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Abstract

In this paper we show that the optimal path of capital accumulation in the usual neoclassical model has at any point of time higher values of the capital stock if the stock of technical knowledge enters the production function as third argument, in comparison to the model without technical knowledge. Moreover the evolution of the stock of technical knowledge is also positively influenced by the accumulation of capital. Sufficient conditions for the existence of a steady state solution and necessary conditions for instability of the model are derived.

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Introduction

In this paper we present a dynamic theory of the firm. Starting point of the analysis is a firm which maximizes the present value of net cash flow subject to constraints such as a neoclassical production function, a capital accumulation equation and an equation for the accumulation of technical knowledge. Hence our approach is decision oriented. Contrary to the usual treatment of neoclassical production functions (cf. Takayama (1985)) the stock of technical knowledge enters the production function as third argument, besides capital and labour. We thus intend to integrate Schumpeterian elements on the microeconomic level of economic theory. As to the stock of technical knowledge we use the usual assumption that it is not exogenously given, but endogenously determined. The firm can influence its level through research and development activities via a production function for technical knowledge. This function is taken from Ramser (1985) or Sato, Nono (1982). The capital stock is influenced by investment which determine the rate of change of the capital stock through an adjustment function. Our goal is to pay special attention to the interrelation between the evolution of the capital stock and the stock of technical knowledge and to derive sufficient conditions assuring the existence of a steady state solution.

The model

The production function depending on capital, labour and the firm's stock of knowledge is written as:

$$Q(t) = Q(K(t), L(t), A(t)) \quad (1),$$

where Q(t) is the rate of output, K(t) is the amount of capital, L(t) is the amount of labour and A(t) is the stock of technical knowledge, all variables measured at time t.

We assume that
$Q(\cdot) > 0$, $Q_i(\cdot) > 0$, $Q_{ij} < 0$, $i=j$ and $Q_{ij} > 0$, $i \neq j$,
$i,j = K, L, A$ and that the production function is strictly concave in its arguments.

The evolution of the capital stock is constrained as follows:

$$\dot{K}(t) = \varphi\,(I(t)) - \delta K(t),\ K(0) = K_0 \quad (2),\ \text{where}$$

$\dot{K}(t)$ = the rate of change of K with respect to time in period t
$I(t)$ = the rate of investment at t
$K(t)$ = the capital stock at t
δ = depreciation rate
$\varphi(\cdot)$ = the function which represents adjustment costs of investment,
with $\varphi(\cdot) > 0$, $\varphi'(\cdot) > 0$, $\varphi''(\cdot) < 0$.
(the prime indicates derivatives)

The production function for technical knowledge is written as:

$$\dot{A}(t) = g\,[A(t),R(t)] - \mu A(t),\ A(0) = A_0 \quad (3),\ \text{with}$$

$\dot{A}(t)$ = the rate of change of technical knowledge with respect to time
$R(t)$ = the rate of research activites at time t
$g(\cdot)$ = the production function for technical advance
μ = depreciation rate of technical knowledge.

We require that:

$g_R(\cdot) > 0$: a rise in the amount of research activities raises the time rate of increase in technology

$g_{RR}(\cdot) < 0$: there are diminishing rate of returns

$g_A(\cdot) > 0$: as A rises, the time rate of rise in A increases for a given R and

$g_{AA}(\cdot) < 0$, $g_{AR}(\cdot) \geq 0$, $g(\cdot) > 0$ and $g(\cdot)$ is concave in A(t) and R(t).

The firm's objective is to maximize its present value profit with respect to the control variables I, L, R over an infinite time horizon. Suppressing time arguments, the control problem is written as:

$$\max_{I,L,R} \int_0^\infty e^{-rt}\ \pi(t)\ dt \quad (4)$$

subject to $\pi = pQ\ (K,L,A) - wL - cI - sR$, (2) and (3).

p is the price of output, w is the wage rate, c cost per unit of investment, s cost per unit of research activities and r is the discount rate. p, w, c, s and r are assumed to be constant over time.

The existence and uniqueness of a solution to this problem is obtained from a standard result (see Seierstad, Sydsaeter (1987), p. 406). Moreover we confine our investigations to interior solutions only which are supposed to exist.

To solve the maximization problem, we define the current-value Hamiltonian as

$$H \equiv [pQ\ (K,L,A) - wL - cI - sR] + \gamma_1\ [g(A,R) - \mu A] + \gamma_2\ [\varphi(I) - \delta K] \quad (5)$$

Let $W(A(t_0),K(t_0)) = \max_{I,L,R} \int_{t_0}^{\infty} e^{-rt}\, \pi(t)\, dt$ and

$V(A(t_0),K(t_0)) = W(A(t_0),K(t_0))/e^{-rt_0}$.

Using this notation,

$$\frac{\partial V}{\partial A}(A_0,K_0) = \gamma_1(A_0,K_0), \quad \frac{\partial V}{\partial K}(A_0,K_0) = \gamma_2(A_0,K_0)$$

Then necessary conditions for the Hamiltonian to take a maximum value are:

$$\frac{\partial H}{\partial L} = pQ_2(\cdot) - w = 0 \qquad (6)$$

$$\frac{\partial H}{\partial I} = \gamma_2 \ \varphi'(\cdot) - c = 0 \qquad (7)$$

$$\frac{\partial H}{\partial R} = \gamma_1 \ g_R(\cdot) - s = 0 \qquad (8)$$

$$\frac{\partial H}{\partial \gamma_1} = \dot{A} = g\ (A,R) - \mu A \qquad (9)$$

$$\frac{\partial H}{\partial \gamma_2} = \dot{K} = \varphi(I) - \delta K \qquad (10)$$

$$\dot{\gamma}_1 = \gamma_1\ [r - g_A(\cdot) + \mu] - pQ_3(\cdot) \qquad (11)$$

$$\dot{\gamma}_2 = \gamma_2\ (r+\delta) - pQ_1(\cdot) \qquad (12)$$

Under the assumptions made above (concavity of $\pi(t)$ in (K,L,A,I,R) and concavity of $\dot{K}$ and $\dot{A}$ in (K,I) and (A,R)

respectively) application of Mangasarian's theorem (see e.g. Seierstad, Sydsaeter (1987) p. 287) shows that the necessary conditons are also sufficient.
Furthermore the following transversality condition must be fulfilled in order to guarantee sufficiency:

$$\lim_{t \to \infty} e^{-rt} [\gamma_1 A + \gamma_2 K] = 0$$

Equations (6), (7) and (8) are the familiar conditions for variable inputs meaning that inputs should be made until its marginal costs equal its marginal products:
(6) requires that labour should be employed until its marginal product equals its cost.
(7) states that investment should be made until its shadow price γ_2 equals its cost c/φ'. The shadow price γ_2 is the marginal value of a unit rise in the state variable K.
Equation (8) says that research activites should be made until the shadow price γ_1 equals the ratio of its cost to its contribution to technical change. The shadow price γ_1 measures the marginal value of the stock of technical knowledge if it rises by one unit. Conditions (9) and (10) simply retrieve the constraints on technical knowledge and on the evolution of capital. Equations (11) and (12) show the dynamic behaviour of γ_1 and γ_2.
(6) can be rewritten as $L = L(K,A,w/p)$ with $\partial L/\partial i = -Q_{Li}/Q_{LL}$, $i=K$, A and write $pQ_1(K,L,A) = pQ_1(K,L(K,A,w/p),A) = f(K,A)$ and $pQ_3(K,L(K,A,w/p),A) = h(A,K)$ (recall that $w/p = \text{const.}$), where $f_1 \leq 0$, and $h_1 \leq 0$ for a given stock of technical knowledge and capital respectively (f_1 and h_1 denote the partial derivatives of $f(\cdot)$ and $h(\cdot)$ with respect to the i-th argument). This follows from the assumptions that $Q(\cdot)$ is concave in its arguments and states that the marginal productivities of capital and technical knowledge do not rise with rising capital and technical knowledge respectively.
From (7) and (8) we get $I = I(\gamma_2, c)$ where $\partial I/\partial \gamma_2 =$

$-\varphi'/(\varphi''\gamma_2)$ and $R = R(A,\gamma_1,s)$ with $\partial R/\partial A = -g_{RA}/g_{RR}$ and $\partial R/\partial\gamma_1 = -g_R/(\gamma_1 g_{RR})$ respectively so that the evolution of our model can completely be caracterized by the system of differential equations (9) to (12).

As equations (9), (10), (11) and (12) form a four-dimensional differential equation system we cannot employ the usual graphical method to find the optimal path describing the dynamic behaviour of A, K, γ_1, γ_2. Therefore we will divide the system into two two-dimensional differential equation systems describing the evolution of the capital stock and the stock of technical knowledge in a $(\gamma_2$-$K)$- and a $(\gamma_1$-$A)$-phase diagram respectively.

The existence and uniqueness of the steady state can easily be checked. This follows again from the concavity assumptions made above.

The evolution of the stock of capital and its shadow price is given by

$$\dot{K} = \varphi(I(\gamma_2,c)) - \delta K$$

$$\dot{\gamma}_2 = \gamma_2 \, (r+\delta) - f(K,A)$$

For a given stock of technical knowledge the phase diagram is the same as the familiar one for the neoclassical production function without technical change (cf. Takayama (1985) p. 700). It can easily be checked that the steady state is stable in the saddle point sense such that there exists a stable manifold. Reachability of the manifold is guaranteed by the concavity of our functions (c.f. Mangasarian (1966); see also Forster (1977)).

As the phase diagram is only valid for a given stock of technical knowledge we now investigate the effect of technical change on the optimal path of K(t). For this purpose we treat the stock of technical knowledge as exogenous to the variational differential system describing the evolution of the optimal stock of capital and the

shadow price of capital over time, that is we neglect repercussion effects arising from a higher capital stock on the stock of technical knowledge, and derive the effect of a once-and-for-all increase in A at any moment t on the path of K(t) and $\gamma_2(t)$.
In order to achieve this goal we use the method developed by Oniki (1973) and employed e.g. by Nagatani (1981).
The relevant variational system describing the effect of a rise in A on $\dot{K}(t)$ and $\dot{\gamma}_2(t)$ is obtained from (14) and (12):

$$\begin{bmatrix} \partial \dot{K}/\partial A \\ \partial \dot{\gamma}_2/\partial A \end{bmatrix} = \begin{bmatrix} -\delta & \varphi'v' \\ -f_1 & r+\delta \end{bmatrix} \begin{bmatrix} \partial K/\partial A \\ \partial \gamma_2/\partial A \end{bmatrix} + \begin{bmatrix} 0 \\ -f_2 \end{bmatrix} \tag{15}$$

From that we can compute the long-run effects (where $\dot{K} = 0$ and $\dot{\gamma}_2 = 0$) on the steady state values K^* and $\gamma_2{}^*$ as:

$$\frac{\partial K^*}{\partial A} = \frac{-f_2\varphi'v'}{f_1\varphi'v' - \delta(r+\delta)} > 0 \quad \text{and}$$

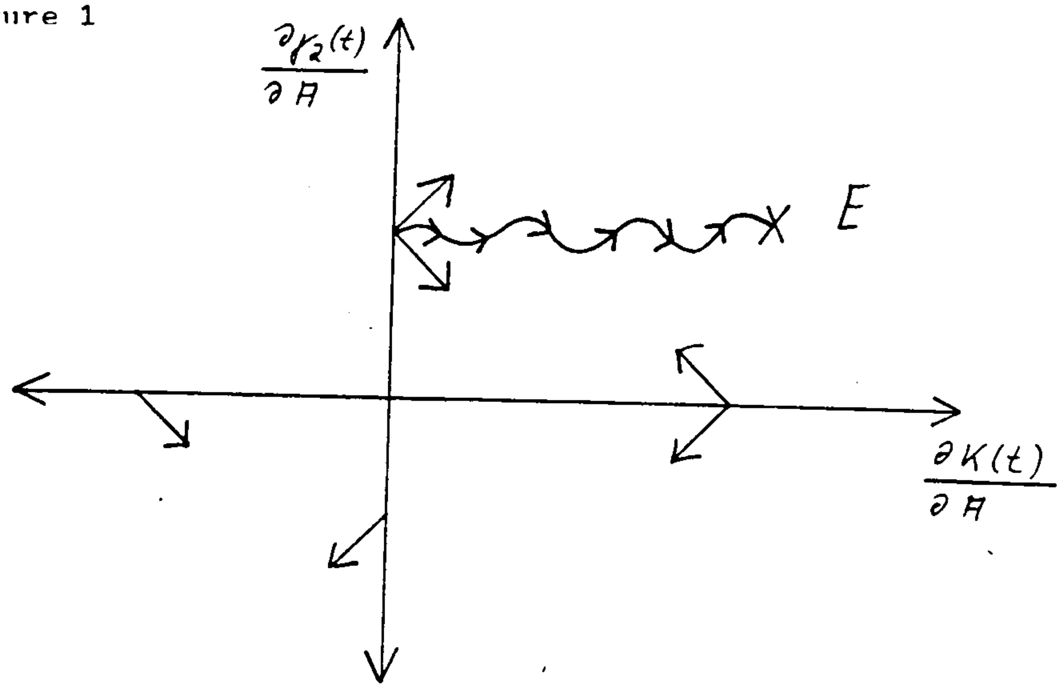
$$\frac{\partial \gamma_2{}^*}{\partial A} = \frac{-\delta f_2}{f_1\varphi'v' - \delta(r+\delta)} > 0$$

From above we know that f_1 is negative. Neglecting repercussion effects from a higher stock of capital on the stock of technical knowledge we can compute f_2 as $f_2 = Q_{13} - Q_{12}Q_{23}/Q_{22} > 0$.
This means that an increase in A leads to a higher stock of capital K^* and a higher shadow price $\gamma_2{}^*$ in the long run.

The second step now is to completely spell out the effect of a change in A on the entire optimal path of K and γ_2 from $t = 0$ to ∞.

We know that a rise in the stock of technical knowledge leads to a higher shadow price $\gamma_2{}^*$ and a higher stock of capital K^* in the long run (point E in figure 1). The effect of a higher stock of technical knowledge on the initial values of K and γ_2 at t = 0 are obtained from (15) if we take into regard that $\partial K_0/\partial A = 0$ as the capital stock at t = 0 is fixed, that is the curve must start from somewhere on the vertical axis. Assuming that $\partial\gamma_2/\partial A < 0$ we see that $\partial\dot{K}/\partial A = \varphi'v'[\partial\gamma_2(0)/\partial A] < 0$ and
$\partial\dot{\gamma}_2/\partial A = (r+\delta)[\partial\gamma_2(0)/\partial A] - f_2 < 0$. This means that the curve must move into the third orthant in figure 2 and can never reach the long run equilibrium at point E.
If $\partial\gamma_2(0)/\partial A > 0$, the effect on K is clear,
$\partial\dot{K}/\partial A=\varphi'v'[\partial\gamma_2/\partial A] > 0$ whereas the effect on γ_2 is ambiguous, $\partial\dot{\gamma}_2/\partial A=(r+\delta)[\partial\gamma_2/\partial A] - f_2 \leq\geq 0$
Thus we see that the curve must start in the first orthant in order to reach point E. The fact that the curve stays in the positive orthant means that the optimal path for a higher value of A has everywhere higher values of K and γ_2 throughout the program. The directions of motion on the horizontal axis are derived if we set $\partial\gamma_2(0)/\partial A = 0$ in (15).

Figure 1



From our result above we can conclude that as long as technical knowledge rises the shadow price of capital γ_2 mounts and thus the capital stock for every $t > 0$.

Proceeding in an analogous way we can compute the effect of a rise in the capital stock on the evolution of the stock of technical knowledge. We now treat the stock of capital as exogenous to the relevant variational differential system describing the evolution of the stock of technical knowledge and the shadow price of technical knowledge.
The dynamic behaviour of A and γ_1 is described by

$$\dot{A} = g(A, R(A, \gamma_1, s)) - \mu A$$

$$\dot{\gamma}_1 = \gamma_1 \; [r - g_A(\cdot) + \mu] - h(A, K)$$

with the Jacobian

$$\begin{bmatrix} g_A-\mu+g_R(\partial R/\partial A) & g_R(\partial R/\partial \gamma_1) \\ -h_1-\gamma_1(g_{AA}+g_{AR}(\partial R/\partial A) & r-g_A+\mu-\gamma_1 g_{AR}(\partial R/\partial \gamma_1) \end{bmatrix}$$

Note that $g_R(\partial R/\partial A) = \gamma_1 g_{AR}(\partial R/\partial \gamma_1)$.

The characteristic roots are given by

$$\lambda_{1,2} = \frac{r}{2} \pm \left[\left(\frac{r}{2}\right)^2 + d - B_2 \right]^{1/2} \quad \text{with}$$

$d = [-g_A+\mu-g_R(\partial R/\partial A)] \cdot [r-g_A+\mu-\gamma_1 g_{AR}(\partial R/\partial \gamma_1)] <> 0$

$B_2 = [h_1+\gamma_1(g_{AA}+g_{AR}(\partial R/\partial A)] \cdot [g_R(\partial R/\partial \gamma_1)] \le 0$ where

$\partial R/\partial A \ge 0$, $\partial R/\partial \gamma_1 > 0$ follows from (8) and $(g_{AA}+g_{AR}(\partial R/\partial A) \ge 0$ results from the concavity assumption of $g(\cdot)$.

From this expression we see that a necessary and sufficient condition for saddle point stability is $d > B_2$.

This condition is always fulfilled if $g_A-\mu+g_R(\partial R/\partial A) < 0$, that is if the depreciation of technical knowledge caused by a marginal increase of A at the steady state exceeds the positive influence of A on the production process for technical knowledge. We will refer to this situation as the unproductive production process for technical knowledge.
If the production process for technical knowledge is productive at the steady state we have to distinguish two further cases. First if $r < g_A-\mu+g_R(\partial R/\partial A)$ that is the discount rate is smaller than the increase in A caused by a marginal rise of A the model is again stable in the saddle point sense. If $r > g_A-\mu+g_R(\partial R/\partial A)$ saddle point stability only holds if $d > B_2$.

For these different cases we now conduct comparative dynamics.

The relevant variational differential system is now obtained from (11) and (16):

$$\begin{bmatrix} \dot{\partial A / \partial K} \\ \dot{\partial \gamma_1 / \partial K} \end{bmatrix} = \begin{bmatrix} g_A - \mu + g_R (\partial R/\partial A) & g_R (\partial R/\partial \gamma_1) \\ -h_1 - \gamma_1 (g_{AA} + g_{AR} (\partial R/\partial A) & r - g_A + \mu - \gamma_1 g_{AR} (\partial R/\partial \gamma_1) \end{bmatrix}$$

$$\cdot \begin{bmatrix} \partial A/\partial K \\ \partial \gamma_1 / \partial K \end{bmatrix} + \begin{bmatrix} 0 \\ -h_2 \end{bmatrix}$$

The long-run effects on the steady state values A^* and $\gamma_1{}^*$ are computed as:

$$\frac{\partial A^*}{\partial K} = \frac{- g_R (\partial R/\partial \gamma_1)\ h_2}{-d + B_2}$$

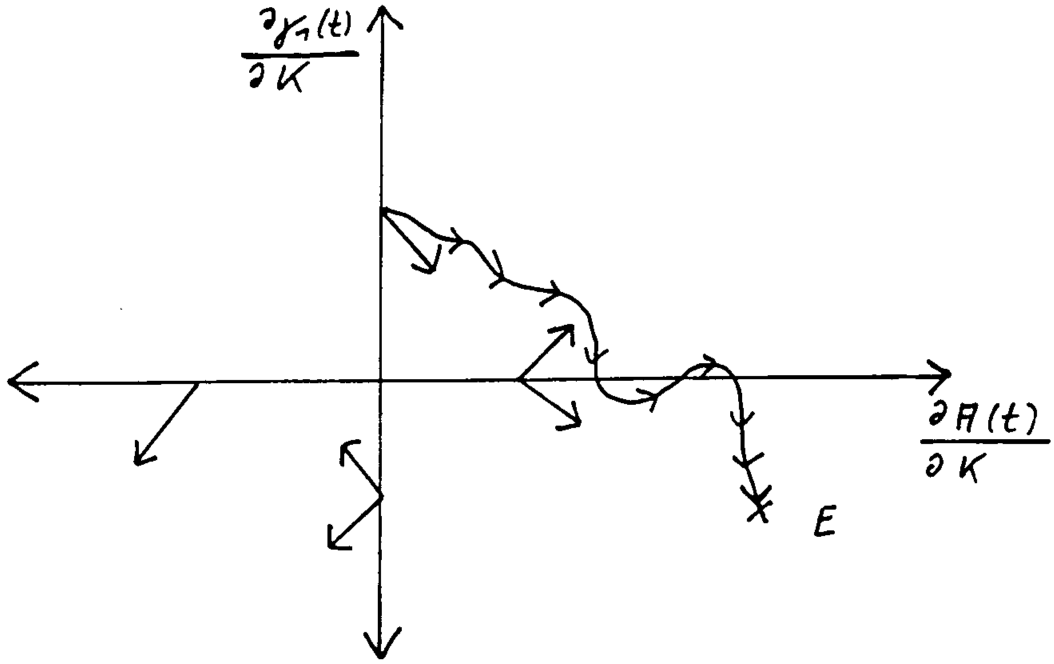
$$\frac{\partial \gamma_1{}^*}{\partial K} = \frac{g_A - \mu + g_R (\partial R/\partial A)}{-d + B_2}$$

In the case of a unproductive production process for technical knowledge we get $\partial A^*/\partial K > 0$, $\partial \gamma_1{}^*/\partial K > 0$ and the comparative dynamic analysis is completely equivalent to the analysis of the influence of technical knowledge on the evolution of capital (see figure 1).

If however the production process for A is productive at the steady state we have $\partial A^*/\partial K > 0$ and $\partial \gamma_1{}^*/\partial K < 0$. That is a higher stock of capital increase the long run steady state value A^* but lowers the shadow price $\gamma_1{}^*$.

For the case $0 < r < g_A - \mu + g_R (\partial R/\partial A)$ the comparative dynamics can be deduced from figure 2.

Figure 2



We see that at t=0 $\partial\gamma_1(0)/\partial K > 0$ and that makes the firm invest more in R&D. Over time the influence on γ_1 becomes negative but now the higher level of A itself raises investment in R&D so that the decline of γ_1 is completely compensated by the higher stock of technical knowledge. So in the long run a higher level of A in connection with a lower shadow price is feasible.

If $r > g_A-\mu+g_R(\partial R/\partial A) > 0$ we only have to change the arrows on the vertical axis and then come to the same result.

Stability analysis

To get insight in the behaviour of the total system at the steady state we compute the Jacobian of the system formed by (9)-(12). This gives

$$J = \begin{bmatrix} -\delta & 0 & \varphi'(\partial I/\partial\gamma_2) & 0 \\ 0 & g_A-\mu+g_R(\partial R/\partial A) & 0 & g_R(\partial R/\partial\gamma_1) \\ -f_1 & -f_2 & r+\delta & 0 \\ -h_2 & -h_1-\gamma_1(g_{AA}+g_{AR}(\partial R/\partial A) & 0 & r-g_A+\mu-\gamma_1 g_{AR}(\partial R/\partial\gamma_1) \end{bmatrix}$$

with the characteristic roots

$$\lambda_{1.2.3.4} = \frac{r}{2} \pm \left[\left(\frac{r}{2}\right)^2 - \frac{K}{2} \pm \frac{1}{2}\left[K^2 - 4 \det J\right]^{1/2} \right]^{1/2} \quad \text{with}$$

$K = -a_{11}^2 - a_{22}^2 + r(a_{11} + a_{22}) - 2a_{12}a_{21} - 2a_{14}a_{32} - a_{13}a_{31} - a_{42}a_{24}$,

with a_{ij} element of the i-th row and j-th column, i,j = 1,...,4 (see Feichtinger, Hartl (1986),pp. 134/135).

Setting

$c = \delta\ (r+\delta) > 0$

$d = [-g_A+\mu-g_R(\partial R/\partial A)]\ [r-g_A+\mu-\gamma_1 g_{AR}(\partial R/\partial\gamma_1)] <> 0$

$B_1 = f_1\ [\varphi'(\partial I/\partial\gamma_2)] \le 0$

$B_2 = [h_1+\gamma_1(g_{AA}+g_{AR}(\partial R/\partial A)]\ [g_R(\partial R/\partial\gamma_2)] \le 0,$

we get

$K = -c - d + B_1 + B_2 <> 0$

$K^2-4\det J = [(c-d)-(B_1-B_2)]^2 + 4\ g_R(\partial R/\partial\gamma_1)\ (\partial I/\partial\gamma_2) f_2 h_2 > 0$

$\det J = B_1 B_2 - B_2 c - B_1 d + cd - g_R(\partial R/\partial\gamma_1)\varphi'(\partial I/\partial\gamma_2) f_2 h_2$

As it can be shown that det J > 0 if d > 0 and det J <> 0 if d < 0 we can derive the following result.

First if $g_A-\mu+g_R(\partial R/\partial A) < 0$, that is the production process for technical knowledge is unproductive at the steady state d is negative and the model is stable in the saddle point sense. Or expressed in another way unproductivity of the production process for technical knowledge is a sufficient but not necessary condition for saddle point stability.

If the production process for technical knowledge is productive at the steady state, i.e. $g_A-\mu+g_R(\partial R/\partial A) > 0$ we have to distinguish between two situations.

If the discount rate r is smaller than the contribution of a marginal unit of A to the increase of technical knowledge, $0 < r < g_A-\mu+g_R(\partial R/\partial A)$ the model is again stable in the saddle point sense, that is this situation is a sufficient but not necessary condition for saddle point stability.

If however the discount rate r exceeds the productivity of the production process for technical knowledge the model may be either saddle point stable or completely unstable (in this case we get det $J < 0$ and $K > 0$ or $K < 0$ so that there are 3 roots with positive real part and 1 root with negative real part). Thus a discount rate greater than the increase of technical knowledge if it rises by one unit is a necessary but not sufficient condition for instability.

Conclusion

The inclusion of technical knowledge in the usual neoclassical production function reveals significant effects.

First a rising stock of technical knowledge influences the time rate of change of the shadow price of capital negatively that is it delays the decline of it and thus makes capital more profitable and increases its long-run desired value. From comparative dynamic analysis we could conclude that the optimal path of K rises with an increase in technical knowledge.

An analogous result was derived for the evolution of the stock of technical knowledge. Here the analysis was more complicated and the phase diagram depended on the productivity of the production process for technical knowledge. But in any case the positive influence of capital lead to a higher value of A whereas the influence on γ_1 may be positive or negative. A lower shadow price γ_1 with a higher stock of capital A can occur because of the selfinforcing mechanism of technical knowledge.

Second, we saw that a sufficient condition for stability is that the production process for technical knowledge is unproductive at the steady state or if the opposite holds that the production process for technical knowledge exceeds the discount rate. If this is not the case instability may be the result. Thus we know that the explicit inclusion of the stock of technical knowledge can give rise to instability.

But there is no economic reason for a steady state solution with a rest point to exist so that a more general notion of long-run behaviour should be formulated. Therefore it is necessary to build a theory that allows more general limit sets than rest points (e.g. limit cycles) or even more general a model with chaotic behaviour.

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