Comment on "Color-Induced Transition to a Nonconventional Diffusion Regime"

In a recent Letter, Tsironis and Grigolini[1] (TG) gave a theoretical account of the diffusion in a one-dimensional bistable system driven by exponentially correlated noise with particular reference to the case of large correlation times τ. The *non-Markovian* process $x(t)$ is *approximated* in their treatment by a nonstationary *Markovian* Fokker-Planck equation (FPE).

The numerical evidence produced by TG to support their approximate theory rests entirely on the explicit integration of such a time-dependent FPE, whose equivalence with the original non-Markovian problem, Eqs. (1) and (2) in Ref. 1, deserves a closer scrutiny. Their FPE describes transient effects due to an initial nonequilibrium preparation. With regard to the stationary limit, $t \to \infty$, the TG approach presents, indeed, some advantages over the Fox theory,[2] but suffers from the same major limitations: (i) $\sigma_{TG}(x) \equiv \sigma(x, t \to \infty)$ is now uniquely defined but always coincides with a stationary solution $\sigma_F(x)$ (also positive semidefinite) of Fox's *bona fide* FPE. For $\tau > \tau_c$ ($\alpha\tau_c \equiv 1$), $\sigma_{TG}(x)$ can be obtained by solving the Fox FPE supplemented with the requirement that $\sigma_F(x)$ is symmetric and prepared identically zero in the critical domain $\tau\phi'(x) > 1$. This critical region is clearly an artifact of the approximation schemes[1,2] as shown in a recent[3] numerical investigation of the original non-Markovian problem. (ii) For $\tau < \tau_c$, the decay time T (=inverse relaxation rate, see Ref. 1) plotted in Fig. 3 of TG is shown to coincide (numerically) with the reciprocal of the smallest nonzero eigenvalue λ_F of the Fox FPE [see Fig. 1(a)]. The decay times $T_F(\tau) = 1/\lambda_F(\tau)$, however, match the exact (numerical) non-Markovian decay times $T(\tau) \equiv 1/\lambda(\tau)$ for *vanishingly small values of τ only.* The disagreement *increases* with decreasing noise intensity D_0 (whereas TG claim that their theory holds for $D_0/V_0 \to 0$). The decay time $T(\tau)$ of the original problem exhibits a clear-cut dependence on τ/D_0 for intermediate values of τ (i.e., $D_0/V_0 \lesssim \alpha\tau < \alpha\tau_c$), contrary to what has been stated in TG. (iii) The approximate theory by TG allows a determination of T also for $\tau > \tau_c$. As shown in Fig. 1(b), the results in TG amount to a smooth continuation of the curve $T_F(t) = 1/\lambda_F(\tau)$ to values of τ larger than τ_c. However, the comparison of TG data with the relevant numerical solution of the original non-Markovian problem[3] [Fig. 1(b)] exhibits a marked discrepancy for $\tau > \tau_c$ as well (e.g., at $\tau = 3.5$ our exact T exceeds T_{TG} by a factor of 3.6).

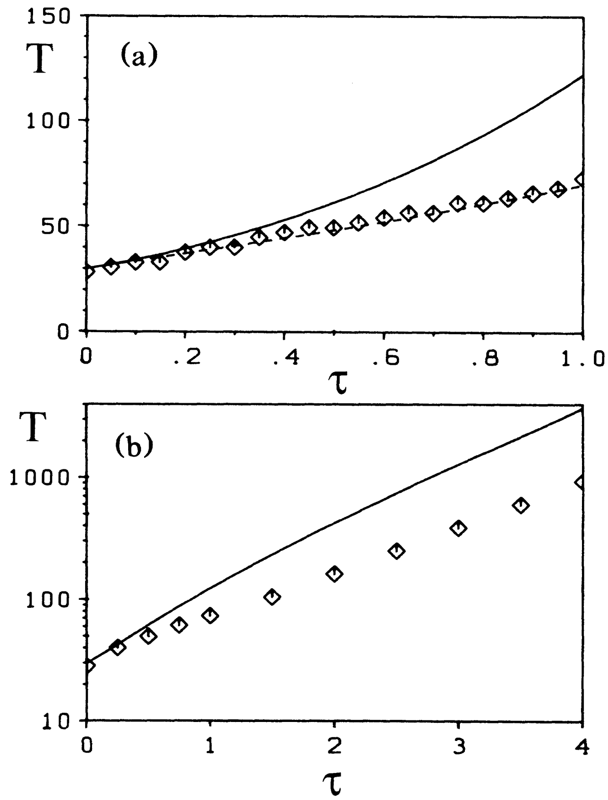

FIG. 1. The TG relaxation times $T(\tau)$ (lozenges) are compared with those of the original, non-Markovian problem (solid line) at $D_0 = 0.1$. The dashed line in (a) denotes $1/\lambda_F(\tau)$. The potential parameters are as in Ref. 1.

We conclude that the numerical study in Ref. 1 requires qualification when being used for comparison with ongoing theoretical work.

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[1]G. P. Tsironis and P. Grigolini, Phys. Rev. Lett. **61**, 7 (1988); our notation is the same.
[2]R. F. Fox, Phys. Rev. A **37**, 911 (1988).
[3]P. Hänggi, P. Jung, and F. Marchesoni, J. Stat. Phys. **54**, 1367 (1989).