UNIVERSITÄT AUGSBURG
Universitätsbibliothek

OPEN ACCESS
OPUS AUGSBURG

Agent-local concurrent epistemic ensembles

Rolf Hennicker, Alexander Knapp, Martin Wirsing

Angaben zur Veröffentlichung / Publication details:

Hennicker, Rolf, Alexander Knapp, and Martin Wirsing. 2027. "Agent-local concurrent epistemic ensembles." In *Leveraging applications of formal methods, verification and validation: rigorous engineering of collective adaptive systems: 13th International Symposium, ISoLA 2026, Kos, Greece, October 24–28, 2026, proceedings, part I*, edited by Tiziana Margaria and Bernhard Steffen, 33–53. Cham: Springer. https://doi.org/10.1007/978-3-032-40108-3_3.

Nutzungsbedingungen / Terms of use:

CC BY 4.0

Dieses Dokument wird unter folgenden Bedingungen zur Verfügung gestellt: / This document is made available under these conditions:
CC-BY 4.0: Creative Commons: Namensnennung
Weitere Informationen finden Sie unter: / For more information see:
https://creativecommons.org/licenses/by/4.0/deed.de



Agent-Local Concurrent Epistemic Ensembles

Rolf Hennicker[1], Alexander Knapp[2(✉)], and Martin Wirsing[1]

[1] Ludwig-Maximilians-Universität München, Munich, Germany
{hennicke,wirsing}@ifi.lmu.de
[2] Universität Augsburg, Augsburg, Germany
alexander.knapp@uni-a.de

Abstract. An epistemic ensemble comprises knowledge-driven agents that acquire, share, and update information about themselves and others. Agents operate on shared or local epistemic states, dynamically modifying knowledge according to protocols specified as epistemic process descriptions. We first consider a global semantic setting where knowledge is represented by indeterminate states and actions, generalising classical Kripke semantics to capture uncertainty in both states and events. Ensemble execution is defined over global configurations that combine the agents with a shared epistemic state. We then introduce a concurrent setting, in which each agent maintains its own knowledge state and actions reflect individual perspectives. A sum operator relates these local views to global states and actions, yielding a bisimulation result: executions in global and concurrent settings correspond to one another. Finally, we refine this model to an agent-local perspective, where local knowledge is expressed via agents' accessibility relations and local epistemic formulæ. This yields a fully local semantics and establishes correspondences between global and local views, including a converse construction for determinate ensembles.

1 Introduction

Ensembles are systems composed of multiple agents executing concurrently in order to achieve a common objective [17,22]. In an epistemic ensemble [14], each agent's behaviour is governed by an epistemic process, capturing how the agent acquires, represents, and updates knowledge about itself and others. Epistemic approaches to such systems have a long tradition, with applications including epistemic planning [3,11], gossip protocols [9], and knowledge puzzles [8,21]. More recently, epistemic logic has been employed to specify behavioural properties of ensembles [20], to support human-robot collaboration [4], and to ensure the safe operation of learning agents [12].

This paper continues our line of research aimed at understanding the behaviour of epistemic ensembles across different semantic environments [14–16,18]. Previous work has focused on global semantic and distributed symbolic

© The Author(s) 2027
T. Margaria and B. Steffen (Eds.): ISoLA 2026, LNCS 17141, pp. 33–53, 2027.
https://doi.org/10.1007/978-3-032-40108-3_3

settings [18]. Here, we move towards distributed semantic environments by introducing and analysing new semantic perspectives on epistemic ensembles. In particular, we include uncertainties in our semantic models which are represented by indeterminate epistemic states.

Our first contribution is the introduction of epistemic ensembles in a concurrent environment. In contrast to the global view, where knowledge is represented by a single indeterminate epistemic state, the concurrent setting assigns each agent its own epistemic state and interprets actions from the perspective of individual agents. We formalise concurrent epistemic states and actions and define a corresponding operational semantics. A key result is that, via a suitable summation construction, executions in the concurrent setting are bisimilar to those in the global setting. In this way, we establish a correspondence between concurrent and global views of knowledge.

We demonstrate that the concurrent setting is, however, too general for capturing truly locally decided but globally consistent behaviours of agents. As our main result we therefore develop agent-local concurrent epistemic ensembles which refine the concurrent model to a strictly local perspective. This is achieved by introducing agent-local epistemic formulæ and showing that, for such formulæ, existential and universal satisfaction coincide at the local level. As a consequence, properties expressed locally by an agent are preserved under the global view and vice versa. The resulting agent-local operational semantics is bisimilar to the global semantics obtained via the summation construction. Conversely, for determinate epistemic ensembles, we construct an agent-local concurrent representation from a global model.

Related Work. Indeterminate epistemic states and actions are commonly referred to as "multi-pointed" in the literature (see, e.g., [19] for actions and [10] for states and actions). The distinction between local and global epistemic states was introduced in [3] (and continued in [4]), where locality is characterised by an accessibility-closed set of designated worlds and events. It is shown that the product update of a local state by a local action yields again a local state. Our notion of locality differs in two respects: we consider only single equivalence classes of designated worlds and events, and we adopt an existential interpretation of preconditions instead of universal applicability. The work of [11] also adopts a multi-pointed setting and introduces a-local and minimal a-local states, the latter corresponding to our notion of agent-local states. Their framework employs perspective shifts to model plan-time uncertainty in cooperative epistemic planning with implicit coordination. While related in spirit, our results go beyond this work; in particular, our key Lem. 2 relating universal and existential satisfaction establishes a property not covered by their Prop. 1.

The present work complements our earlier research, in particular [18], where ensemble behaviour was studied in a global semantic, a global symbolic, and a distributed symbolic environment. Here, we introduce indeterminate semantic environments and refine the global perspective into concurrent and agent-local views. Research on epistemic logic for system modelling and programming has a long tradition, starting with the seminal works on Dynamic Epistemic Logic

(DEL, see [1,21]) and knowledge-based programs [13]. Our work is also based on DEL, but uses a process-oriented description of dynamic system behaviour, in alignment with modern languages for ensembles such as SCEL [7], CARMA [5], and DEECo [6]. However, unlike these languages, which rely on communication primitives such as message passing or predicate-based interaction, our framework is based on epistemic actions as the primary mechanism for modelling knowledge dynamics.

Structure of the Paper. We recall basic definitions of epistemic ensembles in Sect. 2. Global semantic environments, their main properties, and their operational semantics are presented in Sect. 3. Concurrent environments are studied in Sect. 4 and our main results on agent-local processes are presented in Sect. 5. We conclude in Sect. 6.

2 Epistemic Processes and Ensembles

An ensemble consists of a collection of agents which run concurrently to accomplish a certain task. An epistemic ensemble evolves by executing epistemic actions which may inform agents about knowledge of other agents. Each agent's behaviour follows a protocol which is given by an epistemic process description. In this section we recall, along the lines of [18], the syntactic notions for building epistemic ensembles and the generic operational semantics for ensemble execution. It will be instantiated later on in the context of a global and a concurrent semantics knowledge model.

An *epistemic signature* $\Sigma = (\Pi, A)$ consists of a set Π of (atomic) *propositions* and a finite, non-empty set A of *agents*. The set $\mathscr{F}_\Sigma$ of *epistemic formulæ* φ over $\Sigma = (\Pi, A)$ is defined by the following grammar:

$$\varphi ::= p \mid \text{true} \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \mathsf{K}_a\,\varphi \qquad \text{with } p \in \Pi \text{ and } a \in A$$

The epistemic formula $\mathsf{K}_a\,\varphi$ is to be read as "agent a *knows* φ". We write false for $\neg$true, $\varphi_1 \vee \varphi_2$ for $\neg(\neg\varphi_1 \wedge \neg\varphi_2)$, $\varphi_1 \to \varphi_2$ for $\neg\varphi_1 \vee \varphi_2$, and $\varphi_1 \leftrightarrow \varphi_2$ for $(\varphi_1 \to \varphi_2) \wedge (\varphi_2 \to \varphi_1)$.

An *epistemic ensemble signature* $\check{\Sigma} = (\Sigma, \mathrm{S})$ consists of an epistemic signature $\Sigma = (\Pi, A)$ and a set S of *action symbols* (their semantics will be given later in the context of concrete knowledge models). Over such a signature, we consider an epistemic process language for describing the behaviour of agents which participate in an epistemic ensemble. The set $\mathcal{P}_{\check{\Sigma}}$ of *epistemic processes* P over $\check{\Sigma}$ is defined by the grammar

$$P ::= \mathbf{0} \mid \varsigma.P \mid \varphi \supset P \mid P_1 + P_2 \mid \mu X\,.\,P \mid X$$

where $\mathbf{0}$ represents the inactive process, $\varsigma.P$ prefixes P with an action symbol $\varsigma \in \mathrm{S}$, $\varphi \supset P$ is a guarded process with condition $\varphi \in \mathscr{F}_\Sigma$, $P_1 + P_2$ denotes the non-deterministic choice between processes P_1 and P_2, $\mu X\,.\,P$ is a recursive process, and X is a process variable typically used in recursive process definitions.

The (generic) *operational semantics* of epistemic processes is given by *conditional transitions* $\hookrightarrow_{\check{\Sigma}}$ relating a process P via a guard $\widehat{\varphi} \in \mathscr{F}_{\Sigma}$ and an action symbol $\varsigma \in \mathrm{S}$ with another process P'. The transitions are defined inductively by the rules in Table 1, where successive guards (in front of an action symbol) are conjoined and true represents the empty guard.

Table 1. Rules for epistemic processes

$$\varsigma.P \xhookrightarrow{\mathrm{true}:\varsigma}_{\check{\Sigma}} P \qquad \frac{P \xhookrightarrow{\widehat{\varphi}:\varsigma}_{\check{\Sigma}} P'}{\varphi \supset P \xhookrightarrow{\widehat{\varphi}\wedge\varphi:\varsigma}_{\check{\Sigma}} P'}$$

$$\frac{P_\ell \xhookrightarrow{\widehat{\varphi}:\varsigma}_{\check{\Sigma}} P'_\ell}{P_1 + P_2 \xhookrightarrow{\widehat{\varphi}:\varsigma}_{\check{\Sigma}} P'_\ell} \text{ for } \ell \in \{1,2\} \qquad \frac{P\{X \mapsto \mu X\,.\,P\} \xhookrightarrow{\widehat{\varphi}:\varsigma}_{\check{\Sigma}} P'}{\mu X\,.\,P \xhookrightarrow{\widehat{\varphi}:\varsigma}_{\check{\Sigma}} P'}$$

An *epistemic ensemble* over $\check{\Sigma}$ is given by a family $\vec{E} = (a : P_a)_{a\in A}$ such that, for each $a \in A$, P_a is an epistemic process in $\mathcal{P}_{\check{\Sigma}}$. We also consider (sub-)families $(a : P_a)_{a\in G}$ of agent processes with $G \subseteq A$; their *composition* $(a_1 : P_{a_1})_{a_1\in G_1} \| (a_2 : P_{a_2})_{a_2\in G_2}$ for disjoint $G_1, G_2 \subseteq A$ as the family $(a : P_a)_{a\in G_1\cup G_2}$; and, for $i \in A$, the *singleton* $i : P_i$ which stands for $(a : P_a)_{a\in\{i\}}$. The (generic) *operational semantics* of epistemic ensembles is given by *conditional transitions* $\hookrightarrow_{\check{\Sigma}}$ relating an ensemble $\vec{E}$ via an *initiating* agent $i \in A$, a guard $\widehat{\varphi} \in \mathscr{F}_{\Sigma}$, and an action symbol ς with another ensemble $\vec{E}'$ according to the following rule which is based on the rules for processes:

$$i : P_i \parallel \vec{E} \xhookrightarrow{i:\widehat{\varphi}:\varsigma}_{\check{\Sigma}} i : P'_i \parallel \vec{E} \quad \text{if } P_i \xhookrightarrow{\widehat{\varphi}:\varsigma}_{\check{\Sigma}} P'_i \qquad (*)$$

The semantics for processes and ensembles are formulated generically on the basis of syntactic constructs only, not yet considering the evaluation of guards and the effects of actions. They will be instantiated to different concrete semantic environments later on.

Example 1. We consider as a running example a communication protocol for a victim-rescuer scenario inspired by [15]: There are two agents, a victim V and a rescuer R, and two atomic propositions, h and a. The proposition h indicates whether the victim needs help; the proposition a indicates whether an ambulance is on the way. Thus the epistemic signature is $\Sigma_{vr} = (\Pi_{vr}, A_{vr})$ with $\Pi_{vr} = \{\mathrm{h}, \mathrm{a}\}$ and $A_{vr} = \{\mathrm{V}, \mathrm{R}\}$.

For the communication between the agents we use two action symbols $snd^{\mathrm{V}\to\mathrm{R}}_{\mathrm{unrel}}(\mathrm{h})$ and $snd^{\mathrm{R}\to\mathrm{V}}_{\mathrm{rel}}(\mathrm{h})$ indicating an unreliable sending from V to R and a reliable sending from R to V. Unreliable here means, in contrast to reliable, that the sender does not know whether or not the message arrived (and, indeed, it may have not arrived). Thus the epistemic ensemble signature is $\check{\Sigma}_{vr} = (\Sigma_{vr}, \mathrm{S}_{vr})$

with $\mathrm{S}_{vr} = \{snd^{\mathrm{V}\to\mathrm{R}}_{\mathrm{unrel}}(\mathrm{h}), snd^{\mathrm{R}\to\mathrm{V}}_{\mathrm{rel}}(\mathrm{h})\}$. The ensemble $Sys = \mathrm{V} : Vict \parallel \mathrm{R} : Resc$ over S_{vr} then models the protocol with

$$Vict = \mu X \,.\, \big(\neg \mathsf{K}_{\mathrm{V}}\, \mathsf{K}_{\mathrm{R}}\, \mathrm{h} \wedge \mathsf{K}_{\mathrm{V}}\, \mathrm{h} \supset snd^{\mathrm{V}\to\mathrm{R}}_{\mathrm{unrel}}(\mathrm{h}).X\big) + \big(\mathsf{K}_{\mathrm{V}}\, \mathsf{K}_{\mathrm{R}}\, \mathrm{h} \supset stop.\mathbf{0}\big)$$
$$Resc = \mathsf{K}_{\mathrm{R}}\mathrm{h} \supset snd^{\mathrm{R}\to\mathrm{V}}_{\mathrm{rel}}(\mathrm{h}).\mathbf{0}$$

By $snd^{\mathrm{V}\to\mathrm{R}}_{\mathrm{unrel}}(\mathrm{h})$, V repeatedly tells R that it needs help until it is sure that R knows h; when having learned h, R acknowledges this fact in a reliable way to V with $snd^{\mathrm{R}\to\mathrm{V}}_{\mathrm{rel}}(\mathrm{h})$.

3 Epistemic Ensembles in a Global Environment

We start by considering a semantic environment where knowledge is modelled from a global point of view. Knowledge models are given by indeterminate epistemic states and action symbols are interpreted by indeterminate epistemic actions. This generalises the Kripke model semantics considered in epistemic logic [2,13,21], which we used in our previous work (cf. [18]). Moreover, it offers a more abstract scheme that allows to represent uncertainty of both the environmental states and events. Our framework is inspired by the "multi-pointed" approach of [4], where classes of indetermination are considered that are closed under equivalence of accessibility.

3.1 Epistemic States

The classical model underlying knowledge states are *epistemic structures* $K = (W, E, L)$ over $\Sigma = (\Pi, A)$ consisting of a set W of *worlds*, an A-family $E = (E_a \subseteq W \times W)_{a\in A}$ of epistemic *accessibility relations*, and a *labelling* $L : W \to \wp\Pi$ which determines for each world $w \in W$ the set of atomic propositions which hold in w. As in the traditional "logic of knowledge" [13], we assume that the accessibility relations are equivalences such that for any agent a, $(w, w') \in E_a$ expresses that agent a cannot distinguish the two worlds w and w'. The *satisfaction* of an epistemic formula $\varphi \in \mathscr{F}_\Sigma$ by an epistemic structure $K = (W, E, L)$ over Σ *at a world* $w \in W$ is inductively defined as follows:

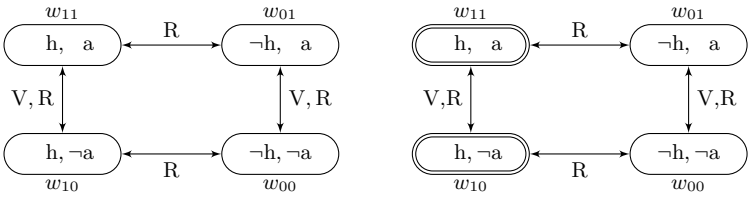
$$K, w \models_\Sigma p \iff p \in L(w)$$
$$K, w \models_\Sigma \text{true}$$
$$K, w \models_\Sigma \neg\varphi \iff \text{not } K, w \models_\Sigma \varphi$$
$$K, w \models_\Sigma \varphi_1 \wedge \varphi_2 \iff K, w \models_\Sigma \varphi_1 \text{ and } K, w \models_\Sigma \varphi_2$$
$$K, w \models_\Sigma \mathsf{K}_a\, \varphi \iff K, w_a \models_\Sigma \varphi \text{ for all } w_a \in W \text{ with} (w, w_a) \in E_a$$

In particular, a knows φ at w if φ holds in all w' which a cannot distinguish from w.

In the traditional setting, an epistemic state is modelled by a pair (K, w_0) where $w_0 \in W$ represents the actual world. We generalise this to *indeterminate*

epistemic states $\mathfrak{N} = (K, W_0)$ with a non-empty set of actually possible worlds $\emptyset \neq W_0 \subseteq W$ modelling the uncertainty about the current world. The class of (indeterminate) epistemic states is denoted by $\mathcal{N}_\Sigma$. An epistemic formula $\varphi \in \mathscr{F}_\Sigma$ *holds universally* in $(K, W_0) \in \mathcal{N}_\Sigma$, written $(K, W_0) \models_\Sigma \varphi$, if, and only if, $K, w_0 \models_\Sigma \varphi$ for all $w_0 \in W_0$; it *holds existentially*, written $(K, W_0) \models_{\exists\Sigma} \varphi$, if, and only if, $K, w_0 \models_\Sigma \varphi$ for some $w_0 \in W_0$.

Example 2. For the epistemic signature Σ_{vr} of Ex. 1 consider the epistemic structure K_{vr} over Σ_{vr} (to the left) and the state $\mathfrak{N}_{vr} = (K_{vr}, \{w_{11}, w_{10}\})$ over K_{vr} (to the right):



We only show a skeleton of the accessibility relations; the full equivalence relations are the reflexive, transitive closure of the edges in the figure. As actually possible worlds we take w_{11} and w_{10}. $\mathfrak{N}_{vr}$ models the situation where h is true and the victim V knows that, formally $\mathfrak{N}_{vr} \models_{\Sigma_{vr}} \mathsf{K}_\mathrm{V}\,\mathrm{h}$, but the rescuer does not know h, formally $\mathfrak{N}_{vr} \models_{\Sigma_{vr}} \neg\mathsf{K}_\mathrm{R}\,\mathrm{h}$, and also $\mathfrak{N}_{vr} \models_{\Sigma_{vr}} \neg\mathsf{K}_\mathrm{V}\,\mathsf{K}_\mathrm{R}\,\mathrm{h}$. Moreover, neither V nor R know the value of a.

Note that an epistemic state could alternatively be modelled by a *set* $\{(K^{(i)}, w_0^{(i)}) \mid i \in I\}$ of "traditional" epistemic states $(K^{(i)}, w_0^{(i)})$ over possibly different epistemic structures $K^{(i)} = (W^{(i)}, E^{(i)}, L^{(i)})$ for some index set I. If the worlds $W^{(i)}$ are pairwise disjoint this set can be turned into a single epistemic state $(K, \{w_0^{(i)} \mid i \in I\})$ with the common epistemic structure $K = (\bigcup_{i\in I} W^{(i)}, \bigcup_{i\in I} E^{(i)}, \bigcup_{i\in I} L^{(i)})$.

3.2 Epistemic Actions

For a concrete interpretation of ensembles the action symbols in an epistemic ensemble signature $\check{\Sigma} = (\Sigma, \mathrm{S})$ have to be assigned some meaning. For this purpose we follow the "action model" approach of [1] but, as for the epistemic models, we generalise the approach to include indetermination. An *epistemic action structure* $U = (Q, F, pre)$ over $\Sigma = (\Pi, A)$ consists of a set Q of *action events*, an A-family $F = (F_a \subseteq Q \times Q)_{a\in A}$ of epistemic *action accessibility relations*, and an *action precondition* function $pre : Q \to \mathscr{F}_\Sigma$. Again we assume that all accessibility relations are equivalence relations. For any agent a, $(q, q') \in F_a$ models that agent a cannot distinguish the two events q and q'. For any event $q \in Q$, the epistemic formula $pre(q)$ determines a condition under which q can happen. The *product update* of an epistemic structure (W, E, L) and an epistemic action structure (Q, F, pre) over $\Sigma = (\Pi, A)$ yields the epistemic structure $(W, E, L) \lhd_\Sigma (Q, F, pre) = (W', E', L')$ with

$W' = \{(w,q) \in W \times Q \mid K, w \models_\Sigma pre(q)\}$,
$E'_a = \{((w,q),(w',q')) \in W' \times W' \mid (w,w') \in E_a,\ (q,q') \in F_a\}$ for all $a \in A$,
$L'(w,q) = L(w)$ for all $(w,q) \in W'$.

In the traditional setting, an epistemic action is modelled by a pair (U, q_0) where $q_0 \in Q$ represents the actual event. We generalise this to *indeterminate* epistemic actions $\mathfrak{c} = (U, Q_0)$ with a non-empty, finite set of actually possible events $\emptyset \neq Q_0 \subseteq Q$. Q_0 modelling the uncertainty about the current event. The *precondition* $pre_\Sigma(\mathfrak{c})$ of $\mathfrak{c}$ is given by $\bigvee_{q_0 \in Q_0} pre(q_0)$. The class of (indeterminate) epistemic actions is denoted by $\mathcal{C}_\Sigma$. The *product update* of an epistemic state $\mathfrak{N} = (K, W_0) \in \mathcal{N}_\Sigma$ by an epistemic action $\mathfrak{c} = (U, Q_0) \in \mathcal{C}_\Sigma$ *exists* if $K, w_0 \models_\Sigma pre(q_0)$ for some $w_0 \in W_0$ and $q_0 \in Q_0$ and is then given by the epistemic state

$$(K, W_0) \lhd_\Sigma (U, Q_0) = (K \lhd_\Sigma U, \{(w_0, q_0) \in W_0 \times Q_0 \mid K, w_0 \models_\Sigma pre(q_0)\}).$$

The formalisation of the existence of a product update expresses that in the current situation an update of $\mathfrak{N}$ by $\mathfrak{c}$ is not impossible; in fact it depends on which of the worlds in W_0 and which of the events in Q_0 are the current ones. The existential quantification differs from the multi-pointed approach [4, Def. 3] that employs the universal view. The relation between these two views is the following:

Lemma 1. *Let $\mathfrak{N} \in \mathcal{N}_\Sigma$ and $\mathfrak{c} \in \mathcal{C}_\Sigma$.*

1. *$\mathfrak{N} \lhd_\Sigma \mathfrak{c}$ exists if, and only if, $\mathfrak{N} \models^{\exists}_\Sigma pre_\Sigma(\mathfrak{c})$.*
2. *If $\mathfrak{N} \models_\Sigma pre_\Sigma(\mathfrak{c})$, then $\mathfrak{N} \lhd_\Sigma \mathfrak{c}$ exists.*

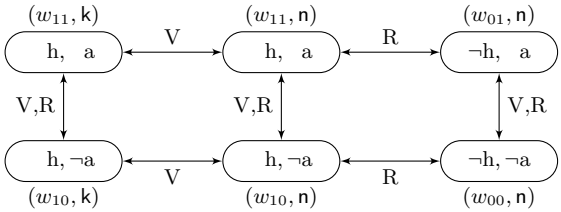
Proof. Let $\mathfrak{N} = (K, W_0)$ and $\mathfrak{c} = (U, Q_0)$ with $U = (Q, F, pre)$. For (1), $(K, W_0) \lhd_\Sigma (U, Q_0)$ exists iff there are $w_0 \in W_0$ and $q_0 \in Q_0$ with $K, w_0 \models_\Sigma pre(q_0)$; iff there is a $q_0 \in Q_0$ with $(K, W_0) \models^{\exists}_\Sigma pre(q_0)$; iff $(K, W_0) \models^{\exists}_\Sigma \bigvee_{q_0 \in Q_0} pre(q_0)$; iff $(K, W_0) \models^{\exists}_\Sigma pre_\Sigma((U, Q_0))$. Furthermore, (2) directly follows from (1) since $\mathfrak{N} \models_\Sigma pre_\Sigma(\mathfrak{c})$ implies $\mathfrak{N} \models^{\exists}_\Sigma pre_\Sigma(\mathfrak{c})$ as $W_0 \neq \emptyset$. □

Example 3. A group announcement of a formula φ (over an arbitrary epistemic signature Σ) to a group $A_* \subseteq A$ of agents is modelled by using the epistemic action structure $U^{grp}(A_*, \varphi)$ shown on the left of the following diagram. The action structure has two events k and n. Event k represents the announcement of φ which should only happen if φ holds and therefore $pre(\mathsf{k}) = \varphi$. Only agents in the group A_* can distinguish this event from event n which models that nothing happened formalised by precondition true. As a particular case we consider the victim-rescuer scenario of Ex. 1 and Ex. 2 and instantiate the group announcement to the special case of private announcement where it is announced to R that V knows that h holds. Thus we consider the epistemic action $\mathfrak{c}_{vr} = (U^{grp}_{vr}(\{\mathrm{R}\}, \mathsf{K}_\mathrm{V}\,\mathrm{h}), \{\mathsf{k}, \mathsf{n}\})$ over the epistemic action structure

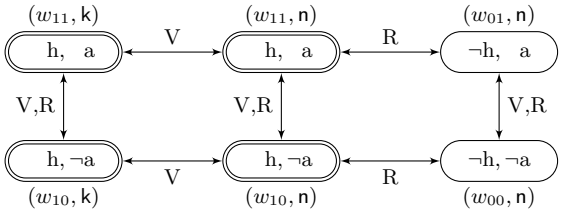
$U^{grp}_{vr}(\{\mathrm{R}\}, \mathsf{K}_\mathrm{V}\,\mathrm{h})$ that specialises $U^{grp}(A_*, \varphi)$ to Σ_{vr} with $A_* = \{\mathrm{R}\}$, $\varphi = \mathsf{K}_\mathrm{V}\,\mathrm{h}$ (to the right):



Note that this action is indeterminate since both events k and n are considered to be actually possible. As for epistemic states we only show skeletons of the action accessibility relations. The product update $K_{vr} \lhd_{\Sigma_{vr}} U^{grp}_{vr}(\{\mathrm{R}\}, \mathsf{K}_\mathrm{V}\,\mathrm{h})$ of the epistemic structure K_{vr} of Ex. 2 by the epistemic action structure $U^{grp}_{vr}(\{\mathrm{R}\}, \mathsf{K}_\mathrm{V}\,\mathrm{h})$ yields



Thus, the product update $\mathfrak{N}_{vr} \lhd_{\Sigma_{vr}} \mathfrak{c}_{vr}$ is given by $(K_{vr} \lhd_{\Sigma_{vr}} U^{grp}_{vr}(\{\mathrm{R}\}, \mathsf{K}_\mathrm{V}\,\mathrm{h}), \{(w_{11}, \mathsf{k}), (w_{10}, \mathsf{k}), (w_{11}, \mathsf{n}), (w_{10}, \mathsf{n})\})$:



The *semantics* of an epistemic action $\mathfrak{c} \in \mathcal{C}_\Sigma$ is given by the relation

$$[\![\mathfrak{c}]\!]_\Sigma = \{(\mathfrak{N}, \mathfrak{N} \lhd_\Sigma \mathfrak{c}) \in \mathcal{N}_\Sigma \times \mathcal{N}_\Sigma \mid \mathfrak{N} \lhd_\Sigma \mathfrak{c} \text{ ex.}\}\ .$$

3.3 Operational Semantics of Ensembles in a Global Environment

To execute an epistemic ensemble, as defined in Sect. 2, on the basis of a global knowledge model we consider *global epistemic ensemble configurations* over $\check{\Sigma}$ which are pairs $(\vec{E}, \mathfrak{N})$ of an ensemble $\vec{E}$ over $\check{\Sigma}$ and an epistemic state $\mathfrak{N} \in \mathcal{N}_\Sigma$ representing global knowledge. The *global ensemble semantics* over $\check{\Sigma}$ w.r.t. a *global epistemic action symbol interpretation* $\alpha : \mathrm{S} \to \mathcal{C}_\Sigma$ is the ternary relation $\rightarrow^{\alpha}_{\check{\Sigma}}$ between configurations, agents and action symbols, and (successor) configurations defined by interpreting the generic operational ensemble semantics of $(*)$, in the global environment of epistemic states:

$$(\vec{E}, \mathfrak{N}) \xrightarrow{i:\varsigma}{}^{\alpha}_{\check{\Sigma}} (\vec{E}', \mathfrak{N}') \quad \text{if } \vec{E} \xrightarrow{i:\widehat{\varphi}:\varsigma}_{\check{\Sigma}} \vec{E}', \quad \mathfrak{N} \models_\Sigma \widehat{\varphi}, \quad \text{and } (\mathfrak{N}, \mathfrak{N}') \in [\![\alpha(\varsigma)]\!]_\Sigma$$

A similar semantics using deterministic epistemic states and actions was given in [18].

Example 4. Based on Ex. 3, we interpret the action symbols of S_{vr} as follows:

$$\begin{aligned}\alpha_{vr}(snd^{\mathrm{V}\to\mathrm{R}}_{\mathrm{unrel}}(\mathrm{h})) &= (\,U^{grp}_{vr}(\{\mathrm{R}\},\mathsf{K}_{\mathrm{V}}\,\mathrm{h}),\{\mathsf{k},\mathsf{n}\}) = \mathfrak{c}_{vr}\ ,\\ \alpha_{vr}(snd^{\mathrm{R}\to\mathrm{V}}_{\mathrm{rel}}(\mathrm{h})) &= (\,U^{grp}_{vr}(\{\mathrm{V},\mathrm{R}\},\mathsf{K}_{\mathrm{R}}\,\mathrm{h}),\{\mathsf{k}\})\ ,\\ \alpha_{vr}(stop) &= (\,U^{grp}_{vr}(\{\mathrm{V},\mathrm{R}\},\mathrm{true}),\{\mathsf{k}\})\ .\end{aligned}$$

Then the first transition of *Sys* in the global environment $\mathfrak{N}_{vr}$ is given by

$$(Sys,\mathfrak{N}_{vr})\xrightarrow[\vec{\Sigma}_{vr}]{\mathrm{V}:snd^{\mathrm{V}\to\mathrm{R}}_{\mathrm{unrel}}(\mathrm{h})\,\alpha_{vr}}(Sys,\mathfrak{N}_{vr}\lhd_{\Sigma_{vr}}\mathfrak{c}_{vr})\ .$$

4 Epistemic Ensembles in a Concurrent Environment

We continue by considering a concurrent semantics environment where knowledge is modelled from a concurrency point of view. This means that each agent participating in an ensemble has its own epistemic state as its knowledge base and action symbols are interpreted by concurrent epistemic actions that comprise each agent's view on the action. We are not aware that such an approach has been considered before. After defining these concurrent notions of epistemic states and actions and the resulting concurrent operational ensemble semantics, we relate concurrent ensembles to a global view by constructing common global states and actions.

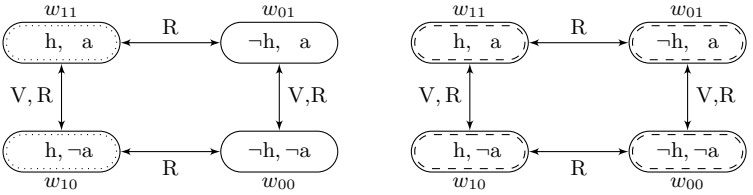
4.1 Concurrent Epistemic States and Actions

A *concurrent epistemic state* $\vec{\mathfrak{N}} = (\mathfrak{N}_a)_{a\in A}$ is given by an A-family of epistemic states $\mathfrak{N}_a = (K, W_{a,0})$ over the same epistemic structure K such that $\bigcap_{a\in A} W_{a,0} \neq \emptyset$. The sets $W_{a,0}$ describe the uncertainty of each agent a about the concrete current worlds. The class of concurrent epistemic states is denoted by $\vec{\mathcal{N}}_\Sigma$.

A *concurrent epistemic action* $\vec{\mathfrak{c}} = (\mathfrak{c}_a)_{a\in A}$ is given by an A-family of epistemic actions $\mathfrak{c}_a = (U, Q_{a,0})$ over the same epistemic action structure U such that $Q_0 = \bigcap_{a\in A} Q_{a,0} \neq \emptyset$; the *precondition* $pre_\Sigma(\vec{\mathfrak{c}})$ of $\vec{\mathfrak{c}}$ is given by $\bigvee_{q_0\in Q_0} pre(q_0)$. As for states, the sets $Q_{a,0}$ describe the uncertainty of each agent a about the current events. The class of concurrent epistemic actions is denoted by $\vec{\mathcal{C}}_\Sigma$.

The *product update* of a concurrent epistemic state $\vec{\mathfrak{N}} = (\mathfrak{N}_a)_{a\in A}$ with $\mathfrak{N}_a = (K, W_{a,0})$ by a concurrent epistemic action $\vec{\mathfrak{c}} = (\mathfrak{c}_a)_{a\in A}$ with $\mathfrak{c}_a = (U, Q_{a,0})$ *exists* if there are $w_0 \in \bigcap_{a\in A} W_{a,0}$ and $q_0 \in \bigcap_{a\in A} Q_{a,0}$ with $K, w_0 \models_\Sigma pre(q_0)$ and is then given by the concurrent epistemic state $\vec{\mathfrak{N}} \vec{\lhd}_\Sigma \vec{\mathfrak{c}} = (\mathfrak{N}_a \lhd_\Sigma \mathfrak{c}_a)_{a\in A}$. Note that all $\mathfrak{N}_a \lhd_\Sigma \mathfrak{c}_a$ indeed exist by the requirements on w_0 and q_0. The converse, however, does not need to hold, as all $\mathfrak{N}_a \lhd_\Sigma \mathfrak{c}_a$ may exist separately but this is not witnessed by a shared world and a shared event.

Example 5. We consider the concurrent epistemic state $\vec{\mathfrak{N}}_{vr} = (\mathfrak{N}_{vr,a})_{a \in A_{vr}}$ over Σ_{vr} given by the epistemic states $\mathfrak{N}_{vr,\mathrm{V}}$ (to the left) and $\mathfrak{N}_{vr,\mathrm{R}}$ (to the right). Dotted lines (on the left) indicate the worlds considered as actually possible by agent V, dashed lines (on the right) those of agent R. In fact, R cannot distinguish any worlds.

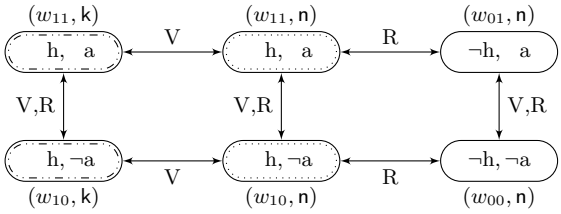


The concurrent epistemic action $\vec{\mathfrak{c}}_{vr} = (\mathfrak{c}_{vr,\mathrm{a}})_{a \in A_{vr}}$ over Σ_{vr} is given by the epistemic actions $\mathfrak{c}_{vr,\mathrm{V}}$ (to the left) and $\mathfrak{c}_{vr,\mathrm{R}}$ (to the right):



Consequently, the precondition of $\vec{\mathfrak{c}}_{vr}$ is $pre_\Sigma(\vec{\mathfrak{c}}_{vr}) = \bigvee_{q \in \{\mathsf{k}\}} pre(q) = \mathsf{K}_\mathrm{V}\,\mathrm{h}$.

The product update of $\vec{\mathfrak{N}}_{vr}$ by $\vec{\mathfrak{c}}_{vr}$ is shown in the following diagram which subsumes two updates, one for the victim and one for the rescuer. The updated victim considers four worlds as actually possible: $(w_{11}, \mathsf{k}), (w_{10}, \mathsf{k}), (w_{11}, \mathsf{n})$, and (w_{10}, n). The updated rescuer considers only the two worlds (w_{11}, k) and (w_{10}, k) as actually possible and therefore it now knows that h is true, i. e., $\mathfrak{N}_{vr,\mathrm{R}} \lhd_{\Sigma_{vr}} \mathfrak{c}_{vr,\mathrm{R}} \models_{\Sigma_{vr}} \mathsf{K}_\mathrm{R}\,\mathrm{h}$. The updated victim does not (yet) know that the rescuer knows this, i. e., $\mathfrak{N}_{vr,\mathrm{V}} \lhd_{\Sigma_{vr}} \mathfrak{c}_{vr,\mathrm{V}} \models_{\Sigma_{vr}} \neg\mathsf{K}_\mathrm{V}\,\mathsf{K}_\mathrm{R}\,\mathrm{h}$. (The shared actually possible worlds are indicated by both dashes and dots.)



The *semantics* of a concurrent epistemic action $\vec{\mathfrak{c}} \in \vec{\mathcal{C}}_\Sigma$ is given by the relation

$$[\![\vec{\mathfrak{c}}]\!]_\Sigma = \{(\vec{\mathfrak{N}}, \vec{\mathfrak{N}}\,\vec{\lhd}_\Sigma\,\vec{\mathfrak{c}}) \in \vec{\mathcal{N}}_\Sigma \times \vec{\mathcal{N}}_\Sigma \mid \vec{\mathfrak{N}}\,\vec{\lhd}_\Sigma\,\vec{\mathfrak{c}}\ \text{ex.}\}\ .$$

4.2 Operational Semantics of Ensembles in a Concurrent Environment

Concurrent epistemic ensemble configurations over $\check{\Sigma}$ add a specific knowledge model to each agent. For defining such configurations, we consider families

$(a : (P_a, \mathfrak{N}_a))_{a\in G}$, now of pairs of agent processes and epistemic states with $G \subseteq A$ such that $P_a \in \mathcal{P}_{\check{\Sigma}}$ and $\mathfrak{N}_a \in \mathcal{N}_\Sigma$ for each $a \in G$. The *composition* $(a_1 : (P_{a_1}, \mathfrak{N}_{a_1}))_{a_1\in G_1} \parallel (a_2 : (P_{a_2}, \mathfrak{N}_{a_2}))_{a_2\in G_2}$ of sub-families for disjoint $G_1, G_2 \subseteq A$ is given by the family $(a : (P_a, \mathfrak{N}_a))_{a\in G_1\cup G_2}$; and the *singleton* $i : (P_i, \mathfrak{N}_i)$ stands for $(a : (P_a, \mathfrak{N}_a))_{a\in\{i\}}$. A *concurrent epistemic ensemble configuration* over $\check{\Sigma}$ is a family $(a : (P_a, \mathfrak{N}_a))_{a\in A}$ such that $(a : P_a)_{a\in A}$ is an epistemic ensemble over $\check{\Sigma}$ and $(\mathfrak{N}_a)_{a\in A}$ a *concurrent epistemic state* in $\vec{\mathcal{N}}_\Sigma$. The *concurrent epistemic ensemble semantics* over $\check{\Sigma}$ w.r.t. a *concurrent epistemic action symbol interpretation* $\vec{\alpha} : \mathrm{S} \to \vec{\mathcal{C}}_\Sigma$ is the ternary relation $\Rightarrow^{\vec{\alpha}}_{\check{\Sigma}}$ between concurrent configurations, agents and action symbols, and (successor) configurations defined by

$$(a : (P_a, \mathfrak{N}_a))_{a\in A} \overset{i:\varsigma}{\Longrightarrow}{}^{\vec{\alpha}}_{\check{\Sigma}} (a : (P'_a, \mathfrak{N}'_a))_{a\in A} \qquad (**)$$
$$\text{if } (a : P_a)_{a\in A} \overset{i:\widehat{\varphi}:\varsigma}{\hookrightarrow}_{\check{\Sigma}} (a : P'_a)_{a\in A}, \quad \mathfrak{N}_i \models_\Sigma \widehat{\varphi}, \quad \text{and}$$
$$((\mathfrak{N}_a)_{a\in A}, (\mathfrak{N}'_a)_{a\in A}) \in [\![\vec{\alpha}(\varsigma)]\!]_\Sigma$$

Example 6. Based on Ex. 5 and abbreviating $U^{grp}_{vr}(\{\mathrm{R}\}, \mathsf{K}_\mathrm{V}\,\mathrm{h})$ by U^1_{vr}, $U^{grp}_{vr}(\{\mathrm{V},\mathrm{R}\}, \mathsf{K}_\mathrm{R}\,\mathrm{h})$ by U^2_{vr}, and $U^{grp}_{vr}(\{\mathrm{V},\mathrm{R}\}, \mathrm{true})$ by U^3_{vr}, we interpret the action symbols of S_{vr} concurrently as follows:

$$\vec{\alpha}_{vr}(snd^{\mathrm{V}\to\mathrm{R}}_{\mathrm{unrel}}(\mathrm{h})) = (\mathrm{V} : (U^1_{vr}, \{\mathsf{k}, \mathsf{n}\}), \mathrm{R} : (U^1_{vr}, \{\mathsf{k}\}))\,,$$
$$\vec{\alpha}_{vr}(snd^{\mathrm{R}\to\mathrm{V}}_{\mathrm{rel}}(\mathrm{h})) = (\mathrm{V} : (U^2_{vr}, \{\mathsf{k}\}), \mathrm{R} : (U^2_{vr}, \{\mathsf{k}\}))\,,$$
$$\vec{\alpha}_{vr}(stop) = (\mathrm{V} : (U^3_{vr}, \{\mathsf{k}\}), \mathrm{R} : (U^3_{vr}, \{\mathsf{k}\}))\,.$$

Remark 1. The concurrent setting does not suffice to let any agent decide on the executability of actions locally. Indeed, if an initiator agent i executes the action symbol ς, i.e., the action $\vec{\mathfrak{c}} = \vec{\alpha}(\varsigma)$ in the concurrent state $\vec{\mathfrak{N}}$, it should have checked that this action is guaranteed to terminate, i.e., it should have checked $pre_\Sigma(\vec{\mathfrak{c}})$.

If agent i checks this precondition existentially it may erroneously deem $\vec{\mathfrak{c}}$ to be executable when it finds *some* satisfying possible world in its knowledge base $\mathfrak{N}_i$ which, however, is not shared with the other agents, invalidating concurrent executability.

If, conversely, agent i uses universal satisfaction and sees that all of its actually possible states of $\mathfrak{N}_i$ satisfy $pre_\Sigma(\vec{\mathfrak{c}})$, then indeed $\vec{\mathfrak{c}}$ is concurrently executable. Nevertheless i cannot really *decide* on executability: If agent i finds that not all of the possible worlds of $\mathfrak{N}_i$ satisfy $pre_\Sigma(\vec{\mathfrak{c}})$, there still may be another agent a for whom the precondition is satisfied universally in $\mathfrak{N}_a$ and thus i now erroneously thinks that $\vec{\mathfrak{c}}$ is not executable.

4.3 Relating Ensembles in Global and Concurrent Environments

We want to compare the execution of ensembles when they are run in global and concurrent environments. To do this we use as a tool the sum operator which

transforms a concurrent epistemic state (a concurrent epistemic action resp.) into a global epistemic state (a global epistemic action resp.). The *sum* of a concurrent epistemic state $\vec{\mathfrak{N}} = (\mathfrak{N}_a)_{a\in A}$ with $\mathfrak{N}_a = (K, W_{a,0})$ for all $a \in A$ is given by $\bigoplus(\mathfrak{N}_a)_{a\in A} = (K, \bigcap_{a\in A} W_{a,0})$. Analogously, the *sum* of a concurrent epistemic action $\vec{\mathfrak{c}} = (\mathfrak{c}_a)_{a\in A}$ with $\mathfrak{c}_a = (U, Q_{a,0})$ for all $a \in A$ is given by $\bigoplus(\mathfrak{c}_a)_{a\in A} = (U, \bigcap_{a\in A} Q_{a,0})$. Note that both sums are well-defined as $\bigcap_{a\in A} W_{a,0} \neq \emptyset \neq \bigcap_{a\in A} Q_{a,0}$. It furthermore holds that $pre_\Sigma(\vec{\mathfrak{c}}) = pre_\Sigma(\bigoplus \vec{\mathfrak{c}})$.

Example 7. $\bigoplus \vec{\mathfrak{N}}_{vr} = (K_{vr}, \{w_{10}, w_{11}\})$ denotes the global sum of the concurrent state $\vec{\mathfrak{N}}_{vr}$ of Ex. 5, which is, in fact, $\mathfrak{N}_{vr,\mathrm{V}}$; and $\bigoplus \vec{\mathfrak{c}}_{vr} = (U^{grp}_{vr}(\{\mathrm{R}\}, \mathsf{K}_{\mathrm{V}}\,\mathrm{h}), \{\mathsf{k}\})$ denotes the global sum of the concurrent actions of Ex. 5, which is $\mathfrak{c}_{vr,\mathrm{R}}$.

The sum operator yields a characterisation of the existence of concurrent updates; the formation of sums is, in fact, compatible with updates:

Proposition 1. *Let* $\vec{\mathfrak{N}} = (\mathfrak{N}_a)_{a\in A} \in \vec{\mathcal{N}}_\Sigma$ *and* $\vec{\mathfrak{c}} = (\mathfrak{c}_a)_{a\in A} \in \vec{\mathcal{C}}_\Sigma$. *Then* $(\bigoplus \vec{\mathfrak{N}}) \lhd_\Sigma (\bigoplus \vec{\mathfrak{c}})$ *exists if, and only if,* $\vec{\mathfrak{N}} \vec{\lhd}_\Sigma \vec{\mathfrak{c}}$ *exists. Moreover, if both sides exist, then*

$$(\bigoplus \vec{\mathfrak{N}}) \lhd_\Sigma (\bigoplus \vec{\mathfrak{c}}) = \bigoplus(\vec{\mathfrak{N}} \vec{\lhd}_\Sigma \vec{\mathfrak{c}})\ .$$

Proof. Let $\mathfrak{N}_a = (K, W_{a,0})$, $\mathfrak{c}_a = (U, Q_{a,0})$ for each $a \in A$, and $W_0 = \bigcap_{a\in A} W_{a,0}$, $Q_0 = \bigcap_{a\in A} Q_{a,0}$ such that $\bigoplus(\mathfrak{N}_a)_{a\in A} = (K, W_0)$, $\bigoplus(\mathfrak{c}_a)_{a\in A} = (U, Q_0)$. Consider

$$W'_0 = \{(w_0, q_0) \in W_0 \times Q_0 \mid K, w_0 \models_\Sigma pre(q_0)\} \quad \text{and}$$
$$W'_{a,0} = \{(w_{a,0}, q_{a,0}) \in W_{a,0} \times Q_{a,0} \mid K, w_{a,0} \models_\Sigma pre(q_{a,0})\} \quad \text{for } a \in A;$$

then $W'_0 = \bigcap_{a\in A} W'_{a,0}$. Now $\bigoplus(\mathfrak{N}_a)_{a\in A} \lhd_\Sigma \bigoplus(\mathfrak{c}_a)_{a\in A} = (K \lhd_\Sigma U, W'_0)$ exists if, and only if, $W'_0 \neq \emptyset$; and $(\mathfrak{N}_a)_{a\in A} \vec{\lhd}_\Sigma (\mathfrak{c}_a)_{a\in A} = (\mathfrak{N}_a \lhd_\Sigma \mathfrak{c}_a)_{a\in A} = ((K \lhd_\Sigma U, W'_{a,0}))_{a\in A}$ exists if, and only if, $\emptyset \neq \bigcap_{a\in A} W'_{a,0} = W'_0$. Furthermore it holds that $\bigoplus((\mathfrak{N}_a)_{a\in A} \vec{\lhd}_\Sigma (\mathfrak{c}_a)_{a\in A}) = (K \lhd_\Sigma U, \bigcap_{a\in A} W'_{a,0})$. □

We thus obtain a bisimulation-style result which shows that execution of ensembles in global and concurrent environments simulate each other when the relationship of the two epistemic states is established by means of the sum operator.

Corollary 1. *Let* $\vec{\alpha} : \mathrm{S} \to \vec{\mathcal{C}}_\Sigma$ *be a concurrent epistemic action symbol interpretation, and define the global epistemic action symbol interpretation* $\bigoplus \vec{\alpha} : \mathrm{S} \to \mathcal{C}_\Sigma$ *by* $(\bigoplus \vec{\alpha})(\varsigma) = \bigoplus \vec{\alpha}(\varsigma)$ *for all* $\varsigma \in \mathrm{S}$. *Let* $(a : (P_a, \mathfrak{N}_a))_{a\in A}$ *and* $(a : (P'_a, \mathfrak{N}'_a))_{a\in A}$ *be concurrent epistemic ensembles,* $i \in A$, *and* $\varsigma \in \mathrm{S}$. *Then the following holds:*

$$(a : (P_a, \mathfrak{N}_a))_{a\in A} \overset{i:\varsigma}{\Longrightarrow}_{\vec{\Sigma}}^{\vec{\alpha}} (a : (P'_a, \mathfrak{N}'_a))_{a\in A} \iff$$
$$((a : P_a)_{a\in A}, \bigoplus(\mathfrak{N}_a)_{a\in A}) \xrightarrow{i:\varsigma}_{\vec{\Sigma}}^{\bigoplus \vec{\alpha}} ((a : P'_a)_{a\in A}, \bigoplus(\mathfrak{N}'_a)_{a\in A})$$

5 Agent-Local Concurrent Epistemic Ensembles

As we have noticed for the semantics of concurrent epistemic actions in Sect. 4.1, it is not possible to decide for an agent *locally* whether or not an action can currently be executed successfully. As discussed in Rem 1, when relying on existential precondition satisfaction the agent would have to resort to a *global* view via the shared states of the sum operator of Sect. 4.3; even when granting more expensive universal checks, some locality principle has to be found defining which agent actually determines which outcome. For such a principle, we build on an idea of [4] where an agent's indetermination is expressed by equivalences of worlds and events using the agent's own, local accessibility relations. However, for specialising the concurrent environment approach to an overall agent-local viewpoint, we strengthen this notion of locality by introducing also agent-local epistemic formulæ and stressing existential satisfaction which requires that for a local state for agent a existential and universal satisfaction of a-epistemic formulæ coincides. We thus obtain a truly agent-local concurrent operational ensemble semantics. After specialising also the relation between global and now agent-local concurrent environments constructing a global view from the concurrent view, we furthermore derive for the classical determinate case a converse relation that constructs an agent-local concurrent view from a global view.

5.1 Agent-Local Epistemic Formulæ, States, and Actions

For each agent $a \in A$, the set $\mathscr{F}_\Sigma\restriction a$ of *a-epistemic formulæ* φ_a having an a-modality as their top operator, is given by the following grammar:

$$\varphi_a ::= \text{true} \mid \neg\varphi_a \mid \varphi_{a,1} \wedge \varphi_{a,2} \mid \mathsf{K}_a\,\varphi \quad \text{where } \varphi \in \mathscr{F}_\Sigma.$$

For any $\varphi \in \mathscr{F}_\Sigma$ we define the set of possible *agents* of φ by $\mathrm{ags}_\Sigma(\varphi) = \{a \in A \mid \varphi \in \mathscr{F}_\Sigma\restriction a\}$. In particular, $\mathrm{ags}_\Sigma(\varphi)$ is either a singleton, or A (if no K_a occurs in φ), or $\emptyset$.

Example 8. For our running example, $\mathrm{ags}_{\Sigma_{vr}}(\mathsf{K}_\mathrm{V}\,\mathrm{h}) = \{\mathrm{V}\} = \mathrm{ags}_{\Sigma_{vr}}(\neg\mathsf{K}_\mathrm{V}\,\mathsf{K}_\mathrm{R}\,\mathrm{h})$ and $\mathrm{ags}_{\Sigma_{vr}}(\mathsf{K}_\mathrm{R}\,\mathrm{h}) = \{\mathrm{R}\}$, but $\mathrm{ags}_{\Sigma_{vr}}(\mathrm{h}) = \emptyset$.

An epistemic state (K, W_0) for $K = (W, E, L)$ is *a-local* for an $a \in A$ if $W_0 = [w_0]_{E_a}$, i.e., the actually possible worlds W_0 are given by the equivalence class of some $w_0 \in W$ w.r.t. E_a. As our key lemma we obtain that for a-local epistemic states existential and universal satisfaction of a-epistemic formulæ coincide:

Lemma 2. *Let $(K, W_0) \in \mathcal{N}_\Sigma$ be a-local. Then for all $\varphi_a \in \mathscr{F}_\Sigma\restriction a$ it holds that $(K, W_0) \models^{\exists}_\Sigma \varphi_a$ if, and only if, $(K, W_0) \models_\Sigma \varphi_a$.*

Proof. The direction from right to left, i.e., that $K, w_0' \models_\Sigma \varphi_a$ for all $w_0' \in W_0$ implies $K, w_0 \models_\Sigma \varphi_a$ for some $w_0 \in W_0$, is clear, since $W_0 \neq \emptyset$. For the converse direction we proceed by structural induction over the structure of φ_a: For true the claim is obvious.

For $\neg\varphi_a$, let $K, w_0 \models_\Sigma \neg\varphi_a$ hold for some $w_0 \in W_0$, i.e., $K, w_0 \not\models_\Sigma \varphi_a$. Assume for a contradiction that $K, w_0' \not\models_\Sigma \neg\varphi_a$, i.e., $K, w_0' \models_\Sigma \varphi_a$, for some $w_0' \in W_0$. Then $K, w_0'' \models_\Sigma \varphi_a$ for all $w_0'' \in W_0$ by the induction hypothesis, but contradicting $K, w_0 \not\models_\Sigma \varphi_a$ for $w_0 \in W_0$. Thus $K, w_0' \not\models_\Sigma \varphi_a$ and hence $K, w_0' \models_\Sigma \neg\varphi_a$ for all $w_0' \in W_0$.

For $\varphi_{a,1} \wedge \varphi_{a,2}$, let $K, w_0 \models_\Sigma \varphi_{a,1} \wedge \varphi_{a,2}$ hold for some $w_0 \in W_0$, i.e., $K, w_0 \models_\Sigma \varphi_{a,1}$ and $K, w_0 \models_\Sigma \varphi_{a,2}$. Then $K, w_0' \models_\Sigma \varphi_{a,1}$ and $K, w_0' \models_\Sigma \varphi_{a,2}$ for all $w_0' \in W_0$ by the induction hypothesis and thus $K, w_0' \models_\Sigma \varphi_{a,1} \wedge \varphi_{a,2}$ for all $w_0' \in W_0$.

For $\mathsf{K}_a\,\varphi$ with $\varphi \in \mathscr{F}_\Sigma$, let $K, w_0 \models_\Sigma \mathsf{K}_a\,\varphi$ for some $w_0 \in W_0$. Let $K = (W, E, L)$. Then $K, w \models_\Sigma \varphi$ for all $w \in W$ with $(w_0, w) \in E_a$. Now let $w' \in W_0$ be arbitrary. We have to show that $K, w' \models_\Sigma \mathsf{K}_a\,\varphi$. Let $w'' \in W$ arbitrary with $(w', w'') \in E_a$. Then both $(w_0, w') \in E_a$ and $(w_0, w'') \in E_a$, as E_a is an equivalence relation, and hence $K, w'' \models_\Sigma \varphi$. Thus $K, w' \models_\Sigma \mathsf{K}_a\,\varphi$. □

In particular, the worlds of the equivalence class of an a-local epistemic state are precisely those equivalent worlds that cannot be distinguished by a-epistemic formulæ:

Corollary 2. *Let $K = (W, E, L)$ be an epistemic structure, $w_0 \in W$, and $a \in A$. Then $[w_0]_{E_a} = \{w \in W \mid (w_0, w) \in E_a \;\&\; \text{f. a. } \varphi_a \in \mathscr{F}_\Sigma|a\text{:}\; K, w_0 \models_\Sigma \varphi_a \iff K, w \models_\Sigma \varphi_a\}$.*

An epistemic action (U, Q_0) for $U = (Q, F, pre)$ is *a-local* for an $a \in A$ if $Q_0 = [q_0]_{F_a}$, i.e., the actually possible events Q_0 are given by the equivalence class of some $q_0 \in Q$ w.r.t. F_a. Updating an a-local epistemic state by an a-local epistemic action yields an a-local epistemic state:

Lemma 3. *Let $\mathfrak{N} \in \mathcal{N}_\Sigma$ be a-local and $\mathfrak{c} \in \mathcal{C}_\Sigma$ be a-local such that $\mathfrak{N} \lhd_\Sigma \mathfrak{c}$ exists. Then $\mathfrak{N} \lhd_\Sigma \mathfrak{c}$ is a-local.*

Proof. Let $\mathfrak{N} = (K, W_0)$, $K = (W, E, L)$, $\mathfrak{c} = (U, Q_0)$, and $U = (Q, F, pre)$ such that $\emptyset \neq W_0' = \{(w_0, q_0) \in W_0 \times Q_0 \mid K, w_0 \models_\Sigma pre(q_0)\}$ and $(K, W_0) \lhd_\Sigma (U, Q_0) = (K \lhd_\Sigma U, W_0')$ with $K \lhd_\Sigma U = (W', E', L')$. Let $(w_0, q_0) \in W_0'$ and $(w, q) \in W'$ with $((w_0, q_0), (w, q)) \in E_a'$, i.e., $K, w_0 \models_\Sigma pre(q_0)$, $K, w \models_\Sigma pre(q)$, $(w_0, w) \in E_a$, and $(q_0, q) \in F_a$. Then $w \in W_0$ and $q \in Q_0$ such that $(w, q) \in W_0'$, since $\mathfrak{N}$ and $\mathfrak{c}$ are a-local. If conversely $(w_0, q_0), (w_0', q_0') \in W_0'$, then $(w_0, w_0') \in E_a$ and $(q_0, q_0') \in F_a$. Thus $((w_0, q_0), (w_0', q_0')) \in E_a'$. Hence, $W_0' = [(w_0, q_0)]_{E_a'}$ for all $(w_0, q_0) \in W_0' \neq \emptyset$. □

Example 9. The epistemic states $\mathfrak{N}_{vr,\mathrm{V}}$ and $\mathfrak{N}_{vr,\mathrm{R}}$ of Ex. 5 are V- and R-local, respectively. The V-equivalence class of $\mathfrak{N}_{vr,\mathrm{V}}$ is $\{w_{10}, w_{11}\}$, the R-equivalence class of $\mathfrak{N}_{vr,\mathrm{R}}$ is $\{w_{00}, w_{01}, w_{10}, w_{11}\}$. Analogously, the epistemic actions $\mathfrak{c}_{vr,\mathrm{V}}$ and $\mathfrak{c}_{vr,\mathrm{R}}$ are V- and R-local, respectively. In particular, $\mathfrak{N}_{vr,\mathrm{V}} \lhd_{\Sigma_{vr}} \mathfrak{c}_{vr,\mathrm{V}}$ and $\mathfrak{N}_{vr,\mathrm{R}} \lhd_{\Sigma_{vr}} \mathfrak{c}_{vr,\mathrm{R}}$ are again V- and R-local, respectively.

5.2 Agent-Local Concurrent Epistemic States and Actions

A concurrent epistemic state $\vec{\mathfrak{L}} = (\mathfrak{L}_a)_{a\in A}$ is *agent-local* if $\mathfrak{L}_a$ is a-local for each $a \in A$. The class of agent-local concurrent epistemic states is denoted by $\vec{\mathcal{L}}_\Sigma$.

An epistemic formula $\varphi \in \bigcup_{a\in A} \mathscr{F}_\Sigma\restriction a$ *universally holds* in $\vec{\mathfrak{L}}$, written $\vec{\mathfrak{L}} \models\!\!\!=_\Sigma \varphi$, if $\mathfrak{L}_a \models_\Sigma \varphi$ for all $a \in \mathrm{ags}_\Sigma(\varphi)$. Note that, indeed, if $|\mathrm{ags}_\Sigma(\varphi)| > 1$, then either $\models_\Sigma \varphi \leftrightarrow \mathrm{false}$ or $\models_\Sigma \varphi \leftrightarrow \mathrm{true}$ by structural induction over φ (see [18, Lem. 9.1]). An epistemic formula $\varphi \in \bigcup_{a\in A} \mathscr{F}_\Sigma\restriction a$ *existentially holds* in $\vec{\mathfrak{L}}$, written $\vec{\mathfrak{L}} \models\!\!\!=_\exists \varphi$, if $\mathfrak{L}_a \models_\exists \varphi$ for all $a \in \mathrm{ags}_\Sigma(\varphi)$.

A concurrent epistemic action $\vec{\mathfrak{e}} = (\mathfrak{e}_a)_{a\in A}$ is *agent-local* if each $\mathfrak{e}_a$ is a-local for all $a \in A$, and $pre_\Sigma(\vec{\mathfrak{e}}) \in \bigcup_{a\in A} \mathscr{F}_\Sigma\restriction a$. The possible *agents* $\mathrm{ags}_\Sigma(\vec{\mathfrak{e}})$ for $\vec{\mathfrak{e}}$ are the agents $\mathrm{ags}_\Sigma(pre_\Sigma(\vec{\mathfrak{e}}))$. The class of agent-local concurrent epistemic actions is denoted by $\vec{\mathcal{E}}_\Sigma$.

Example 10. Consider again the epistemic states $\mathfrak{N}_{vr,\mathrm{V}}$ and $\mathfrak{N}_{vr,\mathrm{R}}$ of Ex. 5 as well as the epistemic actions $\mathfrak{e}_{vr,\mathrm{V}}$ and $\mathfrak{e}_{vr,\mathrm{R}}$. Let $\vec{\mathfrak{L}}_{vr}$ be the A_{vr}-family consisting of $\mathfrak{N}_{vr,\mathrm{V}}$ and $\mathfrak{N}_{vr,\mathrm{R}}$ and let $\vec{\mathfrak{e}}_{vr}$ consist of $\mathfrak{e}_{vr,\mathrm{V}}$ and $\mathfrak{e}_{vr,\mathrm{R}}$. Both families are agent-local.

This notion of agent-locality now indeed allows local, efficient decisions whether an action is executable: The global check of Lem. 1(1) can be localised for concurrent environments, and, in contrast to the global case, existential and universal precondition checking now coincide.

Proposition 2. *Let $\vec{\mathfrak{L}} \in \vec{\mathcal{L}}_\Sigma$ and $\vec{\mathfrak{e}} \in \vec{\mathcal{E}}_\Sigma$.*

1. *$\vec{\mathfrak{L}} \vec{\lhd}_\Sigma \vec{\mathfrak{e}}$ exists if, and only if, $\vec{\mathfrak{L}} \models\!\!\!=_\exists pre_\Sigma(\vec{\mathfrak{e}})$.*
2. *If $\vec{\mathfrak{L}} \models\!\!\!=_\exists pre_\Sigma(\vec{\mathfrak{e}})$, then $\vec{\mathfrak{L}} \models\!\!\!=_\Sigma pre_\Sigma(\vec{\mathfrak{e}})$.*

Proof. For (1), $\vec{\mathfrak{L}} \vec{\lhd}_\Sigma \vec{\mathfrak{e}}$ exists if, and only if, $(\bigoplus \vec{\mathfrak{L}}) \lhd_\Sigma (\bigoplus \vec{\mathfrak{e}})$ exists by Prop. 1; if, and only if, $\bigoplus \vec{\mathfrak{L}} \models\!\!\!=_\exists pre_\Sigma(\bigoplus \vec{\mathfrak{e}})$ holds by Lem. 1(1); if, and only if, $\bigoplus \vec{\mathfrak{L}} \models\!\!\!=_\exists pre_\Sigma(\vec{\mathfrak{e}})$ holds as $pre_\Sigma(\vec{\mathfrak{e}}) = pre_\Sigma(\bigoplus \vec{\mathfrak{e}})$.
(2) holds by Lem. 2, since $pre_\Sigma(\vec{\mathfrak{e}}) \in \bigcup_{a\in A} \mathscr{F}_\Sigma\restriction a$. □

Furthermore, agent-locality is preserved by agent-local updates:

Proposition 3. *Let $\vec{\mathfrak{L}} \in \vec{\mathcal{L}}_\Sigma$ and $\vec{\mathfrak{e}} \in \vec{\mathcal{E}}_\Sigma$ such that $\vec{\mathfrak{L}} \vec{\lhd}_\Sigma \vec{\mathfrak{e}}$ exists. Then $\vec{\mathfrak{L}} \vec{\lhd}_\Sigma \vec{\mathfrak{e}} \in \vec{\mathcal{L}}_\Sigma$.*

Proof. Let $\vec{\mathfrak{L}} \vec{\lhd}_\Sigma \vec{\mathfrak{e}} = (\mathfrak{L}_a \lhd_\Sigma \mathfrak{e}_a)_{a\in A}$. Then, for each $a \in A$, $\mathfrak{L}_a \lhd_\Sigma \mathfrak{e}_a$ is a-local by Lem. 3, since $\mathfrak{L}_a$ and $\mathfrak{e}_a$ are a-local. □

Thus, using Prop. 2(1) and Prop. 3, the semantics $[\![\vec{\mathfrak{e}}]\!]_\Sigma$, given in Sect.4.1 for concurrent actions, can be reformulated for agent-local concurrent actions $\vec{\mathfrak{e}}$ as

$$[\![\vec{\mathfrak{e}}]\!]_\Sigma = \{(\vec{\mathfrak{L}}, \vec{\mathfrak{L}} \vec{\lhd}_\Sigma \vec{\mathfrak{e}}) \in \vec{\mathcal{L}}_\Sigma \times \vec{\mathcal{L}}_\Sigma \mid \vec{\mathfrak{L}} \models\!\!\!=_\exists pre_\Sigma(\vec{\mathfrak{e}})\} \ .$$

5.3 Operational Semantics of Ensembles in an Agent-Local Concurrent Environment

Let $\mathrm{S} = \bigcup_{a\in A} \mathrm{S}_a$ for an A-family $\vec{\mathrm{S}} = (\mathrm{S}_a)_{a\in A}$ of pairwise disjoint sets S_a of *agent action symbols*. For each $\varsigma \in \bigcup \vec{\mathrm{S}} = \bigcup_{a\in A} \mathrm{S}_a$ we set $\mathrm{ags}_{\check{\Sigma}}(\varsigma) = \{a\}$ if $\varsigma \in \mathrm{S}_a$. For any process $P \in \mathcal{P}_{\check{\Sigma}}$ the set $\mathrm{ags}_{\check{\Sigma}}(P)$ of agents that can meaningfully execute P is inductively defined by

$$\begin{aligned} \mathrm{ags}_{\check{\Sigma}}(\mathbf{0}) &= A = \mathrm{ags}_{\check{\Sigma}}(X) & \mathrm{ags}_{\check{\Sigma}}(\mu X\,.\,P) &= \mathrm{ags}_{\check{\Sigma}}(P) \\ \mathrm{ags}_{\check{\Sigma}}(\varsigma.P) &= \mathrm{ags}_{\check{\Sigma}}(\varsigma) \cap \mathrm{ags}_{\check{\Sigma}}(P) & \mathrm{ags}_{\check{\Sigma}}(\varphi \supset P) &= \mathrm{ags}_{\Sigma}(\varphi) \cap \mathrm{ags}_{\check{\Sigma}}(P) \\ \mathrm{ags}_{\check{\Sigma}}(P_1 + P_2) &= \mathrm{ags}_{\check{\Sigma}}(P_1) \cap \mathrm{ags}_{\check{\Sigma}}(P_2) \end{aligned}$$

For executing a process $P \in \mathcal{P}_{\check{\Sigma}}$ a kind of *subject reduction* holds [18, Lem. 2.1]:

Lemma 4. *If* $P \xrightarrow{\widehat{\varphi}:\varsigma}_{\check{\Sigma}} P'$, *then* $\mathrm{ags}_{\check{\Sigma}}(P) \subseteq \mathrm{ags}_{\Sigma}(\widehat{\varphi}) \cap \mathrm{ags}_{\check{\Sigma}}(\varsigma) \cap \mathrm{ags}_{\check{\Sigma}}(P')$.

An *agent-local epistemic ensemble* over $\check{\Sigma}$ with $\mathrm{S} = \bigcup \vec{\mathrm{S}}$ is an epistemic ensemble $(a : P_a)_{a\in A}$ over $\check{\Sigma}$ such that each $a \in \mathrm{ags}_{\check{\Sigma}}(P_a)$ for all $a \in A$. An *agent-local concurrent epistemic action symbol interpretation* for the action symbols in $\bigcup \vec{\mathrm{S}}$ is a function $\vec{\alpha} : \bigcup \vec{\mathrm{S}} \to \vec{\mathcal{E}}_{\Sigma}$ such that $\mathrm{ags}_{\Sigma}(\vec{\alpha}(\varsigma)) \cap \mathrm{ags}_{\check{\Sigma}}(\varsigma) \neq \emptyset$. An *agent-local concurrent epistemic ensemble configuration* over $\check{\Sigma}$ is a family $(a : (P_a, \mathfrak{L}_a))_{a\in A}$ such that $(a : P_a)_{a\in A}$ is an agent-local epistemic ensemble over $\check{\Sigma}$ and $(\mathfrak{L}_a)_{a\in A}$ an agent-local concurrent epistemic state in $\vec{\mathcal{L}}_{\Sigma}$. Specialising the concurrent epistemic ensemble semantics rule (**) to agent-local concurrent epistemic ensemble configurations yields

$$\begin{aligned} &(a : (P_a, \mathfrak{L}_a))_{a\in A} \overset{i:\varsigma}{\Longrightarrow}_{\check{\Sigma}}^{\vec{\alpha}} (a : (P'_a, \mathfrak{L}'_a))_{a\in A} \\ &\quad \text{if } (a : P_a)_{a\in A} \xrightarrow{i:\widehat{\varphi}:\varsigma}_{\check{\Sigma}} (a : P'_a)_{a\in A}, \quad \mathfrak{L}_i \models_{\Sigma} \widehat{\varphi}, \quad \textit{and} \\ &\qquad ((\mathfrak{L}_a)_{a\in A}, (\mathfrak{L}_a{}') \in [\![\vec{\alpha}(\varsigma)]\!]_{\Sigma})_{a\in A} \end{aligned}$$

Remark 2. Let us contrast checking the executability of actions in the concurrent case, as discussed in Rem. 1, and in the agent-local concurrent case. The initiator agent i executing the action symbol ς, i.e., the action $\vec{\alpha}(\varsigma)$, still should have checked that this action is guaranteed to terminate, i.e., it should have checked $pre_{\Sigma}(\vec{\alpha}(\varsigma))$. Differing from the general concurrent case, it now suffices that the precondition is satisfied at *some* actually possible world of i by Prop. 2(1). Moreover, since the precondition then holds in *all* worlds of the initiator by Lem. 2, the agent can be sure that all other agents at least find some world in their knowledge bases where the action succeeds.

5.4 Relating Ensembles in Global and Agent-Local Concurrent Environments

Similarly to Sect. 4.3 we can relate the execution of ensembles when they are run in global and agent-local concurrent environments; see Cor. 5. We use again the

sum operator which now transforms an agent-local concurrent epistemic state (action resp.) into a global epistemic state (action resp.). Additionally we obtain some logical connections between global and agent-local concurrent epistemic states.

For the following proposition the direction from left to right is the important one. It states that any a-local property which holds in the sum holds also locally for agent a.

Proposition 4. *Let $\vec{\mathfrak{L}} = (\mathfrak{L}_a)_{a\in A} \in \vec{\mathcal{L}}_\Sigma$. Then for all $a \in A$ and all $\varphi_a \in \mathscr{F}_\Sigma\restriction a$,*

$$\bigoplus \vec{\mathfrak{L}} \models_\Sigma \varphi_a \iff \mathfrak{L}_a \models_\Sigma \varphi_a \ .$$

Proof. Let $\mathfrak{L}_a = (K, W_{a,0})$ for each $a \in A$ and $\bigoplus \vec{\mathfrak{L}} = (K, W_0)$ with $W_0 = \bigcap_{a\in A} W_{a,0} \neq \emptyset$. Let $a \in A$ and $\varphi_a \in \mathscr{F}_\Sigma\restriction a$. The direction from right to left holds, since $W_0 \subseteq W_{a,0}$. The direction from left to right holds by Lem. 2 as $W_0 \neq \emptyset$. □

This yields *coherence* which expresses that whenever an agent a knows a local property $\varphi_{a'}$ for some agent a' then $\varphi_{a'}$ is indeed valid in the local state of a':

Corollary 3. *Let $\vec{\mathfrak{L}} = (\mathfrak{L}_a)_{a\in A} \in \vec{\mathcal{L}}_\Sigma$, let $a, a' \in A$, and let $\varphi_{a'} \in \mathscr{F}_\Sigma\restriction a'$. Then $\mathfrak{L}_a \models_\Sigma \mathsf{K}_a \varphi_{a'}$ implies $\mathfrak{L}_{a'} \models_\Sigma \varphi_{a'}$.*

Proof. Let $\mathfrak{L}_a \models_\Sigma \mathsf{K}_a \varphi_{a'}$ hold. Then $\bigoplus \vec{\mathfrak{L}} \models_\Sigma \mathsf{K}_a \varphi_{a'}$ by Prop.4. Thus $\bigoplus \vec{\mathfrak{L}} \models_\Sigma \varphi_{a'}$. Hence $\mathfrak{L}_{a'} \models_\Sigma \varphi_{a'}$ again by Prop. 4. □

By Prop. 4, agent-local satisfaction coincides with global satisfaction in the sum of states for agent-local formulæ.

Corollary 4. *Let $\vec{\mathfrak{L}} \in \vec{\mathcal{L}}_\Sigma$ and let $\varphi \in \bigcup_{a\in A} \mathscr{F}_\Sigma\restriction a$. Then $\bigoplus \vec{\mathfrak{L}} \models_\Sigma \varphi \iff \vec{\mathfrak{L}} \models\!\!\models_\Sigma \varphi$.*

Furthermore, the bisimulation result of Cor. 1 refines to the agent-local setting:

Corollary 5. *Let $\vec{\alpha} : \bigcup \vec{\mathrm{S}} \to \vec{\mathcal{E}}_\Sigma$ be an agent-local concurrent epistemic action symbol interpretation, and define the global epistemic action symbol interpretation $\bigoplus \vec{\alpha} : \bigcup \vec{\mathrm{S}} \to \mathcal{C}_\Sigma$ by $(\bigoplus \vec{\alpha})(\varsigma) = \bigoplus \vec{\alpha}(\varsigma)$ for all $\varsigma \in \bigcup \vec{\mathrm{S}}$. Let $(a : (P_a, \mathfrak{L}_a))_{a\in A}$ and $(a : (P'_a, \mathfrak{L}'_a))_{a\in A}$ be agent-local concurrent epistemic ensemble configurations, $i \in A$, and $\varsigma \in \bigcup \vec{\mathrm{S}}$. Then*

$$\begin{aligned}(a : (P_a, \mathfrak{L}_a))_{a\in A} &\overset{i:\varsigma}{\Longrightarrow}_{\vec{\Sigma}}^{\vec{\alpha}} (a : (P'_a, \mathfrak{L}'_a))_{a\in A} \iff \\ &((a : P_a)_{a\in A}, \bigoplus (\mathfrak{L}_a)_{a\in A}) \xrightarrow{i:\varsigma}_{\vec{\Sigma}}^{\bigoplus \vec{\alpha}} ((a : P'_a)_{a\in A}, \bigoplus (\mathfrak{L}'_a)_{a\in A}) \ .\end{aligned}$$

When studying the relationship between global and agent-local concurrent environments our methodology started from a concurrent state and action to which the sum operator was applied to get a global environment. We can, however, also start from a global environment if only a single actual world and event are considered in the global model.

For an $a \in A$ and a singleton epistemic state $(K, \{w_0\})$ with $K = (W, E, L)$ we define its a-projection $(K, \{w_0\})\restriction a$ as $(K, [w_0]_{E_a})$; for a singleton epistemic action $(U, \{q_0\})$ with $U = (Q, F, pre)$ we similarly define $(U, \{q_0\})\restriction a$ as $(U, [q_0]_{F_a})$. We show that $(K, \{w_0\})\restriction a \lhd_\Sigma (U, \{q_0\})\restriction a = ((K, \{w_0\}) \lhd_\Sigma (U, \{q_0\}))\restriction a$ independently of the choice of w_0 and q_0 from their respective a-equivalence classes as long as $K, w_0 \models_\Sigma pre(q_0)$:

Lemma 5. *Let $(K, W_0) \in \mathcal{N}_\Sigma$ and $(U, Q_0) \in \mathcal{C}_\Sigma$ be a-local with $K = (W, E, L)$ and $U = (Q, F, pre)$. Let $w_0 \in W_0$, i.e., $W_0 = [w_0]_{E_a}$, and $q_0 \in Q_0$, i.e., $Q_0 = [q_0]_{F_a}$, with $K, w_0 \models_\Sigma pre(q_0)$. Let $(K, W_0) \lhd_\Sigma (U, Q_0) = (K \lhd_\Sigma U, W_0')$ and $K \lhd_\Sigma U = (W', E', L')$. Then $[(w_0, q_0)]_{E_a'} = W_0'$.*

Proof. It holds that $W' = \{(w, q) \in W \times Q \mid K, w \models_\Sigma pre(q)\}$ and $\emptyset \neq W_0' = \{(w_0', q_0') \in W_0 \times Q_0 \mid K, w_0' \models_\Sigma pre(q_0')\} \subseteq W'$. Now, $(w_0', q_0') \in [(w_0, q_0)]_{E_a'} \subseteq W'$ if, and only if, $K, w_0' \models_\Sigma pre(q_0')$, $(w_0, w_0') \in E_a$, and $(q_0, q_0') \in F_a$; if, and only if, $K, w_0' \models_\Sigma pre(q_0')$, $w_0' \in W_0$ by a-locality, and $q_0' \in Q_0$ again by a-locality; if, and only if, $(w_0', q_0') \in W_0'$. □

This lemma implies that global executions on singleton epistemic states can be localised agent-wise and executed concurrently:

Corollary 6. *Let $\alpha : \bigcup \vec{\mathrm{S}} \to \mathcal{C}_\Sigma$ be a global epistemic action symbol interpretation yielding only singleton epistemic actions, and define the agent-local concurrent epistemic action symbol interpretation $\alpha\restriction : \bigcup \vec{\mathrm{S}} \to \vec{\mathcal{E}}_\Sigma$ by $(\alpha\restriction)(\varsigma) = (\alpha(\varsigma)\restriction a)_{a \in A}$ for all $\varsigma \in \bigcup \vec{\mathrm{S}}$. Let $((a : P_a)_{a \in A}, \mathfrak{K})$ and $((a : P_a')_{a \in A}, \mathfrak{K}')$ be global epistemic ensemble configurations where $\mathfrak{K}$ and $\mathfrak{K}'$ are singleton epistemic states, let $i \in A$ and $\varsigma \in \bigcup \vec{\mathrm{S}}$. Then*

$$((a : P_a)_{a \in A}, \mathfrak{K}) \xrightarrow{i:\varsigma}_{\vec{\Sigma}} ((a : P_a')_{a \in A}, \mathfrak{K}') \iff$$
$$(a : (P_a, \mathfrak{K}\restriction a))_{a \in A} \overset{i:\varsigma}{\Longrightarrow}_{\vec{\Sigma}} (a : (P_a', \mathfrak{K}'\restriction a))_{a \in A} \ .$$

6 Conclusions

Epistemic ensembles consist of agents that manage and update knowledge through shared and local epistemic states. In this work we considered three perspectives: a global semantic setting based on indeterminate states, a concurrent setting with agent-specific knowledge, and an agent-local view capturing indetermination as accessibility equivalences. Only in the agent-local concurrent setting decisions solely require local knowledge. Using a sum operator, we

connected these perspectives as a bisimulation between global and concurrent executions. Thus we provide a unified framework linking global, concurrent, and agent-local views of epistemic ensembles. It complements our former results relating a (determinate) global semantic environment with a symbolic environment defined by a global knowledge base, and a distributed symbolic semantics over local knowledge bases.

In future work, we plan to combine indeterminate epistemic states for modelling uncertainty with our previous results on symbolic executions. In particular, there should be a close correspondence between agent-local concurrency and symbolic distribution [18]. Another direction of future research is extending the epistemic approach to incorporate beliefs, factual changes, and probabilistic behaviour.

Acknowledgements. We would like to thank the anonymous reviewers for valuable and constructive comments.

References

1. Baltag, A., Moss, L.S., Solecki, S.: The logic of public announcements and common knowledge and private suspicions. In: Gilboa, I. (ed.) Proceedings 7th Conference Theoretical Aspects of Rationality and Knowledge (TARK), pp. 43–56. Morgan Kaufmann (1998)
2. Baltag, A., Renne, B.: Dynamic epistemic logic. In: Zalta, E.N., Nodelman, U., Allen, C., Anderson, R.L. (eds.) Stanford Encyclopedia of Philosophy. Stanford University, The Metaphysics Research Lab (2016)
3. Bolander, T., Andersen, M.B.: Epistemic planning for single- and multi-agent systems. J. Appl. Non-Classical Logics **21**(1), 9–34 (2011). https://doi.org/10.3166/jancl.21.9-34
4. Bolander, T., Dissing, L., Herrmann, N.: DEL-based epistemic planning for human-robot collaboration: Theory and implementation. In: Bienvenu, M., Lakemeyer, G., Erdem, E. (eds.) Proc. 18th Intl. Conf. Principles of Knowledge Representation and Reasoning (KR), pp. 120–129 (2021). https://doi.org/10.24963/KR.2021/12
5. Bortolussi, L., et al.: CARMA: collective adaptive resource-sharing Markovian agents. In: Bertrand, N., Tribastone, M. (eds.) Proceedings 13$^{\text{th}}$ Ws. Quantitative Aspects of Programming Languages and Systems (QAPL). EPTCS, vol. 194, pp. 16–31 (2015). https://doi.org/10.4204/EPTCS.194.2
6. Bures, T., Gerostathopoulos, I., Hnetynka, P., Keznikl, J., Kit, M., Plasil, F.: DEECO: an ensemble-based component system. In: Proceedings 16$^{\text{th}}$ ACM SIGSOFT Symposium Component Based Software Engineering (CBSE), pp. 81–90. ACM (2013). https://doi.org/10.1145/2465449.2465462
7. De Nicola, R., Loreti, M., Pugliese, R., Tiezzi, F.: A formal approach to autonomic systems programming: the SCEL language. ACM Trans. Auton. Adapt. Syst. (TAAS) **9**(2), 1–29 (2014). https://doi.org/10.1145/2619998
8. van Ditmarsch, H., van der Hoek, W., Kooi, B.P.: Dynamic epistemic logic and knowledge puzzles. In: Priss, U., Polovina, S., Hill, R. (eds.) Proceedings 15$^{\text{th}}$ International Conferences Conceptual Structures (ICCS). LNCS, vol. 4604, pp. 45–58. Springer (2007). https://doi.org/10.1007/978-3-540-73681-3_4

9. van Ditmarsch, H., van Eijck, J., Pardo, P., Ramezanian, R., Schwarzentruber, F.: Epistemic protocols for dynamic gossip. J. Appl. Log. **20**, 1–31 (2017). https://doi.org/10.1016/J.JAL.2016.12.001
10. van Eijck, J., Orzan, S.: Epistemic verification of anonymity. In: ter Beek, M.H., Gadducci, F. (eds.) Proceedings 2nd International Workshops Views on Designing Complex Architectures (VODCA@FOSAD), pp. 159–174. Electronic Notes in Computer Science, Elsevier (2006). https://doi.org/10.1016/J.ENTCS.2006.08.026
11. Engesser, T., Bolander, T., Mattmüller, R., Nebel, B.: Cooperative epistemic multi-agent planning for implicit coordination. In: Ghosh, S., Ramanujam, R. (eds.) Proceedings 9th Workshops Methods for Modalities (M4M@ICLA), pp. 75–90. EPTCS (2017). https://doi.org/10.4204/EPTCS.243.6
12. Engesser, T., Marre, T.L., Lorini, E., Schwarzentruber, F., Zanuttini, B.: A simple integration of epistemic logic and reinforcement learning. In: Das, S., Nowé, A., Vorobeychik, Y. (eds.) Proceedings 24th International Conferences Autonomous Agents and Multiagent Systems (AAMAS), pp. 686–694. ACM (2025). https://doi.org/10.5555/3709347.3743585
13. Fagin, R., Halpern, J.Y., Moses, Y., Vardi, M.Y.: Reasoning About Knowledge, MIT Press (2003)
14. Hennicker, R., Knapp, A., Wirsing, M.: Epistemic ensembles. In: Margaria, T., Steffen, B. (eds.) Proceedings 11th Intl. Symp. Leveraging Applications of Formal Methods, Verification and Validation (ISoLA). Part III: Adaptation and Learning. Lecture Notes in Computer Science, vol. 13703, pp. 110–126. Springer (2022). https://doi.org/10.1007/978-3-031-19759-8_8
15. Hennicker, R., Knapp, A., Wirsing, M.: Symbolic realisation of epistemic processes. In: Bjørner, N., Heule, M., Voronkov, A. (eds.) Proceedings 25th International Conferences Logic for Programming, Artificial Intelligence and Reasoning (LPAR). EPiC Series in Computing, vol. 100, pp. 390–407 (2024a). https://doi.org/10.29007/H8H3
16. Hennicker, R., Knapp, A., Wirsing, M.: Epistemic ensembles in semantic and symbolic environments. In: Margaria, T., Steffen, B. (eds.) Proceedings 12th International Symposium Leveraging Applications of Formal Methods, Verification and Validation (ISoLA). Part II. Lecture Notes in Computer Science, vol. 15220, pp. 69–87. Springer (2024b). https://doi.org/10.1007/978-3-031-75107-3_5
17. Hölzl, M.M., Rauschmayer, A., Wirsing, M.: Software engineering for ensembles. In: Wirsing, M., Banâtre, J., Hölzl, M.M., Rauschmayer, A. (eds.) Software-Intensive Systems and New Computing Paradigms — Challenges and Visions, Lecture Notes in Computer Science, vol. 5380, pp. 45–63. Springer (2008). https://doi.org/10.1007/978-3-540-89437-7_2
18. Knapp, A., Hennicker, R., Wirsing, M.: Epistemic ensembles in semantic, symbolic, and distributed environments. J. Softw. Tools Techn. Transf. **28**, 167–184 (2026). https://doi.org/10.1007/s10009-026-00855-z
19. Sadzik, T.: Exploring the iterated update universe. Technical report, ILLC, University of Amsterdam (2006). https://eprints.illc.uva.nl/id/eprint/201/1/PP-2006-26.text.pdf. Accessed: 18 Apl 2026
20. Sürmeli, J.: Epistemic logic in ensemble specification. In: Margaria, T., Steffen, B. (eds.) Proceedings 9th International Symposium Leveraging Applications of Formal Methods, Verification and Validation. Distributed Systems (ISoLA), Part II. Lecture Notes in Computer Science, vol. 12477, pp. 329–343. Springer (2020). https://doi.org/10.1007/978-3-030-61470-6_20
21. van Ditmarsch, H., van der Hoek, W., Kooi, B.: Dynamic Epistemic Logic, Synthese Library, vol. 337. Springer (2008)

22. Wirsing, M., Hölzl, M.M., Tribastone, M., Zambonelli, F.: ASCENS: engineering autonomic service-component ensembles. In: Review of Selected Papers 10th International Symposium Formal Methods for Components and Objects (FMCO), pp. 1–24 (2011). https://doi.org/10.1007/978-3-642-35887-6_1

Open Access This chapter is licensed under the terms of the Creative Commons Attribution 4.0 International License (http://creativecommons.org/licenses/by/4.0/), which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license and indicate if changes were made.

The images or other third party material in this chapter are included in the chapter's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the chapter's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.

