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Many-Body Cages: Disorder-Free Glassiness from Flat Bands in Fock Space and Many-Body Rabi Oscillations

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We introduce many-body caging as a novel mechanism for nonthermal behavior in quantum matter. We define many-body cages as eigenstates that, through quantum interference, become localized on a subgraph of the many-body state graph. These many-body cages can lead to the formation of flat bands in the many-body spectrum at characteristic, system-independent energies. These flat bands can realize a novel type of glassy eigenspectrum order in the absence of disorder, which we quantify by a band-overlap order parameter with an intricate, possibly fractal, distribution over the many-body state graph. We further show that these many-body cages exhibit distinctive signatures in experimentally accessible quantities, such as through a nonvanishing long-time memory of the initial condition, and many-body Rabi oscillations set by the characteristic flat-band energies. While our predictions in principle apply to any constrained quantum system, we demonstrate them here for 2D lattice gauge theories and models relevant for current experiments in Rydberg atoms. We expect that these many-body cages offer a promising route to realize nonequilibrium quantum states with novel properties.

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Introduction—Over the past two decades, quantum simulation experiments have established the nonequilibrium real-time dynamics of isolated 2D quantum many-body systems as a central frontier in quantum physics. Driven by developments across diverse platforms, ranging from Rydberg atomic systems [1], ultracold quantum gases [2], trapped ions [3], to superconducting quantum processors [4], these experiments provide access to experimental observations that were previously beyond reach. At long times, coherent quantum dynamics typically gives rise to thermodynamic behavior, as described by the eigenstate thermalization hypothesis (ETH) [5–7]. This raises a fundamental question: can quantum matter exhibit stable nonequilibrium phases with emergent properties that go beyond conventional statistical ensembles?

Many-body localization (MBL) offers a paradigm for such behavior [8–11], primarily in one dimension [12–14], where it supports nonequilibrium phases such as spin glasses and discrete time crystals [15–18], characterized

by eigenstate order and robustness against thermalization. However, identifying generic mechanisms which evade thermalization and host nonequilibrium physics for $d > 1$ remains an open and exciting frontier.

In this Letter, we introduce many-body cages as a new, *disorder-free* route to nonequilibrium many-body quantum dynamics. Many-body cages are eigenstates localized on a subgraph of the many-body state graph through destructive quantum interference. They are inspired by flat-band localization in single-particle spectra, where interference and lattice topology gives rise to compact localized states (CLS) [19–22], and to Aharonov-Bohm cages in single-particle systems [23,24]. We extend this mechanism to many-body systems by showing that the local topology of the many-body state graph can give rise to CLS and degenerate flat bands in the many-body spectrum. Many-body cages constitute a mechanism for nonthermal behavior different from other known ones such as Hilbert-space fragmentation, many-body scars, and disorder-free localization, as we describe in the discussion.

Concretely, across a broad class of constrained quantum systems, we identify many-body cages on the many-body state graph, and find that these form flat bands in the many-body spectrum at characteristic energies determined via a graph-theoretical construction. These many-body flat bands host a novel form of disorder-free glassy order: a many-body-caged spin glass, characterized by a band-overlap order

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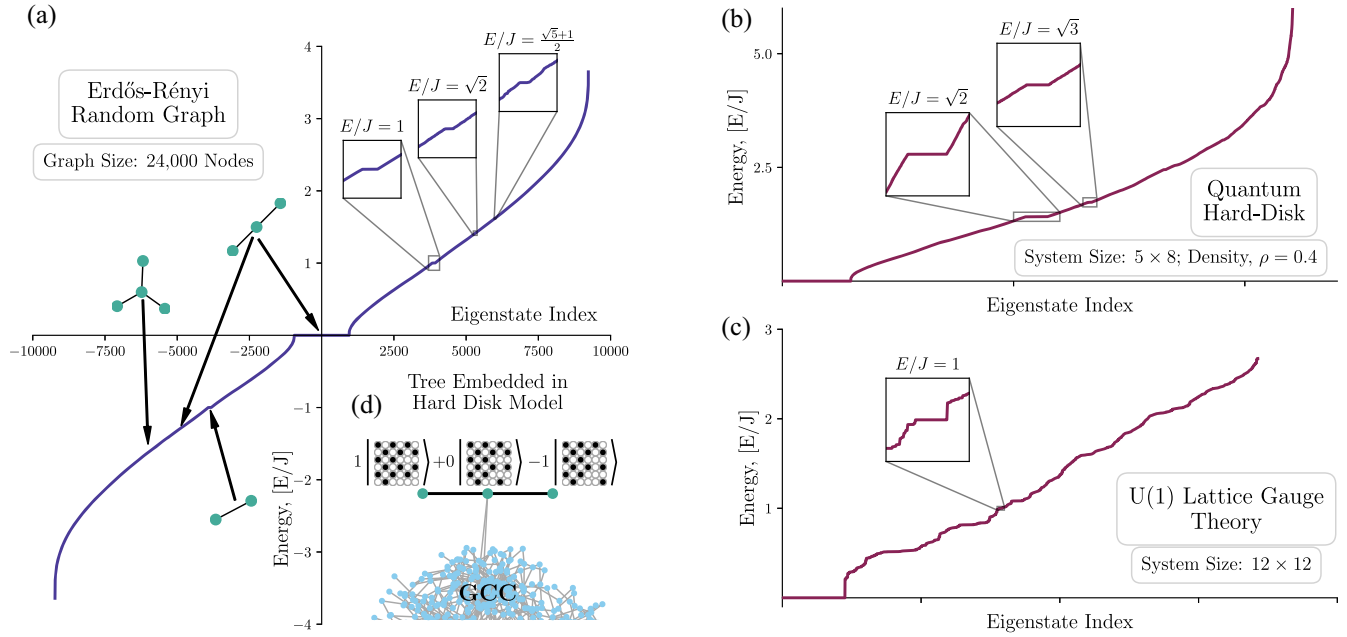


FIG. 1. Many-body flat bands. The sparse Erdős-Rényi random graph (a), state graph of the hard-disk model (b), and the spin-1/2 U(1) lattice gauge theory with a single dynamical charge (c). For (b),(c) only half the spectrum is shown, as it is symmetric around $E = 0$. Flat bands at specific closed-form values can be seen on all three spectra. Examples of tree structures associated with the flat-band energy eigenvalues are illustrated on the spectrum of the Erdős-Rényi random graph. (d) An example of a grafted tree of type T_3 in the hard-disk system.

parameter which quantifies how nonthermal the system is due to many-body cages. Remarkably, in some cases this order displays a structure reminiscent of the fractal devil's staircase and the Farey sequence—a signature of emergent arithmetic complexity.

A particularly distinctive dynamical feature of many-body cages is the appearance of coherent, Rabi oscillations between distinct many-body states, with a frequency proportional to the energy spacing between flat bands. This provides a dynamical fingerprint of the underlying many-body state graph topology.

Many-body cages—To formalize many-body cages and flat bands, we consider the state graph of a quantum many-body system. The nodes of the state graph correspond to many-body basis states (e.g., a Fock state), and every off-diagonal matrix element of the Hamiltonian is represented by an edge linking the two corresponding nodes. We restrict ourselves to Hamiltonians that contain only off-diagonal terms; diagonal contributions, which appear as on-site potentials (self-loops) on the graph, are omitted [see Supplemental Material (SM) for discussion [25]]. Consequently, the problem reduces to a hopping model on the state graph, whose spectrum coincides with that of the underlying microscopic many-body Hamiltonian.

We define many-body cages as states supported only on a subset of the many-body basis states, where they are localised as a result of destructive interference on the many-body state graph. In homogeneous systems, these cages

lead to the formation of degenerate flat bands in the many-body spectrum at characteristic, system-independent energies determined by the eigenspectrum of the underlying local motifs.

The most striking manifestations of many-body cages arise when a local constraint forbids certain many-body basis states. Familiar examples include hard-disk, hard-core dimer, and U(1) gauge models [27–29]. Such constraints prune nodes from the state graph; when the pruning is not so severe as to lead to strong fragmentation [30,31], there still remains a connected component containing a finite fraction of all allowed many-body states, called the giant connected component (GCC). When additional connected components of the state graph exist that have zero weight in the thermodynamic limit [14,30], this is known as weak fragmentation [30,32,33]. Alternatively, strong fragmentation occurs when there is no GCC—i.e., when all connected components of the state graph involve a measure-zero set of nodes in the thermodynamic limit. The transition from weak to strong fragmentation corresponds to a percolation transition [34].

In the following, we focus on the GCC in the weak-fragmentation regime, and neglect contributions from isolated clusters. For concreteness, we examine two constrained models in this regime. First, the quantum hard-disk model [27], which describes particles with an excluded volume on a lattice, where the transition from weak to strong fragmentation is dictated by the particle density.

Second, a class of U(1) lattice gauge theories with dynamical matter, recently shown to host tunable subdiffusive transport [29]. We expand on these two systems and include their Hamiltonians explicitly in the End Matter. We find that the spectra of the GCC in both models display flat bands at a subset of energies,

$$\frac{E}{J} \in \left\{ 0, \pm 1, \pm\sqrt{2}, \pm\frac{\sqrt{5} \pm 1}{2}, \pm\sqrt{3}, \dots \right\}, \quad (1)$$

where J denotes the (uniform) hopping amplitude on the state graph [Figs. 1(b) and 1(c)]. While both models have a local constraint, they differ in that the hard-disk model has kinetic constraints, while the U(1) lattice gauge theory hosts gauge constraints. This highlights the generality of the phenomenon. These energy values are generally modified by the inclusion of diagonal terms and/or nonuniform hopping amplitudes. The stability of these flat bands against such terms on the state graph is discussed in SM [25].

In order to develop a general understanding for the emergence of many-body cages in these model systems we now turn to the more abstract concept of Erdős-Rényi random graphs, which share the key structural features with our model systems. An Erdős-Rényi random graph is constructed by randomly connecting each pair of nodes with probability p , which sets the sparsity of the graph. Close to the percolation threshold, such graphs become increasingly locally treelike [35]. In the weak fragmentation regime, constrained systems show similar qualitative behavior—sparse, nonuniform connectivity.

On such sparsely connected trees of the state graph, one can identify a class of exact CLS of the adjacency matrix of the GCC through the so-called tree-grafting procedure [36,37] [see, e.g., Fig. 1(d)]: consider a finite tree T_i , attached to a graph through a single node, r . If T_i possesses an eigenvector V with eigenvalue λ whose amplitude vanishes on r , then there exists an eigenvector of the GCC W with the same eigenvalue λ that has no support outside of T_i . Consequently, when multiple grafted trees sharing the same eigenvalue λ exist, a flat band at λ emerges [see Fig. 1(a)]. Note that we consider tree grafting only for the purpose of interpretation, and do not graft them explicitly; instead, the systems we consider naturally realize analogous grafts.

The condition for the eigenvector to have zero weight on node r is sufficient, but not necessary. As shown in Refs. [36,37], eigenvalues of finite trees can give rise to CLS even when the corresponding tree eigenvectors have nonzero weight at the attachment site, provided the embedding into the GCC permits a strictly localized solution.

Concretely, small trees have eigenvalues given by Eq. (1) (see Table I for the exact tree eigenvalues), and their localized eigenvectors are decoupled from the rest of the graph [19,20].

TABLE I. All unique trees of up to four nodes and their corresponding eigenenergies.

n	Tree	Graph	Spectrum
1	T_1		$\{0\}$
2	T_2		$\{+1, -1\}$
3	T_3		$\{-\sqrt{2}, 0, \sqrt{2}\}$
4	$T_{4,1}$		$\{-\sqrt{3}, 0, 0, \sqrt{3}\}$
4	$T_{4,2}$		$\{\pm(\sqrt{5}-1)/2, \pm(\sqrt{5}+1)/2\}$

We next corroborate this picture in both microscopic models. The quantum hard-disk model [with a corresponding spectrum displayed in Fig. 1(b)] exhibits nonthermal dynamics solely due to quantum interference [27]. The U(1) lattice gauge theory with dynamical matter [spectrum in Fig. 1(c)], shows distinct flat bands. In both cases the spectra display the flat bands predicted by the tree-grafting picture. All the results shown in the figures of this Letter have been obtained by means of exact diagonalization.

A closer inspection of the energy spectra of our considered models (Fig. 1) reveals that specific flat bands may be suppressed or enhanced (i.e., an increase in the degeneracy) compared to the Erdős-Rényi model. In the Erdős-Rényi model, all tree structures are allowed, while physical constraints in microscopic models, in general, favor or exclude particular trees on the state graph. For instance, for quantum hard disks, the $E/J = \pm 1$ bands are absent, while the $E/J = \pm\sqrt{2}$ bands are prominently enhanced [see Fig. 1(b)].

Many-body cages imply nonthermal behavior: the component of a basis state supported on a many-body cage is localized by destructive interference, thereby violating ETH. Furthermore, we present numerical evidence (see SM [25]) that the degenerate flat bands span a finite fraction of Hilbert space, in both Erdős-Rényi graphs and the quantum hard-disk model.

Many-body-caged spin glass—Many-body cages thus yield quantum states inaccessible in thermal equilibrium. This is encoded in what we call a many-body-caged spin glass, characterized by the following properties: (i) it occurs in the absence of disorder, even in $D > 1$; (ii) the flat bands comprise a finite fraction of Hilbert space. We therefore identify this as eigenspectrum order—a variety of eigenstate order [11]—resulting from many-body caging. This differs from conventional eigenstate order in that it is hosted exclusively on the flat bands of the many-body spectrum.

In systems that exhibit eigenstate order, the order parameter reveals nontrivial structure in a large fraction of eigenstates, including at high energies where such structure would be absent in thermalizing systems [11]. In homogeneous systems, many-body cages form a degenerate manifold. We therefore construct an order parameter (α^{BO}) that characterizes the structure of this manifold in the

many-body basis. Specifically, we evaluate α^{BO} through the overlap between the many-body basis states and the eigenstates comprising the flat band at energy ϵ . In physical systems, the band-overlap order parameter represents how much the system and its local observables remain non-thermal due to many-body cages (see SM for more on the hard-disk model [25]). For a given basis state $|\phi_\ell\rangle$ and energy ϵ , we define

$$\alpha^{BO}(\ell, \epsilon) = \sum_{n|E_n=\epsilon} |\langle \phi_\ell | E_n \rangle|^2, \quad (2)$$

where $|E_n\rangle$ are energy basis eigenstates. This quantity is independent of the explicit choice of eigenbasis in the flat bands (for the two systems we consider, we use the Fock basis).

In systems governed by ETH, $\alpha^{BO}(\ell, \epsilon)$ decays exponentially with system size [7]. In the many-body-caged spin glass, $\alpha^{BO}(\ell, \epsilon)$ remains nonzero. Evaluating $\alpha^{BO}(\ell, \epsilon)$ across all basis states and ordering by the magnitude of α^{BO} reveals rich structure (Fig. 2).

The α^{BO} curves characterize the localization of many-body cages via their overlap with individual basis states; because these cages are compact, i.e., have no exponential tails, the resulting curves encode their underlying motifs. Specifically, those supported on strictly local motifs (e.g., tree structures) often give rise to rational values of the $\alpha^{BO}(\ell, \epsilon)$, reflecting their symmetric support on a finite number of sites. Meanwhile, plateaus in $\alpha^{BO}(\ell, \epsilon)$ indicate multiple similarly structured many-body cages, and a corresponding set of basis states exhibiting comparable longtime dynamics. Thus, $\alpha^{BO}(\ell, \epsilon)$ provides a probe of the structure within the degenerate subspaces. We note that

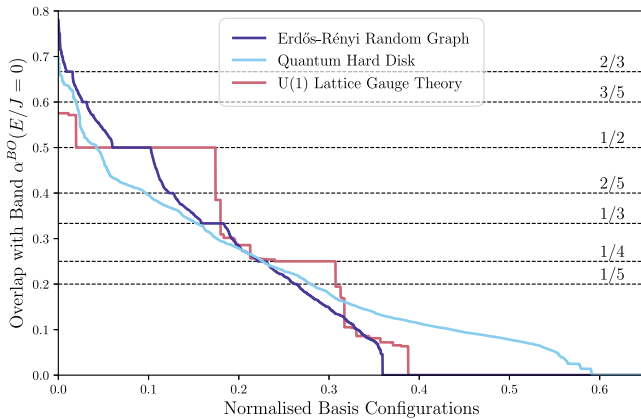


FIG. 2. The band-overlap order parameter $\alpha^{BO}(\ell, 0)$ for the $\epsilon = 0$ flat band. States $|\phi_\ell\rangle$ are ordered by decreasing $\alpha^{BO}(\ell, 0)$. The abscissa shows the corresponding rank index normalised by the Hilbert-space dimension. The curves are shown for the three models in Fig. 1: Erdős-Rényi graph, quantum hard-disk model, and a $U(1)$ lattice gauge theory. The fifth-order Farey sequence is shown in black dashed lines.

these curves can also exhibit noninteger fractal dimensions, discussed further in the End Matter.

Many-body Rabi oscillations—We next explore the spatiotemporal aspect of this eigenspectrum order. In particular, we show that the flat bands give rise to coherent many-body Rabi oscillations at characteristic frequencies (see Fig. 3) which can be detected via, e.g., Loschmidt echoes [38,39]. We define the Loschmidt echo as

$$\mathcal{L}(t) = |\langle \psi | \psi(t) \rangle|^2. \quad (3)$$

Here $|\psi\rangle$ denotes the initial basis state, and $|\psi(t)\rangle \equiv e^{-iHt}|\psi\rangle$ is the time-evolved state with H the underlying microscopic Hamiltonian. For quantum systems satisfying ETH, $\mathcal{L}(t)$ generally becomes exponentially small in system size at late times [7].

The key dynamical signature of many-body-caged spin glasses are long-lived oscillations in $\mathcal{L}(t)$, with a finite

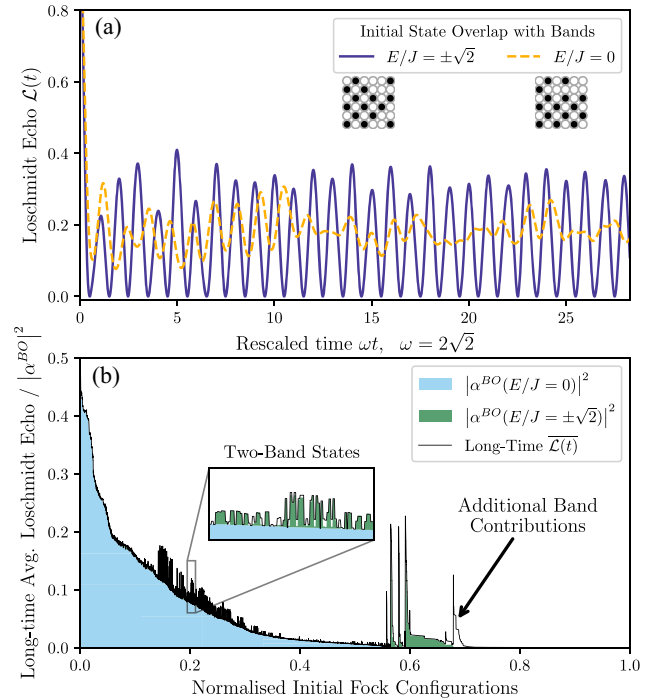


FIG. 3. Dynamical signatures of the many-body-caged spin glass. (a) Loschmidt echo $\mathcal{L}(t)$ in a 6×6 quantum hard-disk model for an initial Fock states overlapping a single ($E/J = 0$) and multiple ($E/J = \pm\sqrt{2}$) flat bands. $\mathcal{L}(t)$ exhibits regular and long-lived many-body Rabi oscillations, absent in the single-band overlap case. (b) Statistics of the long-time behavior of the Loschmidt echo in comparison to the band-overlap order parameters $\alpha^{BO}(\ell, \epsilon)$ for the hard-disk system shown in Fig. 1(b). Initial states $|\phi_\ell\rangle$ are ordered in descending order of $\alpha^{BO}(\ell, 0)$ (shaded light blue region) followed by $\alpha^{BO}(\ell, \sqrt{2})$ (shaded green region). The abscissa shows the corresponding rank index normalised by the Hilbert-space dimension. Initial Fock states with further flat-band overlaps (e.g., $E/J = \pm\sqrt{3}$) correspond to the indicated additional band contributions.

amplitude that is independent of system size. Figure 3(a) shows such many-body Rabi oscillations for an initial state overlapping two flat bands, in a 6×6 hard-disk model: the dynamics takes place in an effectively few-level system (see End Matter) with energies set by the flat-band eigenvalues. This explains the appearance of characteristic frequencies for the case of flat bands due to tree grafting, as well as the absence of these oscillations for a state overlapping a single flat band. Analogous oscillations appear for the hard-disk autocorrelation function; see SM [25].

Figure 3(b) shows the long-time-averaged Loschmidt echo $\bar{\mathcal{L}} = T^{-1} \int_0^T dt' \mathcal{L}(t')$, for all initial basis states for the hard disk model at $Jt = 10^9$, $T = 100$. We compare this to the overlaps $\alpha^{BO}(\epsilon)$ with the dominant flat bands at $\epsilon/J = 0, \pm\sqrt{2}$ for the system size 5×8 considered in Fig. 1.

Overall, the dynamics associated with many-body cages is governed by an effectively few-level system, leading to persistent, system-size-independent oscillations in the Loschmidt echo and related dynamical correlation functions (see SM [25]). The longtime dynamics retain initial-state memory, directly reflected in $\bar{\mathcal{L}}$. This residual signal provides a probe of $\alpha^{BO}(\epsilon)$ from dynamical simulations.

Discussion and outlook—A central aspect of this work is its construction of a theoretical framework of many-body cages and flat bands on the state graph, which interprets and connects nonequilibrium phenomena observed in multiple different models. In this context, several previous works showcase phenomena that naturally fit this framework. Works on the PXP model [13,40–42] note the existence of zero-energy states and investigate zero and/or low-entanglement states in the context of scars. In the single-particle case, quantum percolation [20,43] identifies a similar localization mechanism in real space, while more recent work [44] transposes this to quantum spin ice, which we hypothesize also involves many-body caging. Works on quantum combs identify CLS in real-space tree structures [22], while many-body cages occur purely on the state graph.

Many-body cages represent a distinct phenomenon from previously discovered mechanisms for nonequilibrium phenomena such as Hilbert-space fragmentation (HSF) and disorder free localization (DFL). HSF arises when kinetically or energetically *disconnected* clusters emerge in the state graph, leading to multiple disconnected Krylov subspaces that prevent the system from thermalizing [30,45]. In contrast, many-body cages are localized eigenstates that exist within the fully connected GCC and suppress thermalization through interference, rather than via disconnected Krylov subspaces; while proximity to weak HSF is a favorable setting for their emergence, it is likely not a prerequisite. DFL results from an effective disorder landscape generated by static background charges through the local gauge symmetries, while many-body cages result from local interference effects, independent of the complete system. Finally, quantum many-body scars

are described as measure-zero, *isolated*, nonthermal eigenstates embedded in an otherwise thermal spectrum. Many-body cages, on the other hand, construct discrete flat bands in the spectrum, of potentially finite fraction of Hilbert space. The introduction of disorder or other perturbations could connect the two phenomena through a crossover or transition. This constitutes an interesting future direction, which, however, requires further work to establish rigorously.

The term many-body flat bands has been used for specific many-body states constructed from single-particle flat-band states (see, e.g., [46,47]). In our Letter, many-body flat bands emerge as a property of the many-body state graph, independent of a single-particle picture.

Our Letter not only presents a natural extension of flat-band physics to the many-body state graph, but also gives attainable experimental signatures across multiple platforms—such as Rydberg atoms acting as 2D hard-disk systems or ultracold atom quantum simulation platforms hosting lattice gauge theories [48,49].

Future directions include exploring whether many-body flat bands exhibit a similarly rich zoology to that of single-particle flat band systems, such as topological (kagome [50]) or imbalanced bipartite (Lieb [51] or diluted graphene [52]) lattices. Notably, since bipartite imbalance plays a substantial role in lower bounding the size of the $E = 0$ band, it offers an additional route for introducing many-body cages into the state graph [see Fig. S2(b) in SM [25]]. The high dimensionality of the state graph may offer additional features, as well as potential richer topological features [53,54].

Finally, single-particle (near-)flat bands arising from lattice geometry have long been known to enhance correlations [55], and, e.g., promote superconductivity and other correlated states in Moiré systems [56,57]. In analogy, the statistical mechanics of a microcanonical ensemble, defined by projecting the many-body system onto a many-body flat band, is also a direction worth exploring: what—possibly exotic—properties, instabilities, or other phenomena may be discovered there?

Note added—Three related pieces of work have appeared on the arXiv around the same time as our work [58–60].

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Data availability—The data used to generate all figures in this Letter are available in Zenodo [61].

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End Matter

Quantum hard-disk model—The quantum hard-disk model considered in this Letter is based on the model discussed in Ref. [27]. The model Hamiltonian is

$$H = J \sum_{\langle i,j \rangle} P_i \left(a_i^\dagger a_j + a_j^\dagger a_i \right) P_j, \quad (\text{A1})$$

where $a_i, a_i^\dagger$ are the annihilation and creation operators of hard-core bosons respectively, and the projection operator P_i enforces the hard-disk constraint,

$$P_i = \prod_{j \ni |r_i - r_j| = 1} (1 - n_j), \quad (\text{A2})$$

where $n_j \equiv a_j^\dagger a_j$, and the density of particles in the system is defined as $\rho(N, L) = (N/L_1 L_2)$, with N the number of particles, and L_1, L_2 the extent of the system.

Long-time memory in the hard-disk model can be identified using the autocorrelation function,

$$C(t) = \frac{\frac{1}{L^2} \sum_i \langle (2\hat{n}_i(t) - 1)(2\hat{n}_i(0) - 1) \rangle - C^*}{1 - C^*}, \quad (\text{A3})$$

where $C^* = (2\rho - 1)^2$ ensures that $C(t) \rightarrow 0$ if no long-time memory exists.

As in [27] we consider a square lattice, and enforce a nearest neighbor hard-disk constraint. Finally, all hard-disk systems in this Letter have closed boundary conditions.

U(1) Quantum link model—The U(1) lattice gauge theory used in this Letter is based on the spin-1/2 quantum link model discussed by [62] and modified by [29], named “Stochastic dynamical charge hopping model.” We implement a quantum analog of it.

Choosing a random configuration of background charges on the quantum link model, we then consider the hopping problem of a single dynamical charge, while conserving the Gauss's law constraint. The model Hamiltonian is

$$H = J \sum_{\mathbf{r}, \hat{\mu}} \left(\sigma_{\mathbf{r}}^{-} S_{\mathbf{r}+\hat{\mu}}^{+} \sigma_{\mathbf{r}+\hat{\mu}}^{+} + \text{H.c.} \right), \quad (\text{B1})$$

where $S^{+(-)}$ are the spin raising(lowering) operator representing the gauge spins on the links, $\sigma^{+(-)}$ are the spin raising(lowering) operator representing the dynamical particle on the vertices, $\mathbf{r} = (x, y)$ is the vertex position, and $\hat{\mu} = (\hat{i}, \hat{j})$ is one of the two unit vectors of the lattice.

Fractality in α^{BO} Curves—In the Erdős-Rényi random graph (Fig. 2, dark blue), we observe clear plateaux, occurring at values of α^{BO} corresponding to the Farey sequence of order $n = 5$ (the sequence of all completely reduced fractions between 0 and 1 whose denominators are less than or equal to n ; see SM for system-size scaling [25]). Specifically, $\alpha^{BO}(\varepsilon)$ for a flat band can exhibit a fractal structure; in the Erdős-Rényi graph this manifests as a devil's staircase.

We characterize this fractal behavior by determining the set where the α^{BO} curve varies, computing its fractal dimension D_f via box counting [26] (see SM for details [25]). We present bounds on D_f —rather than exact values—due to uncertainties inherent to the box-counting technique.

For the Erdős-Rényi graph, we find $D_f \leq 0.917 < 1$, signaling a fractal α^{BO} dimension consistent with a devil's staircase. For the hard-disk system (Fig. 2, light blue) we find no clear signature of fractality ($D_f \approx 1$). The lattice gauge theory (Fig. 2, light red) displays clear plateaux, suggesting a fractal dimension of $D_f \leq 0.75$.

Derivation of many-body Rabi oscillations—Since Fock states of the hard-disk model overlap bands symmetrically, we consider a state with substantial overlap in the $\pm E$ bands of a generic system,

$$|\psi(t)\rangle = \sum_n \alpha_n e^{iE_n t} | -E_n \rangle + \sum_n \alpha_n e^{-iE_n t} | E_n \rangle + \text{nonflat}, \quad (\text{D1})$$

where $|E_n\rangle$ is an arbitrary basis in the degenerate band. Since inside the degenerate band a basis may be chosen where the overlap of the basis state, $|\psi\rangle$, is on a single eigenstate, this can be rewritten as

$$|\psi(t)\rangle = a e^{iEt} |\phi_{-E}\rangle + a e^{-iEt} |\phi_{+E}\rangle + \text{nonflat}, \quad (\text{D2})$$

where $a = \sum_n \alpha_n$ and $\phi_{\pm E}$ are the states of the rotated eigenbasis. Assuming the overlap outside the bands is

small, with exponentially decaying fluctuations, the overlap evaluates to

$$\langle \psi(0) | \psi(t) \rangle = 2a^2 \cos(Et) \quad (\text{D3})$$

and the Loschmidt echo

$$\mathcal{L}(t) = 4|a|^4 \cos^2(Et). \quad (\text{D4})$$

Long-time memory—The relation between the band-overlap order parameter, $\alpha^{BO}(\psi, \varepsilon)$, and the Loschmidt echo can be shown analytically. We apply the same procedure as in Eq. (D2) to an initial basis state that overlaps with three bands, $\varepsilon = 0, \pm E$. The state can be written as

$$|\psi(t)\rangle = a e^{iEt} |\phi_{-E}\rangle + b |\phi_0\rangle + a e^{-iEt} |\phi_{+E}\rangle + \text{nonflat}. \quad (\text{E1})$$

The Loschmidt echo then evaluates to

$$\mathcal{L}(t) = 4|a|^4 \cos^2(Et) + 2|a|^2 |b|^2 \cos(Et) + |b|^4, \quad (\text{E2})$$

giving the time-averaged quantity

$$\tilde{\mathcal{L}} \approx 2|a|^4 + |b|^4. \quad (\text{E3})$$

The band-overlap order parameters, defined in Eq. (2), correspond to the overlap of the initial basis state with each band independently. Squaring and summing over all bands yields,

$$\sum_{\varepsilon=0, \pm E} |\alpha^{BO}(\ell, \varepsilon)|^2 = 2|a|^4 + |b|^4, \quad (\text{E4})$$

showing that the two quantities are in agreement.

Furthermore, in the hard-disk model we look at the long-time memory through local observables (Fig. 4), using the autocorrelation function in Eq. (A3).

Band robustness—Overall, perturbations of the Hamiltonian can be distinguished into two different classes.

First, those with diagonal terms in the state graph, which can, e.g., emerge from local potentials in the underlying microscopic Hamiltonian. Introducing an uncorrelated disorder potential into the Erdős-Rényi random graph reveals a reentrant “Anderson” delocalization, which destroys the CLS on the many-body cages and thus the flat bands. This is followed by an MBL phase, to the extent that it exists, as shown in Fig. 5 for the Erdős-Rényi model.

A different kind of diagonal terms can emerge from interactions in the microscopic Hamiltonian. These diagonal terms are not entirely random, but rather realize a

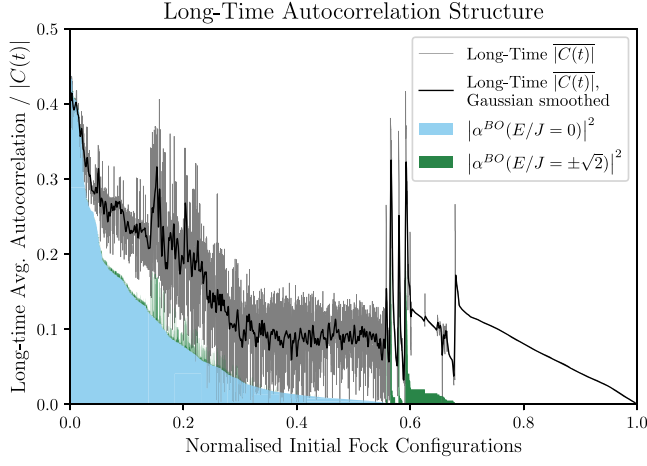


FIG. 4. Statistics of the longtime behavior of the autocorrelation function in comparison with the band-overlap order parameters α^{BO} for the hard-disk model in Fig. 1(b). Fock configuration numbers (abscissa) are divided by the Hilbert space dimension, and are ordered with descending overlap with the $E/J = 0$ flat band, followed by the $E/J = \pm\sqrt{2}$ flat band overlap. The black curve shows the Gaussian smoothed $|C(t)|$ to highlight the overall trend as α^{BO} decreases.

correlated disorder. For judicious choices, this can preserve at least some of the bands. As a simple pedagogical example, if the diagonal terms of a grafted tree on the state graph all have the same on-site values, the many-body cage is preserved, albeit the corresponding eigenvalue will exhibit a shift. For more general soft-core interactions, numerical simulations done on the quantum hard disk model [63] provide evidence that some of the many-body cages can indeed survive. Thus, systems with physically

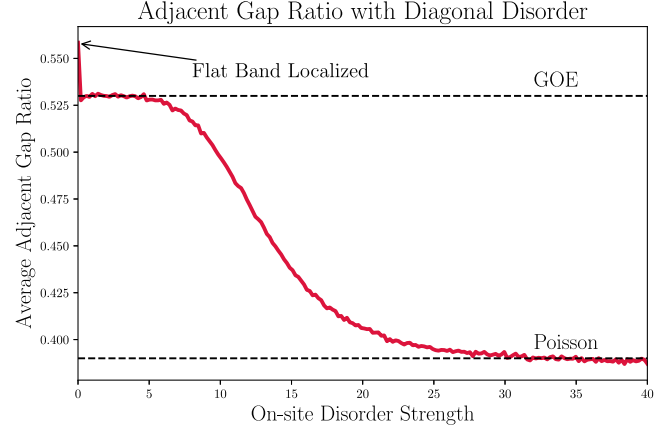


FIG. 5. Reentrant “Anderson” delocalization. Introducing uncorrelated on-site disorder in the sparse Erdős-Rényi graph breaks the many-body cages and destroys the spin glass, recovering GOE level statistics. At higher disorder strength the system undergoes Anderson localization reflected in Poissonian level statistics.

realizable soft-core interactions could still preserve and exhibit our phenomenology.

A second type of perturbation is kinetic disorder, represented as off-diagonal disorder on the state graph. This does not destroy the $E/J = 0$ CLS, but does affect other bands. Since the $E/J = 0$ is the dominant band, and the backbone of the nonthermal behavior of the system, the long-time memory of the system will remain. However, the richness that comes from the interplay of multiple bands can be potentially suppressed. We include detailed calculations of these effects in SM Sec. III [25].