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Probing prethermal nonergodicity through measurement outcomes of monitored quantum dynamics

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Projective measurements are a key element in quantum physics and enable rich phenomena in monitored quantum dynamics. Here, we show that the measurement outcomes, recorded during monitored dynamics, can provide important information about the properties of the monitored dynamical system with prethermal nonergodicity. We demonstrate this for a Floquet model of many-body localization, where we find that the prethermal many-body localized regime becomes unstable against rare measurements, yielding an unusual enhancement of quantum entanglement. Through an unsupervised learning and mutual information analysis on the classical dataset of measurement outcomes, we find that the information loss in the system, reflected by the increased entanglement, is compensated by an emergent structure in this classical dataset. Our findings highlight the crucial role of measurements and corresponding classical outcomes in capturing prethermal nonergodicity, offering a promising perspective for applications to other monitored quantum dynamics.

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Acting in unitary quantum evolution with projective measurements can fundamentally alter the quantum entanglement properties of a system [1–30], and the classical measurement outcomes can potentially offer insights into its nonequilibrium dynamics [31–35]. A prominent example is the measurement-induced phase transition (MIPT) in systems governed by scrambling unitary dynamics, where a disentangled phase characterized by an area-law entanglement entropy (EE) emerges at high measurement rates [1–25]. The MIPT can also manifest as learnability transitions, wherein sufficiently frequent measurements can leak out information about the initial state of the system, enabling its retrieval from the classical measurement record [32–35]. Beyond conventional MIPTs, replacing scrambling unitary dynamics with strictly nonscrambling controlled-Z (CZ) gates introduces a new type of MIPT, where measurements can play an active role in the EE [36,37]. Moreover, the study of MIPT with deeply many-body localized (MBL) unitary evolution [38], as an extremely slow scrambling dynamics that occurs in interacting quantum systems with sufficiently strong disorder [39–42], suggests that the nature of MIPTs is dependent on the ergodicity (or nonergodicity) of the system. Generally, the changes of quantum entanglement induced by measurements are intrinsically linked to a transformation in the information content of the underlying quantum states. This naturally raises two key questions: Where does this information flow to or from, and

how is the corresponding change connected to the ergodic (or nonergodic) properties of the system?

In this Letter, we study the information content embedded in the measurement outcomes recorded during monitored quantum dynamics, and show how this information can be utilized to characterize the dynamical properties of the underlying quantum system with prethermal nonergodicity. A typical example of prethermal nonergodicity is the prethermal MBL regime that occurs in interacting disordered systems at intermediate disorder strengths, where thermalization becomes extremely slow, with the equilibration time increasing exponentially with disorder strength, but is not entirely arrested [43]. Here, we consider the monitored quantum dynamics of a Floquet model of MBL with a broad region of prethermal MBL at finite system sizes [44], interspersed with rare measurements in the Z basis. We find that this prethermal MBL regime becomes unstable in the presence of the rare measurements, leading to an unusual enhancement of quantum entanglement and, consequently information loss. We demonstrate that the change in entanglement properties can manifest as variations in the information content of the classical dataset of measurement outcomes. In particular, by means of a principal component analysis (PCA), we find that in the prethermal MBL regime, the classical dataset of measurement outcomes becomes more structured, exhibiting a lower effective dimension [45,46], in comparison with the well-thermalizing and deep MBL regimes. We further show that this is accompanied by strong temporal correlations between the measurement outcomes with infinite correlation length, as revealed by a mutual information analysis. Our findings suggest that changes in entanglement properties

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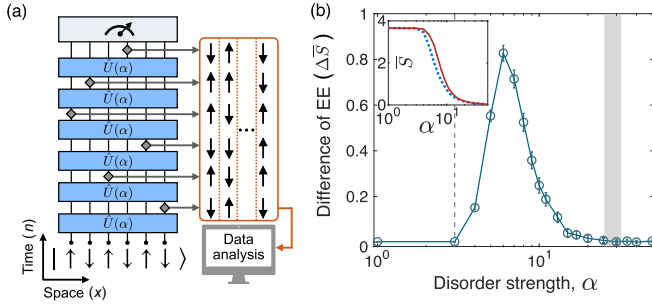


FIG. 1. (a) Schematic illustration of our monitored quantum circuit, whose unitary dynamics is governed by a Floquet model of MBL with gates $\hat{U}(\alpha)$ interspersed with stroboscopic projective measurements in the Z direction. We record the outcome of measurements during the dynamics of the quantum circuit in the form of a dataset, where each column represents the measurement outcomes of a single realization for the monitored quantum circuit. This dataset is then analyzed by means of principal component analysis (PCA) and mutual information. (b) For the system size $L = 12$, the difference of the long-time entanglement entropy (EE) between the purely unitary evolution and the quantum circuit interspersed with rare measurements (at probability $p = 10^{-4}$), i.e., $|\Delta \bar{S}| = |\bar{S}(p = 10^{-4}) - \bar{S}(p = 0)|$ as a function of disorder strength α . Here, the left vertical dashed line highlights an estimation of the crossover point $\alpha \simeq 3$ between the thermal and prethermal MBL regimes, and the right vertical shaded region refers to the boundary between prethermal MBL and deep MBL regions $\alpha \in [25, 32]$ based on Ref. [44]. The inset shows the long-time EE for the unitary dynamics (dotted line) and with rare measurements (solid line) as a function of disorder strength α .

during monitored dynamics can be compensated by information content contained in the dataset of classical measurement outcomes.

Quantum circuits. In the following we consider a one-dimensional system of L qubits with open boundary conditions. The monitored quantum circuit consists of a unitary evolution described by a Floquet model of many-body localization with unitaries $\hat{U}(\alpha)$ where α denotes the disorder strength, and projective measurements in the Z basis through projectors $P_{\pm} = (1 \pm Z)/2$ whose outcomes are given by the Born rule [see Fig. 1(a)]. We will consider these measurements to only occur rarely at a measurement rate $p \simeq 10^{-4}$.

Specifically, the unitary part can be expressed as $\hat{U}(\alpha) = \hat{U}_u[\otimes_{i=1}^L \hat{d}_i]$, which is comprised of single-qubit gates $\hat{d}_i$ created by sampling a two-dimensional (2D) random matrix from the circular unitary ensemble and subsequently diagonalizing it, and two-qubit gates

$$\hat{U}_u = \otimes_{j=1}^{L-1} \exp \left[\left(\frac{i}{\alpha} \right) M_{k_j, k_{j+1}} \right], \quad (1)$$

with α denoting the disorder strength, $M_{k_j, k_{j+1}}$ sampled from Gaussian unitary ensemble, and $k_j \in S_{L-1}$ being a random permutation [44]. Without loss of generality, the initial state $|\psi_0\rangle = |\uparrow\downarrow\cdots\rangle$ for our dynamics is chosen as the Néel state. The level spacing ratio, as a conventional measure for non-ergodicity and therefore MBL, exhibits a crossover point at $\alpha^{(r)} \simeq 6$ [44]. However, recent studies of the prethermal MBL

regime now indicate that the (deep) MBL regime may only occur for much larger disorder strength $\alpha \gtrsim 25$ [43, 44, 47].

Dataset of classical measurement outcomes. During the monitored quantum dynamics illustrated in Fig. 1(a), we collect the individual measurement outcomes (either $\uparrow$ or $\downarrow$) in chronological order yielding for each realization of the dynamics a vector $\mathbf{m} = (m_{n_1}, \dots, m_{n_M}) \in \{\downarrow, \uparrow\}^M$. Notice that this construction is unique in our case, because we work in the limit of a very low measurement rate $p = 10^{-4}$ so that practically excludes two measurements at the same time step. The system size is fixed at $L = 12$, and with a final time of $n_f = 5 \times 10^4$, the total number of measurements is $M = 60$.

For the statistical analysis of the monitored quantum circuit in Fig. 1(a), we generate numerous realizations N_r of the quantum circuit with different random matrices $\hat{d}_i$ and $M_{k_j, k_{j+1}}$, as well as the positions and times of the measurements. We then get one measurement record for each sample $\mathbf{m}$. The full dataset of recorded measurements can be represented by the data matrix [see also Fig. 1(a) for the schematic of the dataset]

$$\mathbf{M} = \begin{bmatrix} \mathbf{m}^{(1)} \\ \mathbf{m}^{(2)} \\ \vdots \\ \mathbf{m}^{(N_r)} \end{bmatrix} = \begin{bmatrix} \downarrow & \uparrow & \cdots & \downarrow \\ \uparrow & \downarrow & \cdots & \uparrow \\ \vdots & \vdots & \ddots & \vdots \\ \downarrow & \uparrow & \cdots & \downarrow \end{bmatrix}_{N_r \times M}, \quad (2)$$

where $\mathbf{m}^{(k)}$ denotes the measurement record of k th realization of the quantum circuit. We note that in the further analysis of the dataset, the elements $\uparrow$ ($\downarrow$) in Eq. (2) can also be replaced by 1 (−1).

Dynamics of entanglement entropy. Before performing the analysis of the classical dataset, let us first discuss our findings for the dynamical properties of the monitored quantum circuit. Specifically, we consider the time evolution of the half-chain entanglement entropy (EE) defined by $S = -\text{Tr}(\rho_A \log \rho_A)$. Here, the reduced density matrix of the subsystem A , encompassing the qubits $i \in \{1, 2, \dots, L/2\}$, is denoted as $\rho_A = \text{Tr}_{\bar{A}} |\psi(n)\rangle \langle \psi(n)|$, which can be obtained by tracing out the complement $\bar{A}$, with $|\psi(n)\rangle = [U(\alpha)]^n |\psi_0\rangle$ being the state at the time step n .

The main results for the long-time dynamics of the EE are summarized in Fig. 1(b), where we plot the difference of the EE at long times between the purely unitary case and the monitored case included rare measurements. Numerically, we adopt a rate of measurements $p = 10^{-4}$, and calculate the dynamics of EE up to time $n \sim 10^4$. To minimize temporal fluctuations we perform a slight time average $\bar{S} = [\int_{n_i}^{n_f} dn S(n)] / (n_f - n_i)$ with the time interval $n \in [n_i, n_f] = [3 \times 10^4, 5 \times 10^4]$ for different disorder strengths α and system size $L = 12$. We denote the such time-averaged EE with rare measurements and unitary case as $\bar{S}(p = 10^{-4})$ and $\bar{S}(p = 0)$, respectively. The difference of EE defined as $|\Delta \bar{S}| = |\bar{S}(p = 10^{-4}) - \bar{S}(p = 0)|$ is shown in Fig. 1(b), indicating that for both the well-thermalizing phase with $\alpha \lesssim 3$ and deep MBL regime with $\alpha \gtrsim 25$, the rare projective measurements with a small rate $p = 10^{-4}$ do not substantially change the EE.

Surprisingly, the case of intermediate values of α , which coincides compellingly well with the recently proposed prethermal MBL regime [43, 44], behaves completely differently. We reveal that the measurements, although vanishingly rare, can significantly increase the EE [see also the inset

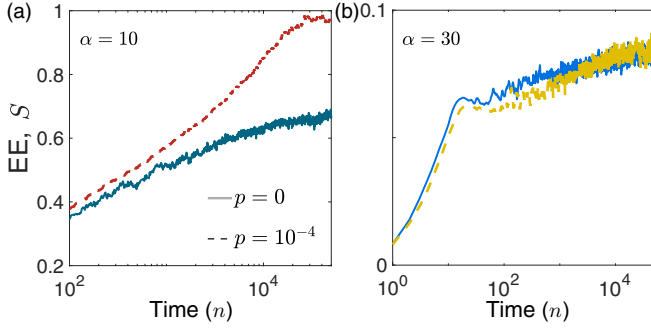


FIG. 2. (a) The entanglement entropy (EE) dynamics for the quantum circuit in Fig. 1(a) with the system size $L = 12$ and disorder strength $\alpha = 10$ for both the purely unitary evolution (with measurement rate $p = 0$) and rare measurements ($p = 10^{-4}$) up to a final stroboscopic time $n = 5 \times 10^4$. (b) is similar to (a) but with $\alpha = 30$. The number of realizations for the monitored quantum circuit is larger than 10^3 .

of Fig. 1(b) for a direct comparison with $\bar{S}(p = 10^{-4})$ and $\bar{S}(p = 0)$. This indicates an instability of the prethermal MBL regime with two noncommuting limits of $n \rightarrow \infty$ and $p \rightarrow 0$. We present a systematic analysis for the finite-size effect for the behavior of $|\Delta \bar{S}|$ in Sec. I of the Supplemental Material (SM) [48], showing that the enhancement of EE by rare measurements can still be observed for larger system sizes.

We now focus on the dynamics of the EE in detail. As shown in Fig. 2(a), at an intermediate value of disorder strength $\alpha = 10$, the dynamics of EE is sensitive to rare measurements, which accelerate its growth and lead to a larger long-time value as compared to the unitary case. In contrast, for a stronger disorder $\alpha = 30$, Fig. 2(b) shows that the effect of rare measurements is less pronounced, and in both the unitary ($p = 0$) and monitored ($p = 10^{-4}$) cases, EE exhibits a slow logarithmic growth, which is similar to the dynamical behaviors of EE in Hamiltonian models of MBL [49–53]. Based on the results of EE, one can see that even with a relatively small system size $L = 12$, the boundary of the deep MBL regime is at much larger α than that estimated by the level spacing ratio $\alpha^{(r)} \simeq 6$. In Sec. II of the SM [48], we discuss the finite-time effect in detail.

Several side remarks are in order. First, if we adopt the forced measurement, so that the outcomes of the measurements are imposed to be $|\downarrow\rangle$ ($|\uparrow\rangle$) for even (odd) sites [54], the enhancement of EE by rare measurements in the prethermal MBL regime is absent (see numerical results in Sec. III of the SM [48]). This indicates that the Born rule plays a key role in the observed instability, which cannot be attributed solely to the breakdown of time periodicity. Second, the enhancement of EE due to measurements restricted to the Z basis is essentially different from the measurement-only dynamics [36], where the measurements in the X basis are necessary to play an active role in the EE. Third, in Sec. IV of the SM [48], we also consider the monitored dynamics of disordered Heisenberg chains, as a standard model for studying many-body localization, with rare measurements. We show that deep MBL regime occurs with $W \gtrsim 15$, significantly exceeding the conventional transition point at $W \simeq 3$, and with $W \lesssim 15$, the

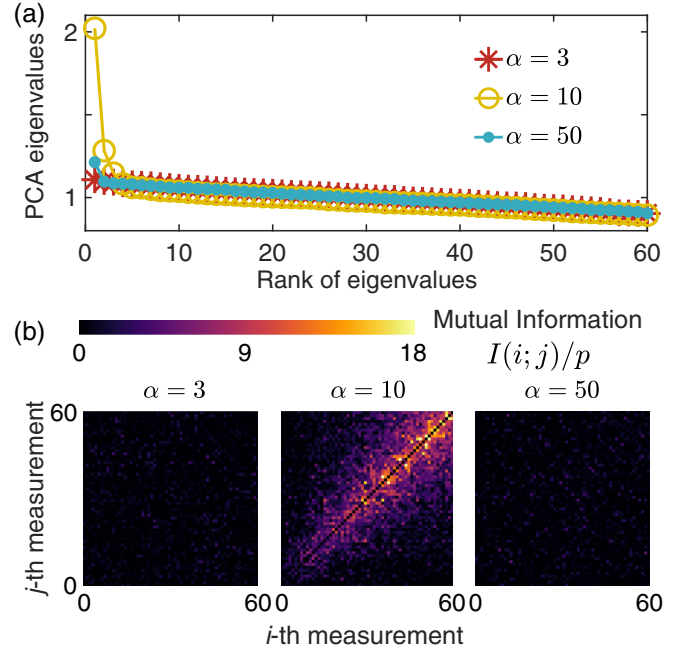


FIG. 3. (a) The PCA eigenvalues for the measurement dataset extracted from the dynamics of the quantum circuit with system size $L = 12$ and three typical values of disorder strength $\alpha = 3, 10$, and 50 . (b) The values of mutual information between the i th and j th measurement outcome $I(i; j)/p$ for $\alpha = 3, 10$, and 50 . Here, we always consider rare measurements with a rate $p = 10^{-4}$.

instability induced by rare measurements with enhanced EE is also observed, revealing a similar broad prethermal MBL regime in the Hamiltonian model.

Analysis of the classical dataset. The enhancement of EE in the prethermal MBL regime suggests an information loss in the nonequilibrium states due to measurements. It is natural to consider that the information can leak into the measurement record. To demonstrate this viewpoint, in the following, we quantify the information content of the classical measurement outcomes.

We start by analyzing the dataset (2) through the PCA, which has been widely employed to detect equilibrium phase transitions [55–57] and MITs [15,58]. With the $M \times N_r$ design matrix $\mathbf{M}$ defined by Eq. (2), the M -dimensional symmetric covariance matrix is

$$\Sigma(\mathbf{M}) = \frac{1}{N_r - 1} \mathbf{M} \mathbf{M}^T. \quad (3)$$

By diagonalizing the matrix $\Sigma(\mathbf{M})$, we can obtain the eigenvalues of $\Sigma(\mathbf{M})$ as the PCA eigenvalues λ_m^i , $i \in \{1, 2, \dots, M\}$. Here, we consider the number of realizations $N_r = 3 \times 10^4$, which is sufficient to obtain robust results (see Sec. V of the SM for more details [48]). We plot the PCA eigenvalues λ_m^i arranged in descending order ($\lambda_m^1 > \lambda_m^2 > \dots > \lambda_m^M$) for the dataset for three typical values of disorder strength $\alpha = 3$ (thermal regime), 10 (prethermal MBL regime), and 50 (deep MBL regime) in Fig. 3(a) with descending order. Notably, for the prethermal MBL case, the first PCA eigenvalue is significantly larger than others. This clearly shows that there is one dominant principle component, and the dataset has a

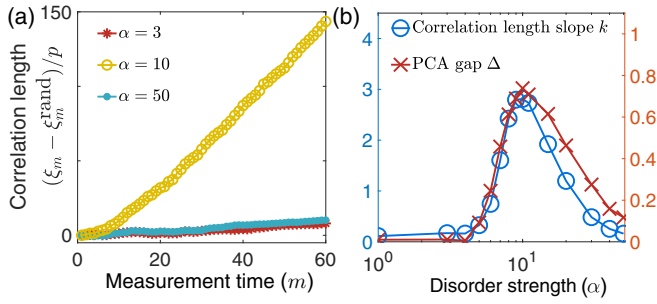


FIG. 4. (a) Growth of the temporal correlation length ξ_m between measurements (normalized by the rate of measurements $p = 10^{-4}$) as a function of the number of measurement m for $\alpha = 3, 10$, and 50 . The dashed line shows the ξ_m/p for the random dataset. (b) The slope k of the correlation length growth between measurements as a function of disorder strength α compared to the PCA gap Δ .

clear structure. However, for both the thermal and deep MBL regime, the dataset is less structural.

To characterize this structure of the dataset, we consider the mutual information between the i th and j th measurements at different times, which can quantify the correlations between the measurement outcomes. It is defined as $I(i; j) = D_{\text{KL}}(p_{ij} || p_i p_j)$ with $p_{i(j)}$ being the marginal distribution of the measurement outcomes at time i (j), p_{ij} being the joint probability distribution, and D_{KL} referring to the Kullback-Leibler divergence. A detailed discussion of its computation is provided in Sec. VI of the SM [48]. In Fig. 3(b), we present the heat maps of the values $I(i; j)/p$ for three typical values of α . In the prethermal MBL regime ($\alpha = 10$), we observe high values of the mutual information, which suggests strong correlations in the measurement outcomes at nearby times, which are absent in the thermal ($\alpha = 3$) and deep MBL regime ($\alpha = 50$).

In the thermal regime, the lack of correlations can be explained by the fact that the information from earlier measurements experiences fast scrambling. Thus, for the rare-measurement limit, the previous measurement does not influence the subsequent one. In the deep MBL regime, the localized nature prevents the information of measurements spreading, and the later measurement outcome will be totally independent of the previous one of the measurements occurs in different sites. The strong correlations of subsequent measurement outcomes with $\alpha = 10$ reveal a more essential role of rare measurements in the prethermal MBL regime.

As a next step, we quantitatively analyze the correlations by computing the “correlation length” defined as

$$\xi_m = \frac{1}{m} \sum_{i \neq j}^m I(i; j), \quad (4)$$

which is actually equal to the average value of the mutual information in the heat map [Fig. 3(b)] for a smaller square of size m . In Fig. 4(a), we plot $(\xi_m - \xi_m^{\text{rand}})/p$, where $p = 10^{-4}$ is the rate of measurements, and ξ_m^{rand} denotes the “correlation length” calculated by the random dataset, i.e., the elements of (2) randomly drawn from $\uparrow$ and $\downarrow$, as a function of the index of measurements m .

For both the thermal ($\alpha = 3$) and deep MBL regime ($\alpha = 50$), the behavior of ξ_m is close to the case of a

random dataset. In contrast, a compellingly more pronounced increase of ξ_m is observed in the prethermal MBL regime ($\alpha = 10$) indicating a divergent temporal correlation length. As the growth is approximately linear, we extract a correlation length slope k by means of a linear fit, which is plotted as a function of disorder strength α in Fig. 4(b). We find that the nonzero slope k and therefore a divergent correlation length aligns directly with the expected range of the prethermal MBL regime.

Lastly, in Fig. 4(b), we also plot the PCA gap $\Delta \equiv \lambda_m^1 - \lambda_m^2$, i.e., the difference between the largest and second largest PCA eigenvalues. It is shown that the dataset structure identified with the PCA aligns with the observed correlation length slope k . The resemblance between Δ and k indicates that the structure of the dataset (2) in the prethermal MBL regime can be reflected by the strong correlations between the measurement outcomes at different times. We also perform the same analysis for the dataset corresponding to larger system sizes, and observe similar phenomena (see Sec. V of the SM [48]).

Based on the analysis of the classical dataset (2), we demonstrate that the changes in the EE induced by measurements can be reflected by the classical measurement outcomes recorded during the dynamics. This provides a more experimental feasible way to observe the instability of prethermal nonergodicity due to rare measurements since directly measuring the entanglement is challenging for monitored dynamics.

Discussion and outlook. We show that rare measurements can induce an instability of the prethermal nonergodicity, reflected by the enhancement of EE during monitored dynamics, indicating that rare measurements can be employed to qualitatively distinguish the prethermal and deep MBL regimes. The study of measurement-only dynamics shows that Z-basis projective measurements cannot enhance EE for the dynamics of the l -bit model, as an effective description of many-body localization [36]. This also suggests that although the prethermal MBL regime persists up to a relatively large disorder strength, it eludes a simple l -bit model description.

For the generality of the results, we observe a similar instability induced by rare measurements for the monitored high-frequency driving Floquet dynamics, where prethermalization can occur [59–62]. One can see Sec. VIII of the SM for more details [48]. The results not only demonstrate that the enhancement of EE induced by rare measurements extends beyond many-body localization, but also show a similarity between the prethermalization due to high-frequency periodic driving and the MBL dynamics with intermediate disorder strengths, revealing a prethermal nature of the regime between the fully ergodic phase and deep MBL regime.

It is worthwhile to demonstrate the instability due to rare measurements on other models of many-body localization, including the disordered kicked Ising model [63] and the quantum sun model [64–66]. Our work paves the way to probe other nonergodic quantum phases of matter, such as the Hilbert space fragmentation [67–70], by subjecting rare measurements to the unitary dynamics. Furthermore, the measurement outcomes recorded during the dynamics can be employed to learn the properties of the system, which is worthwhile to extend to other nonequilibrium phases of

matter related to projective measurements, such as conventional MIPTs. Extending the data analysis to include further information from the measurement outcomes, such as the spatial location, into the classical dataset is an intriguing avenue for future research.

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Data availability. The data that support the findings of this article are openly available [71].

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