K_i}\right)\left(\boldsymbol{\psi}_{i,1}\boldsymbol{\xi}_{i,1,t} + \boldsymbol{\psi}_{i,2}\boldsymbol{\xi}_{i,2,t} + \cdots + \boldsymbol{\psi}_{i,M}\boldsymbol{\xi}_{i,M,t}\right) \\
&+ \left(\mathbf{en}'_{i,t-1} \otimes \mathbf{I}_{K_i}\right)\left(\boldsymbol{\alpha}_{i,1}\boldsymbol{\xi}_{i,1,t} + \boldsymbol{\alpha}_{i,2}\boldsymbol{\xi}_{i,2,t} + \cdots + \boldsymbol{\alpha}_{i,M}\boldsymbol{\xi}_{i,M,t}\right) + \mathbf{u}_{i,t} \\
&= \sum_{m=1}^{M}\left(1 \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\nu}_{i,m}\boldsymbol{\xi}_{i,m,t} + \sum_{m=1}^{M}\left(\mathbf{ex}'_{i,t-1} \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\psi}_{i,m}\boldsymbol{\xi}_{i,m,t} \\
&+ \sum_{m=1}^{M}\left(\mathbf{en}'_{i,t-1} \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\alpha}_{i,m}\boldsymbol{\xi}_{i,m,t} + \mathbf{u}_{i,t} \\
&= \sum_{m=1}^{M}\left(\boldsymbol{\xi}_{i,m,t}1 \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\nu}_{i,m} + \sum_{m=1}^{M}\left(\boldsymbol{\xi}_{i,m,t}\mathbf{ex}'_{i,t-1} \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\psi}_{i,m} \\
&+ \sum_{m=1}^{M}\left(\boldsymbol{\xi}_{i,m,t}\mathbf{en}'_{i,t-1} \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\alpha}_{i,m} + \mathbf{u}_{i,t}.
\end{aligned} \tag{B.2}
$$

Further, we define $\mathbf{Ex}_i = \left(\mathbf{ex}'_{i,-0}, \mathbf{ex}'_{i,-1}, \ldots, \mathbf{ex}'_{i,-P_{exog}}\right)$ with $\mathbf{ex}_{i,-p} = \left(\mathbf{ex}_{i,0-p}, \mathbf{ex}_{i,1-p}, \ldots, \mathbf{ex}_{i,T-p}\right)$, for $p = 0, 1, \ldots, P_{exog}$, and $\mathbf{En}_i = \left(\mathbf{en}'_{i,-1}, \mathbf{en}'_{i,-2}, \ldots, \mathbf{en}'_{i,-P}\right)$ with $\mathbf{en}_{i,-p} = \left(\mathbf{en}_{i,1-p}, \mathbf{en}_{i,2-p}, \ldots, \mathbf{en}_{i,T-p}\right)$, for $p = 1, 2, \ldots, P$. This can be used for $\mathbf{x}_i = \left(\mathbf{x}'_{i,1}, \mathbf{x}'_{i,2}, \ldots, \mathbf{x}'_{i,T}\right)'$ as follows:

$$
\begin{aligned}
\mathbf{x}_i &= \sum_{m=1}^{M}\left(diag\left(\boldsymbol{\xi}_{i,m}\right)\mathbf{1}_T \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\nu}_{i,m} + \sum_{m=1}^{M}\left(diag\left(\boldsymbol{\xi}_{i,m}\right)\mathbf{Ex}'_i \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\psi}_{i,m} \\
&+ \sum_{m=1}^{M}\left(diag\left(\boldsymbol{\xi}_{i,m}\right)\mathbf{En}'_i \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\alpha}_{i,m} + \mathbf{u}_i \\
&= \sum_{m=1}^{M}\left(\boldsymbol{\Xi}_{i,m}\mathbf{1}_T \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\nu}_{i,m} + \sum_{m=1}^{M}\left(\boldsymbol{\Xi}_{i,m}\mathbf{Ex}'_i \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\psi}_{i,m} \\
&+ \sum_{m=1}^{M}\left(\boldsymbol{\Xi}_{i,m}\mathbf{En}'_i \otimes \mathbf{I}_{K_i}\right)\boldsymbol{\alpha}_{i,m} + \mathbf{u}_i,
\end{aligned} \tag{B.3}
$$

with the innovation process:

$$
\mathbf{u}_i \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Omega}_i\right), \text{ with } \boldsymbol{\Omega}_i = \sum_{m=1}^{M}\boldsymbol{\Xi}_{i,m} \otimes \boldsymbol{\Sigma}_{ii,m}. \tag{B.4}
$$

In order to get the regression equations with the corresponding distribution of the innovation process of the different MS-VAR specifications, the regime-invariant parameters simplify the equations. Especially, in case of homoscedastic models, in which the covariances are equal for each regime, the innovation process is given by:

$$\mathbf{u}_i \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Omega}_i\right), \text{ with } \boldsymbol{\Omega}_i = \mathbf{I}_T \otimes \boldsymbol{\Sigma}_{ii}. \tag{B.5}$$

Under the assumption of regime-invariant covariances, it holds $\boldsymbol{\Sigma}_{ii,m} = \boldsymbol{\Sigma}_{ii}$, for all $m = 1, 2, \ldots, M$, and therefore, the corresponding definition of $\boldsymbol{\Omega}_i$ simplifies to:

$$\begin{aligned} \sum_{m=1}^{M} \boldsymbol{\Xi}_{i,m} \otimes \boldsymbol{\Sigma}_{ii,m} &= \sum_{m=1}^{M} \boldsymbol{\Xi}_{i,m} \otimes \boldsymbol{\Sigma}_{ii} \\ &= \left(\sum_{m=1}^{M} \boldsymbol{\Xi}_{i,m}\right) \otimes \boldsymbol{\Sigma}_{ii} \\ &= \mathbf{I}_T \otimes \boldsymbol{\Sigma}_{ii}. \end{aligned} \tag{B.6}$$

The terms of the regression equation corresponding to regime-invariant parameters can be simplified in a similar manner. For example, if the intercept is regime-invariant, the corresponding term simplifies as follows:

$$\begin{aligned} \sum_{m=1}^{M} \left(\boldsymbol{\Xi}_{i,m} \mathbf{1}_T \otimes \mathbf{I}_{K_i}\right) \boldsymbol{\nu}_{i,m} &= \sum_{m=1}^{M} \left(\boldsymbol{\Xi}_{i,m} \mathbf{1}_T \otimes \mathbf{I}_{K_i}\right) \boldsymbol{\nu}_i \\ &= \left(\sum_{m=1}^{M} \left(\boldsymbol{\Xi}_{i,m} \mathbf{1}_T \otimes \mathbf{I}_{K_i}\right)\right) \boldsymbol{\nu}_i \\ &= \left(\left(\left(\sum_{m=1}^{M} \boldsymbol{\Xi}_{i,m}\right) \mathbf{1}_T\right) \otimes \mathbf{I}_{K_i}\right) \boldsymbol{\nu}_i \\ &= \left(\mathbf{1}_T \otimes \mathbf{I}_{K_i}\right) \boldsymbol{\nu}_i. \end{aligned} \tag{B.7}$$

B.1.2 Expectation-Maximization Algorithm

In the following, we show the maximum likelihood (ML) estimator equals to the generalized least squares estimator of the corresponding linear regression model with many observations per cell, where the pseudo observations $(\mathbf{x}_{i,t}, \mathbf{X}_{i,m,t}, \boldsymbol{\xi}_{i,t} = \boldsymbol{\iota}_m)$ are weighted with the smoothed probabilities $\hat{\boldsymbol{\xi}}_{i,m,t|T}\left(\boldsymbol{\lambda}_i^{j-1}\right) = Pr\left(\boldsymbol{\xi}_{i,t} = \boldsymbol{\iota}_m | \mathbf{X}_{i,T}, \boldsymbol{\lambda}_i^{j-1}\right)$. Hereby, we obtain the maximum likelihood estimator for the VAR parameters by using the definition of the residuals and setting the first partial derivative of the log-likelihood function with respect to the VAR parameters $(\boldsymbol{\gamma}_i)$, including the intercept $(\boldsymbol{\nu}_i)$, the autoregressive parameters $(\boldsymbol{\alpha}_i)$ as well as the parameters corresponding to the exogenous variables$(\boldsymbol{\psi}_i)$, equal to zero.

Therefore, using the definitions of Section 3.2.1.5, it holds:

$$
\begin{aligned}
\frac{\partial \ell\left(\boldsymbol{\theta}_i | \mathbf{X}_{i,T}\right)}{\partial \boldsymbol{\gamma}_i} &= -\frac{1}{2}\frac{\partial \mathbf{u}_i\left(\boldsymbol{\gamma}_i\right)' \mathbf{W}_i^{-1} \mathbf{u}_i\left(\boldsymbol{\gamma}_i\right)}{\partial \boldsymbol{\gamma}_i} = -\mathbf{u}_i\left(\boldsymbol{\gamma}_i\right)' \mathbf{W}_i^{-1} \frac{\partial \mathbf{u}_i\left(\boldsymbol{\gamma}_i\right)}{\partial \boldsymbol{\gamma}_i} \\
&= -\left(\mathbf{1}_M \otimes \mathbf{x}_i - \mathbf{X}_i \boldsymbol{\gamma}_i\right)' \mathbf{W}_i^{-1} \frac{\partial}{\partial \boldsymbol{\gamma}_i}\left(\mathbf{1}_M \otimes \mathbf{x}_i - \mathbf{X}_i \boldsymbol{\gamma}_i\right) \\
&= -\left(\mathbf{1}_M \otimes \mathbf{x}_i - \mathbf{X}_i \boldsymbol{\gamma}_i\right)' \mathbf{W}_i^{-1}\left(-\mathbf{X}_i\right) \\
&= \left(\mathbf{1}_M \otimes \mathbf{x}_i\right)' \mathbf{W}_i^{-1} \mathbf{X}_i - \left(\mathbf{X}_i \boldsymbol{\gamma}_i\right)' \mathbf{W}_i^{-1} \mathbf{X}_i \\
&= \left(\mathbf{1}_M \otimes \mathbf{x}_i\right)' \mathbf{W}_i^{-1} \mathbf{X}_i - \boldsymbol{\gamma}_i' \mathbf{X}_i' \mathbf{W}_i^{-1} \mathbf{X}_i \overset{!}{=} \mathbf{0} \\
&\Leftrightarrow \boldsymbol{\gamma}_i' \mathbf{X}_i' \mathbf{W}_i^{-1} \mathbf{X}_i = \left(\mathbf{1}_M \otimes \mathbf{x}_i\right)' \mathbf{W}_i^{-1} \mathbf{X}_i \\
&\Leftrightarrow \boldsymbol{\gamma}_i' = \left(\mathbf{1}_M \otimes \mathbf{x}_i\right)' \mathbf{W}_i^{-1} \mathbf{X}_i \left(\mathbf{X}_i' \mathbf{W}_i^{-1} \mathbf{X}_i\right)^{-1} \\
&\Leftrightarrow \boldsymbol{\gamma}_i = \left(\left(\mathbf{1}_M \otimes \mathbf{x}_i\right)' \mathbf{W}_i^{-1} \mathbf{X}_i \left(\mathbf{X}_i' \mathbf{W}_i^{-1} \mathbf{X}_i\right)^{-1}\right)' \\
&\Leftrightarrow \boldsymbol{\gamma}_i = \left(\left(\mathbf{X}_i' \mathbf{W}_i^{-1} \mathbf{X}_i\right)^{-1}\right)' \mathbf{X}_i' \mathbf{W}_i^{-1'} \left(\mathbf{1}_M \otimes \mathbf{x}_i\right) \\
&\Leftrightarrow \boldsymbol{\gamma}_i = \left(\mathbf{X}_i' \mathbf{W}_i^{-1} \mathbf{X}_i\right)^{-1} \mathbf{X}_i' \mathbf{W}_i^{-1} \left(\mathbf{1}_M \otimes \mathbf{x}_i\right).
\end{aligned}
\tag{B.8}
$$

With help of the definitions of the matrices described in Section 3.2.1.5, we rewrite the maximum likelihood estimator to reduce the computational effort:

$$
\begin{aligned}
\boldsymbol{\gamma}_i =& \left(\mathbf{X}_i' \mathbf{W}_i^{-1} \mathbf{X}_i\right)^{-1} \mathbf{X}_i' \mathbf{W}_i^{-1} \left(\mathbf{1}_M \otimes \mathbf{x}_i\right) \\
=& \left(\left(\mathbf{X}_{i,1}', \mathbf{X}_{i,2}', \ldots, \mathbf{X}_{i,M}'\right) \begin{pmatrix} \hat{\boldsymbol{\Xi}}_{i,1} \otimes \boldsymbol{\Sigma}_{ii,1}^{-1} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \hat{\boldsymbol{\Xi}}_{i,M} \otimes \boldsymbol{\Sigma}_{ii,M}^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{X}_{i,1} \\ \mathbf{X}_{i,2} \\ \vdots \\ \mathbf{X}_{i,M} \end{pmatrix}\right)^{-1} \\
& \left(\mathbf{X}_{i,1}', \mathbf{X}_{i,2}', \ldots, \mathbf{X}_{i,M}'\right) \begin{pmatrix} \hat{\boldsymbol{\Xi}}_{i,1} \otimes \boldsymbol{\Sigma}_{ii,1}^{-1} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \hat{\boldsymbol{\Xi}}_{i,M} \otimes \boldsymbol{\Sigma}_{ii,M}^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{x}_i \\ \mathbf{x}_i \\ \vdots \\ \mathbf{x}_i \end{pmatrix} \\
=& (\mathbf{X}_{i,1}' \left(\hat{\boldsymbol{\Xi}}_{i,1} \otimes \boldsymbol{\Sigma}_{ii,1}^{-1}\right) \mathbf{X}_{i,1} + \mathbf{X}_{i,2}' \left(\hat{\boldsymbol{\Xi}}_{i,2} \otimes \boldsymbol{\Sigma}_{ii,2}^{-1}\right) \mathbf{X}_{i,2} + \ldots \\
& + \mathbf{X}_{i,M}' \left(\hat{\boldsymbol{\Xi}}_{i,M} \otimes \boldsymbol{\Sigma}_{ii,M}^{-1}\right) \mathbf{X}_{i,M})^{-1} (\mathbf{X}_{i,1}' \left(\hat{\boldsymbol{\Xi}}_{i,1} \otimes \boldsymbol{\Sigma}_{ii,1}^{-1}\right) + \mathbf{X}_{i,2}' \left(\hat{\boldsymbol{\Xi}}_{i,2} \otimes \boldsymbol{\Sigma}_{ii,2}^{-1}\right) + \ldots \\
& + \mathbf{X}_{i,M}' \left(\hat{\boldsymbol{\Xi}}_{i,M} \otimes \boldsymbol{\Sigma}_{ii,M}^{-1}\right)) \mathbf{x}_i \\
=& \left(\sum_{m=1}^{M} \mathbf{X}_{i,m}' \left(\hat{\boldsymbol{\Xi}}_{i,m} \otimes \boldsymbol{\Sigma}_{ii,m}^{-1}\right) \mathbf{X}_{i,m}\right)^{-1} \left(\sum_{m=1}^{M} \mathbf{X}_{i,m}' \left(\hat{\boldsymbol{\Xi}}_{i,m} \otimes \boldsymbol{\Sigma}_{ii,m}^{-1}\right)\right) \mathbf{x}_i.
\end{aligned}
\tag{B.9}
$$

If the regressors are identical for each equation $\mathbf{x}_{i,k_i}$, $k_i = 1, 2, \ldots, K_i$, it holds $\mathbf{X}_{i,m} = \bar{\mathbf{X}}_{i,m} \otimes \mathbf{I}_{K_i}$ and it follows:

$$\begin{aligned}
\boldsymbol{\gamma}_i = &\left(\sum_{m=1}^{M} \left(\bar{\mathbf{X}}'_{i,m} \otimes \mathbf{I}_{K_i} \right) \left(\hat{\boldsymbol{\Xi}}_{i,m} \otimes \boldsymbol{\Sigma}_{ii,m}^{-1} \right) \left(\bar{\mathbf{X}}_{i,m} \otimes \mathbf{I}_{K_i} \right) \right)^{-1} \\
&\left(\sum_{m=1}^{M} \left(\bar{\mathbf{X}}'_{i,m} \otimes \mathbf{I}_{K_i} \right) \left(\hat{\boldsymbol{\Xi}}_{i,m} \otimes \boldsymbol{\Sigma}_{ii,m}^{-1} \right) \mathbf{x}_i \right) \\
= &\left(\sum_{m=1}^{M} \left(\bar{\mathbf{X}}'_{i,m} \hat{\boldsymbol{\Xi}}_{i,m} \bar{\mathbf{X}}_{i,m} \right) \otimes \boldsymbol{\Sigma}_{ii,m}^{-1} \right)^{-1} \left(\sum_{m=1}^{M} \left(\bar{\mathbf{X}}'_{i,m} \hat{\boldsymbol{\Xi}}_{i,m} \right) \otimes \boldsymbol{\Sigma}_{ii,m}^{-1} \right) \mathbf{x}_i.
\end{aligned} \tag{B.10}$$

B.1.3 Impulse Response Analysis of Shocks to Exogenous, Global Variables - Monte Carlo Integration

The impulse responses of the commodity-specific variables to shocks in the exogenous variables are calculated recursively, using Equation 3.155, Equation 3.157, and the generalized impulse response functions of exogenous variables to shocks to the exogenous variables, denoted by $\mathbf{GI}_{\mathbf{e}:\mathbf{e}_{k_{exog}}}(n, \boldsymbol{\delta}_{exog}, \Omega_{t-1})$. Since the innovations $\boldsymbol{\varepsilon}_{exog,t}$ are, in general, non-normal, we calculate the generalized impulse response functions via a Monte Carlo integration, similar to the procedure described in Section 3.2.4.2.1. However, we slightly adjust the procedure.

1. Draw with replacement a block of $\tilde{P}_{exog}$ consecutive observations of the exogenous variables from the observed data to get N_{hist} randomly drawn histories $\omega_{t-1}^{n_{hist}}$, $n_{hist} = 1, 2, \ldots, N_{hist}$.

2. Randomly sample with replacement $(N_{IRF} + 1) \times N_{shock}$ values of the K_{exog}-dimensional estimated residuals of the MS-VAR model to get a sequence $\{\boldsymbol{\varepsilon}_{exog,t+n}^{n_{shock}}\}_{n=0}^{N_{IRF}}$ of K_{exog}-dimensional shocks $\boldsymbol{\varepsilon}_{exog,t+n}^{n_{shock}}, n = 0, 1, \ldots, N_{IRF}$, $n_{shock} = 1, 2, \ldots, N_{shock}$. Under the assumption of jointly distributed shocks, if date t' s shock is drawn, all K_{exog} residuals for date t are collected.

3. For a specific n_{shock} as well as n_{hist}, use the $N_{IRF}+1$ random shocks $\{\boldsymbol{\varepsilon}_{exog,t+n}^{n_{shock}}\}$ to compute the realization $\mathbf{e}_{t+n}^{n_{hist},n_{shock}}\left(\boldsymbol{\varepsilon}_{exog,t+n}^{n_{shock}}, \omega_{t-1}^{n_{hist}}\right)$ for $n = 0, 1, \ldots, N_{IRF}$, using Equation 3.153, and iterating on the estimated, nonlinear time series model under consideration from the given initial conditions $\boldsymbol{\varepsilon}_{exog,t+n}^{n_{shock}}, \omega_{t-1}^{n_{hist}}$.

4. Use the same draw of $N_{IRF} + 1$ random shocks $\{\boldsymbol{\varepsilon}_{exog,t+n}^{n_{shock}}\}$, but replace the first shock $\boldsymbol{\varepsilon}_{exog,t+0}^{n_{shock}}$ by $\boldsymbol{\varepsilon}_{exog,t+0}^{n_{shock},\boldsymbol{\delta}_{exog}} = \boldsymbol{\varepsilon}_{exog,t+0}^{n_{shock}} + \boldsymbol{\delta}_{exog}$ to produce a realization of the time series, $\mathbf{e}_{t+n}^{n_{hist},n_{shock},\boldsymbol{\delta}_{exog}}\left(\boldsymbol{\varepsilon}_{exog,t+n}^{n_{shock},\boldsymbol{\delta}_{exog}}, \omega_{t-1}^{n_{hist}}\right)$, for $n = 0, 1, \ldots, N_{IRF}$, based on the initial conditions $\boldsymbol{\varepsilon}_{exog,t+n}^{n_{shock},\boldsymbol{\delta}_{exog}}, \omega_{t-1}^{n_{hist}}$.

5. Repeat steps 3 and 4 N_{shock} times and form the averages for each individual component:

$$\tilde{\mathbf{e}}_{t+n}^{n_{hist},\boldsymbol{\delta}_{exog}}\left(\boldsymbol{\varepsilon}_{exog,t+n}^{\boldsymbol{\delta}_{exog}}, \omega_{t-1}^{n_{hist}}\right) = \frac{1}{N_{shock}} \sum_{n_{shock}=1}^{N_{shock}} \mathbf{e}_{t+n}^{n_{hist},n_{shock},\boldsymbol{\delta}_{exog}}\left(\boldsymbol{\varepsilon}_{exog,t+n}^{n_{shock},\boldsymbol{\delta}_{exog}}, \omega_{t-1}^{n_{hist}}\right), \tag{B.11}$$

$$\tilde{\mathbf{e}}_{t+n}^{n_{hist}}\left(\boldsymbol{\varepsilon}_{exog,t+n}, \omega_{t-1}^{n_{hist}}\right) = \frac{1}{N_{shock}} \sum_{n_{shock}=1}^{N_{shock}} \mathbf{e}_{t+n}^{n_{hist},n_{shock}}\left(\boldsymbol{\varepsilon}_{exog,t+n}^{n_{shock}}, \omega_{t-1}^{n_{hist}}\right).$$

According to Koop et al. (1996), these averages will converge by the law of large numbers to the conditional expectations $\mathbb{E}\left[\mathbf{e}_{t+n} | \boldsymbol{\varepsilon}_{exog,t,s_{exog,t}} = \boldsymbol{\delta}_{exog}, \omega_{t-1}^{n_{hist}}\right]$ and $\mathbb{E}\left[\mathbf{e}_{t+n} | \omega_{t-1}^{n_{hist}}\right]$.

6. The Monte Carlo estimate of the history dependent GIRF is calculated by taking the difference:

$$\begin{aligned}\mathbf{GI}_{\mathbf{e}:\mathbf{e}_{k_{exog}}}\left(\boldsymbol{\varepsilon}_{exog,t+n}, \omega_{t-1}^{n_{hist}}\right) = &\ \tilde{\mathbf{e}}_{t+n}^{n_{hist},\boldsymbol{\delta}_{exog}}\left(\boldsymbol{\varepsilon}_{exog,t+n}^{\boldsymbol{\delta}_{exog}}, \omega_{t-1}^{n_{hist}}\right) \\ &- \tilde{\mathbf{e}}_{t+n}^{n_{hist}}\left(\boldsymbol{\varepsilon}_{exog,t+n}, \omega_{t-1}^{n_{hist}}\right).\end{aligned} \tag{B.12}$$

7. Repeat steps 2 to 6 N_{hist} times and take the average over $\mathbf{GI}_{\mathbf{e}:\mathbf{e}_{k_{exog}}}\left(\boldsymbol{\varepsilon}_{exog,t+n}, \omega_{t-1}^{n_{hist}}\right)$ to get the history independent estimate of the generalized impulse response function $\mathbf{GI}_{\mathbf{e}:\mathbf{e}_{k_{exog}}}(\boldsymbol{\varepsilon}_{exog,t+n}, \omega_{t-1})$. With an increasing number of repetitions the pointwise convergence will be guaranteed by the law of large numbers, according to Koop et al. (1996).

B.1.4 Impulse Response Analysis of Shocks to Exogenous, Global Variables - Bootstrapping

In order to analyze the significance of the generalized impulse response function analysis, we employ an adjusted version of the bootstrap techniques proposed in Ehrmann et al. (2001). Hereby, we create a history for the regimes as well as for the variables within the bootstrapping. As we apply the bootstrapping to generate confidence bounds for our GIRF analysis, the bootstrapping is also regime-dependent, just like the GIRF, which is why the bootstrapping is calculated for the predetermined regime-constellation S of the commodity-specific variables as well as the regime-constellation s_{exog} of the exogenous variables.

In the following, we describe the algorithm for the bootstrapping procedure:

1. Create a history for the regimes $s_{exogt}, t = 1, 2, \ldots, T$. As the regimes are not observable, but the smoothed probabilities represent their best estimate, we assume the history of regimes to correspond to the estimated smoothed probabilities.
2. Calculate the residuals $\hat{\boldsymbol{\varepsilon}}_{exog,t,s_{exog}}, t = 1, 2, \ldots, T$ of the fitted VAR model in Equation 3.153, with the estimated parameters for the prevailing regime-constellations.
3. Draw randomly with replacement a block of $T_{boot} \leq T$ consecutive residuals to get N_{boot} sets of residuals $\boldsymbol{\varepsilon}_{exog}^{n_{boot}} = \left(\boldsymbol{\varepsilon}_{exog,T-T_{boot}}^{n_{boot}}, \boldsymbol{\varepsilon}_{exog,T-T_{boot}+1}^{n_{boot}}, \ldots, \boldsymbol{\varepsilon}_{exog,T}^{n_{boot}}\right)$, $n_{boot} = 1, 2, \ldots, N_{boot}$.
4. Generate N_{boot} bootstrap samples $\mathbf{e}^{n_{boot}} = \left(\mathbf{e}_{T-T_{boot}}^{n_{boot}}, \mathbf{e}_{T-T_{boot}+1}^{n_{boot}}, \ldots, \mathbf{e}_{T}^{n_{boot}}\right)$, according to Equation 3.153, using the resampled, recentered[1] residuals $\boldsymbol{\varepsilon}_{exog}^{n_{boot}}$ as well as the estimated parameters of the fitted VAR model for the prevailing regime-constellations.
5. Estimate the MS-VAR model for a specific bootstrap sample, $\mathbf{e}^{n_{boot}}$ via the EM algorithm, described in Section 3.2.1.5.
6. Calculate the GIRFs $\mathbf{GI}_{\mathbf{e}:\mathbf{e}_{k_{exog}}}(n, \boldsymbol{\delta}_{exog}, \Omega_{t-1})$ for the predefined regime-constellation s_{exog} for the specific bootstrap sample, $\mathbf{e}^{n_{boot}}$, based on the new estimated parameters corresponding to the bootstrap sample and recursively derive the GIRFs of the commodity-specific variables $\mathbf{GI}_{\mathbf{x}:\mathbf{e}_{k_{exog}}}(n, \boldsymbol{\delta}_{exog}, \Omega_{t-1})$, using Equation 3.155, and Equation 3.157.
7. Repeat steps 4 to 6 N_{boot} times.
8. Sort the GIRFs into an ascending order for all time periods $n = 0, 1, \ldots, N_{IRF}$, and calculate the 68% confidence interval by using the 0.16 and 0.84 quantiles of the bootstrap distribution of the GIRFs.

In line with the impulse response analysis of shocks to the endogenous variables, we apply the bootstrap technique for $N_{boot} = 500$ runs, which is sufficiently large to be a good numerical approximation of the distribution of the underlying estimates.

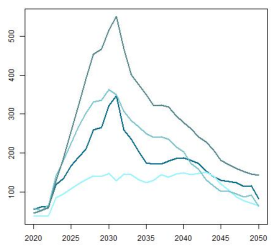
[1]In line with the bootstrapping procedure for the GVAR model, described in Section 3.1.2.1, we follow Dées, di Mauro, Pesaran, and Smith (2007) and recenter the residuals to ensure the bootstrap population mean is zero.

C Data

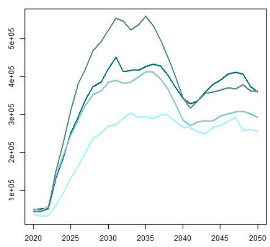
In this section, we provide additional information about the data used in this study. First, we graphically consider the required resource amounts per transformation pathway of the German Energiewende over time. Subsequently, we report the main uses and the largest mining countries of these resources in Table C.1. Further, we present time series plots of the level and logarithmic return data as well as the histogram of the logarithmic returns for all commodity-specific attributes as well as their determinants. In particular, the commodity-specific demand variable represents the approximated global commodity-specific demand, which we estimate by adjusting the U.S. apparent consumption, provided by U.S. Geological Survey (2018), by the ratio of U.S. gross domestic product (U.S. GDP) and world gross domestic product (GDP), drawn from U.S. Bureau of Economic Analysis (2022) and The World Bank (2022a).

C.1 Path-specific Commodity Requirements

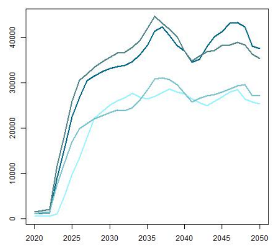
Figure C.1: Required amount per commodity for the considered transformation pathways



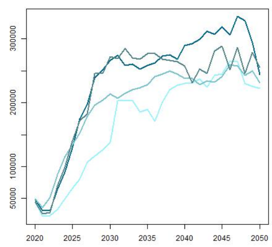
(a) Ag



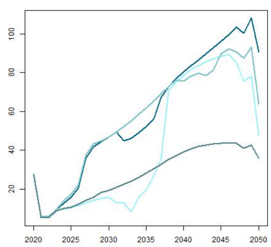
(b) Al



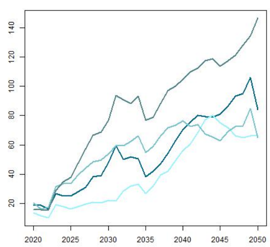
(c) Co



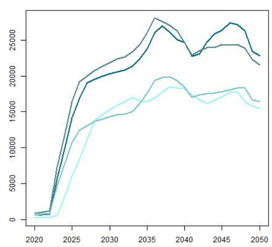
(d) Cu



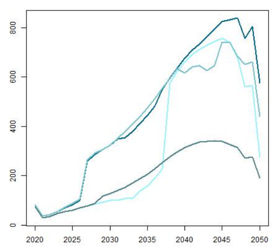
(e) Dy



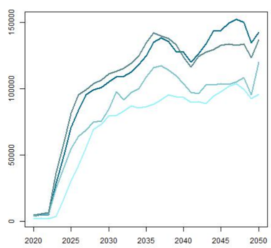
(f) In



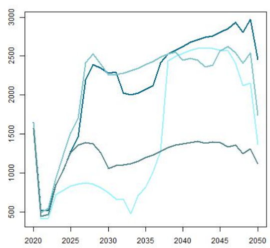
(g) Li



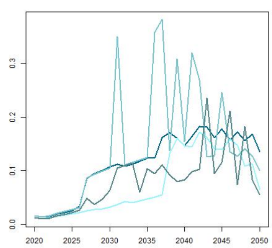
(h) Nd



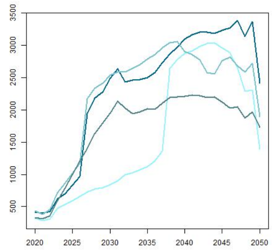
(i) Ni



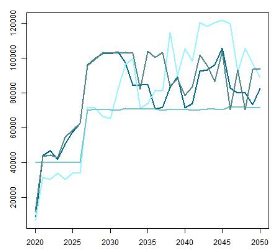
(j) Pb



(k) Pt



(l) Sn



(m) Zn

These figures display the required amount in metric ton per commodity silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), in the period from 2020 to 2050, for the four considered transformation pathways $REMod - REF$, $REMod - SUF$, $REMod - PER$, and $REMod - UNA$.

C.2 Commodity Markets

C.2.1 Main Uses of the Commodities

Table C.1: Main uses of the metals

	Main Use	Largest mining countries
Ag	Electrical and electronics (30%), jewelry and silverware (26%), coins and medals (12%), photography (3%)	Mexico (23%), Peru (14%), China (13%)
Al	Transportation applications (39%), packaging (19%), building (14%), electrical (9%), consumer durables (8%), machinery (8%)	China (56%), India (6%), Russia (6%), Canada (5%)
Co	Superalloys mainly in aircraft gas turbine engines (46%), cemented carbides for cutting and wear-resistant applications (9%), metallic applications (14%), chemical applications (31%)	Congo (71%), Russia (4%), Australia (4%)
Cu	Building construction (43%), electrical and electronic products (20%), transportation equipment (20%), consumer and general products (10%), industrial machinery and equipment (7%)	Chile (28%), Peru (12%), China (8%), United States (7%), Congo (Kinshasa) (7%)
In	Electrically conductive purposes (e.g. flat screens), alloys and solders, compounds, electrical components and semiconductors	China (39%), Republic of Korea (31%)
Li	Batteries (65%), ceramics and glass (18%), lubricating greases (5%), polymer production (3%), continuous casting mold flux powders (3%), air treatment (1%)	Australia (55%), Chile (23%), China (10%), Argentina (18%)
Ni	Stainless and alloy steels, nonferrous alloys and superalloys, electroplating, catalysts and chemicals	Indonesia (30%), Philippines (16%), Russia (10%), New Caledonia (8%)
Pb	Lead-acid battery industry (93%) Ammunition, building-construction materials, lead-acid storage batteries, lead oxides for ceramics, chemicals, glass, and pigments, lead sheet	China (47%), Australia (10%), United States (6%), Peru (6%)
Pt	Catalytic converters, chemical and petroleum refining, dental and medical devices, electronics, jewelry	South Africa (72%), Russia (12%), Zimbabwe (8%)
Rare Earths [1]	Catalysts (75%), metallurgical applications and alloys (5%), ceramics and glass (5%), polishing (5%)	China (63%), United States (12%), Burma (10%) Australia (10%)
Sn	Tinplate (21%), chemicals (17%), solder (14%), alloys (10%), babbitt, brass and bronze, and tinning, (11%)	China (27%), Indonesia (26%), Burma (17%)
Zn	Galvanizing, brass and bronze, zinc-based alloys	China(33%), Peru (12%), Australia (9%), United States (7%), India (6%)

This table displays the main uses as well as the largest producing countries for the considered commodities, silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), the rare earth metals, tin (Sn), and zinc (Zn), based on data of 2019 provided by U.S. Geological Survey (2020d).

[1] U.S. Geological Survey (2020d) only considers rare earth metals aggregated, including dysprosium and neodymium.

C.2.2 Descriptive Statistics of the Level Data

Table C.2: Descriptive statistics of the level data of the commodity-specific variables

			Min.	5% Q.	25% Q.	Med.	Mean	75% Q.	95% Q.	Max.	SD	Skew.	Kurt.	ADF	SW
Ag	HHI	a	794	829	896	936	968	982	1275	1394	123	1.72	2.94	0.47	0.82***
	supply	a	9170	9369	11650	16000	16783	20800	26665	26900	5666	0.34	-1.16	3.60	0.93**
	demand	a	11024	12103	16149	18684	20658	24168	33393	37399	6591	0.75	-0.32	0.33	0.93**
	price$^{\times}$	a	50	68	152	188	308	462	720	1135	240	1.50	1.98	-0.65	0.82***
Al	HHI	a	747	800	890	1173	1426	1515	3116	3502	770	1.47	0.96	2.47	0.76***
	supply$^{\times}$	a	9650	11495	15175	19750	26943	36375	59175	63600	15947	1.01	-0.32	6.58	0.84***
	supply$^{\times}$	m	1517	1704	2066	3133	3330	4557	5390	5722	1295	0.31	-1.37	1.74	0.90***
	demand$^{\times}$	a	9339	12644	16498	19086	18939	21681	24237	26893	3813	-0.29	-0.37	0.66	0.99
	demand$^{\times}$	m	1561	1730	2052	2999	3272	4536	5328	5611	1270	0.35	-1.37	0.99	0.90***
	price	a	594	615	1263	1509	1533	1837	2555	2640	526	0.18	-0.42	-0.31	0.97
	price	m	1168	1320	1525	1757	1837	2043	2703	3070	413	0.90	0.23	-0.58	0.93***
Co	HHI	a	1415	1500	1771	2703	2743	3351	4548	5039	997	0.46	-0.67	-0.07	0.94**
	supply$^{\times}$	a	38	41	52	69	90	120	197	223	52	1.11	0.08	-0.00	0.84***
	demand$^{\times}$	a	17	20	28	33	33	39	45	68	9	0.81	2.13	0.43	0.94*
	price	a	4850	5987	16595	30578	33183	47890	70119	86002	20049	0.56	-0.44	-1.35	0.96.
Cu	HHI	a	947	1007	1302	1423	1386	1542	1616	1833	208	-0.40	-0.38	0.43	0.95
	supply$^{\times}$	a	5900	6630	7630	9750	11695	15400	20055	20400	4530	0.50	-1.14	6.30	0.90***
	supply$^{\times}$	m	749	894	1111	1293	1317	1551	1752	1846	270	0.11	-0.98	0.50	0.97***
	demand$^{\times}$	a	5028	5657	7287	8136	8099	8968	10538	11061	1443	0.01	-0.41	0.31	0.98
	demand$^{\times}$	m	914	1001	1232	1483	1517	1845	2053	2313	343	0.17	-1.10	0.11	0.96***
	price	a	1073	1277	1577	2240	3243	5102	7454	8820	2297	0.99	-0.59	0.03	0.79***
	price	m	1351	1550	2097	5097	4784	6917	8443	9880	2485	0.05	-1.41	0.03	0.91***
Dy[2]	**supply**	a	107	127	207	435	503	706	917	1482	322	0.77	0.13	2.83	0.91***
	demand	a	128	161	241	305	349	435	655	735	150	0.68	-0.31	-0.82	0.94**
	price$^{\times}$	a	249	249	258	318	396	414	751	861	212	1.23	-0.00	-4.35**	0.75**
In	HHI	a	1410	1621	2316	3003	2792	3279	3947	4190	776	-0.29	-0.97	0.09	0.93.
	supply	a	23	33	53	208	326	645	807	968	299	0.60	-1.24	2.26	0.84***
	demand	a	12	31	63	163	242	495	580	651	210	0.62	-1.30	0.94	0.83***
	price$^{\times}$	a	57	80	142	238	310	422	704	961	218	1.01	0.28	-1.24	0.89***
Li	HHI	a	1813	1937	2309	3196	3419	4294	5561	5790	1214	0.39	-1.13	0.37	0.93**
	supply$^{\times}$	a	20	74	109	175	335	375	1357	1929	443	2.75	7.10	1.63	0.58***
	demand	a	3488	4795	8250	9291	9126	10445	12508	14229	2223	-0.40	0.21	-0.52	0.97
	price	a	18	18	46	68	64	79	102	131	27	-0.03	-0.49	0.83	0.96.
Nd	**supply**	a	2628	3119	5090	10659	12340	17311	22492	36358	7901	0.77	0.13	2.83	0.91***
	demand	a	3139	3960	5917	7493	8564	10662	16059	18020	3680	0.68	-0.31	-0.82	0.94**
	price	a	53555	56139	61987	69904	69585	71373	87961	95179	12335	0.78	-0.28	-1.53	0.90
Ni	HHI	a	903	974	1199	1299	1366	1475	2009	2606	346	1.64	3.22	-1.92*	0.84***
	supply$^{\times}$	a	6110	632	795	1025	1269	1543	2494	2790	616	0.98	-0.26	1.79	0.86***
	supply$^{\times}$	m	70	83	103	120	137	175	223	240	45	0.67	-0.81	0.08	0.90***
	demand$^{\times}$	a	329	478	679	760	767	861	1081	1181	178	-0.00	0.15	0.65	0.98
	demand$^{\times}$	m	72	77	94	112	124	146	208	254	40	0.98	0.26	-0.04	0.91***
	price	a	2844	3215	4861	6967	9754	13663	22414	37149	6856	1.69	3.35	-0.96	0.82***
	price	m	3873	5321	7979	12309	13765	16585	29520	49825	7816	1.68	3.85	-0.77	0.86***
Pb	HHI	a	912	929	1211	1823	1846	2478	2931	3310	731	0.29	-1.28	0.30	0.93*
	supply$^{\times}$	a	2710	2873	3200	3435	3627	3705	5022	5440	677	1.24	0.57	0.85	0.84***
	supply$^{\times}$	m	209	229	253	318	323	386	441	506	74	0.29	-1.23	-0.14	0.93***
	demand$^{\times}$	a	3336	3490	4267	5057	5252	6302	7296	8983	1292	0.44	-0.35	1.24	0.96.
	demand$^{\times}$	m	407	457	544	740	738	937	1031	1119	202	0.07	-1.45	0.41	0.92***
	price	a	306	352	545	811	1065	1614	2280	2586	672	0.86	-0.76	0.01	0.84***
	price	m	406	460	613	1683	1446	2091	2583	3683	788	0.15	-1.20	-0.39	0.90***
Pt	HHI	a	4092	4262	4525	5293	5335	6038	6497	6873	793	0.04	-1.29	0.69	0.94**
	supply	a	127	147	214	308	323	421	498	515	116	0.04	-1.34	0.87	0.94*
	demand	a	215	272	418	570	602	696	1065	1301	247	0.80	0.27	-0.61	0.95*
	price$^{\times}$	a	3890	4522	11665	15061	20702	28693	50203	55273	14441	0.91	-0.33	-0.14	0.88***
Sn	HHI	a	1079	1118	1345	1696	1796	2260	2692	3054	545	0.45	-1.04	-0.06	0.93***
	supply$^{\times}$	a	173	180	219	238	241	264	300	318	38	0.16	-0.78	0.28	0.97
	supply	m	15395	17034	19860	25012	24123	27904	30142	37033	4444	-0.08	-0.92	-0.17	0.96***
	demand$^{\times}$	a	138	162	171	183	192	208	247	286	30	0.94	0.71	-0.25	0.93***
	demand	m	18036	19778	22673	28451	27333	30898	33667	38663	4881	-0.26	-0.97	-0.48	0.94***
	price	a	3498	3883	5605	8305	10780	15047	21528	26006	6170	0.69	-0.83	0.03	0.89***
	price	m	3727	4317	5786	13944	13135	19633	23500	32295	7258	0.28	-1.19	0.06	0.90***

[2] Due to data limitations, we exclude the HHI from the analysis of the rare earth metals, dysprosium and neodymium.

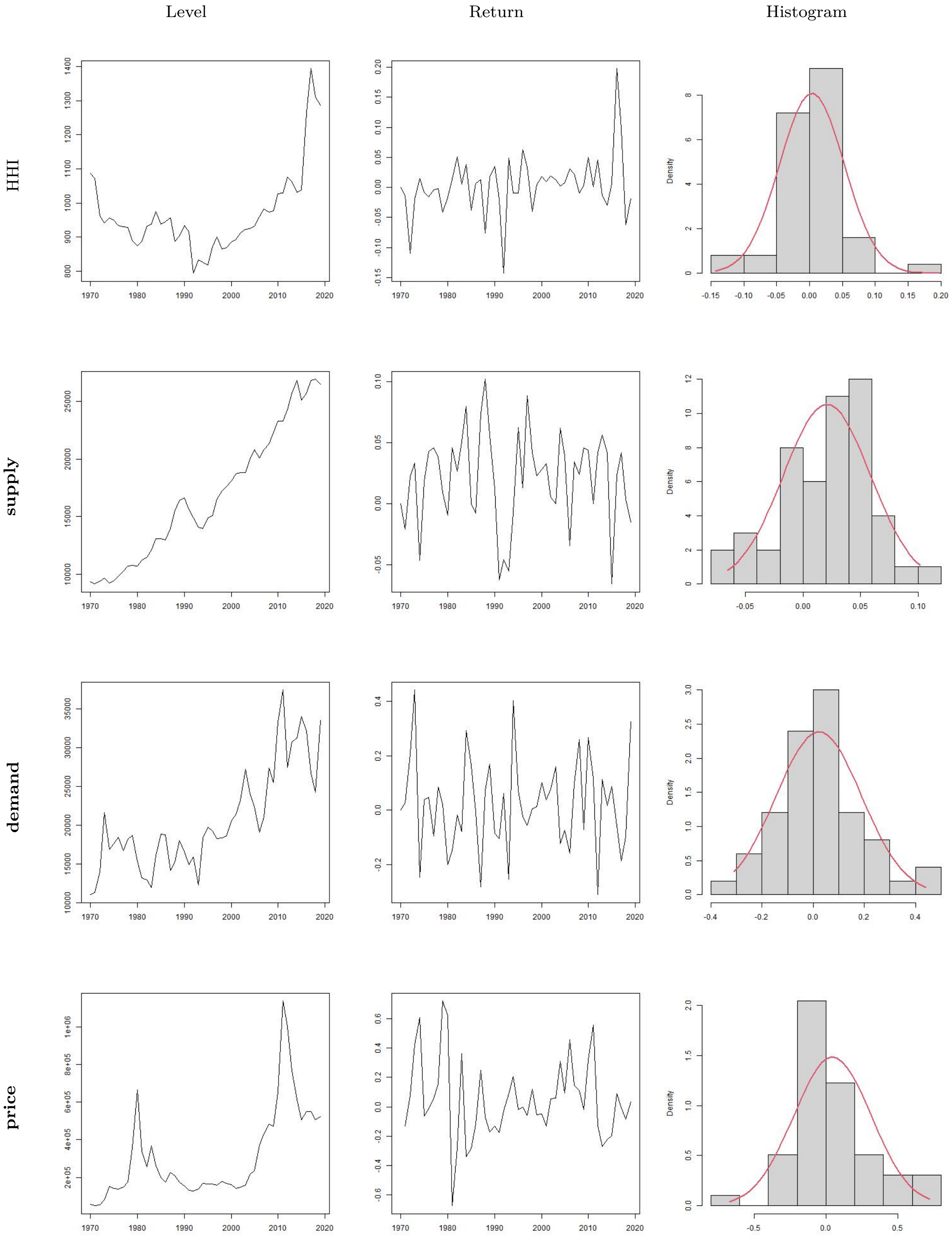
Descriptive statistics of the level data of the commodity-specific variables

			Min.	5% Q.	25% Q.	Med.	Mean	75% Q.	95% Q.	Max.	SD	Skew.	Kurt.	ADF	SW
Zn	HHI	a	794	805	961	1180	1242	1550	1760	1920	344	0.39	-1.23	1.30	0.92*
	supply$^{\times}$	a	5440	5597	6168	7275	8473	10900	12855	13300	2663	0.60	-1.23	3.59	0.85***
	supply$^{\times}$	m	562	598	740	937	889	1037	1154	1270	186	-0.13	-1.25	-0.02	0.94***
	demand$^{\times}$	a	2741	2956	3512	3825	3877	4364	4786	5112	591	-0.04	-0.84	0.31	0.98
	demand$^{\times}$	m	577	608	752	934	922	1093	1202	1262	197	-0.12	-1.26	0.12	0.94***
	price	a	297	467	792	1040	1317	1838	2907	3266	750	1.03	0.22	-0.24	0.88***
	price	m	742	797	1063	1828	1805	2340	3323	4442	807	0.66	-0.22	-0.19	0.93***

This table displays the descriptive statistics (minimum (Min.), 5% quantile (5% Q.), 25% quantile (25% Q.), 75% quantile (75% Q.), 95% quantile (95% Q.), median (Med.), mean (Mean), maximum (Max.), standard deviation (SD), skewness (Skew.) and excess kurtosis (Kurt.)) of the level data of the commodity-specific variables Herfindal-Hirschman index (HHI), supply (**supply**), demand (**demand**) and price (**price**) for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), as well as the results of the test statistics of the augmented Dickey-Fuller (ADF) test and the Shapiro-Wilk (SW) test with corresponding significance level (0.1% (***), 1% (**), 5% (*) and 10% (.)). Hereby, we report the descriptive statistics for the considered time period from 1970 to 2019 for the annual (a) analysis or from 1995 to 2020 in case of the monthly (m) analysis. Moreover, all supply and demand data are displayed in metric tons (or thousand metric tons indicated by $^{\times}$), and all prices are in U.S. dollar per metric ton (or U.S. dollar per thousand metric tons indicated by $^{\times}$).

C.2.3 Development of the commodity-specific Variables over Time[3]

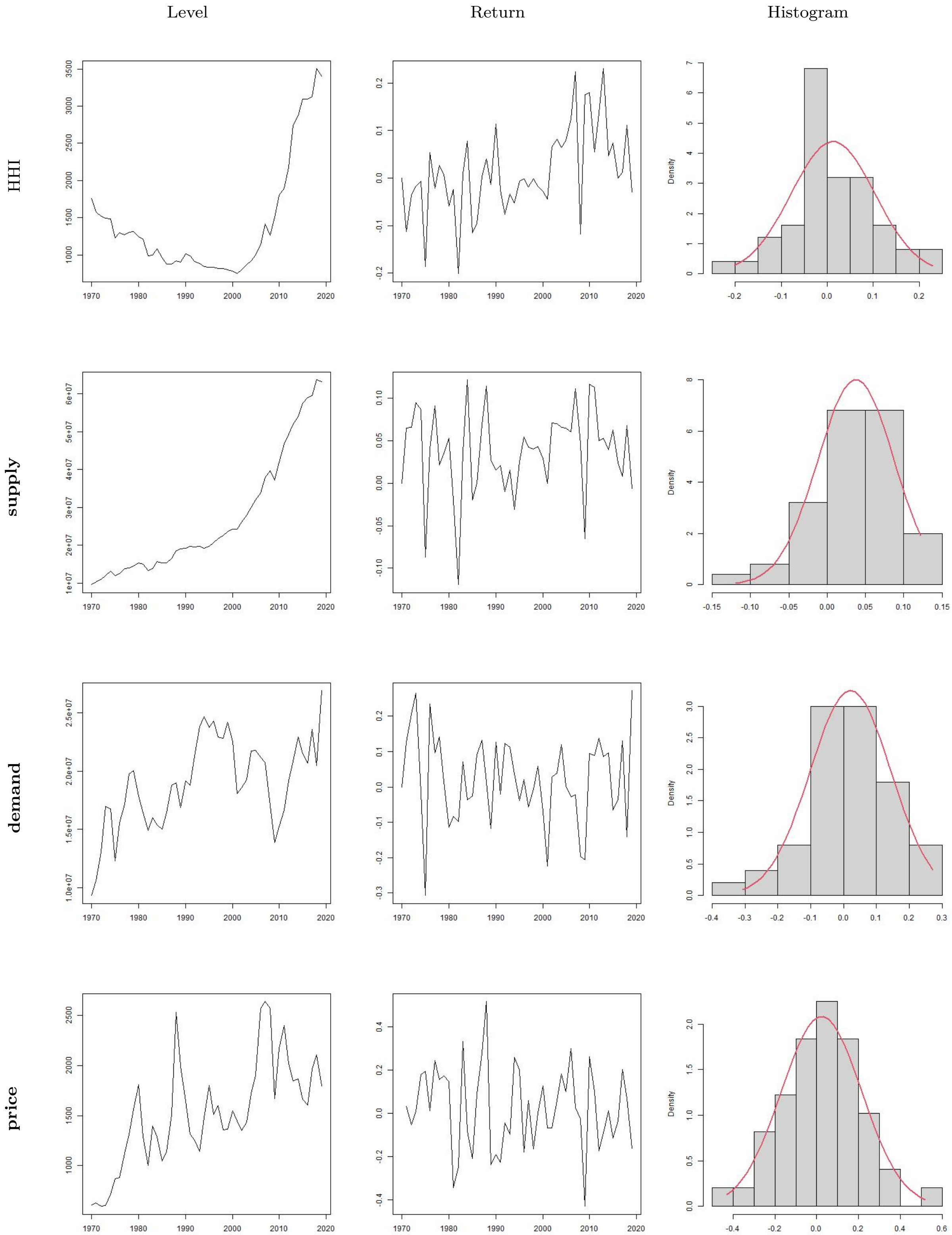
Figure C.2: Time series plots for silver (Ag)



These figures display the histogram of the logarithmic returns and the time series plots of the level as well as of the logarithmic return data of the commodity-specific attributes Herfindal-Hirschman index (HHI), supply (**supply**) in metric tons, demand (**demand**) in metric tons and price (**price**) in U.S. dollar per metric ton of silver (Ag).

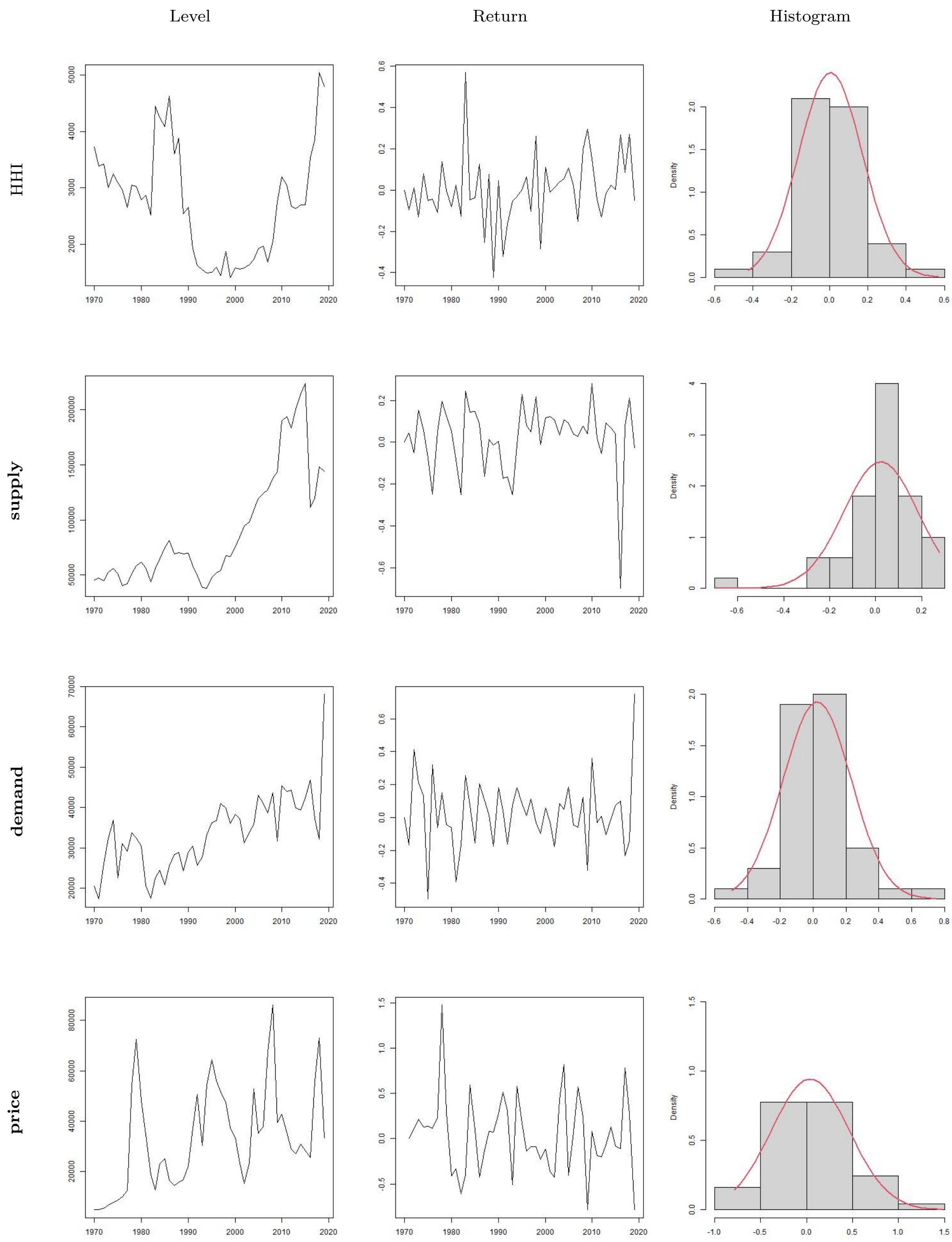
[3] In this section, we only plot the annual prices over the considered sample period from 1970 to 2019.

Figure C.3: Time series plots for aluminum (Al)



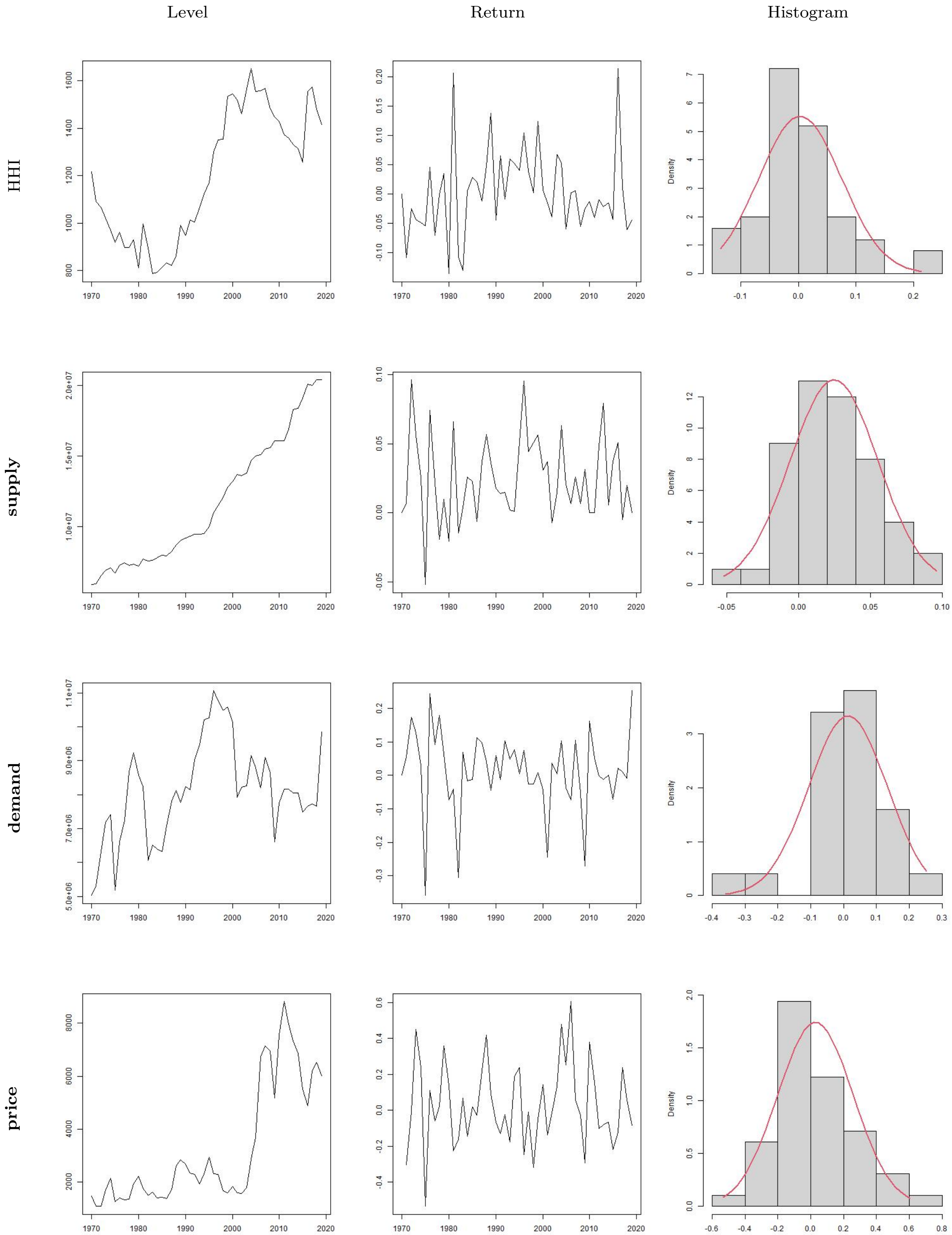
These figures display the histogram of the logarithmic returns and the time series plots of the level as well as of the logarithmic return data of the commodity-specific attributes Herfindal-Hirschman index (HHI), supply (**supply**) in metric tons, demand (**demand**) in metric tons and price (**price**) in U.S. dollar per metric ton of aluminum (Al).

Figure C.4: Time series plots for cobalt (Co)



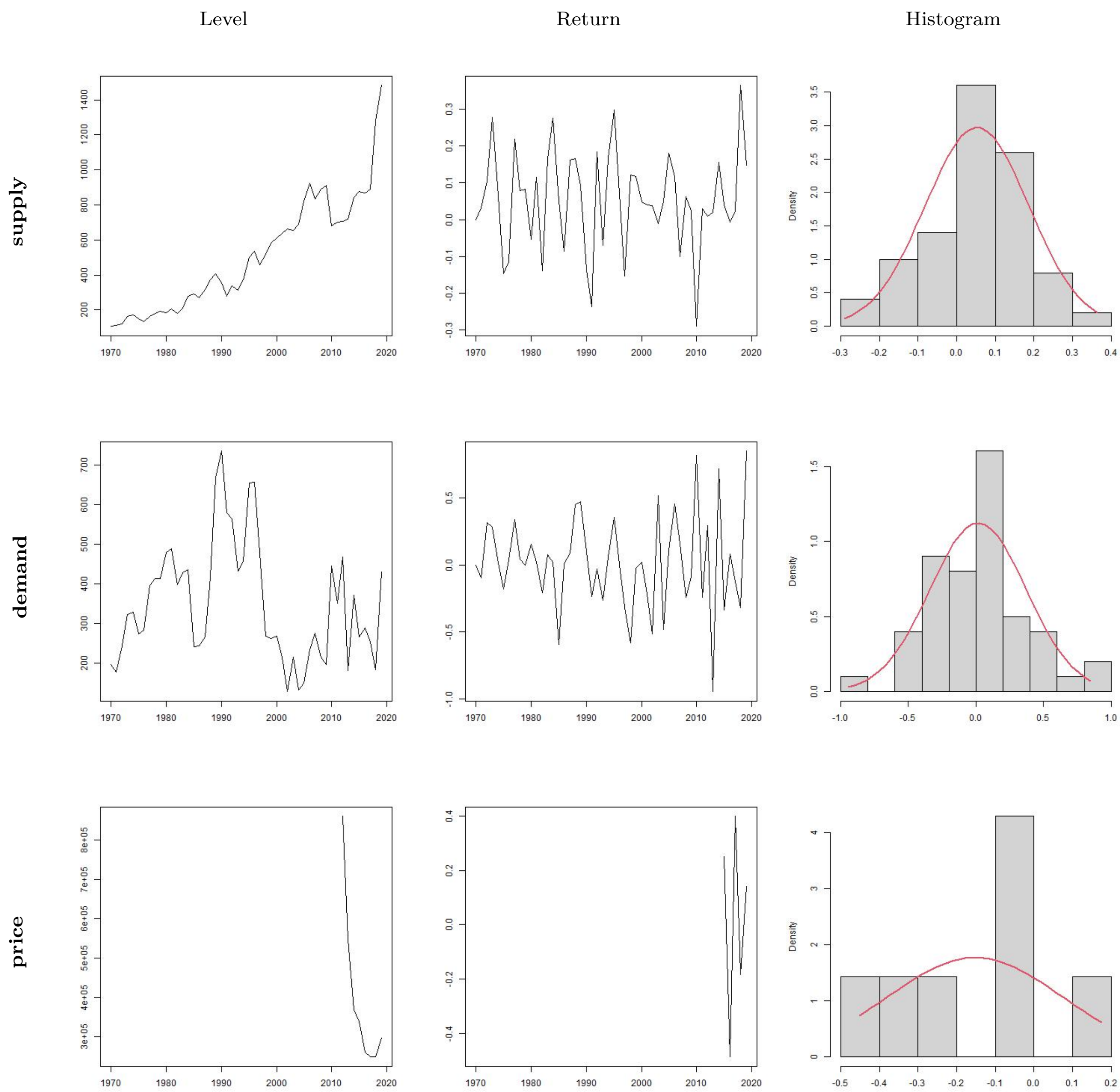
These figures display the histogram of the logarithmic returns and the time series plots of the level as well as of the logarithmic return data of the commodity-specific attributes Herfindal-Hirschman index (HHI), supply (**supply**) in metric tons, demand (**demand**) in metric tons and price (**price**) in U.S. dollar per metric ton of cobalt (Co).

Figure C.5: Time series plots for copper (Cu)



These figures display the histogram of the logarithmic returns and the time series plots of the level as well as of the logarithmic return data of the commodity-specific attributes Herfindal-Hirschman index (HHI), supply (**supply**) in metric tons, demand (**demand**) in metric tons and price (**price**) in U.S. dollar per metric ton of copper (Cu).

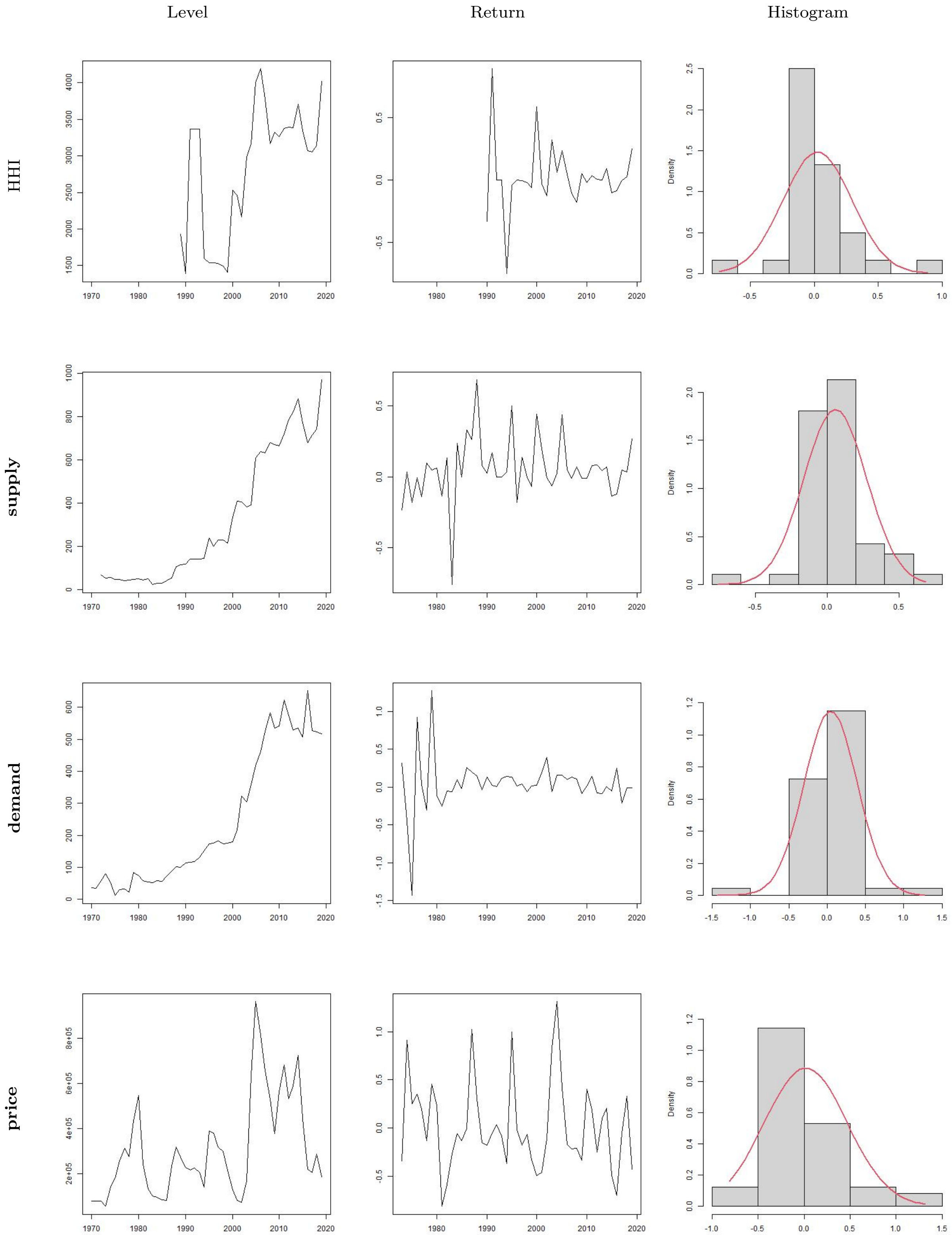
Figure C.6: Time series plots for dysprosium (Dy)



These figures display the histogram of the logarithmic returns and the time series plots of the level as well as of the logarithmic return data of the commodity-specific attributes Herfindal-Hirschman index (HHI), supply (**supply**) in metric tons, demand (**demand**) in metric tons and price (**price**) in U.S. dollar per metric ton of dysprosium (Dy).

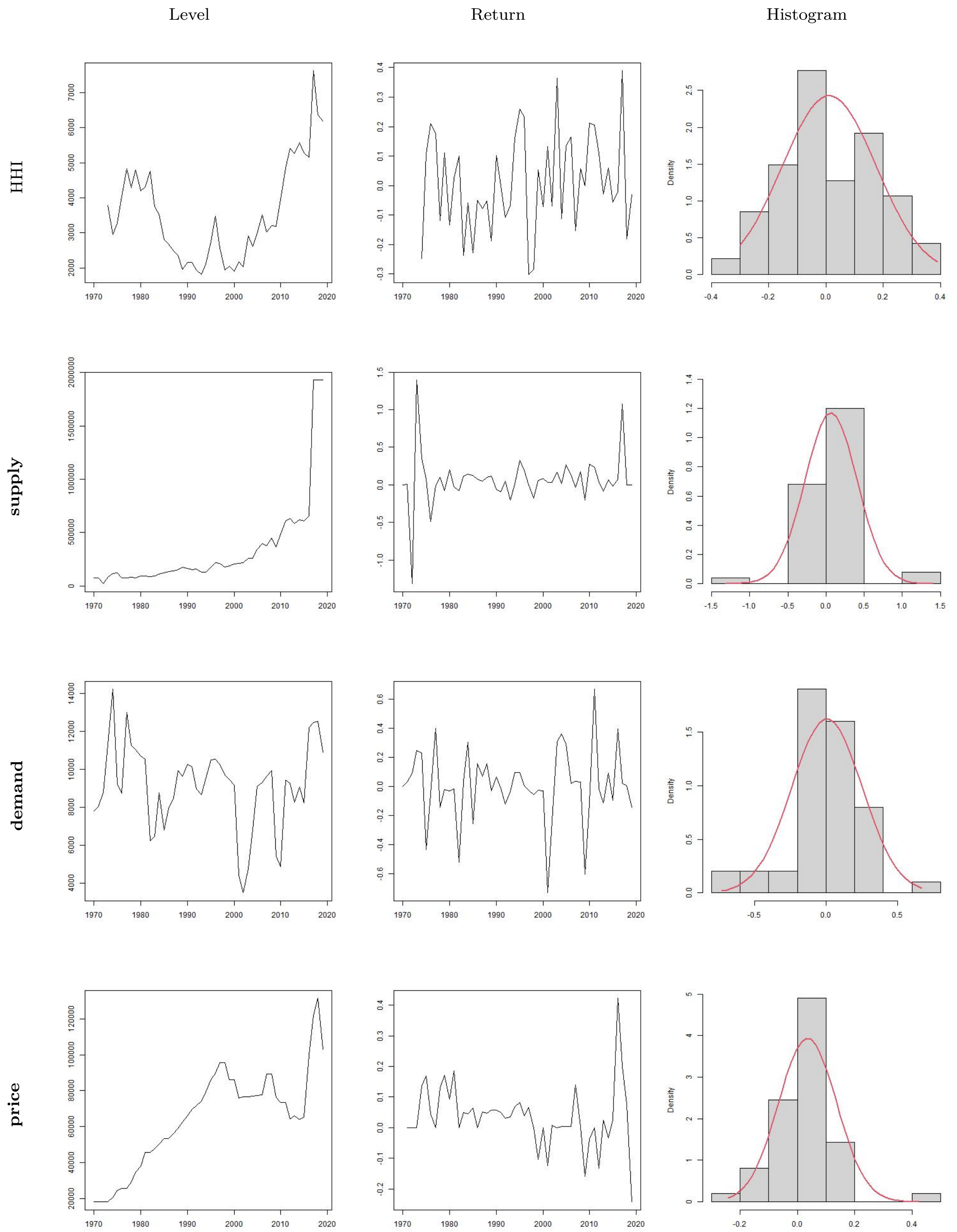
Due to data limitations, we exclude the HHI from the analysis of the rare earth metals, dysprosium and neodymium. In addition, the price is only available since 2012.

Figure C.7: Time series plots for indium (In)



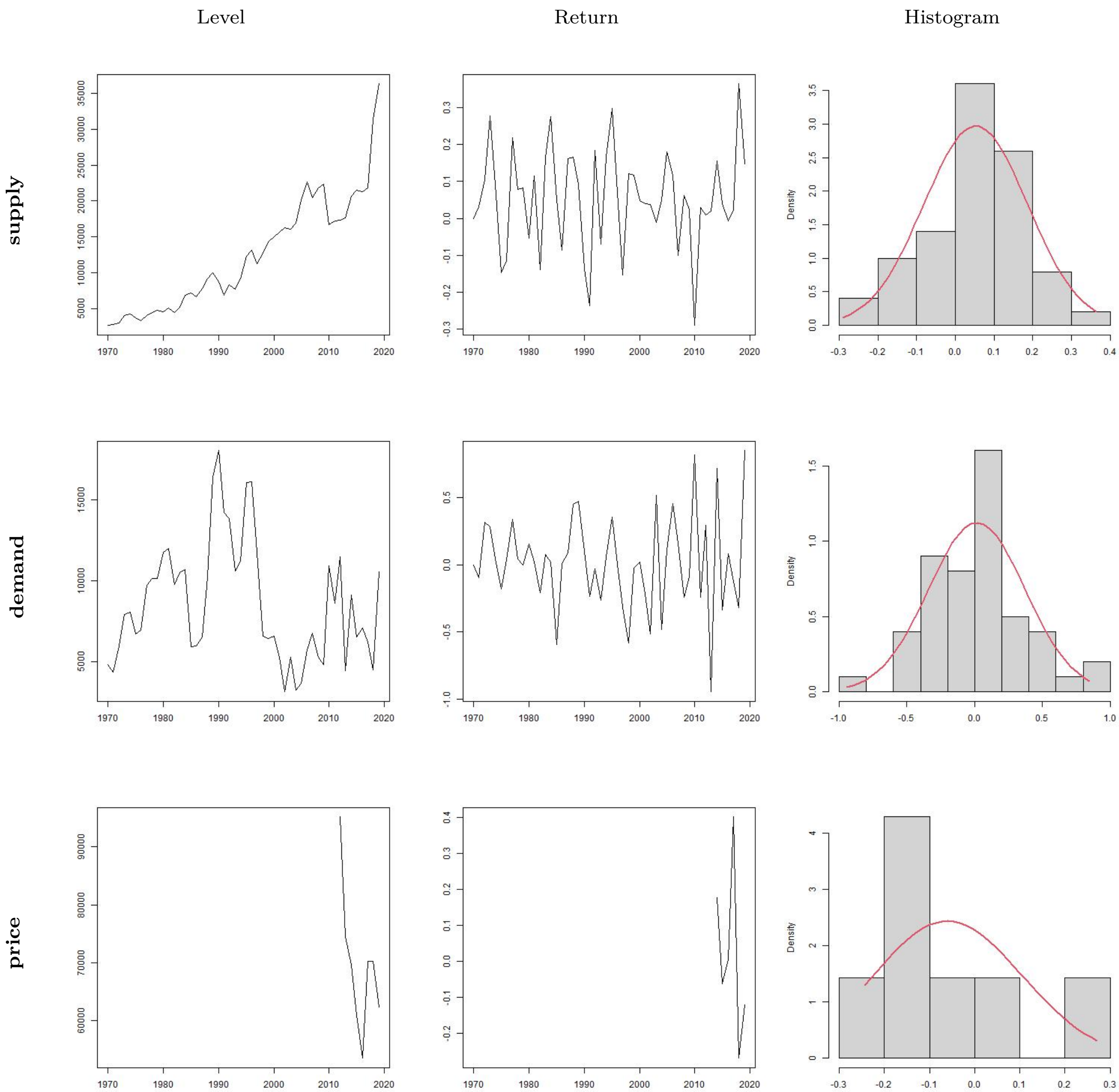
These figures display the histogram of the logarithmic returns and the time series plots of the level as well as of the logarithmic return data of the commodity-specific attributes Herfindal-Hirschman index (HHI), supply (**supply**) in metric tons, demand (**demand**) in metric tons and price (**price**) in U.S. dollar per metric ton of indium (In).

Figure C.8: Time series plots for lithium (Li)



These figures display the histogram of the logarithmic returns and the time series plots of the level as well as of the logarithmic return data of the commodity-specific attributes Herfindal-Hirschman index (HHI), supply (**supply**) in metric tons, demand (**demand**) in metric tons and price (**price**) in U.S. dollar per metric ton of lithium (Li).

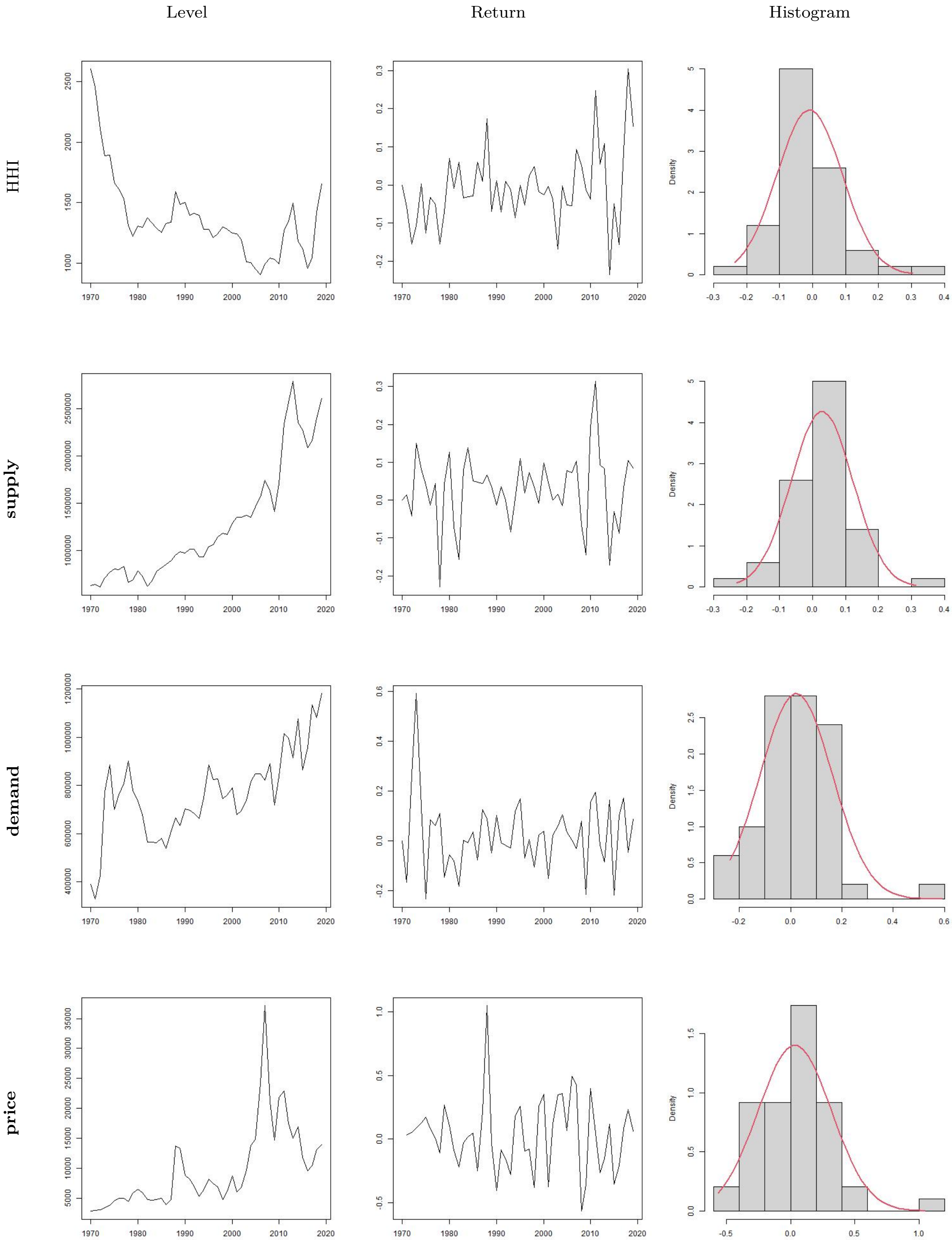
Figure C.9: Time series plots for neodymium (Nd)



These figures display the histogram of the logarithmic returns and the time series plots of the level as well as of the logarithmic return data of the commodity-specific attributes Herfindal-Hirschman index (HHI), supply (**supply**) in metric tons, demand (**demand**) in metric tons and price (**price**) in U.S. dollar per metric ton of neodymium (Nd).

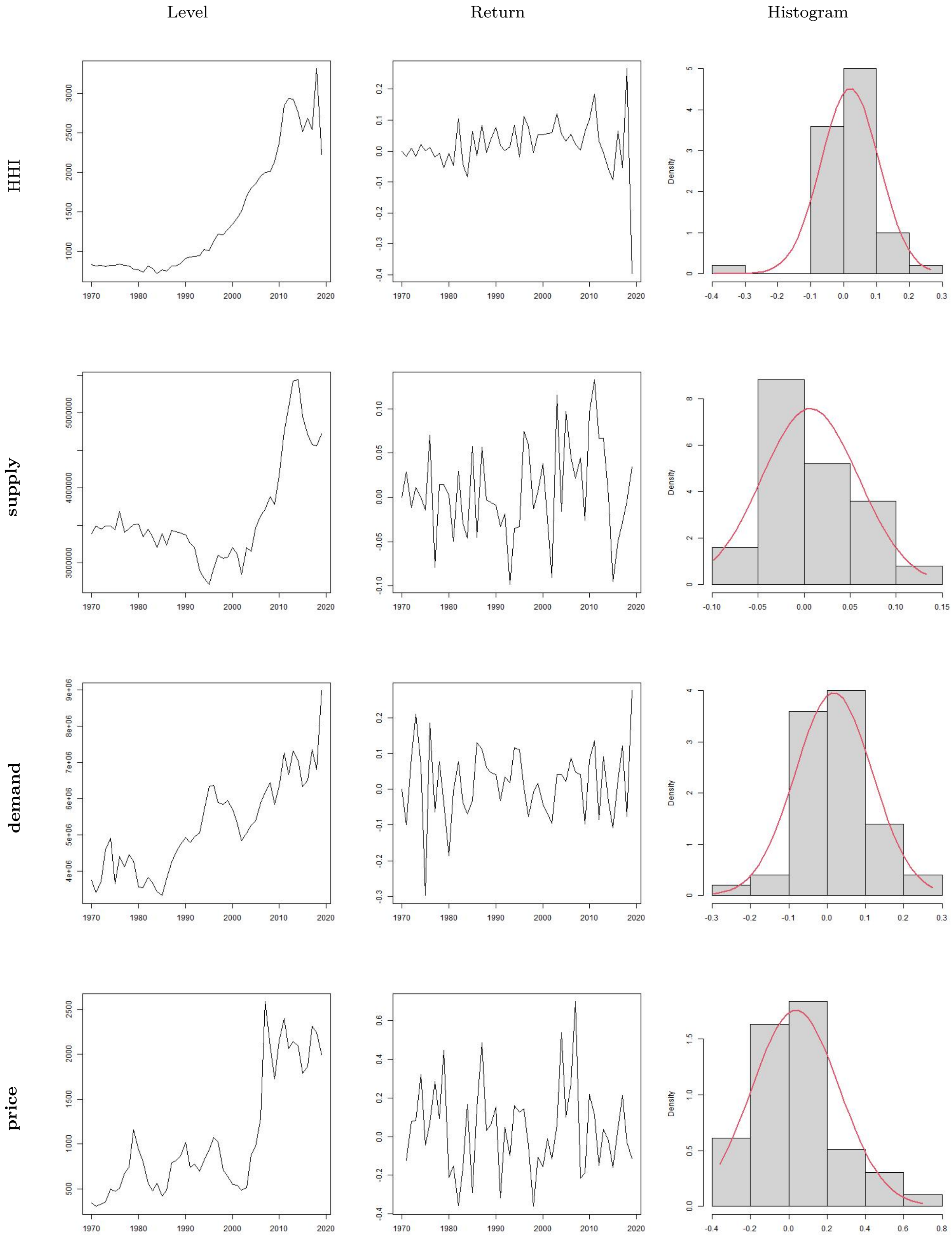
Due to data limitations, we exclude the HHI from the analysis of the rare earth metals, dysprosium and neodymium. In addition, the price is only available since 2012.

Figure C.10: Time series plots for nickel (Ni)



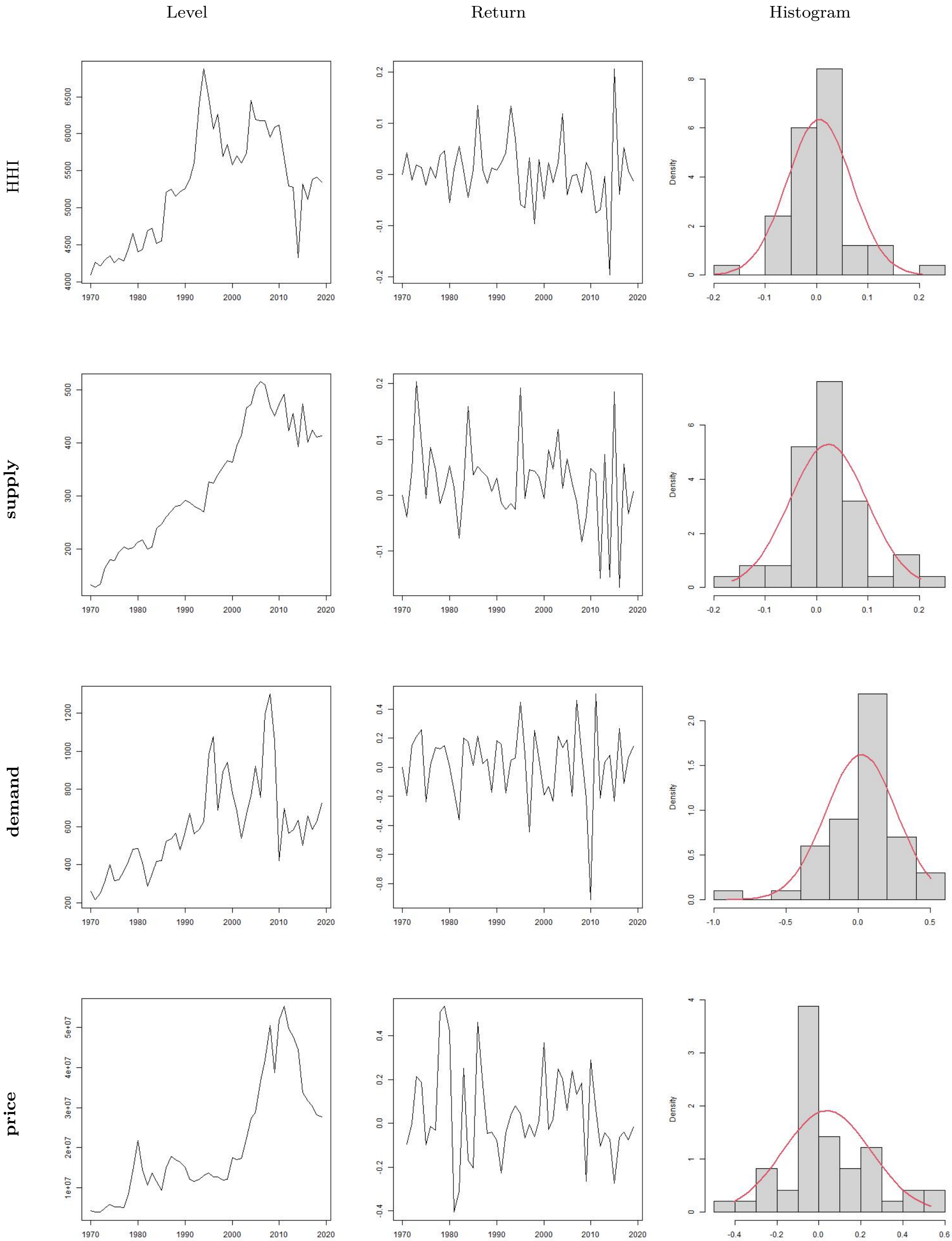
These figures display the histogram of the logarithmic returns and the time series plots of the level as well as of the logarithmic return data of the commodity-specific attributes Herfindal-Hirschman index (HHI), supply (**supply**) in metric tons, demand (**demand**) in metric tons and price (**price**) in U.S. dollar per metric ton of nickel (Ni).

Figure C.11: Time series plots for lead (Pb)



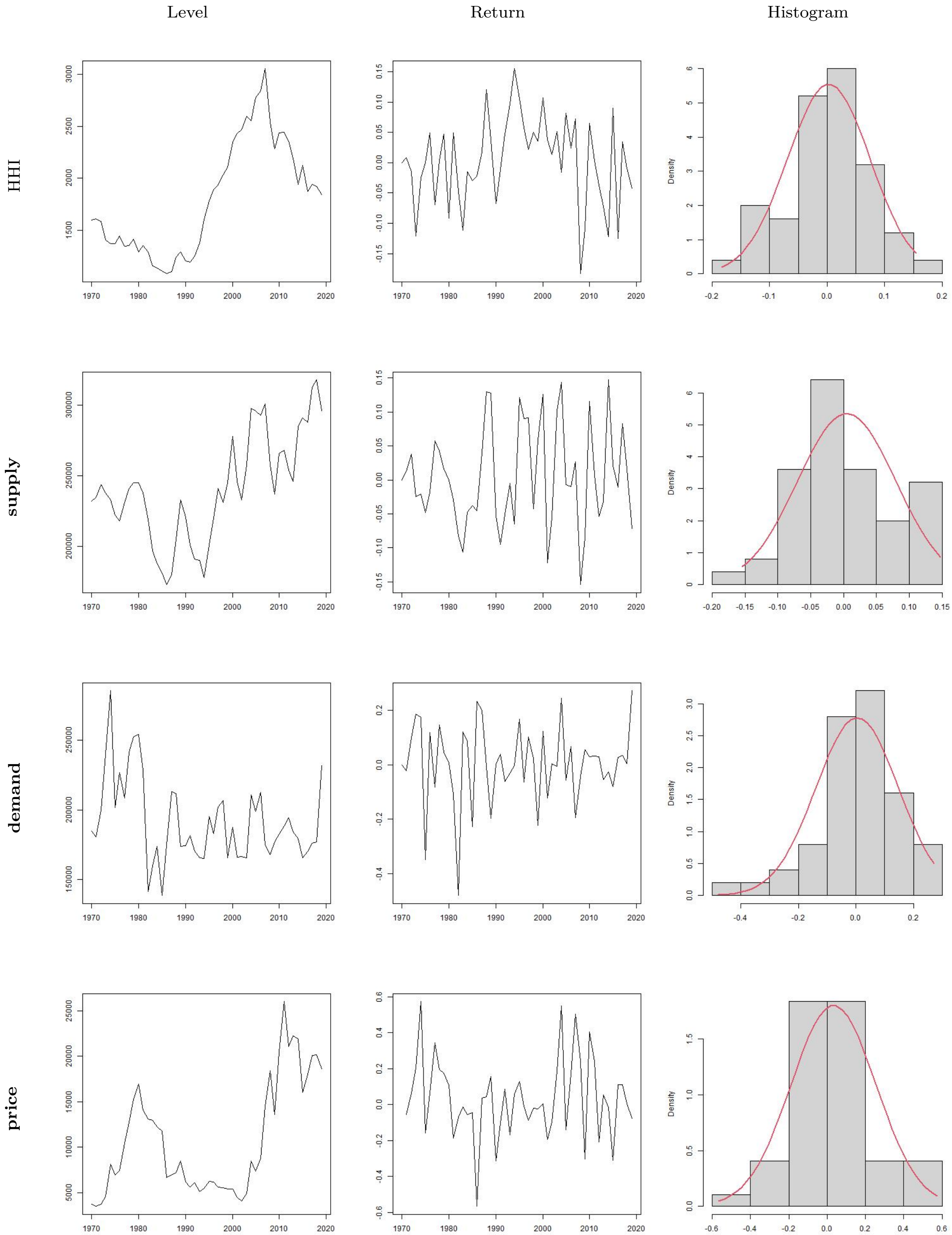
These figures display the histogram of the logarithmic returns and the time series plots of the level as well as of the logarithmic return data of the commodity-specific attributes Herfindal-Hirschman index (HHI), supply (**supply**) in metric tons, demand (**demand**) in metric tons and price (**price**) in U.S. dollar per metric ton of lead (Pb).

Figure C.12: Time series plots for platinum (Pt)



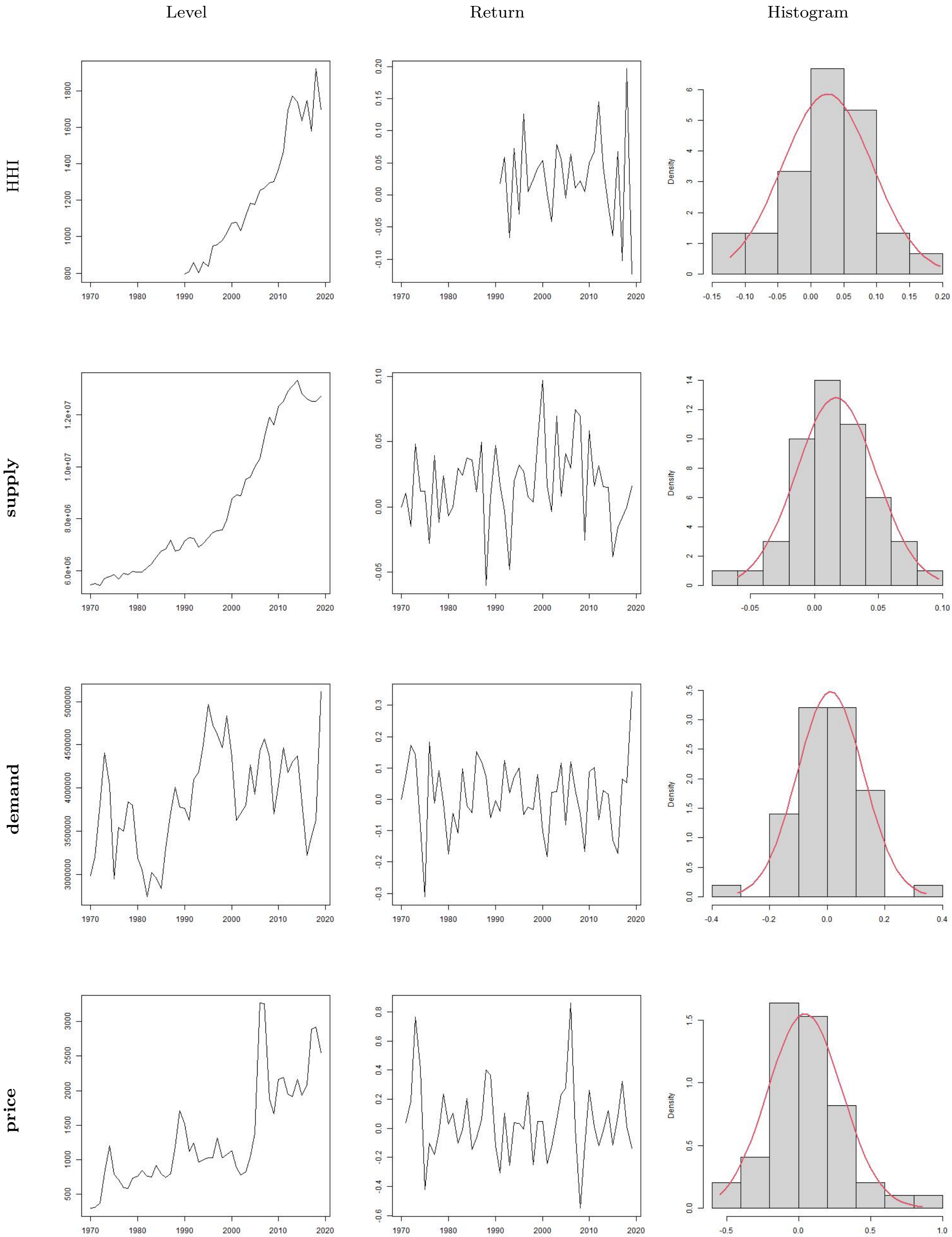
These figures display the histogram of the logarithmic returns and the time series plots of the level as well as of the logarithmic return data of the commodity-specific attributes Herfindal-Hirschman index (HHI), supply (**supply**) in metric tons, demand (**demand**) in metric tons and price (**price**) in U.S. dollar per metric ton of platinum (Pt).

Figure C.13: Time series plots for tin (Sn)



These figures display the histogram of the logarithmic returns and the time series plots of the level as well as of the logarithmic return data of the commodity-specific attributes Herfindal-Hirschman index (HHI), supply (**supply**) in metric tons, demand (**demand**) in metric tons and price (**price**) in U.S. dollar per metric ton of tin (Sn).

Figure C.14: Time series plots for zinc (Zn)



These figures display the histogram of the logarithmic returns and the time series plots of the level as well as of the logarithmic return data of the commodity-specific attributes Herfindal-Hirschman index (HHI), supply (**supply**) in metric tons, demand (**demand**) in metric tons and price (**price**) in U.S. dollar per metric ton of zinc (Zn).

C.3 Determinants of Commodity Prices

C.3.1 Descriptive Statistics of the Level Data of the Determinants of Commodity Prices

Table C.3: Descriptive statistics of the level data of the price determinants

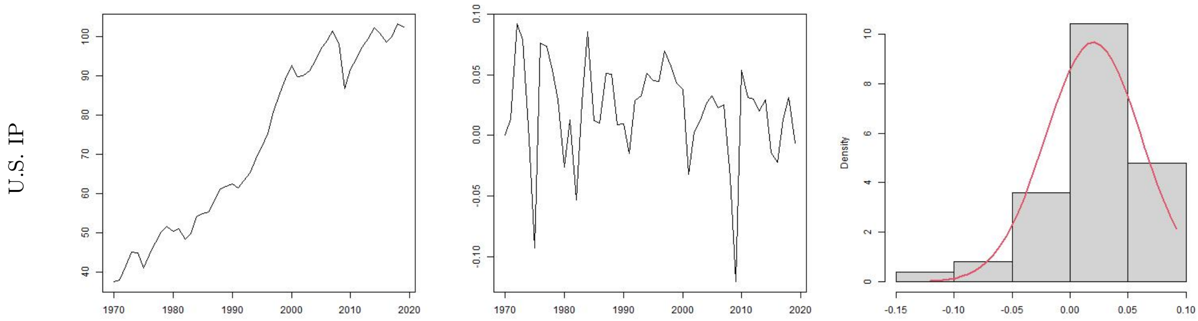
		Min.	5% Q.	25% Q.	Med.	Mean	75% Q.	95% Q.	Max.	SD	Skew.	Kurt.	ADF	SW
U.S. IP	a	37.56	41.34	51.25	70.47	72.85	94.30	101.94	103.18	22.34	-0.04	-1.63	2.75	0.89***
IP	m	859	934	1076	1300	1313	1545	1752	1888	266	0.16	-1.18	0.37	0.95***
U.S. GDP	a	4939	5560	7016	10629	11300	15376	18079	19202	4362	0.17	-1.39	10.00	0.93**
GDP	a	2997	4195	11756	29463	34715	56707	80655	87654	26700	0.61	-1.03	5.60	0.89***
GDPc	a	812	1081	2558	5108	5424	8466	10827	11321	3345	0.40	-1.19	4.26	0.91***
FX	a	76.48	80.58	87.25	96.27	97.59	103.38	123.22	143.01	14.72	1.05	0.97	-0.73	0.92***
FX	m	71.80	76.24	82.06	90.18	91.35	98.29	113.33	120.24	10.84	0.52	-0.22	-0.14	0.96***
FFR	a	0.09	0.12	1.86	5.03	5.16	7.47	11.78	16.38	3.85	0.61	-0.00	-1.29	0.95*
FFR	m	0.05	0.09	0.16	1.74	2.42	5.07	5.85	6.54	2.24	0.43	-1.46	-2.02.	0.84***
SIR	a	0.12	0.25	2.19	5.30	5.29	7.69	11.80	15.91	3.74	0.54	-0.12	-1.28	0.95*
LIR	a	1.80	2.14	4.08	6.28	6.32	7.96	12.00	13.91	3.04	0.48	-0.36	-1.00	0.96.
MB	a	78	92	175	423	975	823	3789	3974	1239	1.45	0.51	2.14	0.69***
CPI	a	17.81	19.71	44.61	68.93	67.66	94.43	111.36	117.24	30.57	-0.08	-1.27	10.39	0.95*
EMP	a	26	26	27	38	35	40	44	45	7	-0.10	-1.73	2.48	0.84***
POP	a	3690	3878	4623	5684	5680	6697	7539	7743	1218	0.03	-1.30	37.90	0.95*
MSCI	a	78.24	104.26	156.82	645.90	783.83	1252.78	1835.01	2151.02	612.10	0.49	-0.94	2.39	0.91***
SPX	a	81.48	90.88	135.96	503.77	821.64	1264.37	2303.48	2937.96	765.28	0.96	0.10	4.29	0.86***
OIL	a	3.35	3.70	17.50	26.01	35.96	50.34	94.68	99.57	27.34	1.01	-0.11	-0.44	0.86***
ND	a	60.00	63.00	147.00	254.00	245.58	345.50	412.65	432.00	115.34	-0.11	-1.33	0.29	0.94*
KOF	a	38.03	39.15	41.74	46.94	48.92	57.64	61.37	61.63	8.20	0.32	-1.52	8.08	0.88***

This table displays the descriptive statistics (minimum (Min.), 5% quantile (5% Q.), 25% quantile (25% Q.), 75% quantile (75% Q.), 95% quantile (95% Q.), median (Med.), mean (Mean), maximum (Max.), standard deviation (SD), skewness (Skew.) and excess kurtosis (Kurt.)) of the determinants U.S. industrial production (U.S. IP) as index with 2017 = 100, world industrial production (IP) in billion U.S. dollars, real U.S. gross domestic product (U.S. GDP) in billions of chained 2012 dollars, world gross domestic product (GDP) in billion U.S. dollars, world gross domestic product per capita (GDPc) in U.S. dollars, U.S. dollar index (FX), Federal Funds Effective Rate (FFR) in %, 3-month U.S. Treasury rate (SIR) in % per annum, 10-year U.S. Treasury rate (LIR) in %, U.S. monetary base (MB) in billion U.S. dollars, U.S. consumer price index (CPI) in %, U.S. employment (EMP) as % of working age population, world population (POP) in billions, MSCI world stock index (MSCI) as annual index level in basis points, Standard & Poor's 500 index (SPX), West Texas Intermediate spot crude oil price (OIL) in U.S. dollar per barrel, global natural disasters (ND) and KOF globalization index (KOF), as well as the results of the test statistics of the augmented Dickey-Fuller (ADF) test and the Shapiro-Wilk (SW) test with corresponding significance level (0.1% (***), 1% (**), 5% (*) and 10% (.)). Hereby, we report the descriptive statistics for the considered time period from 1970 to 2019 for the annual (a) analysis or from 1995 to 2020 in case of the monthly (m) analysis.

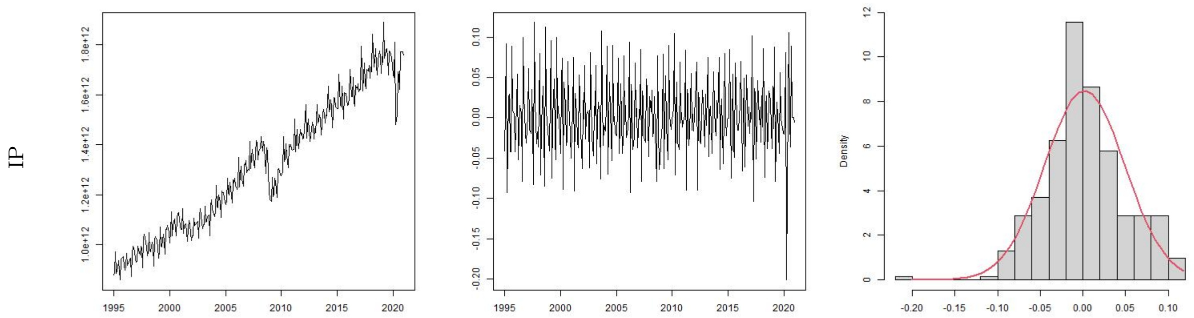
C.3.2 Development of the Determining Factors over Time[4]

Figure C.15: Time series plots of the determining factors

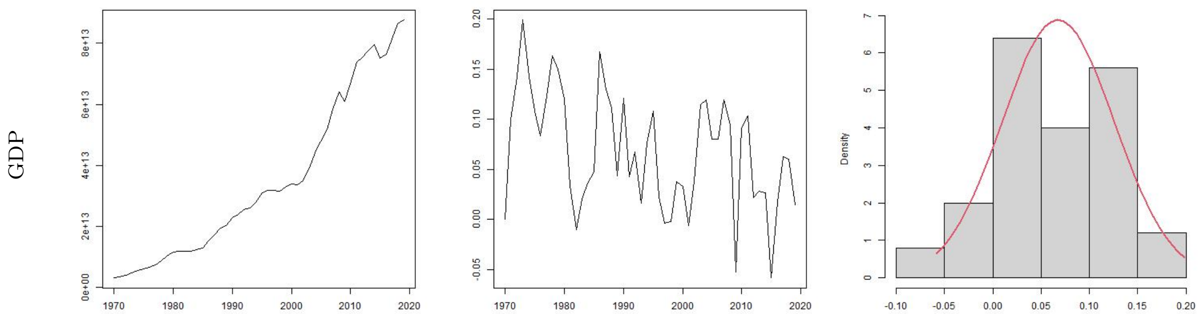
Level Return Histogram



U.S. industrial production (U.S. IP), index 2017 = 100, not seasonally adjusted



monthly world industrial production (IP), in U.S. dollar

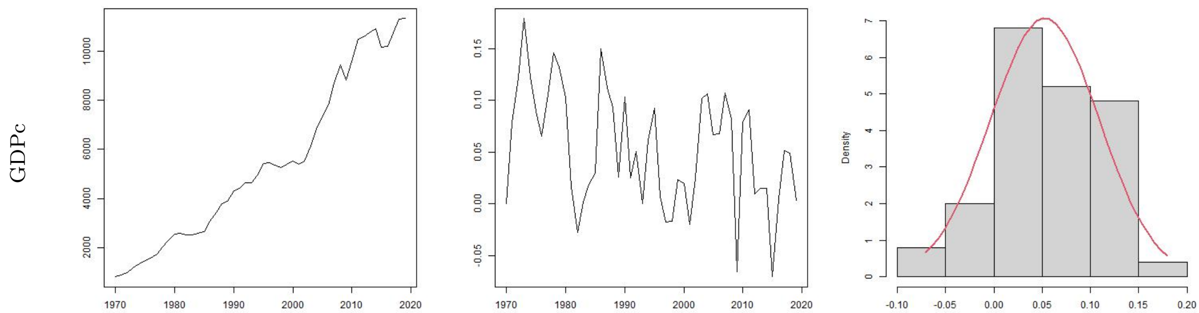


world gross domestic product (GDP), in billions of chained 2012 dollars, seasonally adjusted annual rate

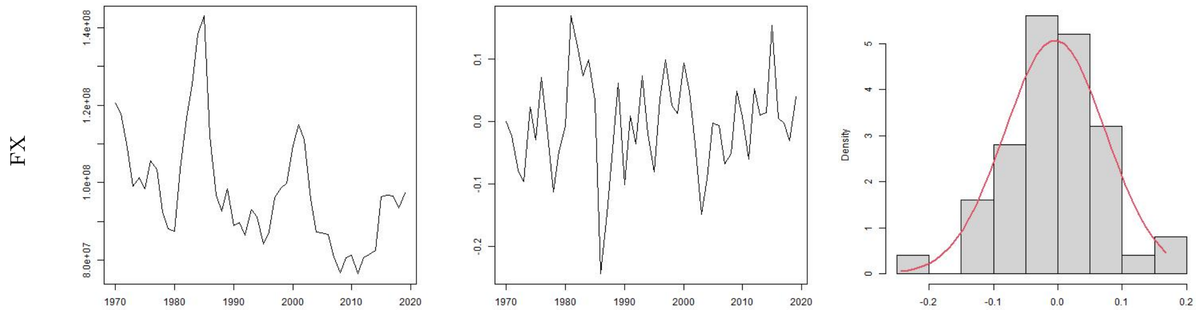
[4]In this section, we only plot the annual variables over the considered sample period from 1970 to 2019, except for the world industrial production, which is available since 1995 in monthly frequency.

Time series plots of the determining factors

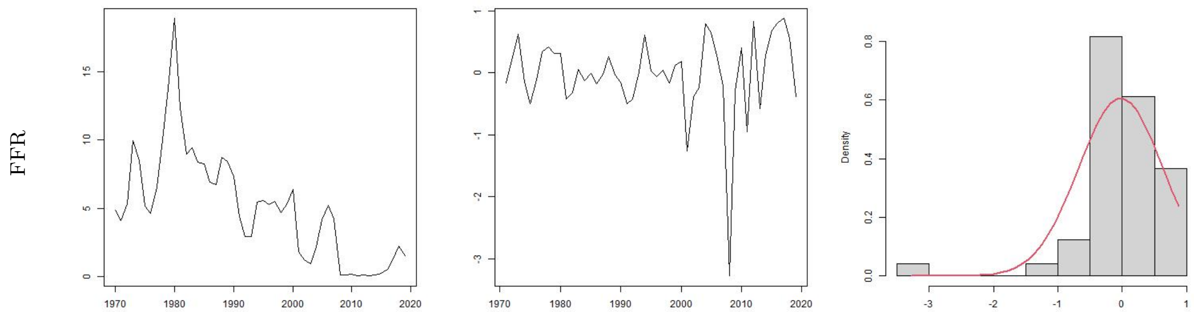
Level Return Histogram



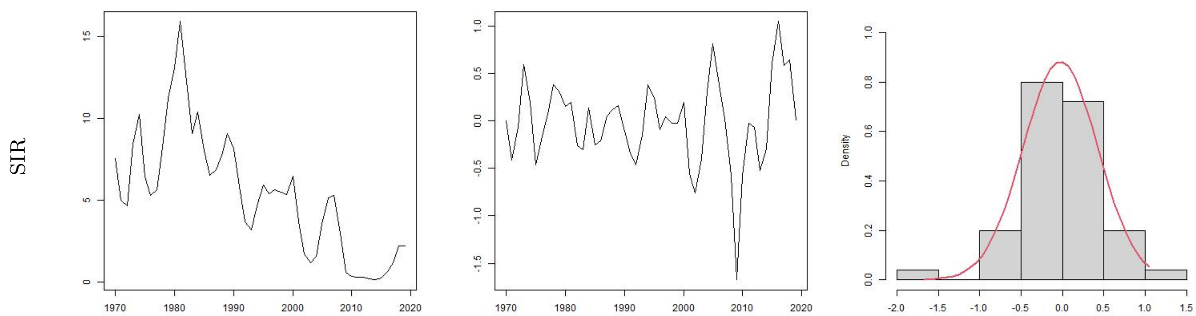
world gross domestic product per capita (GDPc), in current U.S. dollar



U.S. dollar index (FX)



Federal Funds Effective Rate (FFR), in %

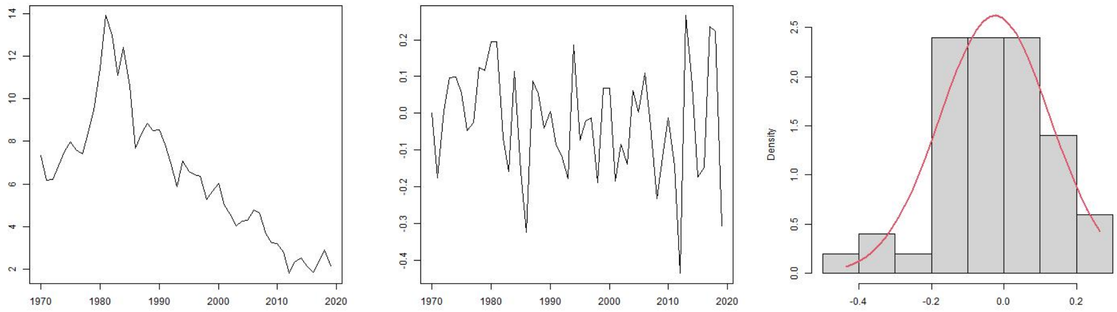


3-month U.S. Treasury rate (SIR), in % per annum

Time series plots of the determining factors

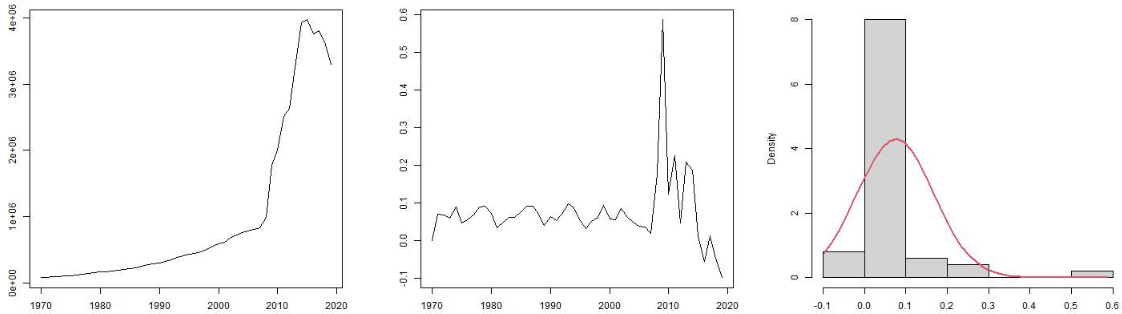
Level | Return | Histogram

LIR



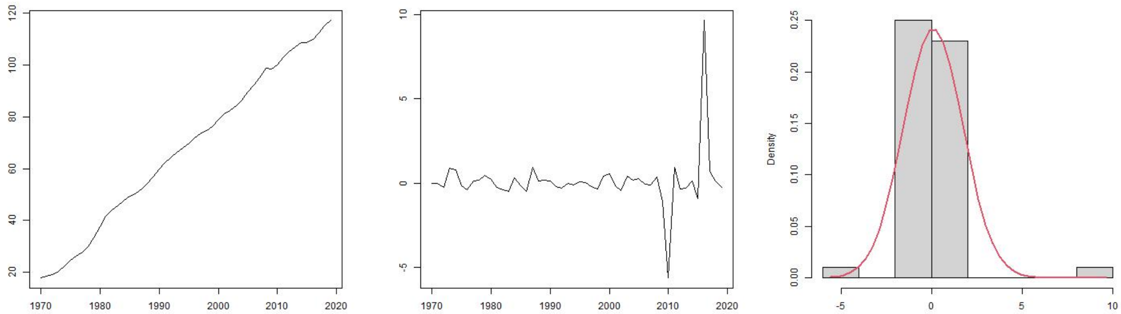
10-year U.S. Treasury rate (LIR), in %

MB



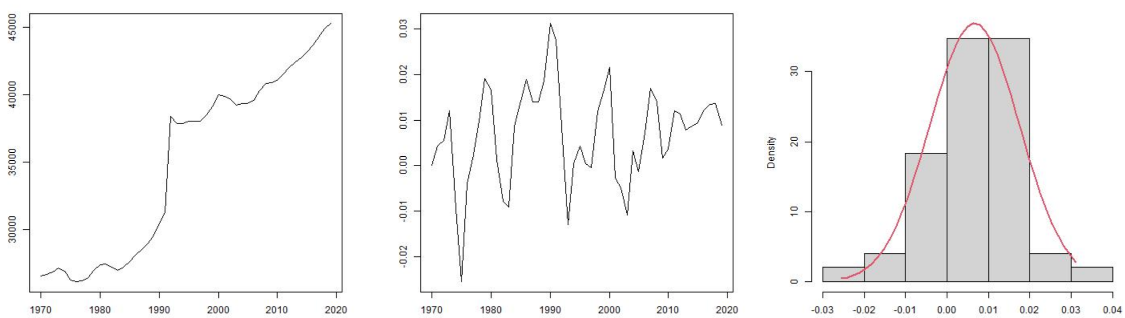
U.S. monetary base (MB), in millions of U.S. dollar

CPI



U.S. consumer price index (CPI), in percent, not seasonally adjusted

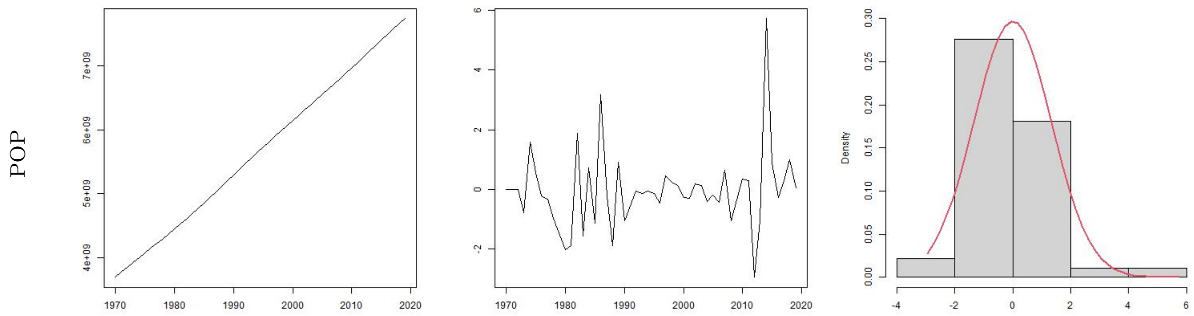
EMP



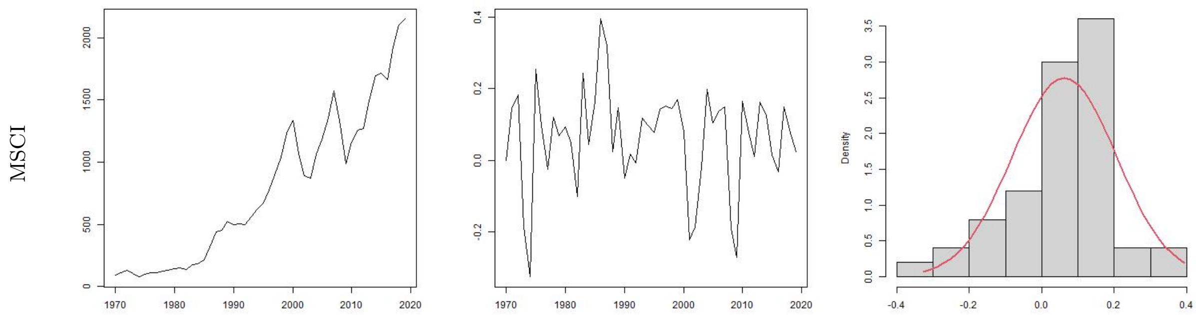
U.S. employment (EMP), displaying the % of working age population $\times 1000$

Time series plots of the determining factors

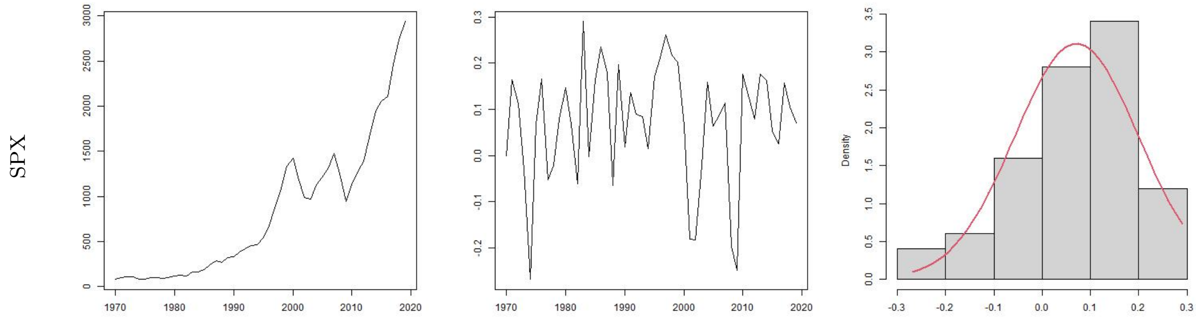
Level Return Histogram



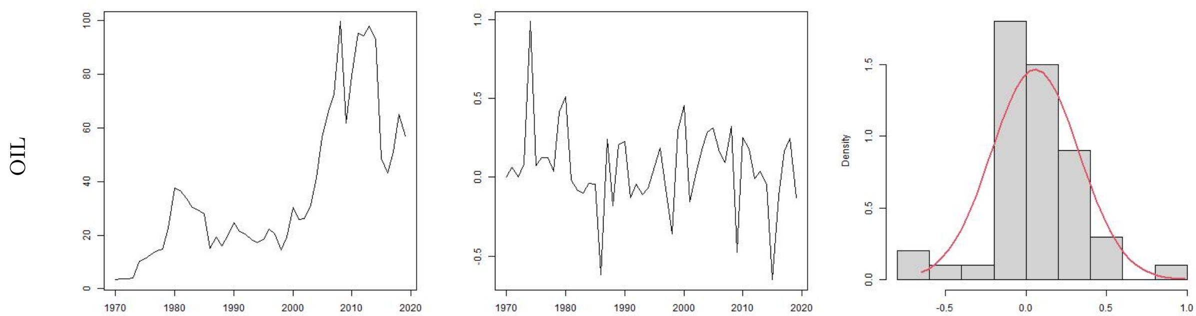
world population (POP)



MSCI world stock index (MSCI), closing price in basis points



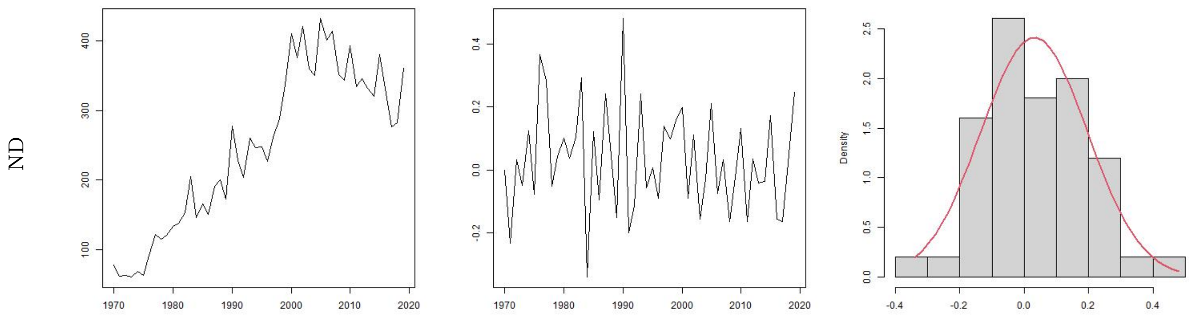
Standard & Poor's 500 index (SPX)



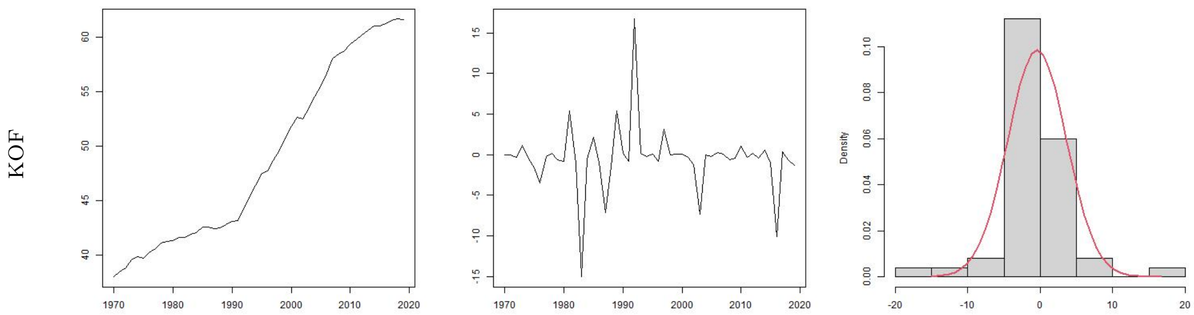
West Texas Intermediate spot crude oil price (OIL), in U.S. dollar per barrel

Time series plots of the determining factors

Level Return Histogram



global natural disasters (ND)



KOF globalization index (KOF)

These figures display the histogram of the logarithmic returns and the time series plots of the level as well as of the logarithmic return data of the price determinants U.S. industrial production (U.S. IP), world industrial production (IP), world gross domestic product (GDP), world gross domestic product per capita (GDPc), U.S. dollar index (FX), Federal Funds Effective Rate (FFR), 3-month U.S. Treasury rate (SIR), 10-year U.S. Treasury rate (LIR), U.S. monetary base (MB), U.S. consumer price index (CPI), U.S. employment (EMP), world population (POP), MSCI world stock index (MSCI), Standard & Poor's 500 index (SPX), West Texas Intermediate spot crude oil price (OIL), global natural disasters (ND), and KOF globalization index (KOF).

D Empirical Results

The scarcity risk assessment of the annual material requirements of four potential transformation pathways of the German energy system is assessed by taking into account the substitutability of the commodities, the future required resource amounts as well as the commodity market structure. Hereby, we model the commodity markets via a (MS-)GVAR model.

First, we provide in Section D.1 (Section D.2) additional information on the exemplary applications of the (MS-)GVAR framework on the industrial metal markets. Besides the test results of the models, we give further insights into the differences in the spillover effects between the calm and volatile regime, as well as between the GVAR and MS-GVAR model. Subsequently, we apply the (MS-)GVAR model in the context of the German Energiewende. Hereby, we provide additional information about these models in Section D.3, in particular, their induced spillover effects. In addition, we also analyze the regime-inferences of the time-varying MS-GVAR models. Moreover, we present further results of the robustness analyses.

D.1 A time-invariant Model for Industrial Metal Markets

Table D.1: Test results for autocorrelation, heteroscedasticity, structural breaks and normality of the individual VAR models, the GVAR models based on the weight matrices supply (**S**), demand (**D**), trading (**T**), and common (**C**), and the VAR model of the exogenous variables

		DW						ARCH-LM		OLS-CUSUM		HZ	
		supply		**demand**		**price**							
		Stat.	p	Stat.	p	Stat.	p	Stat.	p	Stat.	p	Stat.	p
individual VAR	Al	1.81	0.23	1.82	0.24	2.04	0.50	54.85	0.02	1.01	0.26	0.76	0.22
	Cu	2.12	0.62	1.83	0.25	2.20	0.72	19.69	0.99	0.69	0.74	0.55	0.75
	Ni	1.93	0.36	1.99	0.43	2.19	0.68	23.65	0.94	0.83	0.50	0.81	0.15
	Pb	2.09	0.61	1.73	0.17	1.97	0.45	26.18	0.89	0.97	0.31	0.60	0.63
	Sn	2.00	0.48	2.02	0.51	2.06	0.57	36.96	0.42	0.77	0.59	0.90	0.06
	Zn	2.10	0.58	1.55	0.05	1.99	0.44	24.08	0.94	0.56	0.92	0.96	0.03
S	Al	2.07	0.54	2.05	0.51	2.08	0.55	44.46	0.16	0.79	0.57	1.00	0.22
	Cu	2.04	0.49	1.82	0.23	2.03	0.48	43.71	0.18	0.59	0.88	1.00	0.22
	Ni	1.85	0.25	1.96	0.37	2.26	0.75	38.49	0.36	0.52	0.95	1.00	0.22
	Pb	1.85	0.30	1.79	0.24	2.20	0.72	31.21	0.70	0.70	0.72	1.00	0.22
	Sn	1.78	0.20	2.01	0.45	1.96	0.38	28.81	0.80	0.59	0.88	1.00	0.22
	Zn	2.19	0.65	1.93	0.32	1.83	0.22	31.56	0.68	0.75	0.62	1.00	0.22
D	Al	1.86	0.27	2.18	0.66	2.08	0.54	32.97	0.61	0.78	0.58	1.00	0.30
	Cu	2.04	0.49	1.85	0.26	2.02	0.47	41.13	0.26	0.54	0.93	1.00	0.30
	Ni	1.90	0.31	1.90	0.30	2.26	0.74	28.90	0.79	0.55	0.92	1.00	0.30
	Pb	1.88	0.32	1.64	0.10	2.20	0.71	37.14	0.42	0.70	0.72	1.00	0.30
	Sn	1.87	0.29	2.05	0.52	2.02	0.48	24.60	0.92	0.59	0.87	1.00	0.30
	Zn	2.12	0.60	1.82	0.24	1.80	0.22	19.50	0.99	0.81	0.52	1.00	0.30
T	Al	1.71	0.14	2.06	0.52	2.17	0.65	37.42	0.40	0.80	0.55	1.00	0.51
	Cu	2.08	0.56	1.81	0.23	2.08	0.55	44.75	0.15	0.64	0.81	1.00	0.51
	Ni	1.93	0.35	1.94	0.36	2.14	0.61	23.34	0.95	0.63	0.83	1.00	0.51

Test results for autocorrelation, heteroscedasticity, structural breaks and normality of the individual VAR models, the GVAR models based on the weight matrices supply (**S**), demand (**D**), trading (**T**), and common (**C**), and the VAR model of the exogenous variables

		DW						ARCH-LM		OLS-CUSUM		HZ	
		Stat.	p	Stat.	p	Stat.	p	Stat.	p	Stat.	p	Stat.	p
		supply		**demand**		**price**							
T	Pb	1.94	0.39	1.99	0.46	2.02	0.50	33.73	0.58	0.82	0.51	1.00	0.51
	Sn	1.77	0.19	2.05	0.51	1.98	0.42	24.68	0.92	0.58	0.89	1.00	0.51
	Zn	2.13	0.58	1.82	0.21	1.70	0.12	28.45	0.81	1.06	0.21	1.00	0.51
C	Al	1.92	0.35	2.10	0.57	2.10	0.57	40.17	0.29	0.74	0.65	1.00	0.29
	Cu	2.04	0.49	1.82	0.24	2.04	0.49	46.20	0.12	0.57	0.90	1.00	0.29
	Ni	1.91	0.31	1.94	0.35	2.24	0.73	29.21	0.78	0.55	0.92	1.00	0.29
	Pb	1.88	0.32	1.80	0.24	2.18	0.69	32.70	0.63	0.73	0.66	1.00	0.29
	Sn	1.80	0.22	2.03	0.49	1.99	0.43	25.56	0.90	0.58	0.89	1.00	0.29
	Zn	2.16	0.61	1.95	0.35	1.73	0.14	19.97	0.99	0.98	0.30	1.00	0.29
		GDP		FX		FFR							
exog. VAR		1.91	0.35	1.86	0.29	1.83	0.25	46.96	0.10	0.63	0.82	0.66	0.45

This table displays the results of the Durbin-Watson (DW) test for autocorrelation, the multivariate ARCH Lagrange multiplier (ARCH-LM) test for heteroscedasticity, the OLS-cumulative sums of standardized residuals (OLS-CUSUM) test for structural breaks, and the Henze-Zirkler (HZ) test for normality. Hereby, the Durbin-Watson test is applied on each, individual regression equation of the VAR model, corresponding to the commodity-specific supply (**supply**), demand (**demand**), and price (**price**) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), whereas the multivariate ARCH Lagrange multiplier test and the OLS-cumulative sums of standardized residuals test are applied on the commodity-specific VAR models, and the Henze-Zirkler test is applied on the residuals of the (global) vector autoregression ((G)VAR) model. In particular, we report the test results for the individual VAR models, the GVAR models based on the weight matrices supply (**S**), demand (**D**), trading (**T**), and common (**C**), as well as the VAR model of the exogenous variables world gross domestic product (GDP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR), which are estimated on the sample period from 1970 to 2019.

D.2 Time-varying Spillover Effects

D.2.1 Test for Autocorrelation

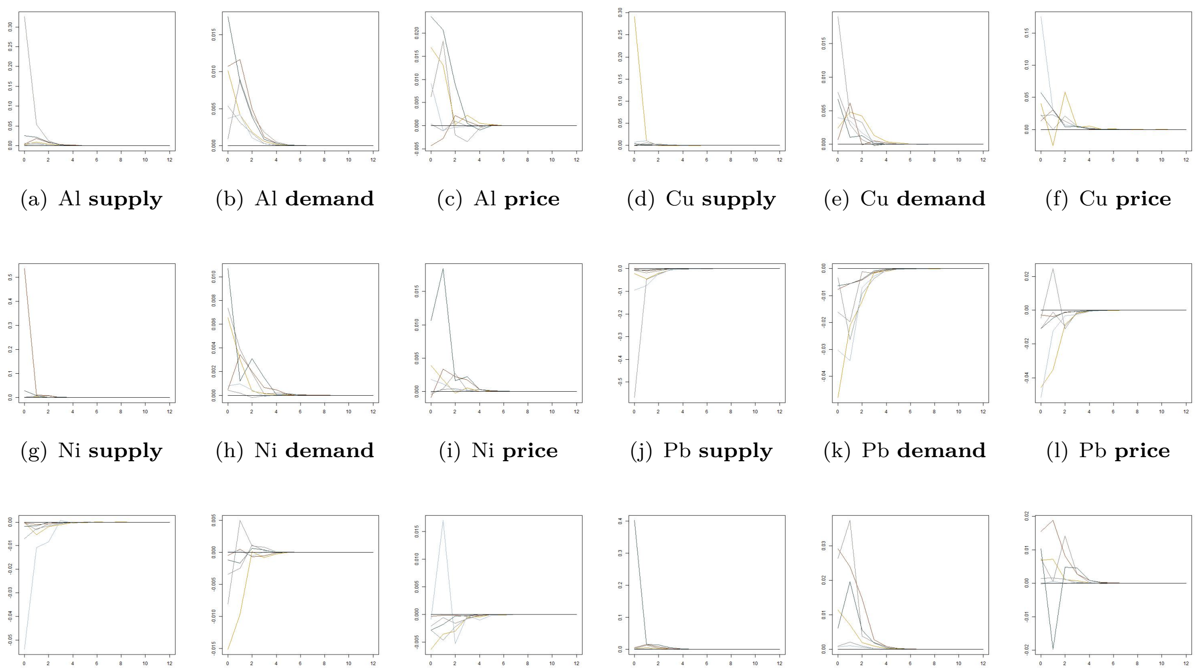
Table D.2: Test results for autocorrelation and normality of the MS-GVAR models

	DW test																		HZ test	
	Al			Cu			Ni			Pb			Sn			Zn				
	supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	Statistics	p-value
D	2.09	2.65	1.90	2.50	2.63	1.69	2.08	2.58	1.76	2.34	2.72	2.01	2.50	2.80	1.82	2.30	2.67	2.02	1.02	0.00
$REMod - REF$	2.18	2.62	1.83	2.43	2.61	1.73	2.10	2.53	1.76	2.44	2.75	2.05	2.60	2.73	1.85	2.41	2.73	1.98	1.02	0.00
$REMod - SUF$	2.13	2.59	1.84	2.39	2.60	1.73	2.08	2.54	1.76	2.43	2.75	2.03	2.58	2.74	1.85	2.48	2.71	1.97	1.02	0.00
$REMod - PER$	2.19	2.62	1.83	2.41	2.61	1.73	2.10	2.53	1.76	2.45	2.76	2.05	2.59	2.74	1.86	2.41	2.73	1.97	1.02	0.00
$REMod - UNA$	2.16	2.60	1.83	2.42	2.58	1.73	2.09	2.52	1.76	2.44	2.73	2.04	2.60	2.77	1.87	2.48	2.72	1.98	1.02	0.00
Exog.		IP			FX			FFR												
1995 - 2020		2.48			2.04			1.40											4.19	0.00
1995 - 2019		2.71			2.04			1.33											2.16	0.00

This table displays the results of the Durbin-Watson (DW) test for autocorrelation, and the Henze-Zirkler (HZ) test for normality. Hereby, the Durbin-Watson test is applied on each, individual regression equation of the MS-GVAR model, corresponding to the commodity-specific supply (**supply**), demand (**demand**), and price (**price**) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), whereas the Henze-Zirkler test is applied on the residuals of the MS-GVAR model. In particular, we report the test results for the MS-GVAR models based on the weight matrix demand (**D**), estimated on the sample period from 1995 to 2020, the weight matrices, representing the dependencies between the commodities within the $REMod - REF$, $REMod - SUF$, $REMod - PER$, and $REMod - UNA$ transformation path, estimated on the sample period from 1995 to 2019, as well as the MS-VAR model of the exogenous variables (Exog.) world industrial production (IP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR), estimated on the sample period from 1995 to 2019 or 1995 to 2020.

D.2.2 Differences in the Spillover Effects between Calm and Volatile Regimes[1]

Figure D.1: Differences in the conditional value at risk of the spillover effects on the **supply** variables in the MS-GVAR model under the volatile vs. the calm regime



(a) Al **supply** (b) Al **demand** (c) Al **price** (d) Cu **supply** (e) Cu **demand** (f) Cu **price**

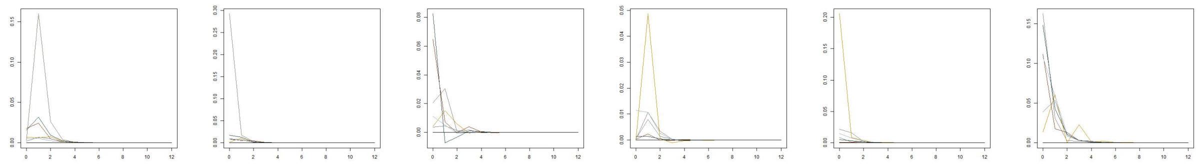
(g) Ni **supply** (h) Ni **demand** (i) Ni **price** (j) Pb **supply** (k) Pb **demand** (l) Pb **price**

(m) Sn **supply** (n) Sn **demand** (o) Sn **price** (p) Zn **supply** (q) Zn **demand** (r) Zn **price**

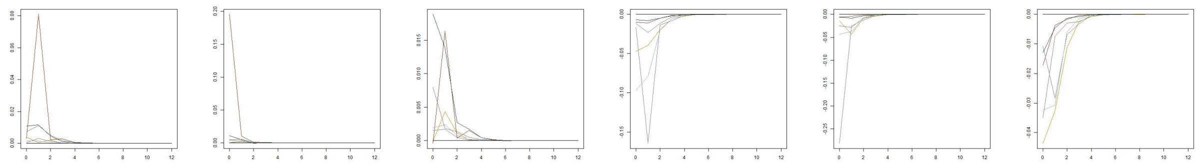
These figures indicate the differences in the conditional value at risk (CoVaR) of the spillover effects from the individual supply (**supply**), demand (**demand**) and price (**price**) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) to the commodity supply of aluminum (Al), copper (Cu), Nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) under the volatile vs. the calm regime.

[1]As all original variables were non-stationary, we base the entire analysis on the logarithmic return data and hence, also the calculation of the conditional value at risk is based on logarithmic returns.

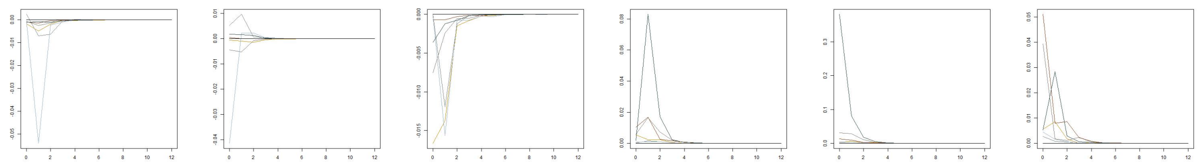
Figure D.2: Differences in the conditional value at risk of the spillover effects on the **demand** variables in the MS-GVAR model under the volatile vs. the calm regime



(a) Al **supply** (b) Al **demand** (c) Al **price** (d) Cu **supply** (e) Cu **demand** (f) Cu **price**



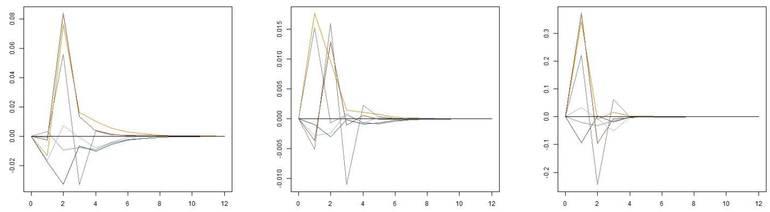
(g) Ni **supply** (h) Ni **demand** (i) Ni **price** (j) Pb **supply** (k) Pb **demand** (l) Pb **price**



(m) Sn **supply** (n) Sn **demand** (o) Sn **price** (p) Zn **supply** (q) Zn **demand** (r) Zn **price**

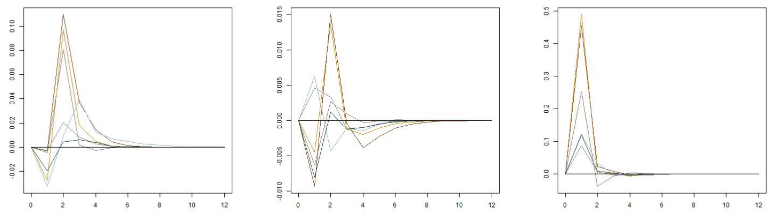
These figures indicate the differences in the conditional value at risk (CoVaR) of the spillover effects from the individual supply (**supply**), demand (**demand**) and price (**price**) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) to the commodity demand of aluminum (Al), copper (Cu), Nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) under the volatile vs. the calm regime.

Figure D.3: Differences in the conditional value at risk of the spillover effects from the exogenous variables to the commodity markets in the MS-GVAR model under the volatile vs. the calm regime



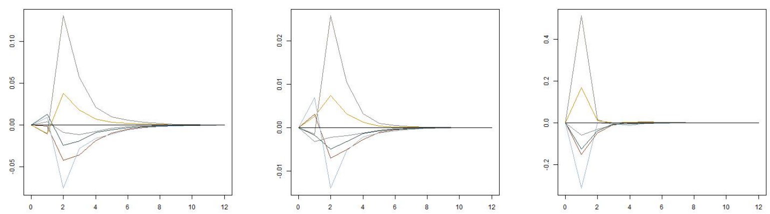
(a) IP (b) FX (c) FFR

supply



(d) IP (e) FX (f) FFR

demand



(g) IP (h) FX (i) FFR

price

These figures indicate the differences in the conditional value at risk (CoVaR) of the spillover effects from the exogenous variables world industrial production (IP), U.S. dollar index (FX), Federal Funds Effective Rate (FFR) to the commodity supply (**supply**), demand (**demand**) and price (**price**) of aluminum (Al), copper (Cu), Nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) under the volatile vs. the calm regime.

D.2.3 Differences in the Spillover Effects of the GVAR Model and the MS-GVAR Model

The GIRF analysis of the MS-GVAR model detects various spillover effects within and between the commodity markets. However, the significance of these effects does not change under the regimes, whereas the reactions of the variables to shocks are generally stronger in the volatile regime. In the following, we examine the differences in the spillover effects of the time-invariant GVAR model, compared to the time-varying MS-GVAR model. For comparability, we estimate the GVAR model, based on the demand matrix, on monthly data from 1995 to 2020.[2] Moreover, we include the impact of macroeconomic factors via the exogenous variables world industrial production (IP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR).

While the results of the Durbin-Watson (DW) test and the OLS-cumulative sums of standardized residuals (OLS-CUSUM) test indicate neither of the underlying individual commodity models suffers from autocorrelation, nor structural breaks, the multivariate ARCH Lagrange multiplier (ARCH-LM) test indicates the commodity-specific models are heteroscedastic, see Table D.3. Since even the inclusion of up to six lags does not lead to homoscedastic results, the time-invariant model is not able to reflect the dependence structure in the monthly data, indicating the need for a time-varying covariance matrix.[3] Moreover, the Henze-Zirkler (HZ) test implies the residuals of the GVAR model are not multivariate normal distributed. In particular, the heteroscedastic residuals lead to too narrow confidence intervals, obtained by the sieve bootstrap procedure, and therefore, the true values of the GIRFs may deviate.[4]

Table D.3: Test results for autocorrelation, heteroscedasticity, structural breaks and normality of the monthly GVAR model

	DW						ARCH-LM		OLS-CUSUM		HZ	
	supply		**demand**		**price**							
	Stat.	p	Stat.	p	Stat.	p	Stat.	p	Stat.	p	Stat.	p
Al	2.05	0.72	2.18	0.96	2.03	0.67	65.54	0.00	0.37	1.00	1.01	0.00
Cu	2.09	0.82	2.22	0.98	1.98	0.50	47.17	0.10	0.63	0.83	1.01	0.00
Ni	2.07	0.76	2.13	0.89	2.00	0.55	86.82	0.00	0.74	0.64	1.01	0.00
Pb	2.03	0.66	2.18	0.95	1.98	0.49	88.28	0.00	0.77	0.60	1.01	0.00
Sn	2.12	0.88	2.19	0.97	2.04	0.69	56.56	0.02	0.59	0.87	1.01	0.00
Zn	2.05	0.72	2.20	0.97	2.01	0.59	86.25	0.00	0.61	0.85	1.01	0.00
	GDP		FX		FFR							
Exog.	1.85	0.10	2.01	0.55	1.98	0.46	189.54	0.00	0.63	0.83	5.51	0.00

This table displays the results of the Durbin-Watson (DW) test for autocorrelation, the multivariate ARCH Lagrange multiplier (ARCH-LM) test for heteroscedasticity, the OLS-cumulative sums of standardized residuals (OLS-CUSUM) test for structural breaks, and the Henze-Zirkler (HZ) test for normality. Hereby, the Durbin-Watson test is applied on each, individual regression equation of the VAR model, corresponding to the commodity-specific supply (**supply**), demand (**demand**), and price (**price**) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), whereas the multivariate ARCH Lagrange multiplier test and the OLS-cumulative sums of standardized residuals test are applied on the commodity-specific VAR models, and the Henze-Zirkler test is applied on the residuals of the GVAR model. In particular, we report the test results for the GVAR model based on the demand weight matrix (**D**), estimated on monthly data from 1995 to 2020, as well as the VAR model of the exogenous variables (Exog.) world gross domestic product (GDP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR).

Table D.4 displays the spillover effects derived from the GVAR model based on monthly data, whereby we indicate significant positive, or negative, responses of the column variables to a

[2] Actually, we include an intercept in case of the GVAR model estimated on monthly data for comparability with the MS-GVAR model.

[3] A further increase of the lag length is due to data limitations not feasible.

[4] Since we aim to compare the GIRF analysis of the GVAR model with the results of the MS-GVAR model, we do not adapt the estimation of the GIRFs for the time-invariant model. However, while the MS-GVAR model allows for a regime-switching covariance matrix, we keep in mind the GVAR model is misspecified.

shock in the row variables by a (+) or (-), respectively. In general, the time-invariant GVAR model only detects few significant spillover effects within and between the commodity markets. However, similar to the GVAR model on annual data in Section 5.1 as well as the MS-GVAR model in Section 5.2, various spillover effects between the prices indicate the model reflects the co-movement between the prices. In particular, the copper price affects the metal prices, but only reacts to shocks in the price of zinc.

In contrast, the strong interrelations between the commodity markets, especially, between the aluminum and copper market, are not reflected in the monthly GVAR model. While the interdependencies between the lead and zinc markets are reflected, caused by their co-production relationship, the supply and demand variables generally do not interact with each other, suggesting that the model does not represent the relationship between supply, demand, and price. In addition, the GVAR model on annual data as well as the MS-GVAR model detect various interdependencies within the individual commodity markets, whereas the GVAR model based on monthly data does not exhibit any significant response in the copper, nickel, lead and zinc markets to shocks in their own market. Only the supply and demand variables of aluminum and tin affect each other.

Overall, the GIRF analysis indicates the GVAR model based on the monthly data is not able to fully reflect the commodity market structure. Moreover, the heteroscedasticity in the commodity markets implies the model is misspecified, whereas the MS-GVAR model controls for time-depending relations and the associated spillover effects reveal the model represents the interdependencies between the commodity markets.

Table D.4: GIRF results of the GVAR model based on monthly data

		Al			Cu			Ni			Pb			Sn			Zn		
		supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
Al	**supply**	+	+																
	demand	+	+																
	price			+															
Cu	**supply**				+														
	demand					+													
	price			+			+												+
Ni	**supply**							+											
	demand								+										
	price									+									
Pb	**supply**										+						+		
	demand											+							
	price												+						+
Sn	**supply**													+	+				
	demand													+	+				
	price			+												+			
Zn	**supply**										+						+		
	demand																	+	
	price						+						+						+

This table displays the results of the GIRF analysis of the GVAR model based on the demand weight matrix (**D**), estimated on monthly data in the period from 1995 to 2020. We analyze the response of the column variables to a shock of the row variables supply (**supply**), demand (**demand**) and price (**price**) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn) and zinc (Zn). Significant positive (+) or negative (-) effects on the 68%- level are displayed.

Similar to the above analyses, we also investigate how global shocks affect the commodity markets. Hereby, we model the monthly exogenous variables, world industrial production (IP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR), via a VAR model with two lags to avoid autocorrelated residuals and base the analysis on monthly data in the period from 1995 to 2020. While the results of the Durbin-Watson (DW) test and the OLS-cumulative sums of standardized residuals (OLS-CUSUM) test suggest the VAR model does not suffer from autocorrelation nor structural breaks at the 5% significance level, the residuals suffer from heteroscedasticity, see Table D.3, indicating the time-invariant model is not able to reflect the dependence structure in the monthly data, in line with the results of the commodity market models. Moreover, the Henze-Zirkler (HZ) test implies the residuals of the GVAR model are not multivariate normal distributed, therefore, the true values of the GIRFs may deviate. However, we only use the spillover effects from the economy to the commodity markets to compare the GVAR model with the MS-GVAR model, therefore, we do not adapt the estimation of the GIRFs for the time-invariant model.

Regarding the impact of global shocks on the commodity markets, the results of the GVAR model based on monthly data in the period from 1995 to 2020 coincides with those of the corresponding annual model, but differ to those of the time-varying model, see Table D.5. In particular, a shock to the global demand, represented by the world industrial production (IP), or the interest rate, represented by the Federal Funds Effective Rate (FFR), cause significantly rising commodity markets, while an increase in the exchange rate reduces the prices. Hereby, the results are contrary to the effects derived from the time-varying MS-GVAR model, since the impact of the lagged values as well as the indirect effects are probably stronger in the time-varying setup, whereas the time-invariant model is less sensitive. However, both models detect a strong impact of the economy to the commodity markets, indicating shocks to the economy significantly affect the supply, demand as well as price of commodities.

Table D.5: GIRF results of the GVAR model based on monthly data for shocks to the exogenous variables

	Al			Cu			Ni			Pb			Sn			Zn		
	supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
IP	+	+	+		+	+	+	+	+	+	+	+	+	+	+	+	+	+
FX	-	-	-		+	-	-		-			-			-	+		-
FFR	+	+	+	+	+	+	+			+	+	+	+	+	+	+	+	+

This table displays the results of the GIRF analysis of the GVAR model based on the demand weight matrix (**D**), estimated on monthly data in the period from 1995 to 2019. We analyze the response of the column variables supply (**supply**), demand (**demand**) and price (**price**) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), to a shock of the row variables world industrial production (IP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR), where significant positive (+) or negative (-) effects are displayed, based on the 68%- level.

In addition to the analysis of the differences between the significance in the GIRFs, we also compare the magnitude as well as the implied risk of the spillover effects. Hereby, we apply the two-sided Wilcoxon-test[5] on the absolute value of the median generalized impulse response functions of the GVAR model against the corresponding spillover effects of the MS-GVAR model under the calm (volatile) regime to examine whether the responses of a shock to a variable significantly differ between the models, see Table D.6 (Table D.7). In general, besides few shocks, the spillover effects significantly differ between the GVAR model and the MS-GVAR model under the calm as well as the volatile regime, indicating the time-invariant analysis provides significantly different spillover effects than the time-varying model. While the MS-GVAR model distinguishes between the responses either under the calm or volatile regime, the GVAR model reflects the overall effects. In contrast, the magnitude of most of the spillover effects from the exogenous variables to the commodity markets is comparable between the GVAR model and

[5]Since we can not ensure the GIRFs follow a normal distribution, we apply the non-parametric Wilcoxon signed rank test (Wilcoxon) test instead of the t-test. However, the results of the t-test are similar.

the MS-GVAR model under the calm and volatile regime, see Table D.8.[6]

Table D.6: Results of the Wilcoxon-test for the assessment of differences in the magnitude of spillover effects of the GVAR model compared to the MS-GVAR model under the calm regime

		Al			Cu			Ni			Pb			Sn			Zn		
		supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
Al	**supply**	78*	79*	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**
	demand	91**	78*	91**	86**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**
	price	66	79*	78*	81*	80*	80*	91**	78*	79*	91**	91**	91**	91**	91**	91**	66	56	79*
Cu	**supply**	91**	91**	91**	68	78*	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**
	demand	91**	80*	91**	91**	78*	91**	91**	91**	91**	91**	67	91**	91**	91**	91**	91**	81*	91**
	price	83**	80*	91**	78*	80*	78*	91**	80*	79*	91**	91**	91**	72.	91**	79*	91**	78*	91**
Ni	**supply**	91**	91**	83**	91**	91**	91**	68	78*	79*	91**	91**	91**	91**	91**	91**	81*	91**	91**
	demand	91**	66	91**	91**	91**	91**	78*	67	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**
	price	70.	80*	76*	91**	91**	91**	78*	72.	67	91**	91**	91**	91**	91**	91**	66	78*	81*
Pb	**supply**	91**	91**	91**	80*	91**	91**	91**	91**	91**	78*	91**	91**	91**	81*	79*	91**	91**	91**
	demand	91**	68	91**	78*	79*	91**	91**	91**	91**	91**	78*	91**	91**	91**	91**	91**	91**	91**
	price	91**	82*	82*	66	66	80*	91**	91**	83**	78*	91**	78*	78*	91**	79*	91**	91**	91**
Sn	**supply**	91**	91**	91**	79*	91**	91**	91**	91**	91**	91**	91**	91**	78*	79*	91**	91**	91**	91**
	demand	91**	91**	91**	79*	83**	91**	91**	91**	91**	91**	66	91**	81*	68	79*	91**	91**	91**
	price	91**	91**	91**	79*	66	80*	91**	91**	91**	91**	91**	91**	78*	82*	78*	91**	91**	91**
Zn	**supply**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	67	79*	80*
	demand	80*	78*	80*	79*	91**	91**	91**	91**	79*	91**	91**	91**	91**	91**	91**	91**	56	91**
	price	67	68	68	91**	91**	91**	84**	78*	91**	91**	91**	91**	91**	91**	91**	78*	80*	67

This table displays the statistics with corresponding significance level (0.1% (***), 1% (**), 5% (*) and 10% (.)) of the two-sided Wilcoxon test to assess whether the magnitude of the spillover effects differs in absolute terms between the GVAR model and the MS-GVAR model under the calm regime. Hereby, we investigate the differences in the spillover effects of the column variables to shocks in the row variables (**supply**), demand (**demand**) and price (**price**) of aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn).

Table D.7: Results of the Wilcoxon-test for the assessment of differences in the magnitude of spillover effects of the GVAR model compared to the MS-GVAR model under the volatile regime

		Al			Cu			Ni			Pb			Sn			Zn		
		supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
Al	**supply**	78*	79*	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	81*	91**	91**
	demand	91**	78*	91**	82*	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**
	price	66	79*	78*	82*	81*	80*	91**	78*	79*	91**	91**	91**	91**	91**	91**	66	57	79*
Cu	**supply**	91**	91**	91**	68	78*	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**
	demand	91**	80*	91**	91**	78*	91**	91**	91**	91**	91**	67	91**	91**	91**	91**	91**	81*	91**
	price	73.	79*	81*	78*	79*	70.	91**	79*	79*	91**	91**	91**	69	91**	78*	91**	78*	91**
Ni	**supply**	91**	91**	83**	91**	91**	91**	67	78*	79*	91**	91**	91**	91**	91**	91**	80*	91**	91**
	demand	91**	66	91**	91**	91**	91**	78*	66	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**
	price	70.	79*	72.	91**	91**	91**	78*	71.	67	91**	91**	91**	91**	91**	91**	66	78*	81*
Pb	**supply**	91**	91**	91**	81*	91**	91**	91**	91**	91**	78*	91**	91**	91**	91**	91**	91**	91**	91**
	demand	91**	74*	91**	78*	82*	91**	91**	91**	91**	91**	78*	91**	91**	91**	91**	91**	91**	91**
	price	91**	91**	83**	66	66	80*	91**	91**	85**	79*	91**	78*	79*	91**	80*	91**	91**	91**
Sn	**supply**	91**	91**	91**	79*	91**	91**	91**	91**	91**	91**	91**	91**	78*	79*	91**	91**	91**	91**
	demand	91**	91**	91**	80*	82*	91**	91**	91**	91**	91**	67	91**	82*	68	79*	91**	91**	91**
	price	91**	91**	91**	80*	66	80*	91**	91**	91**	91**	91**	91**	78*	82*	78*	91**	91**	91**
	supply	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	91**	57	79*	79*

[6]Since we apply the Wilcoxon-test on the absolute values of the GIRFs, we only test for differences in the magnitude of the spillover effects and not for the direction, which deviates between the models.

Results of the Wilcoxon-test for the assessment of differences in the magnitude of spillover effects of the GVAR model compared to the MS-GVAR model under the volatile regime

		Al			Cu			Ni			Pb			Sn			Zn		
		supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
Zn	**demand**	80*	67	80*	79*	91**	91**	91**	91**	69	91**	87**	91**	91**	91**	91**	91**	56	91**
	price	67	67	66	91**	91**	91**	81*	78*	83**	91**	91**	91**	91**	91**	91**	78*	80*	67

This table displays the statistics with corresponding significance level (0.1% (***), 1% (**), 5% (*) and 10% (.)) of the two-sided Wilcoxon test to assess whether the magnitude of the spillover effects differs in absolute terms between the GVAR model and the MS-GVAR model under the volatile regime. Hereby, we investigate the differences in the spillover effects of the column variables to shocks in the row variables (**supply**), demand (**demand**) and price (**price**) of aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn).

Table D.8: Results of the Wilcoxon-test for the assessment of differences in the magnitude of spillover effects from the exogenous variables to the commodity markets of the GVAR model compared to the MS-GVAR model

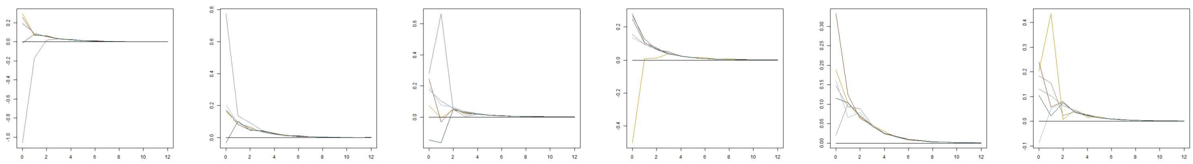
		Al			Cu			Ni			Pb			Sn			Zn		
		supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
calm	IP	38	33	20.	36	45	21.	26	39	21.	23	31	22	13*	43	11*	24	26	9*
	FX	82*	91**	59	55	80*	60	54	56	80*	61	84**	72.	35	50	61	46	40	37
	FFR	62	78*	41	59	80*	59	39	60	21.	80*	81*	48	34	54	44	56	50	10*
volatile	IP	32	32	19.	26	34	21.	15*	29	20.	13*	23	20.	13*	39	0**	22	20.	8**
	FX	81*	91**	57	55	80*	54	52	55	77*	53	84**	71.	33	49	60	29	37	32
	FFR	26	70.	24	56	67	26	20.	45	19.	59	78*	29	24	43	23	42	32	10*

This table displays the statistics with corresponding significance level (0.1% (***), 1% (**), 5% (*) and 10% (.)) of the two-sided Wilcoxon test to assess whether the magnitude of the spillover effects from the exogenous variables world industrial production (IP), U.S. dollar index (FX) and Federal Funds Effective Rate (FFR) to the commodity markets differs in absolute terms between the GVAR model and the MS-GVAR model under the calm and volatile regime. Hereby, we investigate the differences in the spillover effects of the column variables to shocks in the row variables (**supply**), demand (**demand**) and price (**price**) of aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn).

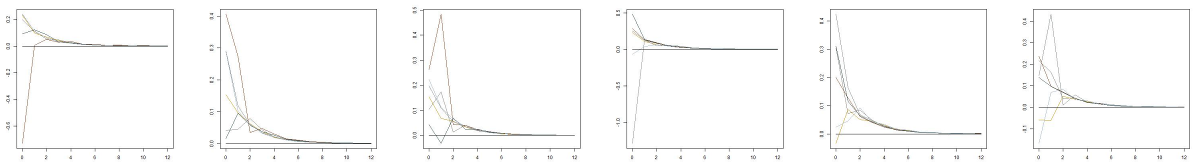
The additional analysis of the conditional value at risk[7] underlines the magnitude of the spillover effects differs between the time-invariant and time-varying analysis. Hereby, Figure D.4, Figure D.7, Figure D.5, Figure D.8, Figure D.6, Figure D.9, Figure D.10, D.11 display the differences in the conditional value at risk for increases in the commodity-specific supply, demand and price variables in response to shocks in the endogenous (exogenous) variables between the GVAR model and the MS-GVAR model under the calm (volatile) regime. Overall, the GVAR model overestimates the spillover risk, as the time-invariant analysis observes a stronger increase in the commodity markets in response to shocks compared to the time-varying MS-GVAR model under the calm as well as volatile regime.

[7] As all original variables were non-stationary, we base the entire analysis on the logarithmic return data and hence, also the calculation of the conditional value at risk is based on logarithmic returns.

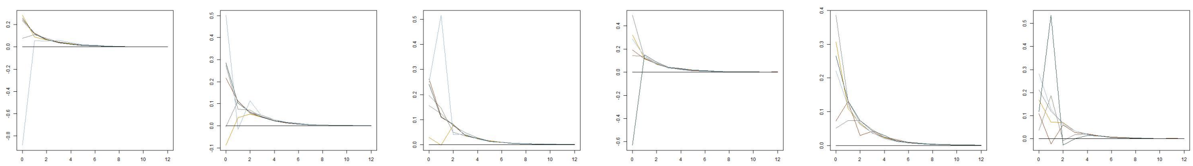
Figure D.4: Differences in the conditional value at risk of the spillover effects on the **supply** variables between the GVAR model vs. the MS-GVAR model under the calm regime



(a) Al **supply** (b) Al **demand** (c) Al **price** (d) Cu **supply** (e) Cu **demand** (f) Cu **price**



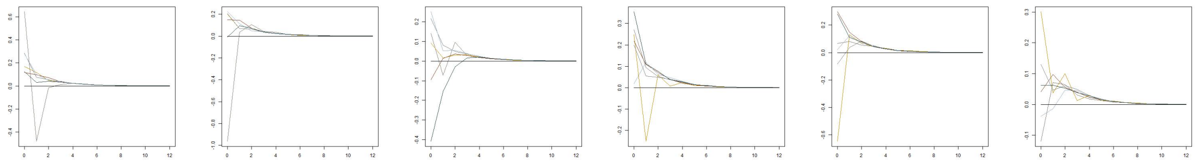
(g) Ni **supply** (h) Ni **demand** (i) Ni **price** (j) Pb **supply** (k) Pb **demand** (l) Pb **price**



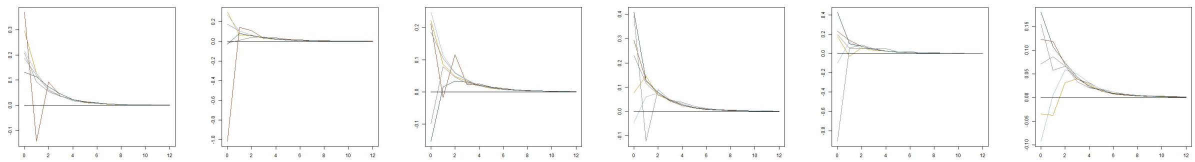
(m) Sn **supply** (n) Sn **demand** (o) Sn **price** (p) Zn **supply** (q) Zn **demand** (r) Zn **price**

These figures indicate the differences in the conditional value at risk (CoVaR) of the spillover effects from the individual supply (**supply**), demand (**demand**) and price (**price**) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) to the commodity supply of aluminum (Al), copper (Cu), Nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) of the GVAR model vs. the MS-GVAR model under the calm regime.

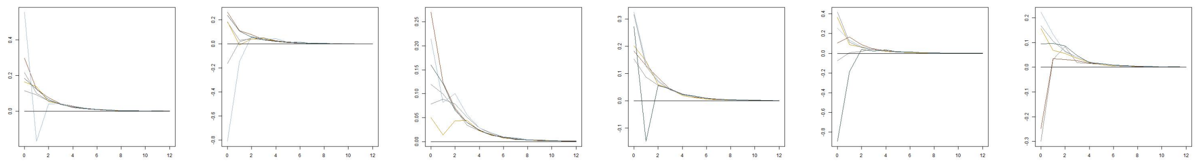
Figure D.5: Differences in the conditional value at risk of the spillover effects on the **demand** variables between the GVAR model vs. the MS-GVAR model under the calm regime



(a) Al **supply** (b) Al **demand** (c) Al **price** (d) Cu **supply** (e) Cu **demand** (f) Cu **price**



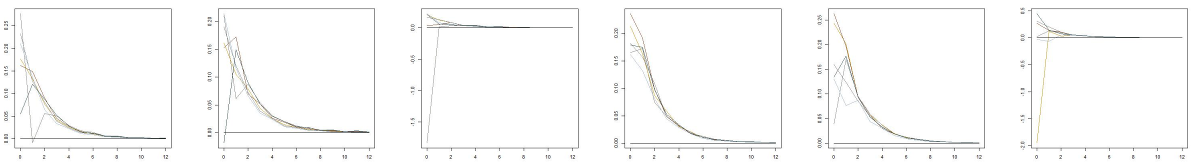
(g) Ni **supply** (h) Ni **demand** (i) Ni **price** (j) Pb **supply** (k) Pb **demand** (l) Pb **price**



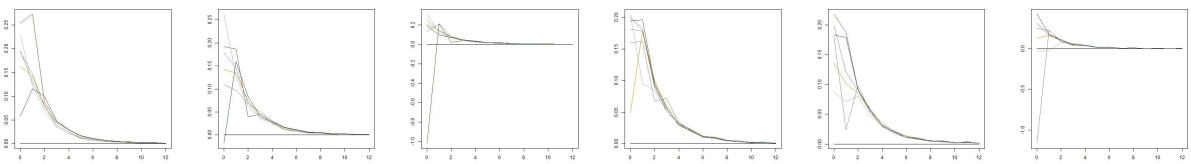
(m) Sn **supply** (n) Sn **demand** (o) Sn **price** (p) Zn **supply** (q) Zn **demand** (r) Zn **price**

These figures the differences in the conditional value at risk (CoVaR) of the spillover effects from the individual supply (**supply**), demand (**demand**) and price (**price**) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) to the commodity demand of aluminum (Al), copper (Cu), Nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) of the GVAR model vs. the MS-GVAR model under the calm regime.

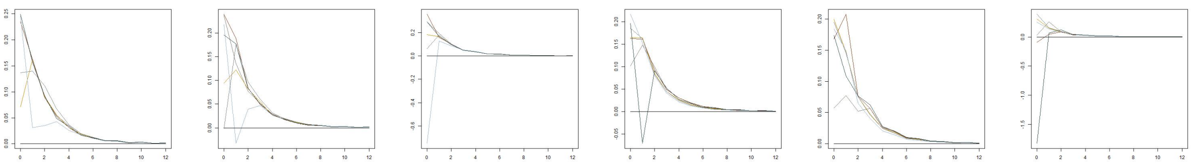
Figure D.6: Differences in the conditional value at risk of the spillover effects on the **price** variables between the GVAR model vs. the MS-GVAR model under the calm regime



(a) Al **supply** (b) Al **demand** (c) Al **price** (d) Cu **supply** (e) Cu **demand** (f) Cu **price**



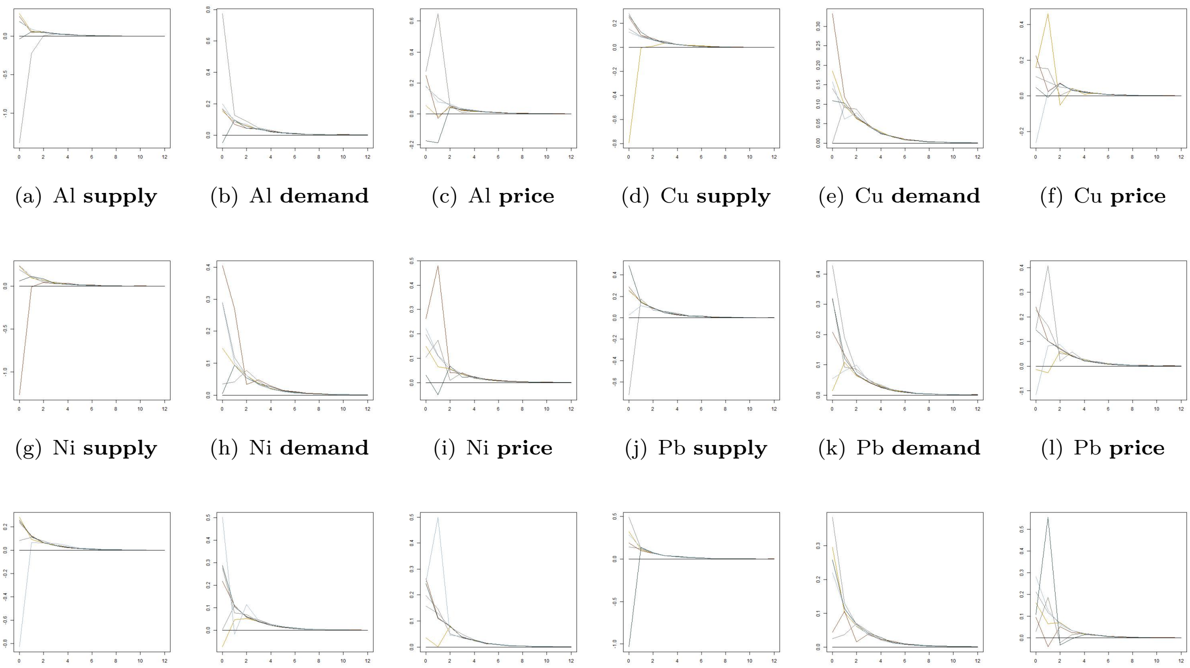
(g) Ni **supply** (h) Ni **demand** (i) Ni **price** (j) Pb **supply** (k) Pb **demand** (l) Pb **price**



(m) Sn **supply** (n) Sn **demand** (o) Sn **price** (p) Zn **supply** (q) Zn **demand** (r) Zn **price**

These figures indicate the differences in the conditional value at risk (CoVaR) of the spillover effects from the individual supply (**supply**), demand (**demand**) and price (**price**) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) to the commodity prices of aluminum (Al), copper (Cu), Nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) of the GVAR model vs. the MS-GVAR model under the calm regime.

Figure D.7: Differences in the conditional value at risk of the spillover effects on the **supply** variables between the GVAR model vs. the MS-GVAR model under the volatile regime



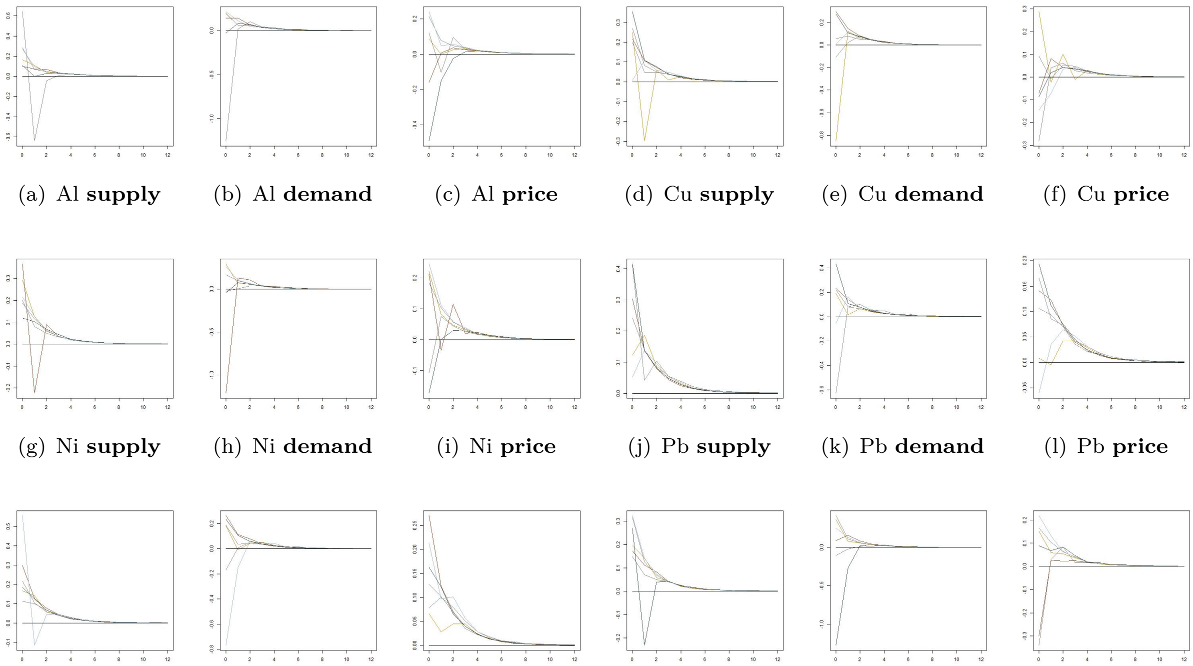
(a) Al **supply** (b) Al **demand** (c) Al **price** (d) Cu **supply** (e) Cu **demand** (f) Cu **price**

(g) Ni **supply** (h) Ni **demand** (i) Ni **price** (j) Pb **supply** (k) Pb **demand** (l) Pb **price**

(m) Sn **supply** (n) Sn **demand** (o) Sn **price** (p) Zn **supply** (q) Zn **demand** (r) Zn **price**

These figures indicate the differences in the conditional value at risk (CoVaR) of the spillover effects from the individual supply (**supply**), demand (**demand**) and price (**price**) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) to the commodity supply of aluminum (Al), copper (Cu), Nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) of the GVAR model vs. the MS-GVAR model under the volatile regime.

Figure D.8: Differences in the conditional value at risk of the spillover effects on the **demand** variables between the GVAR model vs. the MS-GVAR model under the volatile regime



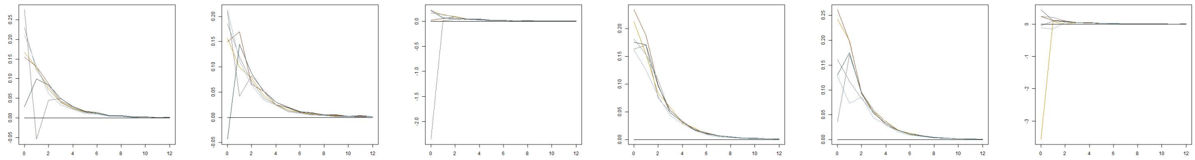
(a) Al **supply** (b) Al **demand** (c) Al **price** (d) Cu **supply** (e) Cu **demand** (f) Cu **price**

(g) Ni **supply** (h) Ni **demand** (i) Ni **price** (j) Pb **supply** (k) Pb **demand** (l) Pb **price**

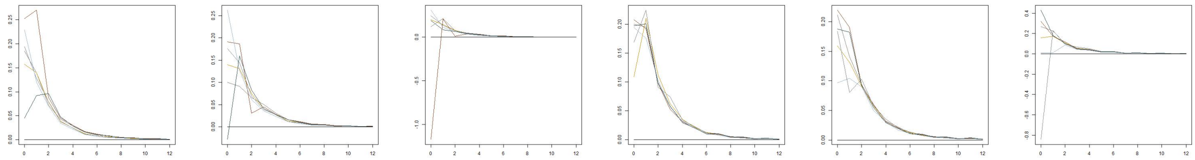
(m) Sn **supply** (n) Sn **demand** (o) Sn **price** (p) Zn **supply** (q) Zn **demand** (r) Zn **price**

These figures indicate the differences in the conditional value at risk (CoVaR) of the spillover effects from the individual supply (**supply**), demand (**demand**) and price (**price**) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) to the commodity demand of aluminum (Al), copper (Cu), Nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) of the GVAR model vs. the MS-GVAR model under the volatile regime.

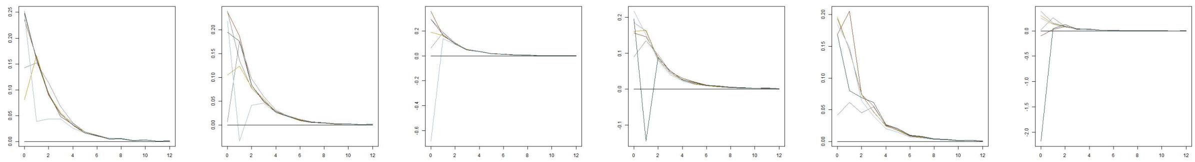
Figure D.9: Differences in the conditional value at risk of the spillover effects on the **price** variables between the GVAR model vs. the MS-GVAR model under the volatile regime



(a) Al **supply** (b) Al **demand** (c) Al **price** (d) Cu **supply** (e) Cu **demand** (f) Cu **price**



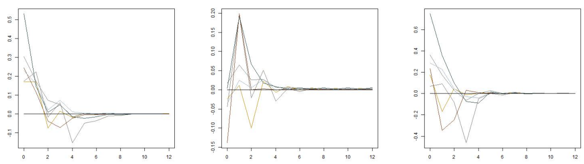
(g) Ni **supply** (h) Ni **demand** (i) Ni **price** (j) Pb **supply** (k) Pb **demand** (l) Pb **price**



(m) Sn **supply** (n) Sn **demand** (o) Sn **price** (p) Zn **supply** (q) Zn **demand** (r) Zn **price**

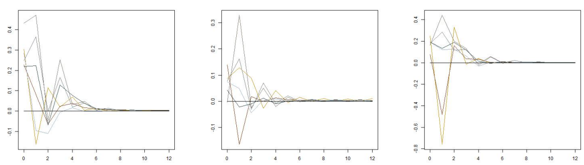
These figures indicate the differences in the conditional value at risk (CoVaR) of the spillover effects from the individual supply (**supply**), demand (**demand**) and price (**price**) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) to the commodity prices of aluminum (Al), copper (Cu), Nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) of the GVAR model vs. the MS-GVAR model under the volatile regime.

Figure D.10: Differences in the conditional value at risk of the spillover effects from the exogenous variables to the commodity markets of the GVAR model vs. the MS-GVAR model under the calm regime



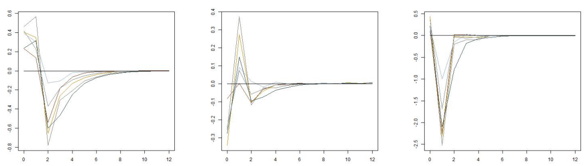
(a) IP (b) FX (c) FFR

supply



(d) IP (e) FX (f) FFR

demand

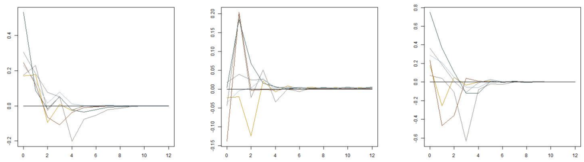


(g) IP (h) FX (i) FFR

price

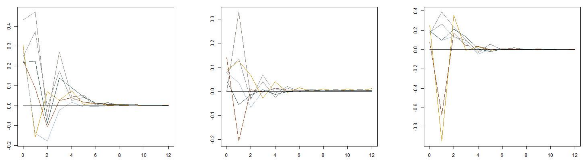
These figures indicate the differences in the conditional value at risk (CoVaR) of the spillover effects from the exogenous variables world industrial production (IP), U.S. dollar index (FX) and Federal Funds Effective Rate (FFR) to the commodity supply (**supply**), demand (**demand**) and price (**price**) of aluminum (Al), copper (Cu), Nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) of the GVAR model vs. the MS-GVAR model under the calm regime.

Figure D.11: Differences in the conditional value at risk of the spillover effects from the exogenous variables to the commodity markets of the GVAR model vs. the MS-GVAR model under the volatile regime



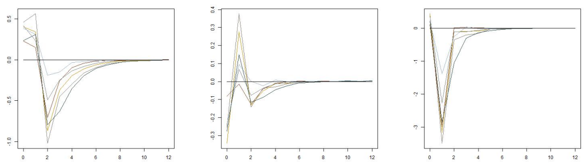
(a) IP (b) FX (c) FFR

supply



(d) IP (e) FX (f) FFR

demand



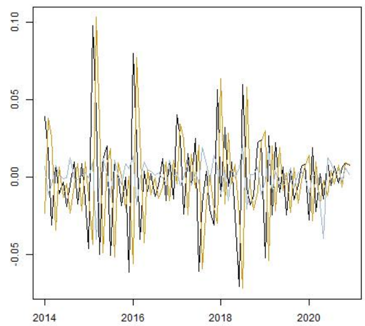
(g) IP (h) FX (i) FFR

price

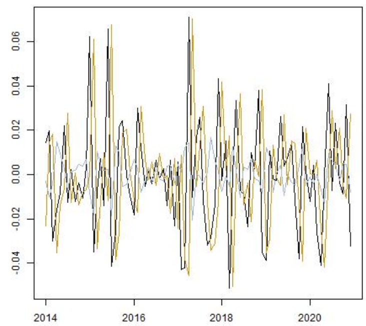
These figures indicate the differences in the conditional value at risk (CoVaR) of the spillover effects from the exogenous variables world industrial production (IP), U.S. dollar index (FX) and Federal Funds Effective Rate (FFR) to the commodity supply (**supply**), demand (**demand**) and price (**price**) of aluminum (Al), copper (Cu), Nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) of the GVAR model vs. the MS-GVAR model under the volatile regime.

D.2.4 Out-of-sample Forecast Performance of the (time-varying) Commodity Market Models[8]

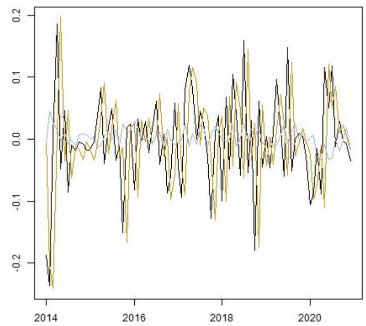
Figure D.12: Observed and predicted supply in the out-of-sample period



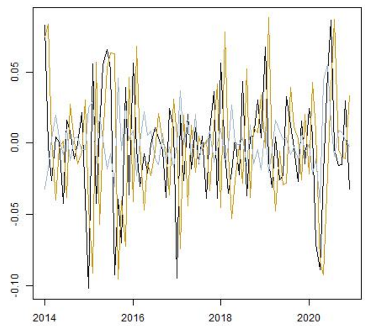
(a) Al **supply**



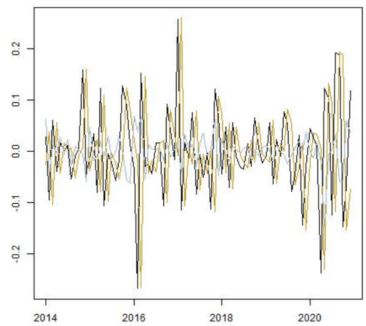
(b) Cu **supply**



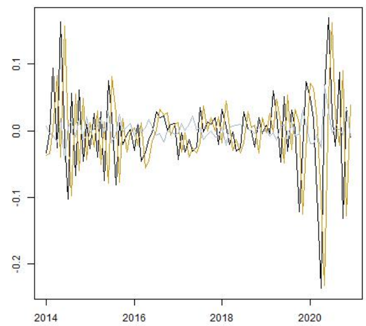
(c) Ni **supply**



(d) Pb **supply**



(e) Sn **supply**

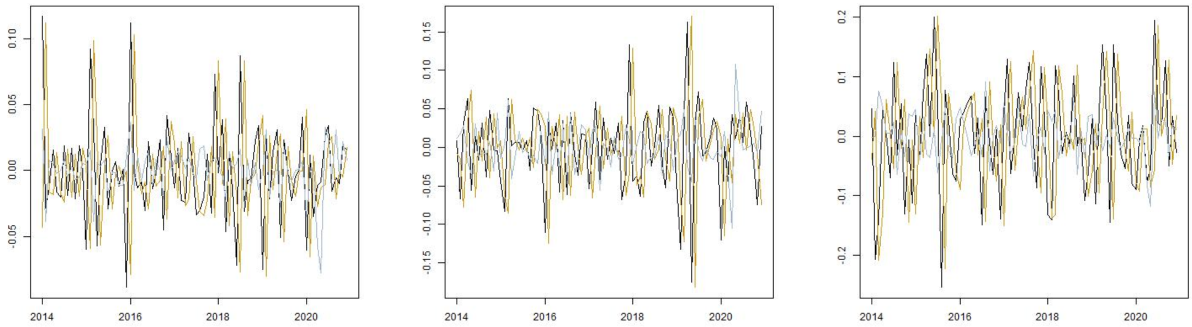


(f) Zn **supply**

These figures compare the observed supply (**supply**) of aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) in the out-of-sample period from January 2014 to December 2020, indicated by the black line, with the estimated supply of the GVAR model and the MS-GVAR model.

[8]As all original variables were non-stationary, we base the entire analysis on the logarithmic return data and hence, also the forecasts are forecasts of logarithmic returns.

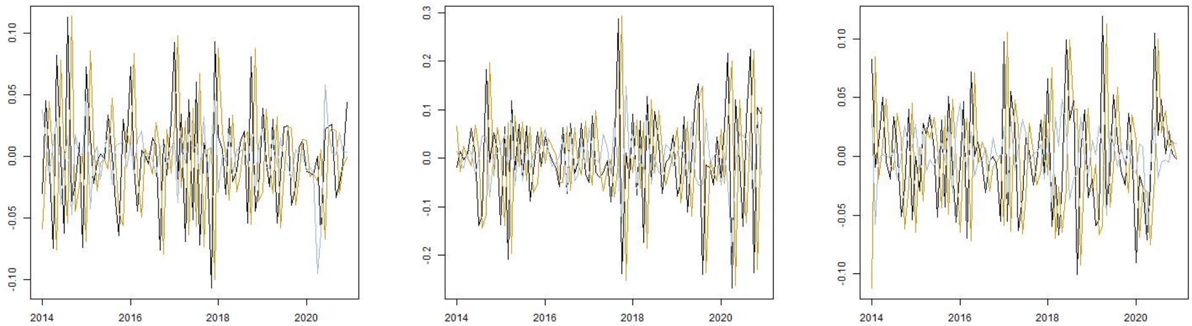
Figure D.13: Observed and predicted demand in the out-of-sample period



(a) Al **demand**

(b) Cu **demand**

(c) Ni **demand**



(d) Pb **demand**

(e) Sn **demand**

(f) Zn **demand**

These figures compare the observed demand (**demand**) of aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) in the out-of-sample period from January 2014 to December 2020, indicated by the black line, with the estimated demand of the GVAR model and the MS-GVAR model.

D.3 Scarcity Risk of the German Energiewende

D.3.1 Results of the Global Commodity Market Models

The objective of this study is to assess and compare the scarcity risk of the annual material requirements of four potential transformation pathways of the German energy system. Hereby, we apply the proposed framework in Section 3, which assesses the scarcity risk of resource-demanding projects, taking into account the substitutability of commodities, the future required resource amounts of the project as well as the historical information available, in the context of the German Energiewende. In particular, we model the commodity markets via the (MS-)GVAR model. Thereby, the interrelations between the commodities reflect their prospective relations induced by the German Energiewende, using the weight matrices in Table 4.27, Table 4.28, Table 4.29 and Table 4.30 (Table 4.31, Table 4.32, Table 4.33 and Table 4.34), representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$ paths, respectively. In the following, we provide more information on the time-invariant GVAR as well as the time-varying MS-GVAR models, used for the risk assessment in Section 5.3.

D.3.1.1 Time-invariant Commodity Market Models

In general, we estimate the GVAR models based on the weight matrices induced by the German Energiewende with one lag for the endogenous variables (commodity-specific supply, demand, and price) as well as exogenous variables (world gross domestic product (GDP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR)) and without intercept, due to data limitations. Hereby, we include the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn).[9] For a comparison of the results derived under the GVAR and MS-GVAR model, we also apply the model only on the industrial metal markets. Moreover, for a better understanding of the relations within the individual commodity markets, we also examine commodity-specific VAR models.

Overall, despite the lithium supply, all variables in the individual as well as the global VAR models do not exhibit any autocorrelation at the 5% significance level, according to the Durbin-Watson (DW) test. Moreover, neither model suffers from heteroscedasticity nor structural breaks, except for the indium market, indicating the time-invariant model is not able to fully display the time-varying relationship between indium's variables, demanding for a time-dependent model. In addition, the Henze-Zirkler (HZ) test implies the residuals of the GVAR model, based on the different weight matrices, are multivariate normal distributed, see Table D.9. The individual VAR models of cobalt, indium, lithium, platinum, and zinc have non-normal errors, which is why the true generalized impulse response functions may deviate from the presented ones. Further, due to the autocorrelation (heteroscedasticity) in the lithium (indium) market, the confidence bounds of the GIRFs may deviate, since the sieve bootstrap procedure assumes independent and identically distributed residuals. However, the GIRF analysis of the GVAR models based on the weight matrices induced by the German Energiewende is only reported to provide better insights into the market structure for the risk assessment, therefore, we do not adjust the specifications.

[9] Due to data limitations, the rare earth metals dysprosium (Dy) and neodymium (Nd) are not included in the GVAR models.

Table D.9: Test results for autocorrelation, heteroscedasticity, structural breaks and normality for the individual VAR models and the GVAR models based on the $REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$ path

		DW						ARCH-LM		OLS-CUSUM		HZ	
		supply		**demand**		**price**							
		Stat.	p	Stat.	p	Stat.	p	Stat.	p	Stat.	p	Stat.	p
indiv. VAR	Ag	1.88	0.30	1.95	0.38	1.93	0.36	35.29	0.50	0.56	0.92	0.72	0.30
	Al	1.81	0.23	1.82	0.24	2.04	0.50	54.85	0.02	1.01	0.26	0.76	0.22
	Co	2.03	0.50	1.89	0.31	2.10	0.59	38.79	0.35	0.65	0.80	0.96	0.03
	Cu	2.12	0.62	1.83	0.25	2.20	0.72	19.69	0.99	0.69	0.74	0.55	0.75
	In	2.11	0.64	2.22	0.77	2.11	0.65	58.92	0.01	0.80	0.55	1.00	0.02
	Li	0.97	0.00	2.19	0.71	1.58	0.07	53.76	0.03	0.74	0.65	0.93	0.04
	Ni	1.93	0.36	1.99	0.43	2.19	0.68	23.65	0.94	0.83	0.50	0.81	0.15
	Pb	2.09	0.61	1.73	0.17	1.97	0.45	26.18	0.89	0.97	0.31	0.60	0.63
	Pt	1.75	0.20	1.88	0.34	1.94	0.42	43.18	0.19	0.78	0.58	0.99	0.02
	Sn	2.00	0.48	2.02	0.51	2.06	0.57	36.96	0.42	0.77	0.59	0.90	0.06
	Zn	2.10	0.58	1.55	0.05	1.99	0.44	24.08	0.94	0.56	0.92	0.96	0.03
$REMod-REF$	Ag	2.03	0.47	2.10	0.55	1.95	0.36	31.73	0.67	0.57	0.90	1.00	0.35
	Al	1.67	0.11	2.09	0.55	2.17	0.66	25.60	0.90	0.66	0.78	1.00	0.35
	Co	2.28	0.80	2.15	0.66	2.15	0.66	22.88	0.96	0.60	0.87	1.00	0.35
	Cu	2.17	0.67	1.68	0.12	1.93	0.36	34.94	0.52	0.58	0.89	1.00	0.35
	In	2.09	0.58	2.27	0.80	2.07	0.56	50.96	0.05	0.79	0.56	1.00	0.35
	Li	1.12	0.00	1.86	0.29	1.72	0.16	33.29	0.60	0.76	0.62	1.00	0.35
	Ni	1.91	0.32	1.99	0.41	2.20	0.68	35.65	0.48	0.45	0.99	1.00	0.35
	Pb	2.09	0.61	1.71	0.18	2.24	0.78	48.27	0.08	0.53	0.94	1.00	0.35
	Pt	1.81	0.23	1.69	0.13	1.90	0.34	33.68	0.58	0.69	0.74	1.00	0.35
	Sn	1.82	0.25	2.11	0.60	1.95	0.40	28.30	0.82	0.89	0.40	1.00	0.35
	Zn	2.12	0.58	1.91	0.31	1.95	0.36	26.54	0.88	0.84	0.48	1.00	0.35
$REMod-SUF$	Ag	2.05	0.50	2.07	0.52	1.90	0.32	29.47	0.77	0.64	0.80	1.00	0.30
	Al	1.63	0.09	2.08	0.53	2.17	0.65	26.44	0.88	0.61	0.85	1.00	0.30
	Co	2.28	0.78	2.16	0.66	2.14	0.64	25.33	0.91	0.66	0.78	1.00	0.30
	Cu	2.19	0.68	1.67	0.11	1.91	0.33	35.84	0.48	0.58	0.89	1.00	0.30
	In	2.09	0.59	2.24	0.75	2.06	0.54	53.72	0.03	0.79	0.56	1.00	0.30
	Li	1.14	0.00	1.84	0.26	1.77	0.19	34.84	0.52	0.76	0.61	1.00	0.30
	Ni	1.91	0.30	1.95	0.35	2.19	0.66	35.87	0.47	0.44	0.99	1.00	0.30
	Pb	2.02	0.52	1.67	0.14	2.18	0.72	49.09	0.07	0.55	0.92	1.00	0.30
	Pt	1.78	0.21	1.65	0.11	1.94	0.38	32.66	0.63	0.69	0.72	1.00	0.30
	Sn	1.82	0.25	2.13	0.63	1.98	0.44	30.02	0.75	0.82	0.51	1.00	0.30
	Zn	2.10	0.57	1.87	0.29	1.94	0.36	26.98	0.86	0.88	0.42	1.00	0.30
$REMod-PER$	Ag	2.02	0.46	2.09	0.53	1.89	0.29	35.29	0.50	0.65	0.80	1.00	0.30
	Al	1.69	0.12	2.07	0.53	2.17	0.66	27.23	0.85	0.68	0.75	1.00	0.30
	Co	2.23	0.75	2.14	0.65	2.15	0.65	22.20	0.97	0.57	0.90	1.00	0.30
	Cu	2.18	0.67	1.71	0.14	1.92	0.35	34.80	0.53	0.57	0.90	1.00	0.30
	In	2.10	0.59	2.22	0.74	2.06	0.54	53.13	0.03	0.81	0.52	1.00	0.30
	Li	1.11	0.00	1.87	0.31	1.72	0.16	31.21	0.70	0.74	0.64	1.00	0.30
	Ni	1.92	0.33	1.98	0.40	2.18	0.65	35.23	0.51	0.45	0.99	1.00	0.30
	Pb	2.07	0.58	1.70	0.16	2.27	0.80	49.39	0.07	0.54	0.93	1.00	0.30
	Pt	1.81	0.23	1.70	0.13	1.91	0.34	33.90	0.57	0.68	0.75	1.00	0.30
	Sn	1.79	0.21	2.12	0.60	1.96	0.41	26.64	0.87	0.87	0.43	1.00	0.30
	Zn	2.11	0.58	1.87	0.29	1.90	0.31	29.44	0.77	0.84	0.48	1.00	0.30
$REMod-UNA$	Ag	2.08	0.53	2.09	0.53	1.92	0.33	30.58	0.72	0.61	0.85	1.00	0.35
	Al	1.66	0.10	2.07	0.51	2.17	0.64	27.27	0.85	0.65	0.79	1.00	0.35
	Co	2.25	0.77	2.16	0.66	2.14	0.64	23.68	0.94	0.61	0.85	1.00	0.35
	Cu	2.19	0.68	1.67	0.11	1.91	0.33	35.76	0.48	0.57	0.90	1.00	0.35
	In	2.10	0.60	2.23	0.75	2.07	0.56	53.00	0.03	0.77	0.60	1.00	0.35
	Li	1.13	0.00	1.85	0.27	1.75	0.18	33.30	0.60	0.75	0.62	1.00	0.35
	Ni	1.92	0.32	1.97	0.38	2.19	0.66	36.59	0.44	0.44	0.99	1.00	0.35
	Pb	2.11	0.63	1.72	0.18	2.27	0.81	48.34	0.08	0.52	0.95	1.00	0.35
	Pt	1.81	0.24	1.71	0.15	1.90	0.34	34.73	0.53	0.71	0.69	1.00	0.35
	Sn	1.78	0.21	2.13	0.62	1.94	0.38	29.34	0.78	0.87	0.44	1.00	0.35

Test results for autocorrelation, heteroscedasticity, structural breaks and normality for the individual VAR models and the GVAR models based on the $REMod - REF$, $REMod - SUF$, $REMod - PER$, and $REMod - UNA$ path

		DW						ARCH-LM		OLS-CUSUM		HZ	
		Stat.	p	Stat.	p	Stat.	p	Stat.	p	Stat.	p	Stat.	p
		supply		**demand**		**price**							
	Zn	2.11	0.56	1.88	0.28	1.97	0.39	25.00	0.92	0.82	0.51	1.00	0.35
$REMod - REF$	Al	1.90	0.31	2.14	0.61	2.08	0.54	35.39	0.50	0.68	0.74	1.00	0.28
	Cu	2.07	0.52	1.80	0.20	2.00	0.43	39.74	0.31	0.59	0.88	1.00	0.28
	Ni	1.86	0.26	1.96	0.37	2.27	0.76	37.53	0.40	0.50	0.96	1.00	0.28
	Pb	1.93	0.40	1.89	0.35	2.17	0.70	38.34	0.36	0.77	0.60	1.00	0.28
	Sn	1.69	0.12	2.16	0.64	1.95	0.38	24.81	0.92	0.58	0.89	1.00	0.28
	Zn	2.16	0.62	1.93	0.33	1.80	0.20	21.39	0.97	0.93	0.35	1.00	0.28
$REMod - SUF$	Al	1.83	0.24	2.16	0.63	2.08	0.53	29.46	0.77	0.73	0.67	1.00	0.28
	Cu	2.09	0.55	1.79	0.20	2.00	0.43	37.14	0.42	0.58	0.89	1.00	0.28
	Ni	1.86	0.26	1.93	0.34	2.27	0.76	33.91	0.57	0.52	0.95	1.00	0.28
	Pb	1.91	0.36	1.86	0.30	2.16	0.68	33.86	0.57	0.76	0.62	1.00	0.28
	Sn	1.70	0.13	2.12	0.59	1.95	0.38	24.18	0.93	0.58	0.89	1.00	0.28
	Zn	2.16	0.62	1.92	0.32	1.80	0.19	20.89	0.98	0.94	0.34	1.00	0.28
$REMod - PER$	Al	1.92	0.33	2.13	0.60	2.08	0.54	36.67	0.44	0.68	0.74	1.00	0.30
	Cu	2.07	0.53	1.80	0.20	2.01	0.44	40.76	0.27	0.58	0.88	1.00	0.30
	Ni	1.87	0.27	1.95	0.36	2.26	0.75	36.23	0.46	0.52	0.95	1.00	0.30
	Pb	1.93	0.39	1.89	0.35	2.16	0.68	37.57	0.40	0.77	0.60	1.00	0.30
	Sn	1.68	0.12	2.15	0.63	1.95	0.38	24.82	0.92	0.58	0.89	1.00	0.30
	Zn	2.16	0.62	1.91	0.30	1.80	0.19	23.09	0.95	0.92	0.37	1.00	0.30
$REMod - UNA$	Al	1.87	0.28	2.13	0.60	2.06	0.50	26.33	0.88	0.74	0.64	1.00	0.21
	Cu	2.10	0.56	1.81	0.21	1.99	0.42	32.97	0.61	0.57	0.90	1.00	0.21
	Ni	1.86	0.26	1.94	0.35	2.26	0.74	37.88	0.38	0.54	0.94	1.00	0.21
	Pb	1.94	0.41	1.91	0.37	2.18	0.70	39.02	0.34	0.77	0.59	1.00	0.21
	Sn	1.64	0.09	2.20	0.69	1.94	0.37	25.25	0.91	0.58	0.89	1.00	0.21
	Zn	2.14	0.61	1.91	0.32	1.82	0.22	19.44	0.99	0.90	0.40	1.00	0.21
		GDP		FX		FFR							
exogenous VAR		1.56	0.07	1.88	0.35	1.96	0.46	18.54	0.99	0.57	0.90	0.83	0.13

This table displays the results of the Durbin-Watson (DW) test for autocorrelation, the multivariate ARCH Lagrange multiplier (ARCH-LM) test for heteroscedasticity, the OLS-cumulative sums of standardized residuals (OLS-CUSUM) test for structural breaks, and the Henze-Zirkler (HZ) test for normality. Hereby, the Durbin-Watson test is applied on each, individual regression equation of the VAR model, corresponding to the commodity-specific supply, demand, and price of the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), whereas the multivariate ARCH Lagrange multiplier test and the OLS-cumulative sums of standardized residuals test are applied on the commodity-specific VAR models, and the Henze-Zirkler test is applied on the residuals of the (G)VAR model. In particular, we report the test results for the individual VAR models, the GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod - REF$, $REMod - SUF$, $REMod - PER$, and $REMod - UNA$ transformation path, as well as the VAR model of the exogenous variables world gross domestic product (GDP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR), which are estimated on the sample period from 1970 to 2019.

In the following, we briefly consider the spillover effects within the individual markets. Thereafter, we examine the interdependencies between the commodity markets, by aggregating the individual models to the GVAR model.

D.3.1.1.1 Individual Commodity Market Models In line with the time-invariant analysis in Section 5.1, we first focus on the individual commodity market models. Therefore, we estimate generalized impulse response function (GIRF) on the individual VAR models to examine the spillover effects within the markets.[10] We summarize the GIRF results, based on the 68% confidence bounds, of each individual commodity market in Table D.10, where we indicate

[10]The VAR models of the industrial metals have already been presented in Section 5.1.1, but are included in this section again for the sake of completeness.

significant positive, or negative, responses of the column variables to a shock in the row variables by a (+) or (-) respectively.

Overall, there are no spillover effects between supply, demand and price in the copper, indium, lithium, platinum, and zinc markets, whereas demand (supply) and price affect each other in the aluminum, cobalt, and nickel (silver, nickel, and tin) markets. Moreover, supply and demand interact to each other for lead and tin. While we expect rising (decreasing) prices in response to a positive shock in the demand (supply), we suppose an increase in the prices cause decreasing (increasing) demand (supply). However, the results indicate a concurrent behavior, probably caused by unobservable indirect effects, included in the GIRF analysis.

Table D.10: GIRF results of the individual, commodity-specific VAR models for all considered metals

	Ag			Al			Co			Cu			In			Li			Ni			Pb			Pt			Sn			Zn		
	supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
supply	+		-	+			+			+			+			+			+		+	+	+		+			+	-	+	+		
demand		+			+	-		+	-		+			+			+			+	-	+	+			+		-	+			+	
price	-		+		-	+		-	+			+			+			+	+	-	+			+			+	+		+			+

This table displays the results of the GIRF analysis of the individual, commodity-specific VAR models, estimated on annual data from 1970 to 2019, showing the response of the column variables to a shock of the row variables supply (**supply**), demand (**demand**) and price (**price**) of the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), where significant positive (+) or negative (-) effects are displayed, based on the 68%- level.

To account for the impact of the economy on commodity prices, we analyze the impact of shocks to the exogenous variables on the individual markets. Hereby, we investigate the spillover effects from the economic activity, the exchange rate or the interest rate to the commodity-specific variables. Therefore, we apply the GIRF analysis on the VAR model of the exogenous variables, world gross domestic product (GDP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR), using Equation 3.17, Equation 3.19, and Equation 3.20.

Table D.11: GIRF results of the individual VAR models for all considered metals for shocks to the exogenous variables

	Ag			Al			Co			Cu			In			Li			Ni			Pb			Pt			Sn			Zn		
	supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
GDP	+	+	+	+	+	+	+	+	+	-	+	+		+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
FX	-	-	-	-	-	-	-	-	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		-	-	-	-	
FFR	+		+		+	+	-	+	+	+	+	+		+		+	+	+	+	+	+		+	+	+		+	+	+				+

This table displays the results of the GIRF analysis of the individual, commodity-specific VAR models, estimated on annual data from 1970 to 2019, showing the response of the column variables supply (**supply**), demand (**demand**) and price (**price**) of the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), to a shock of the row variables world gross domestic product (GDP), U.S. dollar index (FX), and the Federal Funds Effective Rate (FFR), where significant positive (+) or negative (-) effects are displayed, based on the 68%- level.

The results in Table D.11 indicate the commodity markets are affected by shocks in the economy to a similar extent, in line with the results in Section 5.1. While an increase in the economic activity (a contrarian monetary policy), represented by a positive shock in the world gross domestic product (Federal Funds Effective Rate), cause rising commodity markets, an appreciation of the dollar, represented by a positive shock in the U.S. dollar index, leads to a reduction in the commodity markets. The only exception is copper supply which negatively (positively) reacts to the shock in the world gross domestic product (U.S. dollar index), probably caused by copper's role as leading indicator of the global economic situation.

D.3.1.1.2 Global Commodity Market Models The aggregation of the individual VAR models to global VAR models enables for interdependencies between the commodities. Hereby, we reflect the relation between the metals via weight matrices, representing the resource requirements of the German Energiewende, see Section 4.4.4, and apply the models in the risk assessment framework to examine the scarcity risk of the transformation paths. For a better understanding of the underlying GVAR models, we analyze the dynamic properties of the corresponding GVAR models via generalized impulse response functions (GIRFs), according to Equation 3.13, whereby direct as well as indirect effects on the attributes to an innovation of one standard deviation in a certain variable are investigated, see Table D.12 and Table D.13 for the models on the key elements and for the industrial metals, respectively. Our analysis is based on the 68% confidence bounds obtained by a sieve bootstrap procedure with 1000 replications and the recent observations as input variables, as proposed in Dées et al. (2007), in line with the GVAR models presented in Section 5.1.

Overall, the spillover effects are comparable across the GVAR model based on the different weight matrices, representing the resource requirements of the German Energiewende, as the scaled relations derived from the material requirements of the four transformation pathways are similar, see Table D.12. Since the $REMod-REF$ path represents the baseline scenario, we focus on the results of the GVAR model based on the weight matrix $REMod-REF$ in the following.

Table D.12: GIRF results of the GVAR models for the key elements of the German Energiewende

			Ag			Al			Co			Cu			In			Li			Ni			Pb			Pt			Sn			Zn		
			supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
Ag	**supply**	$REMod-REF$	+		-							+																							
		$REMod-SUF$	+		-							+																							
		$REMod-PER$	+		-							+																	-						
		$REMod-UNA$	+		-							+																							
	demand	$REMod-REF$		+													-																		
		$REMod-SUF$		+						+							-																		
		$REMod-PER$		+													-																		
		$REMod-UNA$		+						+							-																		
	price	$REMod-REF$	-		+						-	-			-						+								+	-		+			
		$REMod-SUF$	-		+			+			-	-			-						+								+	-		+			
		$REMod-PER$	-		+						-	-			-						+								+	-					
		$REMod-UNA$	-		+			+			-	-									+								+	-		+			
Al	**supply**	$REMod-REF$				+													+		+					+									
		$REMod-SUF$				+													+		+				-	+						+			
		$REMod-PER$				+													+		+														
		$REMod-UNA$				+													+		+														
	demand	$REMod-REF$		-			+						+							-				+										+	
		$REMod-SUF$		-			+						+							-				+	+									+	
		$REMod-PER$		-			+						+							-				+										+	
		$REMod-UNA$		-			+						+							-				+	+									+	
	price	$REMod-REF$			+			+			-	-											+												
		$REMod-SUF$			+			+			-	-											+						+						
		$REMod-PER$			+			+			-	-											+												
		$REMod-UNA$			+			+			-	-											+												
Co	**supply**	$REMod-REF$							+											-							+		+		+				
		$REMod-SUF$							+											-							+		+		+				
		$REMod-PER$							+											-							+		+		+				
		$REMod-UNA$							+											-							+		+		+				
	demand	$REMod-REF$			+			+		+	-									-									+	-	+				
		$REMod-SUF$			+			+		+	-									-									+	-	+				
		$REMod-PER$		+	+			+		+	-									-									+	-	+				
		$REMod-UNA$			+			+		+	-									-									+	-	+				
	price	$REMod-REF$									+									+	-					+						+			
		$REMod-SUF$									+									+						+						+			

GIRF results of the GVAR models for the key elements of the German Energiewende

			Ag			Al			Co			Cu			In			Li			Ni			Pb			Pt			Sn			Zn		
			supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
		$REMod-PER$									+									+	-					+						+			
		$REMod-UNA$									+									+						+						+			
Cu	**supply**	$REMod-REF$	+		-			-				+																	-						
		$REMod-SUF$	+		-			-				+																	-	+					
		$REMod-PER$	+		-			-				+																	-						
		$REMod-UNA$	+		-			-				+																							
	demand	$REMod-REF$				+	+						+	-				-																	-
		$REMod-SUF$				+	+						+	-				-								+									-
		$REMod-PER$				+	+						+	-				-																	-
		$REMod-UNA$				+	+						+	-				-																	-
	price	$REMod-REF$									-			+	+													-							+
		$REMod-SUF$									-		-	+	+	+				-								-							+
		$REMod-PER$			+						-		-	+	+													-							+
		$REMod-UNA$									-		-	+	+	+				-								-							+
In	**supply**	$REMod-REF$												+	+					-			+												
		$REMod-SUF$												+	+					-			+												
		$REMod-PER$												+	+					-			+												
		$REMod-UNA$												+	+					-			+												
	demand	$REMod-REF$										+		+		+	-	-																	
		$REMod-SUF$										+		+		+		-																	
		$REMod-PER$										+		+		+		-																	
		$REMod-UNA$										+		+		+		-																	
	price	$REMod-REF$		-													+																		-
		$REMod-SUF$		-													+																		-
		$REMod-PER$		-													+																		-
		$REMod-UNA$		-													+																		-
Li	**supply**	$REMod-REF$							+							-		+				+					+								+
		$REMod-SUF$							+				-			-		+				+					+								+
		$REMod-PER$							+							-		+				+					+								+
		$REMod-UNA$							+				-			-		+				+					+								+
	demand	$REMod-REF$				+			-						-				+	+								+				+			
		$REMod-SUF$				+			-						-				+	+								+				+			
		$REMod-PER$				+			-						-				+	+								+				+			
		$REMod-UNA$				+			-						-				+	+								+				+			
	price	$REMod-REF$	+									+			-				+	+						+									
		$REMod-SUF$	+									+			-				+	+						+						+			
		$REMod-PER$	+						-			+							+	+						+									
		$REMod-UNA$	+									+			-				+	+						+									
Ni	**supply**	$REMod-REF$			+						-										+									-					
		$REMod-SUF$	-		+	+					-										+									-					
		$REMod-PER$			+	+					-										+									-					
		$REMod-UNA$	-		+	+					-										+									-					
	demand	$REMod-REF$																+				+			+										
		$REMod-SUF$																+				+			+	-									
		$REMod-PER$																+				+			+										
		$REMod-UNA$																+				+			+										
	price	$REMod-REF$						+						+	+								+							+					+
		$REMod-SUF$						+						+	+					-			+							+					+
		$REMod-PER$						+						+	+					-			+							+					+
		$REMod-UNA$						+						+	+					-			+							+					+
Pb	**supply**	$REMod-REF$																		-	+			+	+		-						+		
		$REMod-SUF$																		-				+	+		-						+		
		$REMod-PER$																						+	+		-						+		
		$REMod-UNA$																		-				+	+		-						+		
	demand	$REMod-REF$					+							+						-		+		+	+							+		+	+
		$REMod-SUF$					+							+						-		+		+	+							+		+	+
		$REMod-PER$												+						-		+		+	+				-			+		+	+
		$REMod-UNA$												+	+					-		+		+	+							+		+	+
		$REMod-REF$				+												-								+									

GIRF results of the GVAR models for the key elements of the German Energiewende

			Ag			Al			Co			Cu			In			Li			Ni			Pb			Pt			Sn			Zn		
			supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
Pb	**price**	$REMod-SUF$																-								+									
		$REMod-PER$																-								+									
		$REMod-UNA$																								+									
Pt	**supply**	$REMod-REF$							+								+	+			+			-			+					-			
		$REMod-SUF$							+								+	+			+			-			+					-			
		$REMod-PER$							+								+	+						-			+					-			
		$REMod-UNA$							+								+	+						-			+					-			
	demand	$REMod-REF$									+			-					+									+							
		$REMod-SUF$									+			-					+									+							
		$REMod-PER$									+								+									+							
		$REMod-UNA$									+			-					+									+							-
	price	$REMod-REF$			+			+				-																	+						
		$REMod-SUF$			+			+				-																	+						
		$REMod-PER$			+			+			-																		+						
		$REMod-UNA$			+			+			-	-																	+						
Sn	**supply**	$REMod-REF$	+		-																-		+							+					
		$REMod-SUF$	+		-					-											-		+							+					
		$REMod-PER$	+		-																-		+							+					
		$REMod-UNA$	+		-																-		+							+					
	demand	$REMod-REF$							+	+													-	-						-	+	-			
		$REMod-SUF$							+	+													-	-			+			-	+	-			
		$REMod-PER$							+	+													-	-			+			-	+	-			
		$REMod-UNA$							+	+													-	-						-	+	-			
	price	$REMod-REF$				+									-	-											-				-	+			
		$REMod-SUF$				+					+				-	-											-				-	+			-
		$REMod-PER$			+	+									-	-											-				-	+			-
		$REMod-UNA$			+	+									-	-											-				-	+			-
Zn	**supply**	$REMod-REF$																						+									+		
		$REMod-SUF$																						+									+		
		$REMod-PER$																						+									+		
		$REMod-UNA$																						+									+		
	demand	$REMod-REF$					+							+						-					+									+	
		$REMod-SUF$					+							+						-					+									+	
		$REMod-PER$					+													-					+									+	
		$REMod-UNA$					+							+						-					+									+	
	price	$REMod-REF$												+			-	+																	+
		$REMod-SUF$												+			-	+					+												+
		$REMod-PER$												+			-	+					+												+
		$REMod-UNA$												+			-	+					+												+

This table displays the results of the GIRF analysis of the GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$ transformation paths and estimated on annual data from 1970 to 2019. We analyze the response of the column variables to a shock of the row variables supply (**supply**), demand (**demand**) and price (**price**) of silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn). Significant positive (+) or negative (-) effects on the 68%- level are displayed.

In analogy to Section 5.1.2, we start with a comparison of the results of the individual VAR models, displayed in Table D.10, to the commodity-specific results of the GVAR models, before we analyze the spillover effects in the cross-commodity dimension in detail. Similar to the GVAR model based on the demand weight matrix, representing the co-consumption of the commodities, the spillover effects in the individual commodity markets change if the individual VAR models are aggregated to the global VAR models, as they account for the interdependencies between the commodities. While the spillover effects in the silver, and lead market coincide to the results of the individual VAR model (and the GVAR model based on the demand weight matrix) and neither model observes interactions in the platinum's and zinc's markets, the significant impacts in the aluminum and nickel (cobalt) markets (partly) vanish once the interdependence between the commodities based on the German Energiewende is included. In contrast, the global models detect the demand and price interact to each other in the copper, indium, and lithium markets. Further, tin's interrelation between supply and price vanish, once the models are aggregated. However, the GVAR model including all eleven commodities indicates instead spillover effects between tin's demand and price.

Besides these responses in the individual markets, various spillover effects in the cross-commodity dimension underline the connectedness of the markets. In particular, the GIRF analysis reveals the strongest spillover effects between supply and supply, demand and demand, as well as price and price, similar to the GVAR models in Section 5.1. Hereby, the fundamentals of aluminum and copper, lead and zinc, as well as silver, cobalt, lithium, lead and platinum highly affect each other. While aluminum and copper are co-consumed in several applications of electrical conduction, automotive and aerospace industries, and lead and zinc are co-mined together from mixed Lead-Zinc ores, the interdependencies between silver, cobalt, lithium, lead and platinum probably originates from their common application in batteries, see Section 4.1. Moreover, shocks to the demand variables lead to significant changes in the supply of the commodities, for example the supply of lead responds to changes in the aluminum and tin demand. However, the supply barely affect the demand of other commodities, indicating the influence of the demand on commodity markets.

Table D.13: GIRF results of the GVAR models for the industrial metals in the context of the German Energiewende

			Al			Cu			Ni			Pb			Sn			Zn		
			supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
Al	**supply**	$REMod-REF$	+						+					+						
		$REMod-SUF$	+						+					+						
		$REMod-PER$	+						+					+						
		$REMod-UNA$	+						+					+						
	demand	$REMod-REF$		+			+					+				-			+	
		$REMod-SUF$		+			+					+							+	
		$REMod-PER$		+			+					+				-			+	
		$REMod-UNA$		+			+					+				-			+	
	price	$REMod-REF$			+	-					+									
		$REMod-SUF$			+	-			+		+									
		$REMod-PER$			+	-					+									
		$REMod-UNA$			+	-					+									
Cu	**supply**	$REMod-REF$			-	+														
		$REMod-SUF$			-	+														
		$REMod-PER$			-	+														
		$REMod-UNA$			-	+														
	demand	$REMod-REF$	+	+			+	-						+				-	+	-
		$REMod-SUF$	+	+			+	-						+			+	-		-
		$REMod-PER$	+	+			+	-						+			+	-		-
		$REMod-UNA$	+	+			+	-						+			+	-	+	-
	price	$REMod-REF$		-			-	+						-						+
		$REMod-SUF$		-			-	+						-	-					+
		$REMod-PER$		-			-	+						-	-					+
		$REMod-UNA$					-	+				+		-	-					+
Ni	**supply**	$REMod-REF$	+						+			+			-					
		$REMod-SUF$	+						+						-					
		$REMod-PER$	+						+			+			-					
		$REMod-UNA$	+						+			+			-					
	demand	$REMod-REF$								+			+	-						
		$REMod-SUF$								+			+	-						
		$REMod-PER$								+			+	-						
		$REMod-UNA$								+			+	-						
	price	$REMod-REF$			+						+				+				+	
		$REMod-SUF$			+						+				+				+	
		$REMod-PER$			+						+				+				+	

GIRF results of the GVAR models for the industrial metals in the context of the German Energiewende

			Al			Cu			Ni			Pb			Sn			Zn		
			supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
		$REMod-UNA$			+						+				+				+	
Pb	**supply**	$REMod-REF$		+								+	+					+		
		$REMod-SUF$		+								+	+					+		
		$REMod-PER$		+								+	+					+		
		$REMod-UNA$		+								+	+					+		
	demand	$REMod-REF$						+				+	+			-			+	
		$REMod-SUF$		+								+	+			-			+	
		$REMod-PER$						+				+	+			-			+	
		$REMod-UNA$						+				+	+			-			+	
	price	$REMod-REF$												+			+			
		$REMod-SUF$												+			+			
		$REMod-PER$					+							+			+			
		$REMod-UNA$												+			+			
Sn	**supply**	$REMod-REF$				+			-		+				+					
		$REMod-SUF$				+					+				+					
		$REMod-PER$				+					+				+					
		$REMod-UNA$				+			-		+				+					
	demand	$REMod-REF$					+		+		-	-			-	+				
		$REMod-SUF$					+				-	-				+				
		$REMod-PER$					+		+		-	-			-	+				
		$REMod-UNA$					+				-	-			-	+				
	price	$REMod-REF$							-					+			+			
		$REMod-SUF$												+			+			
		$REMod-PER$							-					+			+			
		$REMod-UNA$							-					+			+			
Zn	**supply**	$REMod-REF$										+						+		
		$REMod-SUF$										+						+		
		$REMod-PER$										+				-		+		
		$REMod-UNA$										+						+		
	demand	$REMod-REF$								-			+						+	
		$REMod-SUF$		+						-			+						+	
		$REMod-PER$		+						-			+						+	
		$REMod-UNA$		+			+			-			+						+	
	price	$REMod-REF$					-	+			+									+
		$REMod-SUF$					-	+			+									+
		$REMod-PER$					-	+			+									+
		$REMod-UNA$					-	+												+

This table displays the results of the GIRF analysis of the GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$ transformation paths and estimated on annual data from 1970 to 2019. We analyze the response of the column variables to a shock of the row variables supply (**supply**), demand (**demand**) and price (**price**) of aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn). Significant positive (+) or negative (-) effects on the 68%- level are displayed.

Further, similar to the GVAR model based on the demand weight matrix, the prices are significantly affected by the fundamentals. Hereby, the GVAR model for the eleven key resources of the German Energiewende detects various significant responses of the prices to shocks in supply and demand variables, whereas the GVAR model for the six industrial metals induced by the German Energiewende only reveals the impact of demand on prices. However, several spillover effects from prices to the supply suggest shocks to the price lead to significant changes in the production volume, whereas the demand variables react less, indicating the demand variables determine prices, but are affected to a lesser extent.

Overall, the results of the GVAR models based on the different weight matrices are comparable. Hereby, the models detect various spillover effects between the commodity prices, indicating the framework is able to reflect the common behavior in prices. Hereby, aluminum and nickel (copper and zinc) prices react to each other, similar to the GVAR model based on the demand weight matrix. Moreover, silver and platinum, as well as indium and zinc (tin and lead) interact in the models based on the key resources (industrial metals), whereas the GVAR models induced by the German Energiewende do not confirm the interrelation between the aluminum and copper price. However, all commodity prices are interrelated, underlining the importance of jointly modeling commodity markets.

Besides the interdependencies between the commodities, the GVAR frameworks allow for spillover effects from the global economy to commodity markets. Therefore, we examine the effects of global shocks to the commodity markets, similar to the GIRF analysis of the industrial metal markets in Section 5.1.2. In particular, we examine the impacts of a positive shock to each exogenous variable on the commodity markets, using the GIRFs derived recursively, using Equation 3.17, Equation 3.19, and Equation 3.20, based on the VAR model of the exogenous variables, world gross domestic product (GDP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR), see Section 5.1. The resulting GIRFs of the commodity markets derived from the GVAR model based on the key elements of the German Energiewende, and the GVAR model based on the industrial metals are summarized in Table D.14 and Table D.15, respectively.

Table D.14: GIRF results of the GVAR models for the key elements of the German Energiewende for shocks to the exogenous variables

		Ag			Al			Co			Cu			In			Li			Ni			Pb			Pt			Sn			Zn		
		supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
GDP	$REMod-REF$	+	+	+	+	+	+	+	+	+	-	+	+		+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
	$REMod-SUF$	+	+	+	+	+	+	+	+	+	-	+	+		+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
	$REMod-PER$	+	+	+	+	+	+	+	+	+	-	+	+		+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
	$REMod-UNA$	+	+	+	+	+	+	+	+	+	-	+	+		+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
FX	$REMod-REF$	-	-	-	-	-	-	-		-	+	-	-	-		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
	$REMod-SUF$	-	-	-	-	-	-	-	-	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
	$REMod-PER$	-	-	-	-	-	-	-		-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
	$REMod-UNA$	-	-	-	-	-	-	-	-	-	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
FFR	$REMod-REF$		-		+	+	+	-	+	+	+	+	+	+	+		+	+	+	+	+	+	+	+	+		-	+	+	+			+	+
	$REMod-SUF$		-	+	+	+	+	-	+	+	+	+	+	+	+		+	+	+	+	+	+		+	+	+	-	+	+	+			+	+
	$REMod-PER$		-	+	+	+	+	-	+	+	+	+	+	+	+		+	+	+	+	+	+	+	+	+	+	-	+	+	+			+	+
	$REMod-UNA$		-	+	+	+	+	-	+	+	+	+	+	+	+		+	+	+	+	+	+	+	+	+		-	+	+	+			+	+

This table displays the results of GIRF analysis of the GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$ transformation path and estimated on annual data from 1970 to 2019. We analyze the response of the column variables, supply (**supply**), demand (**demand**) and price (**price**) of the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), to a shock of the row variables world gross domestic product (GDP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR), where significant positive (+) or negative (-) effects are displayed, based on the 68%- level.

In line with the results of the GVAR model based on the demand weight matrix, the commodity markets are affected to a similar extent by shocks to the macroeconomic variables, indicating global shocks lead to similar patterns in the metals markets. In particular, the markets increase in response to an increase in the economic activity, represented by a positive shock to the world gross domestic product, indicating a rising global demand cause rising markets. Moreover, an appreciation of the U.S. dollar, represented by a positive shock to the U.S. dollar index, leads to a reduction in the markets. The only exception is copper, as its production volume declines (increases) in response to a shock to the world gross domestic product (U.S. dollar index), similar to the results of the global vector autoregression model based on the demand weight matrix, underling copper's special role. In addition, a contrarian monetary policy, represented by a positive shock to the Federal Funds Effective Rate, cause growing commodity markets, in line with the results of Section 5.1, but contrary to the arguments of Frankel (2008). However, central banks rise the interest rates in periods of increasing prices to reduce the inflation, but the prices often react with some lag, which is why the commodity markets and the interest rates probably exhibit a common behavior in the short-term, see Schischke and Rathgeber (2023).

Table D.15: GIRF results of the GVAR models for the industrial metals in the context of the German Energiewende for shocks to the exogenous variables

		Al			Cu			Ni			Pb			Sn			Zn		
		supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
GDP	$REMod - REF$	+	+	+	-	+	+	+	+	+	+	+	+	+	+	+	+	+	+
	$REMod - SUF$	+	+	+	-	+	+	+	+	+	+	+	+	+	+	+	+	+	+
	$REMod - PER$	+	+	+	-	+	+	+	+	+	+	+	+	+	+	+	+	+	+
	$REMod - UNA$	+	+	+	-	+	+	+	+	+	+	+	+	+	+	+	+	+	+
FX	$REMod - REF$	-	-	-	+	-	-	-	-	-	-	-	-		-	-	-	-	
	$REMod - SUF$	-	-	-	+	-	-	-	-	-	-	-	-		-	-	-	-	
	$REMod - PER$	-	-	-	+	-	-	-	-	-	-	-	-		-	-	-	-	
	$REMod - UNA$	-	-	-	+	-	-	-	-	-	-	-	-		-	-	-	-	
FFR	$REMod - REF$		+	+	+	+	+		+		-	+	+	+	+			+	+
	$REMod - SUF$		+		+	+	+		+		-	+	+	+	+			+	+
	$REMod - PER$		+		+	+	+		+		-	+	+	+	+			+	+
	$REMod - UNA$		+		+	+	+		+		-	+	+	+	+			+	+

This table displays the results of GIRF analysis of the GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod - REF$, $REMod - SUF$, $REMod - PER$, and $REMod - UNA$ transformation path and estimated on annual data from 1970 to 2019. We analyze the response of the column variables, supply (**supply**), demand (**demand**) and price (**price**) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), to a shock of the row variables world gross domestic product (GDP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR), where significant positive (+) or negative (-) effects are displayed, based on the 68%- level.

D.3.1.2 Time-varying Commodity Market Models

To account for the time-varying dependencies in the commodity markets, we include the MS-GVAR models in the risk assessment framework. Hereby, we only consider the industrial metals aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), since the data of these metal markets is available on monthly frequency. In particular, we model the interdependencies between the markets over the period from January 1995 to December 2019, using the weight matrices in Table 4.31, Table 4.32, Table 4.33 and Table 4.34, which represent the dependencies between the commodities within the $REMod - REF$, $REMod - SUF$, $REMod - PER$, and $REMod - UNA$ paths, respectively. In addition, we include the exogenous variables world industrial production (IP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR) to account for the common impact of the economy on prices.

D.3.1.2.1 Model Specification We first determine the specifications of the models via the slightly adjusted model selection procedure of Li and Kwok (2021), described in Section 3.2.1.6, with the information criterion of Hannan and Quinn (1979). In particular, we specify the optimal number of states, up to the predefined maximum number of states, $M_{max} = 3$, the optimal lag length, up to the predefined number of lags, $P_{max} = 2$, as well as the specification out of all considered specifications $spec \in \{MSI, MSIH, MSH, MSA, MSAH\}$.[11] Hereby, the model selection procedure identifies the MSH(2)-VAR(1) model, with regime-dependent covariance matrix, but regime-invariant intercept as well as regime-invariant parameters for the

[11] As the models with regime-dependent exogenous variables lead to unstable MS-GVAR models, we exclude them from the model selection procedure. However, the model selection identified the MSH-VAR model, as the best model, in either case.

endogenous and exogenous variables, performs best for the models based on the $REMod - REF$ and $REMod - UNA$ path. In contrast, the procedure selects the MSIH(3)-VAR(1) (MSH(3)-VAR(2)) model with three states for the $REMod - SUF$ ($REMod - PER$) path. However, the regime inferences are unstable in the models based on three regimes, which is why we apply the MSH(2)-VAR(1) model in either case, in line with the MS-GVAR model based on the demand weight matrix in Section 5.2.1.

Table D.16: Results of the model selection procedure for the MS-GVAR model in the context of the German Energiewende

Model	Nr. States	Nr. Lags	MSI	MSIH	MSH	MSA	MSAH
$REMod - REF$	2	1	2413.82	2354.72	**2349.63**	2522.76	2497.44
		2	2435.32	2379.05	2376.34	2677.69	2620.36
	3	1	2406.31	2356.60	2357.04	2662.77	2627.78
		2	2435.25	2386.36	2360.92	2919.24	2840.18
$REMod - SUF$	2	1	2411.37	2351.56	2346.72	2520.17	2487.48
		2	2432.38	2376.66	2372.78	2672.53	2613.02
	3	1	2403.27	**2344.96**	2355.03	2660.55	2630.05
		2	2433.23	2366.50	2353.88	2916.85	2839.72
$REMod - PER$	2	1	2412.05	2352.63	2347.41	2521.11	2495.27
		2	2433.46	2376.88	2373.98	2672.00	2616.28
	3	1	2404.63	2357.19	2358.46	2660.86	2630.69
		2	2433.81	2364.53	**2328.99**	2918.45	2835.67
$REMod - UNA$	2	1	2417.35	2358.05	**2353.55**	2528.24	2498.22
		2	2438.20	2382.27	2378.59	2677.00	2621.45
	3	1	2409.35	2362.74	2362.93	2666.51	2640.36
		2	2442.71	2378.85	2355.48	2923.71	2838.64

This table displays the model selection results based on the information criterion of Hannan and Quinn (1979), proposed in Section 3.2.1.6, for the different specifications of the MS-GVAR model, based on the weight matrices representing the dependencies between the commodities within the $REMod - REF$, $REMod - SUF$, $REMod - PER$, and $REMod - UNA$ transformation path.

The results of the Durbin-Watson (DW) test indicate neither of the commodity-specific MSH(2)-VAR(1) models suffers from autocorrelation, see Table D.2, indicating the lag length of one, chosen by the model selection procedure, is feasible from a statistical point of view. Moreover, the two states within the models enables to capture and identify calm as well as volatile periods.

D.3.1.2.2 Regime Inferences The MS-GVAR model aggregates the individual, commodity-specific MS-VAR models to one global commodity market model, therefore, the regimes differ between the individual commodity markets. In the following, we first focus on each commodity market and provide the transition probabilities, i.e. the probabilities to switch from state one to state two and vice versa, displayed in Table D.17, as well as the smoothed probabilities, indicating in which state a commodity is located in at a specific point in time, in Figure D.14, Figure D.15, Figure D.16, and Figure D.17,[12] for the models based on the interdependencies inferred from the $REMod - REF$, $REMod - SUF$, $REMod - PER$, and $REMod - UNA$ transformation path, respectively.

Overall, the transition probabilities as well as the regime inferences are comparable between the models based on the different weight matrices as well as to those of the MS-GVAR model based on the demand weight matrix, see Section 5.2.2, indicating the robustness of the MS-GVAR framework to the weight matrix used, and underlining its ability to detect changes in the markets. In particular, the models based on the weights representing the transformation pathways detect the nickel market exhibits a high probability to switch its regimes, while it is more likely to stay in the current regime for the aluminum, copper, lead, tin, and zinc market. Hereby, the MS-GVAR models based on the weight matrices derived from the transformation paths exhibit a higher

[12]These figures show the returns of each individual supply, demand and price variable over the entire sample period. Shaded areas indicate the smoothed probability to be in state two exceeds 50%, hence, it is more likely for the commodity market to be in state two at these points in time.

probability to stay in the second regime for the copper market, compared to the MS-GVAR model based on the demand weight matrix. Moreover, the MS-GVAR models based on the weight matrices derived from the transformation paths indicate the zinc market remains almost surely in its current regime.

Table D.17: Transition probability ,atrices for the individual, commodity-specific MS-VAR models in the context of the German Energiewende

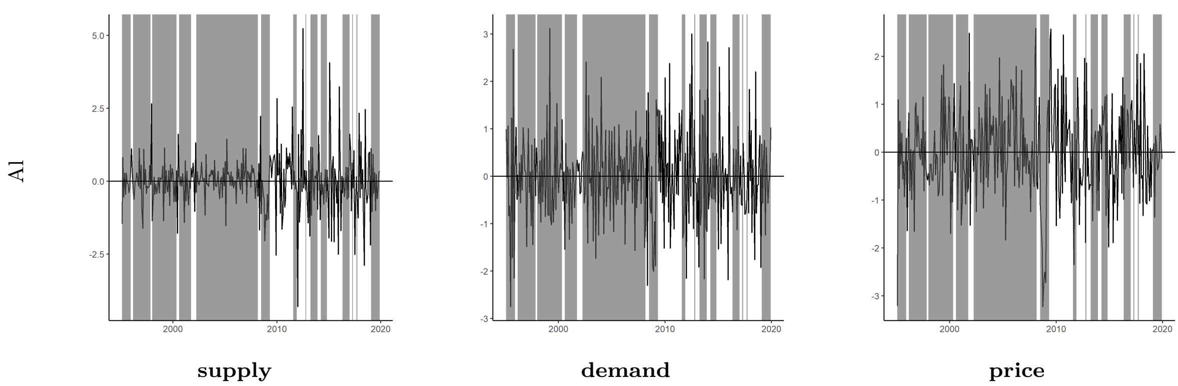
	Al		Cu		Ni		Pb		Sn		Zn	
$REMod - REF$	0.91	0.09	0.90	0.10	0.53	0.47	0.94	0.06	0.91	0.09	0.98	0.02
	0.18	0.82	0.34	0.66	0.59	0.41	0.12	0.88	0.05	0.95	0.02	0.98
$REMod - SUF$	0.91	0.09	0.91	0.09	0.54	0.46	0.94	0.06	0.89	0.11	0.97	0.03
	0.17	0.83	0.32	0.68	0.58	0.42	0.11	0.89	0.06	0.94	0.06	0.94
$REMod - PER$	0.91	0.09	0.90	0.10	0.53	0.47	0.94	0.06	0.90	0.10	0.98	0.02
	0.18	0.82	0.33	0.67	0.59	0.41	0.11	0.89	0.06	0.94	0.03	0.97
$REMod - UNA$	0.91	0.09	0.90	0.10	0.53	0.47	0.94	0.06	0.87	0.13	0.97	0.03
	0.18	0.82	0.34	0.66	0.58	0.42	0.12	0.88	0.08	0.92	0.07	0.93

This table displays the transition probability matrices for the individual, commodity-specific MS-VAR models, based on the weight matrices representing the dependencies between the commodities within the $REMod - REF$, $REMod - SUF$, $REMod - PER$, and $REMod - UNA$ transformation path, of aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn).

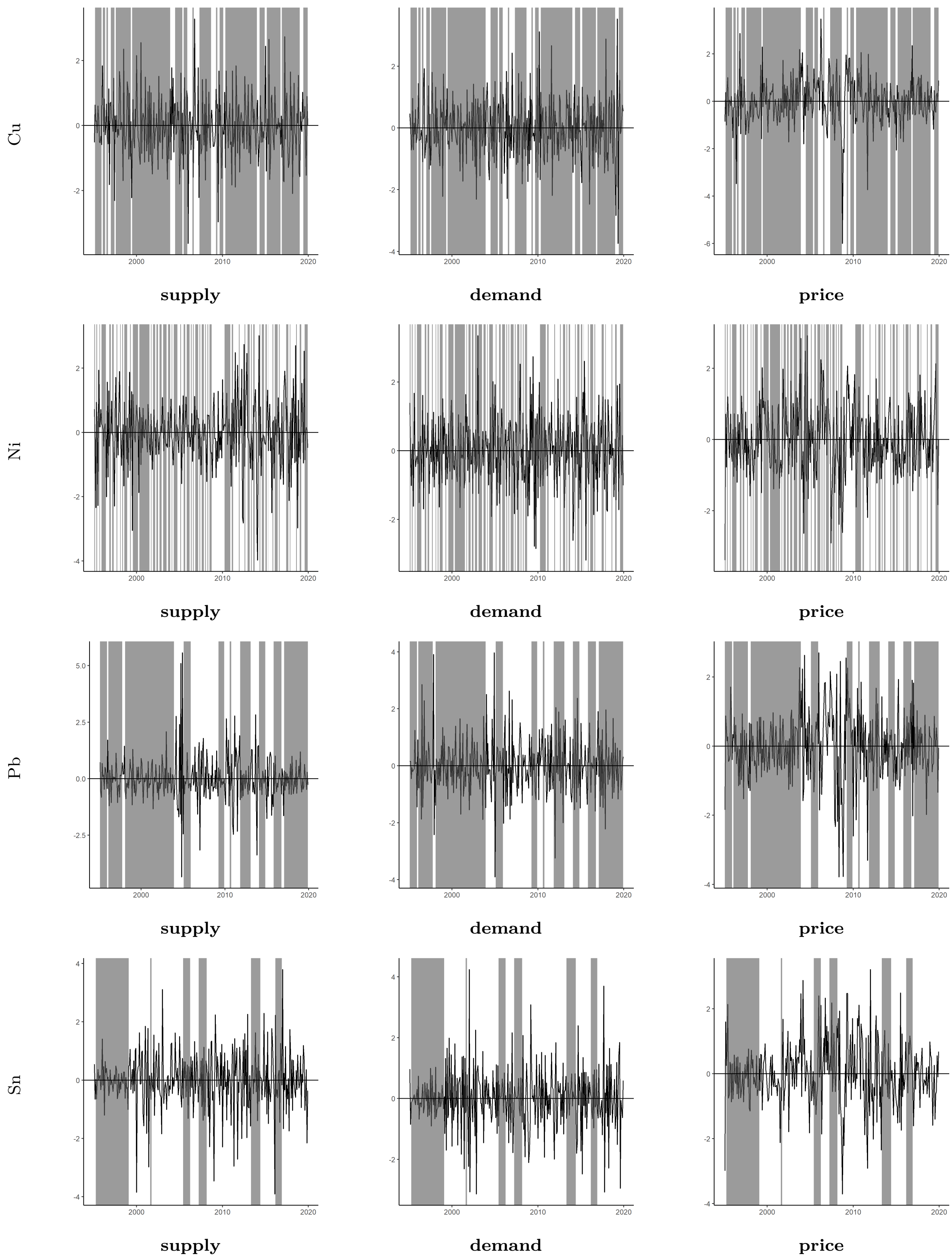
Due to the similar transition probabilities, the regime inferences are comparable between the MS-GVAR models based on the weights induced by the transformation pathways and the MS-GVAR model based on the demand weight matrix, see Figure D.14, Figure D.15, Figure D.16, and Figure D.17. However, copper exhibits slightly longer lasting periods in regime two in the models based on the weights derived from the transformation pathways, caused by its higher probability to stay in regime two. Moreover, tin and especially zinc switch their regimes less often, probably caused by the smaller transition probabilities.

Overall, we observe the commodity markets remain in regime two during periods with high fluctuations in supply, demand, and price. In addition, the distinct descriptive statistics for each regime in Table D.18, Table D.19, Table D.20, and Table D.21, underline the markets exhibit a higher volatility in state two, similar to the MS-GVAR model based on the demand weight matrix, see Section 5.2.2. Moreover, the (interquartile) ranges show the markets take on more extreme values in the second state. Therefore, the first state represents the *calm* and state two the *volatile* period.

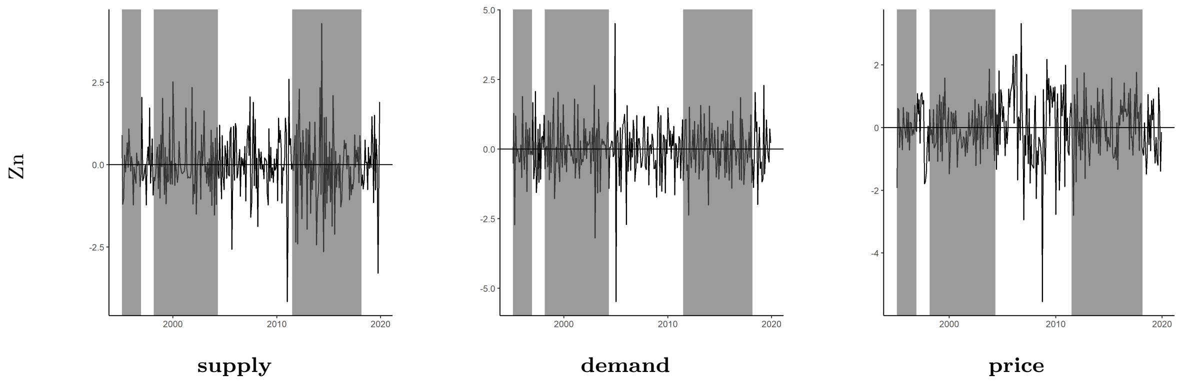
Figure D.14: Regime inferences of the commodity markets, derived from the MS-GVAR model based on the $REMod - REF$ path



Regime inferences of the commodity markets, derived from the MS-GVAR model based on the $REMod - REF$ path

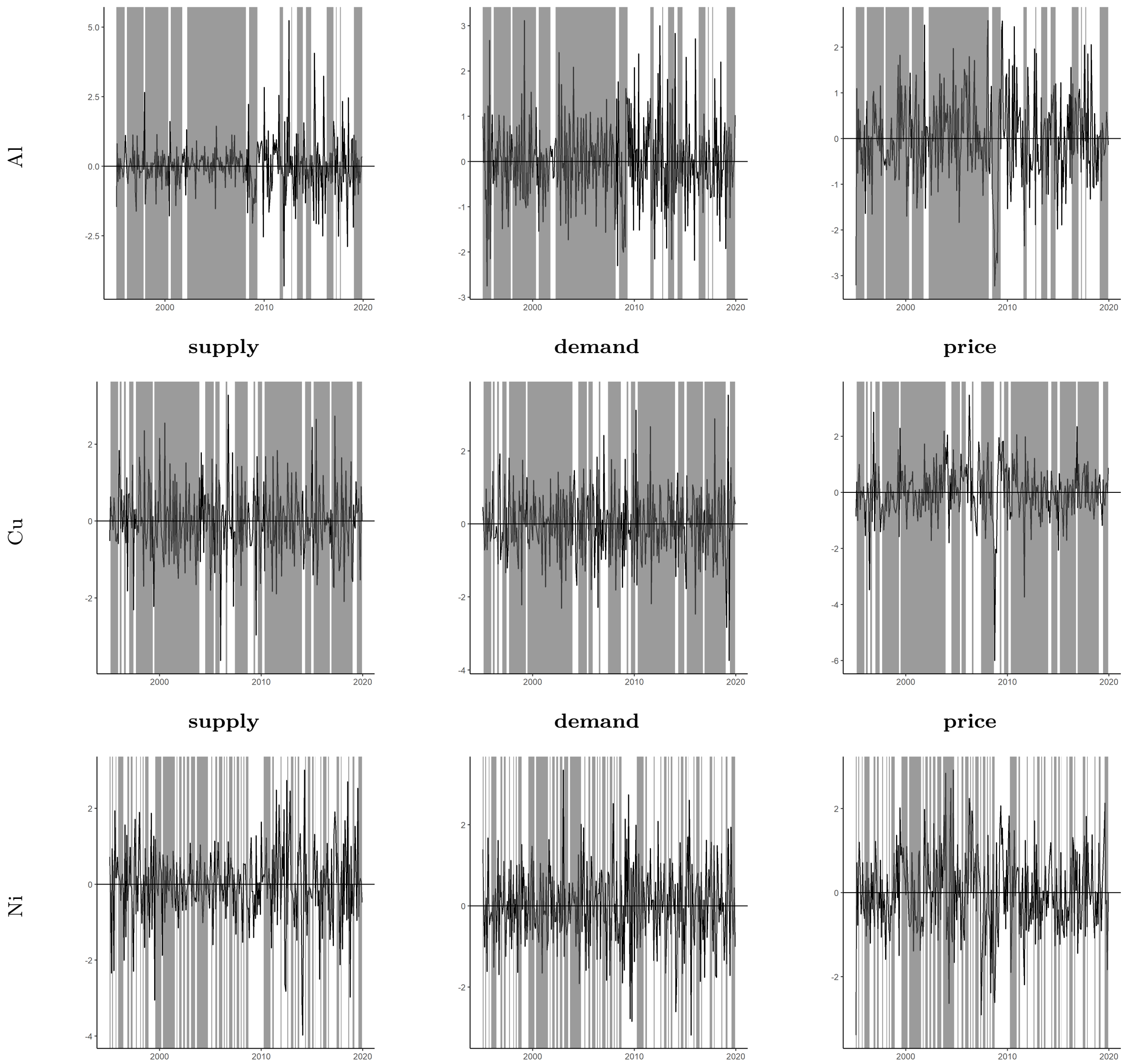


Regime inferences of the commodity markets, derived from the MS-GVAR model based on the $REMod - REF$ path

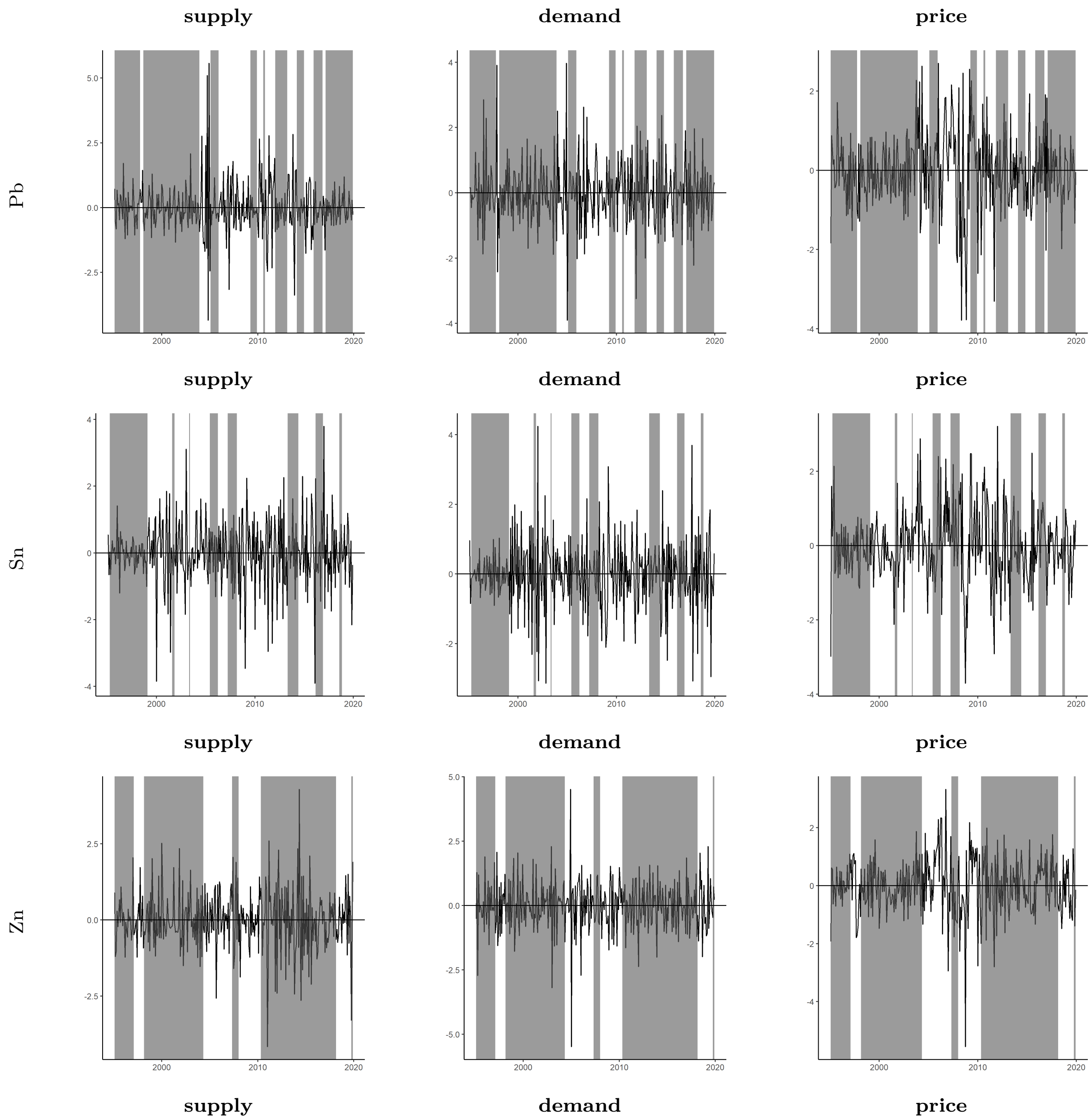


These figures show the logarithmic returns of each individual supply (**supply**), demand (**demand**) and price (**price**) variable of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), over the entire sample period from January 1995 to December 2019. Shaded areas indicate the smoothed probability, derived from the MS-GVAR model based on the $REMod - REF$ path, to be in state one exceeds 50%, hence, it is more likely for the individual commodity market to be in state one at these points in time.

Figure D.15: Regime inferences of the commodity markets, derived from the MS-GVAR model based on the $REMod - SUF$ path

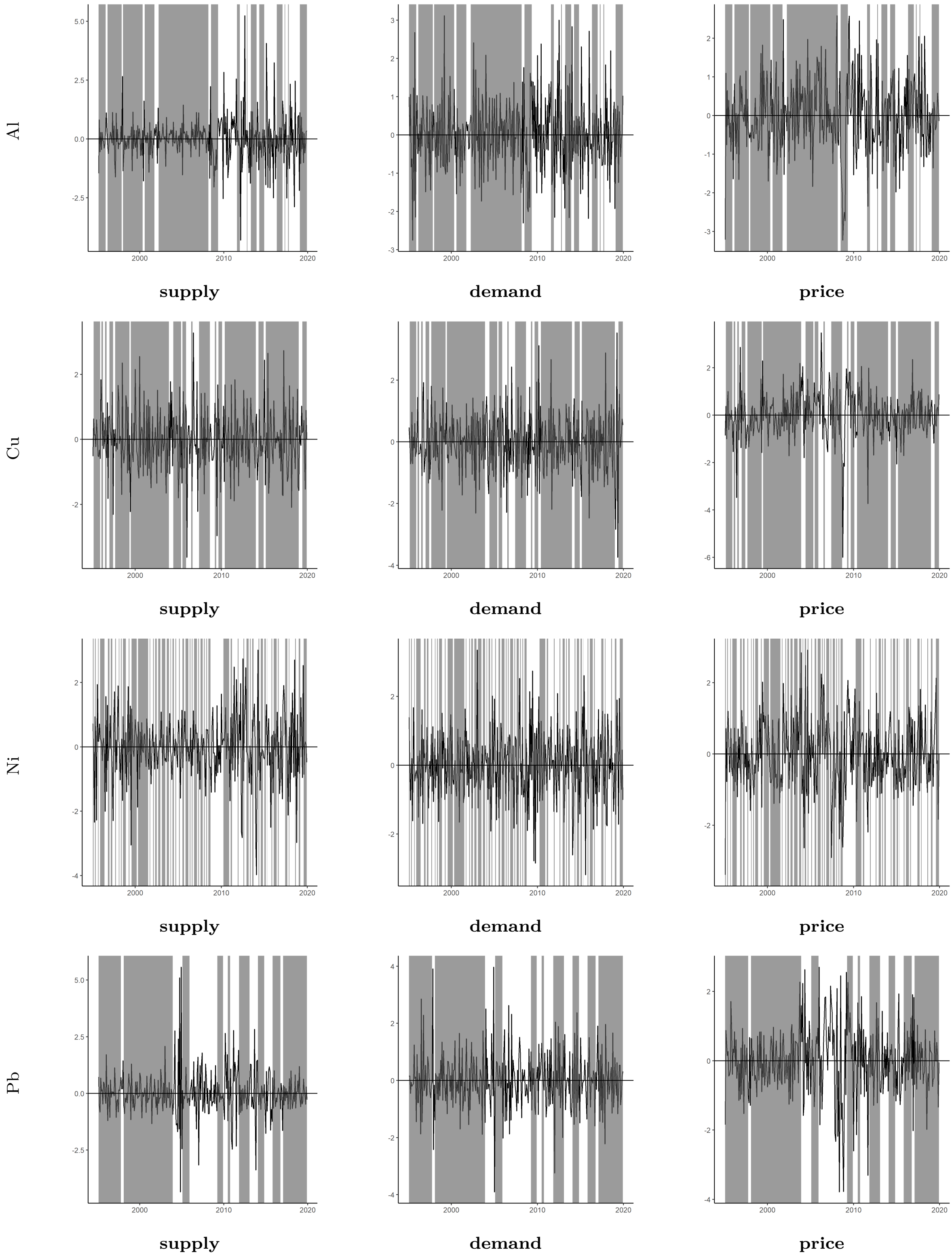


Regime inferences of the commodity markets, derived from the MS-GVAR model based on the $REMod - SUF$ path

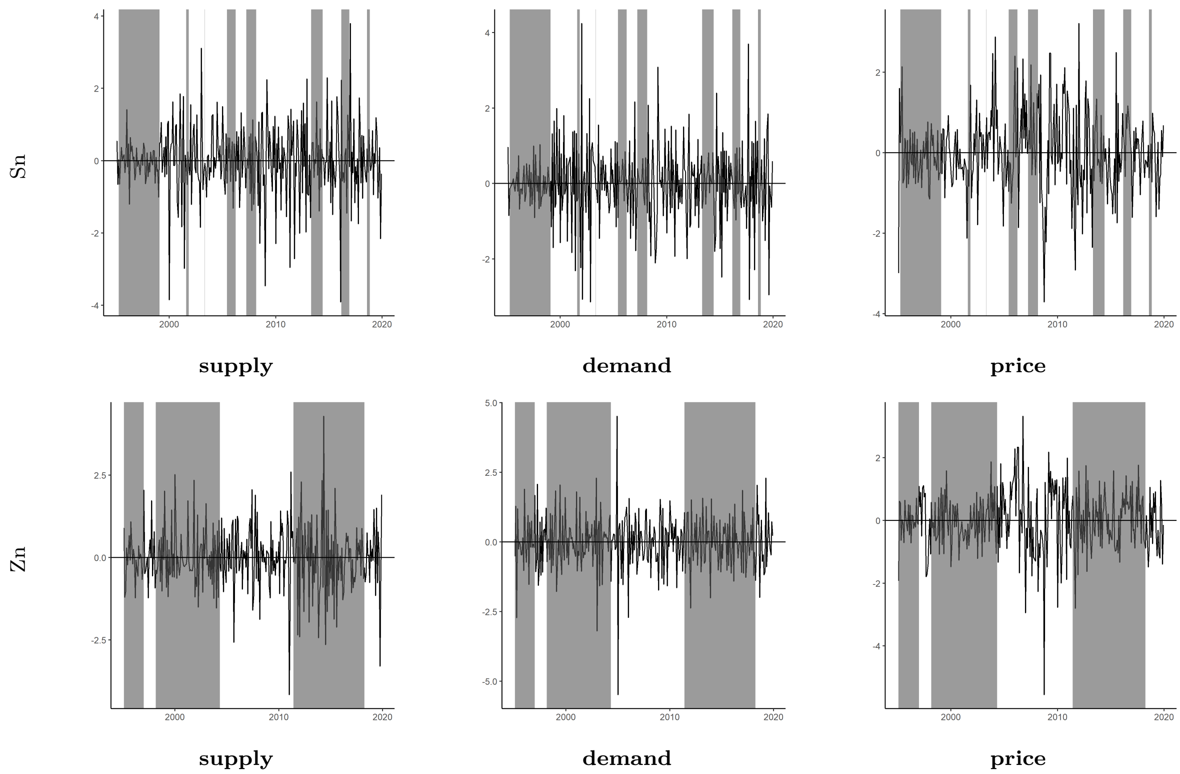


These figures show the logarithmic returns of each individual supply (**supply**), demand (**demand**) and price (**price**) variable of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), over the entire sample period from January 1995 to December 2019. Shaded areas indicate the smoothed probability, derived from the MS-GVAR model based on the $REMod - SUF$ path, to be in state one exceeds 50%, hence, it is more likely for the individual commodity market to be in state one at these points in time.

Figure D.16: Regime inferences of the commodity markets, derived from the MS-GVAR model based on the $REMod - PER$ path

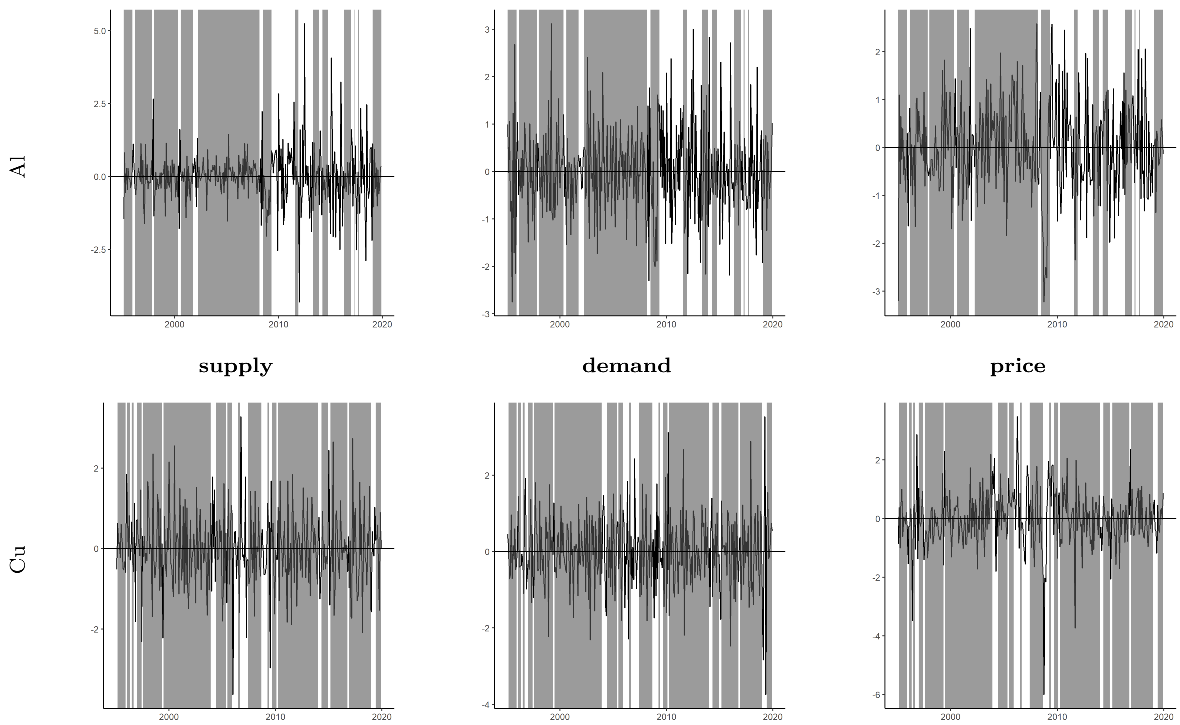


Regime inferences of the commodity markets, derived from the MS-GVAR model based on the $REMod - PER$ path

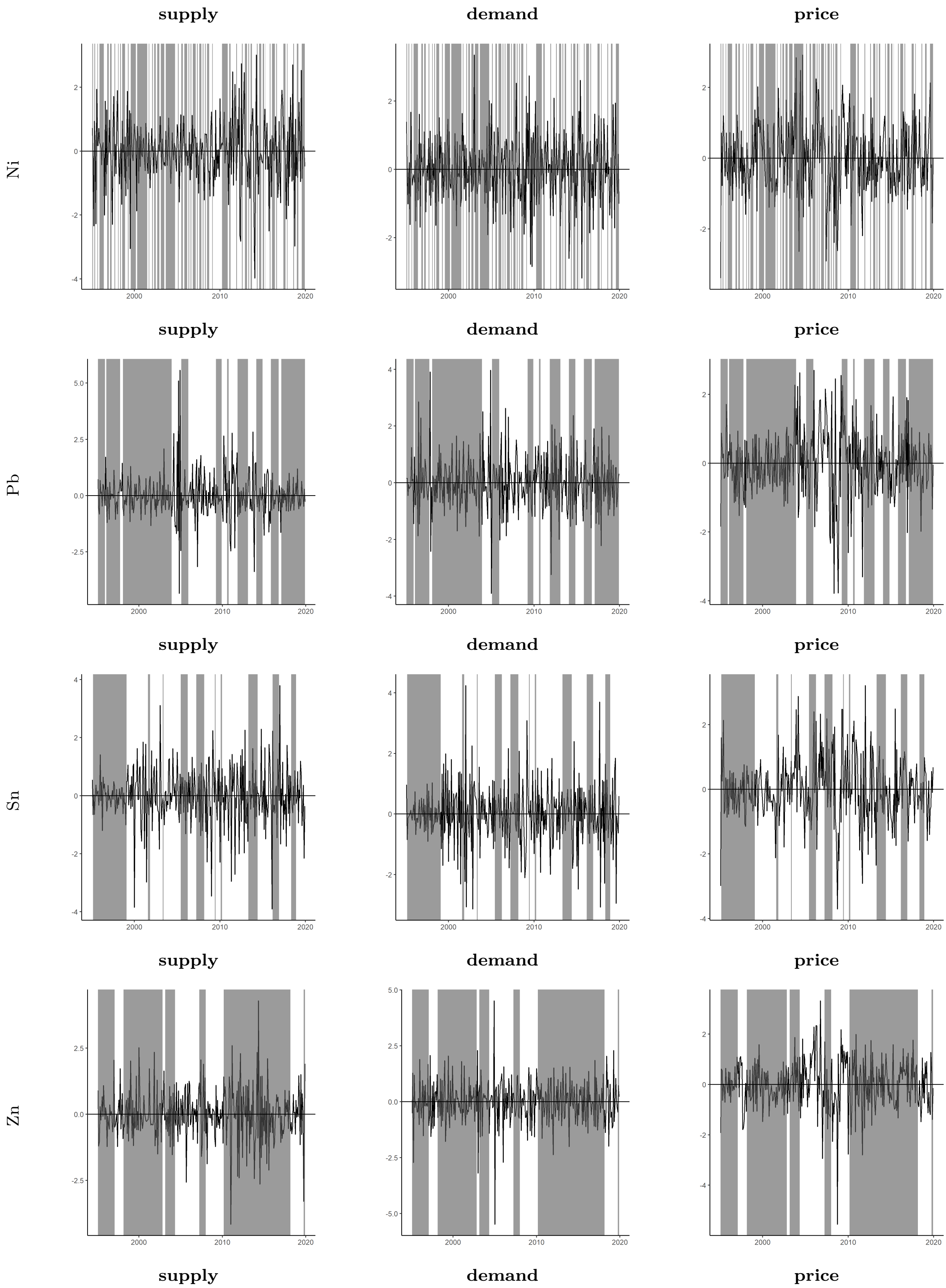


These figures show the logarithmic returns of each individual supply (**supply**), demand (**demand**) and price (**price**) variable of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), over the entire sample period from January 1995 to December 2019. Shaded areas indicate the smoothed probability, derived from the MS-GVAR model based on the $REMod - PER$ path, to be in state one exceeds 50%, hence, it is more likely for the individual commodity market to be in state one at these points in time.

Figure D.17: Regime inferences of the commodity markets, derived from the MS-GVAR model based on the $REMod - UNA$ path



Regime inferences of the commodity markets, derived from the MS-GVAR model based on the $REMod - UNA$ path



These figures show the logarithmic returns of each individual supply (**supply**), demand (**demand**) and price (**price**) variable of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), over the entire sample period from January 1995 to December 2019. Shaded areas indicate the smoothed probability, derived from the MS-GVAR model based on the $REMod - UNA$ path, to be in state one exceeds 50%, hence, it is more likely for the individual commodity market to be in state one at these points in time.

Table D.18: Descriptive statistics of the commodity-specific variables based on the regime inferences of the MS-GVAR model based on the $REMod - REF$ path

		State	Min.	5% Q.	25% Q.	Med.	Mean	75% Q.	95% Q.	Max.	SD	Nr. Obs.
Al	**supply**	1	-0.05	-0.03	-0.01	-0.00	-0.00	0.01	0.02	0.04	0.01	208
		2	-0.10	-0.06	-0.02	0.01	0.00	0.02	0.06	0.13	0.04	92
	demand	1	-0.11	-0.06	-0.02	-0.00	-0.00	0.02	0.06	0.13	0.04	208
		2	-0.09	-0.07	-0.02	-0.00	0.00	0.03	0.09	0.12	0.05	92
	price	1	-0.18	-0.09	-0.03	-0.00	-0.00	0.03	0.08	0.14	0.05	208
		2	-0.11	-0.08	-0.04	-0.00	0.01	0.04	0.11	0.14	0.06	92
Cu	**supply**	1	-0.06	-0.04	-0.01	-0.00	0.00	0.01	0.04	0.07	0.02	251
		2	-0.09	-0.06	-0.02	-0.00	0.00	0.02	0.06	0.09	0.04	49
	demand	1	-0.12	-0.07	-0.03	-0.00	0.00	0.03	0.07	0.13	0.04	251
		2	-0.17	-0.10	-0.05	-0.00	-0.00	0.04	0.10	0.16	0.07	49
	price	1	-0.27	-0.08	-0.04	0.00	-0.00	0.03	0.09	0.16	0.06	251
		2	-0.43	-0.15	-0.08	-0.01	0.00	0.09	0.17	0.25	0.13	49
Ni	**supply**	1	-0.07	-0.05	-0.01	0.00	0.00	0.02	0.05	0.08	0.03	202
		2	-0.24	-0.16	-0.08	-0.03	-0.01	0.07	0.15	0.18	0.10	98
	demand	1	-0.15	-0.11	-0.05	-0.00	-0.00	0.04	0.09	0.16	0.06	202
		2	-0.25	-0.15	-0.07	0.01	0.00	0.07	0.17	0.27	0.11	98
	price	1	-0.33	-0.15	-0.07	0.00	0.00	0.07	0.13	0.28	0.10	202
		2	-0.26	-0.13	-0.07	-0.01	-0.00	0.06	0.18	0.22	0.10	98
Pb	**supply**	1	-0.08	-0.05	-0.02	-0.01	-0.00	0.01	0.05	0.12	0.03	200
		2	-0.25	-0.13	-0.04	0.01	0.01	0.06	0.15	0.32	0.09	100
	demand	1	-0.16	-0.07	-0.02	-0.00	-0.00	0.02	0.07	0.14	0.04	200
		2	-0.19	-0.07	-0.03	-0.00	0.00	0.04	0.09	0.19	0.06	100
	price	1	-0.16	-0.09	-0.04	0.00	0.00	0.04	0.08	0.18	0.06	200
		2	-0.30	-0.19	-0.08	0.00	-0.01	0.07	0.17	0.21	0.11	100
Sn	**supply**	1	-0.10	-0.08	-0.03	0.00	-0.00	0.02	0.06	0.15	0.04	97
		2	-0.27	-0.13	-0.03	-0.00	0.00	0.05	0.11	0.26	0.08	203
	demand	1	-0.08	-0.05	-0.02	0.00	0.00	0.02	0.07	0.08	0.04	97
		2	-0.25	-0.15	-0.05	0.00	-0.00	0.05	0.14	0.34	0.09	203
	price	1	-0.08	-0.06	-0.03	-0.00	0.01	0.03	0.09	0.15	0.05	97
		2	-0.24	-0.12	-0.04	-0.01	-0.00	0.04	0.11	0.20	0.07	203
Zn	**supply**	1	-0.10	-0.06	-0.02	-0.00	-0.00	0.02	0.06	0.17	0.04	180
		2	-0.16	-0.05	-0.02	0.00	0.00	0.02	0.06	0.10	0.04	120
	demand	1	-0.16	-0.07	-0.03	-0.00	-0.00	0.03	0.08	0.12	0.05	180
		2	-0.28	-0.08	-0.03	0.01	0.00	0.04	0.08	0.23	0.06	120
	price	1	-0.21	-0.09	-0.04	-0.00	-0.00	0.04	0.09	0.14	0.06	180
		2	-0.41	-0.13	-0.06	0.01	0.00	0.07	0.13	0.25	0.09	120

This table displays the descriptive statistics (minimum (Min.), 5%, 25%, 75%, 95% quantile (Q.), median (Med.), mean (Mean), maximum (Max.), standard deviation (SD), and number of observations (Nr. Obs.)) of the stationary, commodity-specific variables supply (**supply**), demand (**demand**) and price (**price**) for the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), either in state one or in state two. Hereby, the markets are in state two if the smoothed probability, derived from the MS-GVAR model with weight matrix representing the dependencies between the commodities within the $REMod - REF$ transformation path, to be in state two exceeds 50%.

Table D.19: Descriptive statistics of the commodity-specific variables based on the regime inferences of the MS-GVAR model based on the $REMod - SUF$ path

		State	Min.	5% Q.	25% Q.	Med.	Mean	75% Q.	95% Q.	Max.	SD	Nr. Obs.
Al	**supply**	1	-0.05	-0.03	-0.01	-0.00	-0.00	0.01	0.02	0.04	0.01	207
		2	-0.10	-0.06	-0.02	0.01	0.00	0.02	0.06	0.13	0.04	93
	demand	1	-0.11	-0.06	-0.02	-0.00	-0.00	0.02	0.06	0.13	0.04	207
		2	-0.09	-0.07	-0.02	-0.00	0.00	0.03	0.09	0.12	0.05	93
	price	1	-0.18	-0.09	-0.03	-0.00	-0.00	0.03	0.08	0.14	0.05	207
		2	-0.11	-0.08	-0.04	-0.00	0.00	0.03	0.11	0.14	0.06	93
Cu	**supply**	1	-0.06	-0.04	-0.01	-0.00	0.00	0.02	0.04	0.07	0.02	248
		2	-0.09	-0.06	-0.02	-0.00	0.00	0.02	0.05	0.09	0.03	52
	demand	1	-0.12	-0.07	-0.02	-0.00	0.00	0.03	0.07	0.13	0.04	248
		2	-0.17	-0.09	-0.05	-0.00	-0.00	0.03	0.10	0.16	0.07	52
	price	1	-0.27	-0.08	-0.04	0.00	0.00	0.03	0.09	0.16	0.06	248
		2	-0.43	-0.15	-0.07	-0.02	-0.00	0.08	0.17	0.25	0.12	52
Ni	**supply**	1	-0.06	-0.05	-0.01	0.00	0.00	0.02	0.06	0.08	0.03	203
		2	-0.24	-0.16	-0.08	-0.03	-0.01	0.07	0.15	0.18	0.10	97
	demand	1	-0.15	-0.11	-0.04	0.00	-0.00	0.05	0.09	0.16	0.06	203

Descriptive statistics of the commodity-specific variables based on the regime inferences of the MS-GVAR model based on the $REMod - SUF$ path

		State	Min.	5% Q.	25% Q.	Med.	Mean	75% Q.	95% Q.	Max.	SD	Nr. Obs.
Ni	**demand**	2	-0.25	-0.15	-0.07	0.01	0.00	0.07	0.17	0.27	0.11	97
	price	1	-0.33	-0.16	-0.07	0.00	-0.00	0.06	0.13	0.28	0.10	203
		2	-0.15	-0.13	-0.07	-0.00	0.01	0.07	0.18	0.22	0.09	97
Pb	**supply**	1	-0.08	-0.05	-0.02	-0.00	-0.00	0.01	0.05	0.12	0.03	201
		2	-0.25	-0.13	-0.04	0.01	0.01	0.06	0.15	0.32	0.09	99
	demand	1	-0.16	-0.07	-0.02	-0.00	-0.00	0.02	0.07	0.14	0.04	201
		2	-0.19	-0.07	-0.03	-0.00	0.00	0.03	0.09	0.19	0.06	99
	price	1	-0.16	-0.09	-0.04	0.00	0.00	0.04	0.08	0.18	0.06	201
		2	-0.30	-0.19	-0.08	-0.00	-0.01	0.07	0.17	0.21	0.11	99
Sn	**supply**	1	-0.10	-0.08	-0.02	0.00	0.00	0.02	0.06	0.15	0.04	104
		2	-0.27	-0.13	-0.04	-0.00	0.00	0.05	0.11	0.26	0.08	196
	demand	1	-0.08	-0.05	-0.02	0.00	0.00	0.02	0.07	0.08	0.04	104
		2	-0.25	-0.16	-0.05	0.00	-0.00	0.06	0.15	0.34	0.10	196
	price	1	-0.08	-0.06	-0.03	-0.00	0.01	0.03	0.09	0.15	0.05	104
		2	-0.24	-0.12	-0.04	-0.01	-0.00	0.04	0.11	0.20	0.07	196
Zn	**supply**	1	-0.16	-0.06	-0.02	-0.00	-0.00	0.02	0.07	0.17	0.04	208
		2	-0.10	-0.04	-0.02	0.00	0.00	0.02	0.05	0.07	0.03	92
Zn	**demand**	1	-0.16	-0.07	-0.03	-0.00	-0.00	0.03	0.08	0.12	0.04	208
		2	-0.28	-0.08	-0.04	0.01	0.00	0.04	0.08	0.23	0.06	92
	price	1	-0.21	-0.10	-0.05	-0.00	-0.00	0.04	0.09	0.15	0.06	208
		2	-0.41	-0.13	-0.05	0.01	0.00	0.08	0.15	0.25	0.10	92

This table displays the descriptive statistics (minimum (Min.), 5%, 25%, 75%, 95% quantile (Q.), median (Med.), mean (Mean), maximum (Max.), standard deviation (SD), and number of observations (Nr. Obs.)) of the stationary, commodity-specific variables supply (**supply**), demand (**demand**) and price (**price**) for the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), either in state one or in state two. Hereby, the markets are in state two if the smoothed probability, derived from the MS-GVAR model with weight matrix representing the dependencies between the commodities within the $REMod - SUF$ transformation path, to be in state two exceeds 50%.

Table D.20: Descriptive statistics of the commodity-specific variables based on the regime inferences of the MS-GVAR model based on the $REMod - PER$ path

		State	Min.	5% Q.	25% Q.	Med.	Mean	75% Q.	95% Q.	Max.	SD	Nr. Obs.
Al	**supply**	1	-0.05	-0.03	-0.01	-0.00	-0.00	0.01	0.02	0.04	0.01	208
		2	-0.10	-0.06	-0.02	0.01	0.00	0.02	0.06	0.13	0.04	92
	demand	1	-0.11	-0.06	-0.02	-0.00	-0.00	0.02	0.06	0.13	0.04	208
		2	-0.09	-0.07	-0.02	-0.00	0.00	0.03	0.09	0.12	0.05	92
	price	1	-0.18	-0.09	-0.03	-0.00	-0.00	0.03	0.08	0.14	0.05	208
		2	-0.11	-0.08	-0.04	-0.00	0.01	0.04	0.11	0.14	0.06	92
Cu	**supply**	1	-0.06	-0.04	-0.01	0.00	0.00	0.02	0.04	0.07	0.02	249
		2	-0.09	-0.06	-0.02	-0.00	-0.00	0.01	0.05	0.09	0.03	51
	demand	1	-0.12	-0.07	-0.02	-0.00	0.00	0.03	0.07	0.13	0.04	249
		2	-0.17	-0.09	-0.05	-0.01	-0.00	0.04	0.10	0.16	0.07	51
	price	1	-0.27	-0.08	-0.04	0.00	0.00	0.03	0.09	0.17	0.06	249
		2	-0.43	-0.15	-0.08	-0.02	-0.01	0.07	0.16	0.25	0.12	51
Ni	**supply**	1	-0.07	-0.05	-0.01	0.00	0.00	0.02	0.05	0.08	0.03	200
		2	-0.24	-0.16	-0.08	-0.03	-0.01	0.07	0.15	0.18	0.10	100
	demand	1	-0.14	-0.11	-0.04	-0.00	-0.00	0.05	0.09	0.16	0.06	200
		2	-0.25	-0.15	-0.07	0.01	0.00	0.07	0.17	0.27	0.11	100
	price	1	-0.33	-0.15	-0.07	0.00	0.00	0.07	0.13	0.28	0.10	200
		2	-0.26	-0.13	-0.07	-0.02	-0.00	0.06	0.18	0.22	0.10	100
Pb	**supply**	1	-0.08	-0.05	-0.02	-0.01	-0.00	0.01	0.05	0.12	0.03	202
		2	-0.25	-0.13	-0.04	0.01	0.01	0.06	0.15	0.32	0.09	98
	demand	1	-0.16	-0.07	-0.02	-0.00	-0.00	0.02	0.07	0.14	0.04	202
		2	-0.19	-0.07	-0.03	-0.00	0.00	0.03	0.09	0.19	0.06	98
	price	1	-0.16	-0.09	-0.04	0.00	0.00	0.04	0.08	0.18	0.06	202
		2	-0.30	-0.19	-0.08	-0.00	-0.01	0.06	0.17	0.21	0.11	98
Sn	**supply**	1	-0.10	-0.08	-0.02	0.00	0.00	0.02	0.06	0.15	0.04	103
		2	-0.27	-0.13	-0.04	-0.00	0.00	0.05	0.11	0.26	0.08	197
	demand	1	-0.08	-0.05	-0.02	0.00	0.00	0.02	0.07	0.08	0.04	103
		2	-0.25	-0.16	-0.05	0.00	-0.00	0.06	0.15	0.34	0.10	197
	price	1	-0.08	-0.06	-0.03	-0.00	0.01	0.03	0.09	0.15	0.05	103
		2	-0.24	-0.12	-0.04	-0.01	-0.00	0.04	0.11	0.20	0.07	197

Descriptive statistics of the commodity-specific variables based on the regime inferences of the MS-GVAR model based on the $REMod - PER$ path

		State	Min.	5% Q.	25% Q.	Med.	Mean	75% Q.	95% Q.	Max.	SD	Nr. Obs.
Zn	**supply**	1	-0.10	-0.06	-0.02	-0.00	-0.00	0.02	0.06	0.17	0.04	183
		2	-0.16	-0.05	-0.02	0.00	0.00	0.02	0.06	0.10	0.04	117
	demand	1	-0.16	-0.07	-0.03	-0.00	0.00	0.03	0.08	0.12	0.05	183
		2	-0.28	-0.08	-0.03	0.01	0.00	0.03	0.08	0.23	0.06	117
	price	1	-0.21	-0.08	-0.04	-0.00	-0.00	0.04	0.09	0.14	0.06	183
		2	-0.41	-0.13	-0.06	0.01	0.00	0.07	0.14	0.25	0.10	117

This table displays the descriptive statistics (minimum (Min.), 5%, 25%, 75%, 95% quantile (Q.), median (Med.), mean (Mean), maximum (Max.), standard deviation (SD), and number of observations (Nr. Obs.)) of the stationary, commodity-specific variables supply (**supply**), demand (**demand**) and price (**price**) for the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), either in state one or in state two. Hereby, the markets are in state two if the smoothed probability, derived from the MS-GVAR model with weight matrix representing the dependencies between the commodities within the $REMod - PER$ transformation path, to be in state two exceeds 50%.

Table D.21: Descriptive statistics of the commodity-specific variables based on the regime inferences of the MS-GVAR model based on the $REMod - UNA$ path

		State	Min.	5% Q.	25% Q.	Med.	Mean	75% Q.	95% Q.	Max.	SD	Nr. Obs.
Al	**supply**	1	-0.05	-0.03	-0.01	-0.00	-0.00	0.01	0.02	0.04	0.01	205
		2	-0.10	-0.06	-0.02	0.01	0.00	0.02	0.06	0.13	0.04	95
	demand	1	-0.11	-0.06	-0.02	-0.00	-0.00	0.02	0.06	0.13	0.04	205
		2	-0.09	-0.07	-0.02	-0.00	0.00	0.03	0.09	0.12	0.05	95
	price	1	-0.18	-0.09	-0.03	-0.00	-0.00	0.03	0.08	0.14	0.05	205
		2	-0.11	-0.08	-0.04	-0.00	0.01	0.04	0.11	0.14	0.06	95
Cu	**supply**	1	-0.06	-0.04	-0.01	-0.00	0.00	0.02	0.04	0.07	0.02	252
		2	-0.09	-0.06	-0.02	-0.00	-0.00	0.01	0.06	0.09	0.04	48
	demand	1	-0.12	-0.07	-0.03	-0.00	0.00	0.03	0.07	0.13	0.04	252
		2	-0.17	-0.10	-0.05	-0.00	-0.00	0.04	0.11	0.16	0.07	48
	price	1	-0.27	-0.08	-0.04	0.00	-0.00	0.03	0.09	0.16	0.06	252
		2	-0.43	-0.15	-0.08	0.00	0.00	0.09	0.17	0.25	0.13	48
Ni	**supply**	1	-0.07	-0.05	-0.01	0.00	0.00	0.02	0.06	0.08	0.03	204
		2	-0.24	-0.16	-0.08	-0.03	-0.01	0.07	0.15	0.18	0.10	96
	demand	1	-0.15	-0.11	-0.04	-0.00	-0.00	0.05	0.09	0.17	0.06	204
		2	-0.25	-0.15	-0.07	0.01	0.00	0.07	0.16	0.27	0.11	96
	price	1	-0.33	-0.16	-0.07	0.00	-0.00	0.06	0.13	0.28	0.10	204
		2	-0.15	-0.13	-0.07	-0.01	0.00	0.06	0.18	0.22	0.09	96
Pb	**supply**	1	-0.08	-0.05	-0.02	-0.01	-0.00	0.01	0.05	0.12	0.03	200
		2	-0.25	-0.13	-0.04	0.01	0.01	0.06	0.15	0.32	0.09	100
	demand	1	-0.16	-0.07	-0.02	-0.00	-0.00	0.02	0.07	0.14	0.04	200
		2	-0.19	-0.07	-0.03	-0.00	0.00	0.04	0.09	0.19	0.06	100
	price	1	-0.16	-0.09	-0.04	0.00	0.00	0.04	0.08	0.18	0.06	200
		2	-0.30	-0.19	-0.08	0.00	-0.01	0.07	0.17	0.21	0.11	100
Sn	**supply**	1	-0.10	-0.07	-0.02	-0.00	0.00	0.03	0.06	0.15	0.04	115
		2	-0.27	-0.14	-0.04	-0.00	0.00	0.05	0.11	0.26	0.08	185
	demand	1	-0.08	-0.06	-0.02	0.00	0.00	0.02	0.08	0.13	0.04	115
		2	-0.25	-0.16	-0.06	0.00	-0.00	0.06	0.15	0.34	0.10	185
	price	1	-0.08	-0.05	-0.03	-0.00	0.01	0.03	0.10	0.15	0.05	115
		2	-0.24	-0.12	-0.04	-0.01	-0.00	0.03	0.11	0.20	0.07	185
Zn	**supply**	1	-0.16	-0.06	-0.02	-0.00	-0.00	0.02	0.07	0.17	0.04	208
		2	-0.10	-0.04	-0.02	0.00	0.00	0.02	0.05	0.07	0.03	92
	demand	1	-0.14	-0.06	-0.03	-0.00	0.00	0.03	0.08	0.10	0.04	208
		2	-0.28	-0.08	-0.04	0.01	0.00	0.04	0.09	0.23	0.07	92
	price	1	-0.21	-0.10	-0.05	-0.00	-0.00	0.04	0.10	0.15	0.06	208
		2	-0.41	-0.13	-0.06	0.01	0.00	0.08	0.15	0.25	0.10	92

This table displays the descriptive statistics (minimum (Min.), 5%, 25%, 75%, 95% quantile (Q.), median (Med.), mean (Mean), maximum (Max.), standard deviation (SD), and number of observations (Nr. Obs.)) of the stationary, commodity-specific variables supply (**supply**), demand (**demand**) and price (**price**) for the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), either in state one or in state two. Hereby, the markets are in state two if the smoothed probability, derived from the MS-GVAR model with weight matrix representing the dependencies between the commodities within the $REMod - UNA$ transformation path, to be in state two exceeds 50%.

D.3.1.2.3 Spillover Effects within and between Commodity Markets Similar to the GVAR models based on the weight matrices derived from the transformation paths, we investigate the dynamic spillover effects of the corresponding time-varying MS-GVAR models to provide a better understanding of the models applied in the risk assessment framework. Hereby, we calculate the regime-dependent generalized impulse response functions (GIRFs), according to Ehrmann et al. (2003),[13] based on the 68% confidence bounds, obtained by the adjusted bootstrap procedure of Ehrmann et al. (2001),[14] see Section 3.2.4.2.2, assuming the regime-constellation in December 2019, as we use the models to investigate the scarcity risk of the resource requirements of the transformation pathways from 2020 to 2050.

Table D.22 displays the regime probabilities in December 2019 underlying in the MS-VAR models based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$ transformation paths. Overall, we observe similar results for the different models. Hereby, the regime probabilities of all models indicate the aluminum, copper, and lead markets are in the first, calm state in December 2019, while the tin market is located in its volatile state. Moreover, the regime inferences exhibit an almost equal probability to be either in the calm or volatile state for the nickel market. However, the models based on the $REMod-REF$ and $REMod-PER$ path assign the zinc market to the volatile state, whereas the models based on the $REMod-SUF$ and $REMod-UNA$ path classify zinc to the calm regime, indicating the models differ slightly, probably caused by the lower impact of lead to the other commodities in the $REMod-SUF$ and $REMod-UNA$ weight matrices, see Table 4.32 and Table 4.34.

Table D.22: Regime inference of the individual, commodity-specific MS-VAR models in December 2019

Path	State	Al	Cu	Ni	Pb	Sn	Zn
$REMod-REF$	1	0.84	0.80	0.55	0.93	0.16	0.22
	2	0.16	0.20	0.45	0.07	0.84	0.78
$REMod-SUF$	1	0.84	0.79	0.56	0.93	0.19	0.95
	2	0.16	0.21	0.44	0.07	0.81	0.05
$REMod-PER$	1	0.84	0.79	0.55	0.93	0.18	0.25
	2	0.16	0.21	0.45	0.07	0.82	0.75
$REMod-UNA$	1	0.84	0.80	0.55	0.93	0.27	0.96
	2	0.16	0.20	0.45	0.07	0.73	0.04

This table displays the regime inference in December 2019 underlying in the MS-VAR models based on the weight matrices representing the dependencies between the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn) within the $REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$ transformation path.

Assuming these regime probabilities, we calculate the corresponding regime-dependent generalized impulse response functions for each MS-GVAR model. Hereby, we shock each commodity-specific variable by one standard deviation to analyze the direct as well as the indirect effects of this shock to the remaining variables, in the individual, commodity-specific markets, as well as in the cross-commodity dimension. In addition, we also investigate how shocks to the exogenous variables affect the commodity markets, see Appendix D.3.1.2.4. Table D.23 displays the GIRF results of the endogenous shocks, whereby we indicate significant positive, or negative, responses of the column variables, to a shock in the row variables by a (+), or (-).

Overall, the MS-GVAR models, representing the dependencies between the commodities within the transformation paths, detect various spillover effects within and between the commodity markets, indicating strong interrelations between the metals. In particular, the models imply

[13] We calculate the generalized impulse response functions via a Monte Carlo integration, described in Section 3.2.4.2. Hereby, we draw $N_{hist} = 500$ histories, and $N_{shock} = 500$ shocks.

[14] In particular, we draw $N_{boot} = 500$ times $T_{boot} = 250$ residuals with replacement to generate the bootstrap sample.

more reactions to and from the supply and demand variables, besides the impact of prices between the markets, compared to the MS-GVAR model based on the demand weight matrix, indicating the weights induced by the German Energiewende better reflect the market structure. However, the GIRFs of the MS-GVAR model based on the demand weight matrix are derived under the assumption the commodity markets are all situated either in their calm or volatile regime, whereas the GIRF results presented in Table D.23 assume the regime constellation observed in December 2019. Hereby, the tin (and zinc) market is in its volatile regime, while the regime probabilities indicate the other metal markets are in a calmer period. This mix of calm and volatile periods between the markets may also cause the different spillover effects.

In general, the spillover effects of the MS-GVAR models based on the different weight matrices are comparable, with only few exceptions, due to the similar scaled relations derived from the material requirements of the four transformation pathways, see Table 4.31, Table 4.32, Table 4.33, and Table 4.34 for the corresponding weight matrices. Therefore, we focus on the spillover effects detected by the MS-GVAR model based on the $REMod-REF$ path, as the $REMod-REF$ path represents the baseline scenario.

First of all, we examine the spillover effects within the individual industrial metal markets, followed by an investigation of the impacts in the cross-commodity dimension. While we do not observe any responses in the aluminum market to shocks in its own market, the other metal markets exhibit interrelations between the fundamentals and their prices. In particular, shocks to the individual demand and price significantly affect the supply of copper, nickel, lead, (tin), and zinc. Moreover, changes in the lead price cause an increase in its demand, whereas a positive shock to the zinc demand (supply) lead to decreasing price (demand), which is rather counterintuitive, as we would expect rising prices in response to a demand increase. However, the observed reaction may be caused by unobservable, indirect effects, also captured by the GIRF methodology. In contrast to the few responses in the individual markets observed by the MS-GVAR model based on the demand weight matrix, the models reflecting the relations between the commodities based on the transformation paths indicate strong interrelations between the individual supply, demand and price.

Table D.23: GIRF results of the MS-GVAR models for the industrial metals in the context of the German Energiewende

			Al			Cu			Ni			Pb			Sn			Zn		
			supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
Al	**supply**	$REMod-REF$	+			-		+	-				-	-			-	+		+
		$REMod-SUF$	+			-			-				-				-	+		+
		$REMod-PER$	+			-		+	-				-	-			-	+		+
		$REMod-UNA$	+			-			-				-				-	+		+
	demand	$REMod-REF$		+		+			-		+	+	+		-			+	+	-
		$REMod-SUF$		+		+		+	-		+	+	+		-			+	+	-
		$REMod-PER$		+		+			-		+	+	+		-			+	+	-
		$REMod-UNA$		+		+		+	-		+	+	+		-			+	+	-
	price	$REMod-REF$			+	+	+			+			+	+	+	+		+	+	-
		$REMod-SUF$			+	+	+	+		+		+	+	+	+	+		+	+	-
		$REMod-PER$			+	+	+	+		+		+	+	+	+	+		+	+	-
		$REMod-UNA$			+	+	+	+		+		+	+	+	+	+		+	+	-
Cu	**supply**	$REMod-REF$		-		+							-	-			-	+		+
		$REMod-SUF$	-	-		+							-				-	+		+
		$REMod-PER$		-		+			-				-	-			-	+		+
		$REMod-UNA$	-	-		+							-				-	+		+
	demand	$REMod-REF$				+	+		-		+	+	+		-			+	+	-
		$REMod-SUF$			+	+	+		-		+	+	+		-			+	+	-
		$REMod-PER$				+	+		-		+	+	+		-			+	+	-
		$REMod-UNA$			+	+	+		-		+	+	+		-			+	+	-
	price	$REMod-REF$		+	+	+		+		+		+	+	+	+	+		+	+	-
		$REMod-SUF$		+	+	+		+		+		+	+	+	+	+		+	+	-

GIRF results of the MS-GVAR models for the industrial metals in the context of the German Energiewende

			Al			Cu			Ni			Pb			Sn			Zn		
			supply	demand	price	supply	demand	price	supply	demand	price	supply	demand	price	supply	demand	price	supply	demand	price
		$REMod-PER$		+	+	+		+	-	+		+	+	+	+	+		+	+	-
		$REMod-UNA$		+	+	+		+		+		+	+	+	+	+		+	+	-
Ni	**supply**	$REMod-REF$		-		-		+	+				-	-			-	+		+
		$REMod-SUF$	-	-		-			+				-				-	+		+
		$REMod-PER$		-		-		+	+				-	-			-	+		+
		$REMod-UNA$	-	-		-			+				-				-	+		+
	demand	$REMod-REF$				+			-	+		+	+		-			+	+	-
		$REMod-SUF$				+		+	-	+		+	+		-			+	+	-
		$REMod-PER$				+			-	+		+	+		-			+	+	-
		$REMod-UNA$				+		+	-	+		+	+		-			+	+	-
	price	$REMod-REF$		+	+	+	+	+	-		+	+	+	+	+	+		+	+	-
		$REMod-SUF$		+	+	+	+	+	-		+	+	+	+	+	+		+	+	-
		$REMod-PER$		+	+	+	+	+	-		+	+	+	+	+	+		+	+	-
		$REMod-UNA$		+	+	+	+	+	-		+	+	+	+	+	+		+	+	-
Pb	**supply**	$REMod-REF$		-		-		+	-			+					-	+		+
		$REMod-SUF$	-	-		-			-			+					-	+		+
		$REMod-PER$		-		-		+	-			+					-	+		+
		$REMod-UNA$	-	-		-			-			+					-	+		+
	demand	$REMod-REF$				+			-		+		+		-			+	+	-
		$REMod-SUF$			+	+			-		+		+		-			+	+	-
		$REMod-PER$				+			-		+	+	+		-			+	+	-
		$REMod-UNA$			+	+			-		+		+		-			+	+	-
	price	$REMod-REF$		+	+	+	+	+	-	+		+	+	+	+	+		+	+	-
		$REMod-SUF$		+	+	+	+	+		+		+	+	+	+	+		+	+	-
		$REMod-PER$		+	+	+	+	+	-	+		+	+	+	+	+		+	+	-
		$REMod-UNA$		+	+	+	+	+		+		+	+	+	+	+		+	+	-
Sn	**supply**	$REMod-REF$		-		-			-					-	+		+	+		+
		$REMod-SUF$	-	-		-							-		+			+		+
		$REMod-PER$		-		-		+	-					-	+			+		+
		$REMod-UNA$	-	-		-							-		+			+		+
	demand	$REMod-REF$				+			-		+	+	+			+	-	+	+	-
		$REMod-SUF$			+	+		+	-		+	+	+			+	-	+	+	-
		$REMod-PER$				+		+	-		+	+	+			+	-	+	+	-
		$REMod-UNA$			+	+		+	-		+	+	+			+	-	+	+	-
	price	$REMod-REF$		+	+	+	+			+			+	+	-		+	+	+	-
		$REMod-SUF$		+	+	+	+	+		+			+		-		+	+	+	-
		$REMod-PER$		+	+	+	+	+		+		+	+	+	-		+	+	+	-
		$REMod-UNA$		+	+	+	+	+		+			+		-		+	+	+	-
Zn	**supply**	$REMod-REF$		-		-		+	-				-	-			-	+		
		$REMod-SUF$	-	-		-							-				-	+	-	
		$REMod-PER$		-		-		+	-				-	-			-	+	-	
		$REMod-UNA$	-	-		-							-				-	+	-	
	demand	$REMod-REF$				+			-		+	+	+		-			+	+	-
		$REMod-SUF$			+	+		+	-		+	+	+		-			+	+	
		$REMod-PER$				+			-		+	+	+		-			+	+	-
		$REMod-UNA$			+	+		+	-		+	+	+		-			+	+	
	price	$REMod-REF$		+	+	+	+			+		+	+	+	+	+		+	+	+
		$REMod-SUF$		+	+	+	+	+		+		+	+	+	+	+		+	+	+
		$REMod-PER$		+	+	+	+	+		+		+	+	+	+	+		+	+	+
		$REMod-UNA$		+	+	+	+	+		+		+	+	+	+	+		+	+	+

This table displays the results of the GIRF analysis of the MS-GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$ transformation paths, estimated on monthly data from 1995 to 2019. We analyze the response of the column variables to a shock of the row variables supply (**supply**), demand (**demand**) and price (**price**) of aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn). Significant positive (+) or negative (-) effects on the 68%- level are displayed.

In addition to the responses in the individual markets, shocks to the commodity markets also affect the other industrial metals, underlining the importance of jointly modeling commodity markets. Overall, the GIRF analysis indicates various spillover effects between supply, demand and price in the cross-commodity dimension. In particular, supply, demand and prices interact with each other, similar to the results observed in Section 5.2. Hereby, positive supply shocks lead to decreasing production volumes of aluminum, copper, and nickel, probably caused by substitution effects, whereas the supply of zinc rises. Moreover, lead and zinc demand significantly react to demand shocks, indicating the consumption of lead and zinc depends on the demand for the other commodities. Furthermore, the prices are interrelated. In particular, aluminum, copper, and lead prices increase in response to increasing prices, whereas zinc's price reduces, similar to the GIRF results of the MS-GVAR model based on the demand weight matrix. Due to the common behavior in the commodity prices, prices also affect the supply and demand of the commodities, indicating strong interrelations between the markets. Hereby, the demand (supply) of all metals (except aluminum and nickel) increases in response to a positive price shock. However, the fundamentals also affect prices. While lead and tin prices reduce in response to an increased supply, supply shocks cause increasing copper and zinc prices, probably due to indirect effects. Moreover, the positive reaction in aluminum's, copper's, and nickel's price to demand shocks indicate a higher demand in the metals markets lead to increasing prices. However, zinc's price reduces in response to demand shocks, probably due to the negative reaction of its price to increasing metal prices. Besides the impact on prices, supply and demand also affect each other. Hereby, aluminum and lead demand decrease in response to supply shocks, while demand shocks cause increasing (decreasing) supply of copper, lead, and zinc (nickel, and tin). In general, the demand for one commodity more likely affects the supply of another metal than vice versa, indicating the stronger impact of demand in commodity markets. Overall, the spillover effects within and between the commodity markets are comparable across the models based on the different transformation pathways. Hereby, the results underline the importance of fundamentals on prices as well as on jointly modeling commodity markets.

D.3.1.2.4 Global Spillover Effects to Commodity Markets Besides the spillover effects within and between the commodity markets, we also examine how global shocks, particularly shocks to the global economic activity, the exchange rate and the interest rate, affect the commodity markets under the regime constellation in December 2019. Hereby, we apply the MSH(2)-VAR(1) model for the exogenous variables, world industrial production (IP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR), similar to Section 5.2.3.3, however, we only use data in the period from 1995 to 2019.[15]

The corresponding transition probabilities in Table D.24 indicate a high probability to remain in the current state, whereby it is more likely to stay in the second state, compared to the model based on data from 1995 to 2020 in Section 5.2.3.3, indicating the inclusion of the onset of the COVID-19 pandemic, and the resulting fluctuations in the economy, slightly changes the model. Moreover, the regime inferences presented in Figure D.18 differ. While the model based on the enlarged sample period only detects brief periods in the volatile state, the higher probability to stay in the second regime of the model based on the reduced sample period leads to longer periods in regime two. However, both models suggest the exogenous variables switch their regime to state two during the financial crisis, and the European debt crisis, indicating stressed periods cause a change in the regimes. Hereby, the distinct descriptive statistics for each regime in Table D.25, where we assume the economy to be located either in state one or state two, underlines that especially the Federal Funds Effective Rate exhibits more volatile periods in state two. Overall, the MS-VAR model of the exogenous variables classifies the economy into

[15]The results of the Durbin-Watson (DW) test indicate the MSH(2)-VAR(1) model of the exogenous variables in the reduced sample do not suffer from autocorrelation, see Table D.2, therefore, the lag length of one is feasible from a statistical point of view.

calm and volatile periods, similar to the models presented above.

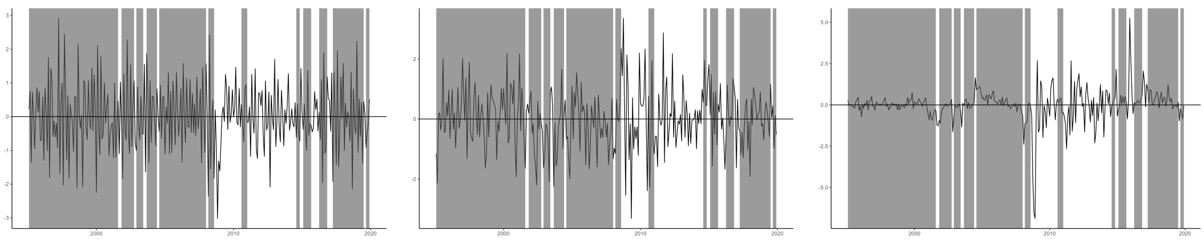
As we apply the MS-GVAR models based on the dependencies induced from the transformation paths to investigate the scarcity risks of the required resources from 2020 to 2050, we examine how global shocks affect the commodity markets in 2019. Therefore, we assume the regime inferences observed in December 2019, presented in Table D.26, and detect the economy is in its calm regime, similar to the aluminum, copper, and lead (zinc) markets.

Table D.24: Transition probability matrices for the individual MS-VAR model of the exogenous variables in the period from 1995 to 2019

Exogenous variables	
0.94	0.06
0.13	0.87

This table displays the transition probability matrix for the individual MS-VAR model of the exogenous variables world industrial production (IP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR) estimated on data in the period from 1995 to 2019.

Figure D.18: Regime inferences of the exogenous variables in the period from 1995 to 2019



(a) IP (b) FX (c) FFR

These figures show the logarithmic returns of the exogenous variables world industrial production (IP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR), over the entire sample period from January 1995 to December 2019. Shaded areas indicate the smoothed probability to be in state one exceeds 50%, hence, it is more likely for the exogenous variables to be in state one at these points in time.

Table D.25: Descriptive statistics of the exogenous variables based on the regime inferences of the MS-VAR model in the period from 1995 to 2019

	State	Min.	5% Q.	25% Q.	Med.	Mean	75% Q.	95% Q.	Max.	SD	Nr. Obs.
IP	1	-0.04	-0.03	-0.01	-0.00	-0.00	0.01	0.03	0.05	0.02	215.00
	2	-0.06	-0.02	-0.01	0.00	0.00	0.01	0.03	0.03	0.02	85.00
FX	1	-0.05	-0.04	-0.01	0.00	-0.00	0.01	0.03	0.05	0.02	215.00
	2	-0.07	-0.04	-0.01	0.00	0.00	0.01	0.05	0.07	0.02	85.00
FFR	1	-0.15	-0.08	-0.01	0.01	0.01	0.04	0.10	0.16	0.05	215.00
	2	-0.90	-0.31	-0.14	-0.01	-0.02	0.12	0.28	0.69	0.23	85.00

This table displays the descriptive statistics (minimum (Min.), 5%, 25%, 75%, 95% quantile (Q.), median (Med.), mean (Mean), maximum (Max.), standard deviation (SD), and number ob observations (Nr. Obs.)) of the stationary, exogenous variables world industrial production (IP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR), being either in state one or in state two, when assuming the markets are in state two if the smoothed probability to be in state two exceeds 50%.

Table D.26: Regime inference of the MS-VAR models for the exogenous variables in December 2019

State	exog
1	0.82
2	0.13

This table displays the regime inference in December 2019 underlying in the MS-VAR models for the exogenous variables.

Using this model for the economy and the associated regime inference in 2019, we disentangle the spillover effects from the exogenous variables to the commodity markets. Overall, the GIRF analysis in Table D.27 reveals shocks to the economy affect each commodity market to a similar extent, in line with the results of the MS-GVAR model based on the demand weight matrix. Hereby, the different induced weight matrices from the transformation paths lead to similar results, indicating the robust effect from the macroeconomic variables to the commodity markets, however the spillover effects slightly differ from the impacts observed above, as the sample period for the models based on the weight matrices induced from the transformation paths exclude the onset of the COVID-19 pandemic, leading to different regime inferences as well as spillover effects.

In particular, similar to the results above, see Table 5.18, the commodity markets are negatively affected by an increase in the world industrial production, except copper and tin demand, as well as zinc supply and price. Hereby, indirect effects from the interest rate, which increases in response to a global demand shock, as well as the negative impact of the lagged world industrial production, because of the negative autocorrelation, cause the observed decrease in the markets.

Furthermore, an appreciation of the U.S. dollar, represented by a positive shock to the U.S. dollar index, generally affects the commodity markets positively. Hereby, the prices, and ultimately, the supply, raise in response to the demand growth caused by the stronger U.S. dollar and the corresponding increase in the demand of consumers holding the U.S. dollar. While the MS-GVAR model based on the demand weight matrix indicates a decrease in nickel's, lead's and zinc's demand as well as in nickel's price in response to the exchange rate shock, the models applied on data in the period from 1995 to 2019 detect also negative reactions in aluminum's price, as well as in lead's and tin's supply, indicating the results based on different weight matrices as well as sample periods slightly differ. Hereby, the reduced demand for consumers holding other currencies, for which a stronger U.S. dollar implies the metals, quoted in U.S. dollars, become more expensive, probably cause the observed decline.

Table D.27: GIRF results of the MS-GVAR models for the industrial metals in the context of the German Energiewende for shocks to the exogenous variables

		Al			Cu			Ni			Pb			Sn			Zn		
		supply	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**	**supply**	**demand**	**price**
IP	$REMod-REF$	-	-	-	-	+	-	-	-	-	-	-	-	-	+	-	+	-	-
	$REMod-SUF$	-	-	-	-	+	-	-	-	-	-	-	-	-	-	-	+	-	+
	$REMod-PER$	-	-	-	-	+	-	-	-	-	-	-	-	-	+	-	+	-	+
	$REMod-UNA$	-	-	-	-	+	-	-	-	-	-	-	-	-	-	-	+	-	+
FX	$REMod-REF$	+	+	-	-	+	+	+	-	-	-	+	+	-	+	-	+	-	+
	$REMod-SUF$	+	+	-	+	+	+	+	-	-	-	+	+	-	+	+	+	-	+
	$REMod-PER$	+	+	-	-	+	+	+	-	-	-	+	+	-	+	+	+	-	+
	$REMod-UNA$	+	+	-	+	+	+	+	-	-	-	+	+	-	+	+	+	-	+
FFR	$REMod-REF$	-	-	-	-	-	-	+	-	+	-	-	-	-	+	+	-	+	+
	$REMod-SUF$	-	-	-	-	-	+	+	-	+	-	-	-	-	+	+	+	+	-
	$REMod-PER$	-	-	-	-	+	+	+	-	+	-	-	-	-	+	+	-	+	-
	$REMod-UNA$	-	-	-	-	-	-	+	-	+	-	-	-	-	+	+	-	+	-

This table displays the results of GIRF analysis of the MS-GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$ transformation path. We analyze the response of the column variables, supply (**supply**), demand (**demand**) and price (**price**) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), to a shock of the row variables world industrial production (IP), U.S. dollar index (FX), and Federal Funds Effective Rate (FFR), where significant positive (+) or negative (-) effects are displayed, based on the 68%- level.

In addition, a positive interest rate shock, representing a contrarian monetary policy, leads to decreasing demand and prices of aluminum, copper, and lead, underlining the theory of Frankel (2008), since the cost of capital for holding a commodity should decrease and the demand for commodities as an alternative asset class should increase in response to an expansionary monetary policy. In contrast, the positive reaction of the nickel, tin and zinc price are in line with the findings of the GVAR model as well as the studies of Hammoudeh et al. (2015), and Schischke and Rathgeber (2023).

In general, the MS-GVAR models, reflecting the dependencies between the commodities in the transformation paths reflect the impact of the fundamentals on prices, as well as spillover effects between the metal markets. Moreover, the GIRF analysis of shocks to the exogenous variables underline the commodity markets are highly affected by the economy.

D.3.2 Robustness Analyses

The scarcity risk assessment framework is based on assumptions of a price threshold, the underlying scenario values, the substitutability, as well as a scaling factor for the commodity amount required. Therefore, we assess the robustness of the framework as well as the corresponding results by calculating the scarcity risk of the resource demands of the transformation paths of the German Energiewende under different assumptions. Hereby, we investigate to what extent our results remain valid if the price threshold, the scenario values, the substitutability, or the scaling factor change. In particular, we focus on one assumption each and present the corresponding expected loss due to scarcity (ES) per commodity, path and scenario in the following.

For a comparison between the models, we restrict the commodity set to the industrial metals, which are considered in the MS-GVAR model. Hereby, the commodity-specific results of the probability of scarcity and expected loss due to scarcity do not change for the logistic regression model, as spillovers between the commodity markets are not reflected. However, due to the interdependencies in the GVAR model, we recalculate the probability of scarcity of the industrial metals for the time-invariant analysis, see Appendix D.3.2.5. Moreover, different price thresholds and scenario values probably lead to different probability of scarcity values for all models which is why we recalculate the probabilities of scarcity in Appendix D.3.2.1 and Appendix D.3.2.2 for the (MS-)GVAR model as well as the logistic regression model. In contrast, the definition of the scaling factor, and the substitutability, only affect the exposure at scarcity, loss given scarcity, and expected loss due to scarcity, respectively. Therefore, we only provide the commodity-specific expected loss due to scarcity values for these robustness analyses in Appendix D.3.2.4, and Appendix D.3.2.3, as the corresponding probability of scarcity is already presented in Section 5.3.1.

D.3.2.1 Robustness Analysis for the Threshold Price

D.3.2.1.1 Results of the Robustness Analysis for the Threshold Price of the reduced Sample

Table D.28: Probability of scarcity per commodity derived from the GVAR models of the robustness analysis for the threshold price for the reduced sample period from 2015 to 2019

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
REMod − REF	2019	Mean	0.42	0.02	0.00	0.06	0.00	0.00	0.29	0.08	0.00	0.04	0.06
		Shock	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	0.94	0.06	1.00	0.01	0.02	1.00	0.98	0.41	1.00	1.00
		Foc. FX	0.87	0.34	0.01	0.88	0.01	0.02	0.97	0.53	0.09	0.77	0.89
		Foc. FFR	0.78	0.19	0.01	0.29	0.02	0.02	0.75	0.31	0.06	0.28	0.35
		Foc. Extr. EA	1.00	1.00	0.67	1.00	0.33	0.34	1.00	1.00	0.98	1.00	1.00
		Foc. Extr. FX	0.95	0.76	0.17	0.99	0.33	0.19	0.99	0.86	0.28	0.96	0.99
		Foc. Extr. FFR	0.90	0.44	0.10	0.56	0.15	0.24	0.90	0.55	0.20	0.61	0.65
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.21	0.01	0.00	0.01	0.00	0.00	0.10	0.02	0.00	0.01	0.03
		Q. 60%	0.94	0.38	0.02	0.38	0.02	0.04	0.94	0.50	0.08	0.42	0.43
		Q. 75%	1.00	1.00	0.75	1.00	0.80	0.88	1.00	1.00	0.96	1.00	1.00
	Mean	Mean	1.00	0.17	0.00	0.42	0.95	0.00	0.66	0.20	0.99	0.21	0.01
		Shock	1.00	1.00	1.00	1.00	1.00	0.97	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	0.18	1.00	1.00	0.00	1.00	1.00	1.00	1.00	0.84
		Foc. FX	1.00	0.79	0.04	1.00	0.99	0.00	0.99	0.76	1.00	0.95	0.54
		Foc. FFR	1.00	0.48	0.04	0.77	0.99	0.00	0.93	0.50	1.00	0.65	0.12
		Foc. Extr. EA	1.00	1.00	0.84	1.00	1.00	0.02	1.00	1.00	1.00	1.00	0.99
		Foc. Extr. FX	1.00	0.94	0.31	1.00	1.00	0.01	1.00	0.94	1.00	0.99	0.93
		Foc. Extr. FFR	1.00	0.73	0.17	0.89	1.00	0.01	0.97	0.71	1.00	0.84	0.37
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
		Q. 40%	0.34	0.00	0.00	0.00	0.11	0.00	0.00	0.00	0.37	0.00	0.00
		Q. 50%	0.92	0.08	0.00	0.10	0.76	0.00	0.21	0.09	0.86	0.05	0.00

Probability of scarcity per commodity derived from the GVAR models of the robustness analysis for the threshold price for the reduced sample period from 2015 to 2019

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
		Q. 60%	1.00	0.74	0.05	0.88	1.00	0.00	0.99	0.75	1.00	0.82	0.15
		Q. 75%	1.00	1.00	0.91	1.00	1.00	0.22	1.00	1.00	1.00	1.00	1.00
REMod – SUF	2019	Mean	0.38	0.02	0.00	0.05	0.00	0.00	0.28	0.05	0.00	0.02	0.06
		Shock	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	0.94	0.06	1.00	0.00	0.01	1.00	0.97	0.38	1.00	1.00
		Foc. FX	0.85	0.31	0.00	0.89	0.01	0.01	0.99	0.49	0.08	0.77	0.90
		Foc. FFR	0.75	0.17	0.01	0.27	0.01	0.01	0.75	0.28	0.06	0.25	0.34
		Foc. Extr. EA	1.00	1.00	0.67	1.00	0.30	0.27	1.00	1.00	0.98	1.00	1.00
		Foc. Extr. FX	0.94	0.77	0.16	0.99	0.31	0.14	1.00	0.85	0.30	0.96	0.99
		Foc. Extr. FFR	0.89	0.41	0.08	0.54	0.13	0.21	0.90	0.52	0.19	0.57	0.63
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.19	0.01	0.00	0.01	0.00	0.00	0.09	0.02	0.00	0.00	0.02
		Q. 60%	0.94	0.29	0.01	0.34	0.01	0.01	0.92	0.44	0.09	0.37	0.38
		Q. 75%	1.00	1.00	0.69	1.00	0.75	0.80	1.00	1.00	0.97	1.00	1.00
	Mean	Mean	0.99	0.16	0.00	0.41	0.94	0.00	0.65	0.17	0.98	0.18	0.00
		Shock	1.00	1.00	1.00	1.00	1.00	0.94	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	0.16	1.00	1.00	0.00	1.00	1.00	1.00	1.00	0.84
		Foc. FX	1.00	0.77	0.03	1.00	0.99	0.00	0.99	0.74	1.00	0.96	0.52
		Foc. FFR	1.00	0.46	0.04	0.77	0.99	0.00	0.93	0.46	1.00	0.62	0.12
		Foc. Extr. EA	1.00	1.00	0.83	1.00	1.00	0.01	1.00	1.00	1.00	1.00	0.98
		Foc. Extr. FX	1.00	0.94	0.28	1.00	1.00	0.00	1.00	0.94	1.00	1.00	0.93
		Foc. Extr. FFR	1.00	0.72	0.15	0.89	1.00	0.00	0.98	0.67	1.00	0.83	0.36
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.37	0.00	0.00	0.00	0.11	0.00	0.01	0.00	0.38	0.00	0.00
		Q. 50%	0.92	0.06	0.00	0.09	0.76	0.00	0.20	0.07	0.86	0.04	0.00
		Q. 60%	1.00	0.68	0.02	0.87	1.00	0.00	0.99	0.68	1.00	0.78	0.13
		Q. 75%	1.00	1.00	0.87	1.00	1.00	0.10	1.00	1.00	1.00	1.00	1.00
REMod – PER	2019	Mean	0.49	0.02	0.00	0.07	0.00	0.00	0.34	0.10	0.00	0.03	0.07
		Shock	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	0.96	0.07	1.00	0.01	0.02	1.00	0.99	0.43	1.00	1.00
		Foc. FX	0.89	0.37	0.01	0.90	0.01	0.02	0.99	0.59	0.08	0.79	0.90
		Foc. FFR	0.82	0.23	0.01	0.31	0.02	0.02	0.80	0.34	0.07	0.31	0.35
		Foc. Extr. EA	1.00	1.00	0.69	1.00	0.31	0.32	1.00	1.00	0.98	1.00	1.00
		Foc. Extr. FX	0.96	0.80	0.19	0.99	0.34	0.19	1.00	0.88	0.31	0.97	0.99
		Foc. Extr. FFR	0.91	0.48	0.11	0.58	0.16	0.25	0.92	0.59	0.21	0.61	0.65
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.25	0.01	0.00	0.02	0.00	0.00	0.10	0.02	0.00	0.01	0.03
		Q. 60%	0.97	0.39	0.01	0.40	0.02	0.04	0.94	0.55	0.09	0.42	0.42
		Q. 75%	1.00	1.00	0.76	1.00	0.76	0.87	1.00	1.00	0.97	1.00	1.00
	Mean	Mean	1.00	0.20	0.00	0.44	0.96	0.00	0.71	0.23	0.99	0.24	0.01
		Shock	1.00	1.00	1.00	1.00	1.00	0.97	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	0.19	1.00	1.00	0.00	1.00	1.00	1.00	1.00	0.86
		Foc. FX	1.00	0.81	0.04	1.00	1.00	0.00	1.00	0.80	1.00	0.97	0.55
		Foc. FFR	1.00	0.53	0.04	0.79	0.99	0.00	0.95	0.54	1.00	0.67	0.13
		Foc. Extr. EA	1.00	1.00	0.85	1.00	1.00	0.02	1.00	1.00	1.00	1.00	0.99
		Foc. Extr. FX	1.00	0.95	0.31	1.00	1.00	0.01	1.00	0.95	1.00	0.99	0.93
		Foc. Extr. FFR	1.00	0.76	0.19	0.90	1.00	0.01	0.98	0.75	1.00	0.86	0.38
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.33	0.00	0.00	0.00	0.13	0.00	0.00	0.00	0.37	0.00	0.00
		Q. 50%	0.93	0.08	0.00	0.10	0.78	0.00	0.22	0.09	0.87	0.05	0.00
		Q. 60%	1.00	0.76	0.04	0.89	1.00	0.00	0.99	0.77	1.00	0.82	0.15
		Q. 75%	1.00	1.00	0.91	1.00	1.00	0.18	1.00	1.00	1.00	1.00	1.00
REMod – UNA	2019	Mean	0.43	0.02	0.00	0.06	0.00	0.00	0.29	0.08	0.00	0.03	0.06
		Shock	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	0.96	0.07	1.00	0.01	0.02	1.00	0.99	0.36	1.00	1.00
		Foc. FX	0.87	0.31	0.01	0.89	0.02	0.01	0.99	0.55	0.07	0.78	0.90
		Foc. FFR	0.78	0.18	0.01	0.29	0.01	0.01	0.76	0.31	0.06	0.28	0.33
		Foc. Extr. EA	1.00	1.00	0.68	1.00	0.30	0.27	1.00	1.00	0.98	1.00	1.00
		Foc. Extr. FX	0.95	0.77	0.18	0.99	0.33	0.15	1.00	0.87	0.28	0.97	0.99
		Foc. Extr. FFR	0.91	0.43	0.09	0.57	0.14	0.24	0.91	0.57	0.19	0.60	0.62
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.20	0.01	0.00	0.01	0.00	0.00	0.08	0.02	0.00	0.00	0.02

Probability of scarcity per commodity derived from the GVAR models of the robustness analysis for the threshold price for the reduced sample period from 2015 to 2019

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
$REMod - UNA$		Q. 60%	0.94	0.29	0.01	0.35	0.01	0.02	0.92	0.47	0.07	0.34	0.37
		Q. 75%	1.00	1.00	0.72	1.00	0.74	0.81	1.00	1.00	0.97	1.00	1.00
	Mean	Mean	1.00	0.17	0.00	0.44	0.96	0.00	0.66	0.20	0.99	0.21	0.01
		Shock	1.00	1.00	1.00	1.00	1.00	0.94	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	0.19	1.00	1.00	0.00	1.00	1.00	1.00	1.00	0.85
		Foc. FX	1.00	0.78	0.04	1.00	1.00	0.00	1.00	0.77	1.00	0.97	0.52
		Foc. FFR	1.00	0.47	0.04	0.78	0.99	0.00	0.95	0.51	1.00	0.66	0.11
		Foc. Extr. EA	1.00	1.00	0.84	1.00	1.00	0.02	1.00	1.00	1.00	1.00	0.98
		Foc. Extr. FX	1.00	0.95	0.30	1.00	1.00	0.00	1.00	0.94	1.00	1.00	0.94
		Foc. Extr. FFR	1.00	0.74	0.17	0.90	1.00	0.00	0.98	0.73	1.00	0.86	0.35
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.36	0.00	0.00	0.00	0.13	0.00	0.00	0.00	0.38	0.00	0.00
		Q. 50%	0.92	0.06	0.00	0.09	0.76	0.00	0.20	0.08	0.85	0.04	0.00
		Q. 60%	1.00	0.67	0.04	0.88	1.00	0.00	0.99	0.72	1.00	0.77	0.11
		Q. 75%	1.00	1.00	0.88	1.00	1.00	0.12	1.00	1.00	1.00	1.00	1.00

This table displays the probability of scarcity (PS) of the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), derived from the GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod - REF$, $REMod - SUF$, $REMod - PER$, and $REMod - UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). Hereby, the probability of scarcity is calculated for the scenarios Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%) of the input variables. The presented results are derived under the robustness test for the threshold prices, in particular, the threshold prices are calculated as the average commodity price of the period from 2015 to 2019.

Table D.29: Probability of scarcity per commodity derived from the MS-GVAR models of the robustness analysis for the threshold price for the reduced sample period from 2015 to 2019

			Al	Cu	Ni	Pb	Sn	Zn
$REMod - REF$	2019	Mean	0.97	1.00	1.00	0.95	0.78	0.74
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	1.00	1.00	0.99	0.99
		Foc. FX	1.00	1.00	1.00	1.00	0.99	0.99
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. FX	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.02	0.12	0.00	0.00	0.00
		Q. 50%	0.14	1.00	1.00	0.34	0.32	0.18
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
	Mean	Mean	1.00	1.00	1.00	1.00	1.00	0.06
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	1.00	1.00	1.00	0.60
		Foc. FX	1.00	1.00	1.00	1.00	1.00	0.69
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	1.00	1.00	1.00	1.00	1.00	0.87
		Foc. Extr. FX	1.00	1.00	1.00	1.00	1.00	0.90
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.62	0.55	0.01	0.07	0.00
		Q. 50%	0.99	1.00	1.00	0.84	0.99	0.00
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
$REMod - SUF$	2019	Mean	0.96	1.00	1.00	0.97	0.75	0.86
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. FX	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. FX	1.00	1.00	1.00	1.00	1.00	1.00

Probability of scarcity per commodity derived from the MS-GVAR models of the robustness analysis for the threshold price for the reduced sample period from 2015 to 2019

			Al	Cu	Ni	Pb	Sn	Zn
REMod – SUF	2019	Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.02	0.12	0.00	0.00	0.00
		Q. 50%	0.13	1.00	1.00	0.33	0.32	0.20
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
	Mean	Mean	1.00	1.00	1.00	1.00	1.00	0.08
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	1.00	1.00	1.00	0.71
		Foc. FX	1.00	1.00	1.00	1.00	1.00	0.79
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	1.00	1.00	1.00	1.00	1.00	0.91
		Foc. Extr. FX	1.00	1.00	1.00	1.00	1.00	0.93
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.63	0.59	0.01	0.07	0.00
		Q. 50%	0.99	1.00	1.00	0.85	1.00	0.00
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
REMod – PER	2019	Mean	0.98	1.00	1.00	0.96	0.76	0.78
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. FX	1.00	1.00	1.00	1.00	0.98	1.00
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. FX	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.02	0.14	0.00	0.00	0.00
		Q. 50%	0.14	0.99	1.00	0.34	0.32	0.18
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
	Mean	Mean	1.00	1.00	1.00	1.00	1.00	0.07
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	1.00	1.00	1.00	0.65
		Foc. FX	1.00	1.00	1.00	1.00	1.00	0.74
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	1.00	1.00	1.00	1.00	1.00	0.89
		Foc. Extr. FX	1.00	1.00	1.00	1.00	1.00	0.93
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.61	0.56	0.02	0.09	0.00
		Q. 50%	0.98	1.00	1.00	0.84	1.00	0.00
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
REMod – UNA	2019	Mean	0.97	1.00	1.00	0.95	0.78	0.83
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. FX	1.00	1.00	1.00	1.00	0.98	1.00
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. Extr. FX	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.02	0.14	0.00	0.01	0.00
		Q. 50%	0.14	0.99	1.00	0.31	0.34	0.22
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
	Mean	Mean	1.00	1.00	1.00	1.00	1.00	0.06
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	1.00	1.00	1.00	0.68
		Foc. FX	1.00	1.00	1.00	1.00	1.00	0.71
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	1.00	1.00	1.00	1.00	1.00	0.91
		Foc. Extr. FX	1.00	1.00	1.00	1.00	1.00	0.93

Probability of scarcity per commodity derived from the MS-GVAR models of the robustness analysis for the threshold price for the reduced sample period from 2015 to 2019

			Al	Cu	Ni	Pb	Sn	Zn
$REMod-UNA$		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
	Mean	Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.63	0.56	0.01	0.08	0.00
		Q. 50%	0.99	1.00	1.00	0.82	1.00	0.01
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00

This table displays the probability of scarcity (PS) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), derived from the MS-GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). Hereby, the probability of scarcity is calculated for the scenarios Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%) of the input variables. The presented results are derived under the robustness test for the threshold prices, in particular, the threshold prices are calculated as the average commodity price of the period from 2015 to 2019.

Table D.30: Estimated coefficients of the logistic regression models of the robustness analysis for the threshold price for the reduced sample period from 2015 to 2019

	Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
U.S. IP			-0.29										
GDP					0.59			0.37					
GDPc	-0.48	0.62	0.95										
FX				-0.14	-0.67	-0.04		-0.76		-0.58	0.18		
FFR				-0.18	-2.47			-1.60					0.51
LIR			0.10				4.20		-0.27	0.39			0.37
CPI							0.90						
MSCI		-0.05											
supply									0.38			-0.02	
OIL	0.49			0.40		0.23					-0.32	0.48	
ND			-0.27										
demand													0.71

This table displays the estimated coefficients of the individual logistic regression models of the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), based on the identified independent variables from the two stage model selection. Hereby, the independent variables are U.S. industrial production (U.S. IP), world gross domestic product (GDP), world gross domestic product per capita (GDPc), U.S. dollar index (FX), Federal Funds Effective Rate (FFR), 10-year U.S. Treasury rate (LIR), U.S. consumer price index (CPI), MSCI world stock index (MSCI), commodity-specific supply (**supply**), West Texas Intermediate spot crude oil price (OIL), global natural disasters (ND), and commodity-specific demand (**demand**). The presented results are derived under the robustness test for the threshold prices, in particular, the threshold prices are calculated as the average commodity price of the period from 2015 to 2019.

Table D.31: Probability of scarcity per commodity derived from the logistic regression models of the robustness analysis for the threshold price for the reduced sample period from 2015 to 2019

	Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
Mean	0.07	0.08	0.03	0.18	0.30	0.31	0.00	0.16	0.25	0.06	0.18	0.07	0.05
Shock	0.18	0.13	0.12	0.33	0.88	0.39	0.79	0.61	0.41	0.18	0.28	0.13	0.34
Extr.	0.39	0.21	0.36	0.51	0.99	0.47	1.00	0.93	0.60	0.41	0.42	0.22	0.82
Foc. EA	0.07	0.08	0.03	0.18	0.42	0.31	0.00	0.21	0.25	0.06	0.18	0.07	0.05
Foc. FX	0.07	0.08	0.03	0.21	0.45	0.32	0.00	0.28	0.25	0.10	0.21	0.07	0.05
Foc. FFR	0.07	0.08	0.03	0.20	0.71	0.31	0.00	0.37	0.25	0.06	0.18	0.07	0.08
Foc. Extr. EA	0.07	0.08	0.03	0.18	0.55	0.31	0.00	0.26	0.25	0.06	0.18	0.07	0.05
Foc. Extr. FX	0.07	0.08	0.03	0.23	0.60	0.33	0.00	0.44	0.25	0.16	0.24	0.07	0.05
Foc. Extr. FFR	0.07	0.08	0.03	0.23	0.93	0.31	0.00	0.63	0.25	0.06	0.18	0.07	0.10
Q. 25%	0.07	0.08	0.02	0.16	0.09	0.30	0.00	0.06	0.17	0.04	0.16	0.07	0.01
Q. 40%	0.07	0.08	0.02	0.17	0.16	0.31	0.00	0.09	0.20	0.04	0.17	0.07	0.03
Q. 50%	0.07	0.08	0.03	0.17	0.23	0.31	0.00	0.12	0.22	0.04	0.17	0.07	0.05
Q. 60%	0.10	0.09	0.04	0.21	0.41	0.33	0.00	0.20	0.28	0.06	0.19	0.09	0.07

Probability of scarcity per commodity derived from the logistic regression models of the robustness analysis for the threshold price for the reduced sample period from 2015 to 2019

	Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
Q. 75%	0.15	0.12	0.09	0.27	0.72	0.36	0.19	0.38	0.40	0.12	0.24	0.11	0.15

This table displays the probability of scarcity (PS) of the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), derived from the logistic regression models based on preselected determinants. Hereby, the probability of scarcity is calculated for the scenarios Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%) of the input variables. The presented results are derived under the robustness test for the threshold prices, in particular, the threshold prices are calculated as the average commodity price of the period from 2015 to 2019.

Table D.32: Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the GVAR models of the robustness analysis for the threshold price for the reduced sample period from 2015 to 2019

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
Mean	2019	$REMod-REF$	3.81	0.19	0.00	1.69	0.00	0.00	25.33	0.11	0.00	0.32	0.46
		$REMod-SUF$	2.57	0.13	0.00	1.22	0.00	0.00	17.12	0.08	0.00	0.22	0.47
		$REMod-PER$	4.33	0.17	0.00	1.44	0.00	0.00	20.04	0.11	0.00	0.31	0.38
		$REMod-UNA$	6.07	0.22	0.00	1.60	0.00	0.00	25.36	0.06	0.00	0.24	0.49
	Mean	$REMod-REF$	9.14	1.44	0.00	11.37	125.45	0.00	57.48	0.28	0.53	1.95	0.04
		$REMod-SUF$	6.17	0.97	0.00	8.22	91.86	0.00	38.85	0.19	0.35	1.33	0.04
		$REMod-PER$	10.41	1.26	0.00	9.66	128.53	0.00	45.47	0.27	0.66	1.88	0.04
		$REMod-UNA$	14.58	1.61	0.00	10.76	191.65	0.00	57.54	0.16	0.38	1.44	0.05
Shock	2019	$REMod-REF$	9.17	8.36	304.32	27.27	131.77	25.23	87.36	1.38	0.53	9.12	7.21
		$REMod-SUF$	6.19	5.67	204.60	19.71	96.49	16.96	59.05	0.94	0.35	6.21	7.33
		$REMod-PER$	10.44	7.34	219.38	23.16	135.01	18.20	69.11	1.36	0.67	8.77	5.91
		$REMod-UNA$	14.62	9.36	311.47	25.80	201.32	25.82	87.45	0.78	0.38	6.72	7.66
	Mean	$REMod-REF$	9.17	8.36	304.32	27.27	131.77	24.53	87.36	1.38	0.53	9.12	7.21
		$REMod-SUF$	6.19	5.67	204.60	19.71	96.49	16.49	59.05	0.94	0.35	6.21	7.33
		$REMod-PER$	10.44	7.34	219.38	23.16	135.01	17.69	69.11	1.36	0.67	8.77	5.91
		$REMod-UNA$	14.62	9.36	311.47	25.80	201.32	25.09	87.45	0.78	0.38	6.72	7.66
Extr.	2019	$REMod-REF$	9.17	8.36	304.32	27.27	131.77	25.23	87.36	1.38	0.53	9.12	7.21
		$REMod-SUF$	6.19	5.67	204.60	19.71	96.49	16.96	59.05	0.94	0.35	6.21	7.33
		$REMod-PER$	10.44	7.34	219.38	23.16	135.01	18.20	69.11	1.36	0.67	8.77	5.91
		$REMod-UNA$	14.62	9.36	311.47	25.80	201.32	25.82	87.45	0.78	0.38	6.72	7.66
	Mean	$REMod-REF$	9.17	8.36	304.32	27.27	131.77	25.23	87.36	1.38	0.53	9.12	7.21
		$REMod-SUF$	6.19	5.67	204.60	19.71	96.49	16.96	59.05	0.94	0.35	6.21	7.33
		$REMod-PER$	10.44	7.34	219.38	23.16	135.01	18.20	69.11	1.36	0.67	8.77	5.91
		$REMod-UNA$	14.62	9.36	311.47	25.80	201.32	25.82	87.45	0.78	0.38	6.72	7.66
Foc. EA	2019	$REMod-REF$	9.17	7.90	19.78	27.27	1.19	0.53	87.36	1.36	0.22	9.12	7.19
		$REMod-SUF$	6.19	5.36	13.30	19.71	0.87	0.36	59.05	0.92	0.14	6.21	7.31
		$REMod-PER$	10.44	6.94	14.26	23.16	1.22	0.38	69.11	1.33	0.27	8.77	5.89
		$REMod-UNA$	14.62	8.85	20.25	25.80	1.81	0.54	87.45	0.76	0.16	6.72	7.63
	Mean	$REMod-REF$	9.17	8.36	54.78	27.27	131.77	0.00	87.36	1.38	0.53	9.12	6.05
		$REMod-SUF$	6.19	5.67	36.83	19.71	96.49	0.00	59.05	0.94	0.35	6.21	6.15
		$REMod-PER$	10.44	7.34	39.49	23.16	135.01	0.00	69.11	1.36	0.67	8.77	4.96
		$REMod-UNA$	14.62	9.36	56.06	25.80	201.32	0.00	87.45	0.78	0.38	6.72	6.42
Foc. FX	2019	$REMod-REF$	8.00	2.83	2.13	24.10	1.84	0.43	85.09	0.74	0.05	7.01	6.44
		$REMod-SUF$	5.40	1.92	1.43	17.42	1.35	0.29	57.51	0.50	0.03	4.78	6.55
		$REMod-PER$	9.11	2.49	1.54	20.47	1.89	0.31	67.31	0.72	0.06	6.74	5.28
		$REMod-UNA$	12.75	3.17	2.18	22.81	2.82	0.44	85.17	0.41	0.03	5.17	6.85
	Mean	$REMod-REF$	9.16	6.61	12.48	27.19	131.11	0.00	86.92	1.05	0.53	8.69	3.91
		$REMod-SUF$	6.18	4.48	8.39	19.65	96.01	0.00	58.75	0.71	0.35	5.92	3.98
		$REMod-PER$	10.43	5.81	8.99	23.09	134.34	0.00	68.76	1.03	0.67	8.35	3.21
		$REMod-UNA$	14.60	7.41	12.77	25.72	200.31	0.00	87.01	0.59	0.38	6.41	4.16
Foc. FFR	2019	$REMod-REF$	7.13	1.62	3.65	7.85	2.50	0.56	65.87	0.43	0.03	2.54	2.55
		$REMod-SUF$	4.82	1.10	2.46	5.68	1.83	0.37	44.52	0.29	0.02	1.73	2.60
		$REMod-PER$	8.13	1.42	2.63	6.67	2.57	0.40	52.11	0.42	0.04	2.45	2.09
		$REMod-UNA$	11.37	1.82	3.74	7.43	3.83	0.57	65.94	0.24	0.02	1.88	2.71
	Mean	$REMod-REF$	9.15	4.00	13.69	21.05	130.32	0.00	81.42	0.69	0.53	5.91	0.89
		$REMod-SUF$	6.18	2.72	9.21	15.22	95.43	0.00	55.03	0.47	0.35	4.02	0.91
		$REMod-PER$	10.42	3.52	9.87	17.88	133.53	0.00	64.41	0.68	0.67	5.68	0.73
		$REMod-UNA$	14.59	4.49	14.02	19.92	199.10	0.00	81.50	0.39	0.38	4.36	0.95
Foc. Extr. EA	2019	$REMod-REF$	9.17	8.36	203.90	27.27	43.09	8.60	87.36	1.38	0.52	9.12	7.21
		$REMod-SUF$	6.19	5.67	137.08	19.71	31.55	5.78	59.05	0.94	0.35	6.21	7.33
		$REMod-PER$	10.44	7.34	146.99	23.16	44.15	6.20	69.11	1.36	0.66	8.77	5.91
		$REMod-UNA$	14.62	9.36	208.69	25.80	65.83	8.80	87.45	0.78	0.38	6.72	7.66
	Mean	$REMod-REF$	9.17	8.36	256.54	27.27	131.77	0.63	87.36	1.38	0.53	9.12	7.12
		$REMod-SUF$	6.19	5.67	172.48	19.71	96.49	0.42	59.05	0.94	0.35	6.21	7.24
		$REMod-PER$	10.44	7.34	184.94	23.16	135.01	0.45	69.11	1.36	0.67	8.77	5.83

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the GVAR models of the robustness analysis for the threshold price for the reduced sample period from 2015 to 2019

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
		$REMod-UNA$	14.62	9.36	262.57	25.80	201.32	0.65	87.45	0.78	0.38	6.72	7.56
Foc. Extr. FX	2019	$REMod-REF$	8.75	6.36	50.52	26.97	43.22	4.90	86.92	1.19	0.15	8.77	7.13
		$REMod-SUF$	5.90	4.31	33.96	19.49	31.65	3.29	58.75	0.81	0.10	5.97	7.25
		$REMod-PER$	9.96	5.59	36.42	22.90	44.28	3.53	68.76	1.17	0.19	8.43	5.85
		$REMod-UNA$	13.95	7.13	51.70	25.52	66.03	5.01	87.01	0.67	0.11	6.47	7.57
	Mean	$REMod-REF$	9.17	7.85	93.12	27.27	131.51	0.28	87.27	1.29	0.53	9.07	6.73
		$REMod-SUF$	6.19	5.32	62.61	19.71	96.30	0.19	58.99	0.88	0.35	6.18	6.85
		$REMod-PER$	10.44	6.89	67.13	23.16	134.74	0.20	69.04	1.27	0.67	8.72	5.52
		$REMod-UNA$	14.62	8.79	95.31	25.80	200.92	0.28	87.36	0.73	0.38	6.69	7.15
Foc. Extr. FFR	2019	$REMod-REF$	8.24	3.64	31.04	15.16	19.77	6.03	78.36	0.77	0.11	5.53	4.71
		$REMod-SUF$	5.56	2.47	20.87	10.96	14.47	4.05	52.97	0.52	0.07	3.77	4.79
		$REMod-PER$	9.39	3.19	22.38	12.88	20.25	4.35	61.99	0.75	0.13	5.32	3.87
		$REMod-UNA$	13.14	4.07	31.77	14.35	30.20	6.17	78.44	0.43	0.08	4.08	5.01
	Mean	$REMod-REF$	9.16	6.14	52.34	24.38	131.24	0.15	85.00	0.98	0.53	7.70	2.68
		$REMod-SUF$	6.18	4.16	35.19	17.62	96.10	0.10	57.45	0.67	0.35	5.25	2.73
		$REMod-PER$	10.43	5.39	37.73	20.70	134.47	0.11	67.24	0.97	0.67	7.41	2.20
		$REMod-UNA$	14.60	6.87	53.57	23.07	200.51	0.15	85.09	0.55	0.38	5.68	2.85
Q. 25%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Mean	$REMod-REF$	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Q. 40%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.00	0.00	0.00	0.00
	Mean	$REMod-REF$	3.12	0.00	0.00	0.00	14.63	0.00	0.26	0.00	0.20	0.00	0.00
		$REMod-SUF$	2.10	0.00	0.00	0.00	10.71	0.00	0.18	0.00	0.13	0.00	0.00
		$REMod-PER$	3.55	0.00	0.00	0.00	14.99	0.00	0.21	0.00	0.25	0.00	0.00
		$REMod-UNA$	4.97	0.00	0.00	0.00	22.35	0.00	0.26	0.00	0.14	0.00	0.00
Q. 50%	2019	$REMod-REF$	1.91	0.07	0.00	0.27	0.00	0.00	9.00	0.03	0.00	0.06	0.19
		$REMod-SUF$	1.29	0.05	0.00	0.20	0.00	0.00	6.08	0.02	0.00	0.04	0.20
		$REMod-PER$	2.17	0.06	0.00	0.23	0.00	0.00	7.12	0.03	0.00	0.06	0.16
		$REMod-UNA$	3.04	0.07	0.00	0.26	0.00	0.00	9.01	0.02	0.00	0.05	0.21
	Mean	$REMod-REF$	8.43	0.66	0.00	2.73	99.49	0.00	18.78	0.12	0.46	0.47	0.01
		$REMod-SUF$	5.69	0.45	0.00	1.97	72.85	0.00	12.70	0.08	0.30	0.32	0.01
		$REMod-PER$	9.60	0.58	0.00	2.32	101.93	0.00	14.86	0.12	0.57	0.46	0.01
		$REMod-UNA$	13.44	0.74	0.00	2.58	151.99	0.00	18.80	0.07	0.33	0.35	0.02
Q. 60%	2019	$REMod-REF$	8.67	3.14	4.56	10.39	2.50	1.01	82.38	0.69	0.05	3.86	3.12
		$REMod-SUF$	5.85	2.13	3.07	7.51	1.83	0.68	55.68	0.47	0.03	2.63	3.17
		$REMod-PER$	9.87	2.75	3.29	8.82	2.57	0.73	65.17	0.68	0.06	3.71	2.56
		$REMod-UNA$	13.82	3.51	4.67	9.83	3.83	1.03	82.46	0.39	0.03	2.84	3.32
	Mean	$REMod-REF$	9.17	6.15	14.00	23.97	131.51	0.03	86.66	1.03	0.53	7.47	1.07
		$REMod-SUF$	6.19	4.17	9.41	17.32	96.30	0.02	58.58	0.70	0.35	5.09	1.09
		$REMod-PER$	10.44	5.40	10.09	20.36	134.74	0.02	68.55	1.02	0.67	7.18	0.88
		$REMod-UNA$	14.62	6.89	14.33	22.68	200.92	0.03	86.75	0.58	0.38	5.51	1.14
Q. 75%	2019	$REMod-REF$	9.17	8.36	228.24	27.27	105.28	22.33	87.36	1.38	0.52	9.12	7.21
		$REMod-SUF$	6.19	5.67	153.45	19.71	77.10	15.01	59.05	0.94	0.34	6.21	7.33
		$REMod-PER$	10.44	7.34	164.54	23.16	107.88	16.10	69.11	1.36	0.65	8.77	5.91
		$REMod-UNA$	14.62	9.36	233.60	25.80	160.85	22.85	87.45	0.78	0.37	6.72	7.66
	Mean	$REMod-REF$	9.17	8.36	277.54	27.27	131.77	5.63	87.36	1.38	0.53	9.12	7.21
		$REMod-SUF$	6.19	5.67	186.59	19.71	96.49	3.78	59.05	0.94	0.35	6.21	7.33
		$REMod-PER$	10.44	7.34	200.08	23.16	135.01	4.06	69.11	1.36	0.67	8.77	5.91
		$REMod-UNA$	14.62	9.36	284.06	25.80	201.32	5.76	87.45	0.78	0.38	6.72	7.66

This table displays the expected loss due to scarcity for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), per path ($REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$), as well as per scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) for the input variables. Hereby, the values are derived from the GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). In particular, the results are derived under the robustness test for the threshold prices, in particular, the threshold prices are calculated as the average commodity price of the period from 2015 to 2019.

Table D.33: Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the MS-GVAR models of the robustness analysis for the threshold price for the reduced sample period from 2015 to 2019

			Al	Cu	Ni	Pb	Sn	Zn
Mean	2019	$REMod-REF$	8.14	27.27	87.36	1.32	7.11	5.33
		$REMod-SUF$	5.52	19.71	59.05	0.89	4.84	5.42
		$REMod-PER$	7.15	23.16	69.11	1.29	6.84	4.37
		$REMod-UNA$	9.12	25.80	87.45	0.74	5.24	5.67
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	0.40
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	0.41
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	0.33
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	0.43
Shock	2019	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
Extr.	2019	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
Foc. EA	2019	$REMod-REF$	8.36	27.27	87.36	1.38	9.01	7.17
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.14	7.29
		$REMod-PER$	7.34	23.16	69.11	1.36	8.66	5.88
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.64	7.61
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	4.30
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	4.37
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	3.52
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	4.56
Foc. FX	2019	$REMod-REF$	8.36	27.27	87.36	1.38	9.01	7.11
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.14	7.23
		$REMod-PER$	7.34	23.16	69.11	1.36	8.66	5.83
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.64	7.55
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	4.95
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	5.03
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	4.05
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	5.25
Foc. FFR	2019	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
Foc. Extr. EA	2019	$REMod-REF$	8.36	27.27	87.36	1.38	9.08	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.19	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.73	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.70	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	6.27
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	6.38
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.14
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	6.66
Foc. Extr. FX	2019	$REMod-REF$	8.36	27.27	87.36	1.38	9.08	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.19	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.73	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.70	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	6.50
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	6.61
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.33
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	6.91
Foc. Extr. FFR	2019	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33

			Al	Cu	Ni	Pb	Sn	Zn
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
Q. 25%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00
	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00
Q. 40%	2019	$REMod-REF$	0.00	0.65	10.83	0.00	0.04	0.00
		$REMod-SUF$	0.00	0.47	7.32	0.00	0.02	0.00
		$REMod-PER$	0.00	0.56	8.57	0.00	0.04	0.00
		$REMod-UNA$	0.00	0.62	10.84	0.00	0.03	0.00
	Mean	$REMod-REF$	0.00	16.85	47.70	0.02	0.64	0.00
		$REMod-SUF$	0.00	12.18	32.24	0.01	0.43	0.00
		$REMod-PER$	0.00	14.31	37.73	0.02	0.61	0.00
		$REMod-UNA$	0.00	15.95	47.75	0.01	0.47	0.00
Q. 50%	2019	$REMod-REF$	1.19	27.16	87.36	0.47	2.90	1.30
		$REMod-SUF$	0.80	19.63	59.05	0.32	1.97	1.32
		$REMod-PER$	1.04	23.07	69.11	0.46	2.79	1.06
		$REMod-UNA$	1.33	25.70	87.45	0.27	2.14	1.38
	Mean	$REMod-REF$	8.24	27.27	87.36	1.16	9.01	0.00
		$REMod-SUF$	5.59	19.71	59.05	0.79	6.14	0.00
		$REMod-PER$	7.24	23.16	69.11	1.14	8.66	0.00
		$REMod-UNA$	9.23	25.80	87.45	0.65	6.64	0.00
Q. 60%	2019	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
Q. 75%	2019	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66

This table displays the expected loss due to scarcity for the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), per path ($REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$), as well as per scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) for the input variables. Hereby, the values are derived from the MS-GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). In particular, the results are derived under the robustness test for the threshold prices, in particular, the threshold prices are calculated as the average commodity price of the period from 2015 to 2019.

Table D.34: Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the logistic regression models of the robustness analysis for the threshold price for the reduced sample period from 2015 to 2019

		Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
Mean	$REMod-REF$	0.67	0.68	9.31	5.03	43.17	41.38	0.01	2.98	21.59	0.09	0.10	0.67	0.39
	$REMod-SUF$	0.45	0.46	6.26	3.64	30.86	30.30	0.01	2.12	14.59	0.06	0.06	0.46	0.40
	$REMod-PER$	0.76	0.59	6.71	4.28	41.58	42.40	0.01	2.76	17.08	0.09	0.12	0.64	0.32
	$REMod-UNA$	1.07	0.76	9.52	4.76	20.58	63.22	0.01	1.28	21.61	0.05	0.07	0.49	0.42

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the logistic regression models of the robustness analysis for the threshold price for the reduced sample period from 2015 to 2019

		Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
Shock	$REMod-REF$	1.68	1.12	35.94	8.91	124.58	51.27	19.96	11.33	36.17	0.25	0.15	1.18	2.44
	$REMod-SUF$	1.14	0.76	24.17	6.44	89.05	37.54	13.42	8.06	24.45	0.17	0.10	0.80	2.48
	$REMod-PER$	1.92	0.99	25.91	7.57	119.98	52.53	14.39	10.48	28.62	0.24	0.19	1.13	2.00
	$REMod-UNA$	2.68	1.26	36.79	8.43	59.40	78.33	20.42	4.85	36.21	0.14	0.11	0.87	2.59
Extr.	$REMod-REF$	3.58	1.80	110.33	13.91	140.55	61.91	25.23	17.18	52.72	0.56	0.22	1.98	5.91
	$REMod-SUF$	2.41	1.22	74.17	10.05	100.46	45.33	16.96	12.22	35.63	0.38	0.15	1.35	6.01
	$REMod-PER$	4.07	1.58	79.53	11.81	135.35	63.43	18.20	15.90	41.70	0.55	0.28	1.91	4.84
	$REMod-UNA$	5.70	2.01	112.92	13.16	67.01	94.58	25.82	7.36	52.77	0.32	0.16	1.46	6.27
Foc. EA	$REMod-REF$	0.67	0.68	9.31	5.03	59.55	41.38	0.01	3.84	21.59	0.09	0.10	0.67	0.39
	$REMod-SUF$	0.45	0.46	6.26	3.64	42.57	30.30	0.01	2.73	14.59	0.06	0.06	0.46	0.40
	$REMod-PER$	0.76	0.59	6.71	4.28	57.35	42.40	0.01	3.56	17.08	0.09	0.12	0.64	0.32
	$REMod-UNA$	1.07	0.76	9.52	4.76	28.39	63.22	0.01	1.65	21.61	0.05	0.07	0.49	0.42
Foc. FX	$REMod-REF$	0.67	0.68	9.31	5.60	63.21	42.45	0.01	5.14	21.59	0.14	0.11	0.67	0.39
	$REMod-SUF$	0.45	0.46	6.26	4.05	45.18	31.09	0.01	3.65	14.59	0.10	0.07	0.46	0.40
	$REMod-PER$	0.76	0.59	6.71	4.75	60.87	43.50	0.01	4.75	17.08	0.14	0.14	0.64	0.32
	$REMod-UNA$	1.07	0.76	9.52	5.30	30.14	64.86	0.01	2.20	21.61	0.08	0.08	0.49	0.42
Foc. FFR	$REMod-REF$	0.67	0.68	9.31	5.57	99.94	41.38	0.01	6.76	21.59	0.09	0.10	0.67	0.54
	$REMod-SUF$	0.45	0.46	6.26	4.02	71.44	30.30	0.01	4.81	14.59	0.06	0.06	0.46	0.55
	$REMod-PER$	0.76	0.59	6.71	4.73	96.25	42.40	0.01	6.26	17.08	0.09	0.12	0.64	0.45
	$REMod-UNA$	1.07	0.76	9.52	5.27	47.65	63.22	0.01	2.90	21.61	0.05	0.07	0.49	0.58
Foc. Extr. EA	$REMod-REF$	0.67	0.68	9.31	5.03	77.27	41.38	0.01	4.88	21.59	0.09	0.10	0.67	0.39
	$REMod-SUF$	0.45	0.46	6.26	3.64	55.23	30.30	0.01	3.47	14.59	0.06	0.06	0.46	0.40
	$REMod-PER$	0.76	0.59	6.71	4.28	74.41	42.40	0.01	4.52	17.08	0.09	0.12	0.64	0.32
	$REMod-UNA$	1.07	0.76	9.52	4.76	36.84	63.22	0.01	2.09	21.61	0.05	0.07	0.49	0.42
Foc. Extr. FX	$REMod-REF$	0.67	0.68	9.31	6.21	84.56	43.54	0.01	8.05	21.59	0.23	0.13	0.67	0.39
	$REMod-SUF$	0.45	0.46	6.26	4.49	60.44	31.88	0.01	5.73	14.59	0.15	0.08	0.46	0.40
	$REMod-PER$	0.76	0.59	6.71	5.27	81.43	44.61	0.01	7.45	17.08	0.22	0.16	0.64	0.32
	$REMod-UNA$	1.07	0.76	9.52	5.87	40.32	66.52	0.01	3.45	21.61	0.13	0.09	0.49	0.42
Foc. Extr. FFR	$REMod-REF$	0.67	0.68	9.31	6.14	131.64	41.38	0.01	11.73	21.59	0.09	0.10	0.67	0.75
	$REMod-SUF$	0.45	0.46	6.26	4.44	94.09	30.30	0.01	8.34	14.59	0.06	0.06	0.46	0.76
	$REMod-PER$	0.76	0.59	6.71	5.22	126.77	42.40	0.01	10.86	17.08	0.09	0.12	0.64	0.61
	$REMod-UNA$	1.07	0.76	9.52	5.81	62.76	63.22	0.01	5.02	21.61	0.05	0.07	0.49	0.79
Q. 25%	$REMod-REF$	0.61	0.64	5.36	4.29	13.37	40.10	0.00	1.13	15.01	0.05	0.09	0.63	0.09
	$REMod-SUF$	0.41	0.43	3.60	3.10	9.56	29.36	0.00	0.80	10.15	0.03	0.06	0.43	0.09
	$REMod-PER$	0.70	0.56	3.86	3.65	12.87	41.09	0.00	1.05	11.88	0.05	0.11	0.61	0.07
	$REMod-UNA$	0.98	0.72	5.48	4.06	6.37	61.26	0.00	0.48	15.03	0.03	0.06	0.47	0.09
Q. 40%	$REMod-REF$	0.63	0.65	6.76	4.51	22.99	40.31	0.00	1.67	17.41	0.05	0.09	0.64	0.20
	$REMod-SUF$	0.42	0.44	4.55	3.26	16.44	29.52	0.00	1.19	11.77	0.04	0.06	0.44	0.20
	$REMod-PER$	0.71	0.57	4.88	3.83	22.14	41.30	0.00	1.55	13.77	0.05	0.11	0.62	0.16
	$REMod-UNA$	1.00	0.73	6.92	4.27	10.96	61.59	0.00	0.72	17.43	0.03	0.06	0.47	0.21
Q. 50%	$REMod-REF$	0.63	0.65	7.90	4.66	32.10	40.45	0.00	2.16	19.17	0.06	0.09	0.65	0.33
	$REMod-SUF$	0.43	0.44	5.31	3.37	22.95	29.62	0.00	1.53	12.95	0.04	0.06	0.44	0.34
	$REMod-PER$	0.72	0.57	5.69	3.96	30.92	41.45	0.00	1.99	15.16	0.06	0.11	0.62	0.27
	$REMod-UNA$	1.01	0.73	8.08	4.41	15.31	61.80	0.00	0.92	19.18	0.03	0.06	0.48	0.35
Q. 60%	$REMod-REF$	0.88	0.79	13.39	5.63	58.60	43.38	0.01	3.65	24.87	0.09	0.10	0.78	0.54
	$REMod-SUF$	0.59	0.53	9.00	4.07	41.89	31.77	0.01	2.59	16.81	0.06	0.07	0.53	0.55
	$REMod-PER$	1.00	0.69	9.65	4.78	56.43	44.45	0.01	3.38	19.67	0.09	0.13	0.75	0.44
	$REMod-UNA$	1.40	0.88	13.70	5.33	27.94	66.28	0.01	1.56	24.89	0.05	0.07	0.57	0.57
Q. 75%	$REMod-REF$	1.40	1.03	28.78	7.36	102.71	47.97	4.87	7.11	35.06	0.17	0.13	1.02	1.06
	$REMod-SUF$	0.94	0.70	19.35	5.32	73.42	35.13	3.27	5.06	23.70	0.11	0.08	0.69	1.08
	$REMod-PER$	1.59	0.91	20.75	6.25	98.91	49.15	3.51	6.58	27.73	0.16	0.16	0.98	0.87

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the logistic regression models of the robustness analysis for the threshold price for the reduced sample period from 2015 to 2019

	Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
$REMod - UNA$	2.23	1.16	29.46	6.96	48.97	73.29	4.98	3.05	35.09	0.09	0.09	0.75	1.13

This table displays the expected loss due to scarcity for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), per path ($REMod - REF$, $REMod - SUF$, $REMod - PER$ and $REMod - UNA$), as well as per scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) for the input variables, derived from the logistic regression model. Hereby, the results are derived under the robustness test for the threshold prices. The threshold prices are calculated as the average commodity price of the period from 2015 to 2019.

D.3.2.1.2 Results of the Robustness Analysis for the Threshold Price of the enlarged Sample

Table D.35: Probability of scarcity per commodity derived from the GVAR models of the robustness analysis for the threshold price for the enlarged sample period from 1995 to 2019

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
REMod − REF	2019	Mean	0.00	0.00	0.00	0.01	0.00	0.99	0.00	0.15	0.00	0.04	0.47
		Shock	1.00	1.00	1.00	1.00	0.96	1.00	1.00	1.00	0.97	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.35	0.47	0.10	0.97	0.00	1.00	0.66	1.00	0.00	1.00	1.00
		Foc. FX	0.18	0.07	0.02	0.47	0.00	1.00	0.23	0.68	0.00	0.79	0.99
		Foc. FFR	0.11	0.04	0.02	0.08	0.00	1.00	0.03	0.42	0.00	0.29	0.81
		Foc. Extr. EA	0.85	0.97	0.76	1.00	0.03	1.00	0.97	1.00	0.23	1.00	1.00
		Foc. Extr. FX	0.54	0.48	0.23	0.93	0.01	1.00	0.80	0.91	0.02	0.96	1.00
		Foc. Extr. FFR	0.38	0.20	0.13	0.28	0.01	1.00	0.23	0.66	0.01	0.62	0.92
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.05	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.00	0.00	0.83	0.00	0.05	0.00	0.01	0.19
		Q. 60%	0.23	0.10	0.02	0.11	0.00	1.00	0.07	0.65	0.00	0.44	0.91
		Q. 75%	0.99	0.99	0.84	1.00	0.17	1.00	1.00	1.00	0.15	1.00	1.00
	Mean	Mean	0.57	0.02	0.00	0.08	0.00	0.00	0.00	0.31	0.03	0.22	0.10
		Shock	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	0.92	0.29	1.00	0.26	0.18	0.84	1.00	0.87	1.00	1.00
		Foc. FX	0.93	0.30	0.06	0.92	0.32	0.13	0.42	0.88	0.17	0.96	0.95
		Foc. FFR	0.86	0.17	0.07	0.33	0.17	0.16	0.08	0.64	0.15	0.66	0.44
		Foc. Extr. EA	1.00	1.00	0.90	1.00	0.80	0.72	0.99	1.00	1.00	1.00	1.00
		Foc. Extr. FX	0.97	0.75	0.39	0.99	0.77	0.48	0.88	0.97	0.49	1.00	0.99
		Foc. Extr. FFR	0.95	0.41	0.24	0.61	0.44	0.61	0.35	0.81	0.35	0.86	0.73
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.31	0.01	0.00	0.02	0.00	0.00	0.00	0.15	0.01	0.06	0.04
		Q. 60%	0.97	0.33	0.08	0.45	0.25	0.24	0.16	0.85	0.24	0.83	0.53
		Q. 75%	1.00	1.00	0.96	1.00	0.99	0.99	1.00	1.00	1.00	1.00	1.00
REMod − SUF	2019	Mean	0.00	0.00	0.00	0.01	0.00	0.99	0.00	0.11	0.00	0.02	0.44
		Shock	1.00	1.00	1.00	1.00	0.96	1.00	1.00	1.00	0.98	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.31	0.44	0.10	0.96	0.00	1.00	0.63	1.00	0.00	1.00	1.00
		Foc. FX	0.15	0.04	0.01	0.45	0.00	1.00	0.17	0.65	0.00	0.78	0.99
		Foc. FFR	0.10	0.04	0.02	0.07	0.00	1.00	0.02	0.38	0.00	0.27	0.79
		Foc. Extr. EA	0.85	0.96	0.75	1.00	0.03	1.00	0.96	1.00	0.21	1.00	1.00
		Foc. Extr. FX	0.51	0.44	0.21	0.93	0.01	1.00	0.81	0.92	0.02	0.97	1.00
		Foc. Extr. FFR	0.35	0.19	0.11	0.26	0.00	1.00	0.20	0.63	0.01	0.58	0.92
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.10	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.00	0.00	0.87	0.00	0.04	0.00	0.00	0.17
		Q. 60%	0.19	0.06	0.01	0.08	0.00	1.00	0.04	0.59	0.00	0.38	0.89
		Q. 75%	0.98	0.98	0.79	1.00	0.11	1.00	1.00	1.00	0.14	1.00	1.00
	Mean	Mean	0.55	0.02	0.00	0.07	0.00	0.00	0.00	0.27	0.02	0.19	0.10
		Shock	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	0.91	0.28	1.00	0.21	0.12	0.83	1.00	0.86	1.00	1.00
		Foc. FX	0.92	0.26	0.05	0.92	0.29	0.09	0.39	0.87	0.18	0.97	0.94
		Foc. FFR	0.85	0.16	0.05	0.32	0.14	0.13	0.06	0.59	0.14	0.64	0.41
		Foc. Extr. EA	1.00	1.00	0.89	1.00	0.77	0.67	0.99	1.00	1.00	1.00	1.00
		Foc. Extr. FX	0.97	0.73	0.36	0.99	0.76	0.41	0.89	0.96	0.49	1.00	0.99
		Foc. Extr. FFR	0.93	0.38	0.20	0.59	0.42	0.60	0.32	0.78	0.35	0.84	0.69
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.31	0.00	0.00	0.01	0.00	0.00	0.00	0.14	0.01	0.04	0.04
		Q. 60%	0.97	0.25	0.05	0.40	0.20	0.12	0.09	0.83	0.23	0.80	0.47
		Q. 75%	1.00	1.00	0.94	1.00	0.99	0.98	1.00	1.00	1.00	1.00	1.00
REMod − PER	2019	Mean	0.01	0.00	0.00	0.01	0.00	0.99	0.00	0.18	0.00	0.04	0.47
		Shock	1.00	1.00	1.00	1.00	0.94	1.00	1.00	1.00	0.97	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.42	0.50	0.12	0.98	0.00	1.00	0.68	1.00	0.00	1.00	1.00
		Foc. FX	0.23	0.07	0.02	0.49	0.00	1.00	0.23	0.73	0.00	0.81	1.00
		Foc. FFR	0.18	0.05	0.03	0.10	0.00	1.00	0.04	0.47	0.00	0.32	0.80
		Foc. Extr. EA	0.89	0.97	0.77	1.00	0.03	1.00	0.97	1.00	0.22	1.00	1.00
		Foc. Extr. FX	0.58	0.49	0.24	0.94	0.01	1.00	0.83	0.93	0.02	0.97	1.00
		Foc. Extr. FFR	0.44	0.23	0.14	0.30	0.00	1.00	0.23	0.70	0.01	0.62	0.92
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Probability of scarcity per commodity derived from the GVAR models of the robustness analysis for the threshold price for the enlarged sample period from 1995 to 2019

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
$REMod-PER$	2019	Q. 40%	0.00	0.00	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.00	0.00	0.87	0.00	0.06	0.00	0.01	0.17
		Q. 60%	0.33	0.11	0.02	0.11	0.00	1.00	0.06	0.69	0.00	0.44	0.90
		Q. 75%	1.00	0.99	0.84	1.00	0.14	1.00	1.00	1.00	0.15	1.00	1.00
	Mean	Mean	0.64	0.02	0.00	0.09	0.00	0.00	0.00	0.36	0.03	0.25	0.12
		Shock	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	0.94	0.30	1.00	0.25	0.18	0.86	1.00	0.88	1.00	1.00
		Foc. FX	0.94	0.32	0.07	0.93	0.33	0.13	0.45	0.91	0.18	0.97	0.94
		Foc. FFR	0.88	0.20	0.07	0.36	0.18	0.18	0.07	0.67	0.16	0.69	0.43
		Foc. Extr. EA	1.00	1.00	0.91	1.00	0.82	0.72	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. FX	0.98	0.77	0.40	0.99	0.80	0.48	0.90	0.97	0.50	1.00	0.99
		Foc. Extr. FFR	0.95	0.45	0.26	0.63	0.45	0.66	0.36	0.83	0.37	0.87	0.71
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.35	0.00	0.00	0.02	0.00	0.00	0.00	0.16	0.01	0.06	0.04
		Q. 60%	0.99	0.35	0.08	0.47	0.24	0.20	0.14	0.88	0.26	0.83	0.52
		Q. 75%	1.00	1.00	0.96	1.00	0.99	0.99	1.00	1.00	1.00	1.00	1.00
$REMod-UNA$	2019	Mean	0.00	0.00	0.00	0.01	0.00	0.99	0.00	0.15	0.00	0.03	0.44
		Shock	1.00	1.00	1.00	1.00	0.95	1.00	1.00	1.00	0.97	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.36	0.46	0.12	0.97	0.00	1.00	0.65	1.00	0.00	1.00	1.00
		Foc. FX	0.18	0.05	0.02	0.47	0.00	1.00	0.20	0.71	0.00	0.79	0.99
		Foc. FFR	0.12	0.04	0.02	0.08	0.00	1.00	0.02	0.43	0.00	0.29	0.78
		Foc. Extr. EA	0.87	0.96	0.77	1.00	0.03	1.00	0.97	1.00	0.21	1.00	1.00
		Foc. Extr. FX	0.55	0.45	0.24	0.94	0.01	1.00	0.83	0.92	0.02	0.97	1.00
		Foc. Extr. FFR	0.38	0.19	0.12	0.26	0.00	1.00	0.22	0.68	0.02	0.61	0.92
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.10	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.00	0.00	0.87	0.00	0.05	0.00	0.00	0.15
		Q. 60%	0.19	0.07	0.01	0.09	0.00	1.00	0.05	0.63	0.00	0.37	0.86
		Q. 75%	0.99	0.98	0.81	1.00	0.10	1.00	1.00	1.00	0.11	1.00	1.00
	Mean	Mean	0.58	0.02	0.00	0.08	0.00	0.00	0.00	0.32	0.02	0.23	0.09
		Shock	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	0.93	0.30	1.00	0.24	0.14	0.85	1.00	0.88	1.00	1.00
		Foc. FX	0.92	0.28	0.06	0.92	0.32	0.10	0.42	0.89	0.17	0.97	0.93
		Foc. FFR	0.86	0.16	0.07	0.34	0.17	0.15	0.07	0.66	0.14	0.68	0.42
		Foc. Extr. EA	1.00	1.00	0.92	1.00	0.80	0.67	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. FX	0.97	0.74	0.38	0.99	0.79	0.43	0.91	0.97	0.46	1.00	0.99
		Foc. Extr. FFR	0.95	0.40	0.24	0.61	0.41	0.64	0.34	0.83	0.33	0.86	0.70
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.30	0.00	0.00	0.02	0.00	0.00	0.00	0.14	0.01	0.04	0.03
		Q. 60%	0.97	0.25	0.06	0.41	0.19	0.13	0.09	0.84	0.21	0.79	0.44
		Q. 75%	1.00	1.00	0.94	1.00	0.99	0.99	1.00	1.00	1.00	1.00	1.00

This table displays the probability of scarcity (PS) of the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), derived from the GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). Hereby, the probability of scarcity is calculated for the scenarios Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%) of the input variables. The presented results are derived under the robustness test for the threshold prices, in particular, the threshold prices are calculated as the average commodity price of the period from 1995 to 2019.

Table D.36: Probability of scarcity per commodity derived from the MS-GVAR models of the robustness analysis for the threshold price for the enlarged sample period from 1995 to 2019

			Al	Cu	Ni	Pb	Sn	Zn
$REMod-REF$	2019	Mean	0.13	0.93	0.02	1.00	0.80	1.00
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.91	1.00	0.53	1.00	0.99	1.00
		Foc. FX	1.00	1.00	0.81	1.00	0.99	1.00
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	0.98	1.00	0.85	1.00	1.00	1.00
		Foc. Extr. FX	1.00	1.00	0.97	1.00	1.00	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00

Probability of scarcity per commodity derived from the MS-GVAR models of the robustness analysis for the threshold price for the enlarged sample period from 1995 to 2019

			Al	Cu	Ni	Pb	Sn	Zn
$REMod-REF$	2019	Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.06
		Q. 50%	0.00	0.16	0.00	0.62	0.34	1.00
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
	Mean	Mean	0.95	1.00	0.20	1.00	1.00	0.93
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	0.87	1.00	1.00	1.00
		Foc. FX	1.00	1.00	0.96	1.00	1.00	1.00
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	1.00	1.00	0.96	1.00	1.00	1.00
		Foc. Extr. FX	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.04	0.00	0.03	0.07	0.00
		Q. 50%	0.10	1.00	0.02	1.00	1.00	0.34
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
$REMod-SUF$	2019	Mean	0.12	0.94	0.02	1.00	0.77	1.00
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.92	1.00	0.55	1.00	0.99	1.00
		Foc. FX	0.99	1.00	0.74	1.00	0.99	1.00
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	0.99	1.00	0.89	1.00	1.00	1.00
		Foc. Extr. FX	1.00	1.00	0.94	1.00	1.00	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.06
		Q. 50%	0.00	0.13	0.00	0.65	0.35	1.00
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
	Mean	Mean	0.91	1.00	0.17	1.00	1.00	0.96
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	0.88	1.00	1.00	1.00
		Foc. FX	1.00	1.00	0.94	1.00	1.00	1.00
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	1.00	1.00	0.97	1.00	1.00	1.00
		Foc. Extr. FX	1.00	1.00	0.98	1.00	1.00	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.03	0.00	0.04	0.08	0.00
		Q. 50%	0.10	1.00	0.01	1.00	1.00	0.40
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
$REMod-PER$	2019	Mean	0.13	0.94	0.02	1.00	0.79	1.00
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.92	1.00	0.55	1.00	0.99	1.00
		Foc. FX	1.00	1.00	0.84	1.00	0.99	1.00
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	0.98	1.00	0.87	1.00	1.00	1.00
		Foc. Extr. FX	1.00	1.00	0.97	1.00	1.00	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.01	0.00	0.06
		Q. 50%	0.00	0.15	0.00	0.63	0.36	1.00
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
	Mean	Mean	0.95	1.00	0.22	1.00	1.00	0.95
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	0.90	1.00	1.00	1.00
		Foc. FX	1.00	1.00	0.97	1.00	1.00	1.00
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	1.00	1.00	0.97	1.00	1.00	1.00
		Foc. Extr. FX	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00

Probability of scarcity per commodity derived from the MS-GVAR models of the robustness analysis for the threshold price for the enlarged sample period from 1995 to 2019

			Al	Cu	Ni	Pb	Sn	Zn
	Mean	Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.03	0.00	0.04	0.09	0.00
		Q. 50%	0.09	1.00	0.02	1.00	1.00	0.33
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
$REMod-UNA$	2019	Mean	0.12	0.93	0.02	1.00	0.81	1.00
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.92	1.00	0.54	1.00	0.99	1.00
		Foc. FX	1.00	1.00	0.84	1.00	0.99	1.00
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	0.99	1.00	0.86	1.00	1.00	1.00
		Foc. Extr. FX	1.00	1.00	0.97	1.00	0.99	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.01	0.09
		Q. 50%	0.00	0.17	0.00	0.61	0.36	1.00
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
	Mean	Mean	0.93	1.00	0.20	1.00	1.00	0.95
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	0.88	1.00	1.00	1.00
		Foc. FX	1.00	1.00	0.97	1.00	1.00	1.00
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	1.00	1.00	0.97	1.00	1.00	1.00
		Foc. Extr. FX	1.00	1.00	0.99	1.00	1.00	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.04	0.00	0.03	0.10	0.00
		Q. 50%	0.09	1.00	0.01	1.00	1.00	0.39
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00

This table displays the probability of scarcity (PS) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), derived from the MS-GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). Hereby, the probability of scarcity is calculated for the scenarios Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%) of the input variables. The presented results are derived under the robustness test for the threshold prices, in particular, the threshold prices are calculated as the average commodity price of the period from 1995 to 2019.

Table D.37: Estimated coefficients of the logistic regression models of the robustness analysis for the threshold price for the enlarged sample period from 1995 to 2019

	Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
U.S. IP			-0.29										
GDP					0.90			0.90					
GDPc	-0.49	0.75	0.95										
FX				0.15	0.14	-0.14		0.14		-0.65	-0.02		
FFR				-0.24	-0.93			-0.93					2.14
LIR			0.10				0.59		-0.02	0.87			0.37
CPI							0.08						
MSCI		-0.21											
supply									2.89			-0.02	
OIL	0.31			0.31		0.34					0.40	0.48	
ND			-0.27										
demand													1.74

This table displays the estimated coefficients of the individual logistic regression models of the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), based on the identified independent variables from the two stage model selection. Hereby, the independent variables are U.S. industrial production (U.S. IP), world gross domestic product (GDP), world gross domestic product per capita (GDPc), U.S. dollar index (FX), Federal Funds Effective Rate (FFR), 10-year U.S. Treasury rate (LIR), U.S. consumer price index (CPI), MSCI world stock index (MSCI), commodity-specific supply (**supply**), West Texas Intermediate spot crude oil price (OIL), global natural disasters (ND), and commodity-specific demand (**demand**). The presented results are derived under the robustness test for the threshold prices, in particular, the threshold prices are calculated as the average commodity price of the period from 1995 to 2019.

Table D.38: Probability of scarcity per commodity derived from the logistic regression models of the robustness analysis for the threshold price for the enlarged sample period from 1995 to 2019

	Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
Mean	0.04	0.06	0.04	0.08	0.01	0.08	0.05	0.01	0.04	0.06	0.09	0.09	0.06
Shock	0.08	0.12	0.15	0.15	0.05	0.11	0.14	0.05	0.74	0.28	0.13	0.13	0.89
Extr.	0.15	0.21	0.39	0.25	0.24	0.17	0.32	0.24	0.99	0.71	0.18	0.20	1.00
Foc. EA	0.04	0.06	0.04	0.08	0.02	0.08	0.05	0.02	0.04	0.06	0.09	0.09	0.06
Foc. FX	0.04	0.06	0.04	0.09	0.01	0.09	0.05	0.01	0.04	0.09	0.09	0.09	0.06
Foc. FFR	0.04	0.06	0.04	0.10	0.02	0.08	0.05	0.02	0.04	0.06	0.09	0.09	0.34
Foc. Extr. EA	0.04	0.06	0.04	0.08	0.04	0.08	0.05	0.04	0.04	0.06	0.09	0.09	0.06
Foc. Extr. FX	0.04	0.06	0.04	0.10	0.01	0.09	0.05	0.01	0.04	0.14	0.09	0.09	0.06
Foc. Extr. FFR	0.04	0.06	0.04	0.13	0.06	0.08	0.05	0.06	0.04	0.06	0.09	0.09	0.79
Q. 25%	0.03	0.04	0.02	0.06	0.00	0.07	0.03	0.00	0.00	0.03	0.08	0.07	0.00
Q. 40%	0.04	0.05	0.03	0.08	0.01	0.07	0.03	0.01	0.04	0.03	0.09	0.08	0.03
Q. 50%	0.04	0.05	0.04	0.08	0.01	0.08	0.04	0.01	0.07	0.05	0.09	0.09	0.06
Q. 60%	0.05	0.07	0.05	0.10	0.01	0.09	0.06	0.01	0.08	0.08	0.10	0.10	0.14
Q. 75%	0.06	0.09	0.10	0.12	0.03	0.10	0.12	0.03	0.13	0.21	0.11	0.12	0.43

This table displays the probability of scarcity (PS) of the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), derived from the logistic regression models based on preselected determinants. Hereby, the probability of scarcity is calculated for the scenarios Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%) of the input variables. The presented results are derived under the robustness test for the threshold prices, in particular, the threshold prices are calculated as the average commodity price of the period from 1995 to 2019.

Table D.39: Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the GVAR models of the robustness analysis for the threshold price for the enlarged sample period from 1995 to 2019

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
Mean	2019	$REMod-REF$	0.04	0.03	0.00	0.30	0.00	24.90	0.17	0.21	0.00	0.33	3.39
		$REMod-SUF$	0.02	0.02	0.00	0.22	0.00	16.74	0.12	0.14	0.00	0.22	3.45
		$REMod-PER$	0.04	0.02	0.00	0.25	0.00	17.96	0.14	0.21	0.00	0.32	2.78
		$REMod-UNA$	0.06	0.03	0.00	0.28	0.00	25.48	0.17	0.12	0.00	0.24	3.60
	Mean	$REMod-REF$	5.21	0.15	0.61	2.10	0.26	0.08	0.17	0.43	0.01	2.04	0.74
		$REMod-SUF$	3.52	0.10	0.41	1.52	0.19	0.05	0.12	0.29	0.01	1.39	0.76
		$REMod-PER$	5.93	0.13	0.44	1.78	0.27	0.05	0.14	0.43	0.02	1.96	0.61
		$REMod-UNA$	8.30	0.17	0.62	1.99	0.40	0.08	0.17	0.24	0.01	1.51	0.79
Shock	2019	$REMod-REF$	9.17	8.36	304.32	27.27	126.10	25.23	87.36	1.38	0.52	9.12	7.21
		$REMod-SUF$	6.19	5.67	204.60	19.71	92.34	16.96	59.05	0.94	0.34	6.21	7.33
		$REMod-PER$	10.44	7.34	219.38	23.16	129.21	18.20	69.11	1.36	0.65	8.77	5.91
		$REMod-UNA$	14.62	9.36	311.47	25.80	192.66	25.82	87.45	0.78	0.37	6.72	7.66
	Mean	$REMod-REF$	9.17	8.36	304.32	27.27	131.77	25.23	87.36	1.38	0.53	9.12	7.21
		$REMod-SUF$	6.19	5.67	204.60	19.71	96.49	16.96	59.05	0.94	0.35	6.21	7.33
		$REMod-PER$	10.44	7.34	219.38	23.16	135.01	18.20	69.11	1.36	0.67	8.77	5.91
		$REMod-UNA$	14.62	9.36	311.47	25.80	201.32	25.82	87.45	0.78	0.38	6.72	7.66
Extr.	2019	$REMod-REF$	9.17	8.36	304.32	27.27	131.77	25.23	87.36	1.38	0.53	9.12	7.21
		$REMod-SUF$	6.19	5.67	204.60	19.71	96.49	16.96	59.05	0.94	0.35	6.21	7.33
		$REMod-PER$	10.44	7.34	219.38	23.16	135.01	18.20	69.11	1.36	0.67	8.77	5.91
		$REMod-UNA$	14.62	9.36	311.47	25.80	201.32	25.82	87.45	0.78	0.38	6.72	7.66
	Mean	$REMod-REF$	9.17	8.36	304.32	27.27	131.77	25.23	87.36	1.38	0.53	9.12	7.21
		$REMod-SUF$	6.19	5.67	204.60	19.71	96.49	16.96	59.05	0.94	0.35	6.21	7.33
		$REMod-PER$	10.44	7.34	219.38	23.16	135.01	18.20	69.11	1.36	0.67	8.77	5.91
		$REMod-UNA$	14.62	9.36	311.47	25.80	201.32	25.82	87.45	0.78	0.38	6.72	7.66
Foc. EA	2019	$REMod-REF$	3.20	3.91	31.35	26.42	0.00	25.23	57.40	1.38	0.00	9.12	7.21
		$REMod-SUF$	2.16	2.65	21.07	19.10	0.00	16.96	38.80	0.94	0.00	6.21	7.33
		$REMod-PER$	3.64	3.43	22.60	22.44	0.00	18.20	45.40	1.36	0.00	8.77	5.91
		$REMod-UNA$	5.10	4.38	32.08	25.00	0.00	25.82	57.45	0.78	0.00	6.72	7.66
	Mean	$REMod-REF$	9.17	7.67	88.56	27.27	34.13	4.52	73.38	1.38	0.46	9.12	7.19
		$REMod-SUF$	6.19	5.20	59.54	19.71	24.99	3.04	49.60	0.94	0.31	6.21	7.32
		$REMod-PER$	10.44	6.73	63.84	23.16	34.97	3.26	58.05	1.36	0.58	8.77	5.90
		$REMod-UNA$	14.62	8.59	90.64	25.80	52.14	4.62	73.46	0.78	0.33	6.72	7.64
Foc. FX	2019	$REMod-REF$	1.60	0.55	4.87	12.79	0.00	25.18	19.83	0.95	0.00	7.16	7.16
		$REMod-SUF$	1.08	0.37	3.27	9.24	0.00	16.93	13.40	0.64	0.00	4.88	7.28
		$REMod-PER$	1.83	0.48	3.51	10.86	0.00	18.16	15.69	0.93	0.00	6.89	5.87
		$REMod-UNA$	2.56	0.62	4.98	12.10	0.00	25.77	19.85	0.53	0.00	5.29	7.60

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the GVAR models of the robustness analysis for the threshold price for the enlarged sample period from 1995 to 2019

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
Foc. FX	Mean	$REMod-REF$	8.49	2.48	19.48	25.06	42.30	3.28	36.43	1.22	0.09	8.73	6.82
		$REMod-SUF$	5.73	1.68	13.09	18.11	30.97	2.21	24.62	0.83	0.06	5.95	6.93
		$REMod-PER$	9.67	2.18	14.04	21.28	43.34	2.37	28.82	1.20	0.12	8.40	5.59
		$REMod-UNA$	13.54	2.78	19.93	23.71	64.62	3.36	36.47	0.68	0.07	6.44	7.24
Foc. FFR	2019	$REMod-REF$	1.05	0.36	5.78	2.24	0.00	25.16	2.80	0.59	0.00	2.65	5.88
		$REMod-SUF$	0.71	0.24	3.89	1.62	0.00	16.91	1.89	0.40	0.00	1.81	5.97
		$REMod-PER$	1.20	0.32	4.17	1.90	0.00	18.14	2.21	0.58	0.00	2.55	4.82
		$REMod-UNA$	1.68	0.40	5.92	2.12	0.00	25.74	2.80	0.33	0.00	1.96	6.24
	Mean	$REMod-REF$	7.85	1.45	21.00	8.97	22.93	3.94	6.90	0.89	0.08	6.02	3.20
		$REMod-SUF$	5.30	0.99	14.12	6.48	16.79	2.65	4.66	0.60	0.05	4.10	3.25
		$REMod-PER$	8.94	1.28	15.14	7.62	23.49	2.84	5.46	0.87	0.10	5.79	2.62
		$REMod-UNA$	12.51	1.63	21.49	8.49	35.03	4.03	6.91	0.50	0.06	4.44	3.40
Foc. Extr. EA	2019	$REMod-REF$	7.83	8.09	232.50	27.24	3.95	25.23	84.65	1.38	0.12	9.12	7.21
		$REMod-SUF$	5.29	5.48	156.31	19.69	2.89	16.96	57.22	0.94	0.08	6.21	7.33
		$REMod-PER$	8.92	7.10	167.61	23.14	4.05	18.20	66.96	1.36	0.15	8.77	5.91
		$REMod-UNA$	12.48	9.05	237.96	25.78	6.04	25.82	84.74	0.78	0.09	6.72	7.66
	Mean	$REMod-REF$	9.17	8.35	275.41	27.27	105.68	18.27	86.66	1.38	0.53	9.12	7.21
		$REMod-SUF$	6.19	5.66	185.16	19.71	77.38	12.28	58.58	0.94	0.35	6.21	7.33
		$REMod-PER$	10.44	7.33	198.54	23.16	108.28	13.17	68.55	1.36	0.67	8.77	5.91
		$REMod-UNA$	14.62	9.35	281.88	25.80	161.46	18.69	86.75	0.78	0.38	6.72	7.66
Foc. Extr. FX	2019	$REMod-REF$	4.92	3.98	71.21	25.44	1.45	25.21	69.98	1.26	0.01	8.78	7.20
		$REMod-SUF$	3.32	2.70	47.88	18.39	1.06	16.95	47.30	0.86	0.01	5.98	7.32
		$REMod-PER$	5.61	3.49	51.34	21.61	1.49	18.18	55.35	1.24	0.01	8.44	5.90
		$REMod-UNA$	7.85	4.46	72.88	24.07	2.21	25.79	70.05	0.71	0.01	6.48	7.65
	Mean	$REMod-REF$	8.94	6.25	118.08	27.10	101.20	12.11	76.88	1.34	0.26	9.08	7.15
		$REMod-SUF$	6.03	4.23	79.38	19.59	74.10	8.14	51.96	0.91	0.17	6.19	7.27
		$REMod-PER$	10.18	5.48	85.12	23.02	103.69	8.73	60.81	1.31	0.33	8.73	5.86
		$REMod-UNA$	14.25	6.99	120.85	25.65	154.61	12.39	76.95	0.75	0.19	6.70	7.60
Foc. Extr. FFR	2019	$REMod-REF$	3.44	1.67	38.65	7.55	1.32	25.23	20.44	0.91	0.01	5.62	6.67
		$REMod-SUF$	2.32	1.13	25.98	5.46	0.96	16.96	13.82	0.62	0.00	3.83	6.78
		$REMod-PER$	3.92	1.47	27.86	6.42	1.35	18.20	16.17	0.89	0.01	5.40	5.47
		$REMod-UNA$	5.48	1.87	39.56	7.15	2.01	25.82	20.46	0.51	0.00	4.14	7.08
	Mean	$REMod-REF$	8.70	3.40	72.43	16.58	58.11	15.37	30.84	1.13	0.19	7.82	5.25
		$REMod-SUF$	5.87	2.31	48.69	11.98	42.55	10.33	20.84	0.77	0.12	5.33	5.34
		$REMod-PER$	9.91	2.99	52.21	14.08	59.54	11.08	24.39	1.11	0.23	7.52	4.30
		$REMod-UNA$	13.87	3.81	74.13	15.69	88.78	15.72	30.87	0.63	0.13	5.77	5.57
Q. 25%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Q. 40%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	1.34	0.00	0.00	0.00	0.00	0.01
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.90	0.00	0.00	0.00	0.00	0.01
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.96	0.00	0.00	0.00	0.00	0.01
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	1.37	0.00	0.00	0.00	0.00	0.02
	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Q. 50%	2019	$REMod-REF$	0.02	0.00	0.00	0.00	0.00	20.94	0.09	0.07	0.00	0.06	1.34
		$REMod-SUF$	0.01	0.00	0.00	0.00	0.00	14.08	0.06	0.05	0.00	0.04	1.36
		$REMod-PER$	0.02	0.00	0.00	0.00	0.00	15.10	0.07	0.07	0.00	0.06	1.10
		$REMod-UNA$	0.03	0.00	0.00	0.00	0.00	21.43	0.09	0.04	0.00	0.05	1.42
	Mean	$REMod-REF$	2.82	0.05	0.00	0.44	0.00	0.03	0.09	0.21	0.01	0.50	0.27
		$REMod-SUF$	1.90	0.03	0.00	0.32	0.00	0.02	0.06	0.14	0.00	0.34	0.28
		$REMod-PER$	3.21	0.04	0.00	0.37	0.00	0.02	0.07	0.21	0.01	0.48	0.22
		$REMod-UNA$	4.49	0.06	0.00	0.41	0.00	0.03	0.09	0.12	0.00	0.37	0.29
Q. 60%	2019	$REMod-REF$	2.15	0.82	6.39	2.97	0.00	25.23	5.94	0.90	0.00	3.99	6.59
		$REMod-SUF$	1.45	0.56	4.30	2.15	0.00	16.96	4.02	0.61	0.00	2.72	6.70
		$REMod-PER$	2.44	0.72	4.61	2.52	0.00	18.20	4.70	0.89	0.00	3.84	5.40
		$REMod-UNA$	3.42	0.92	6.54	2.81	0.00	25.82	5.95	0.51	0.00	2.95	7.00
	Mean	$REMod-REF$	8.92	2.80	24.65	12.22	32.42	5.95	13.72	1.18	0.13	7.54	3.81
		$REMod-SUF$	6.02	1.90	16.57	8.83	23.74	4.00	9.27	0.80	0.09	5.14	3.87
		$REMod-PER$	10.16	2.46	17.77	10.38	33.21	4.29	10.85	1.16	0.16	7.25	3.12
		$REMod-UNA$	14.22	3.14	25.23	11.56	49.52	6.09	13.73	0.66	0.09	5.56	4.04
	2019	$REMod-REF$	9.09	8.30	255.93	27.27	21.87	25.23	87.19	1.38	0.08	9.12	7.21
		$REMod-SUF$	6.13	5.63	172.07	19.71	16.02	16.96	58.93	0.94	0.05	6.21	7.33

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the GVAR models of the robustness analysis for the threshold price for the enlarged sample period from 1995 to 2019

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
Q. 75%		$REMod-PER$	10.35	7.29	184.50	23.16	22.41	18.20	68.97	1.36	0.10	8.77	5.91
		$REMod-UNA$	14.49	9.30	261.95	25.80	33.42	25.82	87.27	0.78	0.06	6.72	7.66
	Mean	$REMod-REF$	9.17	8.36	292.15	27.27	130.72	25.06	87.36	1.38	0.53	9.12	7.21
		$REMod-SUF$	6.19	5.67	196.41	19.71	95.72	16.84	59.05	0.94	0.35	6.21	7.33
		$REMod-PER$	10.44	7.34	210.61	23.16	133.93	18.07	69.11	1.36	0.67	8.77	5.91
		$REMod-UNA$	14.62	9.36	299.01	25.80	199.71	25.64	87.45	0.78	0.38	6.72	7.66

This table displays the expected loss due to scarcity for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), per path ($REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$), as well as per scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) for the input variables. Hereby, the values are derived from the GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). In particular, the results are derived under the robustness test for the threshold prices, in particular, the threshold prices are calculated as the average commodity price of the period from 1995 to 2019.

Table D.40: Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the MS-GVAR models of the robustness analysis for the threshold price for the enlarged sample period from 1995 to 2019

			Al	Cu	Ni	Pb	Sn	Zn
Mean	2019	$REMod-REF$	1.07	25.41	1.40	1.38	7.31	7.21
		$REMod-SUF$	0.73	18.37	0.94	0.94	4.98	7.33
		$REMod-PER$	0.94	21.58	1.11	1.36	7.03	5.91
		$REMod-UNA$	1.20	24.05	1.40	0.78	5.39	7.66
	Mean	$REMod-REF$	7.91	27.27	17.82	1.38	9.12	6.70
		$REMod-SUF$	5.36	19.71	12.05	0.94	6.21	6.82
		$REMod-PER$	6.94	23.16	14.10	1.36	8.77	5.50
		$REMod-UNA$	8.86	25.80	17.84	0.78	6.72	7.12
Shock	2019	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
Extr.	2019	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
Foc. EA	2019	$REMod-REF$	7.61	27.27	46.48	1.38	9.02	7.21
		$REMod-SUF$	5.16	19.71	31.41	0.94	6.15	7.33
		$REMod-PER$	6.68	23.16	36.76	1.36	8.68	5.91
		$REMod-UNA$	8.52	25.80	46.52	0.78	6.66	7.66
	Mean	$REMod-REF$	8.36	27.27	75.65	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	51.14	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	59.85	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	75.73	0.78	6.72	7.66
Foc. FX	2019	$REMod-REF$	8.34	27.27	71.11	1.38	9.01	7.21
		$REMod-SUF$	5.66	19.71	48.07	0.94	6.14	7.33
		$REMod-PER$	7.33	23.16	56.25	1.36	8.66	5.91
		$REMod-UNA$	9.34	25.80	71.18	0.78	6.64	7.66
	Mean	$REMod-REF$	8.36	27.27	84.04	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	56.80	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	66.48	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	84.12	0.78	6.72	7.66

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the MS-GVAR models of the robustness analysis for the threshold price for the enlarged sample period from 1995 to 2019

			Al	Cu	Ni	Pb	Sn	Zn
Foc. FFR	2019	$REMod - REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod - SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod - PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod - UNA$	9.36	25.80	87.45	0.78	6.72	7.66
	Mean	$REMod - REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod - SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod - PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod - UNA$	9.36	25.80	87.45	0.78	6.72	7.66
Foc. Extr. EA	2019	$REMod - REF$	8.21	27.27	74.61	1.38	9.08	7.21
		$REMod - SUF$	5.57	19.71	50.43	0.94	6.19	7.33
		$REMod - PER$	7.21	23.16	59.02	1.36	8.73	5.91
		$REMod - UNA$	9.20	25.80	74.68	0.78	6.70	7.66
	Mean	$REMod - REF$	8.36	27.27	84.04	1.38	9.12	7.21
		$REMod - SUF$	5.67	19.71	56.80	0.94	6.21	7.33
		$REMod - PER$	7.34	23.16	66.48	1.36	8.77	5.91
		$REMod - UNA$	9.36	25.80	84.12	0.78	6.72	7.66
Foc. Extr. FX	2019	$REMod - REF$	8.34	27.27	84.39	1.38	9.08	7.21
		$REMod - SUF$	5.66	19.71	57.04	0.94	6.19	7.33
		$REMod - PER$	7.33	23.16	66.76	1.36	8.73	5.91
		$REMod - UNA$	9.34	25.80	84.47	0.78	6.70	7.66
	Mean	$REMod - REF$	8.36	27.27	87.01	1.38	9.12	7.21
		$REMod - SUF$	5.67	19.71	58.81	0.94	6.21	7.33
		$REMod - PER$	7.34	23.16	68.83	1.36	8.77	5.91
		$REMod - UNA$	9.36	25.80	87.10	0.78	6.72	7.66
Foc. Extr. FFR	2019	$REMod - REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod - SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod - PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod - UNA$	9.36	25.80	87.45	0.78	6.72	7.66
	Mean	$REMod - REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod - SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod - PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod - UNA$	9.36	25.80	87.45	0.78	6.72	7.66
Q. 25%	2019	$REMod - REF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod - SUF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod - PER$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod - UNA$	0.00	0.00	0.00	0.00	0.00	0.00
	Mean	$REMod - REF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod - SUF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod - PER$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod - UNA$	0.00	0.00	0.00	0.00	0.00	0.00
Q. 40%	2019	$REMod - REF$	0.00	0.00	0.00	0.01	0.04	0.42
		$REMod - SUF$	0.00	0.00	0.00	0.00	0.02	0.43
		$REMod - PER$	0.00	0.00	0.00	0.01	0.04	0.34
		$REMod - UNA$	0.00	0.00	0.00	0.00	0.03	0.44
	Mean	$REMod - REF$	0.00	1.09	0.00	0.04	0.66	0.00
		$REMod - SUF$	0.00	0.79	0.00	0.03	0.45	0.00
		$REMod - PER$	0.00	0.93	0.00	0.04	0.63	0.00
		$REMod - UNA$	0.00	1.03	0.00	0.02	0.48	0.00
Q. 50%	2019	$REMod - REF$	0.00	4.25	0.00	0.86	3.08	7.21
		$REMod - SUF$	0.00	3.07	0.00	0.58	2.10	7.33
		$REMod - PER$	0.00	3.61	0.00	0.84	2.96	5.91
		$REMod - UNA$	0.00	4.03	0.00	0.48	2.27	7.66
	Mean	$REMod - REF$	0.80	27.27	1.40	1.38	9.12	2.44
		$REMod - SUF$	0.54	19.71	0.94	0.94	6.21	2.48
		$REMod - PER$	0.70	23.16	1.11	1.36	8.77	2.00
		$REMod - UNA$	0.90	25.80	1.40	0.78	6.72	2.59
Q. 60%	2019	$REMod - REF$	8.36	27.27	87.19	1.38	9.12	7.21
		$REMod - SUF$	5.67	19.71	58.93	0.94	6.21	7.33
		$REMod - PER$	7.34	23.16	68.97	1.36	8.77	5.91
		$REMod - UNA$	9.36	25.80	87.27	0.78	6.72	7.66
	Mean	$REMod - REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod - SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod - PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod - UNA$	9.36	25.80	87.45	0.78	6.72	7.66
Q. 75%	2019	$REMod - REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod - SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod - PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod - UNA$	9.36	25.80	87.45	0.78	6.72	7.66
	Mean	$REMod - REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod - SUF$	5.67	19.71	59.05	0.94	6.21	7.33

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the MS-GVAR models of the robustness analysis for the threshold price for the enlarged sample period from 1995 to 2019

	Al	Cu	Ni	Pb	Sn	Zn
$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66

This table displays the expected loss due to scarcity for the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), per path ($REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$), as well as per scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) for the input variables. Hereby, the values are derived from the MS-GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). In particular, the results are derived under the robustness test for the threshold prices, in particular, the threshold prices are calculated as the average commodity price of the period from 1995 to 2019.

Table D.41: Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the logistic regression models of the robustness analysis for the threshold price for the enlarged sample period from 1995 to 2019

		Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
Mean	$REMod-REF$	0.38	0.50	13.23	2.31	1.48	10.26	1.29	0.19	3.38	0.08	0.05	0.78	0.46
	$REMod-SUF$	0.26	0.34	8.90	1.67	1.06	7.51	0.86	0.14	2.29	0.06	0.03	0.53	0.47
	$REMod-PER$	0.44	0.44	9.54	1.96	1.42	10.51	0.93	0.18	2.67	0.08	0.06	0.75	0.38
	$REMod-UNA$	0.61	0.56	13.54	2.19	0.70	15.67	1.32	0.08	3.38	0.05	0.03	0.57	0.49
Shock	$REMod-REF$	0.74	0.96	44.72	4.07	7.70	15.11	3.49	1.01	64.54	0.39	0.07	1.20	6.39
	$REMod-SUF$	0.50	0.65	30.06	2.94	5.51	11.06	2.35	0.72	43.62	0.27	0.04	0.82	6.49
	$REMod-PER$	0.85	0.84	32.24	3.46	7.42	15.48	2.52	0.93	51.06	0.39	0.09	1.15	5.24
	$REMod-UNA$	1.19	1.08	45.77	3.85	3.67	23.09	3.57	0.43	64.60	0.22	0.05	0.88	6.78
Extr.	$REMod-REF$	1.38	1.75	120.19	6.80	33.85	21.84	8.19	4.42	86.92	0.98	0.10	1.80	7.20
	$REMod-SUF$	0.93	1.18	80.80	4.92	24.19	15.99	5.51	3.14	58.75	0.67	0.06	1.22	7.32
	$REMod-PER$	1.58	1.53	86.64	5.78	32.60	22.38	5.91	4.09	68.76	0.97	0.12	1.73	5.90
	$REMod-UNA$	2.21	1.96	123.01	6.44	16.14	33.37	8.38	1.89	87.01	0.55	0.07	1.33	7.65
Foc. EA	$REMod-REF$	0.38	0.50	13.23	2.31	2.99	10.26	1.29	0.39	3.38	0.08	0.05	0.78	0.46
	$REMod-SUF$	0.26	0.34	8.90	1.67	2.14	7.51	0.86	0.28	2.29	0.06	0.03	0.53	0.47
	$REMod-PER$	0.44	0.44	9.54	1.96	2.88	10.51	0.93	0.36	2.67	0.08	0.06	0.75	0.38
	$REMod-UNA$	0.61	0.56	13.54	2.19	1.43	15.67	1.32	0.17	3.38	0.05	0.03	0.57	0.49
Foc. FX	$REMod-REF$	0.38	0.50	13.23	2.56	1.63	11.25	1.29	0.21	3.38	0.13	0.05	0.78	0.46
	$REMod-SUF$	0.26	0.34	8.90	1.85	1.17	8.24	0.86	0.15	2.29	0.09	0.03	0.53	0.47
	$REMod-PER$	0.44	0.44	9.54	2.17	1.57	11.53	0.93	0.20	2.67	0.13	0.06	0.75	0.38
	$REMod-UNA$	0.61	0.56	13.54	2.42	0.78	17.19	1.32	0.09	3.38	0.07	0.03	0.57	0.49
Foc. FFR	$REMod-REF$	0.38	0.50	13.23	2.83	3.51	10.26	1.29	0.46	3.38	0.08	0.05	0.78	2.43
	$REMod-SUF$	0.26	0.34	8.90	2.04	2.51	7.51	0.86	0.33	2.29	0.06	0.03	0.53	2.48
	$REMod-PER$	0.44	0.44	9.54	2.40	3.38	10.51	0.93	0.42	2.67	0.08	0.06	0.75	2.00
	$REMod-UNA$	0.61	0.56	13.54	2.67	1.67	15.67	1.32	0.20	3.38	0.05	0.03	0.57	2.59
Foc. Extr. EA	$REMod-REF$	0.38	0.50	13.23	2.31	6.00	10.26	1.29	0.78	3.38	0.08	0.05	0.78	0.46
	$REMod-SUF$	0.26	0.34	8.90	1.67	4.29	7.51	0.86	0.56	2.29	0.06	0.03	0.53	0.47
	$REMod-PER$	0.44	0.44	9.54	1.96	5.78	10.51	0.93	0.73	2.67	0.08	0.06	0.75	0.38
	$REMod-UNA$	0.61	0.56	13.54	2.19	2.86	15.67	1.32	0.34	3.38	0.05	0.03	0.57	0.49
Foc. Extr. FX	$REMod-REF$	0.38	0.50	13.23	2.83	1.80	12.33	1.29	0.23	3.38	0.19	0.05	0.78	0.46
	$REMod-SUF$	0.26	0.34	8.90	2.04	1.29	9.03	0.86	0.17	2.29	0.13	0.03	0.53	0.47
	$REMod-PER$	0.44	0.44	9.54	2.40	1.73	12.64	0.93	0.22	2.67	0.19	0.06	0.75	0.38

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the logistic regression models of the robustness analysis for the threshold price for the enlarged sample period from 1995 to 2019

		Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
	$REMod-UNA$	0.61	0.56	13.54	2.67	0.86	18.84	1.32	0.10	3.38	0.11	0.04	0.57	0.49
Foc. Extr. FFR	$REMod-REF$	0.38	0.50	13.23	3.44	8.16	10.26	1.29	1.07	3.38	0.08	0.05	0.78	5.72
	$REMod-SUF$	0.26	0.34	8.90	2.49	5.84	7.51	0.86	0.76	2.29	0.06	0.03	0.53	5.81
	$REMod-PER$	0.44	0.44	9.54	2.92	7.86	10.51	0.93	0.99	2.67	0.08	0.06	0.75	4.69
	$REMod-UNA$	0.61	0.56	13.54	3.26	3.89	15.67	1.32	0.46	3.38	0.05	0.03	0.57	6.07
Q. 25%	$REMod-REF$	0.30	0.37	6.48	1.74	0.57	8.92	0.78	0.07	0.33	0.04	0.04	0.66	0.03
	$REMod-SUF$	0.20	0.25	4.36	1.25	0.41	6.53	0.53	0.05	0.23	0.02	0.03	0.45	0.03
	$REMod-PER$	0.34	0.32	4.67	1.47	0.55	9.14	0.56	0.07	0.26	0.04	0.05	0.63	0.02
	$REMod-UNA$	0.47	0.41	6.64	1.64	0.27	13.62	0.80	0.03	0.34	0.02	0.03	0.49	0.03
Q. 40%	$REMod-REF$	0.34	0.42	9.59	2.11	0.98	9.87	0.85	0.13	3.43	0.04	0.05	0.75	0.20
	$REMod-SUF$	0.23	0.29	6.45	1.53	0.70	7.22	0.57	0.09	2.32	0.03	0.03	0.51	0.20
	$REMod-PER$	0.39	0.37	6.91	1.79	0.94	10.11	0.61	0.12	2.71	0.04	0.06	0.72	0.16
	$REMod-UNA$	0.54	0.47	9.81	2.00	0.47	15.07	0.87	0.05	3.43	0.02	0.03	0.55	0.21
Q. 50%	$REMod-REF$	0.36	0.45	11.45	2.31	1.22	10.36	1.10	0.16	6.27	0.06	0.05	0.80	0.44
	$REMod-SUF$	0.25	0.30	7.70	1.67	0.87	7.59	0.74	0.11	4.24	0.04	0.03	0.55	0.45
	$REMod-PER$	0.41	0.39	8.25	1.96	1.17	10.62	0.79	0.15	4.96	0.06	0.06	0.77	0.36
	$REMod-UNA$	0.58	0.50	11.71	2.18	0.58	15.83	1.13	0.07	6.28	0.04	0.04	0.59	0.47
Q. 60%	$REMod-REF$	0.44	0.55	16.59	2.62	1.81	11.32	1.61	0.24	7.28	0.11	0.05	0.91	1.02
	$REMod-SUF$	0.30	0.37	11.16	1.89	1.29	8.29	1.08	0.17	4.92	0.08	0.04	0.62	1.04
	$REMod-PER$	0.50	0.48	11.96	2.22	1.74	11.60	1.16	0.22	5.76	0.11	0.07	0.87	0.84
	$REMod-UNA$	0.70	0.61	16.98	2.47	0.86	17.29	1.65	0.10	7.28	0.06	0.04	0.67	1.08
Q. 75%	$REMod-REF$	0.58	0.75	29.27	3.20	3.63	12.91	2.91	0.47	11.09	0.29	0.06	1.05	3.11
	$REMod-SUF$	0.39	0.51	19.68	2.32	2.59	9.45	1.95	0.34	7.50	0.20	0.04	0.71	3.16
	$REMod-PER$	0.66	0.66	21.10	2.72	3.50	13.22	2.10	0.44	8.77	0.28	0.08	1.01	2.55
	$REMod-UNA$	0.92	0.84	29.96	3.03	1.73	19.72	2.97	0.20	11.10	0.16	0.04	0.77	3.30

This table displays the expected loss due to scarcity for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), per path ($REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$), as well as per scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) for the input variables, derived from the logistic regression model. Hereby, the results are derived under the robustness test for the threshold prices, in particular, the threshold prices are calculated as the average commodity price of the period from 1995 to 2019.

D.3.2.2 Robustness Analysis for the Scenario Values

D.3.2.2.1 Results of the Robustness Analysis for the Scenario Values of the reduced Sample

Table D.42: Scenario values for the input variables of the robustness analysis for the scenario values for the reduced sample period from 2015 to 2019

			Mean	Shock	Extr.	Foc. EA	Foc. FX	Foc. FFR	Foc. Extr. EA	Foc. Extr. FX	Foc. Extr. FFR	Q. 25%	Q. 40%	Q. 50%	Q. 60%	Q. 75%
GVAR	Ag	**supply**	-0.62	0.47	1.55	-0.62	-0.62	-0.62	-0.62	-0.62	-0.62	-0.95	-0.66	-0.46	-0.25	0.06
		demand	-0.05	1.13	2.32	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.71	-0.57	-0.47	-0.13	0.38
		price	-0.29	0.12	0.53	-0.29	-0.29	-0.29	-0.29	-0.29	-0.29	-0.47	-0.30	-0.19	-0.13	-0.05
	Al	**supply**	-0.14	0.52	1.18	-0.14	-0.14	-0.14	-0.14	-0.14	-0.14	-0.60	-0.41	-0.29	0.02	0.49
		demand	0.08	1.43	2.77	0.08	0.08	0.08	0.08	0.08	0.08	-0.70	-0.56	-0.47	0.07	0.87
		price	-0.16	0.61	1.38	-0.16	-0.16	-0.16	-0.16	-0.16	-0.16	-0.72	-0.47	-0.30	-0.09	0.24
	Co	**supply**	-0.63	1.56	3.74	-0.63	-0.63	-0.63	-0.63	-0.63	-0.63	-0.31	-0.07	0.10	0.19	0.33
		demand	0.40	2.24	4.08	0.40	0.40	0.40	0.40	0.40	0.40	-0.81	-0.18	0.23	0.28	0.35
		price	-0.06	1.30	2.65	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06	-0.35	-0.31	-0.28	0.05	0.54
	Cu	**supply**	-0.15	0.63	1.41	-0.15	-0.15	-0.15	-0.15	-0.15	-0.15	-0.83	-0.44	-0.18	0.05	0.39
		demand	0.22	1.25	2.27	0.22	0.22	0.22	0.22	0.22	0.22	-0.19	-0.10	-0.03	0.00	0.06
		price	-0.24	0.53	1.31	-0.24	-0.24	-0.24	-0.24	-0.24	-0.24	-0.66	-0.56	-0.49	-0.24	0.12
	In	**supply**	-0.16	0.60	1.36	-0.16	-0.16	-0.16	-0.16	-0.16	-0.16	-0.83	-0.38	-0.08	-0.06	-0.02
		demand	-0.17	0.30	0.78	-0.17	-0.17	-0.17	-0.17	-0.17	-0.17	-0.31	-0.24	-0.19	-0.19	-0.18
		price	-0.64	0.27	1.18	-0.64	-0.64	-0.64	-0.64	-0.64	-0.64	-1.13	-1.05	-0.99	-0.66	-0.16
	Li	**supply**	0.46	1.85	3.23	0.46	0.46	0.46	0.46	0.46	0.46	-0.19	-0.19	-0.19	-0.11	0.02
		demand	0.12	0.97	1.82	0.12	0.12	0.12	0.12	0.12	0.12	-0.41	-0.16	-0.00	0.02	0.05
		price	0.60	3.00	5.40	0.60	0.60	0.60	0.60	0.60	0.60	-0.10	0.20	0.39	0.88	1.62
	Ni	**supply**	-0.09	0.76	1.61	-0.09	-0.09	-0.09	-0.09	-0.09	-0.09	-0.63	-0.23	0.04	0.26	0.58
		demand	-0.03	1.05	2.14	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.48	0.08	0.45	0.49	0.55
		price	-0.25	0.59	1.42	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.85	-0.28	0.10	0.13	0.17
	Pb	**supply**	-0.66	0.25	1.15	-0.66	-0.66	-0.66	-0.66	-0.66	-0.66	-1.06	-0.82	-0.65	-0.48	-0.21
		demand	0.30	1.83	3.36	0.30	0.30	0.30	0.30	0.30	0.30	-0.92	-0.32	0.09	0.46	1.02
		price	-0.20	0.45	1.10	-0.20	-0.20	-0.20	-0.20	-0.20	-0.20	-0.66	-0.45	-0.30	-0.17	0.03
	Pt	**supply**	-0.18	1.50	3.18	-0.18	-0.18	-0.18	-0.18	-0.18	-0.18	-0.75	-0.42	-0.21	0.04	0.43
		demand	0.02	0.83	1.64	0.02	0.02	0.02	0.02	0.02	0.02	-0.53	-0.11	0.18	0.31	0.50
		price	-0.64	-0.14	0.35	-0.64	-0.64	-0.64	-0.64	-0.64	-0.64	-0.55	-0.51	-0.49	-0.44	-0.38
	Sn	**supply**	0.03	0.78	1.52	0.03	0.03	0.03	0.03	0.03	0.03	-0.20	0.01	0.14	0.17	0.21
		demand	0.32	1.22	2.12	0.32	0.32	0.32	0.32	0.32	0.32	-0.00	0.09	0.16	0.18	0.21
		price	-0.29	0.49	1.28	-0.29	-0.29	-0.29	-0.29	-0.29	-0.29	-0.50	-0.28	-0.13	0.06	0.35
	Zn	**supply**	-0.85	-0.21	0.44	-0.85	-0.85	-0.85	-0.85	-0.85	-0.85	-1.05	-0.90	-0.81	-0.70	-0.55
		demand	0.18	1.93	3.69	0.18	0.18	0.18	0.18	0.18	0.18	-1.22	-0.27	0.36	0.40	0.47
		price	-0.04	0.68	1.40	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.61	-0.32	-0.13	-0.02	0.15
	exog	GDP	-0.86	-0.01	0.84	-0.01	-0.86	-0.86	0.84	-0.86	-0.86	-0.96	-0.92	-0.90	-0.60	-0.15
		FX	0.47	1.38	2.29	0.47	1.38	0.47	0.47	2.29	0.47	0.02	0.08	0.13	0.30	0.56
		FFR	1.20	1.88	2.57	1.20	1.20	1.88	1.20	1.20	2.57	0.76	0.99	1.14	1.37	1.73
log. Reg.		U.S. IP	-0.48	0.04	0.56	-0.48	-0.48	-0.48	-0.48	-0.48	-0.48	-0.84	-0.73	-0.65	-0.46	-0.17
		GDPc	-0.82	0.06	0.93	-0.82	-0.82	-0.82	-0.82	-0.82	-0.82	-0.90	-0.87	-0.86	-0.55	-0.08
		LIR	-0.06	1.55	3.16	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06	-0.97	-0.87	-0.80	0.16	1.61
		CPI	1.05	3.69	6.33	1.05	1.05	1.05	1.05	1.05	1.05	-0.22	-0.07	0.03	0.16	0.35
		MSCI	-0.11	0.37	0.86	-0.11	-0.11	-0.11	-0.11	-0.11	-0.11	-0.34	-0.30	-0.28	-0.11	0.14
		OIL	-0.57	0.71	1.99	-0.57	-0.57	-0.57	-0.57	-0.57	-0.57	-0.69	-0.67	-0.65	-0.23	0.39
		ND	-0.04	1.07	2.19	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-1.12	-0.49	-0.06	0.30	0.84
MS-GVAR	Al	**supply**	-0.01	1.19	2.39	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.63	-0.41	-0.21	0.25	0.71
		demand	0.01	0.93	1.84	0.01	0.01	0.01	0.01	0.01	0.01	-0.57	-0.24	-0.08	0.09	0.58
		price	0.08	0.94	1.80	0.08	0.08	0.08	0.08	0.08	0.08	-0.46	-0.18	-0.03	0.15	0.60
	Cu	**supply**	-0.07	0.91	1.88	-0.07	-0.07	-0.07	-0.07	-0.07	-0.07	-0.53	-0.20	-0.06	0.11	0.42
		demand	-0.03	1.28	2.59	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.81	-0.18	-0.01	0.41	0.75
		price	0.04	0.73	1.42	0.04	0.04	0.04	0.04	0.04	0.04	-0.49	-0.36	-0.01	0.23	0.46
	Ni	**supply**	0.03	1.20	2.37	0.03	0.03	0.03	0.03	0.03	0.03	-0.82	-0.12	0.11	0.30	0.77
		demand	0.06	1.08	2.10	0.06	0.06	0.06	0.06	0.06	0.06	-0.72	-0.08	0.20	0.38	0.72
		price	0.09	1.00	1.92	0.09	0.09	0.09	0.09	0.09	0.09	-0.72	-0.19	0.02	0.07	0.85
	Pb	**supply**	-0.05	0.48	1.01	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.39	-0.20	-0.03	0.06	0.23
		demand	-0.01	0.89	1.80	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.51	-0.20	0.04	0.22	0.53
		price	-0.03	0.78	1.58	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.54	-0.32	-0.03	0.21	0.50
	Sn	**supply**	-0.07	1.13	2.32	-0.07	-0.07	-0.07	-0.07	-0.07	-0.07	-0.59	-0.32	-0.14	0.08	0.58
		demand	-0.03	1.15	2.33	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.62	-0.31	-0.13	0.10	0.67

Scenario values for the input variables of the robustness analysis for the scenario values for the reduced sample period from 2015 to 2019

			Mean	Shock	Extr.	Foc. EA	Foc. FX	Foc. FFR	Foc. Extr. EA	Foc. Extr. FX	Foc. Extr. FFR	Q. 25%	Q. 40%	Q. 50%	Q. 60%	Q. 75%
MS-GVAR		**price**	-0.00	0.67	1.34	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.48	-0.19	0.07	0.23	0.54
	Zn	**supply**	-0.07	0.80	1.66	-0.07	-0.07	-0.07	-0.07	-0.07	-0.07	-0.65	-0.16	0.00	0.15	0.45
		demand	-0.02	0.93	1.88	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.59	-0.26	-0.09	0.10	0.61
		price	0.07	0.89	1.71	0.07	0.07	0.07	0.07	0.07	0.07	-0.65	-0.22	0.18	0.41	0.65
	exog	IP	0.33	0.98	1.63	0.33	0.33	0.98	0.33	0.33	1.63	-0.04	0.11	0.17	0.34	0.61
		FX	-0.03	0.74	1.51	-0.03	0.74	-0.03	-0.03	1.51	-0.03	-0.45	-0.16	-0.00	0.16	0.51
		FFR	-0.00	1.06	2.12	1.06	-0.00	-0.00	2.12	-0.00	-0.00	-0.69	-0.33	0.01	0.25	0.53

This table displays the scenario values of the (potential) input variables under the different scenarios Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%). Hereby, the endogenous as well as exogenous variables of the annual (monthly) (MS-)GVAR model as well as the commodity-specific determinants of the logistic regression model are displayed. In particular, we report the scenario values of supply (**supply**), demand (**demand**), and price (**price**) of the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), as well as U.S. industrial production (U.S. IP), world industrial production (IP), world gross domestic product (GDP), world gross domestic product per capita (GDPc), U.S. dollar index (FX), Federal Funds Effective Rate (FFR), 10-year U.S. Treasury rate (LIR), U.S. consumer price index (CPI), MSCI world stock index (MSCI), West Texas Intermediate spot crude oil price (OIL), and global natural disasters (ND). Hereby, the values are derived under the robustness test for the scenario values, they are calculated using data in the period from 2015 to 2019.

Table D.43: Probability of scarcity per commodity derived from the GVAR models of the robustness analysis for the scenario values for the reduced sample period from 2015 to 2019

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
REMod − REF	2019	Mean	0.00	0.00	0.01	0.00	0.00	0.18	0.00	0.05	0.00	0.00	0.18
		Shock	1.00	1.00	1.00	1.00	0.96	1.00	1.00	1.00	0.78	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.01	0.15	0.15	0.23	0.00	0.77	0.46	0.56	0.00	0.54	0.98
		Foc. FX	0.01	0.06	0.07	0.12	0.00	0.64	0.27	0.31	0.00	0.20	0.92
		Foc. FFR	0.00	0.02	0.04	0.01	0.00	0.50	0.02	0.13	0.00	0.01	0.33
		Foc. Extr. EA	0.23	0.88	0.79	0.96	0.03	0.97	0.94	0.98	0.06	0.98	1.00
		Foc. Extr. FX	0.15	0.41	0.36	0.72	0.04	0.87	0.79	0.72	0.02	0.73	0.99
		Foc. Extr. FFR	0.02	0.06	0.09	0.04	0.00	0.73	0.08	0.24	0.00	0.06	0.53
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.04
		Q. 60%	0.00	0.04	0.06	0.02	0.00	0.76	0.06	0.22	0.00	0.05	0.60
		Q. 75%	0.52	0.99	0.93	0.98	0.12	1.00	1.00	1.00	0.04	1.00	1.00
	Mean	Mean	0.01	0.01	0.02	0.01	0.02	0.00	0.00	0.10	0.01	0.01	0.05
		Shock	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.17	0.52	0.31	0.72	0.19	0.01	0.65	0.80	0.06	0.81	0.89
		Foc. FX	0.17	0.19	0.15	0.36	0.34	0.02	0.43	0.46	0.06	0.42	0.74
		Foc. FFR	0.06	0.06	0.06	0.04	0.12	0.01	0.05	0.22	0.03	0.04	0.16
		Foc. Extr. EA	0.70	0.98	0.93	0.99	0.72	0.20	0.98	1.00	0.76	1.00	0.99
		Foc. Extr. FX	0.47	0.62	0.50	0.89	0.75	0.19	0.86	0.82	0.18	0.85	0.96
		Foc. Extr. FFR	0.16	0.16	0.17	0.10	0.26	0.05	0.15	0.33	0.06	0.14	0.30
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.01
		Q. 60%	0.10	0.15	0.13	0.07	0.16	0.01	0.13	0.35	0.04	0.13	0.25
		Q. 75%	0.97	1.00	0.99	1.00	0.98	0.77	1.00	1.00	0.74	1.00	1.00
REMod − SUF	2019	Mean	0.00	0.00	0.00	0.00	0.00	0.22	0.00	0.04	0.00	0.00	0.16
		Shock	0.99	1.00	1.00	1.00	0.96	1.00	1.00	1.00	0.77	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.00	0.14	0.13	0.20	0.00	0.81	0.47	0.53	0.00	0.52	0.98
		Foc. FX	0.00	0.04	0.07	0.10	0.00	0.66	0.26	0.28	0.00	0.20	0.92
		Foc. FFR	0.00	0.02	0.04	0.00	0.00	0.55	0.01	0.11	0.00	0.01	0.33

Probability of scarcity per commodity derived from the GVAR models of the robustness analysis for the scenario values for the reduced sample period from 2015 to 2019

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
REMod − SUF	2019	Foc. Extr. EA	0.22	0.90	0.79	0.97	0.02	0.97	0.95	0.97	0.05	0.98	1.00
		Foc. Extr. FX	0.13	0.39	0.34	0.74	0.03	0.87	0.81	0.71	0.01	0.74	0.99
		Foc. Extr. FFR	0.02	0.05	0.08	0.03	0.00	0.78	0.07	0.22	0.00	0.06	0.53
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.05
		Q. 60%	0.00	0.03	0.04	0.01	0.00	0.77	0.03	0.17	0.00	0.03	0.56
		Q. 75%	0.45	0.99	0.89	0.97	0.07	1.00	0.99	1.00	0.03	1.00	1.00
	Mean	Mean	0.00	0.01	0.02	0.00	0.02	0.00	0.00	0.09	0.00	0.00	0.06
		Shock	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.16	0.52	0.33	0.72	0.17	0.00	0.66	0.78	0.06	0.80	0.90
		Foc. FX	0.15	0.17	0.14	0.34	0.33	0.01	0.42	0.43	0.05	0.40	0.72
		Foc. FFR	0.05	0.05	0.06	0.03	0.11	0.00	0.03	0.19	0.02	0.04	0.15
		Foc. Extr. EA	0.69	0.98	0.92	0.99	0.71	0.16	0.98	0.99	0.74	1.00	0.99
		Foc. Extr. FX	0.47	0.61	0.48	0.89	0.75	0.16	0.88	0.81	0.18	0.85	0.96
		Foc. Extr. FFR	0.15	0.14	0.15	0.10	0.25	0.05	0.13	0.31	0.06	0.14	0.28
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.01
		Q. 60%	0.08	0.11	0.11	0.06	0.13	0.00	0.10	0.32	0.03	0.10	0.23
		Q. 75%	0.96	1.00	0.99	1.00	0.97	0.65	1.00	1.00	0.71	1.00	1.00
REMod − PER	2019	Mean	0.00	0.00	0.01	0.00	0.00	0.22	0.00	0.05	0.00	0.00	0.18
		Shock	1.00	1.00	1.00	1.00	0.96	1.00	1.00	1.00	0.80	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.01	0.17	0.15	0.24	0.00	0.79	0.49	0.58	0.00	0.55	0.99
		Foc. FX	0.01	0.06	0.07	0.11	0.00	0.66	0.30	0.33	0.00	0.22	0.90
		Foc. FFR	0.00	0.02	0.04	0.01	0.00	0.55	0.02	0.14	0.00	0.02	0.34
		Foc. Extr. EA	0.24	0.90	0.81	0.96	0.03	0.97	0.94	0.98	0.06	0.99	1.00
		Foc. Extr. FX	0.17	0.42	0.36	0.72	0.05	0.88	0.82	0.75	0.02	0.74	0.99
		Foc. Extr. FFR	0.03	0.07	0.10	0.04	0.00	0.75	0.09	0.25	0.00	0.07	0.51
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.04
		Q. 60%	0.01	0.05	0.05	0.02	0.00	0.77	0.05	0.23	0.00	0.04	0.57
		Q. 75%	0.61	0.99	0.92	0.98	0.10	1.00	1.00	1.00	0.04	1.00	1.00
	Mean	Mean	0.01	0.02	0.02	0.01	0.02	0.00	0.00	0.11	0.01	0.01	0.05
		Shock	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.19	0.54	0.32	0.73	0.20	0.01	0.67	0.81	0.07	0.83	0.90
		Foc. FX	0.17	0.20	0.17	0.37	0.37	0.02	0.46	0.50	0.06	0.43	0.73
		Foc. FFR	0.06	0.06	0.07	0.04	0.13	0.01	0.04	0.22	0.03	0.04	0.16
		Foc. Extr. EA	0.69	0.98	0.92	1.00	0.73	0.19	0.98	1.00	0.76	1.00	0.99
		Foc. Extr. FX	0.49	0.64	0.51	0.89	0.78	0.18	0.89	0.83	0.18	0.84	0.96
		Foc. Extr. FFR	0.17	0.16	0.18	0.11	0.27	0.06	0.16	0.35	0.07	0.14	0.30
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.01
		Q. 60%	0.12	0.15	0.13	0.08	0.16	0.01	0.11	0.37	0.04	0.14	0.26
		Q. 75%	0.99	1.00	0.99	1.00	0.98	0.73	1.00	1.00	0.74	1.00	1.00
REMod − UNA	2019	Mean	0.00	0.00	0.00	0.00	0.00	0.26	0.00	0.06	0.00	0.00	0.16
		Shock	0.99	1.00	1.00	1.00	0.95	1.00	1.00	1.00	0.77	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.01	0.16	0.16	0.23	0.00	0.82	0.49	0.58	0.00	0.55	0.99
		Foc. FX	0.01	0.05	0.08	0.10	0.00	0.68	0.28	0.32	0.00	0.21	0.91
		Foc. FFR	0.00	0.02	0.04	0.01	0.00	0.58	0.02	0.13	0.00	0.02	0.33
		Foc. Extr. EA	0.24	0.90	0.80	0.97	0.02	0.98	0.95	0.98	0.06	0.99	1.00
		Foc. Extr. FX	0.16	0.39	0.35	0.73	0.04	0.88	0.82	0.75	0.02	0.74	0.99
		Foc. Extr. FFR	0.02	0.06	0.09	0.03	0.00	0.79	0.08	0.24	0.00	0.07	0.52
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.04
		Q. 60%	0.00	0.04	0.05	0.02	0.00	0.78	0.04	0.22	0.00	0.04	0.58
		Q. 75%	0.51	0.99	0.90	0.98	0.09	1.00	1.00	1.00	0.04	1.00	1.00
	Mean	Mean	0.01	0.01	0.02	0.01	0.03	0.00	0.00	0.10	0.01	0.01	0.05
		Shock	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.18	0.55	0.35	0.74	0.21	0.01	0.68	0.83	0.07	0.84	0.90
		Foc. FX	0.16	0.17	0.16	0.36	0.37	0.01	0.44	0.48	0.06	0.42	0.74
		Foc. FFR	0.06	0.06	0.08	0.03	0.13	0.00	0.04	0.21	0.03	0.04	0.15

Probability of scarcity per commodity derived from the GVAR models of the robustness analysis for the scenario values for the reduced sample period from 2015 to 2019

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
$REMod-UNA$	Mean	Foc. Extr. EA	0.71	0.99	0.94	1.00	0.73	0.16	0.98	1.00	0.77	1.00	0.99
		Foc. Extr. FX	0.48	0.62	0.50	0.90	0.78	0.19	0.89	0.82	0.18	0.85	0.96
		Foc. Extr. FFR	0.16	0.15	0.17	0.11	0.26	0.06	0.15	0.34	0.06	0.15	0.28
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.01
		Q. 60%	0.08	0.12	0.13	0.06	0.15	0.00	0.11	0.36	0.03	0.13	0.23
		Q. 75%	0.97	1.00	0.99	1.00	0.97	0.69	1.00	1.00	0.70	1.00	1.00

This table displays the probability of scarcity (PS) of the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), derived from the GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). Hereby, the probability of scarcity is calculated for the scenarios Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%) of the input variables. The presented results are derived under the robustness test for the scenario values, in particular, the scenario values are calculated based on data in the period from 2015 to 2019.

Table D.44: Probability of scarcity per commodity derived from the MS-GVAR models of the robustness analysis for the scenario values for the reduced sample period from 2015 to 2019

			Al	Cu	Ni	Pb	Sn	Zn
$REMod-REF$	2019	Mean	1.00	0.96	0.81	1.00	0.30	1.00
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	1.00	1.00	0.87	1.00
		Foc. FX	1.00	1.00	1.00	1.00	0.83	1.00
		Foc. FFR	1.00	1.00	1.00	1.00	0.98	1.00
		Foc. Extr. EA	1.00	1.00	1.00	1.00	0.95	1.00
		Foc. Extr. FX	1.00	1.00	1.00	1.00	0.96	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	0.99	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.01	0.00	0.06
		Q. 50%	0.13	0.06	0.12	0.90	0.04	1.00
		Q. 60%	1.00	1.00	1.00	1.00	0.97	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
	Mean	Mean	1.00	1.00	0.99	1.00	0.90	1.00
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. FX	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. Extr. FX	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.01	0.00	0.04	0.00	0.00
		Q. 50%	0.97	0.93	0.54	1.00	0.35	0.60
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
$REMod-SUF$	2019	Mean	1.00	0.97	0.80	1.00	0.29	1.00
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	1.00	1.00	0.85	1.00
		Foc. FX	1.00	1.00	1.00	1.00	0.80	1.00
		Foc. FFR	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. Extr. EA	1.00	1.00	1.00	1.00	0.95	1.00
		Foc. Extr. FX	1.00	1.00	1.00	1.00	0.94	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.01	0.00	0.09
		Q. 50%	0.12	0.07	0.11	0.91	0.03	1.00
		Q. 60%	1.00	1.00	1.00	1.00	0.96	1.00

Probability of scarcity per commodity derived from the MS-GVAR models of the robustness analysis for the scenario values for the reduced sample period from 2015 to 2019

			Al	Cu	Ni	Pb	Sn	Zn
$REMod-SUF$		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
$REMod-SUF$	Mean	Mean	1.00	1.00	1.00	1.00	0.86	0.99
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. FX	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. FX	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.01	0.00	0.05	0.00	0.00
		Q. 50%	0.94	0.94	0.54	1.00	0.36	0.67
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
$REMod-PER$	2019	Mean	1.00	0.97	0.87	1.00	0.28	1.00
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	1.00	1.00	0.85	1.00
		Foc. FX	1.00	1.00	1.00	1.00	0.83	1.00
		Foc. FFR	1.00	1.00	1.00	1.00	0.98	1.00
		Foc. Extr. EA	1.00	1.00	1.00	1.00	0.95	1.00
		Foc. Extr. FX	1.00	1.00	1.00	1.00	0.96	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	0.99	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.01	0.00	0.07
		Q. 50%	0.13	0.07	0.14	0.91	0.04	1.00
		Q. 60%	1.00	1.00	1.00	1.00	0.97	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
	Mean	Mean	1.00	1.00	1.00	1.00	0.86	0.99
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. FX	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. Extr. FX	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.01	0.00	0.04	0.00	0.00
		Q. 50%	0.97	0.95	0.61	1.00	0.36	0.64
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
$REMod-UNA$	2019	Mean	1.00	0.96	0.82	1.00	0.31	1.00
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	1.00	1.00	0.85	1.00
		Foc. FX	1.00	1.00	1.00	1.00	0.84	1.00
		Foc. FFR	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. Extr. EA	1.00	1.00	1.00	1.00	0.94	1.00
		Foc. Extr. FX	1.00	1.00	1.00	1.00	0.95	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.01	0.00	0.12
		Q. 50%	0.12	0.08	0.12	0.90	0.04	1.00
		Q. 60%	1.00	1.00	1.00	1.00	0.97	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
	Mean	Mean	1.00	1.00	1.00	1.00	0.89	1.00
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. FX	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. Extr. FX	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.01	0.00	0.04	0.00	0.00
		Q. 50%	0.95	0.92	0.59	1.00	0.39	0.67
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00

Probability of scarcity per commodity derived from the MS-GVAR models of the robustness analysis for the scenario values for the reduced sample period from 2015 to 2019

	Al	Cu	Ni	Pb	Sn	Zn
Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00

This table displays the probability of scarcity (PS) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), derived from the MS-GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod - REF$, $REMod - SUF$, $REMod - PER$, and $REMod - UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). Hereby, the probability of scarcity is calculated for the scenarios Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%) of the input variables. The presented results are derived under the robustness test for the scenario values, in particular, the scenario values are calculated based on data in the period from 2015 to 2019.

Table D.45: Probability of scarcity per commodity derived from the logistic regression models of the robustness analysis for the scenario values for the reduced sample period from 2015 to 2019

	Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
Mean	0.03	0.05	0.07	0.05	0.02	0.10	0.05	0.02	0.05	0.06	0.03	0.02	0.12
Shock	0.05	0.09	0.51	0.07	0.07	0.17	0.07	0.07	0.23	0.18	0.08	0.03	0.94
Extr.	0.07	0.18	0.94	0.09	0.25	0.29	0.10	0.25	0.61	0.41	0.19	0.04	1.00
Foc. EA	0.03	0.05	0.07	0.05	0.03	0.10	0.05	0.03	0.05	0.06	0.03	0.02	0.12
Foc. FX	0.03	0.05	0.07	0.05	0.02	0.13	0.05	0.02	0.05	0.10	0.04	0.02	0.12
Foc. FFR	0.03	0.05	0.07	0.05	0.03	0.10	0.05	0.03	0.05	0.06	0.03	0.02	0.31
Foc. Extr. EA	0.03	0.05	0.07	0.05	0.07	0.10	0.05	0.07	0.05	0.06	0.03	0.02	0.12
Foc. Extr. FX	0.03	0.05	0.07	0.05	0.02	0.16	0.05	0.02	0.05	0.16	0.05	0.02	0.12
Foc. Extr. FFR	0.03	0.05	0.07	0.06	0.05	0.10	0.05	0.05	0.05	0.06	0.03	0.02	0.59
Q. 25%	0.03	0.04	0.01	0.04	0.01	0.08	0.04	0.01	0.02	0.04	0.02	0.02	0.01
Q. 40%	0.03	0.04	0.03	0.04	0.01	0.08	0.04	0.01	0.03	0.04	0.02	0.02	0.02
Q. 50%	0.03	0.04	0.04	0.04	0.01	0.09	0.04	0.01	0.05	0.04	0.02	0.02	0.05
Q. 60%	0.04	0.06	0.12	0.05	0.02	0.10	0.04	0.02	0.09	0.06	0.03	0.02	0.23
Q. 75%	0.04	0.08	0.45	0.06	0.05	0.13	0.04	0.05	0.19	0.12	0.05	0.02	0.77

This table displays the probability of scarcity (PS) of the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), derived from the logistic regression models based on preselected covariates. Hereby, the probability of scarcity is calculated for the scenarios Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%) of the input variables. The presented results are derived under the robustness test for the scenario values, in particular, the scenario values are calculated based on data in the period from 2015 to 2019.

Table D.46: Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the GVAR models of the robustness analysis for the scenario values for the reduced sample period from 2015 to 2019

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
Mean	2019	$REMod - REF$	0.00	0.03	1.83	0.00	0.00	4.64	0.09	0.07	0.00	0.03	1.28
		$REMod - SUF$	0.00	0.02	1.23	0.00	0.00	3.12	0.06	0.05	0.00	0.02	1.30
		$REMod - PER$	0.00	0.02	1.32	0.00	0.00	3.35	0.07	0.07	0.00	0.03	1.05
		$REMod - UNA$	0.00	0.03	1.87	0.00	0.00	4.75	0.09	0.04	0.00	0.02	1.36
	Mean	$REMod - REF$	0.08	0.12	5.78	0.22	3.16	0.00	0.35	0.14	0.00	0.05	0.35
		$REMod - SUF$	0.06	0.08	3.89	0.16	2.32	0.00	0.24	0.09	0.00	0.04	0.35
		$REMod - PER$	0.09	0.10	4.17	0.19	3.24	0.00	0.28	0.14	0.01	0.05	0.28
		$REMod - UNA$	0.13	0.13	5.92	0.21	4.83	0.00	0.35	0.08	0.00	0.04	0.37
Shock	2019	$REMod - REF$	9.15	8.36	304.32	27.27	127.16	25.23	87.36	1.38	0.42	9.12	7.21
		$REMod - SUF$	6.18	5.67	204.60	19.71	93.11	16.96	59.05	0.94	0.28	6.21	7.33
		$REMod - PER$	10.42	7.34	219.38	23.16	130.29	18.20	69.11	1.36	0.53	8.77	5.91
		$REMod - UNA$	14.59	9.36	311.47	25.80	194.27	25.82	87.45	0.78	0.30	6.72	7.66
	Mean	$REMod - REF$	9.17	8.36	304.32	27.27	131.77	25.23	87.36	1.38	0.53	9.12	7.21
		$REMod - SUF$	6.19	5.67	204.60	19.71	96.49	16.96	59.05	0.94	0.35	6.21	7.33
		$REMod - PER$	10.44	7.34	219.38	23.16	135.01	18.20	69.11	1.36	0.67	8.77	5.91
		$REMod - UNA$	14.62	9.36	311.47	25.80	201.32	25.82	87.45	0.78	0.38	6.72	7.66

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the GVAR models of the robustness analysis for the scenario values for the reduced sample period from 2015 to 2019

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
Extr.	2019	$REMod-REF$	9.17	8.36	304.32	27.27	131.77	25.23	87.36	1.38	0.53	9.12	7.21
		$REMod-SUF$	6.19	5.67	204.60	19.71	96.49	16.96	59.05	0.94	0.35	6.21	7.33
		$REMod-PER$	10.44	7.34	219.38	23.16	135.01	18.20	69.11	1.36	0.67	8.77	5.91
		$REMod-UNA$	14.62	9.36	311.47	25.80	201.32	25.82	87.45	0.78	0.38	6.72	7.66
	Mean	$REMod-REF$	9.17	8.36	304.32	27.27	131.77	25.23	87.36	1.38	0.53	9.12	7.21
		$REMod-SUF$	6.19	5.67	204.60	19.71	96.49	16.96	59.05	0.94	0.35	6.21	7.33
		$REMod-PER$	10.44	7.34	219.38	23.16	135.01	18.20	69.11	1.36	0.67	8.77	5.91
		$REMod-UNA$	14.62	9.36	311.47	25.80	201.32	25.82	87.45	0.78	0.38	6.72	7.66
Foc. EA	2019	$REMod-REF$	0.06	1.29	44.74	6.22	0.13	19.35	40.62	0.77	0.00	4.91	7.10
		$REMod-SUF$	0.04	0.87	30.08	4.49	0.10	13.01	27.46	0.52	0.00	3.35	7.22
		$REMod-PER$	0.07	1.13	32.25	5.28	0.14	13.96	32.13	0.76	0.00	4.72	5.82
		$REMod-UNA$	0.10	1.44	45.79	5.88	0.20	19.80	40.66	0.43	0.00	3.62	7.54
	Mean	$REMod-REF$	1.59	4.33	95.56	19.77	25.43	0.23	56.61	1.11	0.03	7.43	6.43
		$REMod-SUF$	1.07	2.94	64.24	14.29	18.62	0.15	38.26	0.76	0.02	5.06	6.54
		$REMod-PER$	1.81	3.80	68.89	16.79	26.06	0.16	44.78	1.09	0.04	7.14	5.27
		$REMod-UNA$	2.53	4.85	97.80	18.71	38.85	0.23	56.67	0.62	0.02	5.48	6.83
Foc. FX	2019	$REMod-REF$	0.06	0.51	22.22	3.19	0.13	16.20	23.85	0.43	0.00	1.85	6.60
		$REMod-SUF$	0.04	0.35	14.94	2.31	0.10	10.89	16.12	0.29	0.00	1.26	6.71
		$REMod-PER$	0.07	0.45	16.01	2.71	0.14	11.68	18.87	0.42	0.00	1.78	5.41
		$REMod-UNA$	0.10	0.57	22.74	3.02	0.20	16.57	23.87	0.24	0.00	1.37	7.01
	Mean	$REMod-REF$	1.54	1.56	46.26	9.79	44.93	0.58	37.22	0.63	0.03	3.78	5.31
		$REMod-SUF$	1.04	1.06	31.10	7.08	32.90	0.39	25.15	0.43	0.02	2.58	5.40
		$REMod-PER$	1.75	1.37	33.35	8.31	46.04	0.42	29.44	0.62	0.04	3.64	4.36
		$REMod-UNA$	2.46	1.75	47.34	9.26	68.65	0.59	37.25	0.36	0.02	2.79	5.64
Foc. FFR	2019	$REMod-REF$	0.04	0.17	12.17	0.19	0.00	12.54	1.92	0.19	0.00	0.13	2.41
		$REMod-SUF$	0.02	0.11	8.18	0.14	0.00	8.43	1.30	0.13	0.00	0.09	2.46
		$REMod-PER$	0.04	0.15	8.78	0.16	0.00	9.04	1.52	0.18	0.00	0.12	1.98
		$REMod-UNA$	0.06	0.19	12.46	0.18	0.00	12.83	1.92	0.10	0.00	0.09	2.57
	Mean	$REMod-REF$	0.60	0.48	19.17	1.09	16.47	0.18	4.02	0.30	0.01	0.38	1.15
		$REMod-SUF$	0.40	0.33	12.89	0.79	12.06	0.12	2.72	0.20	0.01	0.26	1.17
		$REMod-PER$	0.68	0.43	13.82	0.93	16.88	0.13	3.18	0.29	0.02	0.37	0.95
		$REMod-UNA$	0.95	0.54	19.62	1.03	25.16	0.18	4.02	0.17	0.01	0.28	1.23
Foc. Extr. EA	2019	$REMod-REF$	2.07	7.40	240.72	26.31	3.43	24.48	82.12	1.35	0.03	8.95	7.21
		$REMod-SUF$	1.40	5.02	161.84	19.02	2.51	16.45	55.51	0.92	0.02	6.10	7.33
		$REMod-PER$	2.36	6.50	173.53	22.35	3.51	17.65	64.96	1.33	0.04	8.61	5.91
		$REMod-UNA$	3.30	8.29	246.37	24.90	5.23	25.04	82.20	0.76	0.02	6.60	7.66
	Mean	$REMod-REF$	6.40	8.21	282.41	27.10	95.14	4.95	85.26	1.38	0.41	9.10	7.14
		$REMod-SUF$	4.32	5.57	189.87	19.59	69.67	3.32	57.63	0.94	0.27	6.20	7.26
		$REMod-PER$	7.29	7.21	203.59	23.02	97.48	3.57	67.45	1.36	0.51	8.75	5.85
		$REMod-UNA$	10.20	9.20	289.04	25.65	145.35	5.06	85.35	0.77	0.29	6.71	7.58
Foc. Extr. FX	2019	$REMod-REF$	1.36	3.45	109.86	19.71	5.67	21.88	69.10	0.99	0.01	6.62	7.14
		$REMod-SUF$	0.92	2.34	73.86	14.25	4.15	14.71	46.71	0.68	0.01	4.51	7.26
		$REMod-PER$	1.55	3.03	79.20	16.74	5.81	15.78	54.66	0.98	0.01	6.36	5.85
		$REMod-UNA$	2.16	3.87	112.44	18.65	8.66	22.38	69.17	0.56	0.01	4.88	7.58
	Mean	$REMod-REF$	4.35	5.18	151.25	24.24	98.43	4.90	74.87	1.14	0.09	7.75	6.91
		$REMod-SUF$	2.93	3.51	101.69	17.52	72.08	3.29	50.60	0.77	0.06	5.28	7.03
		$REMod-PER$	4.95	4.54	109.03	20.59	100.85	3.53	59.22	1.12	0.12	7.45	5.67
		$REMod-UNA$	6.93	5.80	154.80	22.94	150.38	5.01	74.94	0.64	0.07	5.72	7.34
Foc. Extr. FFR	2019	$REMod-REF$	0.16	0.54	28.00	1.06	0.66	18.50	7.25	0.33	0.00	0.59	3.81
		$REMod-SUF$	0.11	0.36	18.82	0.77	0.48	12.43	4.90	0.23	0.00	0.40	3.88
		$REMod-PER$	0.18	0.47	20.18	0.90	0.68	13.34	5.74	0.33	0.00	0.57	3.13
		$REMod-UNA$	0.25	0.60	28.66	1.01	1.01	18.92	7.26	0.19	0.00	0.44	4.05
	Mean	$REMod-REF$	1.44	1.31	52.34	2.84	34.26	1.34	13.10	0.45	0.03	1.29	2.13
		$REMod-SUF$	0.97	0.89	35.19	2.05	25.09	0.90	8.86	0.31	0.02	0.88	2.16
		$REMod-PER$	1.64	1.15	37.73	2.41	35.10	0.96	10.37	0.45	0.04	1.24	1.74
		$REMod-UNA$	2.30	1.47	53.57	2.68	52.34	1.37	13.12	0.26	0.02	0.95	2.26
Q. 25%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Q. 40%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the GVAR models of the robustness analysis for the scenario values for the reduced sample period from 2015 to 2019

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Q. 50%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.43	0.00	0.01	0.00	0.00	0.28
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.29	0.00	0.00	0.00	0.00	0.29
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.31	0.00	0.01	0.00	0.00	0.23
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.44	0.00	0.00	0.00	0.00	0.30
	Mean	$REMod-REF$	0.01	0.01	0.61	0.00	0.13	0.00	0.00	0.03	0.00	0.00	0.07
		$REMod-SUF$	0.01	0.01	0.41	0.00	0.10	0.00	0.00	0.02	0.00	0.00	0.07
		$REMod-PER$	0.01	0.01	0.44	0.00	0.14	0.00	0.00	0.03	0.00	0.00	0.06
		$REMod-UNA$	0.01	0.01	0.62	0.00	0.20	0.00	0.00	0.01	0.00	0.00	0.08
Q. 60%	2019	$REMod-REF$	0.04	0.30	18.26	0.41	0.13	19.25	5.59	0.30	0.00	0.42	4.36
		$REMod-SUF$	0.02	0.20	12.28	0.30	0.10	12.94	3.78	0.20	0.00	0.29	4.44
		$REMod-PER$	0.04	0.26	13.16	0.35	0.14	13.88	4.42	0.30	0.00	0.40	3.58
		$REMod-UNA$	0.06	0.34	18.69	0.39	0.20	19.70	5.60	0.17	0.00	0.31	4.63
	Mean	$REMod-REF$	0.90	1.23	38.95	1.91	21.35	0.20	11.36	0.49	0.02	1.22	1.82
		$REMod-SUF$	0.61	0.83	26.19	1.38	15.63	0.14	7.68	0.33	0.01	0.83	1.85
		$REMod-PER$	1.02	1.08	28.08	1.62	21.87	0.15	8.98	0.48	0.02	1.17	1.49
		$REMod-UNA$	1.43	1.38	39.87	1.81	32.61	0.21	11.37	0.27	0.01	0.90	1.93
Q. 75%	2019	$REMod-REF$	4.74	8.27	283.02	26.69	16.34	25.23	87.19	1.38	0.02	9.11	7.21
		$REMod-SUF$	3.20	5.61	190.28	19.29	11.96	16.96	58.93	0.94	0.01	6.20	7.33
		$REMod-PER$	5.40	7.26	204.03	22.67	16.74	18.20	68.97	1.36	0.02	8.76	5.91
		$REMod-UNA$	7.56	9.26	289.67	25.26	24.96	25.82	87.27	0.78	0.01	6.72	7.66
	Mean	$REMod-REF$	8.89	8.36	301.28	27.27	129.00	19.43	87.36	1.38	0.40	9.12	7.21
		$REMod-SUF$	6.00	5.67	202.55	19.71	94.46	13.06	59.05	0.94	0.26	6.21	7.33
		$REMod-PER$	10.12	7.34	217.19	23.16	132.18	14.01	69.11	1.36	0.50	8.77	5.91
		$REMod-UNA$	14.17	9.36	308.36	25.80	197.09	19.88	87.45	0.78	0.28	6.72	7.66

This table displays the expected loss due to scarcity for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), per path ($REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$), as well as per scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) for the input variables. Hereby, the values are derived from the GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). Hereby, the results are derived under the robustness test for the scenario values, in particular, the scenario values are calculated based on data in the period from 2015 to 2019.

Table D.47: Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the MS-GVAR models of the robustness analysis for the scenario values for the reduced sample period from 2015 to 2019

			Al	Cu	Ni	Pb	Sn	Zn
Mean	2019	$REMod-REF$	8.36	26.29	70.59	1.38	2.72	7.21
		$REMod-SUF$	5.67	19.00	47.71	0.94	1.85	7.33
		$REMod-PER$	7.34	22.33	55.84	1.36	2.61	5.91
		$REMod-UNA$	9.36	24.87	70.66	0.78	2.00	7.66
	Mean	$REMod-REF$	8.36	27.27	86.84	1.38	8.17	7.18
		$REMod-SUF$	5.67	19.71	58.69	0.94	5.56	7.30
		$REMod-PER$	7.34	23.16	68.69	1.36	7.85	5.89
		$REMod-UNA$	9.36	25.80	86.92	0.78	6.02	7.63
Shock	2019	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
Extr.	2019	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
		$REMod-REF$	8.36	27.21	87.19	1.38	7.89	7.21

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the MS-GVAR models of the robustness analysis for the scenario values for the reduced sample period from 2015 to 2019

			Al	Cu	Ni	Pb	Sn	Zn
Foc. EA	2019	$REMod-SUF$	5.67	19.67	58.93	0.94	5.38	7.33
		$REMod-PER$	7.34	23.11	68.97	1.36	7.59	5.91
		$REMod-UNA$	9.36	25.75	87.27	0.78	5.82	7.66
	Mean	$REMod-REF$	8.36	27.27	87.19	1.38	9.01	7.21
		$REMod-SUF$	5.67	19.71	58.93	0.94	6.14	7.33
		$REMod-PER$	7.34	23.16	68.97	1.36	8.66	5.91
		$REMod-UNA$	9.36	25.80	87.27	0.78	6.64	7.66
Foc. FX	2019	$REMod-REF$	8.36	27.27	87.19	1.38	7.55	7.21
		$REMod-SUF$	5.67	19.71	58.93	0.94	5.14	7.33
		$REMod-PER$	7.34	23.16	68.97	1.36	7.26	5.91
		$REMod-UNA$	9.36	25.80	87.27	0.78	5.57	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.02	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.15	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.68	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.66	7.66
Foc. FFR	2019	$REMod-REF$	8.36	27.27	87.36	1.38	8.97	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.11	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.63	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.62	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.08	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.19	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.73	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.70	7.66
Foc. Extr. EA	2019	$REMod-REF$	8.36	27.27	87.19	1.38	8.66	7.21
		$REMod-SUF$	5.67	19.71	58.93	0.94	5.90	7.33
		$REMod-PER$	7.34	23.16	68.97	1.36	8.33	5.91
		$REMod-UNA$	9.36	25.80	87.27	0.78	6.39	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.06	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.17	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.71	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.68	7.66
Foc. Extr. FX	2019	$REMod-REF$	8.36	27.27	87.36	1.38	8.79	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	5.99	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.45	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.48	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.06	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.17	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.71	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.68	7.66
Foc. Extr. FFR	2019	$REMod-REF$	8.36	27.27	87.36	1.38	9.06	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.17	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.71	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.68	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
Q. 25%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00
	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00
Q. 40%	2019	$REMod-REF$	0.00	0.00	0.00	0.01	0.00	0.45
		$REMod-SUF$	0.00	0.00	0.00	0.01	0.00	0.45
		$REMod-PER$	0.00	0.00	0.00	0.01	0.00	0.37
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.47
	Mean	$REMod-REF$	0.00	0.22	0.00	0.06	0.04	0.00
		$REMod-SUF$	0.00	0.16	0.00	0.04	0.02	0.00
		$REMod-PER$	0.00	0.19	0.00	0.06	0.04	0.00
		$REMod-UNA$	0.00	0.21	0.00	0.03	0.03	0.00
Q. 50%	2019	$REMod-REF$	1.09	1.75	10.83	1.25	0.36	7.21
		$REMod-SUF$	0.74	1.26	7.32	0.85	0.25	7.33
		$REMod-PER$	0.95	1.48	8.57	1.23	0.35	5.91
		$REMod-UNA$	1.22	1.65	10.84	0.70	0.27	7.66
	Mean	$REMod-REF$	8.08	25.36	47.17	1.38	3.23	4.31
		$REMod-SUF$	5.48	18.33	31.89	0.94	2.20	4.38
		$REMod-PER$	7.09	21.54	37.32	1.36	3.10	3.53

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the MS-GVAR models of the robustness analysis for the scenario values for the reduced sample period from 2015 to 2019

			Al	Cu	Ni	Pb	Sn	Zn
		$REMod-UNA$	9.05	24.00	47.22	0.78	2.38	4.58
Q. 60%	2019	$REMod-REF$	8.36	27.27	87.36	1.38	8.82	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.01	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.49	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.51	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
Q. 75%	2019	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66

This table displays the expected loss due to scarcity for the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), per path ($REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$), as well as per scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) for the input variables. Hereby, the values are derived from the MS-GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). Hereby, the results are derived under the robustness test for the scenario values, in particular, the scenario values are calculated based on data in the period from 2015 to 2019.

Table D.48: Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the logistic regression models of the robustness analysis for the scenario values for the reduced sample period from 2015 to 2019

		Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
Mean	$REMod-REF$	0.30	0.39	20.31	1.26	2.16	12.84	1.19	0.28	4.52	0.09	0.02	0.17	0.86
	$REMod-SUF$	0.20	0.26	13.65	0.91	1.55	9.40	0.80	0.20	3.06	0.06	0.01	0.11	0.87
	$REMod-PER$	0.34	0.34	14.64	1.07	2.08	13.16	0.86	0.26	3.58	0.09	0.02	0.16	0.70
	$REMod-UNA$	0.47	0.44	20.79	1.20	1.03	19.62	1.22	0.12	4.53	0.05	0.01	0.12	0.91
Shock	$REMod-REF$	0.42	0.79	154.17	1.81	9.51	23.01	1.75	1.24	19.87	0.25	0.04	0.25	6.77
	$REMod-SUF$	0.28	0.53	103.65	1.31	6.80	16.85	1.18	0.88	13.43	0.17	0.03	0.17	6.89
	$REMod-PER$	0.48	0.69	111.14	1.53	9.16	23.58	1.26	1.15	15.72	0.24	0.05	0.24	5.55
	$REMod-UNA$	0.67	0.88	157.79	1.71	4.53	35.16	1.79	0.53	19.89	0.14	0.03	0.18	7.19
Extr.	$REMod-REF$	0.60	1.51	284.99	2.56	35.46	38.62	2.55	4.63	53.61	0.56	0.10	0.36	7.20
	$REMod-SUF$	0.40	1.02	191.60	1.85	25.35	28.28	1.71	3.29	36.23	0.38	0.07	0.24	7.33
	$REMod-PER$	0.68	1.32	205.45	2.17	34.15	39.57	1.84	4.29	42.40	0.55	0.13	0.34	5.91
	$REMod-UNA$	0.96	1.69	291.69	2.42	16.91	59.01	2.61	1.98	53.66	0.32	0.07	0.26	7.65
Foc. EA	$REMod-REF$	0.30	0.39	20.31	1.26	4.58	12.84	1.19	0.60	4.52	0.09	0.02	0.17	0.86
	$REMod-SUF$	0.20	0.26	13.65	0.91	3.27	9.40	0.80	0.43	3.06	0.06	0.01	0.11	0.87
	$REMod-PER$	0.34	0.34	14.64	1.07	4.41	13.16	0.86	0.55	3.58	0.09	0.02	0.16	0.70
	$REMod-UNA$	0.47	0.44	20.79	1.20	2.18	19.62	1.22	0.26	4.53	0.05	0.01	0.12	0.91
Foc. FX	$REMod-REF$	0.30	0.39	20.31	1.29	2.45	16.56	1.19	0.32	4.52	0.14	0.02	0.17	0.86
	$REMod-SUF$	0.20	0.26	13.65	0.93	1.75	12.13	0.80	0.23	3.06	0.10	0.01	0.11	0.87
	$REMod-PER$	0.34	0.34	14.64	1.10	2.36	16.97	0.86	0.30	3.58	0.14	0.03	0.16	0.70
	$REMod-UNA$	0.47	0.44	20.79	1.22	1.17	25.30	1.22	0.14	4.53	0.08	0.01	0.12	0.91
	$REMod-REF$	0.30	0.39	20.31	1.44	4.05	12.84	1.19	0.53	4.52	0.09	0.02	0.17	2.21

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the logistic regression models of the robustness analysis for the scenario values for the reduced sample period from 2015 to 2019

		Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
Foc. FFR	$REMod - SUF$	0.20	0.26	13.65	1.04	2.89	9.40	0.80	0.38	3.06	0.06	0.01	0.11	2.25
	$REMod - PER$	0.34	0.34	14.64	1.22	3.90	13.16	0.86	0.49	3.58	0.09	0.02	0.16	1.81
	$REMod - UNA$	0.47	0.44	20.79	1.36	1.93	19.62	1.22	0.23	4.53	0.05	0.01	0.12	2.35
Foc. Extr. EA	$REMod - REF$	0.30	0.39	20.31	1.26	9.51	12.84	1.19	1.24	4.52	0.09	0.02	0.17	0.86
	$REMod - SUF$	0.20	0.26	13.65	0.91	6.80	9.40	0.80	0.88	3.06	0.06	0.01	0.11	0.87
	$REMod - PER$	0.34	0.34	14.64	1.07	9.15	13.16	0.86	1.15	3.58	0.09	0.02	0.16	0.70
	$REMod - UNA$	0.47	0.44	20.79	1.20	4.53	19.62	1.22	0.53	4.53	0.05	0.01	0.12	0.91
Foc. Extr. FX	$REMod - REF$	0.30	0.39	20.31	1.32	2.77	21.17	1.19	0.36	4.52	0.23	0.03	0.17	0.86
	$REMod - SUF$	0.20	0.26	13.65	0.95	1.98	15.50	0.80	0.26	3.06	0.15	0.02	0.11	0.87
	$REMod - PER$	0.34	0.34	14.64	1.12	2.67	21.69	0.86	0.34	3.58	0.22	0.03	0.16	0.70
	$REMod - UNA$	0.47	0.44	20.79	1.25	1.32	32.34	1.22	0.16	4.53	0.13	0.02	0.12	0.91
Foc. Extr. FFR	$REMod - REF$	0.30	0.39	20.31	1.64	7.50	12.84	1.19	0.98	4.52	0.09	0.02	0.17	4.27
	$REMod - SUF$	0.20	0.26	13.65	1.19	5.36	9.40	0.80	0.70	3.06	0.06	0.01	0.11	4.34
	$REMod - PER$	0.34	0.34	14.64	1.40	7.22	13.16	0.86	0.91	3.58	0.09	0.02	0.16	3.50
	$REMod - UNA$	0.47	0.44	20.79	1.56	3.57	19.62	1.22	0.42	4.53	0.05	0.01	0.12	4.53
Q. 25%	$REMod - REF$	0.29	0.35	4.34	1.13	1.25	10.92	0.99	0.16	1.65	0.05	0.01	0.15	0.04
	$REMod - SUF$	0.19	0.24	2.92	0.82	0.89	8.00	0.66	0.12	1.11	0.03	0.01	0.10	0.04
	$REMod - PER$	0.33	0.31	3.13	0.96	1.21	11.19	0.71	0.15	1.30	0.05	0.02	0.15	0.03
	$REMod - UNA$	0.46	0.39	4.44	1.07	0.60	16.68	1.01	0.07	1.65	0.03	0.01	0.11	0.05
Q. 40%	$REMod - REF$	0.29	0.36	7.85	1.18	1.60	11.19	1.01	0.21	2.96	0.05	0.01	0.17	0.16
	$REMod - SUF$	0.19	0.24	5.28	0.86	1.15	8.19	0.68	0.15	2.00	0.04	0.01	0.11	0.17
	$REMod - PER$	0.33	0.32	5.66	1.01	1.54	11.47	0.73	0.19	2.34	0.05	0.02	0.16	0.13
	$REMod - UNA$	0.46	0.40	8.04	1.12	0.76	17.10	1.03	0.09	2.96	0.03	0.01	0.12	0.17
Q. 50%	$REMod - REF$	0.29	0.37	11.60	1.22	1.89	11.38	1.02	0.25	4.35	0.06	0.01	0.18	0.40
	$REMod - SUF$	0.20	0.25	7.80	0.88	1.35	8.33	0.69	0.18	2.94	0.04	0.01	0.12	0.40
	$REMod - PER$	0.33	0.32	8.36	1.04	1.82	11.66	0.74	0.23	3.44	0.06	0.02	0.17	0.32
	$REMod - UNA$	0.46	0.41	11.88	1.16	0.90	17.38	1.05	0.11	4.35	0.03	0.01	0.13	0.42
Q. 60%	$REMod - REF$	0.33	0.47	35.53	1.37	3.14	13.40	1.04	0.41	7.58	0.09	0.02	0.18	1.64
	$REMod - SUF$	0.22	0.32	23.88	0.99	2.24	9.81	0.70	0.29	5.12	0.06	0.01	0.12	1.67
	$REMod - PER$	0.37	0.41	25.61	1.17	3.02	13.73	0.75	0.38	5.99	0.09	0.02	0.17	1.35
	$REMod - UNA$	0.52	0.53	36.36	1.30	1.50	20.47	1.07	0.18	7.59	0.05	0.01	0.13	1.74
Q. 75%	$REMod - REF$	0.39	0.69	135.69	1.64	6.63	17.03	1.07	0.87	16.46	0.17	0.03	0.19	5.56
	$REMod - SUF$	0.26	0.46	91.23	1.18	4.74	12.47	0.72	0.62	11.12	0.11	0.02	0.13	5.65
	$REMod - PER$	0.45	0.60	97.82	1.39	6.38	17.45	0.77	0.80	13.02	0.16	0.03	0.18	4.56
	$REMod - UNA$	0.62	0.77	138.88	1.55	3.16	26.02	1.10	0.37	16.47	0.09	0.02	0.14	5.91

This table displays the expected loss due to scarcity for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), per path ($REMod - REF$, $REMod - SUF$, $REMod - PER$ and $REMod - UNA$), as well as per scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) for the input variables, derived from the logistic regression model. Hereby, the results are derived under the robustness test for the scenario values, in particular, the scenario values are calculated based on data in the period from 2015 to 2019.

D.3.2.2.2 Results of the Robustness Analysis for the Scenario Values of the enlarged Sample

Table D.49: Scenario values for the input variables of the robustness analysis for the scenario values for the enlarged sample period from 1995 to 2019

			Mean	Shock	Extr.	Foc. EA	Foc. FX	Foc. FFR	Foc. Extr. EA	Foc. Extr. FX	Foc. Extr. FFR	Q. 25%	Q. 40%	Q. 50%	Q. 60%	Q. 75%
GVAR	Ag	**supply**	0.11	0.97	1.82	0.11	0.11	0.11	0.11	0.11	0.11	-0.42	0.06	0.30	0.50	0.54
		demand	0.01	0.88	1.75	0.01	0.01	0.01	0.01	0.01	0.01	-0.56	-0.17	-0.04	0.30	0.47
		price	-0.00	0.74	1.47	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.38	-0.24	-0.17	0.04	0.25
	Al	**supply**	0.19	0.97	1.75	0.19	0.19	0.19	0.19	0.19	0.19	-0.18	0.09	0.23	0.37	0.55
		demand	-0.15	0.80	1.75	-0.15	-0.15	-0.15	-0.15	-0.15	-0.15	-0.63	-0.37	-0.15	0.08	0.55
		price	-0.08	0.79	1.65	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.59	-0.27	-0.05	0.19	0.41
	Co	**supply**	0.18	1.26	2.34	0.18	0.18	0.18	0.18	0.18	0.18	0.07	0.14	0.33	0.38	0.51
		demand	0.02	1.00	1.98	0.02	0.02	0.02	0.02	0.02	0.02	-0.40	-0.26	-0.08	0.14	0.29
		price	-0.14	0.81	1.76	-0.14	-0.14	-0.14	-0.14	-0.14	-0.14	-0.56	-0.36	-0.29	-0.11	0.31
	Cu	**supply**	0.17	1.06	1.96	0.17	0.17	0.17	0.17	0.17	0.17	-0.62	-0.17	0.18	0.39	0.83
		demand	-0.13	0.76	1.64	-0.13	-0.13	-0.13	-0.13	-0.13	-0.13	-0.43	-0.21	-0.11	-0.06	0.18
		price	0.04	1.06	2.07	0.04	0.04	0.04	0.04	0.04	0.04	-0.58	-0.37	-0.18	0.12	0.55
	In	**supply**	0.10	0.92	1.73	0.10	0.10	0.10	0.10	0.10	0.10	-0.30	-0.17	-0.05	0.02	0.14
		demand	-0.02	0.35	0.72	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.31	-0.12	-0.11	0.03	0.22
		price	-0.01	1.08	2.17	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.78	-0.46	-0.28	-0.13	0.42
	Li	**supply**	0.12	0.82	1.52	0.12	0.12	0.12	0.12	0.12	0.12	-0.19	-0.10	-0.04	0.03	0.31
		demand	-0.01	1.14	2.29	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.41	-0.13	-0.01	0.06	0.33
		price	-0.25	1.00	2.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.25	-0.68	-0.35	-0.31	-0.30	0.03
	Ni	**supply**	0.13	1.20	2.27	0.13	0.13	0.13	0.13	0.13	0.13	-0.40	-0.02	0.17	0.49	0.69
		demand	-0.03	0.79	1.60	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.48	-0.14	0.00	0.17	0.55
		price	-0.00	1.06	2.12	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.85	-0.13	0.12	0.30	0.80
	Pb	**supply**	0.27	1.40	2.53	0.27	0.27	0.27	0.27	0.27	0.27	-0.60	-0.14	0.28	0.64	1.12
		demand	0.01	0.91	1.81	0.01	0.01	0.01	0.01	0.01	0.01	-0.83	-0.18	0.04	0.24	0.63
		price	-0.00	1.03	2.06	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.67	-0.33	-0.20	0.07	0.41
	Pt	**supply**	-0.08	1.09	2.26	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.46	-0.17	0.13	0.27	0.43
		demand	-0.06	1.18	2.43	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06	-0.88	-0.18	0.18	0.26	0.67
		price	-0.04	0.72	1.48	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.49	-0.34	-0.21	-0.04	0.46
	Sn	**supply**	0.20	1.34	2.47	0.20	0.20	0.20	0.20	0.20	0.20	-0.63	-0.17	0.14	0.46	1.14
		demand	0.06	0.85	1.64	0.06	0.06	0.06	0.06	0.06	0.06	-0.40	-0.04	0.12	0.18	0.34
		price	0.07	1.08	2.09	0.07	0.07	0.07	0.07	0.07	0.07	-0.55	-0.23	-0.13	0.20	0.62
	Zn	**supply**	0.20	1.25	2.30	0.20	0.20	0.20	0.20	0.20	0.20	-0.42	-0.06	-0.04	0.35	0.75
		demand	-0.05	0.95	1.95	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.66	-0.33	0.09	0.16	0.58
		price	-0.02	0.98	1.98	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.63	-0.19	-0.11	0.01	0.31
	exog	GDP	-0.40	0.48	1.36	-0.40	-0.40	-0.40	-0.40	-0.40	-0.40	-0.90	-0.72	-0.55	-0.13	0.38
		FX	0.09	0.91	1.74	0.09	0.91	0.09	0.09	1.74	0.09	-0.39	0.02	0.15	0.22	0.56
		FFR	-0.00	1.30	2.60	-0.00	-0.00	1.30	-0.00	-0.00	2.60	-0.43	-0.03	0.10	0.35	0.64
log. Reg.		U.S. IP	-0.11	0.83	1.77	-0.11	-0.11	-0.11	-0.11	-0.11	-0.11	-0.42	0.02	0.14	0.24	0.42
		GDPc	-0.36	0.54	1.44	-0.36	-0.36	-0.36	-0.36	-0.36	-0.36	-0.86	-0.68	-0.53	-0.07	0.45
		LIR	-0.15	0.94	2.04	-0.15	-0.15	-0.15	-0.15	-0.15	-0.15	-0.80	-0.48	-0.06	0.08	0.60
		CPI	0.03	1.43	2.82	0.03	0.03	0.03	0.03	0.03	0.03	-0.24	-0.14	-0.04	0.01	0.15
		MSCI	-0.10	0.83	1.77	-0.10	-0.10	-0.10	-0.10	-0.10	-0.10	-0.37	0.10	0.14	0.45	0.59
		OIL	-0.04	0.92	1.87	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.48	-0.12	0.12	0.41	0.68
		ND	-0.10	0.68	1.47	-0.10	-0.10	-0.10	-0.10	-0.10	-0.10	-0.72	-0.38	-0.14	0.01	0.61
MS-GVAR	Al	**supply**	0.00	1.02	2.03	0.00	0.00	0.00	0.00	0.00	0.00	-0.46	-0.16	0.02	0.18	0.42
		demand	0.01	0.98	1.95	0.01	0.01	0.01	0.01	0.01	0.01	-0.58	-0.30	-0.03	0.18	0.58
		price	0.01	1.01	2.01	0.01	0.01	0.01	0.01	0.01	0.01	-0.63	-0.25	-0.03	0.21	0.65
	Cu	**supply**	-0.01	0.99	1.99	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.59	-0.27	-0.04	0.18	0.59
		demand	0.01	1.02	2.03	0.01	0.01	0.01	0.01	0.01	0.01	-0.61	-0.23	0.00	0.24	0.69
		price	0.00	1.02	2.03	0.00	0.00	0.00	0.00	0.00	0.00	-0.54	-0.27	0.01	0.24	0.60
	Ni	**supply**	-0.01	0.98	1.97	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.48	-0.15	0.02	0.21	0.50
		demand	0.00	1.01	2.01	0.00	0.00	0.00	0.00	0.00	0.00	-0.66	-0.21	0.02	0.26	0.64
		price	0.01	1.00	1.99	0.01	0.01	0.01	0.01	0.01	0.01	-0.72	-0.27	0.01	0.24	0.65
	Pb	**supply**	-0.00	1.01	2.02	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.51	-0.21	-0.10	0.07	0.40
		demand	0.00	1.02	2.03	0.00	0.00	0.00	0.00	0.00	0.00	-0.54	-0.24	-0.03	0.14	0.51
		price	-0.00	1.00	2.01	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.59	-0.21	0.00	0.25	0.61
	Sn	**supply**	-0.00	1.02	2.03	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.49	-0.19	-0.02	0.19	0.53
		demand	0.00	1.02	2.04	0.00	0.00	0.00	0.00	0.00	0.00	-0.55	-0.21	0.03	0.14	0.56
		price	0.00	0.99	1.98	0.00	0.00	0.00	0.00	0.00	0.00	-0.55	-0.28	-0.11	0.16	0.55
	Zn	**supply**	-0.01	1.00	2.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.57	-0.22	-0.04	0.15	0.54
		demand	0.01	1.00	2.00	0.01	0.01	0.01	0.01	0.01	0.01	-0.61	-0.20	0.03	0.21	0.66

Scenario values for the input variables of the robustness analysis for the scenario values for the enlarged sample period from 1995 to 2019

		Mean	Shock	Extr.	Foc. EA	Foc. FX	Foc. FFR	Foc. Extr. EA	Foc. Extr. FX	Foc. Extr. FFR	Q. 25%	Q. 40%	Q. 50%	Q. 60%	Q. 75%
MS-GVAR	**price**	0.01	1.02	2.03	0.01	0.01	0.01	0.01	0.01	0.01	-0.64	-0.29	0.05	0.30	0.65
exog	IP	-0.00	1.02	2.04	-0.00	-0.00	1.02	-0.00	-0.00	2.04	-0.23	-0.04	0.06	0.11	0.36
	FX	0.01	1.01	2.01	0.01	1.01	0.01	0.01	2.01	0.01	-0.61	-0.17	0.02	0.23	0.62
	FFR	0.01	1.01	2.01	1.01	0.01	0.01	2.01	0.01	0.01	-0.62	-0.27	-0.11	0.23	0.73

This table displays the scenario values of the (potential) input variables under the different scenarios Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%). Hereby, the endogenous as well as exogenous variables of the annual (monthly) (MS-)GVAR model as well as the commodity-specific determinants of the logistic regression model are displayed. In particular, we report the scenario values of supply (**supply**), demand (**demand**), and price (**price**) of the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), as well as U.S. industrial production (U.S. IP), world industrial production (IP), world gross domestic product (GDP), world gross domestic product per capita (GDPc), U.S. dollar index (FX), Federal Funds Effective Rate (FFR), 10-year U.S. Treasury rate (LIR), U.S. consumer price index (CPI), MSCI world stock index (MSCI), West Texas Intermediate spot crude oil price (OIL), and global natural disasters (ND). Hereby, the values are derived under the robustness test for the scenario values, they are calculated using data in the period from 1995 to 2019.

Table D.50: Probability of scarcity per commodity derived from the GVAR models of the robustness analysis for the scenario values for the enlarged sample period from 1995 to 2019

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
$REMod-REF$	2019	Mean	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03
		Shock	1.00	1.00	1.00	1.00	0.96	1.00	1.00	1.00	0.90	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.00	0.44	0.22	0.51	0.00	0.62	0.72	0.80	0.00	0.87	1.00
		Foc. FX	0.00	0.02	0.01	0.05	0.00	0.39	0.22	0.31	0.00	0.18	0.98
		Foc. FFR	0.00	0.02	0.03	0.00	0.00	0.68	0.02	0.18	0.00	0.03	0.56
		Foc. Extr. EA	0.30	0.93	0.83	0.96	0.04	0.94	0.97	0.98	0.14	0.99	1.00
		Foc. Extr. FX	0.07	0.47	0.32	0.76	0.01	0.80	0.86	0.81	0.00	0.81	1.00
		Foc. Extr. FFR	0.07	0.23	0.26	0.11	0.01	0.94	0.34	0.54	0.00	0.30	0.86
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
		Q. 60%	0.00	0.24	0.12	0.09	0.00	0.97	0.44	0.82	0.00	0.44	1.00
		Q. 75%	0.89	1.00	1.00	1.00	0.41	1.00	1.00	1.00	0.27	1.00	1.00
	Mean	Mean	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Shock	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.21	0.83	0.52	0.91	0.14	0.00	0.90	0.98	0.20	0.99	0.92
		Foc. FX	0.04	0.16	0.07	0.42	0.14	0.00	0.48	0.62	0.01	0.55	0.77
		Foc. FFR	0.06	0.12	0.08	0.05	0.11	0.00	0.07	0.36	0.02	0.10	0.20
		Foc. Extr. EA	0.78	0.99	0.93	0.99	0.69	0.16	0.99	1.00	0.87	1.00	0.99
		Foc. Extr. FX	0.48	0.72	0.52	0.94	0.75	0.08	0.92	0.90	0.19	0.93	0.97
		Foc. Extr. FFR	0.41	0.47	0.43	0.29	0.50	0.16	0.49	0.71	0.16	0.53	0.61
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 60%	0.36	0.81	0.38	0.63	0.32	0.02	0.81	0.99	0.05	0.96	0.95
		Q. 75%	1.00	1.00	1.00	1.00	1.00	0.90	1.00	1.00	1.00	1.00	1.00
$REMod-SUF$	2019	Mean	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03
		Shock	1.00	1.00	1.00	1.00	0.96	1.00	1.00	1.00	0.90	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.00	0.43	0.22	0.50	0.00	0.56	0.72	0.81	0.00	0.87	1.00
		Foc. FX	0.00	0.01	0.01	0.05	0.00	0.33	0.16	0.30	0.00	0.20	0.98
		Foc. FFR	0.00	0.01	0.02	0.00	0.00	0.69	0.01	0.17	0.00	0.02	0.56
		Foc. Extr. EA	0.29	0.93	0.83	0.96	0.04	0.92	0.97	0.98	0.15	0.99	1.00
		Foc. Extr. FX	0.06	0.44	0.31	0.76	0.01	0.75	0.87	0.81	0.00	0.84	1.00

Probability of scarcity per commodity derived from the GVAR models of the robustness analysis for the scenario values for the enlarged sample period from 1995 to 2019

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
REMod – SUF	2019	Foc. Extr. FFR	0.06	0.21	0.21	0.12	0.01	0.96	0.31	0.53	0.00	0.30	0.86
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02
		Q. 60%	0.00	0.15	0.06	0.06	0.00	0.94	0.29	0.78	0.00	0.40	1.00
		Q. 75%	0.84	1.00	1.00	1.00	0.33	1.00	1.00	1.00	0.25	1.00	1.00
	Mean	Mean	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Shock	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.19	0.83	0.52	0.92	0.13	0.00	0.89	0.98	0.20	0.99	0.92
		Foc. FX	0.04	0.14	0.05	0.41	0.15	0.00	0.44	0.61	0.01	0.58	0.77
		Foc. FFR	0.04	0.10	0.06	0.04	0.10	0.00	0.04	0.34	0.02	0.09	0.20
		Foc. Extr. EA	0.77	0.99	0.93	1.00	0.69	0.11	0.99	1.00	0.88	1.00	0.99
		Foc. Extr. FX	0.46	0.71	0.50	0.93	0.77	0.04	0.94	0.91	0.20	0.94	0.97
		Foc. Extr. FFR	0.39	0.45	0.41	0.30	0.47	0.13	0.44	0.70	0.16	0.50	0.61
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 60%	0.29	0.73	0.28	0.59	0.27	0.01	0.71	0.99	0.05	0.94	0.93
		Q. 75%	1.00	1.00	1.00	1.00	1.00	0.80	1.00	1.00	1.00	1.00	1.00
REMod – PER	2019	Mean	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03
		Shock	1.00	1.00	1.00	1.00	0.95	1.00	1.00	1.00	0.91	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.00	0.44	0.21	0.52	0.00	0.62	0.73	0.81	0.00	0.87	1.00
		Foc. FX	0.00	0.02	0.01	0.06	0.00	0.38	0.21	0.34	0.00	0.21	0.99
		Foc. FFR	0.00	0.03	0.02	0.01	0.00	0.72	0.01	0.20	0.00	0.03	0.56
		Foc. Extr. EA	0.32	0.93	0.83	0.96	0.03	0.94	0.98	0.98	0.15	1.00	1.00
		Foc. Extr. FX	0.08	0.48	0.33	0.78	0.01	0.78	0.88	0.82	0.00	0.82	1.00
		Foc. Extr. FFR	0.09	0.25	0.28	0.13	0.01	0.95	0.33	0.56	0.00	0.31	0.87
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
		Q. 60%	0.02	0.25	0.10	0.09	0.00	0.97	0.38	0.85	0.00	0.45	1.00
		Q. 75%	0.92	1.00	1.00	1.00	0.35	1.00	1.00	1.00	0.27	1.00	1.00
	Mean	Mean	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Shock	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.24	0.83	0.52	0.91	0.12	0.00	0.90	0.98	0.20	0.99	0.92
		Foc. FX	0.06	0.17	0.06	0.42	0.16	0.00	0.49	0.66	0.01	0.57	0.76
		Foc. FFR	0.08	0.13	0.08	0.05	0.11	0.00	0.05	0.39	0.02	0.10	0.21
		Foc. Extr. EA	0.79	0.99	0.94	0.99	0.68	0.15	0.99	1.00	0.87	1.00	0.99
		Foc. Extr. FX	0.51	0.73	0.53	0.93	0.77	0.07	0.94	0.92	0.19	0.94	0.97
		Foc. Extr. FFR	0.44	0.50	0.46	0.32	0.49	0.15	0.48	0.72	0.17	0.53	0.60
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 60%	0.45	0.82	0.37	0.64	0.28	0.02	0.78	0.99	0.05	0.95	0.94
		Q. 75%	1.00	1.00	1.00	1.00	1.00	0.88	1.00	1.00	1.00	1.00	1.00
REMod – UNA	2019	Mean	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02
		Shock	1.00	1.00	1.00	1.00	0.95	1.00	1.00	1.00	0.88	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.00	0.42	0.22	0.50	0.00	0.56	0.73	0.80	0.00	0.86	1.00
		Foc. FX	0.00	0.01	0.01	0.05	0.00	0.34	0.19	0.31	0.00	0.18	0.98
		Foc. FFR	0.00	0.02	0.03	0.01	0.00	0.71	0.01	0.17	0.00	0.02	0.54
		Foc. Extr. EA	0.31	0.92	0.84	0.96	0.04	0.92	0.97	0.97	0.15	0.99	1.00
		Foc. Extr. FX	0.07	0.45	0.31	0.77	0.01	0.74	0.88	0.82	0.00	0.82	1.00
		Foc. Extr. FFR	0.07	0.21	0.26	0.12	0.01	0.95	0.33	0.54	0.00	0.30	0.85
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
		Q. 60%	0.00	0.18	0.08	0.07	0.00	0.94	0.33	0.79	0.00	0.40	1.00
		Q. 75%	0.86	1.00	1.00	1.00	0.32	1.00	1.00	1.00	0.22	1.00	1.00
	Mean	Mean	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Shock	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.22	0.83	0.52	0.92	0.13	0.00	0.90	0.98	0.20	0.99	0.93
		Foc. FX	0.04	0.14	0.06	0.40	0.16	0.00	0.46	0.62	0.01	0.55	0.76
		Foc. FFR	0.05	0.09	0.08	0.04	0.10	0.00	0.04	0.35	0.02	0.09	0.18
		Foc. Extr. EA	0.79	0.99	0.94	1.00	0.69	0.11	0.99	1.00	0.88	1.00	0.99
		Foc. Extr. FX	0.48	0.72	0.52	0.94	0.78	0.05	0.94	0.91	0.17	0.94	0.97

Probability of scarcity per commodity derived from the GVAR models of the robustness analysis for the scenario values for the enlarged sample period from 1995 to 2019

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
$REMod-UNA$		Foc. Extr. FFR	0.40	0.45	0.44	0.30	0.47	0.14	0.46	0.72	0.15	0.52	0.58
	Mean	Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 60%	0.32	0.74	0.32	0.59	0.25	0.01	0.74	0.99	0.04	0.92	0.92
		Q. 75%	1.00	1.00	1.00	1.00	1.00	0.82	1.00	1.00	1.00	1.00	1.00

This table displays the probability of scarcity (PS) of the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), derived from the GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). Hereby, the probability of scarcity is calculated for the scenarios Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%) of the input variables. The presented results are derived under the robustness test for the scenario values, in particular, the scenario values are calculated based on data in the period from 1995 to 2019.

Table D.51: Probability of scarcity per commodity derived from the MS-GVAR models of the robustness analysis for the scenario values for the enlarged sample period from 1995 to 2019

			Al	Cu	Ni	Pb	Sn	Zn
$REMod-REF$	2019	Mean	0.00	0.00	0.00	0.02	0.00	0.46
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.13	0.00	0.15	0.43	0.20	0.98
		Foc. FX	0.78	0.19	0.49	0.72	0.32	0.98
		Foc. FFR	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. Extr. EA	0.72	0.14	0.65	0.80	0.66	1.00
		Foc. Extr. FX	0.99	0.74	0.88	0.93	0.75	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.08	0.00	0.83
		Q. 60%	1.00	0.51	0.89	1.00	0.63	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
	Mean	Mean	0.00	0.00	0.00	0.13	0.06	0.01
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.69	0.16	0.42	0.80	0.67	0.47
		Foc. FX	0.98	0.71	0.78	0.92	0.77	0.62
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	0.95	0.60	0.83	0.95	0.91	0.84
		Foc. Extr. FX	1.00	0.93	0.96	0.99	0.95	0.88
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.05	0.04	0.02	0.36	0.09	0.04
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
$REMod-SUF$	2019	Mean	0.00	0.00	0.00	0.02	0.00	0.48
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.13	0.00	0.18	0.45	0.20	0.99
		Foc. FX	0.63	0.20	0.41	0.82	0.36	0.99
		Foc. FFR	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. Extr. EA	0.79	0.17	0.71	0.80	0.65	1.00
		Foc. Extr. FX	0.98	0.72	0.83	0.96	0.76	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.10	0.00	0.89
		Q. 60%	1.00	0.49	0.85	1.00	0.62	1.00

Probability of scarcity per commodity derived from the MS-GVAR models of the robustness analysis for the scenario values for the enlarged sample period from 1995 to 2019

			Al	Cu	Ni	Pb	Sn	Zn
REMod – SUF		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
	Mean	Mean	0.00	0.01	0.00	0.13	0.04	0.01
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.74	0.17	0.48	0.81	0.67	0.56
		Foc. FX	0.97	0.69	0.72	0.95	0.76	0.70
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	0.97	0.58	0.86	0.95	0.91	0.88
		Foc. Extr. FX	1.00	0.93	0.93	1.00	0.92	0.90
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.06	0.05	0.01	0.35	0.10	0.04
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
REMod – PER	2019	Mean	0.00	0.00	0.00	0.02	0.00	0.45
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.12	0.00	0.15	0.42	0.19	0.98
		Foc. FX	0.79	0.21	0.50	0.72	0.38	0.99
		Foc. FFR	1.00	1.00	1.00	1.00	0.98	1.00
		Foc. Extr. EA	0.72	0.14	0.65	0.80	0.66	1.00
		Foc. Extr. FX	0.98	0.75	0.89	0.93	0.78	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.09	0.00	0.86
		Q. 60%	0.99	0.54	0.90	1.00	0.65	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
	Mean	Mean	0.00	0.01	0.00	0.13	0.05	0.01
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.69	0.15	0.41	0.80	0.69	0.51
		Foc. FX	0.98	0.73	0.79	0.92	0.80	0.67
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	0.95	0.57	0.84	0.95	0.91	0.86
		Foc. Extr. FX	1.00	0.93	0.96	0.98	0.93	0.91
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.05	0.04	0.02	0.35	0.09	0.05
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
REMod – UNA	2019	Mean	0.00	0.00	0.00	0.02	0.00	0.44
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.11	0.00	0.16	0.44	0.18	0.99
		Foc. FX	0.77	0.18	0.50	0.75	0.34	0.99
		Foc. FFR	1.00	1.00	1.00	1.00	0.99	1.00
		Foc. Extr. EA	0.75	0.15	0.66	0.80	0.59	1.00
		Foc. Extr. FX	0.99	0.73	0.90	0.94	0.74	1.00
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.00	0.00	0.00	0.08	0.01	0.87
		Q. 60%	1.00	0.47	0.90	1.00	0.61	1.00
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
	Mean	Mean	0.00	0.00	0.00	0.12	0.05	0.01
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.70	0.17	0.44	0.79	0.63	0.49
		Foc. FX	0.98	0.70	0.79	0.93	0.75	0.61
		Foc. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. EA	0.96	0.57	0.83	0.95	0.88	0.85
		Foc. Extr. FX	1.00	0.91	0.96	0.99	0.93	0.90
		Foc. Extr. FFR	1.00	1.00	1.00	1.00	1.00	1.00
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.05	0.05	0.01	0.34	0.11	0.04
		Q. 60%	1.00	1.00	1.00	1.00	1.00	1.00

Probability of scarcity per commodity derived from the MS-GVAR models of the robustness analysis for the scenario values for the enlarged sample period from 1995 to 2019

	Al	Cu	Ni	Pb	Sn	Zn
Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00

This table displays the probability of scarcity (PS) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), derived from the MS-GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). Hereby, the probability of scarcity is calculated for the scenarios Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%) of the input variables. The presented results are derived under the robustness test for the scenario values, in particular, the scenario values are calculated based on data in the period from 1995 to 2019.

Table D.52: Probability of scarcity per commodity derived from the logistic regression models of the robustness analysis for the scenario values for the enlarged sample period from 1995 to 2019

	Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
Mean	0.04	0.06	0.08	0.04	0.01	0.10	0.04	0.01	0.07	0.05	0.03	0.02	0.01
Shock	0.05	0.14	0.41	0.06	0.06	0.16	0.05	0.06	0.31	0.12	0.07	0.03	0.53
Extr.	0.07	0.28	0.84	0.09	0.34	0.25	0.06	0.34	0.74	0.24	0.15	0.06	0.99
Foc. EA	0.04	0.06	0.08	0.04	0.02	0.10	0.04	0.02	0.07	0.05	0.03	0.02	0.01
Foc. FX	0.04	0.06	0.08	0.04	0.01	0.13	0.04	0.01	0.07	0.08	0.04	0.02	0.01
Foc. FFR	0.04	0.06	0.08	0.05	0.02	0.10	0.04	0.02	0.07	0.05	0.03	0.02	0.11
Foc. Extr. EA	0.04	0.06	0.08	0.04	0.03	0.10	0.04	0.03	0.07	0.05	0.03	0.02	0.01
Foc. Extr. FX	0.04	0.06	0.08	0.04	0.01	0.16	0.04	0.01	0.07	0.12	0.06	0.02	0.01
Foc. Extr. FFR	0.04	0.06	0.08	0.07	0.08	0.10	0.04	0.08	0.07	0.05	0.03	0.02	0.53
Q. 25%	0.03	0.04	0.02	0.03	0.00	0.08	0.04	0.00	0.03	0.03	0.02	0.01	0.00
Q. 40%	0.04	0.05	0.04	0.04	0.01	0.10	0.04	0.01	0.05	0.04	0.03	0.02	0.01
Q. 50%	0.04	0.06	0.08	0.04	0.01	0.11	0.04	0.01	0.07	0.05	0.04	0.02	0.02
Q. 60%	0.04	0.09	0.12	0.05	0.01	0.12	0.04	0.01	0.11	0.06	0.04	0.02	0.04
Q. 75%	0.05	0.13	0.32	0.05	0.03	0.14	0.04	0.03	0.17	0.09	0.06	0.03	0.14

This table displays the probability of scarcity (PS) of the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), derived from the logistic regression models based on preselected covariates. Hereby, the probability of scarcity is calculated for the scenarios Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%) of the input variables. The presented results are derived under the robustness test for the scenario values, in particular, the scenario values are calculated based on data in the period from 1995 to 2019.

Table D.53: Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the GVAR models of the robustness analysis for the scenario values for the enlarged sample period from 1995 to 2019

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
Mean	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.19
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.19
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.15
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20
	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Shock	2019	$REMod-REF$	9.17	8.36	304.32	27.27	126.24	25.23	87.36	1.38	0.48	9.12	7.21
		$REMod-SUF$	6.19	5.67	204.60	19.71	92.44	16.96	59.05	0.94	0.32	6.21	7.33
		$REMod-PER$	10.44	7.34	219.38	23.16	129.34	18.20	69.11	1.36	0.60	8.77	5.91
		$REMod-UNA$	14.62	9.36	311.47	25.80	192.86	25.82	87.45	0.78	0.34	6.72	7.66
	Mean	$REMod-REF$	9.17	8.36	304.32	27.27	131.77	25.23	87.36	1.38	0.53	9.12	7.21
		$REMod-SUF$	6.19	5.67	204.60	19.71	96.49	16.96	59.05	0.94	0.35	6.21	7.33
		$REMod-PER$	10.44	7.34	219.38	23.16	135.01	18.20	69.11	1.36	0.67	8.77	5.91
		$REMod-UNA$	14.62	9.36	311.47	25.80	201.32	25.82	87.45	0.78	0.38	6.72	7.66

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the GVAR models of the robustness analysis for the scenario values for the enlarged sample period from 1995 to 2019

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
Extr.	2019	$REMod-REF$	9.17	8.36	304.32	27.27	131.77	25.23	87.36	1.38	0.53	9.12	7.21
		$REMod-SUF$	6.19	5.67	204.60	19.71	96.49	16.96	59.05	0.94	0.35	6.21	7.33
		$REMod-PER$	10.44	7.34	219.38	23.16	135.01	18.20	69.11	1.36	0.67	8.77	5.91
		$REMod-UNA$	14.62	9.36	311.47	25.80	201.32	25.82	87.45	0.78	0.38	6.72	7.66
	Mean	$REMod-REF$	9.17	8.36	304.32	27.27	131.77	25.23	87.36	1.38	0.53	9.12	7.21
		$REMod-SUF$	6.19	5.67	204.60	19.71	96.49	16.96	59.05	0.94	0.35	6.21	7.33
		$REMod-PER$	10.44	7.34	219.38	23.16	135.01	18.20	69.11	1.36	0.67	8.77	5.91
		$REMod-UNA$	14.62	9.36	311.47	25.80	201.32	25.82	87.45	0.78	0.38	6.72	7.66
Foc. EA	2019	$REMod-REF$	0.03	3.65	66.95	13.85	0.00	15.72	63.16	1.11	0.00	7.90	7.20
		$REMod-SUF$	0.02	2.48	45.01	10.01	0.00	10.57	42.69	0.76	0.00	5.38	7.32
		$REMod-PER$	0.03	3.21	48.26	11.76	0.00	11.34	49.96	1.09	0.00	7.60	5.90
		$REMod-UNA$	0.04	4.09	68.52	13.11	0.00	16.08	63.22	0.63	0.00	5.83	7.65
	Mean	$REMod-REF$	1.96	6.97	159.16	24.79	18.05	0.05	78.80	1.35	0.11	9.00	6.65
		$REMod-SUF$	1.32	4.73	107.00	17.92	13.22	0.03	53.26	0.92	0.07	6.13	6.76
		$REMod-PER$	2.24	6.12	114.74	21.05	18.50	0.04	62.33	1.33	0.14	8.65	5.45
		$REMod-UNA$	3.13	7.81	162.90	23.45	27.58	0.05	78.88	0.76	0.08	6.64	7.06
Foc. FX	2019	$REMod-REF$	0.00	0.14	3.96	1.47	0.00	9.92	18.87	0.43	0.00	1.66	7.09
		$REMod-SUF$	0.00	0.10	2.66	1.06	0.00	6.67	12.75	0.29	0.00	1.13	7.21
		$REMod-PER$	0.00	0.12	2.85	1.25	0.00	7.15	14.93	0.42	0.00	1.60	5.81
		$REMod-UNA$	0.00	0.16	4.05	1.39	0.00	10.15	18.89	0.24	0.00	1.22	7.53
	Mean	$REMod-REF$	0.38	1.37	20.09	11.48	18.45	0.10	41.58	0.85	0.01	4.99	5.57
		$REMod-SUF$	0.25	0.93	13.50	8.30	13.51	0.07	28.11	0.58	0.00	3.40	5.66
		$REMod-PER$	0.43	1.20	14.48	9.75	18.90	0.07	32.89	0.84	0.01	4.79	4.56
		$REMod-UNA$	0.60	1.54	20.56	10.86	28.18	0.10	41.62	0.48	0.01	3.68	5.91
Foc. FFR	2019	$REMod-REF$	0.01	0.18	8.83	0.11	0.00	17.13	1.57	0.26	0.00	0.24	4.07
		$REMod-SUF$	0.01	0.12	5.93	0.08	0.00	11.52	1.06	0.17	0.00	0.16	4.13
		$REMod-PER$	0.01	0.15	6.36	0.09	0.00	12.36	1.24	0.25	0.00	0.23	3.33
		$REMod-UNA$	0.01	0.20	9.03	0.10	0.00	17.53	1.57	0.14	0.00	0.17	4.32
	Mean	$REMod-REF$	0.50	0.99	25.26	1.25	14.89	0.03	5.77	0.49	0.01	0.91	1.44
		$REMod-SUF$	0.34	0.67	16.98	0.91	10.90	0.02	3.90	0.34	0.01	0.62	1.47
		$REMod-PER$	0.57	0.87	18.21	1.07	15.26	0.02	4.56	0.49	0.01	0.88	1.18
		$REMod-UNA$	0.80	1.11	25.85	1.19	22.75	0.03	5.77	0.28	0.01	0.67	1.53
Foc. Extr. EA	2019	$REMod-REF$	2.76	7.77	252.89	26.12	4.61	23.82	85.09	1.35	0.08	9.04	7.21
		$REMod-SUF$	1.86	5.27	170.02	18.88	3.38	16.01	57.51	0.92	0.05	6.16	7.33
		$REMod-PER$	3.14	6.82	182.31	22.19	4.73	17.18	67.31	1.33	0.10	8.70	5.91
		$REMod-UNA$	4.40	8.70	258.83	24.72	7.05	24.37	85.17	0.76	0.06	6.67	7.66
	Mean	$REMod-REF$	7.14	8.29	284.24	27.13	90.92	4.11	86.57	1.38	0.47	9.12	7.14
		$REMod-SUF$	4.82	5.62	191.09	19.61	66.58	2.77	58.52	0.94	0.31	6.21	7.26
		$REMod-PER$	8.14	7.28	204.90	23.04	93.16	2.97	68.48	1.36	0.58	8.77	5.86
		$REMod-UNA$	11.39	9.29	290.91	25.67	138.91	4.21	86.66	0.78	0.33	6.72	7.59
Foc. Extr. FX	2019	$REMod-REF$	0.69	3.91	97.38	20.83	1.32	20.14	75.30	1.13	0.00	7.37	7.19
		$REMod-SUF$	0.46	2.65	65.47	15.06	0.96	13.54	50.90	0.77	0.00	5.02	7.32
		$REMod-PER$	0.78	3.43	70.20	17.69	1.35	14.52	59.57	1.11	0.00	7.09	5.90
		$REMod-UNA$	1.10	4.38	99.67	19.71	2.01	20.60	75.38	0.63	0.00	5.44	7.64
	Mean	$REMod-REF$	4.40	5.99	157.94	25.52	98.70	2.09	80.63	1.25	0.10	8.48	6.97
		$REMod-SUF$	2.97	4.06	106.19	18.45	72.27	1.41	54.50	0.85	0.07	5.78	7.09
		$REMod-PER$	5.01	5.26	113.86	21.68	101.12	1.51	63.79	1.22	0.13	8.15	5.72
		$REMod-UNA$	7.02	6.71	161.65	24.15	150.79	2.14	80.71	0.70	0.07	6.25	7.40
Foc. Extr. FFR	2019	$REMod-REF$	0.66	1.94	79.43	3.11	1.71	23.72	29.96	0.75	0.00	2.73	6.19
		$REMod-SUF$	0.45	1.32	53.40	2.25	1.25	15.95	20.25	0.51	0.00	1.86	6.29
		$REMod-PER$	0.75	1.70	57.26	2.64	1.76	17.10	23.70	0.73	0.00	2.63	5.07
		$REMod-UNA$	1.05	2.17	81.29	2.94	2.62	24.27	29.99	0.42	0.00	2.02	6.57
	Mean	$REMod-REF$	3.74	3.90	131.16	7.91	65.62	3.96	42.54	0.98	0.08	4.82	4.41
		$REMod-SUF$	2.53	2.65	88.18	5.72	48.05	2.66	28.76	0.66	0.06	3.29	4.49
		$REMod-PER$	4.26	3.43	94.55	6.72	67.24	2.86	33.66	0.96	0.11	4.64	3.62
		$REMod-UNA$	5.96	4.37	134.24	7.48	100.26	4.05	42.59	0.55	0.06	3.56	4.69
Q. 25%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Q. 40%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the GVAR models of the robustness analysis for the scenario values for the enlarged sample period from 1995 to 2019

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Q. 50%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10
	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Q. 60%	2019	$REMod-REF$	0.03	2.00	38.04	2.40	0.00	24.50	38.35	1.14	0.00	4.04	7.21
		$REMod-SUF$	0.02	1.35	25.57	1.73	0.00	16.47	25.92	0.78	0.00	2.75	7.33
		$REMod-PER$	0.03	1.75	27.42	2.04	0.00	17.67	30.34	1.12	0.00	3.88	5.91
		$REMod-UNA$	0.04	2.24	38.93	2.27	0.00	25.07	38.39	0.64	0.00	2.98	7.66
	Mean	$REMod-REF$	3.29	6.81	115.03	17.10	42.56	0.58	71.02	1.37	0.03	8.71	6.86
		$REMod-SUF$	2.22	4.61	77.34	12.36	31.17	0.39	48.01	0.93	0.02	5.94	6.98
		$REMod-PER$	3.75	5.97	82.93	14.52	43.61	0.42	56.18	1.35	0.03	8.38	5.63
		$REMod-UNA$	5.25	7.62	117.74	16.18	65.03	0.59	71.09	0.77	0.02	6.43	7.29
Q. 75%	2019	$REMod-REF$	8.19	8.36	304.02	27.27	54.29	25.23	87.36	1.38	0.14	9.12	7.21
		$REMod-SUF$	5.53	5.67	204.39	19.71	39.75	16.96	59.05	0.94	0.09	6.21	7.33
		$REMod-PER$	9.33	7.34	219.16	23.16	55.63	18.20	69.11	1.36	0.18	8.77	5.91
		$REMod-UNA$	13.06	9.36	311.16	25.80	82.94	25.82	87.45	0.78	0.10	6.72	7.66
	Mean	$REMod-REF$	9.17	8.36	304.32	27.27	131.77	22.58	87.36	1.38	0.53	9.12	7.21
		$REMod-SUF$	6.19	5.67	204.60	19.71	96.49	15.18	59.05	0.94	0.35	6.21	7.33
		$REMod-PER$	10.44	7.34	219.38	23.16	135.01	16.29	69.11	1.36	0.67	8.77	5.91
		$REMod-UNA$	14.62	9.36	311.47	25.80	201.32	23.11	87.45	0.78	0.38	6.72	7.66

This table displays the expected loss due to scarcity for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), per path ($REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$), as well as per scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) for the input variables. Hereby, the values are derived from the GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). Hereby, the results are derived under the robustness test for the scenario values, in particular, the scenario values are calculated based on data in the period from 1995 to 2019.

Table D.54: Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the MS-GVAR models of the robustness analysis for the scenario values for the enlarged sample period from 1995 to 2019

			Al	Cu	Ni	Pb	Sn	Zn
Mean	2019	$REMod-REF$	0.00	0.00	0.00	0.03	0.00	3.33
		$REMod-SUF$	0.00	0.00	0.00	0.02	0.00	3.39
		$REMod-PER$	0.00	0.00	0.00	0.03	0.00	2.73
		$REMod-UNA$	0.00	0.00	0.00	0.02	0.00	3.54
	Mean	$REMod-REF$	0.02	0.11	0.17	0.18	0.51	0.04
		$REMod-SUF$	0.01	0.08	0.12	0.12	0.35	0.04
		$REMod-PER$	0.01	0.09	0.14	0.17	0.49	0.04
		$REMod-UNA$	0.02	0.10	0.17	0.10	0.38	0.05
Shock	2019	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
Extr.	2019	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
		$REMod-REF$	1.07	0.05	13.28	0.60	1.84	7.06

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the MS-GVAR models of the robustness analysis for the scenario values for the enlarged sample period from 1995 to 2019

			Al	Cu	Ni	Pb	Sn	Zn
Foc. EA	2019	$REMod-SUF$	0.73	0.04	8.98	0.41	1.25	7.18
		$REMod-PER$	0.94	0.05	10.50	0.59	1.77	5.79
		$REMod-UNA$	1.20	0.05	13.29	0.34	1.36	7.50
	Mean	$REMod-REF$	5.74	4.42	36.34	1.10	6.09	3.42
		$REMod-SUF$	3.89	3.19	24.56	0.75	4.15	3.47
		$REMod-PER$	5.04	3.75	28.75	1.08	5.86	2.80
		$REMod-UNA$	6.42	4.18	36.38	0.62	4.49	3.63
Foc. FX	2019	$REMod-REF$	6.49	5.13	43.16	1.00	2.92	7.09
		$REMod-SUF$	4.40	3.71	29.17	0.68	1.99	7.21
		$REMod-PER$	5.70	4.35	34.14	0.98	2.81	5.82
		$REMod-UNA$	7.27	4.85	43.20	0.56	2.15	7.53
	Mean	$REMod-REF$	8.19	19.31	68.32	1.27	7.06	4.44
		$REMod-SUF$	5.55	13.95	46.18	0.86	4.81	4.52
		$REMod-PER$	7.19	16.40	54.04	1.25	6.78	3.64
		$REMod-UNA$	9.18	18.27	68.38	0.71	5.20	4.72
Foc. FFR	2019	$REMod-REF$	8.36	27.27	87.36	1.38	8.99	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.12	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.64	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.63	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.08	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.19	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.73	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.70	7.66
Foc. Extr. EA	2019	$REMod-REF$	6.05	3.93	56.44	1.11	5.98	7.21
		$REMod-SUF$	4.10	2.84	38.15	0.76	4.07	7.33
		$REMod-PER$	5.31	3.33	44.64	1.09	5.75	5.91
		$REMod-UNA$	6.78	3.72	56.49	0.62	4.41	7.66
	Mean	$REMod-REF$	7.96	16.25	72.33	1.31	8.30	6.06
		$REMod-SUF$	5.40	11.75	48.89	0.89	5.65	6.16
		$REMod-PER$	6.99	13.80	57.22	1.29	7.98	4.97
		$REMod-UNA$	8.91	15.38	72.41	0.74	6.12	6.43
Foc. Extr. FX	2019	$REMod-REF$	8.28	20.29	76.53	1.28	6.80	7.19
		$REMod-SUF$	5.61	14.66	51.73	0.87	4.63	7.32
		$REMod-PER$	7.27	17.23	60.54	1.26	6.54	5.90
		$REMod-UNA$	9.27	19.20	76.60	0.72	5.02	7.64
	Mean	$REMod-REF$	8.34	25.30	83.69	1.36	8.62	6.34
		$REMod-SUF$	5.66	18.29	56.57	0.93	5.87	6.45
		$REMod-PER$	7.33	21.49	66.20	1.34	8.29	5.20
		$REMod-UNA$	9.34	23.94	83.77	0.77	6.36	6.74
Foc. Extr. FFR	2019	$REMod-REF$	8.36	27.27	87.36	1.38	9.08	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.19	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.73	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.70	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
Q. 25%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00
	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00
Q. 40%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00
	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00
Q. 50%	2019	$REMod-REF$	0.00	0.00	0.00	0.11	0.04	5.95
		$REMod-SUF$	0.00	0.00	0.00	0.07	0.02	6.06
		$REMod-PER$	0.00	0.00	0.00	0.11	0.04	4.88
		$REMod-UNA$	0.00	0.00	0.00	0.06	0.03	6.32
	Mean	$REMod-REF$	0.43	1.09	1.57	0.49	0.78	0.30
		$REMod-SUF$	0.29	0.79	1.06	0.33	0.53	0.31
		$REMod-PER$	0.38	0.93	1.24	0.48	0.75	0.25

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the MS-GVAR models of the robustness analysis for the scenario values for the enlarged sample period from 1995 to 2019

			Al	Cu	Ni	Pb	Sn	Zn
		$REMod - UNA$	0.49	1.03	1.57	0.28	0.58	0.32
Q. 60%	2019	$REMod - REF$	8.33	13.80	77.75	1.38	5.76	7.21
		$REMod - SUF$	5.65	9.97	52.55	0.94	3.92	7.33
		$REMod - PER$	7.31	11.72	61.51	1.36	5.54	5.91
		$REMod - UNA$	9.33	13.06	77.83	0.78	4.25	7.66
	Mean	$REMod - REF$	8.36	27.27	87.19	1.38	9.12	7.18
		$REMod - SUF$	5.67	19.71	58.93	0.94	6.21	7.30
		$REMod - PER$	7.34	23.16	68.97	1.36	8.77	5.89
		$REMod - UNA$	9.36	25.80	87.27	0.78	6.72	7.63
Q. 75%	2019	$REMod - REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod - SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod - PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod - UNA$	9.36	25.80	87.45	0.78	6.72	7.66
	Mean	$REMod - REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod - SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod - PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod - UNA$	9.36	25.80	87.45	0.78	6.72	7.66

This table displays the expected loss due to scarcity for the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), per path ($REMod - REF$, $REMod - SUF$, $REMod - PER$ and $REMod - UNA$), as well as per scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) for the input variables. Hereby, the values are derived from the MS-GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod - REF$, $REMod - SUF$, $REMod - PER$ and $REMod - UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). Hereby, the results are derived under the robustness test for the scenario values, in particular, the scenario values are calculated based on data in the period from 1995 to 2019.

Table D.55: Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the logistic regression models of the robustness analysis for the scenario values for the enlarged sample period from 1995 to 2019

		Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
Mean	$REMod - REF$	0.35	0.54	24.58	1.08	1.02	13.32	1.02	0.13	5.87	0.07	0.02	0.19	0.09
	$REMod - SUF$	0.23	0.36	16.52	0.78	0.73	9.76	0.69	0.10	3.97	0.05	0.01	0.13	0.09
	$REMod - PER$	0.40	0.47	17.72	0.92	0.99	13.65	0.74	0.12	4.65	0.07	0.02	0.18	0.07
	$REMod - UNA$	0.55	0.60	25.16	1.03	0.49	20.35	1.05	0.06	5.88	0.04	0.01	0.14	0.09
Shock	$REMod - REF$	0.47	1.17	124.03	1.65	8.15	21.47	1.26	1.06	27.41	0.16	0.04	0.31	3.79
	$REMod - SUF$	0.32	0.79	83.39	1.20	5.82	15.72	0.85	0.76	18.53	0.11	0.03	0.21	3.85
	$REMod - PER$	0.54	1.03	89.41	1.41	7.85	22.00	0.91	0.98	21.68	0.16	0.05	0.30	3.10
	$REMod - UNA$	0.75	1.31	126.95	1.57	3.88	32.80	1.29	0.46	27.43	0.09	0.03	0.23	4.02
Extr.	$REMod - REF$	0.63	2.33	256.67	2.50	47.93	33.20	1.54	6.26	64.96	0.34	0.08	0.53	7.14
	$REMod - SUF$	0.43	1.58	172.56	1.81	34.26	24.31	1.04	4.45	43.91	0.23	0.05	0.36	7.26
	$REMod - PER$	0.72	2.05	185.03	2.12	46.15	34.02	1.11	5.79	51.38	0.33	0.10	0.51	5.85
	$REMod - UNA$	1.01	2.61	262.70	2.36	22.85	50.72	1.58	2.68	65.02	0.19	0.06	0.39	7.58
Foc. EA	$REMod - REF$	0.35	0.54	24.58	1.08	2.25	13.32	1.02	0.29	5.87	0.07	0.02	0.19	0.09
	$REMod - SUF$	0.23	0.36	16.52	0.78	1.61	9.76	0.69	0.21	3.97	0.05	0.01	0.13	0.09
	$REMod - PER$	0.40	0.47	17.72	0.92	2.16	13.65	0.74	0.27	4.65	0.07	0.02	0.18	0.07
	$REMod - UNA$	0.55	0.60	25.16	1.03	1.07	20.35	1.05	0.13	5.88	0.04	0.01	0.14	0.09
Foc. FX	$REMod - REF$	0.35	0.54	24.58	1.11	1.15	16.78	1.02	0.15	5.87	0.11	0.02	0.19	0.09
	$REMod - SUF$	0.23	0.36	16.52	0.80	0.82	12.28	0.69	0.11	3.97	0.07	0.02	0.13	0.09
	$REMod - PER$	0.40	0.47	17.72	0.94	1.11	17.19	0.74	0.14	4.65	0.11	0.03	0.18	0.07
	$REMod - UNA$	0.55	0.60	25.16	1.05	0.55	25.63	1.05	0.06	5.88	0.06	0.02	0.14	0.09
	$REMod - REF$	0.35	0.54	24.58	1.39	3.40	13.32	1.02	0.44	5.87	0.07	0.02	0.19	0.76

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the logistic regression models of the robustness analysis for the scenario values for the enlarged sample period from 1995 to 2019

		Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
Foc. FFR	$REMod - SUF$	0.23	0.36	16.52	1.01	2.43	9.76	0.69	0.32	3.97	0.05	0.01	0.13	0.77
	$REMod - PER$	0.40	0.47	17.72	1.18	3.27	13.65	0.74	0.41	4.65	0.07	0.02	0.18	0.62
	$REMod - UNA$	0.55	0.60	25.16	1.32	1.62	20.35	1.05	0.19	5.88	0.04	0.01	0.14	0.81
Foc. Extr. EA	$REMod - REF$	0.35	0.54	24.58	1.08	4.88	13.32	1.02	0.64	5.87	0.07	0.02	0.19	0.09
	$REMod - SUF$	0.23	0.36	16.52	0.78	3.49	9.76	0.69	0.45	3.97	0.05	0.01	0.13	0.09
	$REMod - PER$	0.40	0.47	17.72	0.92	4.70	13.65	0.74	0.59	4.65	0.07	0.02	0.18	0.07
	$REMod - UNA$	0.55	0.60	25.16	1.03	2.33	20.35	1.05	0.27	5.88	0.04	0.01	0.14	0.09
Foc. Extr. FX	$REMod - REF$	0.35	0.54	24.58	1.13	1.29	20.96	1.02	0.17	5.87	0.17	0.03	0.19	0.09
	$REMod - SUF$	0.23	0.36	16.52	0.82	0.92	15.35	0.69	0.12	3.97	0.11	0.02	0.13	0.09
	$REMod - PER$	0.40	0.47	17.72	0.96	1.24	21.48	0.74	0.16	4.65	0.17	0.04	0.18	0.07
	$REMod - UNA$	0.55	0.60	25.16	1.07	0.61	32.03	1.05	0.07	5.88	0.09	0.02	0.14	0.09
Foc. Extr. FFR	$REMod - REF$	0.35	0.54	24.58	1.78	10.86	13.32	1.02	1.42	5.87	0.07	0.02	0.19	3.81
	$REMod - SUF$	0.23	0.36	16.52	1.29	7.76	9.76	0.69	1.01	3.97	0.05	0.01	0.13	3.87
	$REMod - PER$	0.40	0.47	17.72	1.51	10.46	13.65	0.74	1.31	4.65	0.07	0.02	0.18	3.12
	$REMod - UNA$	0.55	0.60	25.16	1.69	5.18	20.35	1.05	0.61	5.88	0.04	0.01	0.14	4.04
Q. 25%	$REMod - REF$	0.30	0.36	7.03	0.92	0.41	10.29	0.98	0.05	2.38	0.04	0.01	0.13	0.01
	$REMod - SUF$	0.20	0.24	4.73	0.66	0.29	7.53	0.66	0.04	1.61	0.03	0.01	0.09	0.01
	$REMod - PER$	0.34	0.32	5.07	0.78	0.40	10.54	0.71	0.05	1.88	0.04	0.02	0.12	0.01
	$REMod - UNA$	0.48	0.40	7.20	0.87	0.20	15.72	1.00	0.02	2.38	0.02	0.01	0.09	0.01
Q. 40%	$REMod - REF$	0.33	0.45	13.14	1.06	0.74	12.80	1.00	0.10	4.40	0.06	0.02	0.16	0.04
	$REMod - SUF$	0.22	0.30	8.83	0.77	0.53	9.37	0.67	0.07	2.98	0.04	0.01	0.11	0.04
	$REMod - PER$	0.37	0.39	9.47	0.90	0.72	13.11	0.72	0.09	3.48	0.06	0.02	0.15	0.04
	$REMod - UNA$	0.52	0.50	13.44	1.01	0.35	19.55	1.02	0.04	4.41	0.03	0.01	0.12	0.05
Q. 50%	$REMod - REF$	0.35	0.50	23.04	1.14	1.00	14.15	1.01	0.13	6.39	0.07	0.02	0.18	0.13
	$REMod - SUF$	0.23	0.34	15.49	0.82	0.71	10.36	0.68	0.09	4.32	0.05	0.01	0.12	0.13
	$REMod - PER$	0.40	0.44	16.61	0.96	0.96	14.49	0.73	0.12	5.05	0.07	0.03	0.18	0.11
	$REMod - UNA$	0.55	0.56	23.58	1.07	0.47	21.61	1.04	0.06	6.39	0.04	0.01	0.13	0.14
Q. 60%	$REMod - REF$	0.39	0.73	37.24	1.25	1.84	15.57	1.02	0.24	9.93	0.08	0.02	0.21	0.25
	$REMod - SUF$	0.27	0.50	25.04	0.90	1.31	11.40	0.69	0.17	6.71	0.06	0.02	0.14	0.26
	$REMod - PER$	0.45	0.64	26.84	1.06	1.77	15.95	0.74	0.22	7.86	0.08	0.03	0.20	0.21
	$REMod - UNA$	0.63	0.82	38.11	1.18	0.88	23.78	1.04	0.10	9.94	0.05	0.02	0.16	0.27
Q. 75%	$REMod - REF$	0.44	1.06	96.65	1.39	3.96	18.36	1.04	0.52	14.51	0.12	0.03	0.29	1.03
	$REMod - SUF$	0.30	0.72	64.98	1.01	2.83	13.44	0.70	0.37	9.81	0.08	0.02	0.19	1.05
	$REMod - PER$	0.51	0.93	69.68	1.18	3.81	18.81	0.75	0.48	11.48	0.12	0.04	0.28	0.84
	$REMod - UNA$	0.71	1.18	98.93	1.32	1.89	28.05	1.06	0.22	14.53	0.07	0.02	0.21	1.09

This table displays the expected loss due to scarcity for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), per path ($REMod - REF$, $REMod - SUF$, $REMod - PER$ and $REMod - UNA$), as well as per scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) for the input variables, derived from the logistic regression model. Hereby, the results are derived under the robustness test for the scenario values, in particular, the scenario values are calculated based on data in the period from 1995 to 2019.

D.3.2.3 Robustness Analysis for the Loss Given Scarcity

Table D.56: Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the GVAR models of the robustness analysis for the loss given scarcity under the assumption neither commodity is substitutable

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
Mean	2019	$REMod-REF$	0.00	0.06	0.56	0.00	0.00	9.35	0.28	0.08	0.00	0.08	5.96
		$REMod-SUF$	0.00	0.04	0.38	0.00	0.00	6.29	0.19	0.05	0.00	0.05	6.06
		$REMod-PER$	0.00	0.05	0.41	0.00	0.00	6.75	0.22	0.07	0.00	0.07	4.88
		$REMod-UNA$	0.00	0.06	0.58	0.00	0.00	9.57	0.28	0.04	0.00	0.06	6.33
	Mean	$REMod-REF$	0.08	0.53	2.82	0.51	0.66	0.00	0.56	0.24	0.00	0.33	0.91
		$REMod-SUF$	0.06	0.36	1.89	0.37	0.48	0.00	0.38	0.16	0.00	0.22	0.93
		$REMod-PER$	0.09	0.47	2.03	0.43	0.68	0.00	0.45	0.24	0.00	0.32	0.75
		$REMod-UNA$	0.13	0.60	2.88	0.48	1.01	0.00	0.56	0.13	0.00	0.24	0.97
Shock	2019	$REMod-REF$	20.84	19.00	563.56	38.95	211.05	61.54	140.90	1.38	0.69	25.32	18.97
		$REMod-SUF$	14.07	12.88	378.88	28.16	154.54	41.37	95.24	0.94	0.45	17.25	19.29
		$REMod-PER$	23.74	16.68	406.26	33.08	216.25	44.38	111.46	1.36	0.86	24.35	15.55
		$REMod-UNA$	33.23	21.28	576.80	36.86	322.44	62.97	141.04	0.78	0.49	18.68	20.15
	Mean	$REMod-REF$	20.84	19.00	563.56	38.95	219.62	61.54	140.90	1.38	0.81	25.32	18.97
		$REMod-SUF$	14.07	12.88	378.88	28.16	160.82	41.37	95.24	0.94	0.53	17.25	19.29
		$REMod-PER$	23.74	16.68	406.26	33.08	225.02	44.38	111.46	1.36	1.01	24.35	15.55
		$REMod-UNA$	33.23	21.28	576.80	36.86	335.53	62.97	141.04	0.78	0.58	18.68	20.15
Extr.	2019	$REMod-REF$	20.84	19.00	563.56	38.95	219.62	61.54	140.90	1.38	0.81	25.32	18.97
		$REMod-SUF$	14.07	12.88	378.88	28.16	160.82	41.37	95.24	0.94	0.53	17.25	19.29
		$REMod-PER$	23.74	16.68	406.26	33.08	225.02	44.38	111.46	1.36	1.01	24.35	15.55
		$REMod-UNA$	33.23	21.28	576.80	36.86	335.53	62.97	141.04	0.78	0.58	18.68	20.15
	Mean	$REMod-REF$	20.84	19.00	563.56	38.95	219.62	61.54	140.90	1.38	0.81	25.32	18.97
		$REMod-SUF$	14.07	12.88	378.88	28.16	160.82	41.37	95.24	0.94	0.53	17.25	19.29
		$REMod-PER$	23.74	16.68	406.26	33.08	225.02	44.38	111.46	1.36	1.01	24.35	15.55
		$REMod-UNA$	33.23	21.28	576.80	36.86	335.53	62.97	141.04	0.78	0.58	18.68	20.15
Foc. EA	2019	$REMod-REF$	0.21	11.06	135.82	22.24	0.00	59.51	111.03	1.33	0.00	22.21	18.97
		$REMod-SUF$	0.14	7.50	91.31	16.08	0.00	40.01	75.05	0.90	0.00	15.13	19.29
		$REMod-PER$	0.24	9.71	97.91	18.89	0.00	42.92	87.83	1.30	0.00	21.35	15.55
		$REMod-UNA$	0.33	12.39	139.01	21.05	0.00	60.89	111.14	0.75	0.00	16.38	20.15
	Mean	$REMod-REF$	7.36	18.34	342.08	38.41	66.32	0.86	132.03	1.38	0.09	25.24	18.84
		$REMod-SUF$	4.97	12.43	229.98	27.76	48.57	0.58	89.24	0.94	0.06	17.20	19.16
		$REMod-PER$	8.38	16.10	246.60	32.62	67.96	0.62	104.44	1.36	0.11	24.28	15.45
		$REMod-UNA$	11.73	20.54	350.12	36.34	101.33	0.88	132.16	0.78	0.06	18.62	20.01
Foc. FX	2019	$REMod-REF$	0.08	1.88	30.43	4.52	0.00	44.99	51.85	0.67	0.00	7.34	18.72
		$REMod-SUF$	0.06	1.28	20.46	3.27	0.00	30.24	35.05	0.46	0.00	5.00	19.04
		$REMod-PER$	0.09	1.65	21.94	3.84	0.00	32.44	41.02	0.66	0.00	7.06	15.35
		$REMod-UNA$	0.13	2.11	31.15	4.28	0.00	46.03	51.90	0.38	0.00	5.42	19.89
	Mean	$REMod-REF$	3.67	7.13	87.35	20.84	77.96	0.55	80.60	1.00	0.03	15.72	16.47
		$REMod-SUF$	2.48	4.83	58.73	15.06	57.09	0.37	54.48	0.68	0.02	10.71	16.74
		$REMod-PER$	4.18	6.26	62.97	17.70	79.88	0.40	63.76	0.98	0.04	15.12	13.50
		$REMod-UNA$	5.85	7.98	89.40	19.72	119.11	0.57	80.68	0.56	0.02	11.60	17.49
Foc. FFR	2019	$REMod-REF$	0.10	1.12	31.56	0.93	0.00	47.82	9.02	0.38	0.00	1.22	13.17
		$REMod-SUF$	0.07	0.76	21.22	0.68	0.00	32.15	6.10	0.26	0.00	0.83	13.39
		$REMod-PER$	0.12	0.98	22.75	0.79	0.00	34.48	7.13	0.37	0.00	1.17	10.80
		$REMod-UNA$	0.17	1.26	32.30	0.88	0.00	48.93	9.03	0.21	0.00	0.90	13.98
	Mean	$REMod-REF$	2.46	4.29	68.19	3.74	44.14	0.43	19.16	0.62	0.02	4.63	5.94
		$REMod-SUF$	1.66	2.91	45.85	2.70	32.32	0.29	12.95	0.42	0.02	3.16	6.04
		$REMod-PER$	2.80	3.77	49.16	3.18	45.23	0.31	15.16	0.61	0.03	4.46	4.87
		$REMod-UNA$	3.92	4.81	69.79	3.54	67.44	0.44	19.18	0.35	0.02	3.42	6.31
Foc. Extr. EA	2019	$REMod-REF$	6.59	18.62	502.69	38.25	6.81	61.36	138.79	1.38	0.06	25.27	18.97
		$REMod-SUF$	4.45	12.62	337.97	27.65	4.99	41.25	93.81	0.94	0.04	17.22	19.29
		$REMod-PER$	7.50	16.35	362.39	32.49	6.98	44.25	109.79	1.36	0.08	24.30	15.55
		$REMod-UNA$	10.50	20.86	514.50	36.20	10.40	62.78	138.93	0.78	0.04	18.64	20.15
	Mean	$REMod-REF$	17.88	19.00	547.22	38.91	182.28	12.99	140.76	1.38	0.69	25.32	18.95
		$REMod-SUF$	12.07	12.88	367.90	28.13	133.48	8.73	95.14	0.94	0.46	17.25	19.27
		$REMod-PER$	20.37	16.68	394.48	33.05	186.77	9.36	111.35	1.36	0.87	24.35	15.54
		$REMod-UNA$	28.51	21.28	560.07	36.82	278.49	13.29	140.90	0.78	0.50	18.68	20.13
Foc. Extr. FX	2019	$REMod-REF$	2.15	10.13	201.19	30.03	2.64	56.07	121.60	1.16	0.00	20.48	18.95
		$REMod-SUF$	1.45	6.87	135.26	21.71	1.93	37.69	82.19	0.79	0.00	13.96	19.27
		$REMod-PER$	2.44	8.89	145.04	25.51	2.70	40.43	96.19	1.14	0.01	19.70	15.54
		$REMod-UNA$	3.42	11.34	205.92	28.42	4.03	57.36	121.72	0.65	0.00	15.11	20.13
	Mean	$REMod-REF$	11.26	15.11	313.90	36.77	173.94	7.51	129.91	1.28	0.13	23.45	18.67
		$REMod-SUF$	7.60	10.24	211.04	26.58	127.37	5.05	87.81	0.87	0.09	15.97	18.98
		$REMod-PER$	12.82	13.26	226.29	31.23	178.22	5.41	102.77	1.26	0.16	22.55	15.31
		$REMod-UNA$	17.94	16.92	321.28	34.80	265.74	7.68	130.04	0.72	0.09	17.30	19.83
	2019	$REMod-REF$	1.25	4.73	116.09	4.28	2.42	57.23	43.96	0.72	0.00	6.76	16.43
		$REMod-SUF$	0.84	3.21	78.05	3.10	1.77	38.48	29.71	0.49	0.00	4.61	16.71

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the GVAR models of the robustness analysis for the loss given scarcity under the assumption neither commodity is substitutable

Scenario	Basis	Path	Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
Foc. Extr. FFR		$REMod-PER$	1.42	4.15	83.69	3.64	2.48	41.27	34.78	0.70	0.00	6.50	13.47
		$REMod-UNA$	1.99	5.30	118.82	4.05	3.69	58.56	44.01	0.40	0.00	4.99	17.45
	Mean	$REMod-REF$	7.86	8.80	212.46	11.96	103.00	7.32	64.11	0.94	0.09	12.28	11.69
		$REMod-SUF$	5.30	5.96	142.84	8.64	75.42	4.92	43.33	0.64	0.06	8.37	11.88
		$REMod-PER$	8.95	7.72	153.16	10.16	105.54	5.28	50.72	0.93	0.11	11.81	9.58
		$REMod-UNA$	12.53	9.85	217.45	11.32	157.36	7.49	64.17	0.53	0.06	9.06	12.41
Q. 25%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Q. 40%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Q. 50%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	1.48	0.14	0.02	0.00	0.00	2.03
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.99	0.10	0.02	0.00	0.00	2.06
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	1.07	0.11	0.02	0.00	0.00	1.66
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	1.51	0.14	0.01	0.00	0.00	2.16
	Mean	$REMod-REF$	0.04	0.29	0.56	0.04	0.00	0.00	0.42	0.09	0.00	0.03	0.36
		$REMod-SUF$	0.03	0.19	0.38	0.03	0.00	0.00	0.29	0.06	0.00	0.02	0.37
		$REMod-PER$	0.05	0.25	0.41	0.03	0.00	0.00	0.33	0.09	0.00	0.02	0.30
		$REMod-UNA$	0.07	0.32	0.58	0.04	0.00	0.00	0.42	0.05	0.00	0.02	0.38
Q. 60%	2019	$REMod-REF$	0.08	2.60	40.01	1.21	0.00	54.90	16.63	0.62	0.00	2.30	15.31
		$REMod-SUF$	0.06	1.76	26.90	0.87	0.00	36.91	11.24	0.42	0.00	1.57	15.57
		$REMod-PER$	0.09	2.29	28.84	1.03	0.00	39.59	13.15	0.61	0.00	2.22	12.55
		$REMod-UNA$	0.13	2.92	40.95	1.14	0.00	56.17	16.64	0.35	0.00	1.70	16.26
	Mean	$REMod-REF$	5.04	7.81	100.31	4.79	58.86	1.35	38.33	0.97	0.03	7.09	7.25
		$REMod-SUF$	3.40	5.29	67.44	3.46	43.10	0.91	25.91	0.66	0.02	4.83	7.37
		$REMod-PER$	5.74	6.86	72.31	4.07	60.31	0.98	30.32	0.96	0.03	6.82	5.94
		$REMod-UNA$	8.04	8.75	102.67	4.53	89.92	1.39	38.36	0.55	0.02	5.23	7.70
Q. 75%	2019	$REMod-REF$	13.40	18.93	530.87	38.80	39.97	61.54	140.90	1.38	0.03	25.30	18.97
		$REMod-SUF$	9.05	12.83	356.91	28.04	29.27	41.37	95.24	0.94	0.02	17.23	19.29
		$REMod-PER$	15.26	16.61	382.70	32.95	40.95	44.38	111.46	1.36	0.04	24.32	15.55
		$REMod-UNA$	21.36	21.20	543.34	36.71	61.07	62.97	141.04	0.78	0.02	18.66	20.15
	Mean	$REMod-REF$	20.72	19.00	558.49	38.95	218.30	47.82	140.90	1.38	0.65	25.32	18.97
		$REMod-SUF$	13.98	12.88	375.47	28.16	159.85	32.15	95.24	0.94	0.43	17.25	19.29
		$REMod-PER$	23.59	16.68	402.61	33.08	223.67	34.48	111.46	1.36	0.81	24.35	15.55
		$REMod-UNA$	33.03	21.28	571.61	36.86	333.52	48.93	141.04	0.78	0.46	18.68	20.15

This table displays the expected loss due to scarcity for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), per path ($REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$), as well as per scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) for the input variables. Hereby, the values are derived from the GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). Hereby, the results are derived under the robustness test for the loss given scarcity, in particular, we assume neither commodity is substitutable, resulting in loss given scarcity values of one.

Table D.57: Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the MS-GVAR models of the robustness analysis for the loss given scarcity under the assumption neither commodity is substitutable

Scenario	Basis	Path	Al	Cu	Ni	Pb	Sn	Zn
Mean	2019	$REMod-REF$	4.79	4.36	14.94	1.26	1.32	18.97
		$REMod-SUF$	3.25	3.15	10.10	0.86	0.90	19.29
		$REMod-PER$	4.20	3.71	11.82	1.24	1.27	15.55

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the MS-GVAR models of the robustness analysis for the loss given scarcity under the assumption neither commodity is substitutable

			Al	Cu	Ni	Pb	Sn	Zn
		$REMod-UNA$	5.36	4.13	14.95	0.71	0.97	20.15
Mean	Mean	$REMod-REF$	18.93	38.41	74.68	1.38	10.89	11.53
		$REMod-SUF$	12.83	27.76	50.48	0.94	7.42	11.73
		$REMod-PER$	16.61	32.62	59.08	1.36	10.47	9.46
		$REMod-UNA$	21.20	36.34	74.75	0.78	8.03	12.25
Shock	2019	$REMod-REF$	19.00	38.95	140.90	1.38	25.32	18.97
		$REMod-SUF$	12.88	28.16	95.24	0.94	17.25	19.29
		$REMod-PER$	16.68	33.08	111.46	1.36	24.35	15.55
		$REMod-UNA$	21.28	36.86	141.04	0.78	18.68	20.15
	Mean	$REMod-REF$	19.00	38.95	140.90	1.38	25.32	18.97
		$REMod-SUF$	12.88	28.16	95.24	0.94	17.25	19.29
		$REMod-PER$	16.68	33.08	111.46	1.36	24.35	15.55
		$REMod-UNA$	21.28	36.86	141.04	0.78	18.68	20.15
Extr.	2019	$REMod-REF$	19.00	38.95	140.90	1.38	25.32	18.97
		$REMod-SUF$	12.88	28.16	95.24	0.94	17.25	19.29
		$REMod-PER$	16.68	33.08	111.46	1.36	24.35	15.55
		$REMod-UNA$	21.28	36.86	141.04	0.78	18.68	20.15
	Mean	$REMod-REF$	19.00	38.95	140.90	1.38	25.32	18.97
		$REMod-SUF$	12.88	28.16	95.24	0.94	17.25	19.29
		$REMod-PER$	16.68	33.08	111.46	1.36	24.35	15.55
		$REMod-UNA$	21.28	36.86	141.04	0.78	18.68	20.15
Foc. EA	2019	$REMod-REF$	18.28	23.68	110.19	1.38	12.91	18.97
		$REMod-SUF$	12.39	17.12	74.48	0.94	8.80	19.29
		$REMod-PER$	16.05	20.12	87.16	1.36	12.42	15.55
		$REMod-UNA$	20.47	22.41	110.30	0.78	9.53	20.15
	Mean	$REMod-REF$	19.00	38.95	136.40	1.38	23.65	18.63
		$REMod-SUF$	12.88	28.16	92.19	0.94	16.11	18.94
		$REMod-PER$	16.68	33.08	107.90	1.36	22.74	15.27
		$REMod-UNA$	21.28	36.86	136.53	0.78	17.45	19.79
Foc. FX	2019	$REMod-REF$	18.96	35.99	130.76	1.38	14.79	18.97
		$REMod-SUF$	12.86	26.02	88.38	0.94	10.07	19.29
		$REMod-PER$	16.65	30.57	103.44	1.36	14.22	15.55
		$REMod-UNA$	21.24	34.06	130.89	0.78	10.91	20.15
	Mean	$REMod-REF$	19.00	38.95	140.62	1.38	23.70	18.48
		$REMod-SUF$	12.88	28.16	95.05	0.94	16.15	18.79
		$REMod-PER$	16.68	33.08	111.24	1.36	22.79	15.15
		$REMod-UNA$	21.28	36.86	140.76	0.78	17.48	19.63
Foc. FFR	2019	$REMod-REF$	19.00	38.95	140.90	1.38	25.07	18.97
		$REMod-SUF$	12.88	28.16	95.24	0.94	17.08	19.29
		$REMod-PER$	16.68	33.08	111.46	1.36	24.11	15.55
		$REMod-UNA$	21.28	36.86	141.04	0.78	18.49	20.15
	Mean	$REMod-REF$	19.00	38.95	140.90	1.38	25.22	18.97
		$REMod-SUF$	12.88	28.16	95.24	0.94	17.18	19.29
		$REMod-PER$	16.68	33.08	111.46	1.36	24.25	15.55
		$REMod-UNA$	21.28	36.86	141.04	0.78	18.60	20.15
Foc. Extr. EA	2019	$REMod-REF$	18.89	34.28	131.04	1.38	21.27	18.97
		$REMod-SUF$	12.81	24.78	88.57	0.94	14.49	19.29
		$REMod-PER$	16.58	29.11	103.66	1.36	20.45	15.55
		$REMod-UNA$	21.15	32.44	131.17	0.78	15.69	20.15
	Mean	$REMod-REF$	19.00	38.95	139.50	1.38	24.61	18.89
		$REMod-SUF$	12.88	28.16	94.29	0.94	16.77	19.21
		$REMod-PER$	16.68	33.08	110.35	1.36	23.67	15.49
		$REMod-UNA$	21.28	36.86	139.63	0.78	18.16	20.07
Foc. Extr. FX	2019	$REMod-REF$	19.00	38.25	139.21	1.38	21.47	18.97
		$REMod-SUF$	12.88	27.65	94.10	0.94	14.63	19.29
		$REMod-PER$	16.68	32.49	110.13	1.36	20.65	15.55
		$REMod-UNA$	21.28	36.20	139.35	0.78	15.84	20.15
	Mean	$REMod-REF$	19.00	38.95	140.62	1.38	24.92	18.86
		$REMod-SUF$	12.88	28.16	95.05	0.94	16.97	19.18
		$REMod-PER$	16.68	33.08	111.24	1.36	23.96	15.46
		$REMod-UNA$	21.28	36.86	140.76	0.78	18.38	20.03
Foc. Extr. FFR	2019	$REMod-REF$	19.00	38.95	140.90	1.38	25.22	18.97
		$REMod-SUF$	12.88	28.16	95.24	0.94	17.18	19.29
		$REMod-PER$	16.68	33.08	111.46	1.36	24.25	15.55
		$REMod-UNA$	21.28	36.86	141.04	0.78	18.60	20.15
	Mean	$REMod-REF$	19.00	38.95	140.90	1.38	25.32	18.97
		$REMod-SUF$	12.88	28.16	95.24	0.94	17.25	19.29
		$REMod-PER$	16.68	33.08	111.46	1.36	24.35	15.55
		$REMod-UNA$	21.28	36.86	141.04	0.78	18.68	20.15
		$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the MS-GVAR models of the robustness analysis for the loss given scarcity under the assumption neither commodity is substitutable

			Al	Cu	Ni	Pb	Sn	Zn
Q. 25%	2019	$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00
	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00
Q. 40%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.27
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.27
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.22
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.28
	Mean	$REMod-REF$	0.00	0.00	0.00	0.01	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.01	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.01	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00
Q. 50%	2019	$REMod-REF$	0.00	0.16	0.56	0.34	0.25	18.48
		$REMod-SUF$	0.00	0.11	0.38	0.23	0.17	18.79
		$REMod-PER$	0.00	0.13	0.45	0.34	0.24	15.15
		$REMod-UNA$	0.00	0.15	0.56	0.19	0.19	19.63
	Mean	$REMod-REF$	3.84	8.80	9.86	0.99	3.39	2.28
		$REMod-SUF$	2.60	6.36	6.67	0.67	2.31	2.31
		$REMod-PER$	3.37	7.48	7.80	0.98	3.26	1.87
		$REMod-UNA$	4.30	8.33	9.87	0.56	2.50	2.42
Q. 60%	2019	$REMod-REF$	19.00	38.95	140.90	1.38	24.31	18.97
		$REMod-SUF$	12.88	28.16	95.24	0.94	16.56	19.29
		$REMod-PER$	16.68	33.08	111.46	1.36	23.38	15.55
		$REMod-UNA$	21.28	36.86	141.04	0.78	17.93	20.15
	Mean	$REMod-REF$	19.00	38.95	140.90	1.38	25.32	18.97
		$REMod-SUF$	12.88	28.16	95.24	0.94	17.25	19.29
		$REMod-PER$	16.68	33.08	111.46	1.36	24.35	15.55
		$REMod-UNA$	21.28	36.86	141.04	0.78	18.68	20.15
Q. 75%	2019	$REMod-REF$	19.00	38.95	140.90	1.38	25.32	18.97
		$REMod-SUF$	12.88	28.16	95.24	0.94	17.25	19.29
		$REMod-PER$	16.68	33.08	111.46	1.36	24.35	15.55
		$REMod-UNA$	21.28	36.86	141.04	0.78	18.68	20.15
	Mean	$REMod-REF$	19.00	38.95	140.90	1.38	25.32	18.97
		$REMod-SUF$	12.88	28.16	95.24	0.94	17.25	19.29
		$REMod-PER$	16.68	33.08	111.46	1.36	24.35	15.55
		$REMod-UNA$	21.28	36.86	141.04	0.78	18.68	20.15

This table displays the expected loss due to scarcity for the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), per path ($REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$), as well as per scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) for the input variables. Hereby, the values are derived from the MS-GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). Hereby, the results are derived under the robustness test for the loss given scarcity, in particular, we assume neither commodity is substitutable, resulting in loss given scarcity values of one.

Table D.58: Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the logistic regression models of the robustness analysis for the loss given scarcity under the assumption neither commodity is substitutable

		Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
Mean	$REMod-REF$	0.74	1.14	40.83	1.66	1.48	22.27	2.55	0.47	12.71	0.08	0.03	0.52	0.72
	$REMod-SUF$	0.50	0.78	27.45	1.20	1.06	16.31	1.71	0.33	8.59	0.05	0.02	0.35	0.73

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the logistic regression models of the robustness analysis for the loss given scarcity under the assumption neither commodity is substitutable

		Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
Mean	$REMod-PER$	0.85	1.00	29.43	1.41	1.42	22.81	1.84	0.44	10.06	0.08	0.03	0.50	0.59
	$REMod-UNA$	1.19	1.28	41.79	1.57	0.70	34.02	2.61	0.20	12.72	0.04	0.02	0.38	0.76
Shock	$REMod-REF$	1.00	2.19	260.29	2.37	7.70	35.04	3.55	2.45	78.55	0.20	0.06	0.81	15.27
	$REMod-SUF$	0.67	1.48	175.00	1.71	5.51	25.66	2.39	1.74	53.10	0.14	0.04	0.55	15.53
	$REMod-PER$	1.14	1.92	187.64	2.01	7.42	35.91	2.56	2.27	62.14	0.20	0.07	0.78	12.52
	$REMod-UNA$	1.59	2.45	266.41	2.24	3.67	53.54	3.63	1.05	78.63	0.11	0.04	0.60	16.22
Extr.	$REMod-REF$	1.33	3.97	509.53	3.35	33.85	53.17	4.91	10.78	132.62	0.44	0.11	1.27	18.93
	$REMod-SUF$	0.90	2.69	342.56	2.42	24.19	38.94	3.30	7.67	89.64	0.30	0.08	0.86	19.25
	$REMod-PER$	1.52	3.48	367.32	2.84	32.60	54.48	3.54	9.98	104.91	0.43	0.14	1.22	15.52
	$REMod-UNA$	2.13	4.44	521.50	3.17	16.14	81.24	5.03	4.62	132.75	0.25	0.08	0.94	20.10
Foc. EA	$REMod-REF$	0.74	1.14	40.83	1.66	2.99	22.27	2.55	0.95	12.71	0.08	0.03	0.52	0.72
	$REMod-SUF$	0.50	0.78	27.45	1.20	2.14	16.31	1.71	0.68	8.59	0.05	0.02	0.35	0.73
	$REMod-PER$	0.85	1.00	29.43	1.41	2.88	22.81	1.84	0.88	10.06	0.08	0.03	0.50	0.59
	$REMod-UNA$	1.19	1.28	41.79	1.57	1.43	34.02	2.61	0.41	12.72	0.04	0.02	0.38	0.76
Foc. FX	$REMod-REF$	0.74	1.14	40.83	1.69	1.63	27.23	2.55	0.52	12.71	0.12	0.03	0.52	0.72
	$REMod-SUF$	0.50	0.78	27.45	1.22	1.17	19.94	1.71	0.37	8.59	0.08	0.02	0.35	0.73
	$REMod-PER$	0.85	1.00	29.43	1.44	1.57	27.90	1.84	0.48	10.06	0.11	0.04	0.50	0.59
	$REMod-UNA$	1.19	1.28	41.79	1.60	0.78	41.59	2.61	0.22	12.72	0.07	0.02	0.38	0.76
Foc. FFR	$REMod-REF$	0.74	1.14	40.83	1.99	3.51	22.27	2.55	1.12	12.71	0.08	0.03	0.52	3.17
	$REMod-SUF$	0.50	0.78	27.45	1.44	2.51	16.31	1.71	0.79	8.59	0.05	0.02	0.35	3.22
	$REMod-PER$	0.85	1.00	29.43	1.69	3.38	22.81	1.84	1.03	10.06	0.08	0.03	0.50	2.60
	$REMod-UNA$	1.19	1.28	41.79	1.89	1.67	34.02	2.61	0.48	12.72	0.04	0.02	0.38	3.37
Foc. Extr. EA	$REMod-REF$	0.74	1.14	40.83	1.66	6.00	22.27	2.55	1.91	12.71	0.08	0.03	0.52	0.72
	$REMod-SUF$	0.50	0.78	27.45	1.20	4.29	16.31	1.71	1.36	8.59	0.05	0.02	0.35	0.73
	$REMod-PER$	0.85	1.00	29.43	1.41	5.78	22.81	1.84	1.77	10.06	0.08	0.03	0.50	0.59
	$REMod-UNA$	1.19	1.28	41.79	1.57	2.86	34.02	2.61	0.82	12.72	0.04	0.02	0.38	0.76
Foc. Extr. FX	$REMod-REF$	0.74	1.14	40.83	1.72	1.80	33.10	2.55	0.57	12.71	0.17	0.04	0.52	0.72
	$REMod-SUF$	0.50	0.78	27.45	1.25	1.29	24.24	1.71	0.41	8.59	0.12	0.03	0.35	0.73
	$REMod-PER$	0.85	1.00	29.43	1.46	1.73	33.92	1.84	0.53	10.06	0.17	0.05	0.50	0.59
	$REMod-UNA$	1.19	1.28	41.79	1.63	0.86	50.57	2.61	0.25	12.72	0.10	0.03	0.38	0.76
Foc. Extr. FFR	$REMod-REF$	0.74	1.14	40.83	2.39	8.16	22.27	2.55	2.60	12.71	0.08	0.03	0.52	9.59
	$REMod-SUF$	0.50	0.78	27.45	1.72	5.84	16.31	1.71	1.85	8.59	0.05	0.02	0.35	9.75
	$REMod-PER$	0.85	1.00	29.43	2.03	7.86	22.81	1.84	2.41	10.06	0.08	0.03	0.50	7.86
	$REMod-UNA$	1.19	1.28	41.79	2.26	3.89	34.02	2.61	1.11	12.72	0.04	0.02	0.38	10.18
Q. 25%	$REMod-REF$	0.67	0.84	9.98	1.37	0.57	18.86	2.39	0.18	3.40	0.05	0.02	0.38	0.05
	$REMod-SUF$	0.45	0.57	6.71	0.99	0.41	13.81	1.60	0.13	2.29	0.03	0.01	0.26	0.05
	$REMod-PER$	0.76	0.73	7.20	1.16	0.55	19.32	1.72	0.17	2.69	0.05	0.02	0.37	0.04
	$REMod-UNA$	1.06	0.94	10.22	1.30	0.27	28.81	2.44	0.08	3.40	0.03	0.01	0.28	0.05
Q. 40%	$REMod-REF$	0.71	0.96	18.45	1.57	0.98	20.96	2.40	0.31	10.70	0.06	0.02	0.45	0.21
	$REMod-SUF$	0.48	0.65	12.40	1.13	0.70	15.35	1.61	0.22	7.23	0.04	0.02	0.31	0.21
	$REMod-PER$	0.81	0.84	13.30	1.33	0.94	21.48	1.73	0.29	8.46	0.06	0.03	0.44	0.17
	$REMod-UNA$	1.14	1.07	18.88	1.49	0.47	32.03	2.46	0.13	10.71	0.03	0.02	0.34	0.22
Q. 50%	$REMod-REF$	0.74	1.02	29.81	1.67	1.22	22.00	2.44	0.39	15.95	0.07	0.03	0.49	0.55
	$REMod-SUF$	0.50	0.69	20.04	1.21	0.87	16.11	1.64	0.28	10.78	0.05	0.02	0.33	0.56
	$REMod-PER$	0.84	0.89	21.49	1.42	1.17	22.54	1.76	0.36	12.62	0.07	0.03	0.47	0.45
	$REMod-UNA$	1.17	1.14	30.51	1.58	0.58	33.60	2.50	0.17	15.97	0.04	0.02	0.36	0.58
Q. 60%	$REMod-REF$	0.80	1.24	62.97	1.81	1.81	23.91	2.49	0.57	20.18	0.09	0.03	0.51	1.83
	$REMod-SUF$	0.54	0.84	42.33	1.31	1.29	17.51	1.67	0.41	13.64	0.06	0.02	0.35	1.86
	$REMod-PER$	0.91	1.09	45.39	1.54	1.74	24.50	1.79	0.53	15.97	0.09	0.04	0.49	1.50
	$REMod-UNA$	1.28	1.39	64.45	1.72	0.86	36.53	2.54	0.25	20.20	0.05	0.02	0.38	1.94
	$REMod-REF$	0.90	1.71	192.65	2.06	3.63	28.05	2.58	1.16	31.36	0.15	0.04	0.69	8.86

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the logistic regression models of the robustness analysis for the loss given scarcity under the assumption neither commodity is substitutable

		Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
Q. 75%	$REMod-SUF$	0.61	1.16	129.52	1.49	2.59	20.54	1.74	0.82	21.20	0.10	0.03	0.47	9.01
	$REMod-PER$	1.02	1.50	138.88	1.75	3.50	28.74	1.86	1.07	24.81	0.15	0.05	0.66	7.27
	$REMod-UNA$	1.43	1.91	197.17	1.95	1.73	42.85	2.64	0.50	31.39	0.08	0.03	0.51	9.41

This table displays the expected loss due to scarcity for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), per path ($REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$), as well as per scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) for the input variables, derived from the logistic regression model. Hereby, the results are derived under the robustness test for the loss given scarcity, in particular, we assume neither commodity is substitutable, resulting in loss given scarcity values of one.

D.3.2.4 Robustness Analysis for the Exposure at Scarcity

D.3.2.4.1 Results of the Robustness Analysis for the Exposure at Scarcity of the reduced Sample

Table D.59: Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the GVAR models of the robustness analysis for the exposure at scarcity for the reduced sample period from 2015 to 2019

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
Mean	2019	$REMod - REF$	0.00	0.02	0.35	0.00	0.00	2.71	0.18	0.08	0.00	0.03	2.28
		$REMod - SUF$	0.00	0.02	0.24	0.00	0.00	1.82	0.12	0.05	0.00	0.02	2.32
		$REMod - PER$	0.00	0.02	0.25	0.00	0.00	1.96	0.14	0.08	0.00	0.02	1.87
		$REMod - UNA$	0.00	0.03	0.36	0.00	0.00	2.78	0.18	0.04	0.00	0.02	2.42
	Mean	$REMod - REF$	0.04	0.21	1.76	0.33	0.40	0.00	0.35	0.25	0.00	0.11	0.35
		$REMod - SUF$	0.02	0.14	1.18	0.24	0.29	0.00	0.24	0.17	0.00	0.08	0.35
		$REMod - PER$	0.04	0.19	1.27	0.28	0.40	0.00	0.28	0.24	0.00	0.11	0.29
		$REMod - UNA$	0.06	0.24	1.80	0.31	0.60	0.00	0.35	0.14	0.00	0.08	0.37
Shock	2019	$REMod - REF$	8.90	7.55	352.23	25.33	126.57	17.85	88.19	1.42	0.47	8.55	7.27
		$REMod - SUF$	6.01	5.12	236.81	18.31	92.68	12.00	59.61	0.97	0.31	5.82	7.39
		$REMod - PER$	10.14	6.63	253.92	21.52	129.68	12.87	69.77	1.40	0.58	8.22	5.96
		$REMod - UNA$	14.20	8.45	360.50	23.97	193.37	18.27	88.28	0.80	0.33	6.31	7.72
	Mean	$REMod - REF$	8.90	7.55	352.23	25.33	131.70	17.85	88.19	1.42	0.55	8.55	7.27
		$REMod - SUF$	6.01	5.12	236.81	18.31	96.44	12.00	59.61	0.97	0.36	5.82	7.39
		$REMod - PER$	10.14	6.63	253.92	21.52	134.94	12.87	69.77	1.40	0.69	8.22	5.96
		$REMod - UNA$	14.20	8.45	360.50	23.97	201.21	18.27	88.28	0.80	0.39	6.31	7.72
Extr.	2019	$REMod - REF$	8.90	7.55	352.23	25.33	131.70	17.85	88.19	1.42	0.55	8.55	7.27
		$REMod - SUF$	6.01	5.12	236.81	18.31	96.44	12.00	59.61	0.97	0.36	5.82	7.39
		$REMod - PER$	10.14	6.63	253.92	21.52	134.94	12.87	69.77	1.40	0.69	8.22	5.96
		$REMod - UNA$	14.20	8.45	360.50	23.97	201.21	18.27	88.28	0.80	0.39	6.31	7.72
	Mean	$REMod - REF$	8.90	7.55	352.23	25.33	131.70	17.85	88.19	1.42	0.55	8.55	7.27
		$REMod - SUF$	6.01	5.12	236.81	18.31	96.44	12.00	59.61	0.97	0.36	5.82	7.39
		$REMod - PER$	10.14	6.63	253.92	21.52	134.94	12.87	69.77	1.40	0.69	8.22	5.96
		$REMod - UNA$	14.20	8.45	360.50	23.97	201.21	18.27	88.28	0.80	0.39	6.31	7.72
Foc. EA	2019	$REMod - REF$	0.09	4.39	84.89	14.46	0.00	17.26	69.50	1.36	0.00	7.50	7.27
		$REMod - SUF$	0.06	2.98	57.07	10.45	0.00	11.61	46.97	0.93	0.00	5.11	7.39
		$REMod - PER$	0.10	3.86	61.19	12.29	0.00	12.45	54.98	1.34	0.00	7.21	5.96
		$REMod - UNA$	0.14	4.92	86.88	13.69	0.00	17.66	69.57	0.77	0.00	5.53	7.72
	Mean	$REMod - REF$	3.14	7.28	213.80	24.98	39.77	0.25	82.64	1.42	0.06	8.52	7.22
		$REMod - SUF$	2.12	4.94	143.74	18.05	29.12	0.17	55.86	0.97	0.04	5.81	7.34
		$REMod - PER$	3.58	6.39	154.13	21.21	40.75	0.18	65.37	1.40	0.07	8.20	5.92
		$REMod - UNA$	5.01	8.16	218.83	23.63	60.77	0.26	82.72	0.80	0.04	6.29	7.66
Foc. FX	2019	$REMod - REF$	0.04	0.75	19.02	2.94	0.00	13.05	32.46	0.69	0.00	2.48	7.17
		$REMod - SUF$	0.02	0.51	12.79	2.12	0.00	8.77	21.94	0.47	0.00	1.69	7.29
		$REMod - PER$	0.04	0.66	13.71	2.50	0.00	9.41	25.67	0.68	0.00	2.38	5.88
		$REMod - UNA$	0.06	0.84	19.47	2.78	0.00	13.35	32.49	0.39	0.00	1.83	7.62
	Mean	$REMod - REF$	1.57	2.83	54.60	13.55	46.75	0.16	50.45	1.03	0.02	5.31	6.31
		$REMod - SUF$	1.06	1.92	36.70	9.80	34.24	0.11	34.10	0.70	0.01	3.62	6.41
		$REMod - PER$	1.78	2.48	39.36	11.51	47.90	0.12	39.91	1.01	0.02	5.11	5.17
		$REMod - UNA$	2.50	3.17	55.88	12.82	71.43	0.16	50.50	0.58	0.01	3.92	6.70
Foc. FFR	2019	$REMod - REF$	0.04	0.45	19.72	0.61	0.00	13.87	5.64	0.39	0.00	0.41	5.04
		$REMod - SUF$	0.03	0.30	13.26	0.44	0.00	9.33	3.82	0.26	0.00	0.28	5.13
		$REMod - PER$	0.05	0.39	14.22	0.52	0.00	10.00	4.47	0.38	0.00	0.39	4.13
		$REMod - UNA$	0.07	0.50	20.19	0.58	0.00	14.19	5.65	0.22	0.00	0.30	5.36
	Mean	$REMod - REF$	1.05	1.71	42.62	2.43	26.47	0.12	11.99	0.64	0.02	1.56	2.27
		$REMod - SUF$	0.71	1.16	28.65	1.76	19.38	0.08	8.11	0.43	0.01	1.07	2.31
		$REMod - PER$	1.20	1.50	30.72	2.07	27.12	0.09	9.49	0.62	0.02	1.50	1.86
		$REMod - UNA$	1.68	1.91	43.62	2.30	40.44	0.13	12.01	0.36	0.01	1.15	2.42
Foc. Extr. EA	2019	$REMod - REF$	2.81	7.40	314.19	24.88	4.08	17.80	86.87	1.42	0.04	8.53	7.27
		$REMod - SUF$	1.90	5.01	211.23	17.98	2.99	11.97	58.72	0.96	0.03	5.81	7.39
		$REMod - PER$	3.20	6.49	226.49	21.13	4.18	12.84	68.72	1.40	0.05	8.21	5.96
		$REMod - UNA$	4.49	8.28	321.57	23.54	6.24	18.21	86.96	0.80	0.03	6.29	7.72
	Mean	$REMod - REF$	7.64	7.55	342.01	25.31	109.31	3.77	88.11	1.42	0.47	8.55	7.26
		$REMod - SUF$	5.16	5.12	229.94	18.29	80.05	2.53	59.55	0.97	0.31	5.82	7.38
		$REMod - PER$	8.70	6.63	246.55	21.49	112.00	2.72	69.70	1.40	0.59	8.22	5.95
		$REMod - UNA$	12.18	8.45	350.05	23.95	167.01	3.85	88.19	0.80	0.34	6.31	7.71
Foc. Extr. FX	2019	$REMod - REF$	0.92	4.02	125.75	19.53	1.58	16.26	76.11	1.20	0.00	6.92	7.26
		$REMod - SUF$	0.62	2.73	84.54	14.12	1.16	10.93	51.44	0.81	0.00	4.71	7.38
		$REMod - PER$	1.04	3.53	90.65	16.59	1.62	11.73	60.21	1.18	0.00	6.65	5.95
		$REMod - UNA$	1.46	4.51	128.70	18.48	2.41	16.64	76.19	0.67	0.00	5.10	7.71
	Mean	$REMod - REF$	4.81	6.00	196.19	23.91	104.31	2.18	81.31	1.31	0.09	7.92	7.15
		$REMod - SUF$	3.25	4.07	131.90	17.28	76.38	1.46	54.96	0.89	0.06	5.39	7.27
		$REMod - PER$	5.48	5.27	141.43	20.31	106.88	1.57	64.32	1.29	0.11	7.61	5.86

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the GVAR models of the robustness analysis for the exposure at scarcity for the reduced sample period from 2015 to 2019

Scenario	Basis	Path	Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
		$REMod-UNA$	7.67	6.72	200.80	22.63	159.36	2.23	81.39	0.74	0.06	5.84	7.59
Foc. Extr. FFR	2019	$REMod-REF$	0.53	1.88	72.56	2.79	1.45	16.60	27.52	0.74	0.00	2.28	6.29
		$REMod-SUF$	0.36	1.27	48.78	2.01	1.06	11.16	18.60	0.50	0.00	1.56	6.40
		$REMod-PER$	0.61	1.65	52.31	2.37	1.48	11.97	21.77	0.72	0.00	2.20	5.16
		$REMod-UNA$	0.85	2.10	74.26	2.64	2.21	16.99	27.54	0.41	0.00	1.68	6.68
	Mean	$REMod-REF$	3.36	3.49	132.79	7.78	61.77	2.12	40.13	0.97	0.06	4.15	4.48
		$REMod-SUF$	2.27	2.37	89.28	5.62	45.23	1.43	27.12	0.66	0.04	2.82	4.55
		$REMod-PER$	3.82	3.07	95.73	6.61	63.29	1.53	31.74	0.95	0.08	3.99	3.67
		$REMod-UNA$	5.35	3.91	135.91	7.36	94.37	2.17	40.17	0.55	0.04	3.06	4.75
Q. 25%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Q. 40%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Q. 50%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.43	0.09	0.02	0.00	0.00	0.78
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.29	0.06	0.02	0.00	0.00	0.79
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.31	0.07	0.02	0.00	0.00	0.64
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.44	0.09	0.01	0.00	0.00	0.83
	Mean	$REMod-REF$	0.02	0.11	0.35	0.03	0.00	0.00	0.26	0.09	0.00	0.01	0.14
		$REMod-SUF$	0.01	0.08	0.24	0.02	0.00	0.00	0.18	0.06	0.00	0.01	0.14
		$REMod-PER$	0.02	0.10	0.25	0.02	0.00	0.00	0.21	0.09	0.00	0.01	0.11
		$REMod-UNA$	0.03	0.13	0.36	0.02	0.00	0.00	0.26	0.05	0.00	0.01	0.15
Q. 60%	2019	$REMod-REF$	0.04	1.03	25.01	0.79	0.00	15.92	10.41	0.64	0.00	0.78	5.86
		$REMod-SUF$	0.02	0.70	16.81	0.57	0.00	10.71	7.03	0.43	0.00	0.53	5.96
		$REMod-PER$	0.04	0.91	18.03	0.67	0.00	11.48	8.23	0.63	0.00	0.75	4.81
		$REMod-UNA$	0.06	1.16	25.60	0.74	0.00	16.29	10.42	0.36	0.00	0.57	6.23
	Mean	$REMod-REF$	2.15	3.10	62.70	3.12	35.30	0.39	23.99	1.00	0.02	2.39	2.78
		$REMod-SUF$	1.45	2.10	42.15	2.25	25.85	0.26	16.21	0.68	0.01	1.63	2.82
		$REMod-PER$	2.45	2.72	45.20	2.65	36.16	0.28	18.98	0.98	0.02	2.30	2.28
		$REMod-UNA$	3.44	3.47	64.17	2.95	53.93	0.40	24.01	0.56	0.01	1.77	2.95
Q. 75%	2019	$REMod-REF$	5.73	7.52	331.80	25.23	23.97	17.85	88.19	1.42	0.02	8.54	7.27
		$REMod-SUF$	3.86	5.10	223.07	18.24	17.55	12.00	59.61	0.97	0.01	5.82	7.39
		$REMod-PER$	6.52	6.60	239.19	21.43	24.56	12.87	69.77	1.40	0.03	8.21	5.96
		$REMod-UNA$	9.13	8.42	339.59	23.87	36.62	18.27	88.28	0.80	0.02	6.30	7.72
	Mean	$REMod-REF$	8.85	7.55	349.06	25.33	130.91	13.87	88.19	1.42	0.44	8.55	7.27
		$REMod-SUF$	5.97	5.12	234.67	18.31	95.86	9.33	59.61	0.97	0.29	5.82	7.39
		$REMod-PER$	10.08	6.63	251.63	21.52	134.13	10.00	69.77	1.40	0.55	8.22	5.96
		$REMod-UNA$	14.11	8.45	357.26	23.97	200.01	14.19	88.28	0.80	0.31	6.31	7.72

This table displays the expected loss due to scarcity for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), per path ($REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$), as well as per scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) for the input variables. Hereby, the values are derived from the GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). Hereby, the results are derived under the robustness test for the exposure at scarcity (EAS), in particular, the exposure at scarcity is calculated as the total required commodity amount scaled by the average commodity world production of the period from 2015 to 2019.

Table D.60: Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the MS-GVAR models of the robustness analysis for the exposure at scarcity for the reduced sample period from 2015 to 2019

Scenario	Basis	Path	Al	Cu	Ni	Pb	Sn	Zn
Mean	2019	$REMod-REF$	1.90	2.84	9.35	1.30	0.44	7.27
		$REMod-SUF$	1.29	2.05	6.32	0.88	0.30	7.39

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the MS-GVAR models of the robustness analysis for the exposure at scarcity for the reduced sample period from 2015 to 2019

			Al	Cu	Ni	Pb	Sn	Zn
Mean		$REMod-PER$	1.67	2.41	7.40	1.28	0.43	5.96
		$REMod-UNA$	2.13	2.68	9.36	0.73	0.33	7.72
	Mean	$REMod-REF$	7.52	24.98	46.74	1.42	3.68	4.42
		$REMod-SUF$	5.10	18.05	31.59	0.97	2.50	4.49
		$REMod-PER$	6.60	21.21	36.98	1.40	3.54	3.62
		$REMod-UNA$	8.42	23.63	46.79	0.80	2.71	4.69
Shock	2019	$REMod-REF$	7.55	25.33	88.19	1.42	8.55	7.27
		$REMod-SUF$	5.12	18.31	59.61	0.97	5.82	7.39
		$REMod-PER$	6.63	21.52	69.77	1.40	8.22	5.96
		$REMod-UNA$	8.45	23.97	88.28	0.80	6.31	7.72
	Mean	$REMod-REF$	7.55	25.33	88.19	1.42	8.55	7.27
		$REMod-SUF$	5.12	18.31	59.61	0.97	5.82	7.39
		$REMod-PER$	6.63	21.52	69.77	1.40	8.22	5.96
		$REMod-UNA$	8.45	23.97	88.28	0.80	6.31	7.72
Extr.	2019	$REMod-REF$	7.55	25.33	88.19	1.42	8.55	7.27
		$REMod-SUF$	5.12	18.31	59.61	0.97	5.82	7.39
		$REMod-PER$	6.63	21.52	69.77	1.40	8.22	5.96
		$REMod-UNA$	8.45	23.97	88.28	0.80	6.31	7.72
	Mean	$REMod-REF$	7.55	25.33	88.19	1.42	8.55	7.27
		$REMod-SUF$	5.12	18.31	59.61	0.97	5.82	7.39
		$REMod-PER$	6.63	21.52	69.77	1.40	8.22	5.96
		$REMod-UNA$	8.45	23.97	88.28	0.80	6.31	7.72
Foc. EA	2019	$REMod-REF$	7.26	15.40	68.97	1.42	4.36	7.27
		$REMod-SUF$	4.92	11.13	46.62	0.96	2.97	7.39
		$REMod-PER$	6.37	13.08	54.56	1.40	4.19	5.96
		$REMod-UNA$	8.13	14.57	69.04	0.80	3.22	7.72
	Mean	$REMod-REF$	7.55	25.33	85.37	1.42	7.99	7.14
		$REMod-SUF$	5.12	18.31	57.70	0.97	5.44	7.26
		$REMod-PER$	6.63	21.52	67.53	1.40	7.68	5.85
		$REMod-UNA$	8.45	23.97	85.46	0.80	5.89	7.58
Foc. FX	2019	$REMod-REF$	7.53	23.41	81.84	1.42	4.99	7.27
		$REMod-SUF$	5.11	16.92	55.32	0.97	3.40	7.39
		$REMod-PER$	6.61	19.88	64.74	1.40	4.80	5.96
		$REMod-UNA$	8.44	22.15	81.92	0.80	3.68	7.72
	Mean	$REMod-REF$	7.55	25.33	88.02	1.42	8.00	7.08
		$REMod-SUF$	5.12	18.31	59.49	0.97	5.45	7.20
		$REMod-PER$	6.63	21.52	69.63	1.40	7.70	5.80
		$REMod-UNA$	8.45	23.97	88.10	0.80	5.90	7.52
Foc. FFR	2019	$REMod-REF$	7.55	25.33	88.19	1.42	8.46	7.27
		$REMod-SUF$	5.12	18.31	59.61	0.97	5.77	7.39
		$REMod-PER$	6.63	21.52	69.77	1.40	8.14	5.96
		$REMod-UNA$	8.45	23.97	88.28	0.80	6.24	7.72
	Mean	$REMod-REF$	7.55	25.33	88.19	1.42	8.52	7.27
		$REMod-SUF$	5.12	18.31	59.61	0.97	5.80	7.39
		$REMod-PER$	6.63	21.52	69.77	1.40	8.19	5.96
		$REMod-UNA$	8.45	23.97	88.28	0.80	6.28	7.72
Foc. Extr. EA	2019	$REMod-REF$	7.50	22.29	82.02	1.42	7.18	7.27
		$REMod-SUF$	5.09	16.11	55.44	0.96	4.89	7.39
		$REMod-PER$	6.59	18.93	64.88	1.40	6.91	5.96
		$REMod-UNA$	8.40	21.09	82.10	0.80	5.30	7.72
	Mean	$REMod-REF$	7.55	25.33	87.31	1.42	8.31	7.24
		$REMod-SUF$	5.12	18.31	59.02	0.97	5.66	7.36
		$REMod-PER$	6.63	21.52	69.07	1.40	7.99	5.93
		$REMod-UNA$	8.45	23.97	87.40	0.80	6.13	7.69
Foc. Extr. FX	2019	$REMod-REF$	7.55	24.88	87.14	1.42	7.25	7.27
		$REMod-SUF$	5.12	17.98	58.90	0.97	4.94	7.39
		$REMod-PER$	6.63	21.13	68.93	1.40	6.97	5.96
		$REMod-UNA$	8.45	23.54	87.22	0.80	5.35	7.72
	Mean	$REMod-REF$	7.55	25.33	88.02	1.42	8.41	7.22
		$REMod-SUF$	5.12	18.31	59.49	0.97	5.73	7.34
		$REMod-PER$	6.63	21.52	69.63	1.40	8.09	5.92
		$REMod-UNA$	8.45	23.97	88.10	0.80	6.21	7.67
Foc. Extr. FFR	2019	$REMod-REF$	7.55	25.33	88.19	1.42	8.52	7.27
		$REMod-SUF$	5.12	18.31	59.61	0.97	5.80	7.39
		$REMod-PER$	6.63	21.52	69.77	1.40	8.19	5.96
		$REMod-UNA$	8.45	23.97	88.28	0.80	6.28	7.72
	Mean	$REMod-REF$	7.55	25.33	88.19	1.42	8.55	7.27
		$REMod-SUF$	5.12	18.31	59.61	0.97	5.82	7.39
		$REMod-PER$	6.63	21.52	69.77	1.40	8.22	5.96

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the MS-GVAR models of the robustness analysis for the exposure at scarcity for the reduced sample period from 2015 to 2019

			Al	Cu	Ni	Pb	Sn	Zn
		$REMod-UNA$	8.45	23.97	88.28	0.80	6.31	7.72
Q. 25%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00
	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00
Q. 40%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.10
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.10
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.08
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.11
	Mean	$REMod-REF$	0.00	0.00	0.00	0.01	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.01	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.01	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00
Q. 50%	2019	$REMod-REF$	0.00	0.10	0.35	0.35	0.09	7.08
		$REMod-SUF$	0.00	0.07	0.24	0.24	0.06	7.20
		$REMod-PER$	0.00	0.09	0.28	0.35	0.08	5.80
		$REMod-UNA$	0.00	0.10	0.35	0.20	0.06	7.52
	Mean	$REMod-REF$	1.52	5.72	6.17	1.02	1.15	0.87
		$REMod-SUF$	1.03	4.14	4.17	0.69	0.78	0.89
		$REMod-PER$	1.34	4.86	4.88	1.00	1.10	0.71
		$REMod-UNA$	1.71	5.42	6.18	0.57	0.85	0.93
Q. 60%	2019	$REMod-REF$	7.55	25.33	88.19	1.42	8.21	7.27
		$REMod-SUF$	5.12	18.31	59.61	0.97	5.59	7.39
		$REMod-PER$	6.63	21.52	69.77	1.40	7.89	5.96
		$REMod-UNA$	8.45	23.97	88.28	0.80	6.05	7.72
	Mean	$REMod-REF$	7.55	25.33	88.19	1.42	8.55	7.27
		$REMod-SUF$	5.12	18.31	59.61	0.97	5.82	7.39
		$REMod-PER$	6.63	21.52	69.77	1.40	8.22	5.96
		$REMod-UNA$	8.45	23.97	88.28	0.80	6.31	7.72
Q. 75%	2019	$REMod-REF$	7.55	25.33	88.19	1.42	8.55	7.27
		$REMod-SUF$	5.12	18.31	59.61	0.97	5.82	7.39
		$REMod-PER$	6.63	21.52	69.77	1.40	8.22	5.96
		$REMod-UNA$	8.45	23.97	88.28	0.80	6.31	7.72
	Mean	$REMod-REF$	7.55	25.33	88.19	1.42	8.55	7.27
		$REMod-SUF$	5.12	18.31	59.61	0.97	5.82	7.39
		$REMod-PER$	6.63	21.52	69.77	1.40	8.22	5.96
		$REMod-UNA$	8.45	23.97	88.28	0.80	6.31	7.72

This table displays the expected loss due to scarcity for the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), per path ($REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$), as well as per scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) for the input variables. Hereby, the values are derived from the MS-GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). Hereby, the results are derived under the robustness test for the exposure at scarcity (EAS), in particular, the exposure at scarcity is calculated as the total required commodity amount scaled by the average commodity world production of the period from 2015 to 2019.

Table D.61: Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the logistic regression models of the robustness analysis for the exposure at scarcity for the reduced sample period from 2015 to 2019

		Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
Mean	$REMod-REF$	0.29	0.35	23.51	1.17	2.16	12.83	0.84	0.28	4.57	0.09	0.02	0.16	0.87
	$REMod-SUF$	0.19	0.24	15.80	0.85	1.55	9.40	0.57	0.20	3.09	0.06	0.01	0.11	0.88

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the logistic regression models of the robustness analysis for the exposure at scarcity for the reduced sample period from 2015 to 2019

		Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
Mean	$REMod-PER$	0.33	0.31	16.95	1.00	2.08	13.15	0.61	0.26	3.61	0.09	0.02	0.15	0.71
	$REMod-UNA$	0.46	0.39	24.06	1.11	1.03	19.61	0.86	0.12	4.57	0.05	0.01	0.12	0.92
Shock	$REMod-REF$	0.41	0.71	178.44	1.68	9.51	23.00	1.24	1.24	20.06	0.25	0.04	0.23	6.83
	$REMod-SUF$	0.28	0.48	119.97	1.21	6.80	16.84	0.83	0.88	13.56	0.17	0.03	0.16	6.94
	$REMod-PER$	0.47	0.62	128.64	1.43	9.16	23.56	0.89	1.15	15.87	0.25	0.05	0.22	5.60
	$REMod-UNA$	0.65	0.79	182.63	1.59	4.53	35.14	1.27	0.53	20.08	0.14	0.03	0.17	7.25
Extr.	$REMod-REF$	0.58	1.36	329.85	2.38	35.46	38.60	1.80	4.63	54.12	0.58	0.10	0.34	7.26
	$REMod-SUF$	0.39	0.92	221.76	1.72	25.35	28.27	1.21	3.29	36.58	0.39	0.07	0.23	7.38
	$REMod-PER$	0.66	1.20	237.79	2.02	34.15	39.55	1.30	4.29	42.81	0.57	0.13	0.32	5.95
	$REMod-UNA$	0.93	1.52	337.60	2.25	16.91	58.98	1.84	1.98	54.17	0.32	0.07	0.25	7.71
Foc. EA	$REMod-REF$	0.29	0.35	23.51	1.17	4.58	12.83	0.84	0.60	4.57	0.09	0.02	0.16	0.87
	$REMod-SUF$	0.19	0.24	15.80	0.85	3.27	9.40	0.57	0.43	3.09	0.06	0.01	0.11	0.88
	$REMod-PER$	0.33	0.31	16.95	1.00	4.41	13.15	0.61	0.55	3.61	0.09	0.02	0.15	0.71
	$REMod-UNA$	0.46	0.39	24.06	1.11	2.18	19.61	0.86	0.26	4.57	0.05	0.01	0.12	0.92
Foc. FX	$REMod-REF$	0.29	0.35	23.51	1.20	2.45	16.55	0.84	0.32	4.57	0.15	0.02	0.16	0.87
	$REMod-SUF$	0.19	0.24	15.80	0.87	1.75	12.12	0.57	0.23	3.09	0.10	0.01	0.11	0.88
	$REMod-PER$	0.33	0.31	16.95	1.02	2.36	16.96	0.61	0.30	3.61	0.15	0.03	0.15	0.71
	$REMod-UNA$	0.46	0.39	24.06	1.14	1.17	25.29	0.86	0.14	4.57	0.08	0.01	0.12	0.92
Foc. FFR	$REMod-REF$	0.29	0.35	23.51	1.34	4.05	12.83	0.84	0.53	4.57	0.09	0.02	0.16	2.23
	$REMod-SUF$	0.19	0.24	15.80	0.97	2.89	9.40	0.57	0.38	3.09	0.06	0.01	0.11	2.27
	$REMod-PER$	0.33	0.31	16.95	1.14	3.90	13.15	0.61	0.49	3.61	0.09	0.02	0.15	1.83
	$REMod-UNA$	0.46	0.39	24.06	1.27	1.93	19.61	0.86	0.23	4.57	0.05	0.01	0.12	2.37
Foc. Extr. EA	$REMod-REF$	0.29	0.35	23.51	1.17	9.51	12.83	0.84	1.24	4.57	0.09	0.02	0.16	0.87
	$REMod-SUF$	0.19	0.24	15.80	0.85	6.80	9.40	0.57	0.88	3.09	0.06	0.01	0.11	0.88
	$REMod-PER$	0.33	0.31	16.95	1.00	9.15	13.15	0.61	1.15	3.61	0.09	0.02	0.15	0.71
	$REMod-UNA$	0.46	0.39	24.06	1.11	4.53	19.61	0.86	0.53	4.57	0.05	0.01	0.12	0.92
Foc. Extr. FX	$REMod-REF$	0.29	0.35	23.51	1.23	2.77	21.16	0.84	0.36	4.57	0.23	0.03	0.16	0.87
	$REMod-SUF$	0.19	0.24	15.80	0.89	1.98	15.49	0.57	0.26	3.09	0.16	0.02	0.11	0.88
	$REMod-PER$	0.33	0.31	16.95	1.04	2.67	21.68	0.61	0.34	3.61	0.23	0.03	0.15	0.71
	$REMod-UNA$	0.46	0.39	24.06	1.16	1.32	32.32	0.86	0.16	4.57	0.13	0.02	0.12	0.92
Foc. Extr. FFR	$REMod-REF$	0.29	0.35	23.51	1.53	7.50	12.83	0.84	0.98	4.57	0.09	0.02	0.16	4.30
	$REMod-SUF$	0.19	0.24	15.80	1.10	5.36	9.40	0.57	0.70	3.09	0.06	0.01	0.11	4.37
	$REMod-PER$	0.33	0.31	16.95	1.30	7.22	13.15	0.61	0.91	3.61	0.09	0.02	0.15	3.53
	$REMod-UNA$	0.46	0.39	24.06	1.44	3.57	19.61	0.86	0.42	4.57	0.05	0.01	0.12	4.57
Q. 25%	$REMod-REF$	0.28	0.32	5.02	1.05	1.25	10.91	0.70	0.16	1.66	0.05	0.01	0.14	0.04
	$REMod-SUF$	0.19	0.22	3.38	0.76	0.89	7.99	0.47	0.12	1.12	0.03	0.01	0.10	0.04
	$REMod-PER$	0.32	0.28	3.62	0.89	1.21	11.18	0.50	0.15	1.32	0.05	0.02	0.14	0.04
	$REMod-UNA$	0.44	0.36	5.14	0.99	0.60	16.67	0.71	0.07	1.66	0.03	0.01	0.10	0.05
Q. 40%	$REMod-REF$	0.28	0.33	9.09	1.10	1.60	11.18	0.71	0.21	2.99	0.05	0.01	0.15	0.17
	$REMod-SUF$	0.19	0.22	6.11	0.80	1.15	8.19	0.48	0.15	2.02	0.04	0.01	0.11	0.17
	$REMod-PER$	0.32	0.29	6.55	0.93	1.54	11.46	0.51	0.19	2.36	0.05	0.02	0.15	0.14
	$REMod-UNA$	0.45	0.36	9.30	1.04	0.76	17.09	0.73	0.09	2.99	0.03	0.01	0.11	0.18
Q. 50%	$REMod-REF$	0.28	0.33	13.43	1.14	1.89	11.37	0.72	0.25	4.39	0.06	0.01	0.16	0.40
	$REMod-SUF$	0.19	0.22	9.03	0.82	1.35	8.33	0.49	0.18	2.97	0.04	0.01	0.11	0.41
	$REMod-PER$	0.32	0.29	9.68	0.97	1.82	11.65	0.52	0.23	3.47	0.06	0.02	0.16	0.33
	$REMod-UNA$	0.45	0.37	13.75	1.08	0.90	17.37	0.74	0.11	4.39	0.03	0.01	0.12	0.42
Q. 60%	$REMod-REF$	0.32	0.43	41.12	1.28	3.14	13.39	0.74	0.41	7.65	0.09	0.02	0.17	1.65
	$REMod-SUF$	0.21	0.29	27.64	0.92	2.24	9.81	0.50	0.29	5.17	0.06	0.01	0.12	1.68
	$REMod-PER$	0.36	0.37	29.64	1.08	3.02	13.72	0.53	0.38	6.05	0.09	0.02	0.16	1.36
	$REMod-UNA$	0.51	0.48	42.08	1.21	1.50	20.46	0.75	0.18	7.66	0.05	0.01	0.13	1.76
	$REMod-REF$	0.38	0.62	157.05	1.52	6.63	17.02	0.76	0.87	16.62	0.17	0.03	0.18	5.60

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the logistic regression models of the robustness analysis for the exposure at scarcity for the reduced sample period from 2015 to 2019

		Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
Q. 75%	$REMod-SUF$	0.26	0.42	105.59	1.10	4.74	12.46	0.51	0.62	11.23	0.12	0.02	0.12	5.70
	$REMod-PER$	0.43	0.54	113.22	1.29	6.38	17.44	0.55	0.80	13.14	0.17	0.03	0.17	4.59
	$REMod-UNA$	0.61	0.69	160.74	1.44	3.16	26.00	0.78	0.37	16.63	0.10	0.02	0.13	5.95

This table displays the expected loss due to scarcity for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), per path ($REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$), as well as per scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) for the input variables, derived from the logistic regression model. Hereby, the results are derived under the robustness test for the exposure at scarcity (EAS), in particular, the exposure at scarcity is calculated as the total required commodity amount scaled by the average commodity world production of the period from 2015 to 2019.

D.3.2.4.2 Results of the Robustness Analysis for the Exposure at Scarcity of the enlarged Sample

Table D.62: Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the GVAR models of the robustness analysis for the exposure at scarcity for the enlarged sample period from 1995 to 2019

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
Mean	2019	$REMod-REF$	0.00	0.04	0.42	0.00	0.00	6.82	0.23	0.09	0.00	0.03	2.71
		$REMod-SUF$	0.00	0.02	0.28	0.00	0.00	4.59	0.16	0.06	0.00	0.02	2.76
		$REMod-PER$	0.00	0.03	0.30	0.00	0.00	4.92	0.19	0.09	0.00	0.03	2.22
		$REMod-UNA$	0.00	0.04	0.43	0.00	0.00	6.98	0.23	0.05	0.00	0.02	2.88
	Mean	$REMod-REF$	0.04	0.33	2.10	0.42	0.55	0.00	0.47	0.30	0.00	0.13	0.41
		$REMod-SUF$	0.03	0.22	1.41	0.31	0.40	0.00	0.32	0.20	0.00	0.09	0.42
		$REMod-PER$	0.05	0.29	1.51	0.36	0.56	0.00	0.37	0.29	0.00	0.12	0.34
		$REMod-UNA$	0.07	0.37	2.15	0.40	0.83	0.00	0.47	0.17	0.00	0.09	0.44
Shock	2019	$REMod-REF$	10.89	11.80	420.09	32.52	175.02	44.89	117.23	1.72	0.46	9.66	8.63
		$REMod-SUF$	7.35	8.00	282.43	23.50	128.16	30.18	79.23	1.17	0.31	6.58	8.78
		$REMod-PER$	12.41	10.36	302.84	27.62	179.33	32.37	92.73	1.69	0.58	9.29	7.08
		$REMod-UNA$	17.37	13.21	429.96	30.77	267.40	45.93	117.34	0.97	0.33	7.13	9.17
	Mean	$REMod-REF$	10.89	11.80	420.09	32.52	182.13	44.89	117.23	1.72	0.55	9.66	8.63
		$REMod-SUF$	7.35	8.00	282.43	23.50	133.36	30.18	79.23	1.17	0.36	6.58	8.78
		$REMod-PER$	12.41	10.36	302.84	27.62	186.61	32.37	92.73	1.69	0.69	9.29	7.08
		$REMod-UNA$	17.37	13.21	429.96	30.77	278.25	45.93	117.34	0.97	0.39	7.13	9.17
Extr.	2019	$REMod-REF$	10.89	11.80	420.09	32.52	182.13	44.89	117.23	1.72	0.55	9.66	8.63
		$REMod-SUF$	7.35	8.00	282.43	23.50	133.36	30.18	79.23	1.17	0.36	6.58	8.78
		$REMod-PER$	12.41	10.36	302.84	27.62	186.61	32.37	92.73	1.69	0.69	9.29	7.08
		$REMod-UNA$	17.37	13.21	429.96	30.77	278.25	45.93	117.34	0.97	0.39	7.13	9.17
	Mean	$REMod-REF$	10.89	11.80	420.09	32.52	182.13	44.89	117.23	1.72	0.55	9.66	8.63
		$REMod-SUF$	7.35	8.00	282.43	23.50	133.36	30.18	79.23	1.17	0.36	6.58	8.78
		$REMod-PER$	12.41	10.36	302.84	27.62	186.61	32.37	92.73	1.69	0.69	9.29	7.08
		$REMod-UNA$	17.37	13.21	429.96	30.77	278.25	45.93	117.34	0.97	0.39	7.13	9.17
Foc. EA	2019	$REMod-REF$	0.11	6.87	101.24	18.57	0.00	43.41	92.37	1.65	0.00	8.48	8.63
		$REMod-SUF$	0.07	4.65	68.07	13.42	0.00	29.18	62.44	1.12	0.00	5.77	8.78
		$REMod-PER$	0.12	6.03	72.98	15.77	0.00	31.30	73.07	1.62	0.00	8.15	7.08
		$REMod-UNA$	0.17	7.69	103.62	17.57	0.00	44.42	92.47	0.93	0.00	6.25	9.17
	Mean	$REMod-REF$	3.85	11.38	255.00	32.06	55.00	0.63	109.84	1.72	0.06	9.64	8.57
		$REMod-SUF$	2.60	7.72	171.44	23.17	40.28	0.42	74.24	1.17	0.04	6.56	8.72
		$REMod-PER$	4.38	9.99	183.82	27.23	56.36	0.45	86.89	1.69	0.07	9.27	7.03
		$REMod-UNA$	6.13	12.75	260.99	30.34	84.03	0.64	109.95	0.97	0.04	7.11	9.10
Foc. FX	2019	$REMod-REF$	0.04	1.17	22.69	3.77	0.00	32.82	43.14	0.83	0.00	2.80	8.52
		$REMod-SUF$	0.03	0.79	15.25	2.73	0.00	22.06	29.16	0.57	0.00	1.91	8.66
		$REMod-PER$	0.05	1.03	16.35	3.20	0.00	23.66	34.13	0.82	0.00	2.70	6.98
		$REMod-UNA$	0.07	1.31	23.22	3.57	0.00	33.58	43.18	0.47	0.00	2.07	9.05
	Mean	$REMod-REF$	1.92	4.42	65.11	17.40	64.65	0.40	67.05	1.24	0.02	6.00	7.49
		$REMod-SUF$	1.29	3.00	43.78	12.57	47.34	0.27	45.32	0.84	0.01	4.09	7.62
		$REMod-PER$	2.18	3.88	46.94	14.78	66.25	0.29	53.04	1.22	0.02	5.77	6.14
		$REMod-UNA$	3.06	4.95	66.64	16.46	98.78	0.41	67.12	0.70	0.01	4.43	7.96
Foc. FFR	2019	$REMod-REF$	0.05	0.70	23.53	0.78	0.00	34.88	7.50	0.47	0.00	0.46	5.99
		$REMod-SUF$	0.04	0.47	15.82	0.56	0.00	23.45	5.07	0.32	0.00	0.32	6.09
		$REMod-PER$	0.06	0.61	16.96	0.66	0.00	25.15	5.93	0.46	0.00	0.45	4.91
		$REMod-UNA$	0.09	0.78	24.08	0.74	0.00	35.69	7.51	0.26	0.00	0.34	6.36
	Mean	$REMod-REF$	1.29	2.67	50.83	3.12	36.61	0.31	15.94	0.77	0.02	1.77	2.70
		$REMod-SUF$	0.87	1.81	34.17	2.26	26.81	0.21	10.78	0.52	0.01	1.20	2.75
		$REMod-PER$	1.46	2.34	36.64	2.65	37.51	0.23	12.61	0.76	0.02	1.70	2.22
		$REMod-UNA$	2.05	2.99	52.03	2.95	55.93	0.32	15.96	0.43	0.01	1.30	2.87
Foc. Extr. EA	2019	$REMod-REF$	3.44	11.56	374.72	31.93	5.65	44.76	115.47	1.72	0.04	9.65	8.63
		$REMod-SUF$	2.32	7.84	251.93	23.08	4.13	30.09	78.05	1.17	0.03	6.57	8.78
		$REMod-PER$	3.92	10.15	270.13	27.12	5.78	32.28	91.34	1.69	0.05	9.28	7.08
		$REMod-UNA$	5.49	12.95	383.53	30.22	8.63	45.79	115.58	0.96	0.03	7.12	9.17
	Mean	$REMod-REF$	9.35	11.80	407.91	32.49	151.16	9.47	117.11	1.72	0.47	9.66	8.62
		$REMod-SUF$	6.31	8.00	274.24	23.48	110.69	6.37	79.16	1.17	0.31	6.58	8.77
		$REMod-PER$	10.64	10.36	294.06	27.59	154.88	6.83	92.64	1.69	0.59	9.29	7.07
		$REMod-UNA$	14.90	13.21	417.49	30.74	230.95	9.69	117.22	0.97	0.34	7.13	9.16
Foc. Extr. FX	2019	$REMod-REF$	1.12	6.29	149.97	25.07	2.19	40.90	101.17	1.45	0.00	7.82	8.62
		$REMod-SUF$	0.76	4.26	100.83	18.12	1.60	27.49	68.38	0.98	0.00	5.33	8.77
		$REMod-PER$	1.28	5.52	108.11	21.29	2.24	29.49	80.03	1.42	0.00	7.52	7.07
		$REMod-UNA$	1.79	7.04	153.50	23.72	3.34	41.84	101.27	0.81	0.00	5.77	9.16
	Mean	$REMod-REF$	5.88	9.38	233.99	30.70	144.24	5.48	108.08	1.59	0.09	8.95	8.49
		$REMod-SUF$	3.97	6.36	157.31	22.19	105.62	3.68	73.05	1.08	0.06	6.10	8.64
		$REMod-PER$	6.70	8.23	168.68	26.07	147.79	3.95	85.50	1.56	0.11	8.61	6.96
		$REMod-UNA$	9.38	10.50	239.49	29.05	220.37	5.60	108.19	0.89	0.06	6.60	9.02
	2019	$REMod-REF$	0.65	2.94	86.54	3.58	2.00	41.75	36.57	0.89	0.00	2.58	7.47
		$REMod-SUF$	0.44	1.99	58.18	2.59	1.47	28.07	24.72	0.61	0.00	1.76	7.60

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the GVAR models of the robustness analysis for the exposure at scarcity for the enlarged sample period from 1995 to 2019

			Ag	Al	Co	Cu	In	Li	Ni	Pb	Pt	Sn	Zn
Foc. Extr. FFR		$REMod-PER$	0.74	2.58	62.39	3.04	2.05	30.11	28.93	0.88	0.00	2.48	6.13
		$REMod-UNA$	1.04	3.29	88.57	3.38	3.06	42.72	36.61	0.50	0.00	1.90	7.94
	Mean	$REMod-REF$	4.11	5.46	158.38	9.98	85.42	5.34	53.34	1.17	0.06	4.69	5.32
		$REMod-SUF$	2.77	3.70	106.48	7.22	62.55	3.59	36.05	0.80	0.04	3.19	5.41
		$REMod-PER$	4.68	4.80	114.17	8.48	87.52	3.85	42.19	1.15	0.08	4.51	4.36
		$REMod-UNA$	6.55	6.12	162.10	9.45	130.50	5.47	53.39	0.66	0.04	3.46	5.65
Q. 25%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Q. 40%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Q. 50%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	1.08	0.12	0.03	0.00	0.00	0.92
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.72	0.08	0.02	0.00	0.00	0.94
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.78	0.09	0.03	0.00	0.00	0.76
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	1.10	0.12	0.02	0.00	0.00	0.98
	Mean	$REMod-REF$	0.02	0.18	0.42	0.03	0.00	0.00	0.35	0.11	0.00	0.01	0.16
		$REMod-SUF$	0.01	0.12	0.28	0.02	0.00	0.00	0.24	0.07	0.00	0.01	0.17
		$REMod-PER$	0.02	0.16	0.30	0.03	0.00	0.00	0.28	0.11	0.00	0.01	0.13
		$REMod-UNA$	0.03	0.20	0.43	0.03	0.00	0.00	0.35	0.06	0.00	0.01	0.17
Q. 60%	2019	$REMod-REF$	0.04	1.62	29.83	1.01	0.00	40.04	13.83	0.77	0.00	0.88	6.97
		$REMod-SUF$	0.03	1.10	20.05	0.73	0.00	26.92	9.35	0.52	0.00	0.60	7.08
		$REMod-PER$	0.05	1.42	21.50	0.86	0.00	28.88	10.94	0.76	0.00	0.85	5.71
		$REMod-UNA$	0.07	1.81	30.53	0.95	0.00	40.97	13.85	0.43	0.00	0.65	7.40
	Mean	$REMod-REF$	2.64	4.85	74.78	4.00	48.81	0.99	31.89	1.21	0.02	2.71	3.30
		$REMod-SUF$	1.78	3.29	50.27	2.89	35.74	0.66	21.55	0.82	0.01	1.84	3.35
		$REMod-PER$	3.00	4.26	53.91	3.40	50.01	0.71	25.22	1.19	0.02	2.60	2.70
		$REMod-UNA$	4.20	5.43	76.53	3.78	74.57	1.01	31.92	0.68	0.01	2.00	3.50
Q. 75%	2019	$REMod-REF$	7.00	11.75	395.73	32.39	33.15	44.89	117.23	1.72	0.02	9.66	8.63
		$REMod-SUF$	4.73	7.97	266.05	23.41	24.27	30.18	79.23	1.17	0.01	6.58	8.78
		$REMod-PER$	7.98	10.32	285.28	27.51	33.96	32.37	92.73	1.69	0.03	9.28	7.08
		$REMod-UNA$	11.17	13.16	405.03	30.65	50.64	45.93	117.34	0.97	0.02	7.12	9.17
	Mean	$REMod-REF$	10.83	11.80	416.31	32.52	181.03	34.88	117.23	1.72	0.44	9.66	8.63
		$REMod-SUF$	7.31	8.00	279.89	23.50	132.56	23.45	79.23	1.17	0.29	6.58	8.78
		$REMod-PER$	12.33	10.36	300.12	27.62	185.49	25.15	92.73	1.69	0.55	9.29	7.08
		$REMod-UNA$	17.26	13.21	426.09	30.77	276.58	35.69	117.34	0.97	0.31	7.13	9.17

This table displays the expected loss due to scarcity for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), indium (In), lithium (Li), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), per path ($REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$), as well as per scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) for the input variables. Hereby, the values are derived from the GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). Hereby, the results are derived under the robustness test for the exposure at scarcity (EAS), in particular, the exposure at scarcity is calculated as the total required commodity amount scaled by the average commodity world production of the period from 1995 to 2019.

Table D.63: Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the MS-GVAR models of the robustness analysis for the exposure at scarcity for the enlarged sample period from 1995 to 2019

		Al	Cu	Ni	Pb	Sn	Zn
2019	$REMod-REF$	2.97	3.64	12.43	1.57	0.50	8.63
	$REMod-SUF$	2.02	2.63	8.40	1.07	0.34	8.78

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the MS-GVAR models of the robustness analysis for the exposure at scarcity for the enlarged sample period from 1995 to 2019

			Al	Cu	Ni	Pb	Sn	Zn
Mean		$REMod-PER$	2.61	3.09	9.83	1.55	0.48	7.08
		$REMod-UNA$	3.33	3.45	12.44	0.88	0.37	9.17
	Mean	$REMod-REF$	11.75	32.06	62.13	1.72	4.16	5.25
		$REMod-SUF$	7.97	23.17	41.99	1.17	2.83	5.34
		$REMod-PER$	10.32	27.23	49.15	1.69	4.00	4.30
		$REMod-UNA$	13.16	30.34	62.19	0.97	3.07	5.57
Shock	2019	$REMod-REF$	11.80	32.52	117.23	1.72	9.66	8.63
		$REMod-SUF$	8.00	23.50	79.23	1.17	6.58	8.78
		$REMod-PER$	10.36	27.62	92.73	1.69	9.29	7.08
		$REMod-UNA$	13.21	30.77	117.34	0.97	7.13	9.17
	Mean	$REMod-REF$	11.80	32.52	117.23	1.72	9.66	8.63
		$REMod-SUF$	8.00	23.50	79.23	1.17	6.58	8.78
		$REMod-PER$	10.36	27.62	92.73	1.69	9.29	7.08
		$REMod-UNA$	13.21	30.77	117.34	0.97	7.13	9.17
Extr.	2019	$REMod-REF$	11.80	32.52	117.23	1.72	9.66	8.63
		$REMod-SUF$	8.00	23.50	79.23	1.17	6.58	8.78
		$REMod-PER$	10.36	27.62	92.73	1.69	9.29	7.08
		$REMod-UNA$	13.21	30.77	117.34	0.97	7.13	9.17
	Mean	$REMod-REF$	11.80	32.52	117.23	1.72	9.66	8.63
		$REMod-SUF$	8.00	23.50	79.23	1.17	6.58	8.78
		$REMod-PER$	10.36	27.62	92.73	1.69	9.29	7.08
		$REMod-UNA$	13.21	30.77	117.34	0.97	7.13	9.17
Foc. EA	2019	$REMod-REF$	11.35	19.77	91.67	1.72	4.93	8.63
		$REMod-SUF$	7.69	14.29	61.96	1.17	3.36	8.78
		$REMod-PER$	9.96	16.79	72.52	1.69	4.74	7.08
		$REMod-UNA$	12.71	18.71	91.76	0.96	3.64	9.17
	Mean	$REMod-REF$	11.80	32.52	113.47	1.72	9.03	8.48
		$REMod-SUF$	8.00	23.50	76.70	1.17	6.15	8.62
		$REMod-PER$	10.36	27.62	89.76	1.69	8.68	6.95
		$REMod-UNA$	13.21	30.77	113.59	0.97	6.66	9.00
Foc. FX	2019	$REMod-REF$	11.77	30.05	108.79	1.72	5.64	8.63
		$REMod-SUF$	7.98	21.72	73.53	1.17	3.85	8.78
		$REMod-PER$	10.34	25.52	86.05	1.69	5.43	7.08
		$REMod-UNA$	13.19	28.43	108.89	0.97	4.16	9.17
	Mean	$REMod-REF$	11.80	32.52	116.99	1.72	9.05	8.41
		$REMod-SUF$	8.00	23.50	79.08	1.17	6.16	8.55
		$REMod-PER$	10.36	27.62	92.55	1.69	8.70	6.89
		$REMod-UNA$	13.21	30.77	117.11	0.97	6.67	8.93
Foc. FFR	2019	$REMod-REF$	11.80	32.52	117.23	1.72	9.57	8.63
		$REMod-SUF$	8.00	23.50	79.23	1.17	6.52	8.78
		$REMod-PER$	10.36	27.62	92.73	1.69	9.20	7.08
		$REMod-UNA$	13.21	30.77	117.34	0.97	7.06	9.17
	Mean	$REMod-REF$	11.80	32.52	117.23	1.72	9.63	8.63
		$REMod-SUF$	8.00	23.50	79.23	1.17	6.56	8.78
		$REMod-PER$	10.36	27.62	92.73	1.69	9.26	7.08
		$REMod-UNA$	13.21	30.77	117.34	0.97	7.10	9.17
Foc. Extr. EA	2019	$REMod-REF$	11.73	28.62	109.02	1.72	8.12	8.63
		$REMod-SUF$	7.95	20.68	73.69	1.17	5.53	8.78
		$REMod-PER$	10.29	24.30	86.24	1.69	7.81	7.08
		$REMod-UNA$	13.13	27.08	109.13	0.96	5.99	9.17
	Mean	$REMod-REF$	11.80	32.52	116.05	1.72	9.39	8.60
		$REMod-SUF$	8.00	23.50	78.44	1.17	6.40	8.74
		$REMod-PER$	10.36	27.62	91.80	1.69	9.03	7.05
		$REMod-UNA$	13.21	30.77	116.17	0.97	6.93	9.13
Foc. Extr. FX	2019	$REMod-REF$	11.80	31.93	115.82	1.72	8.20	8.63
		$REMod-SUF$	8.00	23.08	78.28	1.17	5.58	8.78
		$REMod-PER$	10.36	27.12	91.62	1.69	7.88	7.08
		$REMod-UNA$	13.21	30.22	115.93	0.97	6.05	9.17
	Mean	$REMod-REF$	11.80	32.52	116.99	1.72	9.51	8.58
		$REMod-SUF$	8.00	23.50	79.08	1.17	6.48	8.72
		$REMod-PER$	10.36	27.62	92.55	1.69	9.15	7.03
		$REMod-UNA$	13.21	30.77	117.11	0.97	7.02	9.11
Foc. Extr. FFR	2019	$REMod-REF$	11.80	32.52	117.23	1.72	9.63	8.63
		$REMod-SUF$	8.00	23.50	79.23	1.17	6.56	8.78
		$REMod-PER$	10.36	27.62	92.73	1.69	9.26	7.08
		$REMod-UNA$	13.21	30.77	117.34	0.97	7.10	9.17
	Mean	$REMod-REF$	11.80	32.52	117.23	1.72	9.66	8.63
		$REMod-SUF$	8.00	23.50	79.23	1.17	6.58	8.78
		$REMod-PER$	10.36	27.62	92.73	1.69	9.29	7.08

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the MS-GVAR models of the robustness analysis for the exposure at scarcity for the enlarged sample period from 1995 to 2019

			Al	Cu	Ni	Pb	Sn	Zn
		$REMod-UNA$	13.21	30.77	117.34	0.97	7.13	9.17
Q. 25%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00
	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00
Q. 40%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.12
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.12
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.10
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.13
	Mean	$REMod-REF$	0.00	0.00	0.00	0.01	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.01	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.01	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.01	0.00	0.00
Q. 50%	2019	$REMod-REF$	0.00	0.13	0.47	0.43	0.10	8.41
		$REMod-SUF$	0.00	0.09	0.32	0.29	0.07	8.55
		$REMod-PER$	0.00	0.11	0.37	0.42	0.09	6.89
		$REMod-UNA$	0.00	0.12	0.47	0.24	0.07	8.93
	Mean	$REMod-REF$	2.38	7.35	8.21	1.23	1.30	1.04
		$REMod-SUF$	1.62	5.31	5.55	0.84	0.88	1.05
		$REMod-PER$	2.09	6.24	6.49	1.21	1.25	0.85
		$REMod-UNA$	2.67	6.95	8.21	0.69	0.96	1.10
Q. 60%	2019	$REMod-REF$	11.80	32.52	117.23	1.72	9.28	8.63
		$REMod-SUF$	8.00	23.50	79.23	1.17	6.32	8.78
		$REMod-PER$	10.36	27.62	92.73	1.69	8.92	7.08
		$REMod-UNA$	13.21	30.77	117.34	0.97	6.84	9.17
	Mean	$REMod-REF$	11.80	32.52	117.23	1.72	9.66	8.63
		$REMod-SUF$	8.00	23.50	79.23	1.17	6.58	8.78
		$REMod-PER$	10.36	27.62	92.73	1.69	9.29	7.08
		$REMod-UNA$	13.21	30.77	117.34	0.97	7.13	9.17
Q. 75%	2019	$REMod-REF$	11.80	32.52	117.23	1.72	9.66	8.63
		$REMod-SUF$	8.00	23.50	79.23	1.17	6.58	8.78
		$REMod-PER$	10.36	27.62	92.73	1.69	9.29	7.08
		$REMod-UNA$	13.21	30.77	117.34	0.97	7.13	9.17
	Mean	$REMod-REF$	11.80	32.52	117.23	1.72	9.66	8.63
		$REMod-SUF$	8.00	23.50	79.23	1.17	6.58	8.78
		$REMod-PER$	10.36	27.62	92.73	1.69	9.29	7.08
		$REMod-UNA$	13.21	30.77	117.34	0.97	7.13	9.17

This table displays the expected loss due to scarcity for the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), per path ($REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$), as well as per scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) for the input variables. Hereby, the values are derived from the MS-GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). Hereby, the results are derived under the robustness test for the exposure at scarcity (EAS), in particular, the exposure at scarcity is calculated as the total required commodity amount scaled by the average commodity world production of the period from 1995 to 2019.

Table D.64: Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the logistic regression models of the robustness analysis for the exposure at scarcity for the enlarged sample period from 1995 to 2019

		Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
Mean	$REMod-REF$	0.39	0.71	30.44	1.39	1.48	18.47	1.86	0.19	10.58	0.10	0.02	0.20	0.33
	$REMod-SUF$	0.26	0.48	20.46	1.00	1.06	13.52	1.25	0.14	7.15	0.07	0.01	0.13	0.33
	$REMod-PER$	0.44	0.62	21.94	1.18	1.42	18.92	1.34	0.18	8.37	0.10	0.02	0.19	0.27
	$REMod-UNA$	0.62	0.80	31.15	1.31	0.70	28.21	1.90	0.08	10.59	0.06	0.01	0.15	0.35
Shock	$REMod-REF$	0.52	1.36	194.03	1.98	7.70	29.06	2.59	1.01	65.35	0.25	0.04	0.31	6.95
	$REMod-SUF$	0.35	0.92	130.45	1.43	5.51	21.28	1.74	0.72	44.17	0.17	0.02	0.21	7.06
	$REMod-PER$	0.59	1.19	139.88	1.68	7.42	29.78	1.87	0.93	51.70	0.24	0.05	0.30	5.70
	$REMod-UNA$	0.83	1.52	198.59	1.87	3.67	44.40	2.65	0.43	65.42	0.14	0.03	0.23	7.38
Extr.	$REMod-REF$	0.70	2.46	379.82	2.80	33.85	44.10	3.58	4.42	110.33	0.55	0.08	0.48	8.61
	$REMod-SUF$	0.47	1.67	255.36	2.02	24.19	32.29	2.41	3.14	74.58	0.37	0.05	0.33	8.76
	$REMod-PER$	0.79	2.16	273.81	2.37	32.60	45.18	2.58	4.09	87.28	0.54	0.10	0.47	7.06
	$REMod-UNA$	1.11	2.76	388.74	2.65	16.14	67.37	3.67	1.89	110.44	0.31	0.06	0.36	9.15
Foc. EA	$REMod-REF$	0.39	0.71	30.44	1.39	2.99	18.47	1.86	0.39	10.58	0.10	0.02	0.20	0.33
	$REMod-SUF$	0.26	0.48	20.46	1.00	2.14	13.52	1.25	0.28	7.15	0.07	0.01	0.13	0.33
	$REMod-PER$	0.44	0.62	21.94	1.18	2.88	18.92	1.34	0.36	8.37	0.10	0.02	0.19	0.27
	$REMod-UNA$	0.62	0.80	31.15	1.31	1.43	28.21	1.90	0.17	10.59	0.06	0.01	0.15	0.35
Foc. FX	$REMod-REF$	0.39	0.71	30.44	1.41	1.63	22.58	1.86	0.21	10.58	0.15	0.02	0.20	0.33
	$REMod-SUF$	0.26	0.48	20.46	1.02	1.17	16.53	1.25	0.15	7.15	0.10	0.01	0.13	0.33
	$REMod-PER$	0.44	0.62	21.94	1.20	1.57	23.13	1.34	0.20	8.37	0.14	0.03	0.19	0.27
	$REMod-UNA$	0.62	0.80	31.15	1.34	0.78	34.49	1.90	0.09	10.59	0.08	0.02	0.15	0.35
Foc. FFR	$REMod-REF$	0.39	0.71	30.44	1.66	3.51	18.47	1.86	0.46	10.58	0.10	0.02	0.20	1.44
	$REMod-SUF$	0.26	0.48	20.46	1.20	2.51	13.52	1.25	0.33	7.15	0.07	0.01	0.13	1.47
	$REMod-PER$	0.44	0.62	21.94	1.41	3.38	18.92	1.34	0.42	8.37	0.10	0.02	0.19	1.18
	$REMod-UNA$	0.62	0.80	31.15	1.58	1.67	28.21	1.90	0.20	10.59	0.06	0.01	0.15	1.53
Foc. Extr. EA	$REMod-REF$	0.39	0.71	30.44	1.39	6.00	18.47	1.86	0.78	10.58	0.10	0.02	0.20	0.33
	$REMod-SUF$	0.26	0.48	20.46	1.00	4.29	13.52	1.25	0.56	7.15	0.07	0.01	0.13	0.33
	$REMod-PER$	0.44	0.62	21.94	1.18	5.78	18.92	1.34	0.73	8.37	0.10	0.02	0.19	0.27
	$REMod-UNA$	0.62	0.80	31.15	1.31	2.86	28.21	1.90	0.34	10.59	0.06	0.01	0.15	0.35
Foc. Extr. FX	$REMod-REF$	0.39	0.71	30.44	1.44	1.80	27.45	1.86	0.23	10.58	0.21	0.03	0.20	0.33
	$REMod-SUF$	0.26	0.48	20.46	1.04	1.29	20.10	1.25	0.17	7.15	0.14	0.02	0.13	0.33
	$REMod-PER$	0.44	0.62	21.94	1.22	1.73	28.13	1.34	0.22	8.37	0.21	0.03	0.19	0.27
	$REMod-UNA$	0.62	0.80	31.15	1.36	0.86	41.94	1.90	0.10	10.59	0.12	0.02	0.15	0.35
Foc. Extr. FFR	$REMod-REF$	0.39	0.71	30.44	1.99	8.16	18.47	1.86	1.07	10.58	0.10	0.02	0.20	4.36
	$REMod-SUF$	0.26	0.48	20.46	1.44	5.84	13.52	1.25	0.76	7.15	0.07	0.01	0.13	4.43
	$REMod-PER$	0.44	0.62	21.94	1.69	7.86	18.92	1.34	0.99	8.37	0.10	0.02	0.19	3.58
	$REMod-UNA$	0.62	0.80	31.15	1.88	3.89	28.21	1.90	0.46	10.59	0.06	0.01	0.15	4.63
Q. 25%	$REMod-REF$	0.35	0.52	7.44	1.14	0.57	15.64	1.74	0.07	2.82	0.06	0.01	0.15	0.02
	$REMod-SUF$	0.24	0.35	5.00	0.83	0.41	11.45	1.17	0.05	1.91	0.04	0.01	0.10	0.02
	$REMod-PER$	0.40	0.46	5.36	0.97	0.55	16.03	1.26	0.07	2.23	0.06	0.02	0.14	0.02
	$REMod-UNA$	0.56	0.58	7.62	1.08	0.27	23.90	1.78	0.03	2.83	0.04	0.01	0.11	0.02
Q. 40%	$REMod-REF$	0.37	0.59	13.75	1.31	0.98	17.38	1.75	0.13	8.90	0.07	0.02	0.17	0.09
	$REMod-SUF$	0.25	0.40	9.25	0.95	0.70	12.73	1.18	0.09	6.02	0.05	0.01	0.12	0.09
	$REMod-PER$	0.42	0.52	9.92	1.11	0.94	17.81	1.26	0.12	7.04	0.07	0.02	0.17	0.08
	$REMod-UNA$	0.59	0.67	14.08	1.24	0.47	26.56	1.79	0.05	8.91	0.04	0.01	0.13	0.10
Q. 50%	$REMod-REF$	0.38	0.63	22.22	1.39	1.22	18.24	1.78	0.16	13.27	0.08	0.02	0.19	0.25
	$REMod-SUF$	0.26	0.43	14.94	1.01	0.87	13.36	1.20	0.11	8.97	0.06	0.01	0.13	0.25
	$REMod-PER$	0.44	0.55	16.02	1.18	1.17	18.69	1.28	0.15	10.50	0.08	0.02	0.18	0.20
	$REMod-UNA$	0.61	0.71	22.74	1.32	0.58	27.87	1.82	0.07	13.28	0.05	0.01	0.14	0.27
Q. 60%	$REMod-REF$	0.42	0.77	46.94	1.51	1.81	19.83	1.81	0.24	16.79	0.11	0.02	0.20	0.83
	$REMod-SUF$	0.28	0.52	31.56	1.09	1.29	14.52	1.22	0.17	11.35	0.07	0.01	0.13	0.85
	$REMod-PER$	0.48	0.68	33.84	1.29	1.74	20.32	1.31	0.22	13.28	0.11	0.03	0.19	0.68

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the logistic regression models of the robustness analysis for the exposure at scarcity for the enlarged sample period from 1995 to 2019

		Ag	Al	Co	Cu	Dy	In	Li	Nd	Ni	Pb	Pt	Sn	Zn
	$REMod-UNA$	0.67	0.86	48.04	1.43	0.86	30.30	1.86	0.10	16.81	0.06	0.01	0.14	0.88
Q. 75%	$REMod-REF$	0.47	1.06	143.60	1.72	3.63	23.26	1.88	0.47	26.09	0.18	0.03	0.26	4.03
	$REMod-SUF$	0.32	0.72	96.55	1.24	2.59	17.03	1.27	0.34	17.63	0.13	0.02	0.18	4.10
	$REMod-PER$	0.53	0.93	103.52	1.46	3.50	23.83	1.36	0.44	20.64	0.18	0.03	0.25	3.31
	$REMod-UNA$	0.75	1.19	146.98	1.63	1.73	35.54	1.93	0.20	26.12	0.10	0.02	0.19	4.28

This table displays the expected loss due to scarcity for the commodities silver (Ag), aluminum (Al), cobalt (Co), copper (Cu), dysprosium (Dy), indium (In), lithium (Li), neodymium (Nd), nickel (Ni), lead (Pb), platinum (Pt), tin (Sn), and zinc (Zn), per path ($REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$), as well as per scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) for the input variables, derived from the logistic regression model. Hereby, the results are derived under the robustness test for the exposure at scarcity (EAS), in particular, the exposure at scarcity is calculated as the total required commodity amount scaled by the average commodity world production of the period from 1995 to 2019.

D.3.2.5 Robustness Analysis for the Industrial Metal Markets

Table D.65: Probability of scarcity per commodity derived from the GVAR models of the robustness analysis for the commodity set restricted to the industrial metals

			Al	Cu	Ni	Pb	Sn	Zn
REMod – REF	2019	Mean	0.01	0.01	0.00	0.10	0.00	0.52
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.80	0.81	0.88	0.99	0.95	1.00
		Foc. FX	0.22	0.31	0.41	0.61	0.49	1.00
		Foc. FFR	0.11	0.05	0.05	0.37	0.09	0.84
		Foc. Extr. EA	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. FX	0.66	0.87	0.92	0.88	0.92	1.00
		Foc. Extr. FFR	0.31	0.21	0.26	0.61	0.37	0.95
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.01
		Q. 50%	0.01	0.00	0.00	0.05	0.00	0.23
		Q. 60%	0.23	0.12	0.11	0.55	0.12	0.90
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
	Mean	Mean	0.09	0.04	0.01	0.24	0.03	0.15
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	0.98	1.00	1.00	1.00
		Foc. FX	0.60	0.75	0.71	0.82	0.82	0.95
		Foc. FFR	0.32	0.21	0.12	0.59	0.30	0.49
		Foc. Extr. EA	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. FX	0.88	0.96	0.97	0.95	0.98	0.99
		Foc. Extr. FFR	0.60	0.48	0.43	0.77	0.61	0.74
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.06	0.01	0.00	0.12	0.01	0.06
		Q. 60%	0.54	0.33	0.23	0.78	0.33	0.59
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
REMod – SUF	2019	Mean	0.03	0.02	0.01	0.13	0.01	0.58
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.86	0.85	0.91	1.00	0.96	1.00
		Foc. FX	0.30	0.43	0.53	0.64	0.53	1.00
		Foc. FFR	0.16	0.11	0.11	0.40	0.12	0.87
		Foc. Extr. EA	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. FX	0.71	0.88	0.93	0.88	0.92	1.00
		Foc. Extr. FFR	0.37	0.30	0.36	0.62	0.40	0.95
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.01
		Q. 50%	0.02	0.01	0.00	0.04	0.00	0.29
		Q. 60%	0.35	0.23	0.23	0.59	0.16	0.92
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
	Mean	Mean	0.14	0.09	0.03	0.28	0.04	0.23
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	0.99	1.00	1.00	1.00
		Foc. FX	0.65	0.82	0.76	0.81	0.83	0.95
		Foc. FFR	0.39	0.31	0.22	0.60	0.33	0.56
		Foc. Extr. EA	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. FX	0.91	0.97	0.97	0.95	0.98	0.99
		Foc. Extr. FFR	0.66	0.59	0.51	0.77	0.65	0.78
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.07	0.03	0.02	0.14	0.01	0.11
		Q. 60%	0.63	0.51	0.39	0.80	0.40	0.71
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
REMod – PER	2019	Mean	0.01	0.01	0.00	0.10	0.00	0.53
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.79	0.81	0.88	0.99	0.95	1.00
		Foc. FX	0.21	0.31	0.42	0.61	0.49	1.00
		Foc. FFR	0.10	0.05	0.04	0.38	0.09	0.85
		Foc. Extr. EA	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. FX	0.65	0.87	0.93	0.88	0.92	1.00
		Foc. Extr. FFR	0.30	0.21	0.26	0.61	0.37	0.95
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00

Probability of scarcity per commodity derived from the GVAR models of the robustness analysis for the commodity set restricted to the industrial metals

			Al	Cu	Ni	Pb	Sn	Zn
$REMod-PER$	2019	Q. 40%	0.00	0.00	0.00	0.00	0.00	0.01
		Q. 50%	0.01	0.00	0.00	0.05	0.00	0.24
		Q. 60%	0.22	0.12	0.11	0.57	0.11	0.91
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
	Mean	Mean	0.07	0.04	0.01	0.24	0.03	0.15
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	0.98	1.00	1.00	1.00
		Foc. FX	0.58	0.75	0.71	0.82	0.82	0.95
		Foc. FFR	0.31	0.20	0.12	0.59	0.28	0.50
		Foc. Extr. EA	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. FX	0.88	0.96	0.98	0.95	0.98	0.99
		Foc. Extr. FFR	0.59	0.48	0.42	0.77	0.61	0.76
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.05	0.01	0.00	0.12	0.01	0.06
		Q. 60%	0.52	0.35	0.23	0.77	0.32	0.61
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00
$REMod-UNA$	2019	Mean	0.01	0.01	0.00	0.07	0.00	0.47
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	0.80	0.80	0.88	0.99	0.95	1.00
		Foc. FX	0.21	0.30	0.40	0.55	0.45	1.00
		Foc. FFR	0.10	0.05	0.05	0.32	0.07	0.79
		Foc. Extr. EA	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. FX	0.65	0.86	0.92	0.86	0.91	1.00
		Foc. Extr. FFR	0.30	0.19	0.25	0.57	0.34	0.92
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.01
		Q. 50%	0.01	0.00	0.00	0.03	0.00	0.20
		Q. 60%	0.20	0.09	0.10	0.46	0.07	0.85
		Q. 75%	1.00	0.99	1.00	1.00	1.00	1.00
	Mean	Mean	0.09	0.04	0.01	0.19	0.01	0.14
		Shock	1.00	1.00	1.00	1.00	1.00	1.00
		Extr.	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. EA	1.00	1.00	0.98	1.00	1.00	1.00
		Foc. FX	0.56	0.75	0.70	0.78	0.80	0.94
		Foc. FFR	0.31	0.19	0.12	0.53	0.26	0.46
		Foc. Extr. EA	1.00	1.00	1.00	1.00	1.00	1.00
		Foc. Extr. FX	0.87	0.96	0.97	0.94	0.98	1.00
		Foc. Extr. FFR	0.58	0.48	0.41	0.75	0.58	0.72
		Q. 25%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 40%	0.00	0.00	0.00	0.00	0.00	0.00
		Q. 50%	0.05	0.01	0.01	0.08	0.00	0.06
		Q. 60%	0.49	0.30	0.20	0.73	0.23	0.53
		Q. 75%	1.00	1.00	1.00	1.00	1.00	1.00

This table displays the probability of scarcity (PS) of the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb),tin (Sn), and zinc (Zn), derived from the GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$, and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). Hereby, the probability of scarcity is calculated for the scenarios Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%) of the input variables. The presented results are derived under the robustness test for the restricted commodity set, in particular, only the industrial metals are considered.

Table D.66: Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the GVAR models of the robustness analysis for the commodity set restricted to the industrial metals

			Al	Cu	Ni	Pb	Sn	Zn
Mean	2019	$REMod-REF$	0.12	0.16	0.17	0.14	0.05	3.76
		$REMod-SUF$	0.08	0.12	0.12	0.10	0.03	3.83
		$REMod-PER$	0.10	0.14	0.14	0.14	0.04	3.09
		$REMod-UNA$	0.13	0.15	0.17	0.08	0.03	4.00
	Mean	$REMod-REF$	0.76	1.12	0.79	0.34	0.26	1.10
		$REMod-SUF$	0.52	0.81	0.53	0.23	0.18	1.11
		$REMod-PER$	0.67	0.95	0.62	0.33	0.25	0.90
		$REMod-UNA$	0.85	1.06	0.79	0.19	0.20	1.16
Shock	2019	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
Extr.	2019	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
Foc. EA	2019	$REMod-REF$	6.65	21.98	76.62	1.37	8.69	7.21
		$REMod-SUF$	4.51	15.89	51.79	0.93	5.92	7.33
		$REMod-PER$	5.84	18.67	60.61	1.35	8.35	5.91
		$REMod-UNA$	7.44	20.80	76.69	0.77	6.41	7.66
	Mean	$REMod-REF$	8.35	27.21	85.88	1.38	9.12	7.20
		$REMod-SUF$	5.66	19.67	58.04	0.94	6.21	7.32
		$REMod-PER$	7.33	23.11	67.93	1.36	8.77	5.90
		$REMod-UNA$	9.35	25.75	85.96	0.78	6.72	7.65
Foc. FX	2019	$REMod-REF$	1.81	8.37	35.82	0.84	4.49	7.20
		$REMod-SUF$	1.22	6.05	24.21	0.57	3.06	7.32
		$REMod-PER$	1.59	7.11	28.33	0.83	4.32	5.90
		$REMod-UNA$	2.02	7.92	35.85	0.47	3.32	7.65
	Mean	$REMod-REF$	5.01	20.56	61.94	1.13	7.50	6.82
		$REMod-SUF$	3.40	14.86	41.87	0.77	5.11	6.93
		$REMod-PER$	4.40	17.46	49.00	1.11	7.21	5.59
		$REMod-UNA$	5.61	19.45	62.00	0.64	5.53	7.24
Foc. FFR	2019	$REMod-REF$	0.89	1.42	4.11	0.52	0.82	6.05
		$REMod-SUF$	0.61	1.02	2.78	0.35	0.56	6.15
		$REMod-PER$	0.79	1.20	3.25	0.51	0.79	4.96
		$REMod-UNA$	1.00	1.34	4.11	0.29	0.61	6.42
	Mean	$REMod-REF$	2.68	5.75	10.57	0.81	2.72	3.56
		$REMod-SUF$	1.82	4.16	7.14	0.55	1.85	3.62
		$REMod-PER$	2.36	4.89	8.36	0.80	2.61	2.92
		$REMod-UNA$	3.01	5.44	10.58	0.46	2.00	3.78
Foc. Extr. EA	2019	$REMod-REF$	8.33	27.19	87.01	1.38	9.12	7.21
		$REMod-SUF$	5.65	19.65	58.81	0.94	6.21	7.33
		$REMod-PER$	7.31	23.09	68.83	1.36	8.77	5.91
		$REMod-UNA$	9.33	25.72	87.10	0.78	6.72	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66
Foc. Extr. FX	2019	$REMod-REF$	5.53	23.72	80.63	1.21	8.34	7.21
		$REMod-SUF$	3.75	17.15	54.50	0.82	5.68	7.33
		$REMod-PER$	4.85	20.15	63.79	1.19	8.02	5.91
		$REMod-UNA$	6.19	22.45	80.71	0.68	6.15	7.66
	Mean	$REMod-REF$	7.38	26.20	84.91	1.31	8.91	7.17
		$REMod-SUF$	5.01	18.94	57.40	0.89	6.07	7.29
		$REMod-PER$	6.48	22.26	67.17	1.29	8.56	5.88
		$REMod-UNA$	8.27	24.80	85.00	0.74	6.57	7.62
Foc. Extr. FFR	2019	$REMod-REF$	2.59	5.84	22.54	0.85	3.40	6.88
		$REMod-SUF$	1.76	4.22	15.23	0.58	2.32	7.00
		$REMod-PER$	2.28	4.96	17.83	0.83	3.27	5.64
		$REMod-UNA$	2.90	5.52	22.56	0.48	2.51	7.31
	Mean	$REMod-REF$	5.03	13.14	37.22	1.06	5.60	5.37
		$REMod-SUF$	3.41	9.50	25.15	0.72	3.81	5.46

Commodity-specific expected loss due to scarcity based on the different scenarios, derived from the GVAR models of the robustness analysis for the commodity set restricted to the industrial metals

			Al	Cu	Ni	Pb	Sn	Zn
		$REMod-PER$	4.42	11.16	29.44	1.05	5.38	4.40
		$REMod-UNA$	5.64	12.44	37.25	0.60	4.13	5.70
Q. 25%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00
	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00
Q. 40%	2019	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.09
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.09
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.07
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.09
	Mean	$REMod-REF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-SUF$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-PER$	0.00	0.00	0.00	0.00	0.00	0.00
		$REMod-UNA$	0.00	0.00	0.00	0.00	0.00	0.00
Q. 50%	2019	$REMod-REF$	0.11	0.14	0.09	0.07	0.01	1.65
		$REMod-SUF$	0.07	0.10	0.06	0.05	0.01	1.68
		$REMod-PER$	0.10	0.12	0.07	0.07	0.01	1.35
		$REMod-UNA$	0.12	0.13	0.09	0.04	0.01	1.75
	Mean	$REMod-REF$	0.47	0.35	0.26	0.16	0.07	0.45
		$REMod-SUF$	0.32	0.26	0.18	0.11	0.05	0.45
		$REMod-PER$	0.41	0.30	0.21	0.16	0.07	0.37
		$REMod-UNA$	0.52	0.34	0.26	0.09	0.05	0.47
Q. 60%	2019	$REMod-REF$	1.92	3.27	9.61	0.76	1.06	6.51
		$REMod-SUF$	1.30	2.37	6.50	0.52	0.72	6.62
		$REMod-PER$	1.69	2.78	7.60	0.75	1.02	5.34
		$REMod-UNA$	2.15	3.10	9.62	0.43	0.78	6.91
	Mean	$REMod-REF$	4.50	9.05	20.09	1.07	3.05	4.28
		$REMod-SUF$	3.05	6.54	13.58	0.73	2.08	4.35
		$REMod-PER$	3.95	7.69	15.89	1.05	2.94	3.51
		$REMod-UNA$	5.04	8.57	20.11	0.60	2.25	4.55
Q. 75%	2019	$REMod-REF$	8.34	27.19	87.27	1.38	9.11	7.21
		$REMod-SUF$	5.65	19.65	58.99	0.94	6.20	7.33
		$REMod-PER$	7.32	23.09	69.04	1.36	8.76	5.91
		$REMod-UNA$	9.34	25.72	87.36	0.78	6.72	7.66
	Mean	$REMod-REF$	8.36	27.27	87.36	1.38	9.12	7.21
		$REMod-SUF$	5.67	19.71	59.05	0.94	6.21	7.33
		$REMod-PER$	7.34	23.16	69.11	1.36	8.77	5.91
		$REMod-UNA$	9.36	25.80	87.45	0.78	6.72	7.66

This table displays the expected loss due to scarcity for the commodities aluminum (Al), copper (Cu), nickel (Ni), lead (Pb), tin (Sn), and zinc (Zn), per path ($REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$), as well as per scenario (Mean (Mean), Shock (Shock), Extreme (Extr.), Focus EA (Foc. EA), Focus FX (Foc. FX), Focus FFR (Foc. FFR), Focus Extreme EA (Foc. Extr. EA), Focus Extreme FX (Foc. Extr. FX), Focus Extreme FFR (Foc. Extr. FFR), 25% quantile (Q. 25%), 40% quantile (Q. 40%), 50% quantile (Q. 50%), 60% quantile (Q. 60%), 75% quantile (Q. 75%)) for the input variables. Hereby, the values are derived from the GVAR model based on the weight matrices representing the dependencies between the commodities within the $REMod-REF$, $REMod-SUF$, $REMod-PER$ and $REMod-UNA$ transformation path as well as on the initial basis price level of 2019 or on the average price level of the previous decade (Mean). Hereby, the results are derived under the robustness test for the restricted commodity set, in particular, only the industrial metals are considered.