Journal of Scheduling (2025) 28:329–353
https://doi.org/10.1007/s10951-025-00838-z

Task assignments with rotations and flexible shift starts to improve demand coverage and staff satisfaction in healthcare

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Accepted: 14 January 2025 / Published online: 13 March 2025

Abstract

In recent years, the importance of achieving staffing flexibility to balance supply and demand in unpredictable environments, such as hospitals, has grown. This study focuses on shift design with task rotations for a multi-skilled workforce, specifically in service contexts characterized by pronounced demand variability. We introduce a mathematical programming model designed to identify optimal shift start times with task assignments for both full-time and part-time employees, where workers can rotate between multiple tasks during their shifts. We develop a column generation approach that allows us to solve realistically-sized problem instances. Our analysis, derived from staffing data of a university hospital's radiation oncology department, reveals the model's robust applicability across varying demand landscapes. We demonstrate that incorporating task rotations in the shift design can improve workload balancing when task demands fluctuate considerably. Remarkably, our column generation technique produces optimal integer solutions for realistic problem instances, outperforming the compact mixed-integer formulation which struggles to achieve feasible results. We find that the success of embedding task rotations in shift design decisions is directly influenced by the demand profile, which in turn affects the necessary qualification mix of the workforce.

Keywords Shift design · Flexible task assignment · Task rotation · Column generation

1 Introduction

The increase in service costs and a decline in service quality pose significant challenges in the service industry (Piercy & Rich, 2009; Holtzman 2012). Workforce flexibility has been understood as a means to improve quality, reduce costs, and enhance staff satisfaction. The availability, however, of a flexible workforce is still an ongoing concern (Ağralı et al., 2017). In this context, the concept of task rotations as one component of workforce flexibility emerged to improve organizational performance in the late twentieth century. It is generally defined as allowing workers to rotate between different tasks that require a certain qualification level, as opposed to being required to perform only one task (Casad, 2012; Cristini & Pozzoli, 2010). Task rotation has also been defined as a work design technique in which employees shift periodically and in a planned manner between a range of tasks in their workplace (Jones & James, 2018). Multiple relevant benefits from task rotations have been identified. First, task rotations allow for scheduling fewer workers to cover variable task-specific demand. Second, through on-the-job training, task rotations develop workers' diverse abilities and prevent them from unlearning tasks. Third, the risk for musculoskeletal disorders can be reduced when implementing task rotations with varying exposure levels on certain body parts. (Howarth et al., 2009; Mathiassen, 2006).

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In the healthcare sector, where cost and quality challenges are particularly pronounced, the advantages of implementing task rotations become even more evident. These challenges have been exacerbated by factors such as staff shortages and escalating healthcare expenditures (Bölt, 2017). Notably, nurses constitute the largest workforce segment in hospitals, and their scheduling traditionally adheres to rigid and outdated organizational structures, assigning fixed tasks within predefined shifts (Nancarrow, 2015). Embracing more flexible and sophisticated nurse scheduling practices not only enhances working conditions but also alleviates the healthcare system's burden, ultimately leading to improved quality of care.

The purpose of this paper is to solve a shift design problem with multiple tasks. We consider different demand profiles for tasks that must be performed throughout a working day and workers with various qualifications. The goal is to minimize a combination of the cost associated with the number of workers and the time-dependent costs of overstaffing. We apply the developed optimization model to real-life data of a radiation oncology department and show various benefits of flexibility, i.e., shift flexibility and the type of anticipated task rotations.

This paper offers a multifaceted contribution. First, we introduce a mixed-integer program (MIP) to tackle the problem. The MIP accommodates the use of an extensive pool of workers with varying qualifications, allowing us to determine the optimal shift design and the corresponding workforce composition. Second, we reconfigure this concise model, which is impractical to solve within reasonable timeframes using commercial solvers and devise a column generation (CG) algorithm for its resolution. Furthermore, through our experimental investigation, we unveil a structural property of our formulation that leads to integer solutions for real-world problem instances. Third, we showcase the effectiveness of our solution approach and extract managerial insights regarding the dependency of benefits derived from task rotations and flexible shifts on the demand profile and its intraday variability. We also explore the impact of a flexible workforce by considering increasingly flexible workers in terms of qualifications and daily working hours. Our experimental study centers on the shift design and task assignment problem within a radiation oncology department at a prominent German university hospital. In this context, medical radiation technicians engage in diverse tasks with fluctuating workload profiles. Alongside analyzing the effects of different workforce compositions, we investigate how adjustments in the minimum time intervals between task rotations and the number of daily task rotations influence staffing levels and the occurrence of overstaffed slots. Our findings reveal that the introduction of a single task rotation leads to significant benefits while permitting multiple task rotations yields marginal additional improvements. Furthermore, we conclude that reducing the time intervals between task rotations for multi-skilled workers is advantageous to the greatest extent possible.

The structure of this paper is as follows: Sect. 2 provides an overview of the related literature and covers papers addressing flexible shift design, flexible multi-activity task assignment, and task rotations. In Sect. 3, we present a problem description and the compact formulation of the mathematical model. We describe the model reformulation and the CG-based solution approach in Sect. 4. Section 5 presents our experimental study, the numerical results, as well as the managerial insights that can be derived from the experimental study. Section 6 presents a summary of our work and possibilities for further research. In the Appendix, we provide analyses of additional hospital settings to show the wide applicability of our model.

2 Related literature

Research on rostering and personnel scheduling has garnered significant interest in the field of operations research, which may be driven by economic concerns and the desire for significant cost savings (Van den Bergh et al., 2013). For detailed literature reviews on personnel scheduling, we refer to Ernst et al. (2004), Van den Bergh et al. (2013), and Özder et al. (2020). Research on nurse scheduling specifically has been reviewed by Cheang et al. (2003) and Burke et al. (2004). Katiraee et al. (2021) provide a recent review of the literature on workers' differences in production systems modeling and design, which includes investigations on how worker skill differences impact workload balancing.

In the following section, we discuss research papers on shift scheduling, multi-activity task assignments, as well as task rotations. These topics are the foundational pillars of the underlying problem statement. We present two distinct streams of research that relate to our work. For an overview of the related literature and their modeling approaches as well as important problem features, please refer to Table 1.

2.1 Advanced methodologies for implicit shift scheduling problems

The literature often distinguishes between explicit and implicit shift scheduling approaches. Explicit scheduling assigns resources to predetermined shifts that have exogenous start times and durations. Implicit scheduling endogenously generates variable shift start times and durations. Implicit scheduling is closely related to shift design, as implicit scheduling can be used to design shifts that can subsequently be assigned to workers, rather than directly assigning workers to predefined shifts. In a seminal work,

Table 1 Overview of the related literature

Reference	Modeling technique	Solution approach	Scheduling type	Task rotation modeled	Workforce heterogeneity	Workload balancing	Implicit breaks	Real-world application
Aryanezhad et al. (2009)	IP	LP-metric method & commercial solver	Explicit	X	X	X		
Asensio-Cuesta et al. (2012)	CombiO	Genetic algorithm	Explicit	X	X	X		
Aykin (1996)	IP	Commercial solver	Explicit				X	
Ayough et al. (2020)	MIP	Meta-heuristic	Explicit	X	X			
Bechtold & Jacobs (1990)	MP	Commercial solver	Implicit				X	
Carnahan et al. (2000)	IP	Genetic algorithm	Explicit	X	X			
Côté et al. (2011)	GBP	Commercial solver	Implicit				X	
Dahmen & Rekik. (2015)	IP	Hybrid heuristic	Explicit		X	X		
Dahmen et al. (2018)	IP	Commercial solver	Implicit				X	
Dahmen et al. (2020)	MIP	Heuristics	Explicit	X	X	X		
Hernández-Leandro et al. (2019)	GBP	Matheuristic	Explicit		X	X		
Hochdörffer et al. (2018)	IP	IP-based heuristic	Explicit	X		X		X
Lapègue et al. (2013)	CP	Heuristics	Implicit		X	X		X
Lequy et al. (2012)	IP	Heuristics	Explicit		X	X		
Lian et al. (2018)	IP	Meta-heuristic	Explicit		X	X		
Mossa et al. (2016)	MIP	Not specified	Implicit	X	X	X		X
Quimper & Rousseau (2010)	GBP	Neighborhood search	Explicit		X	X	X	
Rekik et al. (2004)	IP	Benders decomposition	Implicit				X	
Rekik et al. (2010)	MIP	Commercial solver	Implicit				X	partially
Restrepo et al. (2012)	NetBP	Column generation	Implicit				X	X
Rinaldi et al. (2022)	MIP	Heuristics	Explicit		X	X		X
Seçkiner & Kurt (2007)	IP	Simulated annealing	Implicit	X				
Seçkiner & Kurt (2008)	IP	Ant colony optimization	Explicit	X		X		
Thompson (1995)	IP	Commercial solver	Implicit				X	
Our work	**MIP**	**Column generation**	**Implicit**	**X**	**X**	**X**		**X**

CombiO—Combinatorial optimization; CP—Constraint programming; GBP—Grammar-based programming; IP—Integer programming; MP—Mathematical programming; MIP—Mixed-integer programming; NetBP—Network-based programming

Moondra (1976) discusses these concepts in a mathematical problem that accounts for intraday workload variations. The considered shift types include full-time shifts with a defined duration and flexible part-time shifts with a range of minimum and maximum durations. Implicit shift scheduling was extended to include break assignments in early studies such as Bechtold and Jacobs (1990), Thompson (1995), Aykin (1996). Rekik et al. (2004) introduce an implicit model based on a transportation problem to allocate breaks to shifts. They show that the linear programming (LP) relaxation of their model is equivalent to the LP relaxations of other well-established implicit formulations proposed by Aykin (1996) and Bechtolds and Jacobs (1990). Additionally, owing to the similar integrality gap, Dantzig's (1954) set covering model shares equivalence with the model in Aykin (1996) in terms of LP relaxations. However, their implicit approach can only be used in certain contexts when specific properties are met, as further outlined in Rekik et al. (2010), where they propose extensions to enhance flexibility in defining fractionable breaks utilizing forward and backward constraints. Their work is generalized in Dahmen et al. (2015 and 2018) to model a multi-activity shift scheduling problem implicitly. In their subsequent study, they use a two-stage modeling approach to study the impact of a heterogeneous workforce (Dahmen et al., 2020). Hernández-Leandro et al. (2019) also provide a Lagrangian relaxation combined with a matheuristic to solve a multi-activity shift scheduling problem.

The approaches outlined in the literature for multi-activity shift scheduling problems typically fall into two categories. First, there are sequential approaches, where the problem is solved in a stepwise manner, beginning with the construction of generic shifts, followed by the assignment of activities to selected shifts (Lequy et al., 2012). Second, there are integrated methods, where techniques from constraint programming (Côté et al., 2011; Lapègue et al., 2013; Quimper & Rousseau, 2010) or column generation-based approaches (Restrepo et al., 2012) are employed to address the problem. Our presented methodology falls into this second category. Furthermore, several heuristics have been proposed in the literature, including simulated annealing (Seçkiner & Kurt, 2007), ant colony optimization (Seçkiner & Kurt, 2008), metaheuristics (Lian et al., 2018), and LP-metric methods (Aryanezhad et al., 2009).

2.2 Breaks in shift scheduling

As an extension to the previously discussed implicit shift scheduling, researchers have developed models that identify optimal start and end times of breaks as part of their shift scheduling solution. The incorporation of implicit breaks in the shift scheduling problem considerably increases the model complexity, necessitating the development of advanced methodologies to find optimal solutions to realistically-sized problem settings (Dahmen et al. 2015 and 2018). Therefore, it is hardly surprising that the vast majority of studies that do consider implicit breaks are theoretical in nature and do not consider real-world applications (see Table 1), with the exception of Restrepo et al. (2012). In the discussions with the scheduler at our partner hospital, it became quickly apparent that shift assignments had not been coupled with distinct start and end times of breaks previous to our project. Rather, breaks had been considered in the overall duration of shifts, allowing nurses sufficient time to take breaks when possible. The scheduler emphasized that she wanted to maintain this policy, partially in fear of discontent among the nurses whenever breaks could not be granted at the explicitly planned time. Therefore, we do not consider implicit breaks in our scheduling model.

2.3 Task rotations and ergonomic effects

Another relevant area of literature explores task rotation strategies aimed at mitigating ergonomic risks within manufacturing systems (Padula et al., 2017). This line of research predominantly focuses on assembly line environments, where tasks are associated with various factors such as fatigue, stress, and strain on specific body parts. The objective in this context is to optimize the allocation of tasks by identifying combinations that may involve task rotations. The aim is to balance the impact of these tasks on individual workers within a heterogeneous workforce. It is important to note that the ergonomic effects are often nonlinear, as exposure to health risks tends to increase exponentially over time. Consequently, developing effective models for this scenario often necessitates the application of heuristic techniques or linearization approaches. Linearization may involve the transformation of originally nonlinear parameters, while the model itself retains a linear form. For example, Carnahan et al. (2000) tackle the challenge of assigning various lifting tasks to workers with differing fitness levels to mitigate the risk of back injuries. They employ a genetic algorithm to address this issue effectively. In another instance, Mossa et al. (2016) examine musculoskeletal disorders (MSDs) within an industrial context. Their objective is to enhance the production rate of an automotive company while simultaneously reducing the risk of injury. They utilize linearized fatigue factors and formulate a mixed-integer program (MIP), which they successfully solve using a commercial solver. Furthermore, Asensio-Cuesta et al. (2012) present a genetic algorithm to tackle their combinatorial optimization model, intending to design optimal schedules that incorporate task rotations to proactively prevent work-related MSDs. Ayough et al. (2020) develop a variable neighborhood search meta-heuristic to solve a nonlinear MIP that incorporates worker-specific factors such as forgetting and learning into the optimization of takt time and workload balancing.

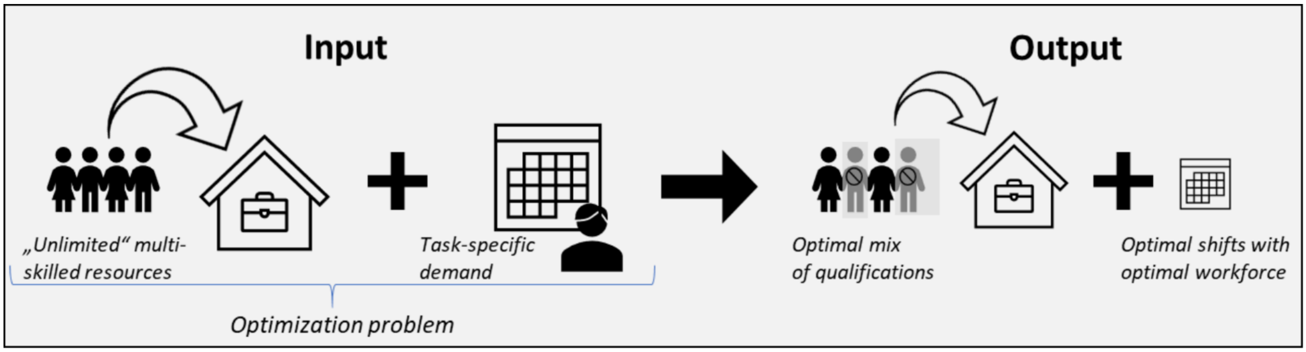

Fig. 1 Inputs and outputs of the optimization model

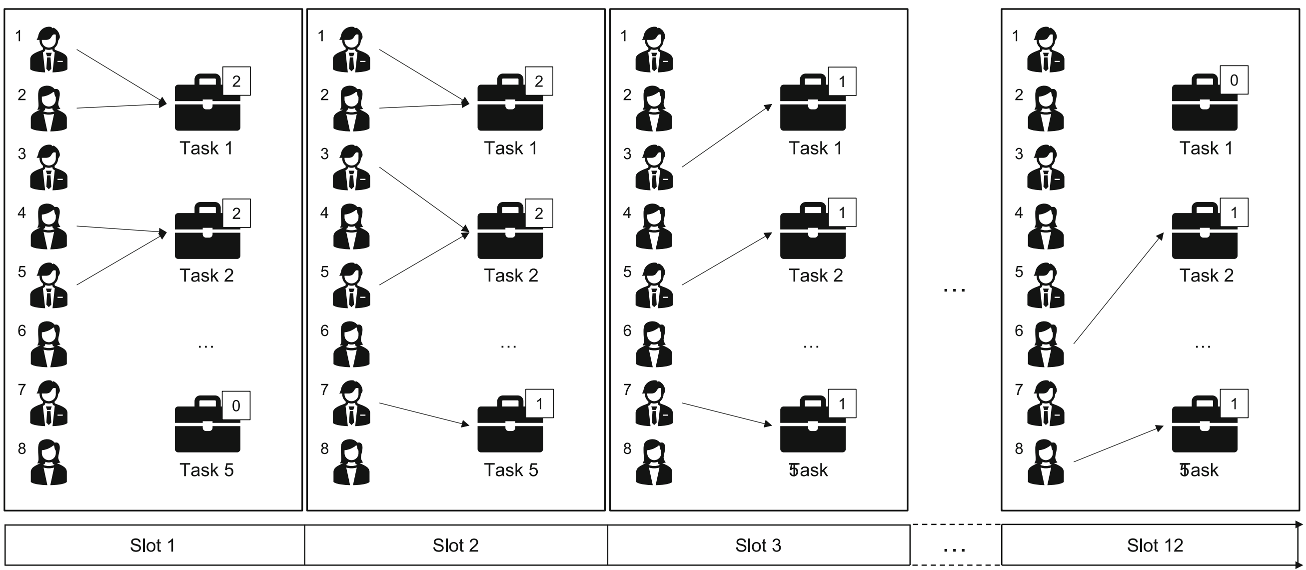

Fig. 2 Visualization for 8 workers, 5 tasks, and 12 time slots

Hochdörffer et al. (2018) create job rotation schedules for a workday in the assembly line of an automotive manufacturer via an IP formulation and an integer programming-based heuristic. They present one of the very few studies where the decision support system was implemented and tested in practice. In a recent manuscript on task assignments in an automotive workshop setup, Rinaldi et al. (2022) bridge the gap between ergonomics assessment and the evaluation of worker performance. The developed MIP is solved with a constructive heuristic procedure. Much like the studies mentioned above, our research also focuses on the critical objective of balancing workloads for employees. Additionally, we employ a technique to linearize nonlinear costs over time, thus establishing a linear MIP framework. It is worth noting that, in contrast to our work, the stream of literature on task rotations and ergonomics typically does not incorporate shift design into its objectives. As a result, our research serves as a bridge between the commonly encountered objectives of workload balance and ergonomic considerations in the context of task rotations and the broader realm of shift design and scheduling literature.

2.4 Literature summary and contribution

The predominant focus of the literature has largely centered on the identification of modeling and solution methodologies aimed at enhancing solvability, specifically in terms of feasibility and speed. Consequently, many studies resort to solving artificial problem instances or utilizing previously published data to benchmark the performance of their methodologies against existing work (see Table 1). In contrast, our study is driven by a genuine real-life challenge encountered within a radiation oncology department at one of Germany's largest hospitals. This unique context enables us to showcase the effectiveness of our column generation-based approach, which has received limited attention in the realm of implicit scheduling encompassing multiple tasks and a diverse workforce, particularly when applied to problems of realistic scale. Moreover, to the best of our knowledge, such an approach has not been used to effectively determine optimal shift types and workforce composition. Notably, the explicit modeling of task rotations, including constraints on the maximum number of rotations and minimum task durations to enable managerial control over desired task assignment flexibility, reveals a noteworthy observation: In realistic problem instances, our column generation approach excels by identifying optimal integer solutions. With small adjustments to the model, we can also find optimal shift-and-task assignments for an existing workforce composition.

3 Problem description

This present work studies the problem of finding the optimal shifts and workforce composition to match supply and demand for multiple tasks in a service environment (see Fig. 1). The fluctuating intraday demand between the various tasks poses significant challenges concerning the resulting over- and understaffing. To address these challenges, the problem includes task rotations, implicit shift design, and workforce flexibility. In the next section, a detailed description of the problem statement is provided, and a mathematical model formulation for solving the problem is proposed.

Problem statement We consider a set of workers N, indexed by n. They can be allocated to a task t during a time slot s throughout the workday. Moreover, the workers possess different qualification levels defined as the binary parameter Q_{nt}, which equals 1 if a worker n can perform task t. These workers can either be part-time or full-time employees, therefore they differ with respect to their worktime W_n. There are no predefined shift start times for workers, and task assignments may vary during a shift. We explicitly model the number of times a worker performs a task during a shift, allowing decision-makers to regulate recurring task assignments. Moreover, we account for the minimum duration required for a worker to engage in a task before a rotation. The working day is split into identically-sized time slots s. We minimize a combination of the costs associated with the number of utilized workers U_n and the time-dependent costs of overstaffing C_{ts}. This time-dependent cost allows the decision-maker to prioritize preventing overstaffing at certain times during the day. Additionally, it is not allowed to have understaffed slots in the final (rotation) pattern.

Figure 2 illustrates this problem. The required number of workers is printed in the top right corner of the briefcase icons. For instance, Task 1 requires two workers in slots 1 and 2, one worker in slot 3, and no worker in the last slot. Some workers may begin their shift in the first time slot (e.g., Worker 1, Worker 2), while others begin their shift in subsequent time slots (e.g., Worker 7 on Slot 2). If a worker's qualification is suitable, they may switch from one task to another during their workday (e.g., Worker 3). Additionally, a worker's shift must consist of consecutive task assignments until the end of the shift, i.e., no idle time is allowed during a shift.

The solution of the mathematical model determines which workers of the different qualification categories are assigned to which tasks in which time slots. Thus, the precise (rotation) pattern of each employed worker is provided. As our study does not focus on the long-term staffing decision but on the assignment and selection problem, we assume that enough workers of each qualification level are available to perform the required tasks. Not all available workers from the workforce pool are necessarily used in a solution. Also, we obtain information about which tasks are overstaffed during which time slot.

Mathematical model formulation. The definition of the sets, parameters, and variables and the mathematical formulation are provided as follows:

Sets	
$\boldsymbol{N}$	Set of workers, index n
$\boldsymbol{T}$	Set of tasks, index t
$\boldsymbol{S}$	Set of slots, index s
$\boldsymbol{K}$	Set of number of task repetitions, index k
Parameters	
D_{ts}	Demand for workers performing task t in slot s
Q_{nt}	1, if worker n is qualified to perform task t, 0 otherwise
W_n	Number of slots worker n has to work
F	Minimum number of slots assigned before a rotation
L_n	Maximum number of tasks (including multiple assignments of the same task) in a shift for worker n
C_{ts}	Penalty for overstaffing for task t in slot s
U_n	Cost for worker n
Decision variables	
x_{ntsk}	1, if worker n is assigned to task t for the k-th time in slot s, 0 otherwise
h_n	1, if worker n is working, 0 otherwise
o_{ts}	Overstaffing in slot s for task t
a_{ntk}	1, if worker n is assigned to task t at least k times, 0 otherwise
b_{ntk}	Start slot of the assignment of worker n to task t for the k-th time
e_{ntk}	First slot after the assignment of worker n to task t for the k-th time
$\underline{b}_n$	Start slot of a shift for worker n
$\overline{e}_n$	First slot after a shift for worker n

$$\min: \sum_{n \in \boldsymbol{N}} U_n \cdot h_n + \sum_{t \in \boldsymbol{T}} \sum_{s \in \boldsymbol{S}} C_{ts} \cdot o_{ts} \tag{1}$$

$$\text{s.t.} \sum_{n \in \boldsymbol{N}} \sum_{k \in \boldsymbol{K}} x_{ntsk} - o_{ts} = D_{ts} \forall t \in \boldsymbol{T},\ s \in \boldsymbol{S} \tag{2}$$

$$\sum_{t \in \boldsymbol{T}} \sum_{s \in \boldsymbol{S}} \sum_{k \in \boldsymbol{K}} x_{ntsk} = W_n \cdot h_n \forall n \in \boldsymbol{N} \tag{3}$$

$$x_{ntsk} \leq Q_{nt} \forall n \in \boldsymbol{N},\ t \in \boldsymbol{T},\ s \in \boldsymbol{S},\ k \in \boldsymbol{K} \tag{4}$$

$$\sum_{t \in \boldsymbol{T}} \sum_{k \in \boldsymbol{K}} x_{ntsk} \leq 1 \forall n \in \boldsymbol{N},\ s \in \boldsymbol{S} \tag{5}$$

$$\sum_{s \in \boldsymbol{S}} x_{ntsk} \geq F \cdot a_{ntk} \forall n \in \boldsymbol{N},\ t \in \boldsymbol{T},\ k \in \boldsymbol{K} \tag{6}$$

$$x_{ntsk} \leq a_{ntk} \forall n \in \boldsymbol{N},\ t \in \boldsymbol{T},\ s \in \boldsymbol{S},\ k \in \boldsymbol{K} \tag{7}$$

$$b_{ntk} \leq s \cdot x_{ntsk} + |\boldsymbol{S}| \cdot (1 - x_{ntsk}) \forall n \in \boldsymbol{N},\ t \in \boldsymbol{T},\ s \in \boldsymbol{S},\ k \in \boldsymbol{K} \tag{8}$$

$$e_{ntk} \geq (s+1) \cdot x_{ntsk} \forall n \in \boldsymbol{N},\ t \in \boldsymbol{T},\ s \in \boldsymbol{S},\ k \in \boldsymbol{K} \tag{9}$$

$$\sum_{s \in \boldsymbol{S}} x_{ntsk} = e_{ntk} - b_{ntk} \forall n \in \boldsymbol{N},\ t \in \boldsymbol{T},\ k \in \boldsymbol{K} \tag{10}$$

$$a_{ntk} \geq a_{nt(k+1)} \forall n \in \boldsymbol{N},\ t \in \boldsymbol{T},\ k \in \boldsymbol{K} \backslash |\boldsymbol{K}| \tag{11}$$

$$x_{ntsk_1} + x_{nt(s+1)k_2} \leq 1 \forall n \in \boldsymbol{N},\ t \in \boldsymbol{T},\ s \in \boldsymbol{S} \backslash |\boldsymbol{S}|,\ k_1 k_2 \in \boldsymbol{K},\ k_1 < k_2 \tag{12}$$

$$\sum_{t \in \boldsymbol{T}} \sum_{k \in \boldsymbol{K}} a_{ntk} \leq L_n \forall n \in \boldsymbol{N} \tag{13}$$

$$b_{ntk} \geq \underline{b}_n \forall n \in \boldsymbol{N},\ t \in \boldsymbol{T},\ k \in \boldsymbol{K} \tag{14}$$

$$e_{ntk} \leq \overline{e}_n \forall n \in \boldsymbol{N},\ t \in \boldsymbol{T},\ k \in \boldsymbol{K} \tag{15}$$

$$\sum_{t \in \boldsymbol{T}} \sum_{s \in \boldsymbol{S}} \sum_{k \in \boldsymbol{K}} x_{ntsk} = \overline{e}_n - \underline{b}_n \forall n \in \boldsymbol{N} \tag{16}$$

$$h_n,\ x_{ntsk},\ a_{ntk} \in \{0; 1\} \forall n \in \boldsymbol{N},\ t \in \boldsymbol{T},\ s \in \boldsymbol{S},\ k \in \boldsymbol{K} \tag{17}$$

$$o_{ts} \geq 0 \forall t \in \boldsymbol{T},\ s \in \boldsymbol{S} \tag{18}$$

$$b_{ntk},\ e_{ntk},\ \underline{b}_n,\ \overline{e}_n \in \mathbb{N}_0 \forall n \in \boldsymbol{N},\ t \in \boldsymbol{T},\ k \in \boldsymbol{K} \tag{19}$$

The first term of the objective function (**1**) represents the cost of the schedule, which is defined as the sum of the expenses U_n for each utilized worker. The costs of a worker may be calculated from the day-to-day costs of a given worker. Commonly, cross-qualified workers are associated with higher costs than workers who are qualified to perform a single task. The second term of the objective function (1) represents the time-dependent overstaffing costs C_{ts}. These costs can be adjusted individually for each slot. Overstaffing might occur for various tasks based on the workers' flexible start times in combination with their shift length W_n and the fluctuating demand for the tasks.

Constraints (2) ensure that the overall demand for each task in each slot is met. Decision variables o_{ts} represent the level of overstaffing for task t in slot s. Understaffed slots are not allowed in this model. Constraints (3) limit the number of working hours for each worker n. Hereby, the length of a shift is given for each worker n. Constraints (4) ensure that worker n can only perform task t if they have the necessary qualification Q_{nt}. Constraints (5) ensure that a worker is assigned to at most one task in each slot. Constraints (6) and (7) force the intervals between rotations for a worker to be at least F slots long. Since multiple rotations within a shift are possible, workers may rotate back to a task that they had already been assigned to previously. Hence, index k indicates the k-th time a worker is assigned to the same task. Constraints (8), (9) and (10) assure that task assignments range over consecutive slots with endogenous start and end slots. A similar modeling idea was introduced by Vijayakumar et al. (2013). Constraints (11) ensure that the rotation of tasks follows the natural order, i.e., if $a_{ntk} = 1$ then $a_{nt(k-1)} = 1$ for al $k > 1$. Constraints (12) disallow workers to "rotate" to the same task without first rotating to a different task. Constraints (13) limit the number of tasks to be performed on a single day. Constraints (14) to (16) ensure that a worker immediately switches from one task to another in case of assigned task rotations. The decision variables $\underline{b}_n$ and ($\overline{e}_n - 1$) denote the earliest and latest working periods of each worker's assigned shift. Domains for the decision variables are defined in (17) to (19).

4 Solution approach

The presented model cannot be solved with a commercial solver for larger instances in our experimental study. The compact formulation, however, can be decomposed by workers n. Consequently, workers with the same worktime (i.e., full-time or part-time) and the same qualifications are grouped into worker types m. The result of the decomposition generates a pattern of task assignments for each worker type. This decomposition reduces the compact model formulation's symmetry significantly. In other words, we use a Dantzig-Wolfe reformulation that relies on column generation (Dantzig & Wolfe, 1960; Desrosiers & Lübbecke, 2005). The reformulation works as follows: the original MIP is divided into a restricted master problem (RMP) that offers a shift schedule structure and m subproblems (SP^{m}) that create the columns for each worker type m. Upon transformation of the mathematical model outlined in Sect. 3, the demand constraints (2) remain in the RMP. The decision variable λ_p^m

defines how many workers of worker type m are assigned to (rotation) pattern p. The updated sets and indices of the RMP are as follows:

Sets

$\boldsymbol{M}$	Set of worker types, index m
$\boldsymbol{T}$	Set of tasks, index t
$\boldsymbol{S}$	Set of slots, index s
$\boldsymbol{P_m}$	Set of (rotation) patterns for worker type m, index p

Parameters

D_{ts}	Demand for workers performing task t in slot s
C_{ts}	Penalty for overstaffing task t in slot s
U^m	Cost of worker type m
X^m_{pts}	1, if worker type m is assigned to perform task t in slot s in pattern p, 0 otherwise

Decision variables

o_{ts}	Overstaffing in slot s for task t
λ^m_p	Number of workers of worker type m assigned to work shift pattern p

$$\min : \sum_{m \in \boldsymbol{M}} \sum_{p \in \boldsymbol{P_m}} U^m \cdot \lambda^m_p + \sum_{t \in \boldsymbol{T}} \sum_{s \in \boldsymbol{S}} C_{ts} \cdot o_{ts} \tag{20}$$

$$\text{s.t.} \sum_{m \in \boldsymbol{M}} \sum_{p \in \boldsymbol{P_m}} X^m_{pts} \cdot \lambda^m_p - o_{ts} = D_{ts} \forall t \in \boldsymbol{T},\ s \in \boldsymbol{S} \tag{21}$$

$$o_{ts},\ \lambda^m_p \geq 0,\ \forall t \in \boldsymbol{T},\ s \in \boldsymbol{S},\ m \in \boldsymbol{M},\ p \in \boldsymbol{P_m} \tag{22}$$

The objective function (20) minimizes the number of employed (rotation) pattern assignments (i.e., workers) and overstaffed slots. Constraints (21) guarantee that the demand for all tasks t during all slots s is satisfied. Domains for the decision variables are defined in (22). The dual value of the demand constraint (21) for task t in slot s is defined as Π_{ts}. Hence, the reduced costs $\overline{c}^m_p$ for each column p in the RMP can be defined according to Eq. (23).

$$\overline{c}^m_p = U^m - \sum_{t \in \boldsymbol{T}} \sum_{s \in \boldsymbol{S}} X^m_{pts} \cdot \Pi_{ts} \tag{23}$$

All columns included in the RMP have positive reduced costs after solving the RMP as a linear program (LP), i.e., $\overline{c}^m_p \geq 0 \forall p \in \boldsymbol{P_m},\ m \in \boldsymbol{M}$. Absent columns might have negative reduced costs, and we need to guarantee for LP-optimality that such columns do not exist. Therefore, we formulate worker-type-specific subproblems SP^m with generic reduced cost functions, i.e., the objective function (24) for SP^m. The updated sets and indices of the SP^m are as follows. Note, worker type-specific parameters have superscript m.

Sets

$\boldsymbol{T}$	Set of tasks, index t
$\boldsymbol{S}$	Set of slots, index s
$\boldsymbol{K}$	Set of the number of times a worker is assigned to a task during their shift, index k

Parameters

Q^m_t	1, if worker type m is qualified to perform task t, 0 otherwise
W^m	Number of slots worker type m has to work
F	Minimum number of slots assigned before a rotation
L^m	Maximum number of task assignments (including multiple assignments of the same task) for worker type m
U^m	Cost for worker type m
Π_{ts}	Dual values of constraints (21) for each task t and slot s

Decision variables

x^m_{tsk}	1, if worker type m is assigned to task t for the k-th time in slot s, 0 otherwise
a^m_{tk}	1, if worker type m is assigned to task t at least k times, 0 otherwise
b^m_{tk}	Start slot of the k-th assignment of worker type m to task t
e^m_{tk}	First slot after the k-th assignment of worker type m to task t
$\underline{b}^m$	Start slot of a shift for worker type m
$\overline{e}^m$	First slot after a shift for worker type m

The mathematical model of the SP^m is shown:

$$\min : U^m - \sum_{t \in \boldsymbol{T}} \sum_{s \in \boldsymbol{S}} \Pi_{ts} \left(\sum_{k \in \boldsymbol{K}} x^m_{tsk} \right) \tag{24}$$

$$\text{s.t.} \sum_{t \in \boldsymbol{T}} \sum_{s \in \boldsymbol{S}} \sum_{k \in \boldsymbol{K}} x^m_{tsk} = W^m \tag{25}$$

$$x^m_{tsk} \leq Q^m_t \forall t \in \boldsymbol{T},\ s \in \boldsymbol{S},\ k \in \boldsymbol{K} \tag{26}$$

$$\sum_{t \in \boldsymbol{T}} \sum_{k \in \boldsymbol{K}} x^m_{tsk} \leq 1 \forall s \in \boldsymbol{S} \tag{27}$$

$$\sum_{s \in \boldsymbol{S}} x^m_{tsk} \geq F \cdot a^m_{tk} \forall t \in \boldsymbol{T},\ k \in \boldsymbol{K} \tag{28}$$

$$x^m_{tsk} \leq a^m_{tk} \forall t \in \boldsymbol{T},\ s \in \boldsymbol{S},\ k \in \boldsymbol{K} \tag{29}$$

$$b^m_{tk} \leq s \cdot x^m_{tsk} + |\boldsymbol{S}| \cdot \left(1 - x^m_{tsk}\right) \forall t \in \boldsymbol{T},\ s \in \boldsymbol{S},\ k \in \boldsymbol{K} \tag{30}$$

$$e^m_{tk} \geq (s+1) \cdot x^m_{tsk} \forall t \in \boldsymbol{T},\ s \in \boldsymbol{S},\ k \in \boldsymbol{K} \tag{31}$$

$$\sum_{s \in \boldsymbol{S}} x^m_{tsk} = e^m_{tk} - b^m_{tk} \forall t \in \boldsymbol{T},\ k \in \boldsymbol{K} \tag{32}$$

$$a^m_{tk} \geq a^m_{\mathrm{t}(k+1)} \forall t \in \boldsymbol{T},\ k \in \boldsymbol{K} \backslash |\boldsymbol{K}| \tag{33}$$

$$x^m_{tsk_1} + x^m_{t(s+1)k_2} \leq 1 \forall t \in \boldsymbol{T},\ s \in \boldsymbol{S} \backslash |\boldsymbol{S}|,\ k_1 k_2 \in \boldsymbol{K},\ k_1 < k_2 \tag{34}$$

$$\sum_{t\in \boldsymbol{T}} \sum_{k\in \boldsymbol{K}} a^m_{tk} \leq L^m \tag{35}$$

$$b^m_{tk} \geq \underline{b}^m \forall t \in \boldsymbol{T},\ k \in \boldsymbol{K} \tag{36}$$

$$e^m_{tk} \leq \overline{e}^m \forall t \in \boldsymbol{T},\ k \in \boldsymbol{K} \tag{37}$$

$$\sum_{t\in \boldsymbol{T}} \sum_{s\in \boldsymbol{S}} \sum_{k\in \boldsymbol{K}} x^m_{tsk} = \overline{e}^m - \underline{b}^m \tag{38}$$

$$x^m_{tsk},\ a^m_{tk} \in \{0;\ 1\} \forall t \in \boldsymbol{T},\ s \in \boldsymbol{S},\ k \in \boldsymbol{K} \tag{39}$$

$$b^m_{tk},\ e^m_{tk},\ \underline{b}^m,\ \overline{e}^m \in \mathbb{N}_0 \forall t \in \boldsymbol{T},\ k \in \boldsymbol{K} \tag{40}$$

The objective function minimizes the generic reduced costs of a new (rotation) pattern for worker type m. The shift-defining constraints (3) through (17) follow the logic in the compact model, except that individual workers are aggregated by worker types m. A new column $\overline{X}^m$ is constructed based on the solution of SP$^{\text{m}}$ with $X^m_{ts} := \sum_{k\in \boldsymbol{K}} x^m_{tsk} \forall t \in \boldsymbol{T}$, $s \in \boldsymbol{S}$ added to the RMP:

$$\overline{X}^m = \begin{bmatrix} U^m \\ X^m_{ts} \end{bmatrix}.$$

Figure 3 illustrates a summary of the CG algorithm. We initialize the RMP with no columns and extremely high understaffing costs to create a first starting solution. After the RMP is solved, the dual variables are passed to the first SP$^{\text{m}}$ (where $m = 1$). Then, SP^1 is solved to optimality, and the (rotation) pattern is added to $\boldsymbol{P_m}$ as column $\overline{X}^m$ to RMP. We resolve the RMP, increment m by 1, and pass the new dual information to SP$^{\text{m}}$. If no negative reduced costs column for SP$^{\text{m}}$ can be found, the corresponding SP$^{\text{m}}$ is temporarily removed from the solution process. If all subproblems are removed, we start over again and repeat the previous process until no negative reduced costs columns can be found.

Structural observation. In an LP with the general form $\{\max cx | Ax \leq b\}$ with $x \geq 0$ and b consisting of only integer values, the matrix A is totally unimodular (TU) if each submatrix has determinants of $\pm$ 1 or 0. By definition, the values of the matrix must have the values $\pm$ 1 or 0 (Hoffmann & Kruskal, 1956). Furthermore, a (0,1) matrix A in which the 1's occur consecutively in each row, a so-called consecutive one's property (C1P), is also TU (Fulkerson & Gross, 1965). The same holds for every column since a transposed TU matrix is also TU. A matrix with multiple blocks of 1's and 0's in between these blocks within a column or row is called gapped C1P or $(\kappa,\ \delta)$-C1P. Hereby, κ presents the number of 1's blocks and δ a separation of these blocks of more than δ 0's. Hereby, the classical C1P problem is equivalent to the (1,0)-C1P problem. For every $\kappa \geq 2,\ \delta \geq 1$, the problem is NP-complete (Maňuch et al., 2012). Applying this property to our problem, a column can be classified as (1,0)-C1P if no rotation is allowed. For one rotation, we have $(2,\ |\boldsymbol{S}| - W^m)$-C1P, and for more than $\alpha \geq 2$ rotations, $(\alpha + 1,\ F)$-C1P in the worst case. Our experimental results show that CG generates integer solutions as most columns exhibit a (1,0)-C1P or $(\kappa,\ \delta)$-C1P with κ very small and δ rather large.

In the following, we discuss the structure of the columns for cases with rotations. For showing the gapped C1P, we assume that the workstations are ordered from 1 to $|\boldsymbol{T}|$, and for each workstation, we have $|\boldsymbol{S}|$ assignment slots indicating a working period (1) or a non-working period (0). Therefore, each column has $|\boldsymbol{S}| \bullet |\boldsymbol{T}|$ entries where at most W^m entries can be 1. We consider two cases (a) without and (b) with multiple assignments to the same workstation to show the theoretical minimum number of 0 s between two consecutive sequences of 1 s. Assume an assignment starts in period 1 at workstation i, then all assignments for any workstation in periods $\left[W^m + 1,\ \ldots,\ |\boldsymbol{P}|\right]$ are forced to 0.

(a) Without multiple assignments to the same workstation. The earliest possible starting period at the following workstation $i + 1$ in the ordered set is in slot $(F + 1)$ which results in $|\boldsymbol{S}|$ 0 s in the column between the two sequences of 1 s. For any other workstation in the ordered set after $i+1$, another set of $|\boldsymbol{S}|$ 0s is added. Conversely, having an assignment starting in slot s on the workstation j at least $s - 1 + (|\boldsymbol{S}| - W^m)$ 0s are in between this assignment and the potential assignment on the previous workstation $j - 1$ in the ordered set. Note, $(|\boldsymbol{S}| - W^m)$ determines the slots with assignments of 0s when starting on any workstation in period 1. For any other workstation in the ordered set before $j - 1$, another set of $|\boldsymbol{S}|$ 0s is added. Therefore, we have at least $(|\boldsymbol{S}| - W^m)$ 0s between two consecutive sequences of 1s in a column. In general, this minimum can only happen if assignments occur in slot s on workstation i and slot $s + W^m$ on the previous workstation $i - 1$ in the ordered sequence of workstations. Otherwise, the number of 0s is much larger.

(b) With multiple assignments to the same workstation. In addition to case (a), we can have multiple assignments on the same workstation. Rotating from workstation i to j (with a minimum length of stay) and then back to i results in a minimum number of F 0s between the two assignments on workstation i.

The probability of having these exceptional cases is very low and decreases with increasing $|\boldsymbol{P}|$ and $|\boldsymbol{W}|$. Generally, at most $[W^m/F]$ sequences of 1s can be seen in any column. In our experimental study, the columns generated exhibit a very large number of 0s between the sequences of 1s. Therefore, we see many integer solutions at the LP relaxation of the column generation procedure.

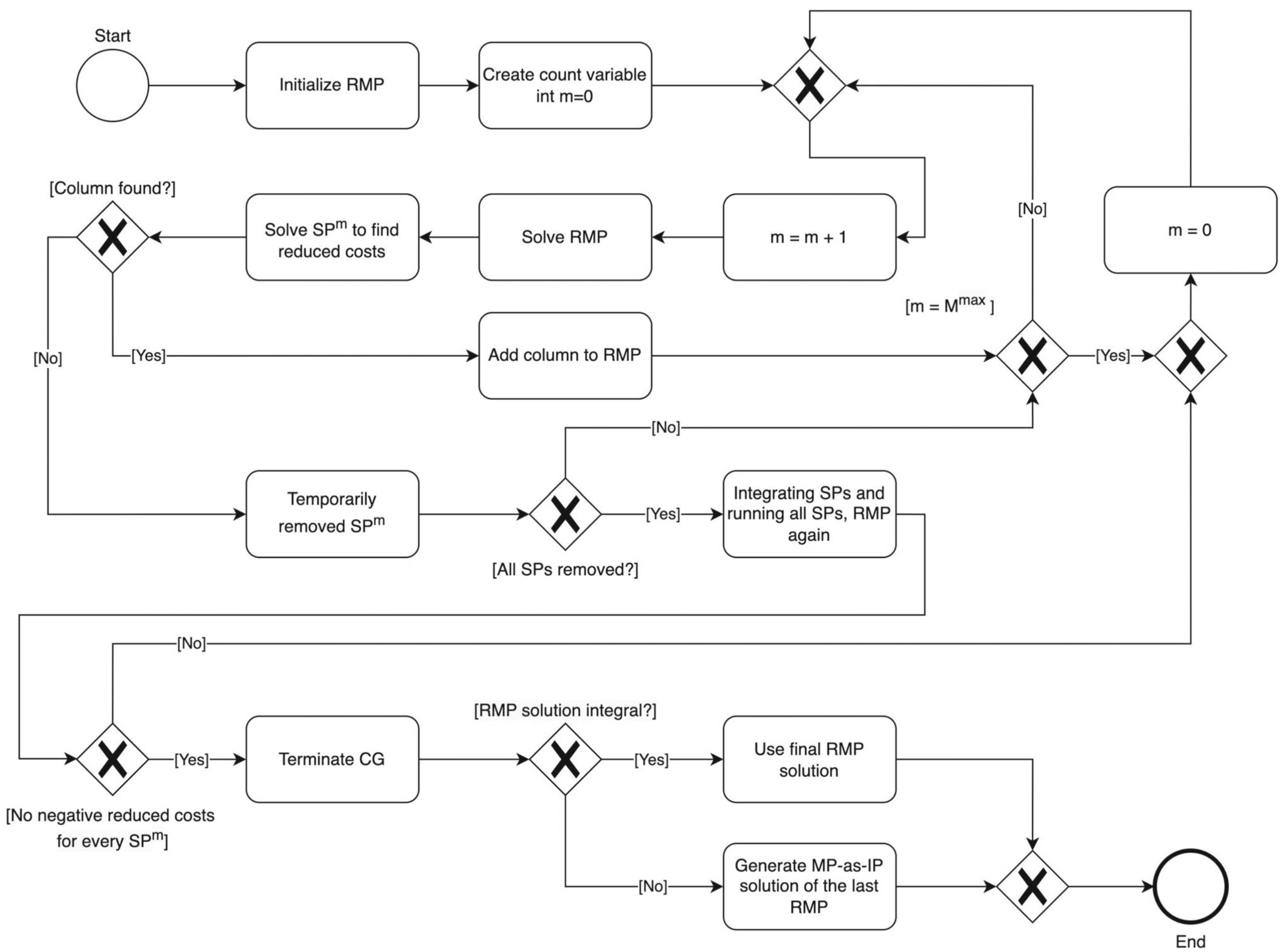

Fig. 3 Column generation (CG) algorithm process flow

5 Experimental study

In this section, we present a task assignment problem in the radiation oncology department of a large German university hospital. Within this department, a diverse team of medical radiation nurses possesses varying sets of qualifications relevant to their work. Experienced nurses can handle various tasks, often acquired through on-the-job training. The department's daily operations involve the completion of numerous tasks, primarily falling into three categories: linear accelerator (LA), computer tomography (CT), and brachytherapy (BT). Due to the standardized nature of these tasks, the planning process for individual patients is typically straightforward. As we aim to find an optimal shift design and the corresponding workforce mix, we assume demand to be deterministic, as infrequent occurrences of patient no-shows will not impact the hospital's responsibility to provide sufficient supply to cover all planned treatments.

The demand for labor to carry out these tasks experiences fluctuations throughout the day. Beyond the distinctions in qualifications, the nursing staff is divided into full-time and part-time employees, adding a layer of complexity to the workforce composition. Distinct costs are associated with varying degrees of expertise, and these expenses are tied to the different pay grades applicable to medical radiation technicians in the German public health system. At the entry-level, certification for a single task is a prerequisite, with standardized expenses set at 100 units. Moving up, nurses at the second level (Nurse Types 4, 5, and 6, as detailed in Table 2) are qualified to perform two tasks. The third-level nurse (Nurse Type 7) can execute any task within their purview.

Notably, part-time nurses incur higher costs compared to their full-time counterparts. The percentage change in costs for these nurses is determined relative to the payment level of base-level nurses. It is important to mention that nurses with qualifications spanning multiple tasks have the flexibility to switch tasks during their shifts.

Additionally, it is worth noting that, for part-time nurses with multiple qualifications, we adopt a modeling approach that employs a sort of "full qualification level" as a simplification strategy to reduce the complexity of subproblems.

Table 2 Parameters of the different nurse types

Nurse type m	Costs U^m	Worktime W^m [slots]	Task qualification Q_t^m		
			LA	**CT**	**BT**
1	100	16	1	0	0
2	100	16	0	1	0
3	100	16	0	0	1
4	112	16	1	1	0
5	112	16	1	0	1
6	112	16	0	1	1
7	120	16	1	1	1
8	75	10	1	0	0
9	75	10	0	1	0
10	75	10	0	0	1
11	90	10	1	1	1

In essence, recognizing that part-time nurses can switch tasks only once, we aggregate them into a single subproblem, encompassing all three types within our CG approach. As our primary focus lies in optimizing shift design and determining the corresponding workforce composition rather than individual scheduling decisions, we operate under the assumption of an ample pool of nurses available across all three qualification levels. This deliberate choice ensures that limited nurse staff availability does not emerge as a bottleneck in our decision-making process. Moreover, we assume that there are 16 available slots for full-time nurses for the three model tasks. In comparison, part-time nurses have 10 slots at their disposal (each slot spanning 20 min) since our analysis strictly considers task-related work during their shifts. Consequently, the actual shifts may extend beyond this timeframe, such as 8 h for full-time nurses or 4 h for part-time nurses. In essence, non-patient-related tasks consume some working time before or after being assigned to specific tasks.

Table 2 and Fig. 4 provide a clear overview of the distinctive attributes associated with each nurse type. Additionally, our model incorporates time-variable overstaffing costs, imposing higher penalties during daytime hours while applying lesser penalties at the beginning and end of the day. These later periods often see nurses engaged in non-patient-related activities, such as documentation, cleaning, and administrative tasks. Furthermore, when overstaffing occurs at the beginning or end of a shift, employees have a greater opportunity to take time off and reduce their accrued overtime hours when, for example, a patient does not show up for treatment or other events make the worker's presence unnecessary. This is especially significant given the prevailing challenge of managing overtime in contemporary work environments. To capture this dynamic, we employ a concave overstaffing cost function, which is adjusted in scale based on the length of the planning horizon. More details can be found in Figure xxxx in the Appendix.

A typical working day at the German university hospital is defined as 36 slots of labor (i.e., 12 h), with varying demands placed for three tasks throughout the day. CT and BT have a low demand in the morning and evening and a high demand during the day, whereas LA has a high demand in the morning and evening and a low demand during the day. Figure 5 depicts the base demand profile for the different tasks, i.e., LA, CT, and BT.

In Sect. 5.1, we explore how the size of the problem instances impacts solver performance in the compact model formulation. Then, we compare the solution quality and runtime to the results from the CG solution approach. In Sect. 5.2, we examine a range of situations using various demand profiles and rules. The model is implemented in Java and uses the CPLEX solver library version 12.10.0.0 with default settings. The computations are performed on a 64-bit Intel Core i7 2.80 GHz CPU with 32GB RAM utilizing 8 threads.

5.1 Comparison of solution approach performance

We adapt the base demand profile to consider smaller problem instances and gain insights into commercial solver performance (which is only capable of handling small instances). We reduce I) the demand levels, II) the number of slots, and III) both the demand levels and the number of slots. A reduction in the number of slots represents a doubling of the slot duration so that the overall worktime remains the same. In total, we have four different demand profiles (BASE-36 slots, BASE-18 slots, LOW-36 slots, and LOW-18 slots), as depicted in Fig. 6 where BASE-36 represents the original demand profile introduced previously. Considering a 36 (18)

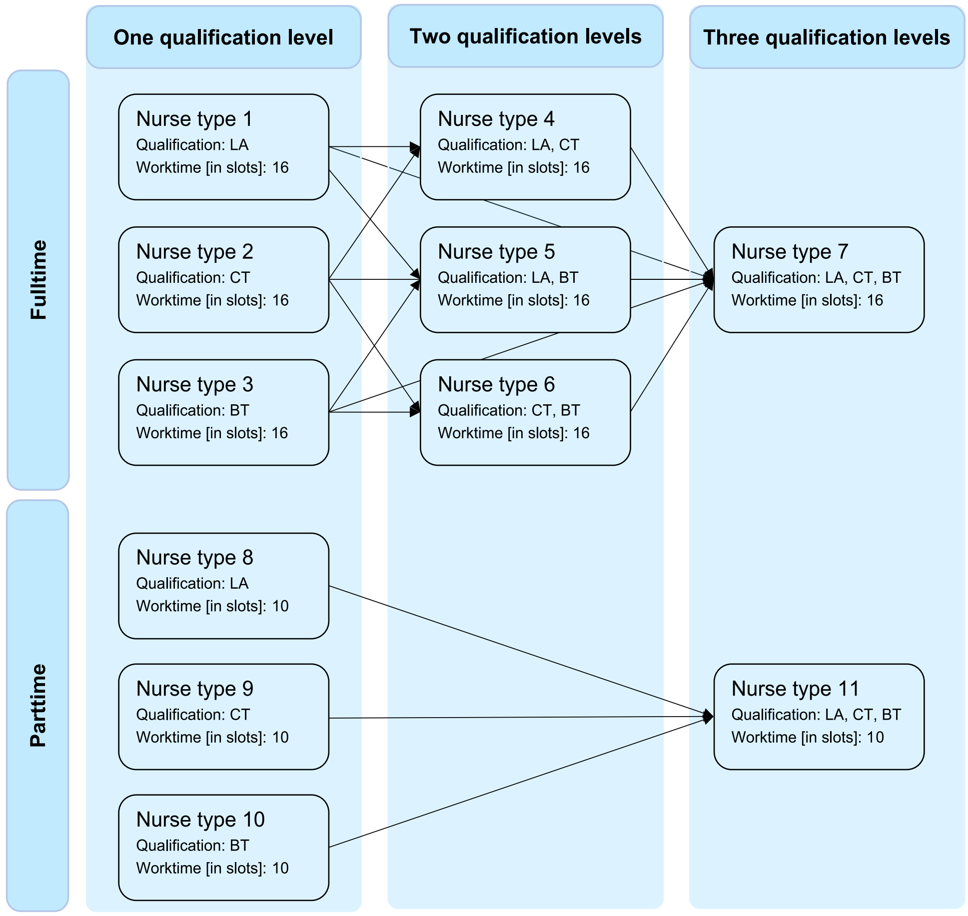

Fig. 4 Qualification level connection between the different nurse types

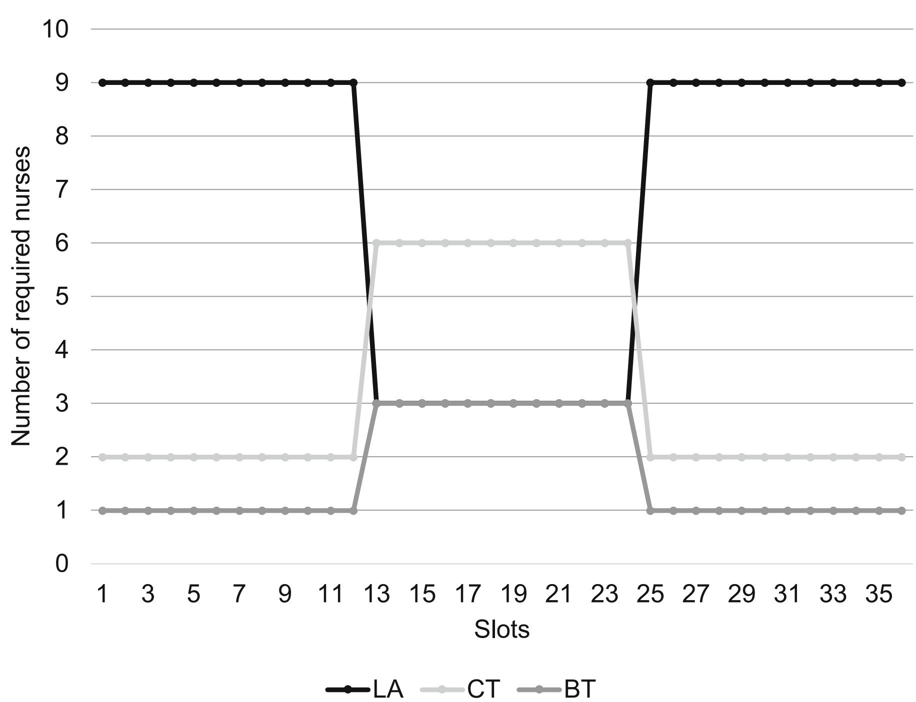

Fig. 5 Base demand profile

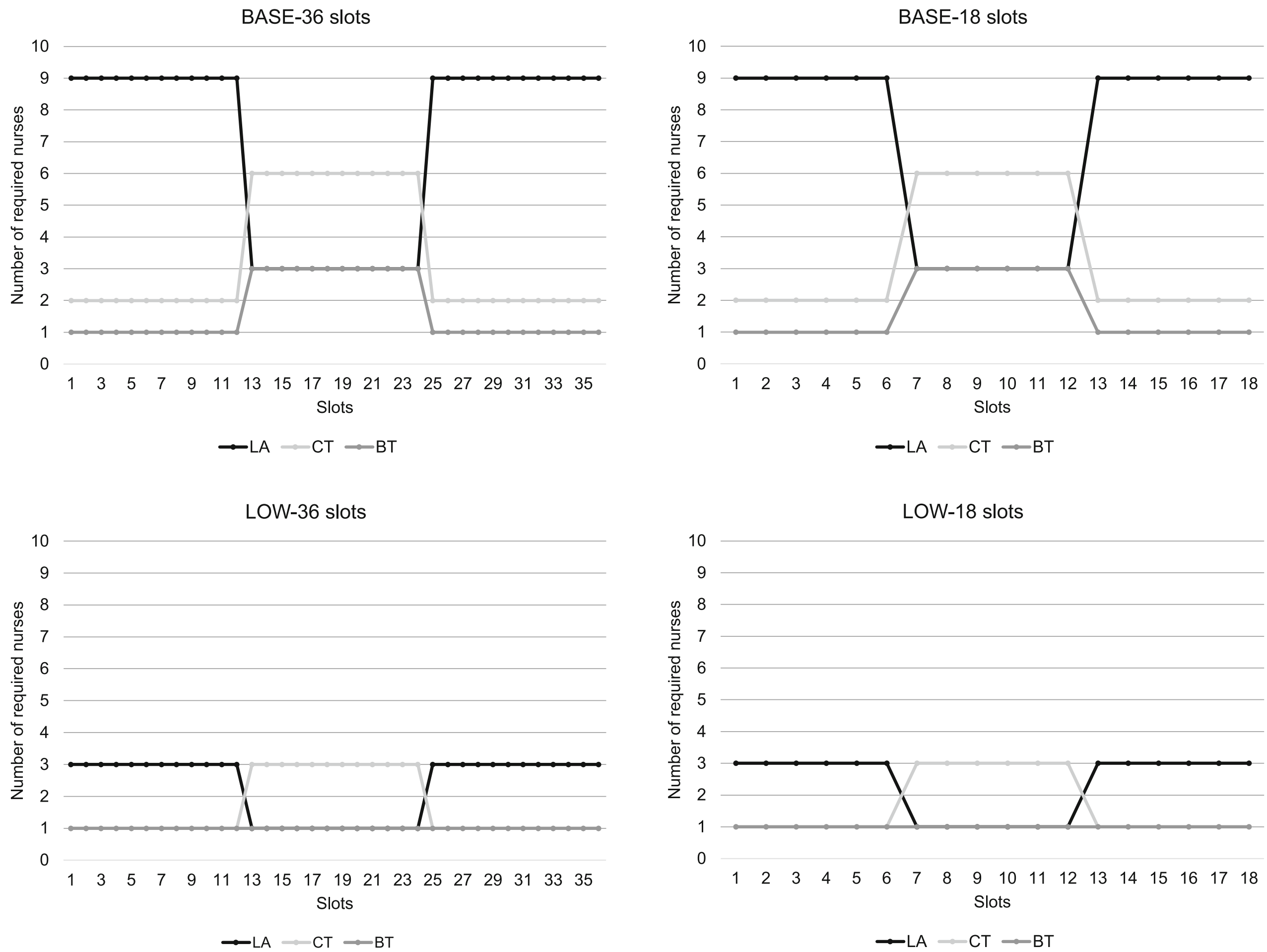

Fig. 6 Demand profiles BASE-36 slots, BASE-18 slots, LOW-36 slots, and LOW-18 slots

slot workday, a total of 432 (216) slots and 180 (90) slots of labor are required for base and low demand, respectively.

In addition to the four demand profiles, the analyzed instances differ regarding the number of considered nurse types, the worktime duration, and the required interval between task rotations, as seen in Table 3. The instances are chosen to represent combinations of the following possible scenarios: What if the staff consists of only full-time nurses with a single qualification, thus making task rotations impossible (Setting A)? What if only full-time nurses with up to the highest qualification level can be assigned (Setting B)? What if we can employ full-time and part-time nurses with up to the highest qualification level (Setting C)? The scenarios are each examined for two levels of the minimum duration between rotations. In total, we have 12 instances.

The MIP solver terminates if no optimal solution is found after 60 min. Table 4 presents the objective function value, solution gap, runtime, the number of overstaffed slots, and the number of used worker types for each instance. Solutions were obtained for instances in Setting A (instances 01, 02, 03, and 04) as well as instances 05 and 09. These instances are the smallest problems in terms of the number of variables and constraints and are hardly representative of real-life settings. For some of the remaining instances, a solution gap of between 10 and 30 percent was attained, while a feasible solution could not be found within one hour in half of the instances. As depicted in Table 4, the solution gap and the runtime performance are highly dependent on the demand load and the considered worker types as well as the number of allowed task rotations. Comparing the solved instances for the LOW-18 slots profile, in Setting C there is a shift from full-time nurses to part-time nurses. Furthermore, the number of overstaffed slots is reduced in instance ID 09 compared to instance ID 01 or 05.

Table 4 comprehensively summarizes the results obtained across all twelve instances. Notably, the CG approach consistently delivers optimal integer solutions for each instance, eliminating the need for further branching strategies. This noteworthy outcome suggests that the columns of the RMP may exhibit the gapped C1P, a concept elaborated upon in Sect. 4 and further substantiated in Sect. 5.2.

Table 3 Parameter settings and problem size of the different instances

Setting	Instance ID	# Worker types m	Demand profile	W^m full/half [slots]	Minimum F [slots]	# Variables	# Constraints
A	01	3	LOW-18	8/-	2	1,440	5,283
	02	3	BASE-18	8/-	2	3,618	13,500
	03	3	LOW-36	16/-	4	2,628	10,251
	04	3	BASE-36	16/-	4	6,588	26,190
B	05	7	LOW-18	8/-	2	5,598	20,970
	06	7	BASE-18	8/-	2	14,310	53,838
	07	7	LOW-36	16/-	4	10,188	40,680
	08	7	BASE-36	16/-	4	26,028	104,436
C	09	11	LOW-18	8/5	2	9,294	34,914
	10	11	BASE-18	8/5	2	23,814	89,694
	11	11	LOW-36	16/10	4	16,908	67,728
	12	11	BASE-36	16/10	4	43,308	173,988

It is essential to highlight that this property is observed across all realistic problem instances encountered in the oncology department and multiple departments within the hospital. Using information gathered in previous consulting projects with the hospital, we derived additional settings in the hospital's pharmacy, emergency department, medical ward, and intensive care unit. The results are omitted from this manuscript for the sake of conciseness. In each setting, allowing for a single task rotation yielded improvements, while additional potential rotations did not have a positive effect. As demonstrated in Sect. 5.2, the integrality property does not hold in a hypothetical scenario characterized by an unrealistically high number of potential rotations, exceptionally low minimum task durations, and extreme fluctuations in task demand.

Comparing the runtimes and quality of the CG approach with the compact formulation, the advantage of the CG approach over the commercial solver is striking. Not only is the CG approach capable of detecting optimal solutions for all realistic instances but all solutions are also obtained in less than one minute. Here, the CG approach finds an integer optimal solution, whereas the commercial solver terminates after one hour and the MIP gap is still significant. Very noticeably, the runtime of CG is affected by the allowed degree of flexibility. Settings with more volatile demand, a higher number of allowed task rotations, and a lower minimum task duration, such as instances 07, 08, 11, and 12, cause the runtime to increase considerably. Moreover, as shown in Sect. 5.2, hypothetical settings with extreme degrees of flexibility and demand variability will cause the runtime to increase to up to an hour. Please refer to Sect. 5.2 for further details.

As shown in Table 4, allowing 11 different nurse types (Setting C) leads to a change in the optimal workforce mix, replacing some of the full-time nurses with part-time nurses. Also, nurses with two or more qualifications are used in the problem since task changes are allowed. Moreover, the number of overstaffed slots decreases to 0 for every demand profile when allowing part-time nurses. Nevertheless, the total number of nurses did not change for each of the three settings.

5.2 Performance analysis of further scenario changes

Instance 12 is the instance that represents the real-life case at the collaborating university hospital. It serves as the basis for the following analyses, where we investigate the impact of different demand profiles on the optimal nurse workforce composition and the resulting number of overstaffed slots. Figure 7 depicts the different demand profiles, which predominantly differ with regard to two aspects: change in overall demand level and change in task-specific demand levels over the day. The latter naturally calls for a higher number of task rotations when possible. Note that all demand profiles have the same total number of 216 slots of task-related demand. Profile 1 is the baseline demand profile. Profile 2 indicates a steady demand for each task throughout the day, while profile 3 shows decreasing demand for one task and increasing demand for the other two. Demand profiles 4 and 5 are continually rising or decreasing for all tasks. Demand for each task fluctuates significantly throughout the day in demand profile 6. Additionally, we created profile 7 to test the performance of our solution approach in a highly unrealistic, hypothetical setting. Here, demand varies drastically between individual periods, and average demand levels for LA change, so using part-time nurses is discouraged. We run profile 7 (named "FLEX") with extreme flexibility parameter values, where the minimum task duration is a single time slot, and the number of maximum rotations is unlimited. Therefore, columns exhibit a $(W^m, 1)$-C1P.

Table 4 Results of the 12 different instances solved by the compact formulation and the CG approach

ID	Obj. value	Gap** [%]	Runtime [sec]	# iterations	Total overstaffing	# full-time	# part-time	Worker Type										
								1	2	3	4	5	6	7	8	9	10	11
01_MIP	1690.9	0	4	–	30	15	–	7	5	3	–	–	–	–	–	–	–	–
02_MIP	4070.0	12.9	3600	–	72	36	–	21	10	5	–	–	–	–	–	–	–	–
03_MIP	2284.4	14	3600	–	60	15	–	7	5	3	–	–	–	–	–	–	–	–
04_MIP	5527.3	26.0	3600	–	144	36	–	21	10	5	–	–	–	–	–	–	–	–
05_MIP	1690.9	31.2	3600	–	30	15	–	7	5	3	0	0	0	0	–	–	–	–
06_MIP	n/a	n/a	n/a	–	n/a	n/a	–	n/a	n/a	n/a	n/a	n/a	n/a	n/a	–	–	–	–
07_MIP	n/a	n/a	n/a	–	n/a	n/a	–	n/a	n/a	n/a	n/a	n/a	n/a	n/a	–	–	–	–
08_MIP	n/a	n/a	n/a	–	n/a	n/a	–	n/a	n/a	n/a	n/a	n/a	n/a	n/a	–	–	–	–
09_MIP	1346.9	14.8	3600	–	3	6	9	2	0	1	3	0	0	0	2	5	1	1
10_MIP	n/a	n/a	n/a	–	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
11_MIP	n/a	n/a	n/a	–	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
12_MIP	n/a	n/a	n/a	–	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
01_CG	1690.9	0.0	0.5	36	30	15	–	7	5	3	–	–	–	–	–	–	–	–
02_CG	4070.0	0.0	0.5	38	72	36	–	21	10	5	–	–	–	–	–	–	–	–
03_CG	2284.4	0.0	1.6	54	60	15	–	7	5	3	–	–	–	–	–	–	–	–
04_CG	5527.3	0.0	1.5	55	144	36	–	21	10	5	–	–	–	–	–	–	–	–
05_CG	1690.9	0.0	3.3	67	30	15	–	7	5	3	0	0	0	0	–	–	–	–
06_CG	4070.0	0.0	2.8	65	72	36	–	21	10	5	0	0	0	0	–	–	–	–
07_CG	2233.7	0.0	44.1	148	60	15	–	5	5	3	2	0	0	0	–	–	–	–
08_CG	5375.3	0.0	45.1	129	144	36	–	15	10	5	4	2	0	0	–	–	–	–
09_CG	1313.0	0.0	5.1	105	0	5	10	0	0	1	4	0	0	0	3	4	2	1
10_CG	3189.0	0.0	6.5	116	0	12	24	0	0	0	8	4	0	0	9	8	4	3
11_CG	1313.0	0.0	10.1	93	0	5	10	1	0	0	4	0	0	0	2	4	3	1
12_CG	3189.0	0.0	19.0	109	0	12	24	0	0	0	8	4	0	0	9	8	4	3

n/a: no solution found after 3,600 s
– : not relevant for this ID
**: For MIP solutions, the gap refers to the optimality gap after one hour (if a solution has been found at that time). For CG solutions, this refers to the gap between the RMP solution and the RMP-as-IP solution

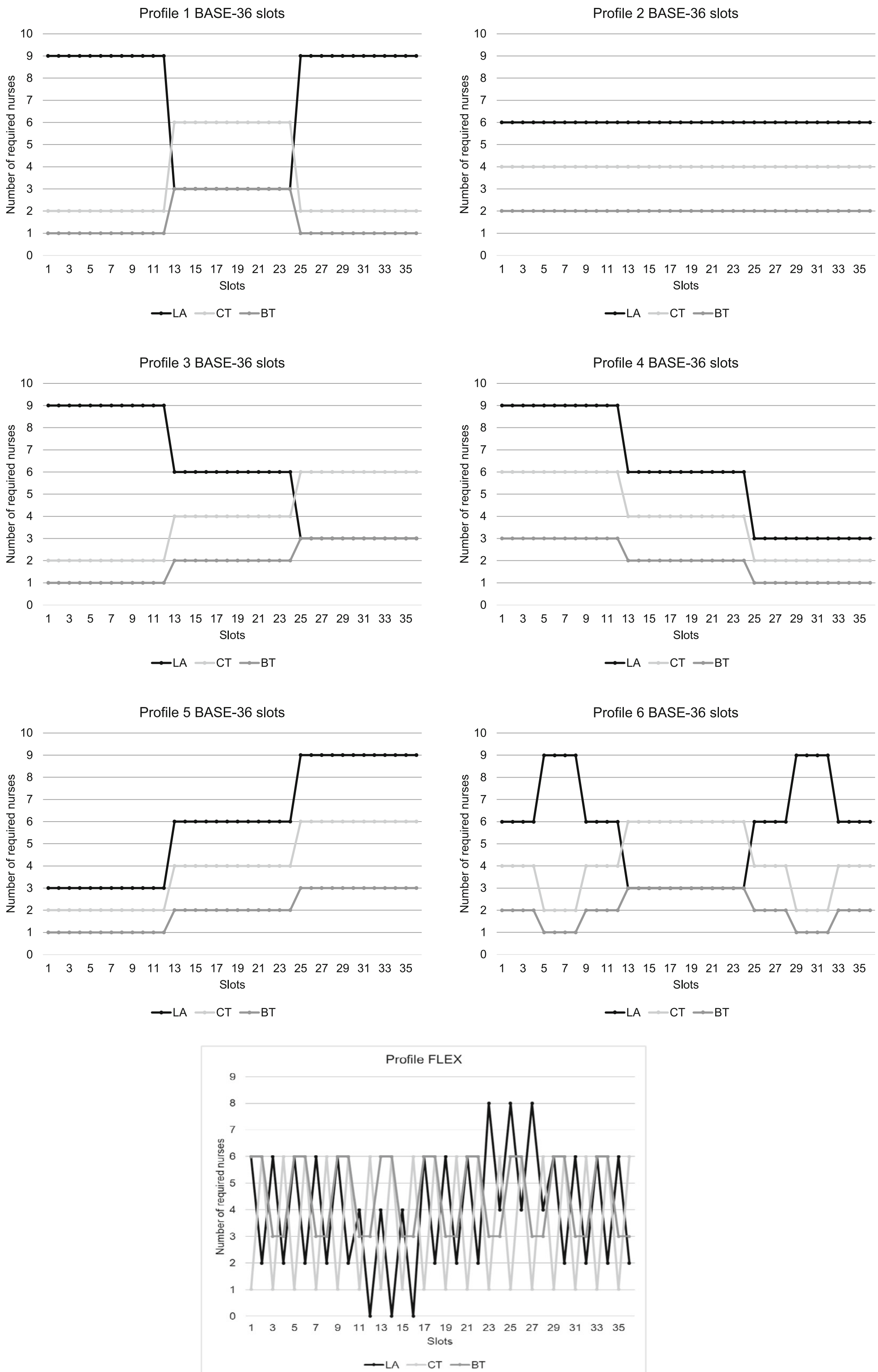

Fig. 7 Different demand profiles

Table 5 Results and nurse mix of all solved instances in Sect. 5.2

ID	Obj. value	Runtime [sec]	# iterations	Total overstaffing	# full-time	# part-time	Worker Type										
							1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
12.P1	3,189.0	18.9	109	0	12	24	0	0	0	8	4	0	0	9	8	4	3
12.P2	3,000.0	4.8	72	0	12	24	6	4	2	0	0	0	0	12	8	4	0
12.P3	3,072.0	7.1	102	0	12	24	3	2	1	4	2	0	0	12	8	4	0
12.P4	3,732.2	25.9	130	36	18	18	9	6	3	0	0	0	0	9	6	3	0
12.P5	3,643.5	10.8	83	36	18	18	9	6	3	0	0	0	0	9	6	3	0
12.P6	3,279.0	35.7	169	0	12	24	0	0	0	8	4	0	0	3	8	4	9
13.P1	4,375.7	4.0	78	72	24	12	15	6	3	0	0	0	0	6	4	2	0
14.P1	3,189.0	250.1	255	0	12	24	0	0	0	8	4	0	0	9	8	4	3
15.P1	4,324.7	29.6	135	72	24	12	12	6	3	2	1	0	0	9	2	1	0
16.P1	4,324.7	18.2	113	72	24	12	12	6	3	2	1	0	0	9	2	1	0
13.P2	3,000.0	1.6	55	0	12	24	6	4	2	0	0	0	0	12	8	4	0
14.P2	3,000.0	13.0	69	0	12	24	6	4	2	0	0	0	0	12	8	4	0
15.P2	3,000.0	5.6	84	0	12	24	6	4	2	0	0	0	0	12	8	4	0
16.P2	3,000.0	2.9	66	0	12	24	6	4	2	0	0	0	0	12	8	4	0
13.P3	3,687.8	2.5	67	36	18	18	9	6	3	0	0	0	0	9	6	3	0
14.P3	2,712.0	56.5	123	0	12	24	3	2	1	4	2	0	0	12	8	4	0
15.P3	3,636.8	16.3	100	36	18	18	9	4	2	2	1	0	0	6	8	4	0
16.P3	3,636.8	15.4	95	36	18	18	9	4	2	2	1	0	0	6	8	4	0
13.P4	3,732.2	2.4	64	36	18	18	9	6	3	0	0	0	0	9	6	3	0
14.P4	3,732.2	115.1	114	36	18	18	9	6	3	0	0	0	0	9	6	3	0
15.P4	3,732.2	21.6	81	36	18	18	9	6	3	0	0	0	0	9	6	3	0
16.P4	3,732.2	11.0	79	36	18	18	9	6	3	0	0	0	0	9	6	3	0
13.P5	3,643.5	2.3	76	36	18	18	9	6	3	0	0	0	0	9	6	3	0
14.P5	3,643.5	56.2	103	36	18	18	9	6	3	0	0	0	0	9	6	3	0
15.P5	3,643.5	12.9	87	36	18	18	9	6	3	0	0	0	0	9	6	3	0
16.P5	3,643.5	14.4	105	36	18	18	9	6	3	0	0	0	0	9	6	3	0
13.P6	4,204.2	4.8	88	60	12	30	3	6	3	0	0	0	0	18	8	4	0
14.P6	3,189.0	318.5	254	0	12	24	0	0	0	8	4	0	0	9	8	4	3
15.P6	3,703.8	28.8	139	36	18	18	3	6	3	4	2	0	0	12	4	2	0
16.P6	3,703.8	6.8	78	36	18	18	3	6	3	4	2	0	0	12	4	2	0
FLEX	5,850.5	3648.0	290	144	29	14	3	1	4	9	9	3	0	1	3	4	6

The first 6 entries of Table 5 provide the results of the various instances using the demand profiles shown in Fig. 7. The resulting instance ID is a combination of the instance ID 12 from Table 3 and the respective demand profile, i.e., ID 12.P1 is the base profile from Sect. 5.1, while ID 12.P2 uses profile 2 from Fig. 6. Even with varying demand profiles, the algorithm has no difficulty solving the instances. The runtimes for demand profiles 1 through 6 are within a 30-s range, and the final RMP solutions are all integer solutions. Therefore, no branching is required after the CG converges. Despite at least one task rotation being allowed in all settings, they are not used in the optimal assignments for demand profiles 2, 4, and 5. Here, a task rotation would not reduce overstaffed slots due to the constant, decreasing, or increasing demand for all tasks throughout the day. If, however, the demand for one task increases, while the demand for another task decreases, task rotations are an appealing means to reduce overstaffing. Consequently, the benefit of nurses with higher qualification levels rises, and their higher wage costs are (more than) offset by the reduction in overstaffing they provide. This is seen in instances 12.P1, 12.P3, and 12.P6. The highest need for multi-qualified nurses with a high level of expertise arises with profile 6 because it is the profile with the highest frequency of changes in demand levels. Mainly, in total 42 nurses are used on this working day to cover the demand.

However, overstaffed slots are unavoidable in profiles 4 and 5 since more full-time nurses are used.

In the demand profile FLEX, our solution approach's performance decreases. While CG does find a feasible solution, it takes just over an hour to do so. Moreover, the optimal values for some variables are finally non-integer. However, it is worth mentioning that the LP-as-IP solution of the final RMP is only 0.01% worse than the LP solution (5850.5 vs. 5849.7). Future research may aim to investigate under what circumstances the gapped C1P in the generated columns collapses enough to cause these issues. For instance, investigating the order of the workstation assignments in each column might be interesting.

Following, we analyze the effects of the minimum duration between task rotations as well as the maximum number of allowed task rotations during a shift. Therefore, five distinct settings are defined for each of the previously defined six demand profiles. First, we evaluate how the solution changes if no task rotations (Instance ID 13), one task rotation (Instance ID 12), or up to two task rotations (Instance ID 14) are permitted during a shift. Second, we vary how long a nurse must remain on a task before a task rotation is possible, a limit that management may want to impose to account for, e.g., setup times or continuity of care. This so-called residence time is set to 4 slots (Instance ID 12), 6 slots (instance ID 15), and 8 slots (Instance ID 16). With residence times of 6 and 8 slots, it is implied that only full-time workers may change their task once, whose shifts last 16 slots while part-time employees only work 10 slots each day and therefore cannot perform a task rotation. Table 6 offers a summary of the analyzed settings.

Each of the four new settings is evaluated using each of the six distinct profiles as shown in Fig. 7. These 24 additional solutions are also shown in Table 5. The results are organized by profile since we need to compare the distinct settings for each profile. Most cases may be solved in less than one minute, with a few requiring up to six minutes. Again, all solutions of the final RMP result in integer solutions.

Again, there is no effect of allowing task rotations on the optimal nurse mix or the number of overstaffed slots for demand profiles 2, 4, or 5, the ones with constant, decreasing, or increasing demand for all tasks. Assigning task rotations is unnecessary as demand for tasks does not shift between tasks. The lack of required task rotations means that it is optimal to employ nurses with single-task qualifications, as the higher cost of nurses with higher qualification levels cannot be offset by reducing the number of overstaffed slots through task rotations. In general, the minimum demand for a task over a shift-long period is covered by full-time nurses, while part-time nurses cover additional demand for intervals.

Allowing one or two rotations results in the assignment of nurses with single and multiple qualification levels with demand profiles 1, 3, and 6. However, even allowing two feasible rotations does not result in the assignment of full-time nurses who are qualified for all tasks. The impact of considering a single task rotation on the objective function value is significant when facing demand profiles 1, 3, and 6, with objective function values dropping between 20 and 30%. Considering a potential second task rotation yields an additional 10% (3%) reduction in profile 3 (profile 6). When assigning task rotations for multi-qualified nurses is optimal, we often see a shift in the nurse workforce composition. Then, the number of full-time nurses drops while the number of part-time nurses increases. Overall, we can conclude that more than one assigned task rotation is only beneficial for demand profiles with very large demand variability, where demand shifts from one task to another multiple times during the day.

In the second comparison, the minimum time required to stay at each activity before switching is increased. This number increases from 4 to 6 to 8 slots. If the time slots are expanded to 6 or 8, the maximum number of switches for each full-time nurse is restricted to one, as a full-time nurse works a total of 16 slots. Increasing this residency period leads to an increase from 0 to 36 overstaffed slots for profiles 3 and 6 (identical for 6 or 8 slots) and 72 overstaffed slots for profile 1, as well as a shift from multi-qualified nurses to nurses with a single qualification level. Consequently, fewer switches are performed.

Managerial Insights. We can conclude that the incorporation of task rotations in the determination of optimal shift design and workforce composition is only meaningful in settings with highly variable demand, where demand shifts between tasks frequently. Over a typical workday, it is usually sufficient to include a single rotation for realistic demand settings. In general, these demand settings call for a higher share of part-time nurses in the workforce. Additionally, we observe that it is preferable to minimize the required time between rotations spent on a particular task, particularly when demand levels change frequently. However, if switching between tasks is permitted too early, additional effects that are outside of the scope of our study may occur. For example, productivity may drop due to the tasks' setup times, and learning may be hindered. When demand for the various tasks is constant or growing or decreasing in parallel, task rotations are not necessary.

6 Conclusion

The present work describes a problem that aims to identify optimal shifts and the corresponding workforce mix when flexible full- and part-time workers are allocated to multiple tasks with fluctuating demand over a workday. The assignments may include potential rotations between tasks for individual workers, and shift start times are endogenous.

Table 6 Possible variations of parameters and the reference to the instance ID

		Possible rotations ($L^m - 1$)		
		0	1	2
Minimum residence time F at a task [slots]	4	13	12	14
	6	redundant	15	Not possible
	8	redundant	16	Not possible

Additionally, workers differ with regard to their qualification levels.

Our approach commences with the formulation of the problem as a concise mixed-integer program, where we proceed to generate solutions across a spectrum of scenarios. We rely on data sourced from a radiation oncology department in Germany, a clinical setting characterized by the need to complete numerous tasks throughout the day, with the demand for each task fluctuating over time.

In the course of our experimental investigation, it becomes evident that a commercial solver like CPLEX struggles to deliver optimal solutions, and in most realistically-sized instances, it fails to provide any solutions within a reasonable timeframe, often exceeding one hour. Consequently, we introduce a column generation solution strategy, leveraging problem decomposition based on nurse types. Our findings demonstrate the efficacy of this approach, as all instances of realistic scale are solved within a matter of minutes, with the majority being resolved in mere seconds. Notably, the CG approach consistently yields integer solutions for all practical instances, thereby eliminating the need for additional branching strategies. However, we also show that this property breaks down in an extreme hypothetical setting with maximum flexibility and demand variability, which causes the runtime to dramatically increase. Nevertheless, the MP-as-IP solution of the final RMP is still virtually similar to its LP solution. Utilizing the CG method, we conduct analyses for a variety of demand profiles and sensitivity analyses on the most interesting input parameters. For example, we evaluate the impact of the maximum number of changes permitted during a nurse's shift as well as the minimum time before a task rotation is allowed. Moreover, we provide analyses of other hospital settings in the Appendix.

The proposed method is shown to be advantageous for settings with a flexible workforce and variable demand for multiple tasks. This may have a favorable effect on the health of employees since they do not execute the same activity all day long. However, this strategy may not be ideal for situations that need simple and rapid task changes. The potential problem of additional setup times caused by task rotations is not addressed in our study. This may also impact productivity and can be addressed in future research. Methodologically, it may be worthwhile to further study the gapped C1P and the necessary conditions for it to break down, resulting in the decreasing performance of our column generation approach. The formulation of the compact model might be inefficient and exploring different formulations could be useful. For example, using binary representations of some integer decision variables could be a wise choice. If the model was to be used to support staffing or rostering decisions, it would have to be expanded to a weekly or monthly time horizon.

Appendix

Overstaffing penalty function

The penalty function C_{ts} for time-dependent overstaffing is (Fig. 8):

$$C_{ts} := \frac{-\left(s - \frac{1}{3} * |S|\right)^2}{|S|} + \frac{|S|}{2} = \lim_{s \to |S|} \frac{|S|}{18}$$

Additional hospital settings

To illustrate the wide applicability of our model that, we compiled further demand profiles for additional hospital departments (i.e., pharmacy, emergency department, medical ward, intensive care unit), drawing from our experience from prior consulting projects. Here, we would like to present the demand profiles and the results for these settings. Each hospital setting is solved for 9 instances with different combinations of the allowed number of possible rotations and the minimum time spent per task, as explained in Table 7.

Note: While the demand fluctuation in these settings resembles the observed fluctuation, overall demand levels have been scaled for the sake of comparability. Moreover, we chose the same time span from 7 a.m. to 6 p.m. to be covered in all departments to make the results comparable, even though these units operate 24/7 (with very low worker demand at night times).

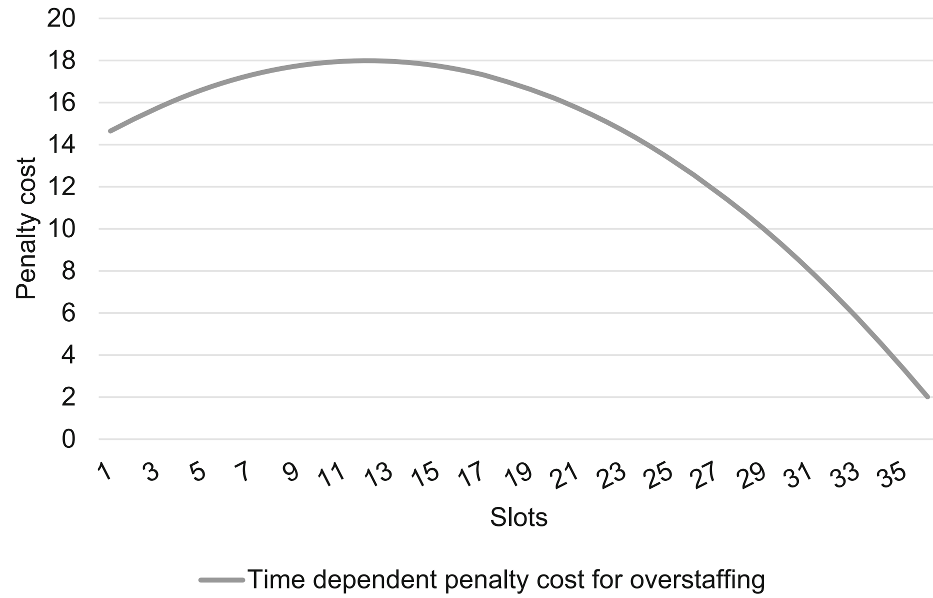

Fig. 8 Penalty function for the time-dependent overstaffing

Table 7 Variations of parameters and corresponding instance IDs in additional settings

		Possible rotations ($L^m - 1$)				
		0	1	2	3	4
Minimum time F at a task [slots]	1	redundant	ID2	ID6	ID8	ID10
	2	ID1	ID3	ID7	ID9	Not possible
	3	redundant	ID4	Not possible	Not possible	Not possible
	4	redundant	ID5	Not possible	Not possible	Not possible

Demand profiles for additional settings

In the *hospital pharmacy*, we include the following activities in our numerical study: prescription filling, medication delivery, and administrative tasks. In the *emergency department*, the activities are treatment administration, patient monitoring and discharge, and patient triage. In the *medical ward* setting, we include patient care, charting, and coordination. Lastly, we study complex procedures, basic care, and medication administration in the *intensive care unit* setting. The demand profiles are provided in Fig. 9.

Results for each instance of the additional settings

Here, we present the results for the 10 instances in each of the additional settings, see Tables 8, 9 10 and 11. Overall, we see that our model is widely applicable. However, in the additional settings, the maximum number of allowed rotations and the required minimum duration at a task play a rather insignificant role. In three of the four settings, we see the biggest, and often only, improvement when moving from 0 to 1 maximum rotations. In the medical ward, the solutions are the same for all instances. This can mostly be attributed to the lower demand variability over the day compared to our original study.

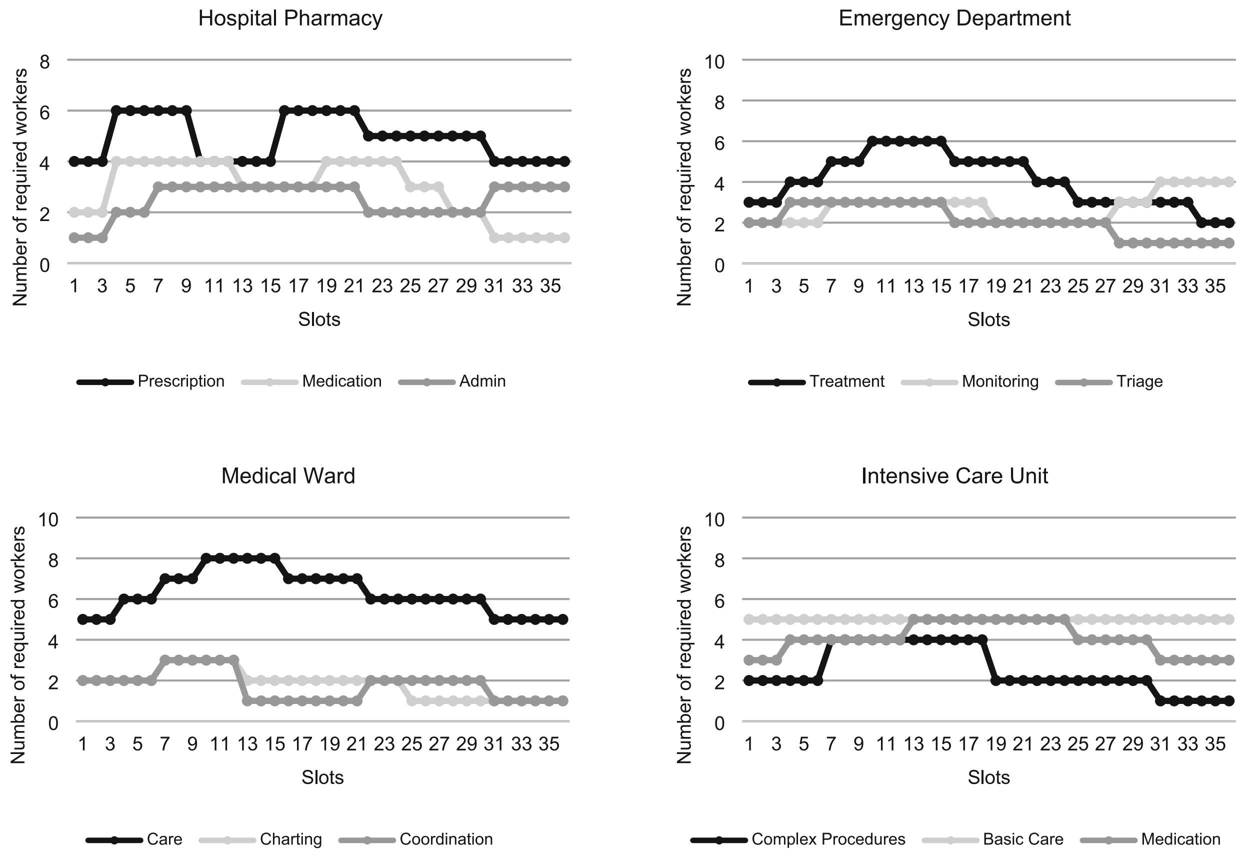

Fig. 9 Demand profiles in the additional hospital settings

Table 8 Results for the pharmacy setting

ID	max rotations	min shift	CG solution	MPasIP solution	% BB to CG	Iterations	Solution time [sec]	Prescription full	Admin full	Medication full	Prescription half	Admin half	Medication half	Overstaffed slots
ID1	0	2	2415.2	2415.2	0%	32	24	3	3	3	3	3	3	11
ID2	1	1	2318.4	2318.4	0%	72	72	3	4	3	3	1	2	7
ID3	1	2	2318.4	2318.4	0%	61	54	3	4	3	3	1	2	7
ID4	1	3	2333.7	2333.7	0%	46	39	3	3	3	3	3	2	7
ID5	1	4	2407.9	2407.9	0%	34	24	3	3	4	3	2	2	11
ID6	2	1	2318.4	2318.4	0%	80	100	3	4	3	3	1	2	7
ID7	2	2	2318.4	2318.4	0%	65	67	3	4	3	3	1	2	7
ID8	3	1	2318.4	2318.4	0%	64	70	3	4	3	3	1	2	7
ID9	3	2	2318.4	2318.4	0%	64	64	3	4	3	3	1	2	7
ID10	4	1	2318.4	2318.4	0%	75	112	3	4	3	3	1	2	7

Table 9 Results for the emergency department setting

ID	max rotations	min shift	CG solution	MPasIP solution	% BB to CG	Iterations	Solution time [sec]	Treatment full	Monitoring full	Triage full	Treatment half	Monitoring half	Triage half	Overstaffed slots
ID1	0	2	1948.0	1948.0	0%	40	29	5	2	3	3	5	1	9
ID2	1	1	1880.5	1880.5	0%	93	106	5	2	2	3	5	2	5
ID3	1	2	1880.5	1880.5	0%	75	81	5	2	2	3	5	2	5
ID4	1	3	1880.5	1880.5	0%	49	42	5	2	2	3	5	2	5
ID5	1	4	1948.0	1948.0	0%	45	37	5	2	3	3	5	1	9
ID6	2	1	1880.5	1880.5	0%	110	153	5	2	2	3	5	2	5
ID7	2	2	1880.5	1880.5	0%	59	65	5	2	2	3	5	2	5
ID8	3	1	1880.5	1880.5	0%	76	89	5	2	2	3	5	2	5
ID9	3	2	1880.5	1880.5	0%	67	76	5	2	2	3	5	2	5
ID10	4	1	1880.5	1880.5	0%	89	155	5	2	2	3	4	3	5

Table 10 Results for the medical ward setting

ID	max rotations	min shift	CG solution	MPasIP solution	% BB to CG	Iterations	Solution time [sec]	Care full	Charting full	Coordination full	Care half	Charting half	Coordination half	Overstaffed slots
ID1	0	2	2193.7	2193.7	0%	37	27	6	2	2	7	2	2	5
ID2	1	1	2193.7	2193.7	0%	63	56	6	2	2	7	2	2	5
ID3	1	2	2193.7	2193.7	0%	62	56	6	2	2	7	2	2	5
ID4	1	3	2193.7	2193.7	0%	42	32	6	2	2	7	2	2	5
ID5	1	4	2193.7	2193.7	0%	46	37	6	2	2	7	2	2	5
ID6	2	1	2193.7	2193.7	0%	99	145	6	2	2	7	2	2	5
ID7	2	2	2193.7	2193.7	0%	59	69	6	2	2	7	2	2	5
ID8	3	1	2193.7	2193.7	0%	84	116	6	2	2	7	2	2	5
ID9	3	2	2193.7	2193.7	0%	79	107	6	2	2	7	2	2	5
ID10	4	1	2193.7	2193.7	0%	91	157	6	2	2	7	2	2	5

Table 11 Results for the intensive care unit setting

ID	max rotations	min shift	CG solution	MPasIP solution	% BB to CG	Iterations	Solution time [sec]	Complex full	Basic care full	Medication full	Complex half	Basic care half	Medication half	Overstaffed slots
ID1	0	2	2236.4	2236.4	0%	35	28	3	5	5	2	5	3	5
ID2	1	1	2229.7	2229.7	0%	70	74	4	5	5	1	5	2	5
ID3	1	2	2229.7	2229.7	0%	51	48	4	5	5	1	5	2	5
ID4	1	3	2229.7	2229.7	0%	44	41	4	5	5	1	5	2	5
ID5	1	4	2229.7	2229.7	0%	38	31	4	5	5	1	5	2	5
ID6	2	1	2229.7	2229.7	0%	76	100	4	5	5	1	5	2	5
ID7	2	2	2229.7	2229.7	0%	62	76	4	5	5	1	5	2	5
ID8	3	1	2229.7	2229.7	0%	68	84	4	5	5	1	5	2	5
ID9	3	2	2229.7	2229.7	0%	76	103	4	5	5	1	5	2	5
ID10	4	1	2229.7	2229.7	0%	88	151	4	5	5	1	5	2	5

Funding Open access funding provided by Technical University of Denmark.

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