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Type-2 Fuzzy Clustering and a Type-2 Fuzzy Inference Neural Network for the Prediction of Short-Term Interest Rates

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Abstract

The following paper discusses the use of a hybrid model for the prediction of short-term US interest rates. The model consists of a differential evolution-based fuzzy type-2 clustering with a fuzzy type-2 inference neural network, after input preprocessing with multiple regression analysis. The model was applied to forecast the US 3-Month T-bill rates. Promising model performance was obtained as measured using root mean square error.

Keywords: Multiple Regression Analysis, Differential Evoultion, Type-2 Fuzzy Systems, Interest Rate Forecasting

1. Introduction

1.1. Structure

Interest rate forecasting is one of the more challenging problems in the finance domain given their dynamic and uncertain nature. Various types of conventional statistical techniques have been used during the first generation of models that were used for interest rate forecasting. One popular approach, traditional time series forecast modelling, is an important prediction technique that attempts to explain the input-output relationship based on collected historical data. An important advantage of using these types of conventional techniques is their ability to explain underlying relationships when there is a small amount of data available. However, due to the uncertainty, non-linearity, and complexity, it is difficult to model problems such as interest rate prediction with conventional linear models. Researchers tend to develop models that can capture and handle the complex nature and dependencies of the

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time series variables without considering the structural assumptions, such as homoscedasticity, linearity, and normality, among others. This has caused the number of nonparametric or nonlinear models to rapidly increase over the last few years. For instance, a Markov switching model was applied by Hamilton [1] to forecast the US 3-Month T-bill rates, revealing a methodology that outperforms the linear models. Das [2] explains the nonlinear nature of interest rates by associating them with stochastic jumps, whereas Naik and Lee [3] explained this complex relationship by considering the business cycle. Pfann et al. [4] and Granger and Terasvirta [5] analysed the term structure of interest rates by considering the nonlinearities in the data.

The complex structure of the interest rate markets, the errors in collecting historical data, time lags, the reciprocal dependency of the input variables, and the need for a relevant linguistic variable expression play a critical role in interest rate market prediction, even though the prediction ability of these factors tend to decrease when using traditional statistical models. Most of the traditional models, such as ARIMA, GARCH, VAR, and Bayesian VAR have difficulties dealing with the non-linear, non-stationary and dynamic environment of the interest rate markets. With the need for more advanced models to overcome current difficulties, the use of computational intelligence, in particular, artificial neural networks (ANN), has increased as a result of their ability to deal with non-linear problems given that ANN time series modeling is not impacted by irregular sampling and short time series. Such benefits have allowed ANNs to be accepted as a powerful technique for modeling dynamic nonlinear systems. Kang [6] found that ANN forecasting models perform well, even with small sample sizes (lower than 50), whereas traditional Box-Jenkins models (ARIMA) often require greater than 50 data points for successful forecasting. This approach is suitable for many empirical data sets where no guidance is available to suggest an appropriate data generating process [7]. ANNs, such as fuzzy inference neural networks, are considered to be strong techniques for environments that cannot be modeled easily, even with available historical input-output numerical data. As mentioned, interest rate prediction is one example of such an environment due to its multidimensional and non-stationary nature, making it difficult to model unless using very sophisticated mathematical expressions.

Different types of the uncertainties related to the interest rate markets and their associated factors should be considered during model building in order to get more precise prediction results. Models based on type-2 fuzzy sets, originally proposed by Zadeh [8], deal with uncertainty directly and efficiently due to their three-dimensional membership functions, thereby producing robust and stable results. At the same time it is necessary to take into consideration that the implementation of type-2 fuzzy logic leads to a higher computational complexity in comparison with type-1 sets given that they utilize three-dimensional membership functions [9].

1.2. Research Purpose

The purpose of the following research is the implementation of a differential evolution-based fuzzy type-2 clustering method with a fuzzy type-2 inference neural network, after input pre-processing with regression analysis, in order to predict future interest rate values, particularly 3-month T-bill rates. In previous work, LeRoy [10] determined that regardless of whether or not past T-bill rates play a role for predicting future T-bill rate returns, their movement is not random, causing failure of the random walk hypothesis. Similarly, Larrain [11] found that past returns can help determine future interest rates, and that the relationship of the lagged interest rates and future returns are nonlinear, but these lagged interest rates are not the only determinants of future returns.

As with normal equity returns, work in the interest rate prediction area is based on either technical factors, which refer to the past interest rate values, or fundamental factors, which imply that economic and financial variables are determinants of the future returns. The research in this paper considers a unique combination of fundamental factors combined with historical interest rate returns (technical component) to produce better results.

2. Proposed Model and Empirical Results

Type-2 fuzzy neural networks can take advantage of the capabilities of fuzzy clustering by generating a type-2 fuzzy rule base, resulting in a small rule set [12]. The networks then optimize the membership functions of the type-2 fuzzy sets present in the antecedent and consequent parts of the If-Then rules. The clustering is realized with the aid of differential evolution. The goal of the research is to propose a hybrid methodology that consists of multiple regression analysis (MRA), type-2 fuzzy clustering, and a type-2 fuzzy neural network for predicting 3-month T-bill rates. More specifically (see Figure 1), the model initially pre-processes data with regression analysis to determine the variables with strong prediction ability. The model then generates a type-2 fuzzy "If-Then" rule base from data by making use of clustering augmented by the mechanisms of evolutionary optimization, and then adjusts (optimizes) the parameters of the initial rule-base on the basis of a type-2 fuzzy neural inference system using the DE optimization (DEO). In this study, the authors are confined to the optimization framework of differential evolution.

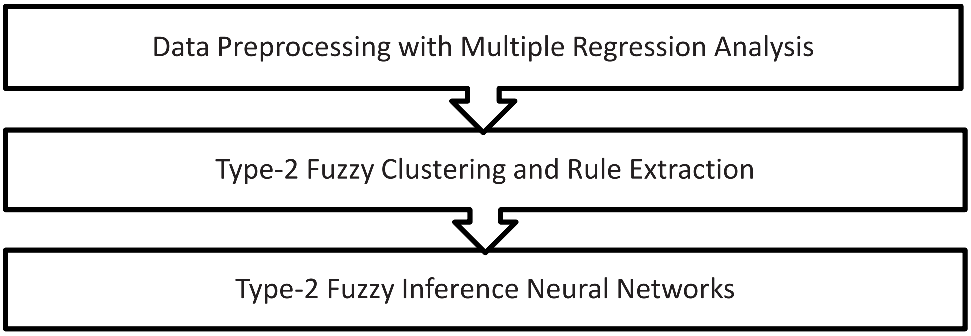

Figure 1: Proposed Model

2.1. Multiple Regression Analysis

The volatile factors that influence interest rates and their dynamic environment requires a good understanding of which variables have the strongest ability to describe the characteristics of interest rates. Hurst exponent analysis and principal component analysis are considered effective methods for input variable pre-processing. Multiple regression analysis (MRA) is implemented in the proposed interest rate prediction model in order to better understand which input variables are significant and might be influencing the output. MRA considers the relationship degree between the included inputs and the output. The variables that are more strongly correlated with the output are retained in the system and used in the next steps for prediction. The indicators are entered to the regression analysis in a specified order and the contribution of each entered variable is assessed. The specified order requires information about theoretical links and previous experiences. If a variable does not improve the prediction ability of the model, it is removed from the list of candidates. Such iteration allows the set of optimal variables to be identified after testing all candidate variables.

Multiple regression analysis was performed on the following 36 financial and economical variables to reduce the input variable size and explore the link between diverse input variables and the 3-month T-bill rate for the next month: PP_{t-1}, CP_{t-1}, IP_{t-1}, $M1_{t-1}$, $M2_{t-2}$, $T3_t$, $T6_t$, $T12_t$, $T60_t$, $T120_t$, $CD1_t$, $CD3_t$, $CD6_t$, AAA_t, BAA_t, DIV_t, $T1_t$, $SP500_{t-1}$, DY_t, GNP_t, $TE1_t$, $TE2_t$, $TE3_t$, $TE4_t$, $TE5_t$, $TE6_t$, FFR_t, $DE1_t$, $DE2_t$, $DE3_t$, $DE4_t$, $DE5_t$, $DE6_t$, $DE7_t$, r^2_t, and r^3_t. A description of the data, obtained from the database of the Federal Reserve Bank of St. Louis and Thompson Reuters DATASTREAM, is provided in the Appendix. Quarterly data was used, and covers the period of June 1960 to January 2011, for a total of 208 data points. The multiple regression analysis was conducted in the statistical software SPSS, Version 19.0. According to the multiple regression analysis, the following variables are identified to have a strong link with the 3-month T-bill rate: $M2_{t-2}$ (Money Supply), GNP_t (Gross National Product), CP_{t-1} (Consumer Price Index), FFR_t (Federal Funds Rate), SP 500_{t-1} (Standard and Poor 500 Market Index), r^2_t (Squared 3-month T-bill rate of the current month), and r^3_t (Cubed 3-month T-bill rate of the current month).

R	R-Square	Adjusted R-Square	Standard Error of the Estimate	Observations
0.9683	0.9377	0.9351	0.7548	208

Table 1: The Results of Multiple Regression Analysis

The value of R is the measure of the correlation between the 3-month T-bill rate and the predicted value of the variables mentioned above. R-Square is the numerical square of this value and shows the proportion of the variance in the 3-month T-bill rate accounted for by the selected input variables. The value of the R-Square (0.9377) suggests that the model is a good fit and that the combination of the selected variables can be used as input to the next phases of the proposed hybrid model (see Table 1).

2.2. Type-2 Fuzzy Clustering

As previous experiments have shown, using a type-2 fuzzy clustering method provides better location of the cluster centers and subsequently results in a better fuzzy rule model, ultimately allowing the model to capture more uncertainty while delivering higher robustness against data imprecision. The objective function is as follows (with n data vectors, $P = \{p_1, p_2, \ldots, p_n\}$ inputs; prototype v_j of the j^{th} cluster generated by the fuzzy clustering; membership degree u_{ij} of the i^{th} data belonging to the j^{th} cluster represented by the prototype v_j) [13]:

$$J_{m1} = \sum_{i=1}^{n}\sum_{j=1}^{c} u_{ij}^{m_1} \left\| \mathbf{p}_i - \mathbf{v}_j^{(1)} \right\| \to \min \quad , \quad J_{m2} = \sum_{i=1}^{n}\sum_{j=1}^{c} u_{ij}^{m_2} \left\| \mathbf{p}_i - \mathbf{v}_j^{(2)} \right\| \to \min \tag{1}$$

subject to constraints:

$$0 < \sum_{i=1}^{n} u_{ij} < n \quad (j = 1, 2, ..., c) \quad \text{and} \quad \sum_{j=1}^{c} u_{ij} = 1 \quad (i = 1, 2, ..., n)$$

The vector $\tilde{\mathbf{v}}_i$ is formed as:

$$\tilde{\mathbf{v}}_i = [\min(\mathbf{v}_i^{(1)}, \mathbf{v}_{Ind_i}^{(2)}), \max(\mathbf{v}_i^{(1)}, \mathbf{v}_{Ind_i}^{(2)})] \text{ where } Ind_i = \arg\min_j \left\| \mathbf{v}_i^{(1)}, \mathbf{v}_j^{(2)} \right\| \tag{2}$$

For the proposed approach, DE is used for objective function minimization since it acts as a global search algorithm and is expected to be more advantageous than standard fuzzy c-means for the case of a large number of highly dimensional data vectors [14]. For simulation, the initial DEO fuzzy type-2 clustering data are: number of clusters = 7, max iteration = 1000, exponent = 2, population size = 200. The clusters found by the DE-based clustering are shown in Table 2:

Cluster 1	**Cluster 2**	**Cluster 3**	**Cluster 4**	**Cluster 5**	**Cluster 6**	**Cluster 7**
0.0904201	0.2086109	0.0103873	0.7671320	1.9397363	0.4394652	0.2267071
0.0368449	0.1082193	0.0037804	0.6918620	2.7293430	0.3055322	0.1128614
6.1333193	0.5588744	8.2932517	1.5077672	1.5988195	2.7827404	3.7709589
1.2408362	0.3995932	1.3218029	0.5877308	0.6015189	0.7524985	0.9590778
1.9016772	0.3717830	2.1692072	0.7739771	0.8761585	1.1694665	1.5513402
1.2408362	0.3995932	1.3218029	0.5877308	0.6015189	0.7524985	0.9590778
0.2757177	0.4598274	0.0554584	0.9802734	1.6110595	0.7284304	0.4993491

Table 2: Clusters Found by DE

The linguistic hedges approach [15] can then be used to derive the corresponding interpretable linguistic model. An example of the fuzzy type-2 If-Then model (rules) discovered by fuzzy clustering is shown below:

IF $M2_{t-2}$ is about A1 AND GNP_t is about A2 AND GNP_t is A3 AND FFR_t is about A4 AND
FFR_t is about A5 AND SP 500_{t-1} is about A6 AND r^2_t is about A7 AND r^3_t is about A8
THEN r_{t+1} is about A9

2.3. Type-2 Fuzzy Inference Neural Network

The type-2 fuzzy inference neural network structure proposed by Aliev [12] is shown in Figure 2. Layer 1 consists of fuzzifiers that map inputs to type-2 fuzzy terms used in the rules. Layer 2 comprises nodes representing these rules, with each rule node performing the Min operation on the outputs (interval valued membership degrees) of the incoming links from the previous layer. Layer 3 consists of output term membership functions of type-1. Layer 4 computes the fuzzy output signal for the output variables. Layer 5 realizes the defuzzification using the center-of-gravity (COG) defuzzification.

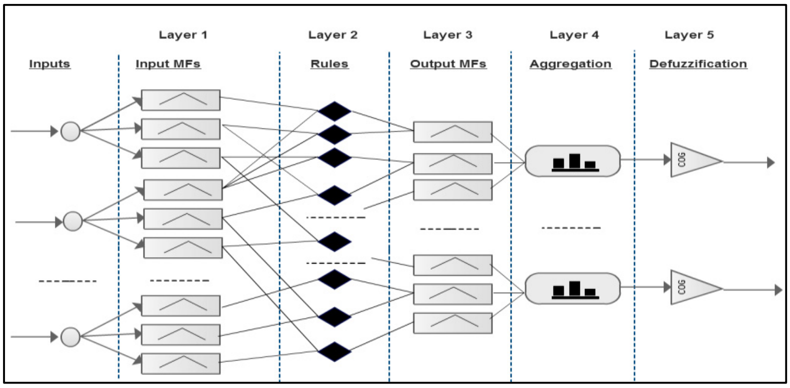

Figure 2: Fuzzy Inference Neural Network

Type-2 neural network parameters (initiated during the clustering procedure) are adjusted by the DE algorithm on the training series (90% of all data). The experimental results indicate that the artificial neural network hybrid model with differential evolution optimization-based fuzzy clustering and fuzzy inference achieves a very good performance with RMSE = 0.7952, much better than was achieved (RMSE = 0.9377) without using type-2 fuzzy clustering and a type-2 fuzzy inference neural network [16].

3. Conclusion

This research analyzed the implementation of the hybrid interest rate prediction model that consists of multiple regression analysis, type-2 fuzzy clustering, and a type-2 fuzzy inference neural network. In the first stage the input data was preprocessed with multiple regression analysis and the selected variables with strong prediction ability were provided as input to the second stage that included differential evolution optimization-based type-2 fuzzy clustering. Multiple regression analysis retained $M2_{(t-2)}$ (Money Supply), $GNP_{(t)}$ (Gross National Product), $CPI_{(t-1)}$ (Consumer Price Index), $FFR_{(t)}$ (Federal Funds Rate), SP $500_{(t-1)}$ (Standard and Poor 500 Market Index) and two non-linear forms of the interest rates as input variables. As seen from the variables above, the proposed hybrid model distinguishes itself from previous interest prediction modeling approaches by using both fundamental and technical factors. At the final stage the interest rate values of the following period were predicted via a type-2 fuzzy inference neural network that deals with nonlinear, non-stationary and uncertain environments more efficiently than other conventional statistical methods.

Appendix

- SP: Nominal Standard & Poor's 500 index at the close of the last trading day of each month. Source: CSI
- T1: Annualized average of bid and ask yields on the one-month T-bill rate on the last trading day of the month. This refers to the shortest maturity T-bill not less than one month in maturity. Source: CRSP tapes, the Fama risk-free rates
- T3: 3-month T-bill rate, secondary market, averages of business days, discount basis. Source: H.15 Release-Federal Reserve Board of Governors (FED B of G)
- T6: 6-month T-bill rate, secondary market, averages of business days, discount basis. Source: H.15 Release, FED B of G
- T12: 1-year T-bill rate, secondary market, averages of business days, discount basis. Source: H.15 Release, FED B of G
- T60: 5-year T-bill constant maturity rate, secondary market, averages of business days. Source: H.15 Release, FED B of G
- T120: 10-year T-bill constant maturity rate, secondary market, averages of business days. Source: H.15 Release, FED B of G
- r^2: Squared 3-month T-bill rate of the current month
- r^3: Cubed 3-month T-bill rate of the current month
- CD1: 1-month certificate of deposit rate, averages of business days. Source: H.15 Release, FED B of G

- CD3: 3-month certificate of deposit rate, averages of business days. Source: H.15 Release, FED B of G
- CD6: 6-month certificate of deposit rate, averages of business days. Source: H.15 Release, FED B of G
- AAA: Moody's seasoned Aaa corporate bond yield, averages of business days. Source: The Federal Reserve Bank of St. Louis
- BAA: Moody's seasoned Baa corporate bond yield, averages of business days. Source: The Federal Reserve Bank of St. Louis
- PP: Producer Price Index: Finished Goods. Source: US Department of Labor, Bureau of Labor Statistics (BLS)
- CP: Consumer Price Index: CPI for All Urban Consumers. Source: US Department of Labor, BLS
- M1: Money Stock. Source: H.6 Release, FED B of G
- M2: Money Supply. Source: H.6 Release, FED B of G
- IP: Industrial Production Index: Market Groups and Industry Groups. Source: G.17 Statistical Release, FED Statistical Release
- FFR: Federal Funds Rate
- DIV: Nominal dividends per share per month, S&P 500 portfolio, Source: Annual dividend record/Standard and Poor's Corporation
- GNP: Gross National Product
- TE1: Term spread between T120 and T1, calculated as TE1 = T120 - T1
- TE2: Term spread between T120 and T3, calculated as TE2 = T120 - T3
- TE3: Term spread between T120 and T6, calculated as TE3 = T120 - T6
- TE4: Term spread between T120 and T12, calculated as TE4 = T120 - T12
- TE5: Term spread between T3 and T1, calculated as TE5 = T3 - T1
- TE6: Term spread between T6 and T1, calculated as TE6 = T6 - T1
- DE1: Default spread between BAA and AAA, calculated as DE1 = BAA - AAA
- DE2: Default spread between BAA and T120, calculated as DE2 = BAA - T120
- DE3: Default spread between BAA and T12, calculated as DE3 = BAA - T12
- DE4: Default spread between BAA and T6, calculated as DE4 = BAA - T6
- DE5: Default spread between BAA and T3, calculated as DE5 = BAA - T3
- DE6: Default spread between BAA and T1, calculated as DE6 = BAA - T1
- DE7: Default spread between CD6 and T6, calculated as DE7 = CD6 - T6
- DY: Dividend yield on the S&P 500 portfolio, calculated as $DY_t = DIV_t / SP_t$

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