Modeling techniques to enhance credit portfolio risk measurement

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Contents

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Kevin Jakob. Estimating correlation parameters in credit portfolio models under time-varying and nonhomogeneous default probabilities. *Journal of Credit Risk*, 18(4), 2022. DOI: 10.21314/JCR.2022.010

Kevin Jakob and Matthias Fischer. Quantifying the impact of different copulas in a generalized creditrisk+ framework an empirical study. *Dependence Modeling*, 2(1), 2014. DOI:10.2478/demo-2014-0001

Matthias Fischer and Kevin Jakob. pTAS distributions with application to risk management. *Journal of Statistical Distributions and Applications*, 3:1–18, 2016. DOI: 10.1186/s40488-016-0049-9

Johanna Eckert, Kevin Jakob, and Matthias Fischer. A credit portfolio framework under dependent risk parameters: probability of default, loss given default and exposure at default. *Journal of Credit Risk*, 12(1), 2016. DOI: 10.21314/JCR.2016.202

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Chapter 1

Introduction

Since it is in the nature of banking that sometimes borrowers are unable to serve their loans, credit risk is one of the oldest risks financial institutions have to deal with. So-called non-performing loans where one of the most critical issues the ECB and other supervisors around the world were focused during the time of the financial crises and the following years. But still today in light of the COVID19 pandemic, the Ukraine crisis or the climate crisis credit risk and the creditworthiness of loans is one of the biggest threats within the financial system (see e.g. [1]). Therefore, the individual creditworthiness of counterparties is analyzed and monitored on an ongoing basis with the help of rating models by different rating agencies and financial institutions using their own models. However, in general a single counterparty does not represent a threat to a (well diversified) financial institute or the overall systems. But risks increase when economic dependencies come into play and the default of one counterparty leads to the another or when multiple counterparties are affected by the same (macro)economic shock. At this stage, rating models are still useful for the analysis of individual risks but to analyze the credit risk for a whole financial institute (or system) based on its portfolio one needs credit portfolio models taking dependencies and different uncertainties into account.

Quantitative credit portfolio models arise in the 1990s, were also two of the most famous credit portfolio models, namely the Credit Metrics (see [16]) and the Credit Risk$^+$ (see [16]) model come up. However, the possibility to quantify credit risk with the help of measures like value at risk (VaR) also led to a deceptive sense of security. In the years before the financial crises, portfolio models were used for analysis of asset backed securities (ABS) which then were one of the main triggers of the crisis. Besides other issues also disadvantages of the statistical models used contributed to the crises (e.g. [15] or [3]). Today, the topic of model risk is discussed very intensively and banks are requested by supervisors to independently validate their models and to take uncertainties in the form of model risks into account (see [2]).

Within this thesis, we analyze different modeling techniques to enhance and improve credit portfolio risk measurement based on the two mentioned credit portfolio models. This includes the estimation and modeling of the dependencies between counterparties as well as the stochastic modeling of other risk parameters besides creditworthiness and their dependency among each other and finally we present some helpful tools to work and explore credit portfolio risk together with investigations for faster approximation techniques.

1.1 Default Dependency Modeling and Parameter Estimation

One of the main challenges why credit portfolio models are used is the modeling of dependency between different counterparties, which can arise due to equal industry or geographical affiliation. Even if counterparties do not belong to the same industry or country, dependencies must be considered due to overall connection of national economics. Against this background one generally distinguishes between

intra-sector dependency, describing the connection of different counterparties belonging to the same sector (e.g. industry and/or country) and

inter-sector dependency, which describes the interaction between different sectors.

If every counterparty $i = 1, ..., N$ of a portfolio with $N \in \mathbb{N}$ counterparties is mapped to one (or several) sector(s) $k(i) = 1, ..., K$ out of K different sectors and all sectors are in turn connected via the inter-sector dependency, then also all counterparties are connected to each other even if they are belonging to different sectors. Since the direct modeling of thousands of counterparties within a credit portfolio is still not feasible (due to data and capacity restrictions), this is the basic approach the most credit portfolio models are based on. Mathematically, this is just a kind of hidden factor model, where the sectors or more precisely their state is represented by $k = 1, ..., K$ hidden factors S_k each following a sector specific one-dimensional distribution $\mathcal{L}_k$. Together with the so called link-function $f\left(PD_i, \boldsymbol{S}\right)$,which describes the connection of a counterparties creditworthiness, i.e. its individual probability of default PD_i, to the factor $\boldsymbol{S} = (S_1, ..., S_K)$ this determines the intra-sector dependency by assigning a conditional default probability PD_i^S to each counterparty, depending on the state of $\boldsymbol{S}$, i.e. $PD_i^S := f\left(PD_i, \boldsymbol{S}\right)$. Generally, the distribution class $\mathcal{L}$ of S_k is given by the type of the credit portfolio model (e.g. gamma distribution within standard Credit Risk$^+$ or a standard normal distribution within Credit

Metrics) but the parameters, which heavily influences the intra-sector dependency have to be estimated based on observed data. Within [10] we propose two new estimators for dependency parameters within a credit portfolio model, which overcome different shortcomings of already known and widely applied estimators like the assumption of an infinitely portfolio size or homogeneity of the portfolio with respect to the creditworthiness of counterparties and the composition of the portfolio over time. Within a simulation study we compare the new estimator's performance to others and show that, in the context of practical applications, the new estimators often outperform their competitors.

Beyond that, the work of [6] follows the question about the influence of the marginal distribution of S_k on risk figures. Here we focus on a Credit Risk$^+$ model where within the original setup of [16] S_k follows a gamma distribution due to analytical properties and performance issues. Within this framework, the family of positive tempered α-stable (pTAS) distributions is an excellent choice to enhance the standard model because they generalize the originally used gamma distribution. We summarize theoretical properties of the pTAS family including parameter estimation methods and apply them together with other distribution candidates (i.e. Weibull and Lognormal distribution) to a test portfolio. Since the pTAS distribution is more flexible and well suited to model light and semi-heavy tailed data distributions we could show that it can be useful to quantify risk figures more accurate and therefore to account for aspects of model risk. Beyond that, we also successfully apply the pTAS distribution in the context of operational risk management.

On the other hand the inter-sector dependency is subject to the studies presented in [12]. The multivariate distribution $\boldsymbol{\mathcal{L}}_K$ of the sector variables $\boldsymbol{S} = (S_1, ..., S_K)$ is also determined by the type of portfolio model and must be parameterized based on historical data. Within our work, we explicitly model different dependency structures with the help of different copulas (i.e. elliptical copulas, (hierarchical) Archimedean, generalized hyperbolic copulas). We also explicitly focus on asymmetric copulas, which are able to model an increasing dependency in times of a crises. Additionally, we present results on the implicit copula of a generalized Credit Risk$^+$ framework and quantify aspects of model risk, i.e. the impact of different copulas on risk figures.

1.2 Modeling Risk Parameter Dependencies

Aspects of model risk and uncertainty do not only affect the counterparties default probabilities but also other risk figures like exposure and the loss given default. More generally, the loss L_i due to a counterparties i default can be described by $L_i = EAD_i\, LGD_i\, D_i$, where $D_i \sim \text{Bern}\left(PD_i^S\right)$ is an indicator variable for the default event, EAD_i describes the exposure (outstanding amount) at default and $LGD_i \in [0, 1]$ the loss given default, which describes the share of the exposure which cannot be recovered in case of a default. Sometimes this is also expressed by the recovery rate $RR_i = 1 - LGD_i$. When the first credit portfolio models came up, the LGD_i was treated as a deterministic variable. Within [7] we develop an analytically framework to incorporate stochastic LGDs into an extended version of the Credit Risk$^+$ model, the common background vector (CBV) model by [5] and calculate risk figures by means of a saddle point approximation. Additionally, we switch from an analytically to a simulation-based framework to include not only stochastic but also dependent LGDs. Like the sector variables S_k determining the dependence structure between default probabilities, we introduce other sector variables L_k to model the distribution and dependency between LGDs. Therefore we couple S_k and L_k together with the help of additional background variables. Finally, the marginal distribution of LGDs is described by a beta distribution. Within this setup, the LGDs are stochastic and they are correlated among each other and with default probabilities. We extend this aspect of parameter dependencies further more within [4]. Here we enhance a standard portfolio model of CreditMetrics type to model dependency between EAD, LGD (with separate modeling of secured and unsecured part) and PD. For parameter estimation we use a multivariate extension of the selection model of [9]

1.3 Working with Credit Portfolio Models

Finally, we also present some tools and results to support working with credit portfolio models. Within [13] we introduce the GCPM (generalized credit portfolio model) R-package. The package is freely available via CRAN and includes different types of portfolio models, i.e. the link functions of Credit Risk$^+$ and the Credit Metrics type models are included. In case of Credit Risk$^+$ one can choose between an analytical (standard) version and a simulation-based framework. Using the latter one, analytical assumption like Poison distributed defaults or gamma distributed sector variables S_k can be relaxed and various aspects of model risk can be quantified. Furthermore, the package allows to include special simulation techniques like pooling or importance sampling to speed up calculation in case of simulation-based models. Reducing calculation time is also the aim of [11]. Here we apply different approximation methods to a credit portfolio to accelerate the quantification. We use classical regression models or Taylor approximation as well as methods based on artificial intelligence like neuronal networks, support vector machines or various tree-based models. We analyze the approximation performance within three different use cases, which include the value at risk approximation on portfolio level under constant or changing model parameters and the approximation of counterparty contributions to value at risk.

Chapter 2

Estimating correlation parameters in credit portfolio models under time-varying and nonhomogeneous default probabilities

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Chapter 3

Quantifying the impact of different copulas in a generalized CreditRisk+

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Chapter 4

pTAS distributions with application to risk management

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Chapter 5

A Credit Portfolio Framework Under Dependent Risk Parameters: Probability of Default, Loss Given Default and Exposure at Default

Johanna Eckert, Kevin Jakob, and Matthias Fischer. A credit portfolio framework under dependent risk parameters: probability of default, loss given default and exposure at default. *Journal of Credit Risk*, 12(1), 2016. DOI: 10.21314/JCR.2016.202

Chapter 6

Modeling stochastic recovery rates and dependence between default rates and recovery rates within a generalized credit portfolio framework

Matthias Fischer, Christoph Köstler, and Kevin Jakob. Modeling stochastic recovery rates and dependence between default rates and recovery rates within a generalized credit portfolio framework. *Journal of Statistical Theory and Practice*, 10:342–356, 2016. DOI: 10.1080/15598608.2016.1141733

Chapter 7

GCPM: A Flexible Package to Explore Credit Portfolio Risk

Kevin Jakob and Matthias Fischer. GCPM: A flexible package to explore credit portfolio risk. *Austrian Journal of Statistics*, 45(1):25–44, 2016. DOI: 10.17713/ajs.v45i1.87

Chapter 8

Fast Approximation Methods for Credit Portfolio Calculations

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Chapter 9

Summary and Outlook

Within the previous chapters, we analyzed several aspects of model risk and uncertainties related to credit portfolio risk. This includes the modeling of dependency structure between different counterparties by means of various copulas to model the multivariate distribution function of sectors or by changing the marginal sector distribution as well as uncertainty of parameter estimation. Furthermore, we also focused on the multivariate distribution and dependency of different risk parameters like PD, LGD and EAD. We proposed new modeling and estimation techniques to overcome crucial shortcoming and difficulties. Although, enhancing established models is necessary to improve risk management it is not always sufficient. Because, if model complexity rises this mostly comes with the cost of more model parameters which must be estimated. In case of market risk applications, where lots of data are available on a daily or even more frequent basis this would be no problem. However, for credit risk in most cases data availability depends on the default event of a counterparty which especially in case of non-retail portfolios is a rare event. Therefore, even if it is not possible to exactly quantify credit risk it is important to pay attention to model restrictions and to be aware of the sensitivities. Against this background we analyzed the impact on risk figures with respect to the mentioned model risks and uncertainties. We could show that some of them, e.g. the choice of the sector copula (see [12]) or the dependency structure between risk parameters (see [4]) really can have a significant impact on model outcome and therefore on credit risk management.

Since many aspects of model risk are still not analyzed extensively enough or even still unknown there are lots of aspects for further research, e.g. treatment of inhomogeneous dependencies within one sector or weaker dependency structures between various subsidiaries within a group or the problem of changing model parameters over time. However, depending on available data, eventually some of them can be solved by further model enhancements but some of them might be inaccessible for exact quantification. However, even if model risks cannot be quantified exactly it is still the task of researchers to highlight them and the job of practitioners to address them adequately within risk management even under greater uncertainty.

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