Risk Preferences Beyond Expected Utility Theory:

Theoretical and Experimental Approaches

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Risk Preferences Beyond Expected Utility Theory

Theoretical and Experimental Approaches

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Contents

List of Tables

List of Figures

Part I

Introduction

Risk preferences of economic agents are of fundamental importance for present day economic analysis. The presence of uncertainty is undeniably a key factor in shaping economic behavior. The canonical model of expected utility theory (EUT) and several structural assumptions concerning the form of the value function have served as the working horses of theoretical and applied microeconomics for over half a century.

From a choice-theoretic perspective, the central element of EUT is that probabilities enter linear into the function determining choices. In applying EUT, it has become customary to additionally assume unifying properties concerning the value function. The value function evaluates outcomes and is multiplied with the respective probabilities to generate overall decision utility. I will give a short exemplary discussion of some of these customary assumptions on the value function.

Inside EUT, derivatives of the value function have been linked to economic behavior even before an axiomatic characterization of expected utility has been provided. A positive first derivative is equivalent to positive marginal utility. The combination of a positive first and a negative second derivative is equivalent to risk aversion. These assumptions constitute basic building blocks of every textbook on microeconomics. An important common feature of these equivalences is that explicitly or implicitly it is assumed that the sign of the derivative of a certain order is constant over the whole range of outcomes.

Assumptions concerning higher derivatives of the value function are less customary in textbooks, but have nevertheless been applied extensively in economic models. One of the economically most important forms of behavior is that of saving. From a microeconomic view, the motives to save have been identified as a desire to smooth consumption over time, and to prepare for unfavorable contingencies in the future. The latter motive is undoubtedly linked to uncertainty. If the individual wants to save more if the future is uncertain, this higher level of saving is denoted as precautionary saving. It has been clear since Leland (1968) and Sandmo (1970) that the first two derivatives of the value function cannot explain such a behavior. Instead, a positive third derivative as an additional assumption is a sufficient and necessary condition for precautionary saving to exist. Similarly, a negative fourth derivative has been linked to risk attitudes under a stochastically independent background risk. The literature on these higher-order risk preferences in EUT models has developed substantially in the last two decades. Nevertheless, some basic puzzles concerning the empirical validity of such models remain.

Another set of assumptions concerning the value function deals with the question of how several influences of behavior interact to form the utility according to which decisions are made. In case of more than one influence on utility this is termed multi-attribute utility. The utility depends on several arguments or attributes.

A very classical example of multi-attribute utility is how levels of consumption in different future points in time shape the utility function that describes behavior in the present. Expected utility theory per se does not restrict the functional form of such inter-temporal utility functions beyond the linearity in probabilities. However, it seems fair to state that the customary application in expected utility puts a lot more structure on the inter-temporal utility function. To assume a (quasi-)additive structure of inter-temporal utility has become so common that its explicit statement is often omitted.

In addition to inter-temporal aspects, multi-attribute utility has been immensely productive in answering questions on how several goals at a given point in time can jointly influence the overall decision utility. In contrast to inter-temporal utility, it has been precisely the omission of an additive structure that allowed the fruitful analysis of the complex interplay between different goals. Although economists sometimes assume additive separability in order to obtain tractability in complicated models, it is without dispute that this comes at the cost of a lower degree of realism.

Expected utility has normatively desirable features that makes it a relatively undisputed favorite for prescriptive decision making. It ensures a high degree of consistency and rationality. However, in terms of a descriptive theory that aims at describing behavior of typical economic agents, its strengths become its weaknesses. The empirical findings of systematic violations of basic axioms of expected utility by economic agents have subsequently called into question, whether reasonable predictions of behavior should be based on the hypothesis of expected utility maximization.

There are two meaningful responses to the perception of the failure of expected utility theory as a descriptive theory. The first response is to develop detailed models of economic behavior that extend expected utility. The second response is the development of methods that do not require the assumption of a specific model of behavior.

The new alternative theoretical models should incorporate insights on what determines economic behavior from empirical studies, experimental evidence, psychology and common sense. They should be built subsequently, based on the work of other researchers and they should try to further improve these models. Part II of this thesis aims to contribute to this endeavor. I briefly outline the theoretical modifications of expected utility that have advanced the search for an improved model.

A radical approach would be to abandon the assumption that behavior can be described by the outcome of an optimization process. In place of the hypothesis of optimization steps the assumption that individuals follow a set of heuristics. While the psychological evidence for simple heuristics may be favorable in many situations, this approach comes at a huge cost. If one allows for many different simple and situation-specific heuristics, one leaves behind the ambition to find a relatively universal explanation of human behavior. The question which heuristic should be applied in which situation becomes tedious and to a certain degree arbitrary. Also, out of context predictions seem unobtainable. If a heuristic is used to describe behavior in a well-observable setting and this explanation is very context-specific, difficulties arise in extending this theory to other contexts. However, these out of sample predictions are the ultimate goal of any theory.

A less radical deviation from the standard model is to account for the fact that individuals might act altogether in accordance with the optimization of a objection function, but in doing so make systematic mistakes. These biases are often occurring in well-defined classes of decision problems. Examples are inter-temporal or inter-contextual inconsistencies. These can often be described as self-control problems. Here, the individual might want to commit to an action that she anticipates she will have difficulties in following through in other circumstances.

A fruitful extension of the expected utility model has been to maintain the formal structure, but to define the exact content of the argument of the value function more carefully. In textbook models, the argument of the value function is predominantly stated as wealth, disposable income, or the consumption of narrowly defined goods and services. However, by incorporating a wider range of objectives into the value function, the basic structure of EUT can be maintained and the descriptive accuracy highly enhanced. Examples for helpful enhancements are the incorporation of emotions, aspirations, habits, and feelings of fairness, equity and justice into the value function. An extension used in in chapter 2 of this thesis is the notion that the expectation of future consumption and changes in this expectation can be carriers of utility in the present. I will refer to such models as anticipatory utility.

The extensions of EUT discussed so far have contributed to a certain degree in improving the standard model. However, the vast majority of extensions have taken the approach to make less substantial changes. Instead, they have investigated which small departures in the structural assumptions lead to better predictions. These changes in the structure can be distinguished according to whether the focus is on the influence of probabilities or on the shape of the value function.

The predominant focus of theoretical economic research on decision making in the 1980s and 1990s that was seeking to extend EUT attempted to include nonlinear influences of probabilities. These extensions tried to capture the perception that individuals have difficulties in assessing probabilities which are very small. This was achieved by applying a probability weighting function to every probability before aggregating over the states of nature. Early models based on a longer tradition in psychology took a dual approach were each probability is transformed by a function independently of the corresponding outcome. It was soon noted (e.g. by Fishburn, 1978) that this leads to violations of stochastic dominance, which discredited the model. A solution was found by Quiggin (1982), who introduced rank-dependent expected utility which allows a non-linear influence of probabilities without violating stochastic dominance. The central questions in this field seem to be solved. Therefore, recent studies focus on the precise parametrical characterization of the probability weighting function. The remaining obstacles concerning probabilities are likely to stem from the fact that individuals fail to execute Bayesian updating, or lack the capability to form subjective probabilities in the first place.

Only in recent years the importance of the shape of the value function was recognized by a broad group of researchers. Of special importance to the shape of the value function is the concept of a reference point. The reference point is the notion that an important aspect leading to a decision is the comparison of the alternatives in relation to a mental anchor. If a potential outcome exceeds this reference point it is perceived as a gain, if it falls below the reference point it induces sentiments of loss.

The influence of reference points is well-established in psychology. It originates from the field of psychophysics, which can be defined as the quantitative analysis of the relationship between physical stimuli and the resulting sensations. A classical example is an experiment where subjects first place one hand in cold and one hand in hot water. After some time, both hands are placed into water of intermediate temperature. Although both hands are now exposed to an identical temperature, the subject thinks that the hand which was placed in the hot water before, is now exposed to a lower temperature, and vice versa. The reason is that the reference temperature is different for both hands, and the present temperature is partly evaluated by a comparison to this reference temperature.

In the economics literature, the role of the stimuli is played by the extent that wants are met. Early economics literature on reference points were Markowitz (1952) and Kahneman and Tversky's (1979) prospect theory. In these early models the reference point is assumed to be a real-valued variable. The influence of such a reference point on the shape of the value function then can be twofold.

First, the reference point determines the reflection point of the value function. The reflection point captures the idea that the second derivative of the value function does not need to have a constant sign. In the model by Kahneman and Tversky (1979), the value function is concave above the reference point and convex below the reference point.[1] This would capture the idea that individuals are risk-seeking in losses and risk-averse in gains. The empirical assessment of whether individuals are really risk-seeking in losses is still mixed and seems to depend on the context. The assumption of a reflection point itself is compatible with arbitrary levels of differentiability of the value function.

Second, the reference point also determines loss aversion. If the value function is represented by a graph, loss aversion is a concave kink at the reference point. By allowing for a kink, one abandons the property of differentiability over the whole range of the value function. Often it is assumed that the value function is differentiable at all other levels than the reference point. Therefore, calculus can still be used when analyzing behavior concerning situations with outcomes entirely strictly above (or entirely strictly below) the reference point.

The literature on reference points has developed substantially in recent years. Two major developments can be identified. First, the reference point is no longer generally assumed to be a single point. In recent models (Sugden 2003; Delquié and Cillo, 2006; Kőszegi and Rabin, 2006, 2007, 2009) the reference point is a whole distribution potential outcomes are compared to. This captures the idea that individuals can make multiple comparisons in order to evaluate the desirability of an outcome. Second, the reference point was endogenized by Kőszegi and Rabin (2006, 2007, 2009). This is of high importance since an arbitrary exogenous reference point was one of the main obstacles raised by critics of such models. Kőszegi and Rabin derive how recent expectations can shape the reference point.

A more detailed overview over the recent development on reference points, its applications and empirical assessment is provided in chapter 1 of part II of this thesis. Nevertheless, it seems appropriate to stress that existing models of reference-dependent preferences focused almost entirely on risk aversion and behavior that follows directly from risk aversion. Some of

[1]In the model of Markowitz (1952) the reverse is true for small deviations from the reference point. However, he assumes as well that for larger deviations from the reference point the value function becomes locally concave in gains and locally convex in losses. This would indicate the existence of three reflection points. Also, the classical value function of Friedman and Savage (1948) exhibits two reflection points, although they do not refer to the concept of a reference point.

the results in Kőszegi and Rabin (2007) can be interpreted as statements on behavior under background risk and Kőszegi and Rabin (2009) include a short discussion of precautionary behavior. However, no systematic treatment of higher-order risk preferences was attempted so far. The existing results were derived only in narrowly defined models. I therefore consider it worthwhile to fill this gap.

Eeckhoudt and Schlesinger (2006) defined higher-order risk preferences in terms of simple gamble definitions. Chapter 1 in part II of this thesis employs these model-independent definitions in a systematic way to study higher-order risk preferences in models of reference-dependent preferences. This theoretical application is joint work with Johannes Maier from the University of Munich. We find that in the framework of Kőszegi and Rabin (2006, 2007), individuals exhibit even- but never odd-order risk attitudes. We further use these results to explain empirical patterns of seemingly sub-optimal behavior concerning precautionary saving and insurance demand, that have been described as puzzles in the literature.[2] In order to show that our results only obtain when expectations shape the reference point, we also analyze alternative models and find that they deliver other or even opposite patterns of higher-order risk preferences.

Kőszegi and Rabin (2009) extend their framework of endogenously defined reference-dependent preferences to the realm of anticipatory utility. They also provide a multi-attribute formulation of their model. However, in chapter 2 of part II of this thesis I will argue that the precise form of the model proposed suppresses the major advantages of multi-attribute utility. Kőszegi and Rabin's (2009) definition of reference-dependence restricts utility to be additively separable.

In chapter 2, I will outline an alternative formulation that follows the same basic intuitions as Kőszegi and Rabin (2009), but which is also applicable if utility is not additively separable in arguments. It is shown that for utility functions with a single dimension, the two models coincide. However, if the utility function has several dimensions, the two models differ,

[2]See, for instance, the overview article by Caroll and Kimball (2008) on empirical evidence on precautionary saving falling short of optimal levels in traditional models. See Gollier (2003) on an apparent excess of insurance demand in the presence of buffer stock saving.

even if the utility function is additive separable. Therefore, although my model has a larger domain of applicability, it is not a mere generalization of the previous model. Two examples are discussed to show that this difference with additive-separable utility applies to practically relevant situations. First, only in my model, simultaneous changes in different dimensions of consumption that would not result in an overall effect on pure consumption utility, do not have an effect on reference-dependent anticipatory utility. Second, an update in beliefs over the correlation of risks affecting different dimensions of pure consumption utility, generates a change in decision utility only in my model.

The in-depth engagement in the advancement of theoretical economic models raises a general methodological question regarding the final goal of this task. It is closely related to the fraction of individuals one seeks to model by a given model. Are we searching for a unifying theory to describe the vast majority of individuals in any setting? Or would we be satisfied with a tractable set of theories where each theory would describe a subset of individuals? For example, Harrison, Humphrey and Verschoor (2010) find in a series of experiments that about half of their subjects can be described relatively accurately by EUT, while the other half generally acts in accordance with prospect theory (PT).

If heterogeneity of individuals basic preference structure is accepted, we face the serious problem of how to construct methods of preference measurement that can be applied to the whole set of individuals. Statements, such as that one subject is more risk averse than another one, are difficult, if the applied method relies on assumptions specific to EUT or PT. General propositions on behavior, especially comparing individuals, become hard to obtain.

Even if we do not accept that individuals may be fundamentally different in their basic preference structure, we might face a similar problem. The constant advancement of theoretical models can make a method of measurement based on a specific formulation of a theory rapidly out-dated. Empirical assessments of preferences building on different specific models may be incomparable. Also, the challenge arises of how to deal with uncertainty of the modeler on which is the right model. Should he conduct a multitude of preference elicitations, each tailored to a different model?

A way out of both of these dilemmas is the development of model-independent tools. By these, I refer to methods that do not require the assumption of a specific model of behavior. Instead, they apply to a wide range of models that have been proposed. This does not mean that we make no assumptions at all. We always need some assumptions, otherwise we are not in the position to measure anything. However, the required assumptions are of a general nature and are shared by a wide range of models. If the method is based only on these general assumption, its results can be informatively applied in all these models. Part III of this thesis, which is based on joint work with Johannes Maier, contributes to this literature in the form of two experimental studies.

In the analysis of risk preferences, the most fundamental model-independent measure concerns risk aversion. A theoretical concept of risk aversion that does not rely on a specific model would be, for example, whether an individual prefers a sure payment of the expected value of a lottery over the lottery itself. There is a vast amount of experimental work that can be interpreted in this sense. Therefore, we consider the issues as settled with respect to this qualitative direction of risk aversion.

However, the measurement of the intensities of risk aversion is model-dependent for the prevailing experimental methods. This is despite the fact that there also exist well established model-independent definitions of an increase in risk.[3] Building on these, we can define an individual as more risk averse than another one if she is ready to accept more increases in risk as a trade-off to a certain reward.[4] This establishes a definition of being more risk averse that can be applied in any commonly proposed model. In chapter 3 in part III of this thesis I discuss our model-independent methodology to measure risk aversion in the laboratory. We implemented the method in an experiment jointly with the risk aversion elicitation method of Holt and Laury (2002). Nowadays, this is the risk aversion elicitation method most widely used in economic experiments, although it is based on specific assumptions of EUT. By implementing both methods simultaneously in our experiment, we can compare

[3]The most prominent definition of increasing risk is the concept of mean-preserving spreads by Rothschild and Stiglitz (1971) which will also be applied by us.

[4]Such a definition is closely related to the work of Diamond and Stiglitz (1974).

the results of both methods in a within-subject analysis. We show that the differences in the two methods are not only of a methodological nature, but also that they yield substantially different results.

Although the experimental evidence on risk aversion is vast, on higher-order risk preferences it is quite scarce. The few experimental studies that exist are very recent. Therefore, there are no established experimental methods to measure higher-order risk preferences. We provide a contribution to this emerging field of study that I discuss in chapter 4 in part III of this thesis. The experimental studies can be based on the model-independent definitions of higher-order risk preferences by Eeckhoudt and Schlesinger (2006). Therefore, they meet the criterion of model-independent measurement outlined above.

Deck and Schlesinger (2010) is so far the only published experiment on higher-order risk preferences. They find evidence for prudence (downside risk aversion), but at the same time against temperance (outer risk aversion) in their subject pool. Within EUT the commonly assumed parametric value functions (such as those of constant absolute or relative risk aversion) cannot exhibit this combination. In contrast, such a combination is compatible with proposed parameterizations of prospect theory. Deck and Schlesinger (2010) therefore interpret their findings as support of prospect theory.

A key feature of many non-EUT models including prospect theory is the distinction between gains and losses. In experiments, gains and losses are most often only differentiated by the framing of subjects decisions. This is mainly due to institutional constraints that prevent researchers to inflict real monetary losses on subjects. However, framing effects are elusive and considered by some economic methodologists as non-informative for economic questions. Therefore, we specifically designed our experiment to induce real monetary losses on subjects. To do so, we implemented a design that separates the experiment over two dates, several weeks apart. At the second date we were allowed to inflict real losses on the subjects, confined by their earnings of the first date.

We found that subjects generally are prudent and temperate. This general pattern holds for decisions in gains as well as in losses. Our findings of temperance are in contrast to the results of Deck and Schlesinger (2010) and therefore lead to diverging conclusions. Also, in our experiment, risk preferences of different orders are highly correlated.

In part IV of this thesis I provide a brief conclusion of the presented results. Also, an outlook is given on how the taken route of research can be traveled further in the direction of more general concepts of uncertainty than risk.

Part II

Theoretical Approaches Based on Reference Dependence

Chapter 1

Higher-Order Risk Preferences*

1.1 Introduction

While the well-known second-order effect of risk aversion describes a preference for less uncertainty, higher-order risk preferences characterize how this preference changes under different circumstances. For instance, the third-order effect of prudence plays an important role for changes in risk aversion due to variations in wealth, and the fourth-order effect of temperance is crucial for changes in risk aversion in the face of a background risk. The theoretical literature on these higher orders is well-developed (building on Kimball, 1990, 1992) since the importance of prudence for phenomena like precautionary saving was recognized early (Leland, 1968). While numerous empirical studies find supportive evidence for the presence of precautionary saving, they also show that it is usually too low to be consistent with those theoretical models and common assumptions about risk aversion.[1] Also, contrary to observation, optimal precautionary saving should theoretically lead to substantial crowding out of insurance demand (Gollier, 2003).

*This chapter is based on joint work with Johannes Maier. Both authors contributed equally to this work.

[1]For instance, the test of Dynan (1993, p1104) "...yields a fairly precise estimate of a small precautionary motive; in fact, the estimate is too small to be consistent with widely accepted beliefs about risk aversion." In an overview article Carroll and Kimball (2008, pp583-584) conclude that "...estimates of relative risk aversion imply precautionary saving motives much stronger than those that have been used empirically to match observed wealth holdings. This discrepancy remains unresolved."

Until now, theoretical models analyzing higher-order risk preferences were constrained to the framework of expected utility theory (EUT) in which they correspond to the derivatives of the utility function. In contrast, in order to resolve various empirical puzzles concerning second-order risk preferences, a prominent strand of literature (starting with Kahneman and Tversky, 1979) evolved that stresses the dependence of preferences on a reference point. The latest advancement in this literature was made by Kőszegi and Rabin (2006, 2007) who endogenize recent expectations as the reference point. We analyze higher-order risk preferences within their model of reference-dependent preferences. We further show that our results can explain seemingly sub-optimal levels of precautionary saving and insurance demand. As a robustness analysis, it is also shown that alternative models, like disappointment, regret, or exogenous reference points yield different results and cannot resolve these empirical puzzles concerning higher-order risk preferences.

Section 1.2 presents the two concepts that are merged in this chapter and gives the theoretical background that is needed in order to understand how we derive our results. In section 1.2.1 we follow Eeckhoudt and Schlesinger (2006) and define lotteries that represent higher-order risk preferences independently of a specific model. It is their gamble representation that allows us to analyze higher orders in the model of Kőszegi and Rabin (2007) which is presented in section 1.2.2. In order to make our results comprehensible as well as comparable to each other, we transform choices such that they reflect preference relations as if the reference point was zero. This method of transformation is explained in section 1.2.3.

Our results are presented in section 1.3. We show that individuals with expectation-based reference-dependent preferences are neither prudent nor imprudent while second- and fourth-order effects are still present. More specifically, using common functional forms of gain-loss utility we show that individuals are risk-averse and intemperate, but they are indifferent toward the third order *independent* of the functional form of gain-loss utility. While risk aversion is also predicted by classical EUT models, third- and fourth-order behavior differs substantially under reference dependence. We generalize these results for risk preferences of arbitrary order n and show that those of odd orders (except order one) are absent while those of even orders are still present. The intuition for this result is based on the fact that odd-order risk preferences can always be thought of as a decision on the location of risk(s), whereas even-order risk preferences are concerned with the aggregation of risk(s). Due to the anticipation of choices, the location does not matter for sensations of gains and losses.

In section 1.4 some of the possible economic implications of our results are discussed. In general our results show that it may be highly problematic to derive optimal behavior that rests on preferences of higher orders from measures of risk aversion. With reference-dependent preferences combinations of certain risk attitudes are possible that cannot be consistently derived within EUT. More specifically, we show that empirical findings on seemingly sub-optimal amounts of precautionary saving under classical EUT can be attributed to the optimal behavior of individuals with expectation-based reference-dependent preferences. The reason is that reference dependence contributes positively to risk aversion, but leaves prudence unaffected. Within classical EUT models optimal precautionary saving is directly derived from measures of risk aversion. However, if individuals have reference-dependent preferences instead, measured risk aversion will indeed be higher but not the optimal amount of precautionary saving. Also, the theoretically puzzling existence of costly insurance when capital markets are well developed and individuals can accumulate buffer-stock wealth is less surprising for such individuals. Under reference dependence second-order effects become relatively more important than third-order effects.

In section 1.5 we consider several other behavioral models in order to show that the same results do not obtain under alternative specifications of the reference point. Section 1.5.1 considers disappointment models where the reference point is the expectational mean of the chosen alternative (Bell, 1985; Loomes and Sugden, 1986). Here, we do not observe the absence of odd-order risk preferences while even-order risk preferences are similar to those of expectation-based reference dependence. In recent empirical work it has been difficult to distinguish between these two models of reference dependence. Our third-order result therefore suggests a new way how to differentiate between expectation-based reference dependence and models of disappointment. In section 1.5.2 we analyze higher orders in models of regret (Bell, 1982, 1983; Loomes and Sugden, 1982) where the reference point is the alternative that was *not* chosen. It is shown that decisions taken to avoid regret are unaffected by even-order effects, but odd-order effects still exist. This suggests that regret influences higher-order risk preferences in the opposite way as expectation-based reference dependence. Risk preferences of any order (except order one) differ between these two models, but regret induces similar odd-order risk preferences as disappointment models. Models with reference points shaped by expectations and regret are usually analyzed separately rather than jointly in a unified framework. However, our analysis shows the close relationship they share. Risk preferences of a particular order are only affected by either expectations or regret as the reference point, but never by both. Lastly, we analyze exogenous reference points, such as the status quo, in section 1.5.3. Here, results vary with the various reference points considered and attitudes concerning gains and losses. However, just as in the case of disappointment and regret, exogenous reference point do not yield the same results as expectation-based reference dependence on higher-order risk preferences. Our robustness analysis of section 1.5 therefore shows that alternative specifications of the reference point imply different results and cannot explain empirical puzzles that have been found with respect to higher orders.

Section 1.6 is devoted to the conclusion where we summarize our results and discuss further extensions. All proofs appear in the appendix of section 1.7.

1.2 Theoretical Background

1.2.1 Higher Orders

Within EUT it has long been recognized that risk preferences are not exhaustively described via the concept of risk aversion alone. Already Leland (1968) and Sandmo (1970) identified the importance of a positive third derivative of the utility function for motives of precautionary saving. Since Kimball (1990) the feature of preferences that is necessary and sufficient for such precautionary behavior is termed prudence.[2] An equivalent concept to prudence, termed downside risk aversion, was established by Menezes, Geiss and Tressler (1980). It describes an aversion to mean-variance preserving transformations that shift risk from the right to the left of a wealth distribution. Afterward, an extensive literature developed that discussed also higher derivatives of utility functions and their implications for economically important decisions under uncertainty. For instance, negative fourth derivatives of the utility function were shown to be crucial for risk aversion to increase with the existence of a zero-mean background risk (see Gollier and Pratt, 1996; or Eeckhoudt, Gollier and Schlesinger, 1996). This fourth-order property of the utility function was termed temperance by Kimball (1992) and outer risk aversion by Menezes and Wang (2005). Eeckhoudt and Schlesinger (2008) show that temperance is necessary and sufficient for precautionary saving to increase as the *downside* risk of future income rises.

Outside standard EUT, preferences of higher orders have not been considered explicitly in the literature yet. Building on the concept of n^{th}-degree risk by Ekern (1980), Eeckhoudt and Schlesinger (2006), however, provided definitions of all higher-order effects in terms of preferences over simple lotteries.[3] This representation is quite general since it is independent of a specific model. The authors showed that when applied in an EUT framework their definitions correspond to the common definitions in terms of derivatives used so far. Because we are interested in higher-order risk preferences under reference dependence, we employ these gamble definitions in our analysis.

Denote initial wealth by $y \in \mathbb{R}$.[4] Let $k \in \mathbb{R}$ be a sure reduction in wealth with $k > 0$. $\tilde{\varepsilon}_i$ are symmetric random variables which are non-degenerate, independent of all other random

[2]Despite being not necessary *and* sufficient, prudence has also been shown to play an important role in other economic applications, like for instance rent-seeking games (Treich, 2010), auctions (Eső and White, 2004), bargaining (White, 2008), principal-agent monitoring (Fagart and Sinclair-Desagné, 2007), global commons problems (Bramoullé and Treich, 2009), inventory management (Eeckhoudt, Gollier and Schlesinger, 1995), or optimal prevention (Eeckhoudt and Gollier, 2005; or Courbage and Rey, 2006). It has also been used in a more normative context to derive an optimal ecological discount rate (Gollier, 2010) or to justify the so-called precautionary principle (Gollier, Jullien and Treich, 2000; and Gollier and Treich, 2003).

[3]These lotteries have also been used in recent experiments to investigate higher-order risk preferences empirically (see Deck and Schlesinger, 2010; Ebert and Wiesen, 2010; or chapter 4 of this thesis.

[4]The non-randomness of initial wealth is only used for simplicity.

variables that affect wealth, and have $\mathbb{E}[\tilde{\varepsilon}_i] = 0$.[5] Now, we define the following standard gambles, where each element denotes an outcome and each outcome of a specific gamble is realized with equal probability.[6] Such defined outcomes can be either deterministic or stochastic themselves.

$$\begin{aligned}
B_1 &\equiv [y] & A_1 &\equiv [y-k] \\
B_2 &\equiv [y] & A_2 &\equiv [y+\tilde{\varepsilon}_1] \\
B_3 &\equiv [y-k; y+\tilde{\varepsilon}_1] & A_3 &\equiv [y; y-k+\tilde{\varepsilon}_1] \\
&= [A_1; B_1+\tilde{\varepsilon}_1] & &= [B_1; A_1+\tilde{\varepsilon}_1] \\
B_4 &\equiv [y+\tilde{\varepsilon}_1; y+\tilde{\varepsilon}_2] & A_4 &\equiv [y; y+\tilde{\varepsilon}_1+\tilde{\varepsilon}_2] \\
&= [A_2; B_2+\tilde{\varepsilon}_2] & &= [B_2; A_2+\tilde{\varepsilon}_2] \\
B_n &\equiv [A_{n-2}; B_{n-2}+\tilde{\varepsilon}_{\text{int}(\frac{n}{2})}] & A_n &\equiv [B_{n-2}; A_{n-2}+\tilde{\varepsilon}_{\text{int}(\frac{n}{2})}].
\end{aligned}$$

Intuitively, B_1 vs. A_1 is equivalent to the question whether more is preferred to less. B_2 vs. A_2 is the comparison between a certain level of wealth and a risky alternative with an identical mean. B_3 vs. A_3 is equivalent to the question whether one prefers to add an unavoidable random variable to the higher or lower outcome. It is also equivalent to the question whether one prefers to accept a sure reduction in wealth in the certain or uncertain state. B_4 vs. A_4 can be seen as the question whether one prefers to aggregate or disaggregate two independent random variables.

Following Eeckhoudt and Schlesinger (2006), preferences are then

$$\begin{array}{llll}
\text{monotone} & \Leftrightarrow \; B_1 \succsim A_1 & \forall y, k & (\Leftrightarrow u'(x) \geq 0 \text{ in EUT models}) \\
\text{risk-averse} & \Leftrightarrow \; B_2 \succsim A_2 & \forall y, \tilde{\varepsilon}_1 & (\Leftrightarrow u''(x) \leq 0 \text{ in EUT models}) \\
\text{prudent} & \Leftrightarrow \; B_3 \succsim A_3 & \forall y, \tilde{\varepsilon}_1, k & (\Leftrightarrow u'''(x) \geq 0 \text{ in EUT models}) \\
\text{temperate} & \Leftrightarrow \; B_4 \succsim A_4 & \forall y, \tilde{\varepsilon}_1, \tilde{\varepsilon}_2 & (\Leftrightarrow u''''(x) \leq 0 \text{ in EUT models})
\end{array}$$

[5]Except the assumption of symmetry, all $\tilde{\varepsilon}_i$ are identically defined in Eeckhoudt and Schlesinger (2006).

[6]We defined all gambles having a mean of y if $k = 0$ instead of 0 as in Eeckhoudt and Schlesinger (2006).

or, more generally

$$\text{risk apportioning of order } n \;\Leftrightarrow\; B_n \succsim A_n \quad \forall y, \tilde{\varepsilon}_1, \ldots, \tilde{\varepsilon}_{\mathrm{int}(\frac{n}{2})}, k$$
$$\big(\Leftrightarrow u^n(x) \leq (\geq)\ 0 \text{ if } \tfrac{n}{2} \in (\notin)\ \mathbb{N} \text{ in EUT models}\big)$$

where $u^n(\cdot)$ denotes the n^{th} derivative of $u(\cdot)$. Note that all commonly used utility functions in EUT, like for instance those exhibiting constant relative or absolute risk aversion, have derivatives that alternate in signs with $u'(\cdot)$ being positive (see Brockett and Golden, 1987), a property termed mixed risk aversion by Caballé and Pomansky (1996).

1.2.2 Reference Dependence

Reference-dependent preferences were established in economics via Kahneman and Tversky's (1979) prospect theory. It introduced four novel aspects: reference dependence, loss aversion, diminishing sensitivity concerning gains and losses and non-linear probability weighting. Our main focus is on the first feature. Some results will be completely independent of whether the second and third features are present while other results will vary with their existence. The fourth feature of probability weighting will be absent from the whole analysis. In this respect we follow the models we discuss in this chapter.

Focusing on reference dependence immediately leads to the question of what the reference point is. In the traditional literature on prospect theory the reference point is a real numbered variable that is completely exogenous. It is a simple point which every potential outcome is compared to, and it is not influenced by the decision-maker in any way. Often, the status quo was advocated as a candidate for this reference point (e.g. Kahneman and Tversky, 1979; or Tversky and Kahneman, 1991). However, even this early literature noted that this might not be the relevant comparison in many important situations. "For example, an unexpected tax withdrawal from a monthly pay check is experienced as a loss, not as a reduced gain." (Kahneman and Tversky, 1979, p286). Later, considerations that expectations may play a role were explicitly included into formal analysis (see Bell, 1985; Loomes and Sugden, 1986;

Sugden, 2003; Delquié and Cillo, 2006; and Kőszegi and Rabin, 2006, 2007, 2009).

In early models of disappointment (see Bell, 1985; and Loomes and Sugden, 1986) individuals compare potential outcomes to the mean of the choice. Every option that can be chosen has therefore its specific reference point which is thus endogenous. An advancement of disappointment models has been proposed by Kőszegi and Rabin (2006, 2007). Here, the reference point is not simply the mean but rather the full distribution of outcomes resulting from the chosen alternative. We refer to such reference points as expectational references. This specification is motivated by the observation that individuals can make multiple comparisons to evaluate an outcome. It also accounts for the fact that individuals realize uncertainties and incorporate them into their reference point.

Expectational references have since been applied in order to explain various important phenomena. For instance, Heidhues and Kőszegi (2008) use them in a model of price competition to explain sticky prices. Herweg (2010) shows that the flat-rate bias when choosing optimal tariffs can be explained when consumers have expectational references. Heidhues and Kőszegi (2010) use expectational references in order to explain why "sale" prices in addition to regular prices only exist in certain environments. Herweg, Müller and Weinschenk (2010) analyze a moral hazard setting with expectational references in order to explain binary payment schemes. Closely related, Macera (2010) shows in a two-period model that contracts deferring all present incentives into future payments, like e.g. yearly productivity bonuses combined with a present fixed wage, are optimal with expectational references. Lange and Ratan (2010) study bidding behavior of agents with expectational references in different auction formats and explain why in laboratory experiments but not necessarily in the field overbidding should be observed.

Empirical work also points toward expectations as the relevant reference point. Meng (2009) uses expectations as the reference point in order to explain the disposition effect, i.e. the observation that stock market investors tend to keep their loosing assets for too long and sell their winning ones too soon, and further finds expectations as best estimate of investors' reference point from individual trading data. Card and Dahl (forthcoming) show that the rate of family violence when the local professional football team loses depends on the extent to which losing the game was expected. Mas (2006) shows that the larger the difference is between requested (by the union) and received wages the more police performance declines. Pope and Schweitzer (forthcoming) show that expectation-based loss aversion is present among professional golfers suggesting that it is a bias which survives experience, competition and large stakes. Post et al. (2008) analyze game show data and find that behavior of contestants is consistent with reference points shaped by expectations. Crawford and Meng (forthcoming) (building upon previous work by Camerer et al. 1997; and Farber, 2005, 2008) propose and estimate a labor supply model of New York City cabdrivers with reference-dependent preferences where income- as well as working hour-targets are both determined by rational expectations. Using new data of New York City cabdrivers and using a natural experiment Doran (2009) finds that a permanent wage increase causes hours worked to remain constant, a finding consistent with expectations as the reference point.

Labor supply decisions have also been tested in a laboratory experiment by Abeler et al. (forthcoming). They manipulate subjects' rational expectations and find that effort provision is affected in the way predicted by expectation-based reference-dependent preferences. Ericson and Fuster (2010) show in an exchange and in a valuation experiment that the reference point is determined by expectations rather than by the status quo and discuss why some researchers have not found endowment effects in different settings. Knetsch and Wong (2009) also conduct exchange experiments and suggest that expectations shape the reference point. A recent experiment by Gill and Prowse (forthcoming) finds that expectations are the relevant reference point when subjects compete in a real effort tournament. Finally, Loomes and Sugden (1987), Choi et al. (2007), or Hack and Lammers (2009) find supportive evidence for expectations as the reference point in risky choice experiments.

All the above mentioned literature is concerned with first- or second-order effects of reference-dependent preferences. In this chapter we are rather interested in higher-order effects. To our knowledge there has been no attempt to consider higher orders under reference dependence. We follow Kőszegi and Rabin (2007) and define overall decision utility $V(\cdot)$ as

$$\begin{aligned} V(F|G) &= M(F) + U(F|G) \\ &= \int m(x)dF(x) + \iint u(x|r)dG(r)dF(x). \end{aligned} \tag{1.1}$$

The outcome $x \in \mathbb{R}$ and the reference point $r \in \mathbb{R}$ are independently drawn from the probability distributions F and G, respectively. Pure consumption utility depending solely on the outcome x is denoted by $m(x)$ and thus the expected pure consumption utility is $M(F) = \int m(x)dF(x)$. The function

$$u(x|r) = u(m(x) - m(r))$$

captures sentiments of gains and losses that occur if the outcome x is realized and the reference point was r. The expected level of gain-loss utility is denoted by $U(F|G) = \iint u(x|r)dG(r)dF(x)$. The assumption of independently distributed x and r captures the notion that the evaluation of all possible wealth outcomes is based on comparing each of them to all possibilities in the support of the reference lottery. What we refer to as expectational references (Kőszegi and Rabin, 2006, 2007, 2009) equates the reference and the outcome distribution and therefore $G = F$.[7]

Some of our results in the following section hold independently of the functional form of gain-loss utility $u(\cdot)$. The remaining results depend on its specific shape. For the latter cases we will consider u functions that are usually proposed in the literature. This includes standard (weakly) concave as well as loss-averse S-shaped u functions. S-shaped u functions are defined as being (weakly) concave in gains and (weakly) convex in losses (i.e. $u''(\tau) \leq 0 \ \forall \tau > 0$ and $u''(\tau) \geq 0 \ \forall \tau < 0$). Following Köbberling and Wakker (2005) we refer to loss aversion as a kink at the reference point zero (i.e. $\lim_{\tau \to 0} \frac{u'(-|\tau|)}{u'(|\tau|)} \equiv \lambda > 1$). Furthermore, when S-shaped utility is assumed we refer to loss-averse u functions as the standard case where

[7] Kőszegi and Rabin (2006, 2007, 2009) further develop an equilibrium concept that restricts which choices can be rationally expected. We will discuss this at the end of section 1.3.

utility is diminishing faster or equally in gains than in losses (i.e. $u''(\tau) \leq -u''(-\tau)\ \forall \tau > 0$). This includes the case where $u(\cdot)$ is piecewise linear with loss aversion. Often results differ for strict formulations of the functional characteristics above and the limiting cases. A limiting case of S-shaped u functions without loss aversion are symmetric S-shaped u functions. A linear $u(\cdot)$ is a limiting case of both weakly concave and S-shaped functions without loss aversion. Both limiting cases have in common that they are (two-fold rotational) symmetric around the reference point. This property is crucial in explaining why their results differ compared to their strict (and asymmetric) counterparts.[8] Note that both strictly concave and loss-averse S-shaped u functions share the common feature that losses loom larger than corresponding gains (i.e. $u'(\tau) < u'(-\tau)\ \forall \tau > 0$).[9] Throughout it is assumed that $u(\tau)$ is strictly increasing, continuous and sufficiently often differentiable for all τ ($\forall \tau \neq 0$ if $\lambda > 1$), and that $u(0) = 0$.

While we consider various general functional forms of $u(\cdot)$, we again follow Kőszegi and Rabin (2007) in that we restrict $m(\cdot)$ to be linear (i.e. $m'(\cdot) > 0, m''(\cdot) = 0$).[10] The reason for this assumption is twofold. First, in models where $m(\cdot)$ is assumed to be concave instead usually strong restrictions on the functional form of $u(\cdot)$ are needed to keep the model tractable.[11] Since we are interested in the effect of reference dependence on higher-order risk

[8]A loss-averse S-shaped u function is the typical assumption in the literature on reference dependence. The reason why we additionally discuss concave as well as (two-fold rotational) symmetric u functions is the following. In section 1.5 we consider alternative models, like disappointment or regret theory, in order to show that the results on expectational references cannot be derived using such alternative models. These alternative models can also be captured by (1.1) but with a different specification concerning the reference point. A common assumption in the regret literature is a concave u function. In their disappointment theories, Loomes and Sugden (1986) assume a u function that is convex in gains and concave in losses but has the property of being (two-fold rotational) symmetric, while Bell (1985) assumes a piecewise linear $u(\cdot)$. Since there exists no a priori reason to assume that a specific functional form is limited to a specific reference scenario we rather consider all possibilities throughout.

[9]While evidence for this feature in the literature on reference dependence is vast, for evidence that regret looms larger than rejoicing, see e.g. Inman, Dyer and Jia (1997) and Camille et al. (2004).

[10]With the exception of their proposition 9, this assumption is made throughout by Kőszegi and Rabin (2007) and for some results in Kőszegi and Rabin (2006). Other more applied models in the literature impose this assumption as well (see e.g. Heidhues and Kőszegi, 2010; Lange and Ratan, 2010; Meng, 2009; or Gill and Prowse, forthcoming). In the context of regret, Bleichrodt, Cillo and Diecidue (2010) show in an experiment that $m(\cdot)$ is *not* significantly different from linear. In the disappointment models of Bell (1985) and Loomes and Sugden (1986) $m(\cdot)$ is assumed to be linear as well.

[11]These models restrict attention to piecewise linear gain-loss utility (see Heidhues and Kőszegi, 2008, 2010; Crawford and Meng, forthcoming; Meng, 2009; Herweg, Müller and Weinschenk, 2010; Herweg, 2010; Lange and Ratan, 2010; Macera, 2010; and some results in Kőszegi and Rabin, 2006, 2007).

attitudes, we rather consider different functional forms that directly affect gain-loss utility $u(\cdot)$. Second and more importantly, higher orders are already well analyzed within classical EUT models. Assuming a concave m function would simply reintroduce higher-order effects of EUT into the gain-loss function and would therefore dilute the pure effect of reference dependence on higher-order risk attitudes.[12] Moreover, reference dependence is especially considered as important in situations where classical consumption utility is almost linear. In order to generate reasonable risk taking behavior, Rabin (2000) shows that consumption utility is compatible with risk neutrality for amounts as high as a monthly salary.

1.2.3 Higher Orders under Reference Dependence

For notational convenience we will use $F|G \succsim F'|G'$ interchangeably to $V(F|G) \geq V(F'|G')$ throughout the chapter. Since we are interested in higher-order risk preferences under reference dependence, we apply the gamble definitions of section 1.2.1 to the outlined model (1.1) in the following way.

Definition 1 *A reference-dependent individual is said to be risk apportioning of order n (its opposite, neither nor) if and only if $B_n|G \succ (\prec, \sim)\ A_n|G'$, or synonymously if and only if $V(B_n|G) > (<, =)\ V(A_n|G')$, for all $y, k, \tilde{\varepsilon}_i$.*

Expectational references equate the reference distribution to the distribution of the chosen alternative, and we can therefore further specify the reference point.

Definition 2 *If the reference point is determined by expectational references, $G = B_n$ and $G' = A_n$.*

Because we only have to consider pairwise choices in our analysis, it is often useful to transform the comparison of $B_n|G$ vs. $A_n|G'$ into a comparison that is easier to comprehend but is behaviorally equivalent to the original comparison. We will systematically use two transformations that simplify comparisons. First, we will transform every gamble given a

[12] For instance, Macera (2010) needs to restrict consumption utility to be not too concave such that the reference-dependent effect still dominates the consumption-utility effect.

specific reference point distribution into an equivalent gamble with a hypothetical reference point of 0. Second, we will reduce both gambles that are compared to the components in which these two gambles differ. Both transformations make it much easier to derive behavioral predictions in cases where our results depend on the gain-loss utility u, since its functional characteristics are formulated in relation to the reference point 0. They further ensure that our results are comparable to each other.

For an illustration consider a situation in which either B_2 or A_2 has to be chosen. In this hypothetical situation the assumed reference point distribution is $[y + \tilde{\varepsilon}_1' + \tilde{\varepsilon}_2']$ if B_2 is chosen and the assumed reference point distribution is $[y; y + \tilde{\varepsilon}_2']$ if A_2 is chosen.[13] Under which conditions in the original comparison, which is

$$\begin{aligned} & B_2|[y + \tilde{\varepsilon}_1' + \tilde{\varepsilon}_2'] && \succ (\prec, \sim) && A_2|[y; y + \tilde{\varepsilon}_2'] \\ \Leftrightarrow \quad & [y]|[y + \tilde{\varepsilon}_1' + \tilde{\varepsilon}_2'] && \succ (\prec, \sim) && [y + \tilde{\varepsilon}_1]|[y; y + \tilde{\varepsilon}_2'], \end{aligned} \tag{1.2}$$

a certain preference relation holds may be hard to conceive intuitively. However, we can restate (1.2) by an equivalent comparison in which the reference point of both alternatives is $[0]$. This makes an intuitive assessment possible. Using the fact that $\tilde{\varepsilon}_i$ is symmetric we can say that

$$M([y]) + U([\tilde{\varepsilon}_1' + \tilde{\varepsilon}_2']|[0]) \quad > (<, =) \quad M([y + \tilde{\varepsilon}_1]) + \tfrac{1}{2}U([\tilde{\varepsilon}_1]|[0]) + \tfrac{1}{2}U([\tilde{\varepsilon}_1 + \tilde{\varepsilon}_2']|[0]) \tag{1.3}$$

is equivalent to (1.2). Since $m(\cdot)$ is linear, $\mathbb{E}[\tilde{\varepsilon}_i] = 0$, and hence $M([y]) = M([y + \tilde{\varepsilon}_1])$ and $M([\tilde{\varepsilon}_1]) = M([\tilde{\varepsilon}_1 + \tilde{\varepsilon}_2])$, (1.3) in turn is equivalent to

[13] $\tilde{\varepsilon}_i'$ and $\tilde{\varepsilon}_i$ are identically and independently distributed random variables. Note that this is a direct implication of the assumption that reference and outcome distributions are independent. We denote all random variables stemming from the reference point with a $'$ to clarify the fact that $\tilde{\varepsilon}_i + \tilde{\varepsilon}_i'$ denotes the convolution of two random variables rather than the sum of outcomes.

$$\begin{aligned} M([\tilde{\varepsilon}_1' + \tilde{\varepsilon}_2']) + U([\tilde{\varepsilon}_1' + \tilde{\varepsilon}_2']|[0]) \quad &> (<,=) \quad \tfrac{1}{2}M([\tilde{\varepsilon}_1]) + \tfrac{1}{2}U([\tilde{\varepsilon}_1]|[0]) \\ &\qquad + \tfrac{1}{2}M([\tilde{\varepsilon}_1 + \tilde{\varepsilon}_2']) + \tfrac{1}{2}U([\tilde{\varepsilon}_1 + \tilde{\varepsilon}_2']|[0]) \qquad (1.4) \\ \Leftrightarrow \quad [\tilde{\varepsilon}_1' + \tilde{\varepsilon}_2']|[0] \quad &\succ (\prec,\sim) \quad [\tilde{\varepsilon}_1; \tilde{\varepsilon}_1 + \tilde{\varepsilon}_2']|[0]. \end{aligned}$$

Although this comparison is already easier to conceive than the original comparison, we can further simplify (1.4) by eliminating components that are common in both gambles and can therefore not contribute to a preference of one gamble over the other, so that

$$\begin{aligned} M([\tilde{\varepsilon}_1' + \tilde{\varepsilon}_2']) + U([\tilde{\varepsilon}_1' + \tilde{\varepsilon}_2']|[0]) \quad &> (<,=) \quad M([\tilde{\varepsilon}_1]) + U([\tilde{\varepsilon}_1]|[0]) \\ \Leftrightarrow \quad [\tilde{\varepsilon}_1' + \tilde{\varepsilon}_2']|[0] \quad &\succ (\prec,\sim) \quad [\tilde{\varepsilon}_1]|[0]. \end{aligned}$$

Clearly, the question whether an individual prefers the convolution of $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ over $\tilde{\varepsilon}_1$ given the choice-independent reference point of 0 is more intuitive than the original comparison in (1.2). For most commonly used functional forms of $u(\cdot)$ an answer could be derived easier when the problem is stated in such residual gambles, whose preference relation is nevertheless equivalent to that of the original gambles.

We denote these residual gambles by $[B_n|G\langle A_n|G'\rangle]\,|[0]$ and $[A_n|G'\langle B_n|G\rangle]\,|[0]$, respectively, and apply them to the comparisons of B_n and A_n for all $n \geq 2$, where $M(B_n) = M(A_n)$ and thus only the reference-dependent part differs. Generally, $[B_n|G\langle A_n|G'\rangle]\,|[0]$ contains all components that occur in $B_n|G$ with higher probability than in $A_n|G'$. Similarly, $[A_n|G'\langle B_n|G\rangle]\,|[0]$ only contains those components that have a higher probability in $A_n|G'$ than in $B_n|G$. We derive the densities of the residual gambles by applying the following procedure. The probability density function of $[B_n|G\langle A_n|G'\rangle]\,|[0]$ is $f_{BA}(x) = \max\{[f_B(x) - f_A(x)]/f; 0\}$ where $f_B(x)$ is the density of $B_n|G$, $f_A(x)$ is the density of $A_n|G'$, and $f = \int \max\{f_B(x) - f_A(x); 0\}dx$. Similarly, the density of $[A_n|G'\langle B_n|G\rangle]|[0]$ is $f_{AB}(x) = \max\{[f_A(x) - f_B(x)]/f; 0\}$. These transformations ensure that the probability mass of the residual gambles is one without changing relative densities. While such definitions may seem notationally cumbersome they precisely indicate how a residual gamble was derived. A special case arises if both gambles contain the same components with identical probabilities. The relation in the original comparison is then equivalent to the comparison of any arbitrary gamble C with itself, formally $C|[0]$ is compared to $C|[0]$. Any such situation is then characterized by indifference without any further assumptions concerning the functional form of $u(\cdot)$.

1.3 Results

Our first result states that preferences are monotone. Note that the component of gain-loss utility does not influence this preference in any way, so the result is solely driven by pure consumption utility. Intuitively, in risk-less decisions, no feelings of gain or loss can arise when the outcome was expected.

Proposition 1 *If preferences depend on reference points formed by expectations, individuals have monotone preferences, that is* $B_1|B_1 \succ A_1|A_1$.

In contrast to proposition 1, our results for all orders higher than one will be driven solely by the gain-loss utility component of overall decision utility (as formally expressed in lemma 1 in the appendix). Intuitively, because of the linearity of $m(\cdot)$ the expectational mean of two options is all that matters in terms of pure consumption utility, and it is easily verified that $\mathbb{E}[B_n] = \mathbb{E}[A_n]$ for all $n \geq 2$ holds.

Our second result states the sufficient and necessary condition for second-order risk preferences. It replicates findings of the existing literature.

Proposition 2 *If preferences depend on reference points formed by expectations, individuals are risk-averse (risk-seeking, risk-neutral), that is* $B_2|B_2 \succ (\prec, \sim)\ A_2|A_2$, *if and only if* $[0]|[0] \succ (\prec, \sim)\ [\tilde{\varepsilon}_1 + \tilde{\varepsilon}_1']|[0]$.

Proposition 2 specifies risk aversion in a framework of expectational references. Since in the comparison of $B_2|B_2$ vs. $A_2|A_2$ densities of similar elements can be reduced as described in the previous section, we can derive the last equivalence. Thus, $[B_2|B_{2\langle A_2|A_2\rangle}]|[0]$ equals $[0]|[0]$ and $[A_2|A_{2\langle B_2|B_2\rangle}|[0]$ is equal to $[\tilde{\varepsilon}_1 + \tilde{\varepsilon}_1']|[0]$. Our result resembles the classical definition of risk aversion if the choice is transformed such that the hypothetical reference point is $[0]$. An individual who dislikes the convolution of any symmetric and zero-mean $\tilde{\varepsilon}_1$ is classified as being risk-averse.

As an illustrative example, suppose $\tilde{\varepsilon}_1 = [-\varepsilon_1; \varepsilon_1]$ with $\varepsilon_1 > 0$. Then, if an individual chooses the gamble B_2 and this is also his expectation, the choice would result in the utility of $M([y]) + U([0]|[0])$ since the deterministic outcome of y is always correctly anticipated. However, if this individual chooses the gamble A_2 and this is also his expectation, four combinations of outcomes and reference points will be considered. With probability 1/2 the outcome is $y + \varepsilon_1$. But only in half of these cases this outcome was also anticipated and reduces to 0. In the other half of these cases $y - \varepsilon_1$ was expected instead, which yields the sensation of a gain of $2\varepsilon_1$. Also, with probability 1/2 the outcome is $y - \varepsilon_1$. In half of these cases this bad outcome was anticipated and reduces to 0. In the other half of these cases the good outcome $y + \varepsilon_1$ was anticipated, yielding the sensation of a loss of $2\varepsilon_1$. Thus, the choice of A_2 would deliver a utility of $M([y-\varepsilon_1; y+\varepsilon_1]) + U([0; 0; -2\varepsilon_1; 2\varepsilon_1]|[0])$. Since for a linear m function it follows that $M([y]) = M([y-\varepsilon_1; y+\varepsilon_1])$, this individual prefers B_2 over A_2 if and only if $u(0) > \frac{1}{2}u(0) + \frac{1}{4}u(-2\varepsilon_1) + \frac{1}{4}u(2\varepsilon_1)$. This can be further simplified to the condition $u(0) > \frac{1}{2}u(-2\varepsilon_1) + \frac{1}{2}u(2\varepsilon_1)$. In gamble notation, $B_2|B_2 = [0]|[0] \succ [0; 0; -2\varepsilon_1; 2\varepsilon_1]|[0] = A_2|A_2$ reduces to $[B_2|B_{2\langle A_2|A_2\rangle}]|[0] = [0]|[0] \succ [2\tilde{\varepsilon}_1]|[0] = [A_2|A_{2\langle B_2|B_2\rangle}]|[0]$.

It directly follows from proposition 2 that for u functions which are usually proposed in the literature individuals are risk-averse.

Corollary 1 *If preferences depend on reference points formed by expectations, individuals with*

(i) $u''(\cdot) < (>, =)\ 0$ *over the whole range are risk-averse (risk-seeking, risk-neutral);*

(ii) loss-averse S-shaped $u(\cdot)$ *are risk-averse;*

(iii) $u(\cdot)$ *being (two-fold rotational) symmetric around the reference point are risk-neutral.*

The next result is concerned with third-order risk preferences.

Proposition 3 *If preferences depend on reference points formed by expectations, individuals are never prudent or imprudent, that is* $B_3|B_3 \sim A_3|A_3$*. This holds for all formulations of* $u(\cdot)$*.*

Proposition 3 constitutes a crucial result of our analysis. Since $B_3|B_3$ always contains the same elements with identical probabilities as $A_3|A_3$, both choices are equally valued regardless of the shape of $u(\cdot)$. Therefore, reference-dependent utility does not contribute to third-order risk preferences despite its effect on the second order.

For an illustration consider again an example where $\tilde{\varepsilon}_1 = [-\varepsilon_1; \varepsilon_1]$. If the choice of an individual is B_3 and he was expecting this choice, his utility will be determined by comparing every possible outcome to every possible reference point. Thus, $U(B_3|B_3) = \frac{3}{8}u(0) + \frac{1}{8}u(-k-\varepsilon_1) + \frac{1}{8}u(-k+\varepsilon_1) + \frac{1}{8}u(k-\varepsilon_1) + \frac{1}{8}u(k+\varepsilon_1) + \frac{1}{16}u(-2\varepsilon_1) + \frac{1}{16}u(2\varepsilon_1)$. However, if the individual chooses A_3 and he was expecting this to be his choice his utility will be identical to the expression above and hence $U(B_3|B_3) = U(A_3|A_3)$. Since also $M(B_3) = M(A_3)$ holds, we can conclude that $V(B_3|B_3) = V(A_3|A_3)$ for all functional forms of $u(\cdot)$.

The result of proposition 3 is in stark contrast to the usual properties of models in EUT. As an example, functional forms exhibiting constant or decreasing absolute risk aversion imply prudence in EUT. Within EUT only rarely used utility functions, like those with quadratic utility, never exhibit prudence or imprudence. In contrast, assuming dependence on expectational references eliminates any third-order risk preference. Since this holds for arbitrary u functions, proposition 3 has very general implications that are discussed in section 1.4. We will show that under reference dependence precautionary saving is lower and insurance demand higher than in a classical EUT model of pure consumption utility.

In the next proposition we consider fourth-order risk preferences.

Proposition 4 *If preferences depend on reference points formed by expectations, individuals are temperate (intemperate, neither nor), that is* $B_4|B_4 \succ (\prec, \sim)\ A_4|A_4$*, if and only if* $[\tilde{\varepsilon}_1 + \tilde{\varepsilon}_1'; \tilde{\varepsilon}_2 + \tilde{\varepsilon}_2']|[0] \succ (\prec, \sim)\ [0; \tilde{\varepsilon}_1 + \tilde{\varepsilon}_1' + \tilde{\varepsilon}_2 + \tilde{\varepsilon}_2']|[0]$*.*

Proposition 4 specifies temperance when preferences depend on expectational references. It is similar to proposition 2 in that it resembles the general definition of temperance in case the choice is transformed such that the reference point is [0]. Since densities of identical elements in $B_4|B_4$ and $A_4|A_4$ can be reduced in comparison, $[B_4|B_{4\langle A_4|A_4\rangle}]|[0]$ equals $[\tilde{\varepsilon}_1+\tilde{\varepsilon}_1';\tilde{\varepsilon}_2+\tilde{\varepsilon}_2']|[0]$ and $[A_4|A_{4\langle B_4|B_4\rangle}]|[0]$ is equal to $[0;\tilde{\varepsilon}_1+\tilde{\varepsilon}_1'+\tilde{\varepsilon}_2+\tilde{\varepsilon}_2']|[0]$.

Unlike in propositions 2 and 3, fourth-order risk preferences differ for various shapes of $u(\cdot)$ that have been proposed in the literature.

Corollary 2 *If preferences depend on reference points formed by expectations, individuals with*

(i) $u''''(\cdot) < (>,=)\ 0$ *over the whole range are temperate (intemperate, neither nor);*

(ii) loss-averse S-shaped $u(\cdot)$ *are intemperate;*

(iii) $u(\cdot)$ *being (two-fold rotational) symmetric around the reference point are neither temperate nor intemperate.*

As case *(ii)* of corollary 2 is the standard assumption in the literature on reference dependence, we find a clear difference in the predictions of standard EUT with commonly used utility functions and loss-averse models concerning temperance. The implication of proposition 4, that typical loss-averse u functions always induce intemperance, is in line with Kőszegi and Rabin (2007, p1060) who show that in their model with a piecewise linear loss-averse u function and a linear m function "...behavior also approaches risk neutrality with even moderate amounts of background risk."[14]

To receive some intuition for case *(ii)* of corollary 2 consider the example where $\tilde{\varepsilon}_1 = [-\varepsilon_1;\varepsilon_1]$, $\tilde{\varepsilon}_2 = [-\varepsilon_2;\varepsilon_2]$ and $\varepsilon_1 = \varepsilon_2$. Then, if the choice is B_4 and this was also expected, the

[14] Kőszegi and Rabin (2007, p1052) "...identify common ways in which the decision maker becomes less risk-averse if she had been expecting, or is now facing, more risk." This is in line with our results not only for surprise situations (comparing their proposition 1 with our Table 1.3 in section 1.5.3), but also for expectational references. More specifically, in their proof of proposition 5 (p1068) they show that, for some lotteries G and F, $U(F|F) \le U(G+F|G+F) - U(G|G)$ holds. Rewriting this inequality as $U(F|F)+U(G|G) \le U(G+F|G+F)$ offers the direct interpretation that a person satisfying this inequality would rather aggregate than disaggregate the two lotteries. This is exactly the interpretation of intemperance. Moreover, by applying $F = y+\tilde{\varepsilon}_1$, $G = y+\tilde{\varepsilon}_2$, and $u(0) = 0$ it can be shown that this inequality corresponds to (weakly) choosing $[A_4|A_{4\langle B_4|B_4\rangle}]|[0]$ over $[B_4|B_{4\langle A_4|A_4\rangle}]|[0]$.

individual will experience with probability 1/4 a gain of $2\varepsilon_1$ and with probability 1/4 a loss of the same size. With probability 1/2 he will experience neither a gain nor a loss. If the individual chooses A_4 instead and also expected this choice, he will face a gain of $2\varepsilon_1$ with probability 1/8 and a gain of $4\varepsilon_1$ with probability 1/32. However, with probability 1/8 he will also face a loss of $2\varepsilon_1$ and with probability 1/32 a loss of $4\varepsilon_1$. With probability 11/16 he will experience neither a gain nor a loss. Note that choosing B_4 results in feelings of loss with a higher probability than choosing A_4. At a first glance, A_4 may still seem less attractive because the maximal potential loss is higher. However, because of the weakly decreasing sensitivity in losses, a loss of $4\varepsilon_1$ can have at most twice the effect as a loss of $2\varepsilon_1$. Hence, the danger of a larger loss with A_4 is always overcompensated by the smaller probability of experiencing a loss.

Propositions 3 and 4 are special cases of the following theorem that identifies risk preferences of arbitrary order n when reference points are formed by expectations.

Theorem 1 *If preferences depend on reference points formed by expectations, individuals are,*

(i) for even orders ($\frac{n}{2} \in \mathbb{N}$) and $n \geq 4$, risk apportioning of order n (its opposite, neither nor), that is $B_n|B_n \succ (\prec, \sim)\ A_n|A_n$, if and only if

$$[B_n|B_{n\langle A_n|A_n\rangle}]|[0] = \left[A_{n-2}|A_{n-2\langle B_{n-2}|B_{n-2}\rangle}; B_{n-2}|B_{n-2\langle A_{n-2}|A_{n-2}\rangle} + \tilde{\varepsilon}_{(\frac{n}{2})} + \tilde{\varepsilon}'_{(\frac{n}{2})}\right]|[0]$$
$$\succ (\prec, \sim)$$
$$\left[B_{n-2}|B_{n-2\langle A_{n-2}|A_{n-2}\rangle}; A_{n-2}|A_{n-2\langle B_{n-2}|B_{n-2}\rangle} + \tilde{\varepsilon}_{(\frac{n}{2})} + \tilde{\varepsilon}'_{(\frac{n}{2})}\right]|[0] = [A_n|A_{n\langle B_n|B_n\rangle}]|[0];$$

(ii) for odd orders ($\frac{n}{2} \notin \mathbb{N}$) and $n \geq 3$, never risk apportioning of order n or its opposite, that is $B_n|B_n \sim A_n|A_n$. This holds for all formulations of $u(\cdot)$.

In order to get an intuition for the general structure of the results of propositions 1 to 4 and theorem 1, consider first the case where $n = 1$. If a certain outcome is expected, it will not generate any feelings of gains or losses. Hence, any first-order effect is solely driven by the difference in pure consumption utility. However, for $n \geq 2$ the expected value of the pure consumption utility of B_n and A_n is identical because of the linearity of $m(\cdot)$. Therefore, for

$n \geq 2$ only the gain-loss component of utility determines the assessment of the two gambles. With $n = 2$ (see proposition 2), expecting a risk does not eliminate the risk in outcomes. This is because both random variables are realized independently. Thus, it is possible that a high expectation coincides with a low outcome (generating a sensation of loss) or that a low expectation is exceeded by a high outcome (generating a sensation of gain). In contrast to the second order, for $n = 3$ the question is not whether to take or avoid a risk but rather whether to add an unavoidable risk in a high- or low-wealth state. Since the decision is anticipated, location is irrelevant for feelings of gains and losses. Therefore, individuals do not care where to add the risk. Unlike the third order, with $n = 4$ the question is not whether to add risk either to a high or to a low state, but rather whether to add it to a certain or uncertain state. Generally, aggregating or disaggregating risk(s) affects gain-loss utility and this is captured by even-order risk preferences. By contrast, odd-order risk preferences of orders higher than one can always be thought of as a decision on the location of risk(s). With expectational references the choice of location becomes irrelevant.

More formally, for even orders $-B_n \succ (\prec, \sim) \; -A_n$ if and only if $B_n \succ (\prec, \sim) \; A_n$ since all $\tilde{\varepsilon}_i$ are symmetric and there is no reduction of k. Thus, an individual prefers $B_n|B_n$ over $A_n|A_n$ if and only if he is risk apportioning of order n. For odd orders $\tilde{\varepsilon}_i$ are still symmetric, but there is now always a reduction of k which reverses into an additional k when B_n and A_n become negative. Thus, we get that $-B_n \succ (\prec, \sim) \; -A_n$ if and only if $B_n \prec (\succ, \sim) \; A_n$. Because the two effects with expectational references are of equal magnitude and opposing direction they cancel out and an individual is therefore always indifferent between $B_n|B_n$ and $A_n|A_n$ for odd orders.

While risk preferences of even orders with expectational references resemble those without reference dependence for a given functional form of $u(\cdot)$, they substantially differ for odd orders. In fact, risk preferences of odd orders are completely absent with expectational references. In EUT risk preferences of any order n depend on those of order $n-1$ and are therefore strongly related. If the $(n-1)^{th}$ derivative of $u(\cdot)$ is known, the n^{th} derivative can be directly computed. In contrast, theorem 1 shows that with reference dependence this is no longer the case. Especially when the evaluation of an odd order delivered indifference, one cannot conclude that the next-higher order delivers indifference as well. This is of general importance for the empirical derivation of higher-order risk preferences via measures of risk aversion. We discuss such implications of theorem 1 for the relationship between risk aversion and precautionary saving in section 1.4.

Kőszegi and Rabin (2007) describe in which circumstances reference points that are formed by expectations seem appropriate. These can be separated into two broad categories. First, some decisions are anticipated well before they have to be made. Then, the individual also anticipates how he will decide. Therefore, at the time the decision is made the individual has already expected his actual choice. Consistent choices in these situations are *preferred personal equilibria* (PPE) as defined by the authors. An example for an economically interesting scenario would be the decision over savings. The individual may think about saving well before doing so and then has already incorporated this behavior into his reference point when conducting the transaction. Second, some decisions are real choices that cannot be reversed but whose actual outcome is realized much later. Here, individuals' references have time to adjust to the choice already made. Decision-makers anticipate the adjustment before deciding. According to Kőszegi and Rabin (2007) all choices are then *choice-acclimating personal equilibria* (CPE). This seems relevant in situations where one can commit to choices well in advance of events. Interesting examples include many insurance and pension decisions. Here, contracts are often binding and are agreed upon years before the risks are realized.

Both scenarios are analytically identical since the reference distribution is the same as the chosen outcome distribution. Thus, our results are applicable to both equlibrium concepts. However, the two concepts differ with respect to the existence of such personal equilibria. All choices are equilibria if commitment is possible and CPE is the relevant concept.[15] By contrast, if commitment is not possible but decisions have been anticipated not all decisions

[15]Recent experiments by Abeler et al. (forthcoming) and Gill and Prowse (forthcoming) support CPE as the relevant concept. The latter even find that reference points adjust essentially instantaneously to choices made.

can rationally be expected and constitute a PPE. In these cases equilibria existence is an important issue because rational agents cannot trick themselves to expect a later choice they have no incentive to follow. In other words, if the agent wanted to deviate from his expectation once he makes the decision, this expectation could not sustain in the first place. All choices that can be rationally expected are defined as *unacclimated personal equilibria* (UPE) as introduced by Kőszegi and Rabin (2006, 2007). The UPE that maximize ex-ante decision utility $V(\cdot)$ are PPE and are therefore the chosen alternatives. In our context this translates into the following existence condition

$$\begin{aligned} B_n|B_n \text{ is an UPE} \quad &\Leftrightarrow \quad B_n|B_n \succsim A_n|B_n \quad \text{and} \\ A_n|A_n \text{ is an UPE} \quad &\Leftrightarrow \quad A_n|A_n \succsim B_n|A_n. \end{aligned}$$

1.4 Implications

Results of the previous section may have strong implications in situations where decisions under uncertainty cannot solely be explained via risk aversion. Then, higher-order risk preferences are needed to explain phenomena like precautionary saving, precautionary labor, or the influence of background risk. Until now, in attempts to explain these phenomena only the effect of pure consumption utility in EUT models has been considered. Applying the preceding analysis allows us to investigate the influence of reference dependence in such settings. In the following, we will discuss two implications in more detail, the optimal amount of precautionary saving and the interaction of precautionary saving and insurance demand.

Often quantitative measures of one order were used to predict behavior concerning a higher order. For example, measures of risk aversion were thought to determine exactly the optimal amount of precautionary saving because both were solely derived from the functional form of consumption utility. As reported by Browning and Lusardi (1996) or Carroll and Kimball (2008) for instance, precautionary saving is often empirically observed but to a lesser extent than thought to be optimal. "Structural models that match broad features of consumption and saving behavior tend to produce estimates of the degree of prudence that are less than those obtained from theoretical models in combination with risk aversion estimates from survey evidence." (Carroll and Kimball, 2008, p584). Also, Ballinger, Palumbo and Wilcox (2003) found in a laboratory experiment simulating life-cycle decisions that precautionary saving falls short of optimal theoretical predictions. When utility is partly dependent on expectational references, the reference-dependent component contributes positively to risk aversion but leaves prudence unaffected. Therefore, such a reference-dependent model would predict less precautionary saving than the standard model. This is consistent with the empirical and experimental literature.

Closely related and building on the well-known finding that saving and insurance are substitutes[16] is what Gollier (2003) called the 'insurance puzzle'. When financial markets are complete, individuals should prefer to cope with risk via buffer-stock saving rather than by purchasing costly insurance. In other words, allowing for precautionary saving leads theoretically to a crowding out of insurance demand in dynamic life-cycle models. "The bottom line is similar to the one underlying the literature on the equity premium puzzle: the theory cannot easily explain why people are so reluctant to accept risk." (Gollier, 2003, p21). Again, assuming preferences that depend partly on expectational references can serve as an alternative explanation to market imperfection. Intuitively, since insurance demand is driven by second-order effects but precautionary saving by third-order effects, the reference-dependent component generates additional insurance demand but does not affect precautionary saving decisions.

To illustrate these implications of our results we analyze a simple two-period setting ($T = 2$) without discounting and interest on savings. As *benchmark* model we consider a standard EUT model, where individuals maximize expected life-time utility $V^b = \sum_{t=1}^{T} V_t^b$ with $V_t^b = \int c(x_t)dF(x_t)$. x_t is consumption in period t and $c(\cdot)$ denotes pure consumption

[16] To our knowledge, the first formal analysis showing that these two are substitutes was by Moffet (1975) within a simple two-period model. For a more general treatment without additive separable utility, refer to Dionne and Eeckhoudt (1984).

utility that is assumed to be strictly increasing and strictly concave.

To facilitate comparisons between implications of such a benchmark model and a reference-dependent model we specify the latter as a *hybrid* model. This hybrid model slightly deviates from the reference-dependent model before in that the pure consumption utility part has now the same properties as our standard EUT benchmark model while the gain-loss part stays unchanged and still has the properties of the previous analysis. More formally, individuals maximize $V^h = \sum_{t=1}^{T} V_t^h$ with $V_t^h = \int c(x_t)dF(x_t) + \iint u(x_t - r_t)dG(r_t)dF(x_t)$, where r_t denotes the reference point in t. The reason why we allow the pure consumption utility part to be concave in wealth in the hybrid model is that this leads to an equivalence of both models in risk-less situations. Within both models we will compare behavior in risky and risk-less situations. As we are interested in the implications of reference dependence for risk preferences we would like to keep behavior in risk-less situations constant between the two models. In order to isolate the pure effects of reference dependence and to avoid reintroducing classical EUT effects into gain-loss utility we still assume that the gain-loss part depends on absolute differences of outcomes and reference points.

Consumption in period 1 is income y which is potentially reduced by saving k^s and an insurance premium. The insurance premium consists of the premium of an actuarially fair insurance k^i plus proportional loading λ. Thus, the final consumption in period 1 is $x_1 = y - k^s - (1+\lambda)k^i$ and the distribution representing the beliefs over x_1 is simply $F(x_1) = [y - k^s - (1+\lambda)k^i]$. Consumption in period 2 is the same income y plus a random component $\tilde{\varepsilon}_1$ whose two realizations occur with equal probability, $\tilde{\varepsilon}_1 = [-\varepsilon_1; \varepsilon_1]$. Savings k^s increase outcomes in both states but insurance only in the bad state. Since the premium of an actuarially fair insurance was defined as k^i, the amount of indemnity is $2k^i$ when the probability of the bad state is $1/2$. Therefore, x_2 can take on two different values, $\underline{x}_2 = y - \varepsilon_1 + k^s + 2k^i$ and $\overline{x}_2 = y + \varepsilon_1 + k^s$. The beliefs over x_2 are then the distribution $F(x_2) = [\underline{x}_2; \overline{x}_2]$. In the following, we will restrict insurance to have either partial or full indemnity but rule out over-insurance, hence $k^i \leq \varepsilon_1$, which also implies $\underline{x}_2 \leq \overline{x}_2$.

To highlight the implications for the optimal amount of precautionary saving, consider first the case where no insurance is available, thus $k^i = 0$. In the benchmark case

$$V^b = c(y - k^s) + \tfrac{1}{2}c(y - \varepsilon_1 + k^s) + \tfrac{1}{2}c(y + \varepsilon_1 + k^s), \tag{1.5}$$

where the first term in (1.5) denotes consumption utility in period 1 and the last two terms are expected consumption utility in period 2. In the hybrid model

$$V^h = c(y - k^s) + \tfrac{1}{2}c(y - \varepsilon_1 + k^s) + \tfrac{1}{2}c(y + \varepsilon_1 + k^s) + \tfrac{1}{4}u(-2\varepsilon_1) + \tfrac{1}{4}u(2\varepsilon_1). \tag{1.6}$$

Again, the first three terms in (1.6) denote pure consumption utility in periods 1 and 2. There is no gain-loss utility in period 1 since there is no uncertainty resolved. Gain-loss utility in period 2 is captured by the last two terms. Since choosing the optimal k^s in (1.5) and (1.6) means solving identical first-order conditions we obtain the result that the decision for precautionary saving is not affected by the reference-dependent part of utility $V^h(\cdot)$. If preferences are reference dependent instead of reference independent, the extent of risk aversion will be higher[17] as long as the slope of $u(\cdot)$ is steeper in losses than in corresponding gains ($u'(-\tau) > u'(\tau)\ \forall \tau > 0$) which is met for strictly concave as well as loss-averse S-shaped u functions. But at the same time an identical amount of precautionary saving would be optimal. This shows that seemingly sub-optimal levels of precautionary saving may well be optimal if preferences are reference dependent. In contrast to classical models of EUT, this result shows that predicted behavior under reference dependence is in line with the empirical findings.

Only at first glance this results seems to be in contrast to a result of Kőszegi and Rabin (2009). In their proposition 8 they show that a piecewise linear gain-loss utility contributes to precautionary saving. However, their result rests on the assumption of concave consumption utility entering gain-loss utility and thereby reintroducing classical EUT effects into the gain-loss part. Indeed, with such a specification it can be shown that optimal precautionary

[17] For example, it can be easily verified that the risk premium of an individual with $V^h(\cdot)$ is higher than with $V^b(\cdot)$.

saving is higher under reference dependence compared to the benchmark case as long as the slope of $u(\cdot)$ is steeper in losses than in corresponding gains ($u'(-\tau) > u'(\tau)\ \forall \tau > 0$).[18] However, since reference dependence increases risk aversion with this specification as well, this does neither contradict our result nor the empirical pattern observed.[19]

We next analyze precautionary saving and insurance jointly. Our analysis encompasses the case where insurance is costly. That is, it may have a positive loading factor λ. However, in the following we restrict λ to the more realistic cases where it is below 1, hence $0 \leq \lambda < 1$. Then, in the benchmark model

$$\begin{aligned} V^b &= c(y - k^s - (1+\lambda)k^i) + \tfrac{1}{2}c(y - \varepsilon_1 + k^s + 2k^i) + \tfrac{1}{2}c(y + \varepsilon_1 + k^s), \\ \partial V^b/\partial k^s &= -c'(y - k^s - (1+\lambda)k^i) + \tfrac{1}{2}c'(y - \varepsilon_1 + k^s + 2k^i) + \tfrac{1}{2}c'(y + \varepsilon_1 + k^s) \\ &= 0, \qquad (1.7) \\ \partial V^b/\partial k^i &= -(1+\lambda)c'(y - k^s - (1+\lambda)k^i) + c'(y - \varepsilon_1 + k^s + 2k^i) = 0. \qquad (1.8) \end{aligned}$$

As shown in the appendix all second partial derivatives are strictly negative and the Hessian is negative definite. We denote the solution of (1.7) and (1.8) by $(\hat{k}^s, \hat{k}^i)$.

By combining (1.7) and (1.8) we get the following condition:

$$(1-\lambda)c'(y - \varepsilon_1 + \hat{k}^s + 2\hat{k}^i) = (1+\lambda)c'(y + \varepsilon_1 + \hat{k}^s). \qquad (1.9)$$

Although (1.9) cannot be solved explicitly, due to the strict concavity of $c(\cdot)$ we receive that $\hat{k}^i < \varepsilon_1$ for $\lambda > 0$ and $\hat{k}^i = \varepsilon_1$ for $\lambda = 0$.

Gollier (2003) simulated a life-cycle model for an infinite time horizon and found that

[18] At the optimal level of precautionary saving in the benchmark case $\partial V^h/\partial k^s > 0$ since the additional terms in the first-order condition of the hybrid model compared to the benchmark case are $\frac{1}{4}u'[c(y+\epsilon_1+k^s) - c(y-\epsilon_1+k^s)][c'(y+\epsilon_1+k^s) - c'(y-\epsilon_1+k^s)] + \frac{1}{4}u'[c(y-\epsilon_1+k^s) - c(y+\epsilon_1+k^s)][c'(y-\epsilon_1+k^s) - c'(y+\epsilon_1+k^s)]$.

[19] Among the studies reviewed by Browning and Lusardi (1996) there are even numerous articles where no precautionary saving was measured at all. Another recent example is Jappelli, Padula and Pistaferri (2008). Lee and Sawada (2007, p196) stress that "while a growing number of theoretical studies point out the importance of precautionary saving, the existing evidence suggests that precautionary saving motives may not be empirically important." Such empirical findings even point toward a reference-dependent model with linear consumption utility in both the reference-dependent and the reference-independent part. This is the model we analyzed in the previous section.

demand for costly insurance completely vanishes with the opportunity of precautionary savings. He termed this result the 'insurance puzzle' because it is clearly at odds with empirical observations. His explanation for this puzzle rests on market imperfections. In the following, we show that it can also be explained by reference-dependent preferences, even if markets are complete. This is because saving and insurance are weaker substitutes under reference dependence than in the benchmark model. Consider our hybrid model where

$$\begin{aligned}
V^h &= c(y-k^s-(1+\lambda)k^i)+\tfrac{1}{2}c(y-\varepsilon_1+k^s+2k^i)+\tfrac{1}{2}c(y+\varepsilon_1+k^s) \\
&\quad +\tfrac{1}{4}u(-2\varepsilon_1+2k^i)+\tfrac{1}{4}u(2\varepsilon_1-2k^i), \\
\partial V^h/\partial k^s &= -c'(y-k^s-(1+\lambda)k^i)+\tfrac{1}{2}c'(y-\varepsilon_1+k^s+2k^i)+\tfrac{1}{2}c'(y+\varepsilon_1+k^s) \\
&= 0, \qquad (1.10) \\
\partial V^h/\partial k^i &= -(1+\lambda)c'(y-k^s-(1+\lambda)k^i)+c'(y-\varepsilon_1+k^s+2k^i) \\
&\quad -\tfrac{1}{2}u'(2\varepsilon_1-2k^i)+\tfrac{1}{2}u'(-2\varepsilon_1+2k^i)=0. \qquad (1.11)
\end{aligned}$$

Again, in the appendix it is shown that all elements of the Hessian are strictly negative and that the Hessian is negative definite. This holds for $u(\cdot)$ being (weakly) concave, (piecewise) linear, or (loss-averse) S-shaped. Combining (1.10) and (1.11) yields

$$\begin{aligned}
(1-\lambda)c'(y-\varepsilon_1+k^s+2k^i) &= (1+\lambda)c'(y+\varepsilon_1+k^s) \\
&\quad -u'(2\varepsilon_1-2k^i)+u'(-2\varepsilon_1+2k^i). \qquad (1.12)
\end{aligned}$$

Using the assumptions that the slope of $u(\cdot)$ is steeper in losses than in corresponding gains, $k^i \leq \varepsilon_1$, and the strict concavity of $c(\cdot)$, we receive from (1.12) that in the optimum $k^i < \varepsilon_1$ for $\lambda > 0$ and $k^i = \varepsilon_1$ for $\lambda = 0$. In the following, we first analyze the case of positive loading ($\lambda > 0$) and then discuss actuarially fair insurance ($\lambda = 0$).

For a comparison of the solutions of both models consider the optimality conditions (1.10) and (1.11) at $(\hat{k}^s, \hat{k}^i)$ which would be the solution if preferences were not reference dependent. Indeed, $(\hat{k}^s, \hat{k}^i)$ is one possible solution of (1.10) because condition (1.10) equals condition

(1.7). Using $u'(-\tau) > u'(\tau) \forall \tau > 0$ and $\hat{k}^i < \varepsilon_1$ we find that $(\hat{k}^s, \hat{k}^i)$ cannot be a solution of (1.11) since $\partial V^h(\hat{k}^s, \hat{k}^i)/\partial k^i > 0$. Our goal is to show that the optimal demand for insurance is higher and optimal saving is lower with reference-dependent preferences than without.[20] We denote by (dk^s, dk^i) the amounts that the optimal solution of (1.10) and (1.11) deviates from $(\hat{k}^s, \hat{k}^i)$.

As mentioned above, condition (1.10) is met at $(\hat{k}^s, \hat{k}^i)$. This implies that any change in k^s or k^i has to be compensated by a change of the other variable. By totally differentiating (1.10) we establish the explicit relationship between these changes as

$$dk^s = -\left(\frac{\partial^2 V^h}{\partial k^s \partial k^i} \Big/ \frac{\partial^2 V^h}{(\partial k^s)^2}\right) dk^i. \tag{1.13}$$

We show in the appendix that the term in brackets is positive. Therefore, in order to fulfill condition (1.10) dk^s and dk^i need to have opposite signs as long as they are not both zero.

Our analysis of (1.11) showed that $\partial V^h(\hat{k}^s, \hat{k}^i)/\partial k^i > 0$. Hence, in the hybrid model $\partial V^h/\partial k^i$ has to be lower in the optimum than at $(\hat{k}^s, \hat{k}^i)$. Both an increase in k^s or k^i would decrease $\partial V^h/\partial k^i$, but (1.13) showed that their changes must have opposing signs. The overall effect of an increase in one variable and a decrease in the other has to be negative,

$$d\left(\frac{\partial V^h}{\partial k^i}\right) = \frac{\partial^2 V^h}{\partial k^s \partial k^i} dk^s + \frac{\partial^2 V^h}{(\partial k^i)^2} dk^i < 0. \tag{1.14}$$

In order to show that only an increase in k^i and a decrease in k^s can accomplish this we insert (1.13) in (1.14) and receive

$$\frac{dk^i}{\frac{\partial^2 V^h}{(\partial k^s)^2}} \left(\frac{\partial^2 V^h}{(\partial k^i)^2}\frac{\partial^2 V^h}{(\partial k^s)^2} - \left[\frac{\partial^2 V^h}{\partial k^s \partial k^i}\right]^2\right) < 0. \tag{1.15}$$

In the proof of proposition 5 we show that the term in brackets in (1.15), which is the determinant of the Hessian matrix, is positive and that $\partial^2 V^h/(\partial k^s)^2 < 0$. Hence, it follows that $dk^i > 0$ and in combination with (1.13) that $dk^s < 0$ in the optimum.

[20] There is no reason for a change in saving other than for precautionary motives since gain-loss utility vanishes without uncertainty. All changes in saving are therefore changes in precautionary saving.

Proposition 5 *Individuals with preferences depending on reference points formed by expectations have a higher demand for insurance and save less than individuals without reference-dependent preferences.*

Proposition 5 holds for all u functions with a slope that is steeper in losses than in corresponding gains ($u'(-\tau) > u'(\tau)\ \forall \tau > 0$) and with sensitivity that is diminishing faster or equally in gains than in losses ($u''(\tau) \geq -u''(-\tau)\ \forall \tau > 0$). As mentioned before, these conditions are met for u functions that are strictly concave or loss-averse S-shaped, including the limiting case of $u(\cdot)$ being piecewise linear with loss aversion. Proposition 5 further shows that reference dependence can be an alternative explanation for the insurance puzzle that does not rest on market imperfections. We derived this result for costly insurance, i.e. $0 < \lambda < 1$. For actuarially fair insurance ($\lambda = 0$) it can be easily shown that both models yield the identical result of full insurance and the absence of a precautionary motive for saving.[21]

Although this section focused on implications concerning the third and second order, our results may have implications concerning higher orders as well. As an example, many financial decisions, such as demand for insurance or risky assets, are taken in the face of exogenous background risk. It is known that the fourth-order concept of temperance is necessary for risk aversion to increase with increasing background risk and all commonly used utility functions in EUT models exhibit this feature. If, by contrast, preferences are reference dependent, our analysis of the previous section showed that despite higher risk aversion, its sensitivity to background risk would be reduced under standard assumptions on gain-loss utility. A formalization of these ideas will be the subject of future work.

1.5 Alternative Models

It is important to analyze whether our results for Kőszegi and Rabin's (2007) model of expectational references can in fact *not also* be derived by using alternative reference

[21] From $\lambda = 0$ it follows that $\hat{k}^i = \varepsilon_1$ and $\hat{k}^s = -\varepsilon_1$. Hence, $V^b = V^h$ in the optimum.

points. Only then, we may be able to conclude that it is their concept of expectational references which is not only capable of resolving empirical puzzles concerning first and second orders, but also concerning higher-order risk preferences. As alternative models we consider disappointment models, models of regret, and reference-dependent models with exogenous reference points like the status quo. All these alternative models can be captured by (1.1) as well but are characterized by a different assumption concerning the reference point G.

Rather than using the full outcome distribution representing expectations as the reference point, individuals may also assess choices relative to the mean of these expectations. This is the assumption of disappointment models (Bell, 1985; Loomes and Sugden, 1986), where outcomes above the mean are perceived as elation and outcomes below as disappointment. So, in models of disappointment it is assumed that the reference point is the mean of the chosen alternative's distribution, thus $G = \mathbb{E}[F]$. Assuming such a reference point may seem appropriate in situations where information or framing focuses on the mean outcome. For instance, buyers of bonds may have clear expectations on their mean performance leading to the mean as a natural source of reference for the evaluation of their actual performance.

With disappointment as well as with expectational references, the alternatives not chosen do not have any impact on the final evaluation of a chosen option and thus every alternative's appeal can be described entirely by its own attributes. By contrast, in models of regret sentiments (see Bell, 1982, 1983; and Loomes and Sugden, 1982) possible outcomes of the chosen gamble are compared to possible outcomes of the gambles not chosen. This kind of behavior is motivated in the literature by an ex-ante anticipation of ex-post utility that is driven by regret (and rejoice) feelings. With the notion of regret employed in the following analysis all combinations of outcomes are evaluated by a regret function and weighted by their joint probabilities. This coincides with the initial models of Bell (1982, 1983) and Loomes and Sugden (1982) in case the individual anticipates prior to her decision that she will finally learn the true outcome that would have resulted had she decided differently than she has. This qualification is a direct consequence of the emphasis these models put on the resolution of foregone alternatives for the presence of regret feelings. In order for our representation (1.1) to be in line with these regret models an assumption needed is that stochastic components of alternatives are realized independently.[22] Since in models of regret the alternative *not* chosen is the relevant reference distribution and since in the definitions of higher orders only pairwise choices were applied, the relevant reference distribution under regret is unambiguous in our context. If we denote the distribution of the alternative not chosen by F', models of regret assume $G = F'$.

Although the theoretical literature has advanced toward endogenous reference points like regret, disappointment, and most recently, expectational references, first models of reference-dependent preferences (see Kahneman and Tversky, 1979) started with exogenous reference points and they still seem to be relevant for specific decision scenarios. For instance, in surprise situations, where it was not anticipated that a decision has to be made, the status quo may seem to be a relevant state of comparison. Also, in experiments subjects may not anticipate what will happen in the experiment and are unlikely to endogenize the rules of the game in the short time until they have to make their decisions. This is, however, not a necessary experimental feature. If the reference point is meant to be endogenous, this could be achieved by careful experimental design and a longer time horizon. With exogenous reference points, G can be chosen arbitrarily but we will focus on the most relevant ones in our analysis.

The following definition specifies how the reference point is formed in these alternative models and therefore complements (1.1) and definition 1 for the analysis of this section.

Definition 3 *If the reference point is determined (i) by disappointment, $G = \mathbb{E}[B_n]$ and $G' = \mathbb{E}[A_n]$; (ii) by regret, $G = A_n$ and $G' = B_n$; and if the reference point is given (iii) exogenously, then $G = G'$.*

If the reference point is given exogenously (case *(iii)* in definition 3) and is stochastic rather than deterministic, our analysis is formally very similar to classical models of background risk

[22] For this reason our results do not carry over to cases where the outcome of the chosen alternative determines the counterfactual outcome (e.g. the toss of a coin). Nevertheless, despite our qualifications of stochastic independence and full resolution of information we perceive the remaining field of applicability as substantial. In particular, decisions made in a market environment are often characterized by independence and full information disclosure (e.g. the decision to buy a particular financial asset or a treasury bond).

(Ross, 1981; Kihlstrom, Romer and Williams, 1981). In these models, preferences toward risk are analyzed in the presence of a given independent background risk. A stochastic, exogenous reference point is analytically equivalent to a background risk since it enters additively into the u function. This analogy does not carry over to endogenous reference points (definition 2 and cases *(i)* and *(ii)* in definition 3). Here, the actual choice influences the reference point. Thus, the reference point entering $u(\cdot)$ is in these cases not independent of the choice taken. Despite the fact that once the reference point is determined it is as if realized independently of the outcome of the choice, the ex-ante distribution of the reference point depends on the actual choice.

1.5.1 Disappointment

Just as in the case of expectational references, disappointment references are in principle endogenous since they depend on the chosen alternative. However, in our analysis for $n \geq 2$ the expectational mean of B_n is always identical to the expectational mean of A_n and it can therefore also be considered as exogenous. For the same reason, the results of this section also apply to models of regret in which the reference point is the expectational mean of the foregone alternative. This is the usual assumption in regret models where the foregone alternative is not realized or the individual does not receive feedback on its outcomes (see Bell, 1983; or Krähmer and Stone, 2008).[23] In the following, we report results for risk preferences when reference points are formed by the expectational mean. For the gambles used in our analysis the mean is y for even orders and $y - \frac{k}{2}$ for odd orders, except order one. For the first order the mean is identical to the full distribution since everything is certain and there is no risk, hence $B_1 = \mathbb{E}[B_1] = y$ and $A_1 = \mathbb{E}[A_1] = y - k$. We therefore receive for the first order the exact same result as with expectational references, namely that individuals with disappointment references have monotone preferences. Proposition 6 summarizes our

[23] Note that in these models it is assumed that $U(F|G) = \int u(m(x) - \int m(r)dG(r))dF(x)$. This is also the assumption of the disappointment model of Loomes and Sugden (1986). However, with $m(\cdot)$ being linear, which is assumed by Loomes and Sugden (1986) as well, this is equivalent to our formulation and that of Bell (1985) where $U(F|G) = \int u(m(x) - m(\int r\ dG(r)))dF(x)$.

results with disappointment references for all orders.

Proposition 6 *If preferences depend on reference points formed by the expectational mean, individuals*

(i) have monotone preferences, that is $B_1|\mathbb{E}[B_1] \succ A_1|\mathbb{E}[A_1]$*;*

(ii) are risk-averse (risk-seeking, risk-neutral), that is $B_2|\mathbb{E}[B_2] \succ (\prec, \sim)\ A_2|\mathbb{E}[A_2]$*, if and only if* $[0]|[0] \succ (\prec, \sim)\ [\tilde{\varepsilon}_1]|[0]$*;*

(iii) are prudent (imprudent, neither nor), that is $B_3|\mathbb{E}[B_3] \succ (\prec, \sim)\ A_3|\mathbb{E}[A_3]$*, if and only if* $[-\frac{k}{2}; \frac{k}{2} + \tilde{\varepsilon}_1]|[0] \succ (\prec, \sim)\ [-\frac{k}{2} + \tilde{\varepsilon}_1; \frac{k}{2}]|[0]$*;*

(iv) are (for $n \geq 4$*) risk apportioning of order* n *(its opposite, neither nor), that is* $B_n|\mathbb{E}[B_n] \succ (\prec, \sim)\ A_n|\mathbb{E}[A_n]$*, if and only if*

$$\begin{aligned} & \left[A_{n-2}|\mathbb{E}[A_{n-2}]; B_{n-2}|\mathbb{E}[B_{n-2}] + \tilde{\varepsilon}_{int(\frac{n}{2})}\right] | [0] \\ \succ (\prec, \sim) \quad & \left[B_{n-2}|\mathbb{E}[B_{n-2}]; A_{n-2}|\mathbb{E}[A_{n-2}] + \tilde{\varepsilon}_{int(\frac{n}{2})}\right] | [0] \quad . \end{aligned}$$

Concerning orders higher than one, the mean is not equal to the full distribution and therefore results generally differ between expectational and disappointment references (cases *(ii)* – *(iv)* in proposition 6). However, for the second order (case *(ii)* in proposition 6) we find that if an individual is classified as being risk-averse (risk-seeking, risk-neutral) with disappointment references and a given u function, he will also be risk-averse (risk-seeking, risk-neutral) with the same u function and expectational references. As with expectational references an individual with disappointment references is therefore risk-averse if $u(\cdot)$ is strictly concave or loss-averse S-shaped and risk-neutral if $u(\cdot)$ is (two-fold rotational) symmetric. Corollary 1 also applies to disappointment references.

So, concerning the first two orders expectational and disappointment references yield the same behavior. This is one reason why it has been difficult to discriminate between these two forms of expectations as the relevant reference point in the empirical literature. Neither of the empirical studies reviewed in section 1.2.2 has been able to distinguish them. Even recent controlled laboratory experiments that directly test expectations as the reference point, like for instance Abeler et al. (forthcoming), Gill and Prowse (forthcoming), or Ericson and Fuster (2010), cannot differentiate between expectational and disappointment references.

However, with respect to the third order disappointment yields a different prediction than expectational references (case *(iii)* in proposition 6).[24]

Corollary 3 *If preferences depend on reference points formed by the expectational mean, individuals with*

(i) $u'''(\cdot) > (<, =)\ 0$ *over the whole range are prudent (imprudent, neither nor);*

(ii) loss-averse S-shaped $u(\cdot)$ *are never prudent.*

This result shows that by using the third order it may well be possible to discriminate between expectational and disappointment references. One advantage of the gamble definitions used to define higher-order risk preferences is that they are easily implemented into choice situations in experiments. Future experiments on expectations as the relevant reference point could therefore use this result in order to be able to distinguish between expectational and disappointment references. Moreover, this result further shows that the implications we derived for expectational references, i.e. the effects on precautionary saving and insurance demand, cannot be derived with disappointment references for all functional forms of $u(\cdot)$, since they heavily relied on the general absence of third-order risk preferences.

There is again no difference in predicted behavior between these two reference points for fourth-order risk preferences. More specifically, if an individual is temperate (intemperate, neither nor) with disappointment references for a given u function, he will also be temperate (intemperate, neither nor) with expectational references under this u function. In fact, individuals with disappointment references are temperate, that is $B_4|\mathbb{E}[B_4]$ is preferred to $A_4|\mathbb{E}[A_4]$ if and only if $[\tilde{\varepsilon}_1; \tilde{\varepsilon}_2]|[0]$ is preferred to $[0; \tilde{\varepsilon}_1 + \tilde{\varepsilon}_2]|[0]$. This preference holds if

[24] Note that case *(ii)* in corollary 3 can be further specified: Individuals are imprudent if $u(\cdot)$ is loss-averse S-shaped without the limiting case of piecewise linear, since then individuals like gambles in losses and dislike gambles in gains. If $u(\cdot)$ is piecewise linear with loss aversion, individuals are neither prudent nor imprudent.

$u(\cdot)$ globally satisfies $u''''(\cdot) < 0$. But individuals are intemperate if $u(\cdot)$ is loss-averse S-shaped. Moreover, individuals are neither temperate nor intemperate for $u(\cdot)$ being (two-fold rotational) symmetric around the reference point. Again, corollary 2 not only holds with expectational but also with disappointment references. So, as in the case of second-order risk preferences, the same behavior is predicted for both reference points using common assumptions on $u(\cdot)$.

Our higher-order results under disappointment (cases *(ii)* – *(iv)* in proposition 6) after our reference-point transformation resemble those of Eeckhoudt and Schlesinger (2006). They further show that common assumptions on $u(\cdot)$ generate contrary predictions on third-order effects compared to both standard EUT and expectational references, and on fourth-order effects compared to EUT. However, just as with EUT and expectational references, they predict risk aversion as well as monotone preferences. Furthermore, similar to our findings for expectational references, the significance of higher-order derivatives no longer applies in general. For strictly concave u functions we still need the third and fourth derivative in order to characterize prudence and temperance respectively. But in cases of $u(\cdot)$ being loss-averse S-shaped we only need the first two derivatives in order to describe the preference relations of those higher orders.

In all models where some form of expectations shape the reference point, we can additionally ask whether these expectations form rationally, because only choices followed through can reasonably be expected in the first place. We provided conditions in section 1.3 that have to be fulfilled for a choice to be a UPE under expectational references. This is only relevant in decision scenarios where the CPE concept does not apply. If the CPE concept can be applied, any choice is in fact an equilibrium. This also holds for disappointment references. However, if CPE cannot be used, we call the disappointment counterpart of UPE *disappointment equilibria* (DE). The DE that maximize ex-ante decision utility $V(\cdot)$ are the *preferred disappointment equilibria* (PDE). Then,

$$B_n|\mathbb{E}[B_n] \text{ is a DE} \;\Leftrightarrow\; B_n|\mathbb{E}[B_n] \succsim A_n|\mathbb{E}[B_n] \quad \text{and}$$
$$A_n|\mathbb{E}[A_n] \text{ is a DE} \;\Leftrightarrow\; A_n|\mathbb{E}[A_n] \succsim B_n|\mathbb{E}[A_n].$$

Since $\mathbb{E}[B_n] = \mathbb{E}[A_n]$ (for $n \geq 2$), we can further say that if and only if $B_n|\mathbb{E}[B_n]$ is strictly preferred to $A_n|\mathbb{E}[A_n]$, choosing B_n is the only DE and thus PDE. Similarly, if and only if $A_n|\mathbb{E}[A_n]$ is strictly preferred to $B_n|\mathbb{E}[B_n]$, choosing A_n is the only DE and thus PDE. This holds for all but the first order. For the first order DE conditions are identical to the UPE conditions of expectational references since $\mathbb{E}[B_1] = B_1$ and $\mathbb{E}[A_1] = A_1$. Here, $B_1|B_1 = B_1|\mathbb{E}[B_1]$ is the only UPE and DE and thus PPE and PDE.

1.5.2 Regret

As already discussed, reference points need not necessarily be formed by the expectation of the choice made but can also be driven by regret feelings. Then, ex-ante expectations of the foregone alternative functions as the reference point for the choice. A prevalent question in the regret literature is how individuals evaluate choices when there are several alternatives initially. This is not discussed in our context because the definition of higher orders applied here is over binary alternatives only. In the following, we derive how regret references affect higher-order risk attitudes.

Proposition 7 *If preferences depend on reference points formed by regret, individuals have monotone preferences, that is $B_1|A_1 \succ A_1|B_1$.*

While proposition 7 is a standard result which is satisfied by all other models as well, our next result differs substantially from previous findings.

Proposition 8 *If preferences depend on reference points formed by regret, individuals are always risk-neutral, that is $B_2|A_2 \sim A_2|B_2$. This holds for all formulations of $u(\cdot)$.*

This proposition applies to all u functions, since $M(B_2) = M(A_2)$ and $U(B_2|A_2)$ always contains the same elements with identical probabilities as $U(A_2|B_2)$. For instance, consider again the simple case where $\tilde{\varepsilon}_1 = [-\varepsilon_1; \varepsilon_1]$. If the individual chooses the gamble B_2 over A_2 she will receive y with certainty. Her foregone alternative would have given her $y - \varepsilon_1$ in half of the cases and $y + \varepsilon_1$ in the other half of the cases. With regret references, choosing B_2 over A_2 exposes her to the regret of not receiving ε_1 with probability 1/2. But it also gives her with probability 1/2 the chance of rejoicing having not lost the same amount. Therefore, choosing B_2 would give her the utility of $M([y]) + U([-\varepsilon_1; \varepsilon_1])$. By contrast, if the individual chooses A_2 over B_2, with equal probability she will receive $y - \varepsilon_1$ or $y + \varepsilon_1$ and will always compare it to the foregone certain alternative y. Thus, choosing A_2 would give her the utility $M([y - \varepsilon_1; y + \varepsilon_1]) + U([-\varepsilon_1; \varepsilon_1])$. Since $U(B_2|A_2) = U(A_2|B_2) = \frac{1}{2}u(-\varepsilon_1) + \frac{1}{2}u(\varepsilon_1)$ the individual is indifferent between B_2 and A_2 for all possible shapes of $u(\cdot)$.

Proposition 8 is a surprising result, so it may require some further explanation. One assumption that leads to this result is that $m(\cdot)$ is linear. Empirical evidence for this assumption in the context of regret can be found in Bleichrodt, Cillo and Diecidue (2010). However, even if we allow $m(\cdot)$ to be concave or convex instead, it can be shown that second-order risk attitudes with regret references are solely driven by the shape of $m(\cdot)$. As long as $u(\cdot)$ and $m(\cdot)$ are strictly increasing, a strictly concave (convex, linear) m function implies risk aversion (risk seeking, risk neutrality).[25] This shows that regret references do in this case only contribute to risk aversion because EUT effects are reintroduced into the gain-loss function. Behavior under risk is still completely driven by the shape of pure consumption utility. Since we are interested in the pure effect of regret references without confounding effects of standard EUT preferences, proposition 8 still constitutes an important result. The other assumption that leads to this result is that outcome and reference distributions are stochastically independent. This is also the reason why some models of regret, in which outcomes and foregone alternatives are compared state-wise, conclude that regret leads to

[25] $B_2|A_2 \succ (\prec, \sim)\ A_2|B_2 \Leftrightarrow u(\delta_2) - u(-\delta_2) > (<, =)\ u(\delta_1) - u(-\delta_1)$ where $\delta_1 \equiv m(y + \varepsilon_1) - m(y)$ and $\delta_2 \equiv m(y) - m(y - \varepsilon_1)$. For all strictly increasing m functions $\delta_1, \delta_2 > 0$. Since $\delta_2 > (<, =)\ \delta_1 \Leftrightarrow m''(\cdot) < (>, =)\ 0$, for strictly increasing u functions it holds that $B_2|A_2 \succ (\prec, \sim)\ A_2|B_2 \Leftrightarrow m''(\cdot) < (>, =)\ 0$.

risk-averse behavior (see e.g. Bleichrodt, Cillo and Diecidue, 2010).

Unlike second-order effects, third-order effects are not eliminated by regret references.

Proposition 9 *If preferences depend on reference points formed by regret, individuals are prudent (imprudent, neither nor), that is* $B_3|A_3 \succ (\prec, \sim) A_3|B_3$*, if and only if* $[-k; k+\tilde{\varepsilon}_1+\tilde{\varepsilon}_1']|[0] \succ (\prec, \sim) [k; -k+\tilde{\varepsilon}_1+\tilde{\varepsilon}_1']|[0]$.

Proposition 9 shows under which conditions individuals are prudent if their reference point is determined by regret. Since $M(B_3) = M(A_3)$ holds and in the comparison of $B_3|A_3$ vs. $A_3|B_3$ densities of similar elements can be reduced this choice is therefore equivalent to $[B_3|A_{3\langle A_3|B_3\rangle}]|[0] = [-k; k+\tilde{\varepsilon}_1+\tilde{\varepsilon}_1']|[0]$ vs. $[k; -k+\tilde{\varepsilon}_1+\tilde{\varepsilon}_1']|[0] = [A_3|B_{3\langle B_3|A_3\rangle}]|[0]$. Similar to the definition of prudence by Eeckhoudt and Schlesinger (2006), an individual in proposition 9 is prudent if she prefers to face risk in the higher wealth rather than in the lower wealth state (given her reference point was zero).

Whether individuals with reference points formed by regret are in fact prudent or imprudent crucially depends on the functional form of $u(\cdot)$. If $u'''(\cdot) > (<, =)\ 0$ over the whole range, individuals are prudent (imprudent, neither nor). However, if $u(\cdot)$ is loss-averse S-shaped individuals are never prudent. Corollary 3 also applies to third-order risk preferences under regret. Moreover, if an individual is classified as being prudent (imprudent, neither nor) with disappointment references and a given u function, he will also be prudent (imprudent, neither nor) with regret references under this u function.

Since the usual assumption in the regret literature is that $u'''(\cdot) > 0$, we again observe a clear difference in predicted behavior between expectational and regret references concerning third-order risk preferences.[26] Moreover, since these two reference points yielded already different predictions concerning second-order risk preferences, the implications for precautionary saving and insurance demand could clearly not have been generated by regret

[26]Note that some authors (see e.g. Loomes and Sugden, 1982) use a function $Q(m(\alpha) - m(\beta)) \equiv m(\alpha) - m(\beta) + u(m(\alpha) - m(\beta)) - u(m(\beta) - m(\alpha))$ in order to state that the act is chosen where α obtains in a certain state of the world and β would have obtained by a different choice (if and only if the weighted value of $Q(\cdot)$ is positive). With this formulation, it is usually assumed that $Q(\cdot)$ is convex, which corresponds to $u(\cdot)$ being decreasingly concave (see Bleichrodt, Cillo and Diecidue, 2010). This is equivalent to the assumption that $u'(\cdot) > 0, u''(\cdot) < 0, u'''(\cdot) > 0$.

references.

Unlike for third-order effects, we again receive a result that is independent of the functional form of $u(\cdot)$ for fourth-order effects.

Proposition 10 *If preferences depend on reference points formed by regret, individuals are never temperate or intemperate, that is $B_4|A_4 \sim A_4|B_4$. This holds for all formulations of $u(\cdot)$.*

Proposition 10 shows that reference-dependent preferences with reference points determined by regret can neither be temperate nor intemperate. Thus, there exists no preference to aggregate or disaggregate $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$, two stochastically independent, symmetric and zero-mean random variables.

When looking at second- to fourth-order risk preferences, an interesting pattern can be observed when our results for different endogenous reference points are compared. Regret and expectational references do impose different risk attitudes for all these orders. However, disappointment references impose risk preferences that can be considered as in between expectational and regret references. For a given functional form of gain-loss utility, predicted behavior of disappointment references is generically different compared to expectational but not regret references with respect to the third order, and it is different compared to regret but not expectational references concerning the second and fourth order. This pattern generalizes to any arbitrary order n. By comparing case *(iv)* of proposition 6 to theorem 1 one can easily see that if an individual is risk apportioning of order n with $\frac{n}{2} \in \mathbb{N}$ (its opposite, neither nor) for a given u function under disappointment, she will also be risk apportioning of order n (its opposite, neither nor) under this u function with expectational references. This holds, however, only for even but not for odd orders. In contrast, a similar relationship between disappointment and regret only holds for odd but not for even orders. Comparing case *(iv)* of proposition 6 to theorem 2 shows that if an individual is risk apportioning of order n with $\frac{n}{2} \notin \mathbb{N}$ (its opposite, neither nor) for a given u function under disappointment, she will also be risk apportioning of order n (its opposite, neither nor) with this u function under regret.

Theorem 2 generalizes our results under regret for arbitrary order n.

Theorem 2 *If preferences depend on reference points formed by regret and $n \geq 4$, individuals are*

(i) for odd orders ($\frac{n}{2} \notin \mathbb{N}$) risk apportioning of order n (its opposite, neither nor), that is $B_n|A_n \succ (\prec, \sim)\ A_n|B_n$, if and only if

$$[B_n|A_{n\langle A_n|B_n\rangle}]\,|[0] = \left[A_{n-2}|B_{n-2\langle B_{n-2}|A_{n-2}\rangle}; B_{n-2}|A_{n-2\langle A_{n-2}|B_{n-2}\rangle} + \tilde{\varepsilon}_{(\frac{n-1}{2})} + \tilde{\varepsilon}'_{(\frac{n-1}{2})}\right]|[0]$$
$$\succ (\prec, \sim)$$
$$\left[B_{n-2}|A_{n-2\langle A_{n-2}|B_{n-2}\rangle}; A_{n-2}|B_{n-2\langle B_{n-2}|A_{n-2}\rangle} + \tilde{\varepsilon}_{(\frac{n-1}{2})} + \tilde{\varepsilon}'_{(\frac{n-1}{2})}\right]|[0] = [A_n|B_{n\langle B_n|A_n\rangle}]\,|[0];$$

(ii) for even orders ($\frac{n}{2} \in \mathbb{N}$) never risk apportioning of order n or its opposite, that is $B_n|A_n \sim A_n|B_n$. This holds for all formulations of $u(\cdot)$.

Considering $n = 1$ (proposition 7) we showed that under regret first-order effects are present. The same result was obtained for expectational (proposition 1) as well as disappointment references (case *(i)* in proposition 6). However, with these reference points the result was solely driven by pure consumption utility. By contrast, under regret the gain-loss component of utility contributes to this result as well, since choosing a high and comparing it to a low outcome yields the sensation of rejoice, while choosing the low outcome and comparing it to the high outcome yields a feeling of regret. With $n = 2$, choosing an uncertain outcome and comparing it to a certain outcome with an equal mean can result in sensations of both rejoice and regret. However, choosing a certain outcome and comparing it to the risky alternative yields exactly the same feelings of rejoice and regret. Individuals with regret references therefore do not care whether to take or avoid a risk. Risk enters gain-loss utility either via the outcome or via the reference point. In general, questions of aggregating or disaggregating risk(s) become irrelevant with regret preferences since they enter the evaluation through either the avoided or the chosen alternative. This eliminates even-order risk preferences. Nevertheless, location still matters and makes odd-order effects relevant.

As for theorem 1, a similar, more formal intuition also applies to theorem 2. Since for even orders $-B_n \succ (\prec, \sim)\ -A_n$ if and only if $B_n \succ (\prec, \sim)\ A_n$ (due to the symmetry of $\tilde{\varepsilon}_i$ and the absence of a reduction of k) and with regret references the magnitude of both these opposing effects is equal, an individual is now always indifferent between $B_n|A_n$ and $A_n|B_n$ for even n. For odd orders note that $-B_n \succ (\prec, \sim) \; -A_n$ if and only if $B_n \prec (\succ, \sim) \; A_n$ (again due to the existence of a k reduction in addition to symmetric $\tilde{\varepsilon}_i$). Thus, an individual now prefers $B_n|A_n$ to $A_n|B_n$ if and only if she is risk apportioning of order n.

Comparing theorem 2 to 1 highlights the fact that reference points formed by regret in a way induce opposite risk preferences as those formed by expectations. With regret, preferences of even orders vanish while those of odd orders may well be present. By contrast, with reference points formed by expectations preferences of odd orders are absent and those of even orders are still present. One implication is that reference-dependent risk preferences of any order are only affected by either expectations or regret. This suggests that both forms of endogenous reference points rather supplement than collide with each other. Our findings stress the advantage of jointly analyzing regret and expectational references in a unified framework. Until now, both streams of the literature have been analyzed separately despite the fact that they addressed similar questions.

1.5.3 Exogenous Reference Points

Unlike endogenous reference points like expectational, disappointment, or regret references, exogenous reference points are usually of an ad hoc nature. This is because almost every assumption on exogenous reference points can be supported by some reasoning. In the following, we concentrate on the intuitively most appealing and most frequently discussed reference points.

Assuming the status quo is not only very popular, but it is also easily assessed. Furthermore, it is undeniably exogenous at a given point in time. For these reasons the status quo is advantageous if limited information is available on processes prior to the decision at hand. Also, it does not require any assumption on how individuals form their beliefs over future states. Only the minimum assumption of an individual's awareness of her current state is necessary. Independently of these considerations the status quo may be the natural reference point when decisions come as a surprise. In our analysis wealth is normalized such that the status quo is 0.[27]

Although the status quo is often an obvious candidate for the reference point in experiments, it seems highly sensitive to framing effects. In general, many framing situations are possible and could support various exogenous reference points. If, for instance, an experiment was actually conducted over the gambles we used in our definitions, the reference point could be influenced in several ways. However, we focus firstly on situations where the framing of the choice reduces complexity. These are scenarios in which the subject decides in which situation she accepts an additional 'harm', i.e. $-k$ or $\tilde{\varepsilon}_i$ (see Eeckhoudt and Schlesinger, 2006). As in the experiment of Deck and Schlesinger (2010), which tests for prudence and temperance in the laboratory, a subject has to decide whether to take an unavoidable risk in a high- or low-wealth state (prudence) or in a certain or uncertain state (temperance). Formally, she has to add $\tilde{\varepsilon}_1$ to one of the outcomes in the gamble $[y; y-k]$ (prudence) or $[y; y+\tilde{\varepsilon}_2]$ (temperance) and these gambles may then act as the subject's reference point. This is, however, only the case if the reference point adjusts sufficiently fast. An equally complex framing to measure prudence, which has been used in the experiments of Deck and Schlesinger (2010) and Ebert and Wiesen (2010), would be the task of assigning a sure reduction of k to one of the outcomes in the gamble $[y; y+\tilde{\varepsilon}_1]$. Again, this gamble may then act as subjects' reference point.[28] Secondly, y is the single element that is present in all possible states of the world. It may therefore act as a mental anchor and thus be a natural point of comparison.

In Tables 1.1 to 1.3 we summarize our results for these reference scenarios. The first column defines the assumed exogenous reference point. We computed $B_n|\cdot$ and $A_n|\cdot$ given these reference points with $n \in \{2, 3, 4\}$. In the second column we state what the comparison of these gambles reduces to, namely $[B_n| \cdot {\scriptstyle\langle A_n|\cdot\rangle}]|[0]$ vs. $[A_n| \cdot {\scriptstyle\langle B_n|\cdot\rangle}]|[0]$. If and only if the former gamble is preferred to the latter, $B_n|\cdot \succ A_n|\cdot$. In general, our findings for exogenous

[27]Other deterministic reference points could be generated by past outcomes (habit formation) or self-selected goals (aspiration levels). We do not analyze these scenarios in our static framework.

[28]Note that it is not required to implement the higher-order gamble definitions in such compound ways. In our experiment presented in chapter 4 all gambles were presented in a non-compound format which would suggest using 0 as the reference point even if the adjustment happens fast.

Table 1.1: Second-order effects with exogenous references

reference point	equivalent to evaluating	globally linear	linear w/ loss aversion	globally standard	symmetric S-shape	S-shape (w/ loss aversion)
$\cdot\|[0]$	$[y]\|[0]$ vs. $[y+\tilde{\varepsilon}_1]\|[0]$	$B_2\|\cdot \sim A_2\|\cdot$	$B_2\|\cdot \succsim^1 A_2\|\cdot$	$B_2\|\cdot \succ A_2\|\cdot$	$B_2\|\cdot \overset{\succ}{\underset{\prec}{\sim}}^2 A_2\|\cdot$	$B_2\|\cdot \overset{\succ}{\underset{\prec}{\sim}}^3 A_2\|\cdot$
$\cdot\|[y]$	$[0]\|[0]$ vs. $[\tilde{\varepsilon}_1]\|[0]$	$B_2\|\cdot \sim A_2\|\cdot$	$B_2\|\cdot \succ A_2\|\cdot$	$B_2\|\cdot \succ A_2\|\cdot$	$B_2\|\cdot \sim A_2\|\cdot$	$B_2\|\cdot \succ A_2\|\cdot$

Notes:
1) $B_2|\cdot \succ A_2|\cdot$ if $-\varepsilon_1 < y < \varepsilon_1$; $B_2|\cdot \sim A_2|\cdot$ if $\varepsilon_1 \leq y$ or $y \leq -\varepsilon_1$; thus determined over the full range.
2) $B_2|\cdot \succ A_2|\cdot$ if $0 < y$; $B_2|\cdot \sim A_2|\cdot$ if $y = 0$; $B_2|\cdot \prec A_2|\cdot$ if $y < 0$; thus determined over the full range.
3) $B_2|\cdot \succ A_2|\cdot$ if $\varepsilon_1 \leq y$; $B_2|\cdot \prec A_2|\cdot$ if $y \leq -\varepsilon_1$; generally ambiguous if $-\varepsilon_1 < y < \varepsilon_1$.

reference points depend on the functional form of the evaluation function $u(\cdot)$. As in the previous sections, we again distinguish between common assumptions about $u(\cdot)$ and present their corresponding results in columns three to seven.

Table 1.2: Third-order effects with exogenous references

reference point	equivalent to evaluating	globally linear	linear w/ loss aversion	globally standard	symmetric S-shape	S-shape (w/ loss aversion)
$\cdot\|[0]$	$[y-k; y+\tilde{\varepsilon}_1]\|[0]$ vs. $[y; y-k+\tilde{\varepsilon}_1]\|[0]\|[0]$	$B_3\|\cdot \sim A_3\|\cdot$	$B_3\|\cdot \overset{\succ}{\underset{\prec}{\sim}}^1 A_3\|\cdot$	$B_3\|\cdot \succ A_3\|\cdot$	$B_3\|\cdot \overset{\succ}{\underset{\prec}{\sim}}^2 A_3\|\cdot$	$B_3\|\cdot \overset{\succ}{\underset{\prec}{\sim}}^2 A_3\|\cdot$
$\cdot\|[y]$	$[-k; \tilde{\varepsilon}_1]\|[0]$ vs. $[0; -k+\tilde{\varepsilon}_1]\|[0]$	$B_3\|\cdot \sim A_3\|\cdot$	$B_3\|\cdot \prec A_3\|\cdot$	$B_3\|\cdot \succ A_3\|\cdot$	$B_3\|\cdot \prec A_3\|\cdot$	$B_3\|\cdot \prec A_3\|\cdot$
$\cdot\|[y; y+\tilde{\varepsilon}_1]$	$[-k; 2\tilde{\varepsilon}_1]\|[0]$ vs. $[0; -k+2\tilde{\varepsilon}_1]\|[0]$	$B_3\|\cdot \sim A_3\|\cdot$	$B_3\|\cdot \prec A_3\|\cdot$	$B_3\|\cdot \succ A_3\|\cdot$	$B_3\|\cdot \prec A_3\|\cdot$	$B_3\|\cdot \prec A_3\|\cdot$
$\cdot\|[y; y-k]$	$[-k; k+\tilde{\varepsilon}_1]\|[0]$ vs. $[k; -k+\tilde{\varepsilon}_1]\|[0]$	$B_3\|\cdot \sim A_3\|\cdot$	$B_3\|\cdot \precsim^3 A_3\|\cdot$	$B_3\|\cdot \succ A_3\|\cdot$	$B_3\|\cdot \prec A_3\|\cdot$	$B_3\|\cdot \prec A_3\|\cdot$

Notes:
1) $B_3|\cdot \sim A_3|\cdot$ if $y \leq -\varepsilon_1$, or $\varepsilon_1 + k \leq y$; $B_3|\cdot \prec A_3|\cdot$ if $y = 0$; generally ambiguous if $-\varepsilon_1 < y < \varepsilon_1 + k \wedge y \neq 0$, but $B_3|\cdot \succ A_3|\cdot$ if $k < y < \varepsilon_1$, $B_3|\cdot \sim A_3|\cdot$ if $\varepsilon_1 < y < k - \varepsilon_1$, and $B_3|\cdot \prec A_3|\cdot$ if $k - \varepsilon_1 < y < 0$.
2) $B_3|\cdot \succ (\sim, \prec)\ A_3|\cdot$ if either $y \leq -\varepsilon_1$ or $\varepsilon_1 + k \leq y$, and additionally $u'''(\cdot) > (=, <)\ 0$; generally ambiguous if $-\varepsilon_1 < y < \varepsilon_1 + k$, or the sign of $u'''(\cdot)$ is not constant over the full range.
3) $B_3|\cdot \sim A_3|\cdot$ if $\varepsilon_1 \leq k$; $B_3|\cdot \prec A_3|\cdot$ if $k < \varepsilon_1$; thus determined over the full range.

In column three $u(\cdot)$ is globally linear. In the fourth column we consider piecewise linear gain-loss utility with a unique kink at 0 inducing loss aversion. In the fifth column we analyze a standard u function that has derivatives that alternate in sign with $u'(\cdot)$ being positive. Column six considers a function that exhibits identical diminishing sensitivity for losses and gains. In column seven we report results for S-shaped u functions that are asymmetric, either due to loss aversion or faster diminishing sensitivity in gains than in losses, or both. This functional form is the traditional assumption made in prospect theory. All results in Tables

1.1 to 1.3 are formally derived for two-outcome $\tilde{\varepsilon}_i$, hence $\tilde{\varepsilon}_i = [-\varepsilon_i, \varepsilon_i]$.

Table 1.3: Fourth-order effects with exogenous references

reference point	equivalent to evaluating	globally linear	linear w/ loss aversion	globally standard	symmetric S-shape	S-shape (w/ loss aversion)
$\cdot\|[0]$	$[y+\tilde{\varepsilon}_1; y+\tilde{\varepsilon}_2]\|[0]$ vs. $[y; y+\tilde{\varepsilon}_1+\tilde{\varepsilon}_2]\|[0]$	$B_4\|\cdot \sim A_4\|\cdot$	$B_4\|\cdot \underset{\precsim}{\succ}^{1} A_4\|\cdot$	$B_4\|\cdot \succ A_4\|\cdot$	$B_4\|\cdot \underset{\precsim}{\succ}^{2} A_4\|\cdot$	$B_4\|\cdot \underset{\precsim}{\succ}^{2} A_4\|\cdot$
$\cdot\|[y]$	$[\tilde{\varepsilon}_1; \tilde{\varepsilon}_2]\|[0]$ vs. $[0; \tilde{\varepsilon}_1+\tilde{\varepsilon}_2]\|[0]$	$B_4\|\cdot \sim A_4\|\cdot$	$B_4\|\cdot \prec A_4\|\cdot$	$B_4\|\cdot \succ A_4\|\cdot$	$B_4\|\cdot \sim A_4\|\cdot$	$B_4\|\cdot \prec A_4\|\cdot$
$\cdot\|[y; y+\tilde{\varepsilon}_2]$	$[2\tilde{\varepsilon}_2; \tilde{\varepsilon}_1]\|[0]$ vs. $[0; 2\tilde{\varepsilon}_2+\tilde{\varepsilon}_1]\|[0]$	$B_4\|\cdot \sim A_4\|\cdot$	$B_4\|\cdot \prec A_4\|\cdot$	$B_4\|\cdot \succ A_4\|\cdot$	$B_4\|\cdot \sim A_4\|\cdot$	$B_4\|\cdot \prec A_4\|\cdot$

Notes:
1) $B_4|\cdot \sim A_4|\cdot$ if $\varepsilon_1+\varepsilon_2 \leq y$, $y=0$, or $y \leq -\varepsilon_1-\varepsilon_2$; $B_4|\cdot \succ A_4|\cdot$ if $-\varepsilon_1-\varepsilon_2 < y \leq -\max\{\varepsilon_1,\varepsilon_2\}$, or $\max\{\varepsilon_1,\varepsilon_2\} < y < \varepsilon_1+\varepsilon_2$; generally ambiguous if $-\max\{\varepsilon_1,\varepsilon_2\} < y < \max\{\varepsilon_1,\varepsilon_2\}$.
2) $B_4|\cdot \succ (\sim, \prec)\ A_4|\cdot$ if either $y \leq -\varepsilon_1-\varepsilon_2$ or $\varepsilon_1+\varepsilon_2 \leq y$, and additionally $u''''(\cdot) < (=, >)\ 0$; generally ambiguous if $-\varepsilon_1-\varepsilon_2 < y < \varepsilon_1+\varepsilon_2$, or the sign of $u''''(\cdot)$ is not constant over the full range.

As can be seen from Tables 1.1 to 1.3 results are rather ambiguous in case the reference point is the status quo. This is due to the fact that individuals can end up making gains or losses depending on the specific parameter combinations of y, k, ε_1, and ε_2. In the other reference scenarios initial wealth y cancels out since it is part of both the reference point and the gambles evaluated. Results for these reference scenarios are therefore more straightforward.[29] Generally, they vary widely with the functional form considered. Nevertheless, a few tendencies seem prevalent. While not stated explicitly, exogenous reference points also yield monotone preferences. As already extensively discussed in the existing literature, we further find that loss aversion induces risk aversion. But, more surprisingly, loss aversion also induces imprudence and intemperance in many cases. Assuming a S-shaped u function does only generate risk aversion in the asymmetric case (column seven). Such a requirement is not needed for the third order. Here, an S-shaped u function always induces imprudence in cases where y is an element of the reference point. These results clearly show that exogenous reference points do not yield the same predictions on third-order risk preferences as expectational references. Again, the implications of expectational references we derived for precautionary saving and insurance demand would therefore not obtain with exogenous

[29] Note that in case of a fixed, real-numbered reference point, as in the first two rows of Tables 1.1 to 1.3, our reference-dependent model reduces to a general EUT model if the functional form of $u(\cdot)$ does not depend on the reference point (see Wakker, 2005).

reference points.

1.6 Conclusion

Models of reference-dependent preferences have been proposed to resolve empirical puzzles concerning second-order risk preferences. There has been no attempt to consider higher orders, like the third-order effect of prudence or the fourth-order effect of temperance, in these models in order to see whether they can resolve empirical puzzles concerning higher orders as well. This chapter is a first attempt in this direction. We first analyzed higher-order risk preferences in Kőszegi and Rabin's (2006, 2007) model of expectation-based reference dependence. Risk preferences of odd orders (except order one) were generally found to be absent while even-order risk attitudes still exist in their model. These results show that higher-order risk preferences under reference dependence follow a completely different pattern than in classical EUT models as they do not depend on the previous order. For example, if a certain order delivers indifference in EUT, the next higher order has to deliver indifference as well. This is not the case under reference dependence. Therefore, our results have strong implications in situations where characteristics of one order are used to derive statements on a succeeding order. We considered precautionary saving and optimal demand for insurance as illustrating examples for the implications of our results. In a simple model we showed that seemingly sub-optimal amounts of precautionary saving under EUT may well be optimal under reference dependence. Also, the importance of insurance is less mitigated by buffer-stock saving with such preferences. These implications were formally derived in a two-period model. In order to verify whether the observed empirical patterns also match the optimal behavior of individuals under reference dependence quantitatively, a dynamic analysis would be necessary. However, this would additionally require a dynamic model of belief formation concerning own behavior in all future periods. Such a task is left for future research.

In a second step, we further showed that alternative behavioral models cannot resolve these empirical puzzles as they predict different patterns of risk attitudes toward higher orders. In this robustness analysis we considered other endogenous as well as exogenous reference points. While exogenous reference points often seem to be of an ad-hoc nature, endogenous reference points generally account for the fact that individuals anticipate how their behavior will have a feedback on their preferences. With reference points being endogenously formed by the alternative choice, as in models of regret, risk preferences toward even orders are absent while those of odd orders are present. With reference points being endogenously formed by the mean of expectations, as in models of disappointment, results can be considered as in between expectational and regret references. Note that all these models predict monotone preferences. However, only concerning higher *even* orders expectation-based reference dependence and disappointment models yield the same predicted behavior and only with respect to higher *odd* orders models of regret and disappointment yield the same behavior. Regret models and expectation-based reference dependence never induce the same behavior as they yield opposite patterns of higher-order risk attitudes.

Together with our results on exogenous reference points, which generally differ from expectation-based reference dependence, our robustness analysis clearly showed that higher-order risk preferences under expectation-based reference dependence follow a different pattern than under alternative assumptions on the reference point. Alternative behavioral models can therefore in general not resolve the same empirical puzzles that can be resolved with expectation-based reference dependence. Besides this observation, comparing our results on endogenous reference points offers further interesting insights. On the one hand, they emphasize a new way, for instance via third-order risk preferences, to discriminate between expectation-based reference dependence and models of disappointment in laboratory experiments. This has not been possible in the past and the gamble definitions of Eeckhoudt and Schlesinger (2006) we used in our analysis are especially advantageous in this respect. On the other hand, they show that models of regret and expectation-based reference dependence cannot influence risk preferences of a certain order simultaneously. Thus, risk preferences of a particular order can only be influenced by either expectational or regret references. The notion that expectations and regret influence human decision making under uncertainty in distinctively different ways is also suggested by neurological research. For instance, Camille et al. (2004) and Coricelli et al. (2005) find that counterfactual thinking on the alternative choice is activated in a different brain region than counterfactual thinking on one's choice taken.

Although we analyzed the implications of our results for two specific examples only, i.e. precautionary saving and insurance demand, they may well be of significance in other areas as well where higher orders have shown to play an important role. For instance, they could lead to further insights for asset demand under background risk or for precautionary labor supply. They might even influence normative criteria like the precautionary principle that is frequently used in international treaties and environmental regulation. Future research should therefore analyze further applications where our results on reference-dependent risk preferences of higher orders may be helpful to yield better predictions than those of classical EUT models.

1.7 Appendix: Proofs

Proof. **[Proposition 1]** Since $U(B_1|B_1) = u(m(y) - m(y)) = u(0)$ and $U(A_1|A_1) = u(m(y-k) - m(y-k)) = u(0)$, it follows that $U(B_1|B_1) = U(A_1|A_1)$. Then, $V(B_1|B_1) = M(B_1) + U(B_1|B_1) = m(y) + u(0)$ and $V(A_1|A_1) = M(A_1) + U(A_1|A_1) = m(y-k) + u(0)$. Since $m(\cdot)$ is strictly increasing and $k > 0$, it follows that $V(B_1|B_1) > V(A_1|A_1)$ and thus $B_1|B_1 \succ A_1|A_1$. ■

Lemma 1 *$M(B_n) = M(A_n)$ holds for all $n \geq 2$. It therefore holds for all $n \geq 2$ that*

$$B_n|C \succ (\prec, \sim)\ A_n|C \Leftrightarrow V(B_n|C) > (<, =)\ V(A_n|C) \Leftrightarrow U(B_n|C) > (<, =)\ U(A_n|C)$$

where C is any arbitrary gamble.

Proof. **[Lemma 1]** From $\mathbb{E}[\tilde{\varepsilon}_1] = 0$, the linearity of $m(\cdot)$, and the definitions of B_2, A_2, B_3, A_3, it follows that $M(B_2) = M(A_2)$ and $M(B_3) = M(A_3)$. Using these results, $\mathbb{E}[\tilde{\varepsilon}_i] = 0\ \forall i$, and the linearity of $m(\cdot)$, it follows from the definitions of B_n and A_n that $M(B_n) = M(A_n)$

also holds for $n \geq 4$. Thus, for all $n \geq 2$ and C being any arbitrary gamble it holds that $B_n|C \succ (\prec, \sim)\ A_n|C \Leftrightarrow V(B_n|C) > (<, =)\ V(A_n|C) \Leftrightarrow U(B_n|C) > (<, =)\ U(A_n|C)$. ■

Proof. [Proposition 2] From lemma 1 it follows that $B_2|B_2 \succ (\prec, \sim)\ A_2|A_2 \Leftrightarrow$ $U(B_2|B_2) > (<, =)\ U(A_2|A_2)$. Evaluating $B_2|B_2$ by $U(\cdot)$ means getting $B_2 = y$ and also expecting $B_2 = y$. Therefore,

$$U(B_2|B_2) = u(m(y) - m(y)) = u(0).$$

Evaluating $A_2|A_2$ by $U(\cdot)$ means getting $A_2 = y + \tilde{\varepsilon}_1$ and also expecting $A_2 = y + \tilde{\varepsilon}'_1$. Therefore,

$$\begin{aligned} U(A_2|A_2) &= \iint u(m(y + \tilde{\varepsilon}_1) - m(y + \tilde{\varepsilon}'_1))dF(\tilde{\varepsilon}_1)dF(\tilde{\varepsilon}'_1) \\ &= \iint u(\tilde{\varepsilon}_1 + \tilde{\varepsilon}'_1)dF(\tilde{\varepsilon}_1)dF(\tilde{\varepsilon}'_1), \end{aligned}$$

where the last equality holds because of the symmetry of all $\tilde{\varepsilon}_i$ and the linearity of $m(\cdot)$. From $\mathbb{E}[\tilde{\varepsilon}_1] = 0$, $\mathbb{E}[\tilde{\varepsilon}'_1] = 0$ and the stochastic independence of $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}'_1$ it follows that $\mathbb{E}[\tilde{\varepsilon}_1 + \tilde{\varepsilon}'_1] = 0$. From $\mathbb{E}[\tilde{\varepsilon}_1 + \tilde{\varepsilon}'_1] = 0$ and the linearity of $m(\cdot)$ it follows that $M([\tilde{\varepsilon}_1 + \tilde{\varepsilon}'_1]) = M([0])$. Thus, we can state that

$$V(B_2|B_2) > (<, =)\ V(A_2|A_2) \Leftrightarrow V([0]|[0]) > (<, =)\ V([\tilde{\varepsilon}_1 + \tilde{\varepsilon}'_1]|[0]),$$

and therefore

$$B_2|B_2 \succ (\prec, \sim)\ A_2|A_2 \Leftrightarrow [0]|[0] \succ (\prec, \sim)\ [\tilde{\varepsilon}_1 + \tilde{\varepsilon}'_1]|[0].$$

Alternatively, we can also state that both gambles reduce to

$$[B_2|B_{2\langle A_2|A_2\rangle}]\,|[0] = [0]|[0] \quad \text{and} \quad [A_2|A_{2\langle B_2|B_2\rangle}]\,|[0] = [\tilde{\varepsilon}_1 + \tilde{\varepsilon}'_1]|[0]$$

when compared with each other. ■

Proof. [Proposition 3] Evaluating $B_3|B_3$ by $U(\cdot)$ delivers

$$\begin{aligned} U(B_3|B_3) &= \iint \left[\tfrac{1}{4}u(m(y-k)-m(y-k)) + \tfrac{1}{4}u(m(y-k)-m(y+\tilde{\varepsilon}_1')) \right. \\ &\quad \left. +\tfrac{1}{4}u(m(y+\tilde{\varepsilon}_1)-m(y-k)) + \tfrac{1}{4}u(m(y+\tilde{\varepsilon}_1)-m(y+\tilde{\varepsilon}_1'))\right] dF(\tilde{\varepsilon}_1)dF(\tilde{\varepsilon}_1') \\ &= \iint \left[\tfrac{1}{4}u(0) + \tfrac{1}{4}u(-k+\tilde{\varepsilon}_1') + \tfrac{1}{4}u(k+\tilde{\varepsilon}_1) + \tfrac{1}{4}u(\tilde{\varepsilon}_1+\tilde{\varepsilon}_1')\right] dF(\tilde{\varepsilon}_1)dF(\tilde{\varepsilon}_1'), \end{aligned}$$

where the last equality holds because of the symmetry of all $\tilde{\varepsilon}_i$ and the linearity of $m(\cdot)$. Evaluating $A_3|A_3$ by $U(\cdot)$ delivers

$$\begin{aligned} U(A_3|A_3) &= \iint \left[\tfrac{1}{4}u(m(y)-m(y)) + \tfrac{1}{4}u(m(y)-m(y-k+\tilde{\varepsilon}_1')) \right. \\ &\quad \left. +\tfrac{1}{4}u(m(y-k+\tilde{\varepsilon}_1)-m(y)) + \tfrac{1}{4}u(m(y-k+\tilde{\varepsilon}_1)-m(y-k+\tilde{\varepsilon}_1'))\right] \\ &\quad dF(\tilde{\varepsilon}_1)dF(\tilde{\varepsilon}_1') \\ &= \iint \left[\tfrac{1}{4}u(0) + \tfrac{1}{4}u(k+\tilde{\varepsilon}_1') + \tfrac{1}{4}u(-k+\tilde{\varepsilon}_1) + \tfrac{1}{4}u(\tilde{\varepsilon}_1+\tilde{\varepsilon}_1')\right] dF(\tilde{\varepsilon}_1)dF(\tilde{\varepsilon}_1'), \end{aligned}$$

where the last equality holds because of the symmetry of all $\tilde{\varepsilon}_i$. Since $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_1'$ are identically distributed we can conclude that $U(B_3|B_3) = U(A_3|A_3)$ regardless of the shape of $u(\cdot)$. Lemma 1 showed that $M(B_3) = M(A_3)$. From $M(B_3) = M(A_3)$ and $U(B_3|B_3) = U(A_3|A_3)$ it follows that $V(B_3|B_3) = V(A_3|A_3)$ and thus $B_3|B_3 \sim A_3|A_3$. Alternatively, we can also state that both gambles reduce to

$$[B_3|B_{3\langle A_3|A_3\rangle}]\,|[0] = [C]|[0] \quad \text{and} \quad [A_3|A_{3\langle B_3|B_3\rangle}]\,|[0] = [C]|[0] \tag{1.16}$$

when compared with each other, where C denotes any arbitrary gamble. ■

Proof. [Proposition 4] Follows directly from theorem 1. ■

Proof. [Theorem 1] To prove both part (i) and part (ii) of theorem 1 (where we use lemma 1, $\tilde{\varepsilon}_i$ being symmetric with $\mathbb{E}[\tilde{\varepsilon}_i] = 0\ \forall i$, and the linearity of $m(\cdot)$) it helps to show first that for all $n \geq 3$

$$[B_n|B_{n\langle A_n|A_n\rangle}]\,|[0] = \Big[B_{n-2}|B_{n-2\langle A_{n-2}|A_{n-2}\rangle}; A_{n-2}|A_{n-2\langle B_{n-2}|B_{n-2}\rangle} + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})} + \tilde{\varepsilon}'_{\text{int}(\frac{n}{2})}\Big]\,|[0], \tag{1.17}$$

$$[A_n|A_{n\langle B_n|B_n\rangle}]\,|[0] = \Big[A_{n-2}|A_{n-2\langle B_{n-2}|B_{n-2}\rangle}; B_{n-2}|B_{n-2\langle A_{n-2}|A_{n-2}\rangle} + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})} + \tilde{\varepsilon}'_{\text{int}(\frac{n}{2})}\Big]\,|[0]. \tag{1.18}$$

To show this, note that $[B_n|B_{n\langle A_n|A_n\rangle}]|[0]$ and $[A_n|A_{n\langle B_n|B_n\rangle}]|[0]$ can be expressed as gambles containing only B_n and A_n.

$$B_n|B_n = [B_n - B_n]\,|[0] \quad \text{and} \quad A_n|A_n = [A_n - A_n]\,|[0]$$
$$\Rightarrow [B_n|B_{n\langle A_n|A_n\rangle}]|[0] = [B_n - B_n]\,|[0] \quad \text{and} \quad [A_n|A_{n\langle B_n|B_n\rangle}]|[0] = [A_n - A_n]\,|[0],$$

and therefore

$$[B_{n-2}|B_{n-2\langle A_{n-2}|A_{n-2}\rangle}]|[0] = [B_{n-2} - B_{n-2}]\,|[0] \tag{1.19}$$
$$\text{and} \quad [A_{n-2}|A_{n-2\langle B_{n-2}|B_{n-2}\rangle}]|[0] = [A_{n-2} - A_{n-2}]\,|[0]. \tag{1.20}$$

However, $[B_n|B_{n\langle A_n|A_n\rangle}]|[0]$ and $[A_n|A_{n\langle B_n|B_n\rangle}]|[0]$ can also be expressed as gambles containing only B_{n-2} and A_{n-2} since $B_n = [A_{n-2}; B_{n-2} + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}]$ and $A_n = [B_{n-2}; A_{n-2} + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}]$.

$$\begin{aligned}
B_n|B_n &= \Big[A_{n-2} - A_{n-2}; A_{n-2} - B_{n-2} + \tilde{\varepsilon}'_{\text{int}(\frac{n}{2})}; B_{n-2} - A_{n-2} + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}; \\
&\quad B_{n-2} - B_{n-2} + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})} + \tilde{\varepsilon}'_{\text{int}(\frac{n}{2})}\Big]\,|[0] \\
\text{and} \quad A_n|A_n &= \Big[B_{n-2} - B_{n-2}; B_{n-2} - A_{n-2} + \tilde{\varepsilon}'_{\text{int}(\frac{n}{2})}; A_{n-2} - B_{n-2} + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}; \\
&\quad A_{n-2} - A_{n-2} + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})} + \tilde{\varepsilon}'_{\text{int}(\frac{n}{2})}\Big]\,|[0]
\end{aligned}$$

$$\Rightarrow \quad [B_n|B_{n\langle A_n|A_n\rangle}]|[0] = \Big[A_{n-2} - A_{n-2}; B_{n-2} - B_{n-2} + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})} + \tilde{\varepsilon}'_{\text{int}(\frac{n}{2})}\Big]\,|[0] \tag{1.21}$$
$$\text{and} \quad [A_n|A_{n\langle B_n|B_n\rangle}]|[0] = \Big[B_{n-2} - B_{n-2}; A_{n-2} - A_{n-2} + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})} + \tilde{\varepsilon}'_{\text{int}(\frac{n}{2})}\Big]\,|[0], \tag{1.22}$$

because two of the elements cancel out when the two gambles are compared with each other. Finally, inserting (1.19) and (1.20) into (1.21) and (1.22) delivers (1.17) and (1.18). Part (i) of theorem 1 follows directly from (1.17) and (1.18) when we note that for $\frac{n}{2} \in \mathbb{N} \Rightarrow \text{int}(\frac{n}{2}) = \frac{n}{2}$.

In order to prove part (ii) of theorem 1 ($\frac{n}{2} \notin \mathbb{N}$) we again use (1.17) and (1.18). Suppose $n = 5$, then

$$\begin{aligned} [B_5|B_{5\langle A_5|A_5\rangle}]|[0] &= [A_3|A_{3\langle B_3|B_3\rangle}; B_3|B_{3\langle A_3|A_3\rangle} + \tilde{\varepsilon}_2 + \tilde{\varepsilon}_2']\,|[0] && (1.23) \\ \text{and} \quad [A_5|A_{5\langle B_5|B_5\rangle}]|[0] &= [B_3|B_{3\langle A_3|A_3\rangle}; A_3|A_{3\langle B_3|B_3\rangle} + \tilde{\varepsilon}_2 + \tilde{\varepsilon}_2']\,|[0]. && (1.24) \end{aligned}$$

By inserting (1.16) into (1.23) and (1.24) and canceling $\tilde{\varepsilon}_2$ and $\tilde{\varepsilon}_2'$ in both gambles we then get

$$\begin{aligned} &[B_5|B_{5\langle A_5|A_5\rangle}]|[0] = [C]|[0] \quad \text{and} \quad [A_5|A_{5\langle B_5|B_5\rangle}]|[0] = [C]|[0]. && (1.25) \\ &\Rightarrow \qquad B_5|B_5 \quad \sim \quad A_5|A_5. \end{aligned}$$

Since (1.25), by the same logic it must also hold that $[B_7|B_{7\langle A_7|A_7\rangle}]|[0] = [C]|[0]$ and $[A_7|A_{7\langle B_7|B_7\rangle}]|[0] = [C]|[0]$. Continuing this reasoning yields

$$\begin{aligned} &[B_n|B_{n\langle A_n|A_n\rangle}]|[0] = [C]|[0] \quad \text{and} \quad [A_n|A_{n\langle B_n|B_n\rangle}]|[0] = [C]|[0] \\ &\Rightarrow \qquad B_n|B_n \quad \sim \quad A_n|A_n \end{aligned}$$

for $\frac{n}{2} \notin \mathbb{N}$ and C being any arbitrary gamble. ■

Proof. [Proposition 5] First, we show that the second-order conditions are met for the optimal values in the benchmark model. The second partial derivatives of $V^b(\cdot)$ are

$$\begin{aligned} \frac{\partial^2 V^b}{(\partial k^i)^2} &= (1+\lambda)^2 c''(y - k^s - (1+\lambda)k^i) + 2c''(y - \varepsilon_1 + k^s + 2k^i) < 0, \\ \frac{\partial^2 V^b}{(\partial k^s)^2} &= c''(y - k^s - (1+\lambda)k^i) + \tfrac{1}{2}c''(y - \varepsilon_1 + k^s + 2k^i) + \tfrac{1}{2}c''(y + \varepsilon_1 + k^s) < 0, \\ \frac{\partial^2 V^b}{\partial k^s \partial k^i} &= (1+\lambda)c''(y - k^s - (1+\lambda)k^i) + c''(y - \varepsilon_1 + k^s + 2k^i) < 0. \end{aligned}$$

Then, the determinant of the Hessian in the benchmark case reduces to

$$
\begin{aligned}
|\mathcal{H}^b| &= \tfrac{1}{2}\left(1+\lambda^2\right) c''(y-\varepsilon_1+k^s+2k^i)c''(y-k^s-(1+\lambda)k^i) \\
&\quad +\tfrac{1}{2}c''(y+\varepsilon_1+k^s)\left[(1+\lambda)^2 c''(y-k^s-(1+\lambda)k^i)+2c''(y-\varepsilon_1+k^s+2k^i)\right] > 0.
\end{aligned}
$$

Now, we show that the second-order conditions are met for the values solving the first-order conditions in the hybrid model. Then conditions (1.13) and (1.15) in fact have to hold in the optimum. The second partial derivatives of $V^h(\cdot)$ are

$$
\begin{aligned}
\tfrac{\partial^2 V^h}{(\partial k^i)^2} &= (1+\lambda)^2 c''(y-k^s-(1+\lambda)k^i)+2c''(y-\varepsilon_1+k^s+2k^i) \\
&\quad +u''(2\varepsilon-2k^i)+u''(-2\varepsilon+2k^i) < 0, && (1.26)\\
\tfrac{\partial^2 V^h}{(\partial k^s)^2} &= c''(y-k^s-(1+\lambda)k^i)+\tfrac{1}{2}c''(y-\varepsilon_1+k^s+2k^i)+\tfrac{1}{2}c''(y+\varepsilon_1+k^s) \\
&< 0, && (1.27)\\
\tfrac{\partial^2 V^h}{\partial k^s \partial k^i} &= (1+\lambda)c''(y-k^s-(1+\lambda)k^i)+c''(y-\varepsilon_1+k^s+2k^i) < 0, && (1.28)
\end{aligned}
$$

where we used the assumption of $u''(\tau) \leq -u''(-\tau)\ \forall \tau > 0$ to derive (1.26). The determinant of the Hessian in the hybrid case reduces to

$$
\begin{aligned}
|\mathcal{H}^h| &= \tfrac{1}{2}\left(1+\lambda^2\right) c''(y-\varepsilon_1+k^s+2k^i)c''(y-k^s-(1+\lambda)k^i) \\
&\quad +\tfrac{1}{2}c''(y+\varepsilon_1+k^s)\left[(1+\lambda)^2 c''(y-k^s-(1+\lambda)k^i)+2c''(y-\varepsilon_1+k^s+2k^i)\right] \\
&\quad +\tfrac{\partial^2 V^h}{(\partial k^s)^2}\left[u''(2\varepsilon-2k^i)+u''(-2\varepsilon+2k^i)\right] > 0, && (1.29)
\end{aligned}
$$

where we again used the assumption $u''(\tau) \leq -u''(-\tau)\ \forall \tau > 0$ as well as (1.27). We can now conclude from (1.15) that $dk^i > 0$ has to hold if we use (1.27) and (1.29). Using this result it follows from (1.13), (1.27), and (1.28) that $dk^s < 0$ which completes the proof. ■

Proof. **[Proposition 6]** Parts (i)-(iv) of proposition 6 will be proved subsequently. Part (i) follows directly from the proof of proposition 1 since $B_1 = \mathbb{E}[B_1]$ and $A_1 = \mathbb{E}[A_1]$.

For proving part (ii), first note that from lemma 1 it follows that $B_2|\mathbb{E}[B_2] \succ (\prec, \sim)$

$A_2|\mathbb{E}[A_2] \Leftrightarrow U(B_2|\mathbb{E}[B_2]) > (<,=)\, U(A_2|\mathbb{E}[A_2])$. Evaluating $B_2|\mathbb{E}[B_2]$ by $U(\cdot)$ means getting $B_2 = y$ and expecting $\mathbb{E}[B_2] = y$. Thus,

$$U(B_2|\mathbb{E}[B_2]) = u(m(y) - m(y)) = u(0).$$

Evaluating $A_2|\mathbb{E}[A_2]$ by $U(\cdot)$ means getting $A_2 = y + \tilde{\varepsilon}_1$ and expecting $\mathbb{E}[A_2] = y$ since $\mathbb{E}[\tilde{\varepsilon}_1'] = 0$. Therefore,

$$\begin{aligned} U(A_2|\mathbb{E}[A_2]) &= \int u(m(y+\tilde{\varepsilon}_1) - m(\int (y + \tilde{\varepsilon}_1') dF(\tilde{\varepsilon}_1'))) dF(\tilde{\varepsilon}_1) \\ &= \int u(m(y+\tilde{\varepsilon}_1) - m(y)) dF(\tilde{\varepsilon}_1) = \int u(\tilde{\varepsilon}_1) dF(\tilde{\varepsilon}_1), \end{aligned}$$

where the last equality holds because of the linearity of $m(\cdot)$. From $\mathbb{E}[\tilde{\varepsilon}_1] = 0$ and $m(\cdot)$ being linear it follows that $M([\tilde{\varepsilon}_1]) = M([0])$. We can therefore state that

$$V(B_2|\mathbb{E}[B_2]) > (<,=)\, V(A_2|\mathbb{E}[A_2]) \Leftrightarrow V([0]|[0]) > (<,=)\, V([\tilde{\varepsilon}_1]|[0]),$$

and hence

$$B_2|\mathbb{E}[B_2] \succ (\prec,\sim)\, A_2|\mathbb{E}[A_2] \Leftrightarrow [0]|[0] \succ (\prec,\sim)\, [\tilde{\varepsilon}_1]|[0].$$

For proving part (iii) we first note that evaluating $B_3|\mathbb{E}[B_3]$ by $U(\cdot)$ delivers

$$\begin{aligned} U(B_3|\mathbb{E}[B_3]) &= \int \Big[\tfrac{1}{2} u(m(y-k) - m(\int (\tfrac{1}{2}(y-k) + \tfrac{1}{2}(y+\tilde{\varepsilon}_1')) dF(\tilde{\varepsilon}_1'))) \\ &\quad + \tfrac{1}{2} u(m(y+\tilde{\varepsilon}_1) - m(\int (\tfrac{1}{2}(y-k) + \tfrac{1}{2}(y+\tilde{\varepsilon}_1')) dF(\tilde{\varepsilon}_1'))) \Big] dF(\tilde{\varepsilon}_1) \\ &= \int \left[\tfrac{1}{2} u(m(y-k) - m(y - \tfrac{k}{2})) + \tfrac{1}{2} u(m(y+\tilde{\varepsilon}_1) - m(y - \tfrac{k}{2})) \right] dF(\tilde{\varepsilon}_1) \\ &= \int \left[\tfrac{1}{2} u(-\tfrac{k}{2}) + \tfrac{1}{2} u(\tfrac{k}{2} + \tilde{\varepsilon}_1) \right] dF(\tilde{\varepsilon}_1), \end{aligned}$$

where the second equality holds because $\mathbb{E}[\tilde{\varepsilon}_1'] = 0$ and the last equality holds since $m(\cdot)$ is linear. Evaluating $A_3|\mathbb{E}[A_3]$ by $U(\cdot)$ yields

$$\begin{aligned}
U(A_3|\mathbb{E}[A_3]) &= \int \Big[\tfrac{1}{2}u(m(y-k+\tilde{\varepsilon}_1) - m(\int (\tfrac{1}{2}(y-k+\tilde{\varepsilon}_1') + \tfrac{1}{2}y)dF(\tilde{\varepsilon}_1'))) \\
&\quad + \tfrac{1}{2}u(m(y) - m(\int (\tfrac{1}{2}(y-k+\tilde{\varepsilon}_1') + \tfrac{1}{2}y)dF(\tilde{\varepsilon}_1')))\Big]\, dF(\tilde{\varepsilon}_1) \\
&= \int \left[\tfrac{1}{2}u(m(y-k+\tilde{\varepsilon}_1) - m(y-\tfrac{k}{2})) + \tfrac{1}{2}u(m(y) - m(y-\tfrac{k}{2}))\right] dF(\tilde{\varepsilon}_1) \\
&= \int \left[\tfrac{1}{2}u(-\tfrac{k}{2}+\tilde{\varepsilon}_1) + \tfrac{1}{2}u(\tfrac{1}{2})\right] dF(\tilde{\varepsilon}_1),
\end{aligned}$$

where again the second equality holds because $\mathbb{E}[\tilde{\varepsilon}_1'] = 0$ and the last equality holds since $m(\cdot)$ is linear. Lemma 1 showed that $M(B_3) = M(A_3)$. From $\mathbb{E}[\tilde{\varepsilon}_1] = 0$ and the linearity of $m(\cdot)$ it follows that $M([-\frac{k}{2}; \frac{k}{2}+\tilde{\varepsilon}_1]) = M([-\frac{k}{2}+\tilde{\varepsilon}_1; \frac{k}{2}])$, and we can therefore state that

$$V(B_3|\mathbb{E}[B_3]) > (<,=)\ V(A_3|\mathbb{E}[A_3]) \Leftrightarrow V([-\tfrac{k}{2}; \tfrac{k}{2}+\tilde{\varepsilon}_1]|[0]) > (<,=)\ V([-\tfrac{k}{2}+\tilde{\varepsilon}_1; \tfrac{k}{2}]|[0]),$$

and hence

$$B_3|\mathbb{E}[B_3] \succ (\prec,\sim)\ A_3|\mathbb{E}[A_3] \Leftrightarrow [-\tfrac{k}{2}; \tfrac{k}{2}+\tilde{\varepsilon}_1]|[0] \succ (\prec,\sim)\ [-\tfrac{k}{2}+\tilde{\varepsilon}_1; \tfrac{k}{2}]|[0].$$

In order to prove part (iv) note that from the definitions of B_n and A_n it follows that $\mathbb{E}[B_n] = \mathbb{E}[A_n] = y$ if $\frac{n}{2} \in \mathbb{N} \wedge n \geq 2$ and that $\mathbb{E}[B_n] = \mathbb{E}[A_n] = y - \frac{k}{2}$ if $\frac{n}{2} \notin \mathbb{N} \wedge n \geq 3$. Consider first the case where $\frac{n}{2} \in \mathbb{N} \wedge n \geq 4$. Then, $B_n|\mathbb{E}[B_n] = [B_n - y]$ and $A_n|\mathbb{E}[A_n] = [A_n - y]$ holds for all $n \geq 2$ with $\frac{n}{2} \in \mathbb{N}$. Since $B_n = [A_{n-2}; B_{n-2} + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}]$ and $A_n = [B_{n-2}; A_{n-2} + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}]$ we get for any arbitrary $n \geq 4$ with $\frac{n}{2} \in \mathbb{N}$ that

$$\begin{aligned}
B_n|\mathbb{E}[B_n] &= [A_{n-2} - y; B_{n-2} - y + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}]|[0] \\
&= [A_{n-2}|\mathbb{E}[A_{n-2}]; B_{n-2}|\mathbb{E}[B_{n-2}] + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}]|[0], \\
\text{and} \quad A_n|\mathbb{E}[A_n] &= [B_{n-2} - y; A_{n-2} - y + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}]|[0] \\
&= [B_{n-2}|\mathbb{E}[B_{n-2}]; A_{n-2}|\mathbb{E}[A_{n-2}] + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}]|[0].
\end{aligned}$$

Now consider the other case where $\frac{n}{2} \notin \mathbb{N} \wedge n \geq 5$. Then, $B_n|\mathbb{E}[B_n] = [B_n - (y - \frac{k}{2})]$ and $A_n|\mathbb{E}[A_n] = [A_n - (y - \frac{k}{2})]$ holds for all $n \geq 3$ with $\frac{n}{2} \notin \mathbb{N}$. Since $B_n = [A_{n-2}; B_{n-2} + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}]$ and $A_n = [B_{n-2}; A_{n-2} + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}]$ we get for any arbitrary $n \geq 5$ with $\frac{n}{2} \notin \mathbb{N}$ that

$$\begin{aligned} B_n|\mathbb{E}[B_n] &= [A_{n-2} - (y - \tfrac{k}{2}); B_{n-2} - (y - \tfrac{k}{2}) + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}]|[0] \\ &= [A_{n-2}|\mathbb{E}[A_{n-2}]; B_{n-2}|\mathbb{E}[B_{n-2}] + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}]|[0], \\ \text{and} \quad A_n|\mathbb{E}[A_n] &= [B_{n-2} - (y - \tfrac{k}{2}); A_{n-2} - (y - \tfrac{k}{2}) + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}]|[0] \\ &= [B_{n-2}|\mathbb{E}[B_{n-2}]; A_{n-2}|\mathbb{E}[A_{n-2}] + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}]|[0]. \end{aligned}$$

Lemma 1 showed that $M(B_n) = M(A_n)\ \forall n \geq 2$. From the fact that $\mathbb{E}[\tilde{\varepsilon}_i] = 0$ always holds and the linearity of $m(\cdot)$ it follows that $M([A_{n-2}|\mathbb{E}[A_{n-2}]; B_{n-2}|\mathbb{E}[B_{n-2}] + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}]) = M([B_{n-2}|\mathbb{E}[B_{n-2}]; A_{n-2}|\mathbb{E}[A_{n-2}] + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}])$, and we can therefore state that

$$\begin{aligned} & V(B_n|\mathbb{E}[B_n]) > (<,=)\ V(A_n|\mathbb{E}[A_n]) \\ \Leftrightarrow\ & V([A_{n-2}|\mathbb{E}[A_{n-2}]; B_{n-2}|\mathbb{E}[B_{n-2}] + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}]|[0]) \\ & > (<,=)\ V([B_{n-2}|\mathbb{E}[B_{n-2}]; A_{n-2}|\mathbb{E}[A_{n-2}] + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}]|[0]), \end{aligned}$$

and hence

$$\begin{aligned} & B_n|\mathbb{E}[B_n] \succ (\prec,\sim)\ A_n|\mathbb{E}[A_n] \\ \Leftrightarrow\ & [A_{n-2}|\mathbb{E}[A_{n-2}]; B_{n-2}|\mathbb{E}[B_{n-2}] + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}]|[0] \\ & \succ (\prec,\sim)\ [B_{n-2}|\mathbb{E}[B_{n-2}]; A_{n-2}|\mathbb{E}[A_{n-2}] + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}]|[0]. \end{aligned}$$

■

Proof. [Proposition 7] Since $U(B_1|A_1) = u(m(y) - m(y-k)) = u(k)$ and $U(A_1|B_1) = u(m(y-k) - m(y)) = u(-k)$ (because of the linearity of $m(\cdot)$), it follows that $V(B_1|A_1) = M(B_1) + U(B_1|A_1) = m(y) + u(k)$ and $V(A_1|A_1) = M(A_1) + U(A_1|B_1) = m(y-k) + u(-k)$. Since $m(\cdot)$ and $u(\cdot)$ are strictly increasing and $k > 0$ it follows that $V(B_1|A_1) > V(A_1|B_1)$ and therefore $B_1|A_1 \succ A_1|B_1$. ■

Proof. [Proposition 8] Evaluating $B_2|A_2$ by $U(\cdot)$ delivers

$$U(B_2|A_2) = \int u(m(y) - m(y + \tilde{\varepsilon}_1'))dF(\tilde{\varepsilon}_1') = \int u(\tilde{\varepsilon}_1')dF(\tilde{\varepsilon}_1').$$

Evaluating $A_2|B_2$ by $U(\cdot)$ delivers

$$U(A_2|B_2) = \int u(m(y + \tilde{\varepsilon}_1) - m(y))dF(\tilde{\varepsilon}_1) = \int u(\tilde{\varepsilon}_1)dF(\tilde{\varepsilon}_1).$$

The last equalities in both cases hold due to the symmetry of all $\tilde{\varepsilon}_i$ and the linearity of $m(\cdot)$. Since $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_1'$ are identically distributed we can conclude that $U(B_2|A_2) = U(A_2|B_2)$ regardless of the shape of $u(\cdot)$. Using $M(B_2) = M(A_2)$ from lemma 1 and $U(B_2|A_2) = U(A_2|B_2)$, it follows that $V(B_2|A_2) = V(A_2|B_2)$ and thus $B_2|A_2 \sim A_2|B_2$. Alternatively, we can also state that both gambles reduce to

$$[B_2|A_{2\langle A_2|B_2\rangle}]\,|[0] = [C]|[0] \quad \text{and} \quad [A_2|B_{2\langle B_2|A_2\rangle}]\,|[0] = [C]|[0] \tag{1.30}$$

when compared with each other, where C denotes any arbitrary gamble. ■

Proof. [Proposition 9] From lemma 1 we know that $B_3|A_3 \succ (\prec, \sim)\ A_3|B_3 \Leftrightarrow$ $U(B_3|A_3) > (<, =)\ U(A_3|B_3)$. Evaluating $B_3|A_3$ by $U(\cdot)$ yields

$$\begin{aligned} U(B_3|A_3) \;&=\; \iint \left[\tfrac{1}{4}u(m(y-k) - m(y)) + \tfrac{1}{4}u(m(y-k) - m(y-k+\tilde{\varepsilon}_1'))\right. \\ &\quad \left. +\tfrac{1}{4}u(m(y+\tilde{\varepsilon}_1) - m(y)) + \tfrac{1}{4}u(m(y+\tilde{\varepsilon}_1) - m(y-k+\tilde{\varepsilon}_1'))\right] dF(\tilde{\varepsilon}_1)dF(\tilde{\varepsilon}_1') \\ &=\; \iint \left[\tfrac{1}{4}u(-k) + \tfrac{1}{4}u(\tilde{\varepsilon}_1') + \tfrac{1}{4}u(\tilde{\varepsilon}_1) + \tfrac{1}{4}u(k+\tilde{\varepsilon}_1+\tilde{\varepsilon}_1')\right] dF(\tilde{\varepsilon}_1)dF(\tilde{\varepsilon}_1'), \end{aligned}$$

where the last equality holds because of the symmetry of all $\tilde{\varepsilon}_i$ and the linearity of $m(\cdot)$.

Evaluating $A_3|B_3$ by $U(\cdot)$ delivers

$$\begin{aligned} U(A_3|B_3) &= \iint \left[\tfrac{1}{4}u(m(y)-m(y-k))+\tfrac{1}{4}u(m(y)-m(y+\tilde{\varepsilon}_1'))\right. \\ &\quad \left.+\tfrac{1}{4}u(m(y-k+\tilde{\varepsilon}_1)-m(y-k))+\tfrac{1}{4}u(m(y-k+\tilde{\varepsilon}_1)-m(y+\tilde{\varepsilon}_1'))\right] \\ &\quad dF(\tilde{\varepsilon}_1)dF(\tilde{\varepsilon}_1') \\ &= \iint \left[\tfrac{1}{4}u(k)+\tfrac{1}{4}u(\tilde{\varepsilon}_1')+\tfrac{1}{4}u(\tilde{\varepsilon}_1)+\tfrac{1}{4}u(-k+\tilde{\varepsilon}_1+\tilde{\varepsilon}_1')\right] dF(\tilde{\varepsilon}_1)dF(\tilde{\varepsilon}_1'), \end{aligned}$$

where again the last equality holds because of the symmetry of all $\tilde{\varepsilon}_i$ and the linearity of $m(\cdot)$. Comparing $U(B_3|A_3)$ to $U(A_3|B_3)$ yields

$$\begin{aligned} U(B_3|A_3) \;&> (<,=)\; U(A_3|B_3) \Leftrightarrow \\ &\iint \left[\tfrac{1}{2}u(-k)+\tfrac{1}{2}u(k+\tilde{\varepsilon}_1+\tilde{\varepsilon}_1')\right] dF(\tilde{\varepsilon}_1)dF(\tilde{\varepsilon}_1') \\ > (<,=) \;&\iint \left[\tfrac{1}{2}u(k)+\tfrac{1}{2}u(-k+\tilde{\varepsilon}_1+\tilde{\varepsilon}_1')\right] dF(\tilde{\varepsilon}_1)dF(\tilde{\varepsilon}_1'). \end{aligned}$$

From $\mathbb{E}[\tilde{\varepsilon}_1]=0$ and $\mathbb{E}[\tilde{\varepsilon}_1']=0$ it follows that $\mathbb{E}[k+\tilde{\varepsilon}_1+\tilde{\varepsilon}_1']=k$ and $\mathbb{E}[-k+\tilde{\varepsilon}_1+\tilde{\varepsilon}_1']=-k$. With $m(\cdot)$ being linear it follows that $M([-k;k+\tilde{\varepsilon}_1+\tilde{\varepsilon}_1'])=M([k;-k+\tilde{\varepsilon}_1+\tilde{\varepsilon}_1'])$. Thus, we can state that

$$V(B_3|A_3) > (<,=)\; V(A_3|B_3) \Leftrightarrow V([-k;k+\tilde{\varepsilon}_1+\tilde{\varepsilon}_1']|[0]) > (<,=)\; V([k;-k+\tilde{\varepsilon}_1+\tilde{\varepsilon}_1']|[0]),$$

and thus

$$B_3|A_3 \succ (\prec,\sim)\; A_3|B_3 \Leftrightarrow [-k;k+\tilde{\varepsilon}_1+\tilde{\varepsilon}_1']|[0] \succ (\prec,\sim)\; [k;-k+\tilde{\varepsilon}_1+\tilde{\varepsilon}_1']|[0].$$

Alternatively, we can also state that both gambles reduce to

$$[B_3|A_{3\langle A_3|B_3\rangle}]\,|[0] = [-k;k+\tilde{\varepsilon}_1+\tilde{\varepsilon}_1']|[0] \quad \text{and} \quad [A_3|B_{3\langle B_3|A_3\rangle}]\,|[0] = [k;-k+\tilde{\varepsilon}_1+\tilde{\varepsilon}_1']|[0]$$

when compared with each other. ■

Proof. [**Proposition 10**] Follows directly from theorem 2. ■

Proof. [**Theorem 2**] In the following proof of theorem 2 we again first show that a generalization of the first part holds and then that this generalization implies both parts of the theorem (where we use lemma 1, $\tilde{\varepsilon}_i$ being symmetric with $\mathbb{E}[\tilde{\varepsilon}_i] = 0\ \forall i$, and the linearity of $m(\cdot)$). The general part is

$$[B_n|A_{n\langle A_n|B_n\rangle}]\,|[0] = \left[B_{n-2}|A_{n-2\langle A_{n-2}|B_{n-2}\rangle}; A_{n-2}|B_{n-2\langle B_{n-2}|A_{n-2}\rangle} + \tilde{\varepsilon}_{\mathrm{int}(\frac{n}{2})} + \tilde{\varepsilon}'_{\mathrm{int}(\frac{n}{2})}\right]|[0], \tag{1.31}$$

$$[A_n|B_{n\langle B_n|A_n\rangle}]\,|[0] = \left[A_{n-2}|B_{n-2\langle B_{n-2}|A_{n-2}\rangle}; B_{n-2}|A_{n-2\langle A_{n-2}|B_{n-2}\rangle} + \tilde{\varepsilon}_{\mathrm{int}(\frac{n}{2})} + \tilde{\varepsilon}'_{\mathrm{int}(\frac{n}{2})}\right]|[0]. \tag{1.32}$$

To show this, note that $[B_n|A_{n\langle A_n|B_n\rangle}]\,|[0]$ and $[A_n|B_{n\langle B_n|A_n\rangle}]\,|[0]$ can be expressed as gambles containing only B_n and A_n.

$$B_n|A_n = [B_n - A_n]\,|[0] \quad \text{and} \quad A_n|B_n = [A_n - B_n]\,|[0]$$
$$\Rightarrow [B_n|A_{n\langle A_n|B_n\rangle}]\,|[0] = [B_n - A_n]\,|[0] \quad \text{and} \quad [A_n|B_{n\langle B_n|A_n\rangle}]\,|[0] = [A_n - B_n]\,|[0],$$

and therefore

$$[B_{n-2}|A_{n-2\langle A_{n-2}|B_{n-2}\rangle}]\,|[0] = [B_{n-2} - A_{n-2}]\,|[0] \tag{1.33}$$
$$\text{and} \quad [A_{n-2}|B_{n-2\langle B_{n-2}|A_{n-2}\rangle}]\,|[0] = [A_{n-2} - B_{n-2}]\,|[0]. \tag{1.34}$$

But $[B_n|A_{n\langle A_n|B_n\rangle}]\,|[0]$ and $[A_n|B_{n\langle B_n|A_n\rangle}]\,|[0]$ can also be expressed as gambles containing only B_{n-2} and A_{n-2} as $B_n = [A_{n-2}; B_{n-2} + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}]$ and $A_n = [B_{n-2}; A_{n-2} + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}]$.

$$\begin{aligned}
B_n|A_n &= \Big[A_{n-2} - B_{n-2}; A_{n-2} - A_{n-2} + \tilde{\varepsilon}'_{\text{int}(\frac{n}{2})}; B_{n-2} - B_{n-2} + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}; \\
&\quad B_{n-2} - A_{n-2} + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})} + \tilde{\varepsilon}'_{\text{int}(\frac{n}{2})}\Big] \,|[0] \\
\text{and} \quad A_n|B_n &= \Big[B_{n-2} - A_{n-2}; B_{n-2} - B_{n-2} + \tilde{\varepsilon}'_{\text{int}(\frac{n}{2})}; A_{n-2} - A_{n-2} + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})}; \\
&\quad A_{n-2} - B_{n-2} + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})} + \tilde{\varepsilon}'_{\text{int}(\frac{n}{2})}\Big] \,|[0] \\
\Rightarrow \quad [B_n|A_{n\langle A_n|B_n\rangle}] \,|[0] &= \Big[A_{n-2} - B_{n-2}; B_{n-2} - A_{n-2} + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})} + \tilde{\varepsilon}'_{\text{int}(\frac{n}{2})}\Big] \,|[0] \qquad (1.35) \\
\text{and} \quad [A_n|B_{n\langle B_n|A_n\rangle}] \,|[0] &= \Big[B_{n-2} - A_{n-2}; A_{n-2} - B_{n-2} + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})} + \tilde{\varepsilon}'_{\text{int}(\frac{n}{2})}\Big] \,|[0], \qquad (1.36)
\end{aligned}$$

since $\tilde{\varepsilon}_{\text{int}(\frac{n}{2})}$ and $\tilde{\varepsilon}'_{\text{int}(\frac{n}{2})}$ are valued equally and therefore two of the elements cancel out when the two gambles are compared. Finally, inserting (1.33) and (1.34) into (1.35) and (1.36) delivers (1.31) and (1.32). Part (i) of theorem 2 follows directly from (1.31) and (1.32) when we note that for $\frac{n}{2} \notin \mathbb{N} \Rightarrow \text{int}(\frac{n}{2}) = \frac{n-1}{2}$.

To prove part (ii) of theorem 2 ($\frac{n}{2} \in \mathbb{N}$) we again use (1.31) and (1.32). Suppose $n = 4$, then

$$\begin{aligned}
[B_4|A_{4\langle A_4|B_4\rangle}] \,|[0] &= \Big[A_2|B_{2\langle B_2|A_2\rangle}; B_2|A_{2\langle A_2|B_2\rangle} + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})} + \tilde{\varepsilon}'_{\text{int}(\frac{n}{2})}\Big] \,|[0] \qquad (1.37) \\
\text{and} \quad [A_4|B_{4\langle B_4|A_4\rangle}] \,|[0] &= \Big[B_2|A_{2\langle A_2|B_2\rangle}; A_2|B_{2\langle B_2|A_2\rangle} + \tilde{\varepsilon}_{\text{int}(\frac{n}{2})} + \tilde{\varepsilon}'_{\text{int}(\frac{n}{2})}\Big] \,|[0]. \qquad (1.38)
\end{aligned}$$

By inserting (1.30) into (1.37) and (1.38) and canceling $\tilde{\varepsilon}_2$ and $\tilde{\varepsilon}'_2$ in both gambles we then get

$$[B_4|A_{4\langle A_4|B_4\rangle}]|[0] = [C]|[0] \quad \text{and} \quad [A_4|B_{4\langle B_4|A_4\rangle}]|[0] = [C]|[0]. \qquad (1.39)$$

$$\Rightarrow \qquad B_4|A_4 \quad \sim \quad A_4|B_4.$$

Since (1.39), by the same logic it must also hold that $[B_6|A_{6\langle A_6|B_6\rangle}]|[0] = [C]|[0]$ and

$[A_6|B_{6\langle B_6|A_6\rangle}]|[0] = [C]|[0]$. Continuing this reasoning yields

$$
\begin{aligned}
[B_n|A_{n\langle A_n|B_n\rangle}]|[0] = [C]|[0] \quad \text{and} \quad [A_n|B_{n\langle B_n|A_n\rangle}]|[0] = [C]|[0] \\
\Rightarrow \qquad\qquad B_n|A_n \quad \sim \quad A_n|B_n
\end{aligned}
$$

for $\frac{n}{2} \in \mathbb{N}$ and C being any arbitrary gamble. $\blacksquare$

Chapter 2

Multi-Attribute Anticipatory Utility

2.1 Introduction

In this chapter I will introduce a model of multi-attribute reference-dependent anticipatory utility. It is developed along similar notions as the model of Kőszegi and Rabin (2009). Also, the deviations of the two models can be expressed in terms of easily identifiable structural assumptions. Therefore, I will give a thorough comparison of these two alternative models as I proceed in the analysis. The crucial difference of the two models is how anticipatory utility is generated if the pure consumption utility function depends on several arguments.

The notion that the anticipation of future events can generate feelings in the presence that in turn constitute experience utility has been first expressed in a formal model by Loewenstein (1987). He models deterministic streams of consumption and the resulting amounts of direct and anticipatory utility. However, Caplin and Leahy (2001) emphasized that the influence of anticipation may be especially pronounced in settings of uncertainty. We can think of the anxiety a patient faces before receiving a medical diagnosis or the suspense a gambler experiences before learning whether his wager pays off. Although in both situations the envisioned events might not have a high probability, they can nevertheless have a huge influence on the utility experienced by the individuals. Both models of Loewenstein (1987) and Caplin and Leahy (2001) have in common that the absolute level of anticipated future direct consumption utility is the basis of anticipatory utility.

While utility from anticipation may appear like a reasonable concept at first glance, it raises additional issues when considering the specifics of modeling. The assumption that the expected absolute value of the consumption of a later period equals anticipatory utility yields fairly strong implications in some settings. Contrary to such a model we can imagine for instance that the effects of anticipation can wear out very quickly as the individual adjusts to the new mind-set of future events. Similarly, especially those anticipations create strong feelings of excitement that were subject to recent change in the expectation of the future.

An alternative choice of modeling utility from anticipations would be that changes in expectations are carriers of utility. Consider the following example introduced by Matthey (2008). Assume that an individual gets a note from his employer that because of a change in the pay scale he will receive a raise in his monthly wage, starting in three months. Further assume that this comes as an unexpected surprise to him. A few days later he learns that the notification was an unintended mistake and the change in the pay scale was not implemented. Even if the individual did not exercise any real options in the last days due to the erroneous belief of a future pay raise we could imagine that the overall effect on experienced utility is non-zero. In classical models of expected utility (EUT) and non-EUT where utility solely depends on physical consequences there cannot be any effect on utility. If we would include the absolute value of the anticipated future level of consumption, as in Caplin and Leahy (2001), we would receive the prediction that the individual is better off, because for a few days he was elated by the outlook on a better future. According to such a theory, the utility from expectation after he learned that there will be no pay-raise is the same as in a situation where he never thought there would be a raise. However, we may intuitively think that the individual is worse off in the situation where he erred. The reason is that we think he suffers from disappointment after he learns that he overestimated his future wealth. Although we might acknowledge that at the point of time receiving the good news he might have been delighted, we are tempted to think that the latter disappointment may prevail.

The model by Kőszegi and Rabin (2009) captures such situations and many similar ones. The precise model might seem very complex at the outset but nevertheless it delivers clear-cut predictions in many important scenarios.

One example is that the model generates specific preferences for information, even if this information does not aid in making real options. According to the model, individuals prefer to acquire information clustered rather than in small pieces, they prefer to receive information sooner rather than later and they dislike false information even if it represents good news and is eventually corrected. Experimental evidence of such informational preferences can be found in Gneezy and Potters (1997), Bellemare et al. (2005) and Haigh and List (2005).

A second example of a prediction of the model of Kőszegi and Rabin (2009) is that individuals receive substantial utility from an increase in wealth. Wealth does not only increase consumption when it is used to purchase consumption goods, as in traditional models. It also immediately increases the anticipation of such a higher level of future consumption. This increase in the expected consumption constitutes an additional source of utility.

The model of Kőszegi and Rabin (2009) also has the advantage that many existing models of established carriers of utility can be incorporated and are special cases of the more general model. A specific parameter choice also allows to reduce the model to the more simple one by Kőszegi and Rabin (2006)[1] where only final outcomes are compared with recent expectations. An early draft of Kőszegi and Rabin (2006) that contains extensive additional discussions on separability of utility that are important to some aspects of the present analysis is Kőszegi and Rabin (2004). In subsequent versions of the working paper and in the final article, those considerations were omitted. In risk-less situations the model of Kőszegi and Rabin (2009) reduces to classical consumption utility with all properties of the standard model of consumption in such situations.

These former models with endogenous reference points assume that reference-dependent utility only originates from the comparison of final outcomes with recent expectations. The

[1] A variant of Kőszegi and Rabin (2006) that is slightly less closely related to Kőszegi and Rabin (2009) is Kőszegi and Rabin (2007). It does not treat the multi-attribute utility case and assumes risk-less utility to be defined over wealth instead of consumption.

model of Kőszegi and Rabin (2009) instead allows additionally for utility stemming from changes in the expectations on future consumption. An article treating similar questions as Kőszegi and Rabin (2009) but in a less formal way is Matthey (2008). Matthey (2008) does not consider multi-attribute utility and is therefore less relevant to the central points of my analysis. Another restriction of generality is that Matthey (2008) considers only binary choices over outcome distributions of a single future period.

The model of Kőszegi and Rabin (2009) allows for arbitrary many dimensions of pure consumption utility in riskless situations. These dimensions are also the relevant dimensions across which changes in beliefs are assessed and valued. The extension of the model to multi-attribute utility is important since the stated aim of the model of Kőszegi and Rabin (2009) is to incorporate classical models of risky choice. However, with the chosen formal specification the pure consumption utility has to be additive separable in all dimensions in the model of Kőszegi and Rabin (2009). This restricts the model to multi-attribute applications were the dimensions of utility are valued independently of each other.

I propose an alternative formal model along the same intuitions of Kőszegi and Rabin (2009). However, my model is specified in a way that allows the multiple attributes of the pure consumption utility to have any functional form. The model accomplishes this because the level at which a comparison is evaluated and at which feelings of gain or loss are generated is the overall level of pure consumption utility. In the original model by Kőszegi and Rabin (2009) this is applied instead at the level of the pure consumption utility inflicted by the amount of consumption of a single dimension.

Therefore, the scope of applicability is broader than that of the previous model. The analysis of reference-dependent anticipatory utility can also be applied to situations in which intuition or empirical evidence suggests that dimensions of consumption contribute in a non-additive way. Since all situations that can be analyzed by the model of Kőszegi and Rabin (2009) can also be analyzed in my model, the latter covers an unambiguously larger range of situations.

I show that in case of single-dimensioned consumption both models are behaviorally identical. Since in the article of Kőszegi and Rabin (2009) all formally stated results and propositions were derived for the case of single-dimensionality, all results of Kőszegi and Rabin (2009) apply in my model as well.

The cases were both models are applicable are characterized by additively separable pure consumption utility. However, I show that if pure consumption utility is additively separable the two models do not necessarily coincide in their predictions. Thus, my model does not represent a mere generalization of Kőszegi and Rabin (2009) but rather an alternative model with a broader domain of applicability.

To show that the differences between the proposed model and the model of Kőszegi and Rabin (2009) that occur with additively separable pure consumption utility do not constitute an obscure technicality, I discuss two practically relevant cases were predictions differ. These two examples could also be used as a device to discriminate between the two models since they show in which instances the two models yield different but empirically testable predictions.

The first example shows that two distinctive dimensions of consumption are in a certain way more substitutable in my model compared to the model of Kőszegi and Rabin (2009). In my model, simultaneous changes in expectations on different dimensions of consumption that would not result in an overall effect on pure consumption utility do not have an effect on reference-dependent anticipatory utility. In contrast, in the model by Kőszegi and Rabin (2009) in such a situation the individual would still dislike these simultaneous changes since the feeling of gain in one dimension of utility does not outweigh the feeling of loss in the other dimension.

The second example shows that my model is sensitive to a wider range of changes in beliefs. An update in beliefs over the correlation of risks affecting different dimensions of pure consumption utility generates a change in utility in my model. In the model of Kőszegi and Rabin (2009) such a change in the beliefs has no effect since only the marginal distributions of the beliefs are considered.

The outline of the chapter is as follows. In section 2.2 I outline the basic features that both models have in common. In section 2.3 I state the additional assumptions that define the model of Kőszegi and Rabin (2009). Section 2.4 formally defines my alternative model. Those three sections define the two compared models up to the functional form of what I will call disaggregate gain-loss utility. Since this functional form is again common to both models I will discuss it in section 2.5. Since the main motivation to formulate an alternative model is to allow for a non-restrictive specification of multi-attribute pure consumption utility I discuss this matter in section 2.6. I formally show that all differences between the two models disappear if consumption has only a single dimension in section 2.7. In section 2.8 I show that for multi-attribute utility the two models yield diverging predictions even if pure consumption utility is additively separable. Section 2.9 concludes. The appendix of section 2.10 discusses why I did not choose two alternative adjustments of the Kőszegi and Rabin (2009) model that would have also allowed pure consumption utility to be multi-attributed. All lengthy proofs of propositions and lemmas are relegated to the appendix of section 2.11.

Continuing Example

For the sake of illustration I will discuss at the end of many sections what the outlined theoretical concepts imply in a simple continuing example. This example assumes that both the consumption of material goods and the current state of health contribute to pure consumption utility. This has been one of the classical settings where the analysis of multi-attribute utility has led to valuable insights. Although the example can hint at some potential applications, it is not meant to demonstrate all aspects of the models.

2.2 Common Features of All Models

In this section I discuss the features that all considered models share. In particular I will show how decision utility depends on what I will call aggregate gain-loss utility. The specific structure of this aggregate gain-loss utility will distinguish the different models.

The period in which a decision has to be made is denoted by s. The utility function according to which the decision is made is U^s. I will call U^s decision utility henceforth. It is assumed that decisions under uncertainty are taken such that the expectation of U^s is maximized. The time horizon taken into account when making the decision is T. Decision utility is based on the experience utility in present and future periods. Experience utility in period t is denoted by u_t. The experience utility of the relevant periods leads to decision utility according to

$$U^s = \sum_{t=s}^{T} u_t. \tag{2.1}$$

Experience utility u_t consists of both direct utility from consumption in period t and utility from an update of expectations,

$$u_t = m(c_t) + \sum_{\tau=t}^{T} \gamma_{t,\tau} N\left(\mathcal{F}_{t,\tau} \middle| \mathcal{F}_{t-1,\tau}\right). \tag{2.2}$$

The vector of consumption levels of the K goods that influence utility in period t is c_t. Formally,

$$c_t = \left(c_t^1, \ldots, c_t^K\right). \tag{2.3}$$

Direct consumption utility over this vector of consumption levels is $m(\cdot)$ and satisfies

$$\partial m / \partial c_t^k > 0 > \partial^2 m / \left[\partial c_t^k\right]^2 \quad \forall\ k, t. \tag{2.4}$$

Changes in expectations that concern consumption levels of periods far away may have weaker influences on utility than those concerning periods less far away. This notion is captured by the weights $\gamma_{t,\tau}$. Those weights have the properties

$$0 \le \gamma_{0,\tau} \le \gamma_{1,\tau} \le \cdots \le \gamma_{\tau,\tau} \equiv 1 \quad \forall\ \tau. \tag{2.5}$$

$N\left(\mathcal{F}_{t,\tau} \middle| \mathcal{F}_{t-1,\tau}\right)$ is the aggregate gain-loss utility that comes into effect in period t and originates in the beliefs concerning period τ. $\mathcal{F}_{t,\tau}$ is the belief held in period t over the consumption vector in period τ, c_τ. Here I define beliefs as general as possible. It consists of an conception of the joint distribution function of all K dimensions of utility in a given period τ. I denote beliefs over the joint distribution over the bivariate distribution of two dimensions k and l as $\mathcal{F}_{t,\tau}^{k,l}$. This could be extended to more than two dimensions. The beliefs over the distribution of all K variables then would be $\mathcal{F}_{t,\tau}^{1,\ldots,K} \equiv \mathcal{F}_{t,\tau}$. The precise way in which those beliefs influence aggregate gain-loss utility differs among the considered models and will be discussed in the next two sections.

Summarizing the parts that all models discussed in this chapter have in common yields

$$U^s \;=\; \sum_{t=s}^{T} \left\{ m(c_t) + \sum_{\tau=t}^{T} \gamma_{t,\tau} N\left(\mathcal{F}_{t,\tau} \middle| \mathcal{F}_{t-1,\tau}\right) \right\}. \tag{2.6}$$

Additionally, all models discussed in this chapter also have the later defined assumptions (A0) to (A4) in common. These five assumptions concern the functional form of the disaggregate gain-loss utility function $\mu(\cdot)$. Since this function will enter aggregate gain-loss utility differently for the considered models I will discuss these additional common features in section 2.5 after I formally introduced both models.

Continuing Example

Suppose $K = 4$. Define c_1 as the level of health the decision maker enjoys. To discuss a more tangible example, think of health as the ability to use his legs. The levels of c_2, c_3 and c_4 denote the consumption levels of material goods and services. While the utility derived from good 3 is independent of health, the appeal of both good 2 and good 4 vary with different health levels. Good 2 is especially valuable if the level of health is good, such as going to a skiing vacation. In contrast, good 4 is especially valuable if the level of health is low, such as a wheelchair. Define consumption levels such that $c_t^k \geq 0 \; \forall \; k, t$. Define $m(c_t) = \ln\left(c_t^1 c_t^2 + c_t^4\right) + \ln\left(c_t^3\right)$. Then $\partial m / \partial c_t^k > 0 > \partial^2 m / \left[\partial c_t^k\right]^2$ for all k and t and therefore condition (2.4) is met. The cross-derivatives are negative between good 1 and 4 and between 2 and 4. The cross-derivative between good 1 and 2 is positive. All cross-derivatives involving good 3 are zero. The precise terms of all second order derivatives can be extracted from Table 2.1. The table should be consulted as follows. The nominator of every second-

Table 2.1: Second order derivatives in the continuing example

$\partial^2 m/\cdot$			$\frac{-1}{[c_t^1 c_t^2 + c_t^4]^2}$	$\cdot/\partial c_t^4$
		$\frac{-1}{[c_t^3]^2}$	0	$\cdot/\partial c_t^3$
	$\frac{-[c_t^1]^2}{[c_t^1 c_t^2 + c_t^4]^2}$	0	$\frac{-c_t^1}{[c_t^1 c_t^2 + c_t^4]^2}$	$\cdot/\partial c_t^2$
$\frac{-[c_t^2]^2}{[c_t^1 c_t^2 + c_t^4]^2}$	$\frac{c_t^4}{[c_t^1 c_t^2 + c_t^4]^2}$	0	$\frac{-c_t^2}{[c_t^1 c_t^2 + c_t^4]^2}$	$\cdot/\partial c_t^1$
$\cdot/\partial c_t^1$	$\cdot/\partial c_t^2$	$\cdot/\partial c_t^3$	$\cdot/\partial c_t^4$	

order derivative is always $\partial^2 m$. The denominator of the second-order derivative depicted in a certain cell is always the product of the terms at the margins to the right of the cell and below the cell. For instance, the only entry in the first row, namely $-1/[c_t^1 c_t^2 + c_t^4]^2$, is the second-order derivative $\partial^2 m/[\partial c_t^4]^2$.

2.3 Original Kőszegi and Rabin (2009) Model

In this section I discuss the properties of the model by Kőszegi and Rabin (2009) that distinguish it from my model and from the two adjusted models discussed in the appendix in section 2.10. Together with the definitions of the previous section the present section delivers a complete description of the formal structure of the model of Kőszegi and Rabin (2009). The only exception is that a theory on the formation of beliefs is absent. While Kőszegi and Rabin (2006, 2007, 2009) successively develop such a theory, I do not incorporate it here because of two reasons. First, the theory on belief formation does not influence the discussed differences in the considered models in any way. Second, the discussed differences also apply if any other theory on the formation of beliefs would be used.

Equations (2.1) to (2.5) hold, thus also equation (2.6) holds. Features of the model of Kőszegi and Rabin (2009) that are not common to my models are

$$m(c_t) = \sum_{k=1}^{K} m^k\left(c_t^k\right) \tag{2.7}$$

$$m^k\left(c_t^k\right) : \text{any function satisfying} \quad \frac{\partial m^k}{\partial c_t^k} > 0 > \frac{\partial^2 m^k}{\left[\partial c_t^k\right]^2} \quad \forall\, t, k \tag{2.8}$$

$$\mathcal{F}_{t,\tau} = \left(\mathcal{F}_{t,\tau}^1, \ldots, \mathcal{F}_{t,\tau}^K\right) \tag{2.9}$$

$$N\left(\mathcal{F}_{t,\tau} \middle| \mathcal{F}_{t-1,\tau}\right) = \sum_{k=1}^{K} N^k\left(\mathcal{F}_{t,\tau}^k \middle| \mathcal{F}_{t-1,\tau}^k\right) \tag{2.10}$$

$$N^k\left(\mathcal{F}_{t,\tau}^k \middle| \mathcal{F}_{t-1,\tau}^k\right) = \int_0^1 \mu\left(m^k\left(c_{\mathcal{F}_{t,\tau}^k}(p)\right) - m^k\left(c_{\mathcal{F}_{t-1,\tau}^k}(p)\right)\right) dp \tag{2.11}$$

From equations (2.7) and (2.8) it follows that equation (2.4) holds. The assumption that pure consumption utility is additively separable in all arguments will be one of the main differences of the original model of Kőszegi and Rabin (2009) and my alternative model. It is not a straightforward task to generalize this pure consumption utility function for more general functional forms. The reason is that pure consumption utility enters not only directly into the experience utility of every period but also additionally via the aggregate gain-loss utility function. It is the specification of this aggregate gain-loss utility function that requires pure consumption utility to be additively separable in the original model of Kőszegi and Rabin (2009).

Equations (2.10) and (2.11) jointly define the aggregate gain-loss utility function in the model of Kőszegi and Rabin (2009). Equation (2.10) states that aggregate gain-loss utility is also separable across the K dimensions of the consumption vector. Equation (2.11) defines aggregate gain-loss utility for an arbitrary dimension k. It depends on changes in the beliefs concerning this dimension.

$\mathcal{F}_{t,\tau}^k$ is the belief held in period t over the consumption level of good k in period τ, that is c_τ^k. In the model of Kőszegi and Rabin (2009), the condition $\mathcal{F}_{t,\tau} = (\mathcal{F}_{t,\tau}^1, \ldots, \mathcal{F}_{t,\tau}^K)$ holds.

While the definition of $\mathcal{F}_{t,\tau}$ in the preceding section contained the beliefs over the joint probabilities of consumption levels of all dimensions, the element $\mathcal{F}^k_{t,\tau}$ merely contains the beliefs over the marginal distribution of the consumption level of dimension k. Therefore, equation (2.9) constitutes a restriction on what information contained in the beliefs is of relevance in the model.

As expressed in (2.11), the disaggregate gain-loss utility function $\mu(\cdot)$ evaluates the differences in pure consumption utility of a consumption level that can be expected according to two beliefs at a certain percentile. The functional form of $\mu(\cdot)$ will be discussed in detail in section 2.5. Here, I will now investigate the precise composition of the argument of $\mu(\cdot)$. This is of some importance since it constitutes one of the key differences of the model of Kőszegi and Rabin (2009) and my alternative model proposed in the succeeding section.

$c_{\mathcal{F}}(p)$ is consumption in percentile p given belief $\mathcal{F}$. If $F(z)|\mathcal{F}$ denotes the cumulative distribution function of the real-valued variable z given belief $\mathcal{F}$, then $c_{\mathcal{F}^k_{t,\tau}}(p)$ is the value of c^k_τ that is defined by $F(c^k_\tau)|\mathcal{F}^k_{t,\tau} \geq p$ and $F(c)|\mathcal{F}^k_{t,\tau} < p \ \forall \ c < c^k_\tau$. Figure 2.1 illustrates the composition of the argument of the gain-loss utility function $\mu(\cdot)$. In each panel two examples of cumulative distribution functions over c^k_τ are sketched out, one according to the beliefs held in period $t-1$ and one according to the beliefs held in period t.

In panel A of figure 2.1 the change in beliefs is a deterministic positive shift in c^k_τ, that is, c^k_τ increases by a constant amount at each $p \in \,]0\,;1[\,$. Therefore, at each percentile the change of beliefs is perceived as a gain. Note, however, that the amount of the gain is not constant for all percentiles p because at each percentile p the consumption level c^k_τ is evaluated by the function $m^k(\cdot)$ and the difference is taken over these levels of pure consumption utility. For illustration consider the percentile $p = \tilde{p}$. At this percentile the argument of $\mu(\cdot)$ is $\grave{m} - \acute{m} > 0$, since $\grave{m} = m^k(c_{\mathcal{F}^k_{t,\tau}}(\tilde{p}))$ and $\acute{m} = m^k(c_{\mathcal{F}^k_{t-1,\tau}}(\tilde{p}))$.

In panel B of figure 2.1 the change in beliefs has a more complex pattern. For some percentiles the change results in a sensation of gain. For example, at the percentile $p = \tilde{p}$ the change in beliefs delivers again a gain of $\grave{m} - \acute{m}$. However, at other percentiles the change in beliefs now induces sentiments of loss. An instance for the latter case is percentile $p = \check{p}$.

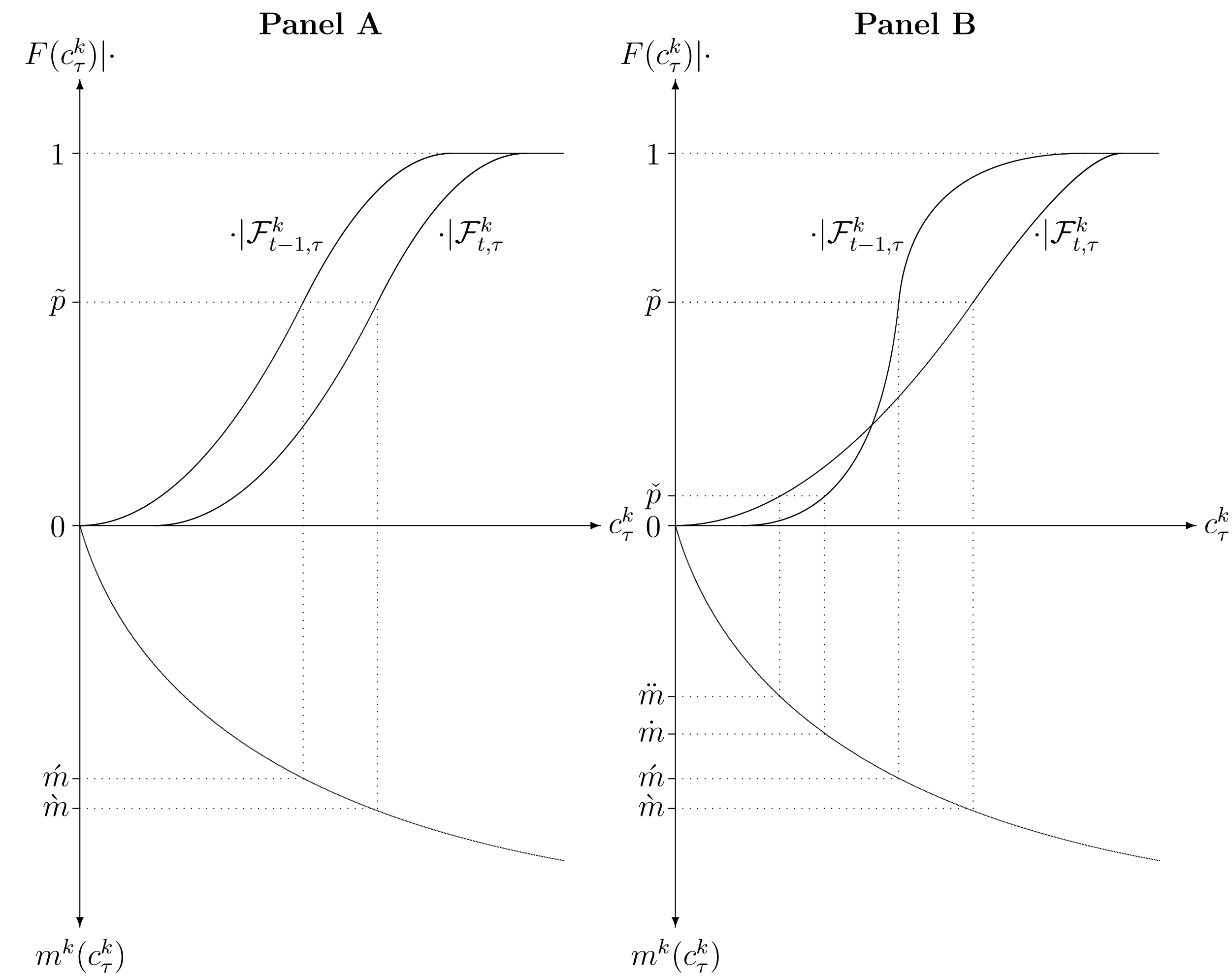

Figure 2.1: The argument of the gain-loss function $\mu(\cdot)$

At this lower percentile the argument of $\mu(\cdot)$ is $\ddot{m} - \dot{m} < 0$, since $\ddot{m} = m^k(c_{\mathcal{F}_{t,\tau}^k}(\check{p}))$ and $\dot{m} = m^k(c_{\mathcal{F}_{t-1,\tau}^k}(\check{p}))$.

I will denote decision utility according to the original model of Kőszegi and Rabin (2009) by $\bar{U}^s$. By incorporating the assumptions (2.7) to (2.11) into (2.6), the model of Kőszegi and Rabin (2009) can be summarized as

$$\bar{U}^s = \sum_{t=s}^{T} \left\{ \sum_{k=1}^{K} \left[m^k\left(c_t^k\right)\right] + \sum_{\tau=t}^{T} \left[\gamma_{t,\tau} \sum_{k=1}^{K} \int_0^1 \mu\left(m^k\left(c_{\mathcal{F}_{t,\tau}^k}(p)\right) - m^k\left(c_{\mathcal{F}_{t-1,\tau}^k}(p)\right)\right) dp \right] \right\}. \quad (2.12)$$

2.4 An Alternative Model

This section introduces my alternative model. It shares the features discussed in section 2.2 with the original model by Kőszegi and Rabin (2009). Thus, equations (2.1) to (2.5) hold, and subsequently also equation (2.6) holds.

I will now state the features of my model that are not common to the other models of the previous section and of appendix 2.10.

$$m(c_t) \quad : \quad \text{any function satisfying} \quad \frac{\partial m}{\partial c_t^k} > 0 > \frac{\partial^2 m}{\left[\partial c_t^k\right]^2} \quad \forall\, t, k \tag{2.13}$$

$$N(\mathcal{F}_{t,\tau}|\mathcal{F}_{t-1,\tau}) \quad = \quad \int_0^1 \mu\left(m_{\mathcal{F}_{t,\tau}}(p) - m_{\mathcal{F}_{t-1,\tau}}(p)\right) dp \tag{2.14}$$

Equation (2.13) states that any function satisfying condition (2.4) can be employed to describe pure consumption utility in my model. This is in contrast to the restriction to additively separable functions expressed in (2.7) and (2.8) in the model of Kőszegi and Rabin (2009). In section 2.6 I will argue that this feature allows my model to be applied in a significantly broader range of situations.

Equation (2.14) defines how aggregate gain-loss utility is comprised in my model. In contrast to equations (2.10) and (2.11) it does not rely on a separation of aggregate gain-loss utility across the dimensions of the consumption vector. Instead, it relies on differences in the pure consumption utility determined by all K dimensions jointly. These differences in pure consumption utility according to two beliefs are also evaluated at a certain percentile by the disaggregate gain-loss utility function $\mu(\cdot)$. For a discussion of the functional form of $\mu(\cdot)$ I again refer to section 2.5. I will now illustrate the precise composition of the argument of $\mu(\cdot)$ in my model.

By $m_{\mathcal{F}}(p)$ I denote pure consumption utility in percentile p given belief $\mathcal{F}$. For notational convenience, I define $m_t = m(c_t)$. I can now formally state that $m_{\mathcal{F}_{t,\tau}}(p)$ is the value of m_τ that is defined by $F(m_\tau)|\mathcal{F}_{t,\tau} \geq p$ and $F(m|\mathcal{F}_{t,\tau}) < p\ \forall\ m < m_\tau$.

Note also, that I made no restriction on the beliefs $\mathcal{F}_{t,\tau}$ that enter into my model. All features of the K-dimensional joint cumulative distribution function over the consumption levels could have an influence on pure consumption utility. Thus, all these influences could play a role in the formation of gain-loss utility in my model.

To give an impression on what information is neglected if one simply defines beliefs as the vector of marginal distributions $\mathcal{F}_{t,\tau} = (\mathcal{F}_{t,\tau}^1, \ldots, \mathcal{F}_{t,\tau}^K)$ as in Kőszegi and Rabin (2009), consider an example where $K = 2$, $c_\tau^k \in \{\underline{c}_\tau^k, \bar{c}_\tau^k\}\ \forall\ \tau, k$ and $\underline{c}_\tau^k \neq \bar{c}_\tau^k\ \forall\ \tau, k$. In Table 2.2

Table 2.2: Content of beliefs in an example with two dimensions

	$\bar{c}_\tau^2$	$\underline{c}_\tau^2$	
$\bar{c}_\tau^1$	$\Pr(\bar{c}_\tau^1, \bar{c}_\tau^2)$	$\Pr(\bar{c}_\tau^1, \underline{c}_\tau^2)$	$\Pr(\bar{c}_\tau^1, \cdot\ )$
$\underline{c}_\tau^1$	$\Pr(\underline{c}_\tau^1, \bar{c}_\tau^2)$	$\Pr(\underline{c}_\tau^1, \underline{c}_\tau^2)$	$\Pr(\underline{c}_\tau^1, \cdot\ )$
	$\Pr(\ \cdot\ , \bar{c}_\tau^2)$	$\Pr(\ \cdot\ , \underline{c}_\tau^2)$	1

the joint probabilities are spelled out inside the box. The set of those joint probabilities contains all information on the bivariate distribution of c_τ^1 and c_τ^2. This general formulation of beliefs now captures all appreciation and information concerning possible values of these joint probabilities held in period t. To the right of and below the box I stated the marginal probabilities concerning a single dimension of utility. Assuming that $\mathcal{F}_{t,\tau} = (\mathcal{F}_{t,\tau}^1, \mathcal{F}_{t,\tau}^2)$ rules out all influences of beliefs over the joint distribution of consumption levels of different dimensions that are not captured in the marginal distributions.

I will denote decision utility according to my proposed model by $\hat{U}^s$. By incorporating assumptions (2.13) and (2.14) into (2.6) my model can be written as

$$\hat{U}^s = \sum_{t=s}^{T} \left\{ m(c_t) + \sum_{\tau=t}^{T} \left[\gamma_{t,\tau} \int_0^1 \mu\left(m_{\mathcal{F}_{t,\tau}}(p) - m_{\mathcal{F}_{t-1,\tau}}(p) \right) dp \right] \right\}. \tag{2.15}$$

2.5 Functional Form of Gain-Loss Utility

In sections 2.3 and 2.4 we saw that in both the original model of Kőszegi and Rabin (2009) and in my alternative model aggregate gain-loss utility depends on a function representing disaggregate gain-loss utility $\mu(\cdot)$. However, not only is the way this gain-loss utility function $\mu(\cdot)$ is integrated in the aggregated gain-loss utility function $N(\mathcal{F}_{t,\tau}|\mathcal{F}_{t-1,\tau})$ different between the two models. Also, the construction of the argument of $\mu(\cdot)$ was shown to be different.

Despite these differences, the functional form of $\mu(\cdot)$ across all models is identical and is therefore discussed separately in this section. The functional form of $\mu(\cdot)$ will not be parameterized. Instead, it will be defined more broadly by the common set of assumptions (A0) - (A4),

(A0) $\mu(x)$ is continuous $\forall\ x$, $\mu(x)$ is two times differentiable $\forall\ x \neq 0$, and $\mu(0) = 0$

(A1) $\mu(x)$ is strictly increasing

(A2) $\mu(y) + \mu(-y) < \mu(x) + \mu(-x) \quad \forall\ \ y > x \geq 0$

(A3) $\mu''(x) \leq 0 \quad \forall\ \ x > 0$ and $\mu''(x) \geq 0 \quad \forall\ \ x < 0$

(A4) $\lim\limits_{x \to 0} \frac{\mu'(-|x|)}{\mu'(|x|)} \equiv \lambda > 1.$

The literature on the functional form of the gain-loss is too vast to allow for an exhaustive discussion here. However, almost all concepts originate in the classical prospect theory of Kahneman and Tversky (1979). Bowman, Minehart and Rabin (1999) first introduced the precise formal definitions (A0) to (A4) as I use them here. They were subsequently used by Kőszegi and Rabin (2006, 2007, 2009) and many others.

Let us discuss assumptions (A0) to (A4) in isolation first. Assumption (A0) states continuity since a function with gaps would easily yield counterfactual predictions. The assumption of (two-times) differentiability ($\forall\ x \neq 0$) is a technicality allowing the application of calculus within domains. The assumption that $\mu(0) = 0$ ensures that in a situation of no changes, no gain-loss utility occurs.

Assumption (A1) ensures that a larger gain is perceived as more favorable than a smaller gain, a smaller loss is preferred to a larger loss, and to experience a gain is better than a loss.

Assumption (A2) ensures loss aversion in the large. Thus, an individual always strictly dislikes an increase in a loss more than it likes the increase in a gain if the initial gain equals the initial loss and the considered magnitudes of the changes are equal.

Assumption (A3) ensures that diminishing sensitivity (weakly) holds. Thus, if a gain is increased, the marginal utility stemming from this improvement decreases with the initial value of the gain or stays constant. Similarly, if an initial loss is increased the marginal disutility stemming from this deterioration decreases with the initial value of the loss or stays constant.

Assumption (A4) ensures that loss aversion in the small holds. This condition formally states that the function $\mu(\cdot)$ has a concave kink at $x = 0$. On this property, Köbberling and Wakker (2005) provide the most concise discussion.

Although assumptions (A0) to (A4) constitute the baseline model, in order to derive some results, additionally one or several stronger assumption will have to be made.

(A2') $-\mu''(-x) \geq \mu''(x) \quad \forall \ x > 0$

(A3') $\mu''(x) = 0 \quad \forall \ x \neq 0$

(A3°) $\lim\limits_{x \to -\infty} \mu'(x) \geq \lim\limits_{x \to 0} \mu'(|x|)$

Assumption (A2') is a stronger version of assumption (A2) while assumptions (A3') and assumption (A3°) are stronger versions of assumption (A3).

To clarify the roles of these stronger assumption in relation to each other and in relation to the original assumptions (A0) to (A4), we can state the following relationships

$\mu(\cdot)$ with (A0), (A1), (A2), (A3'), (A4) $\Rightarrow$ $\mu(\cdot)$ with (A0), (A1), (A2), (A3°), (A4).
$\mu(\cdot)$ with (A0), (A1), (A2), (A3°), (A4) $\Rightarrow$ $\mu(\cdot)$ with (A0), (A1), (A2), (A3), (A4).
$\mu(\cdot)$ with (A0), (A1), (A2'), (A3), (A4) $\Rightarrow$ $\mu(\cdot)$ with (A0), (A1), (A2), (A3), (A4).

Assuming (A0), (A1), (A2), (A3'), (A4) defines the special case where $\mu(\cdot)$ is a piecewise linear function. Assumption (A3') was already introduced by Bowman, Minehart and Rabin (1999) as a special case and used by Kőszegi and Rabin (2006) and Kőszegi and Rabin (2009) to derive some of their central results.

Let us now discuss some joint implications of assumptions (A0) to (A4). The exemption of differentiability at point $x = 0$ in assumption (A0) is necessary to allow for loss aversion in the small as expressed in assumption (A4) to occur.

Assumptions (A2) and (A3) jointly require that gain-loss disutility in losses does not diminish too fast in relation to the diminishing of gain-loss utility in gains. A sufficient but not necessary condition is that disutility diminishes less fast in losses than utility diminishes in equally strong gains. Another sufficient but not necessary condition is that the slope of $\mu(\cdot)$ in the domain of losses becomes never smaller than the amount of loss aversion in the small as expressed as λ in assumption (A4).

In figure 2.2 I sketch $\tilde{\mu}(\cdot)$ with $\tilde{\mu} = \tilde{\mu}_+$ for $x \geq 0$ and $\tilde{\mu} = \tilde{\mu}_-$ for $x < 0$ as one possible function that meets all assumptions (A0) to (A4). The line $\bar{\mu}_+$ is the tangent to $\tilde{\mu}_+$ at $x = 0$. The line $\bar{\mu}_-$ is the tangent to $\tilde{\mu}_-$ as x approaches 0 from below. The function $\bar{\mu}(\cdot)$ (with $\bar{\mu} = \bar{\mu}_+$ for $x \geq 0$ and $\bar{\mu} = \bar{\mu}_-$ for $x < 0$) itself is another possibility of meeting assumptions (A0) to (A4). In contrast to $\tilde{\mu}(\cdot)$, the function $\bar{\mu}(\cdot)$ additionally meets assumption (A3').

Finally, I will derive two lemmas concerning implications of the functional form of $\mu(\cdot)$ that will be of help when discussing the differences in the predictions of the two models. Consider a variable ω with the K outcomes $\omega^1, \ldots, \omega^K$. Call ω non-degenerate if there exists a $k \in \{1, \ldots, K\}$ for which $\omega^k \neq 0$. Call ω symmetrically distributed around 0 if $\#\left\{\omega^k \middle|\, \omega^k = x\right\} = \#\left\{\omega^k \middle|\, \omega^k = -x\right\}\ \forall\, x \in \mathbb{R}$, where $\#\{\cdot\}$ denotes the cardinality of a set. Then I can state the following two lemmas.

Lemma 2 *If $\mu(\cdot)$ satisfies (A0), (A1), (A2), (A3), (A4) and ω is non-degenerate and symmetrically distributed around 0 it holds that* $\sum\limits_{k=1}^{K}\left[\mu\left(\omega^k\right)\right] < 0 = \mu\left(\sum\limits_{k=1}^{K}\left[\omega^k\right]\right)$.

Proof. **[Lemma 2]** Is provided in the appendix of section 2.11 ■

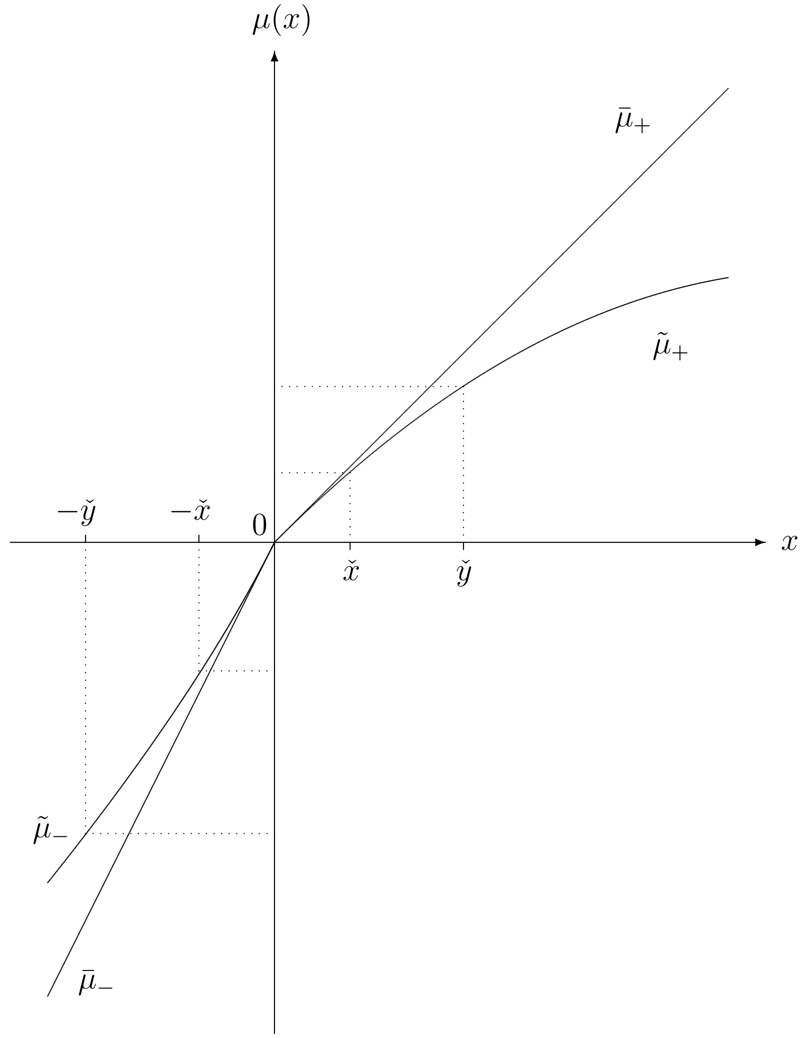

Figure 2.2: Functional form of the gain-loss function $\mu(\cdot)$

Lemma 3 *If $\mu(\cdot)$ satisfies (A0), (A1), (A2), (A3°), (A4) and ω^k is non-degenerate and $\sum_{k=1}^{K}\left[\omega^k\right] = 0$ it holds that $\sum_{k=1}^{K}\left[\mu\left(\omega^k\right)\right] < 0 = \mu\left(\sum_{k=1}^{K}\left[\omega^k\right]\right)$.*

Proof. [Lemma 3] Is provided in the appendix of section 2.11 ■

The sum of outcomes of a variable that is distributed symmetrically around zero is always zero. Therefore the lemmas illustrate that we have to either (lightly) restrict $\mu(\cdot)$ or the risky situation to ensure the sum over the gain-loss utilities of ω^k is smaller than the gain-loss utility of the sum over ω^k.

2.6 Generality of Pure Consumption Utility

In this section I will consider how sever the restriction to additively separable pure consumption is that the model of Kőszegi and Rabin (2009) requires. First, I discuss what exactly should be considered as a separate dimension of consumption. Second, I will investigate under which conditions an additively separable utility function can represent preferences over these consumption vectors.

In a very early draft of Kőszegi and Rabin (2006), namely in Kőszegi and Rabin (2004), the authors discuss the matter of separability at some length and devote an entire section on the matter of how a dimension of consumption should be defined. In subsequent versions of the working paper and in the final articles of Kőszegi and Rabin (2006) and Kőszegi and Rabin (2009) a discussion of the separability of consumption utility and the definition of dimensions is almost completely absent.

Kőszegi and Rabin (2004) argue that their categories of consumption should be best understood as hedonic dimensions. Examples of such hedonic dimensions could be basic nutrition, security, entertainment or shelter. A single traded good or service can potentially contribute to several of these dimensions. Also, different goods can satisfy the same need and the joint contributions of these goods to a single hedonic dimension can be viewed as a single level of consumption. In Kőszegi and Rabin (2004) the dimensions of consumption are precisely defined by the fact that trade-offs within such a dimension are not subject to sentiments of gain or loss.

On the one hand, we could adopt this definition. Then, my model could be thought of as the analysis of reference-dependent anticipatory utility within these hedonic dimensions and the original model of Kőszegi and Rabin (2009) could be perceived as the analysis of reference-dependent anticipatory utility across these hedonic dimensions. This could be appropriate if we would primarily be interested in composing a theoretically formulated psychological model.

However, most often economists are not interested in fitting psychological models as an end in itself. In contrast, we propose models in order to predict or describe observable behavior of economic agents in response to exogenous parameters. If we focus on private households, the classical example is to investigate how the demand functions of such a household depend on prices. Hedonic dimensions of utility themselves rarely have observable prices, there is no explicit price of security for instance. In contrast, traded goods and services have explicitly stated prices. I therefore perceive the models to be of higher value to the analysis of economic phenomena if we understand consumption in the traditional way as the usage or exhaustion of tradable commodities instead of hedonistic attributes. Although not hinted at by Kőszegi or Rabin anywhere, similar considerations might have led to the omission of the discussion on the definition of dimensions of consumptions in later versions of Kőszegi and Rabin (2004).

If we accept that our theories deal with consumption levels of tradeable commodities, we are left with the question under which conditions several of these commodities can be aggregated into a single index. The classical composite commodity theorem of Hicks (1936) states that only if a group of prices moves in parallel, the corresponding group of commodities can be treated as a single good. If we accept that several dimensions of consumption exist with prices that do not move in parallel, we have to investigate under which conditions an additively separable utility function is a valid representation of preferences.

One approach would be to discuss applications that are compatible with additively separable preferences and instances where no additively separable representation can be applied. An example of the latter would be that, if we restrict pure consumption utility to be additively separable, none of the goods can be complementary to any other good. Additionally, no single good can be an inferior good (see e.g. Deaton and Muellbauer, 1980, pp138-139).

In contrast, I give a necessary and sufficient condition for the validity of an additively separable representation of preferences. Following Gorman (1968), for $K \geq 3$ the following equivalence holds

$$\begin{aligned} & m(c_t) = M\left(\sum_{k=1}^{K} m^k(c_t^k)\right),\ M' > 0 \\ \Leftrightarrow\ & m \ \text{ is completely separable in } \{c_t^1, \ldots, c_t^k, \ldots, c_t^K\} \end{aligned} \tag{2.16}$$

with $M(\cdot)$ denoting an arbitrary increasing function. The utility function m is completely separable in a set if the preference relations within every subset is independent of the values of all remaining attributes not in the subset.[2] See also Blackorby, Primont and Russell (1998, p72), and Keeney and Raiffa (1993, pp111-116), for a discussion on this result. The latter text substitutes the term of separability with the term of preferential independence. A set of attributes is preferentially independent of other attributes if the conditional preferences in the set do not depend on the value of the other attributes.

Whether preferences are completely separable in the arguments of the utility function finally constitutes an empirical question. On a critique of additive separable utility functions based on empirical findings, see Deaton (1974). He concludes that "*the assumption of additive preferences is almost certain to be invalid in practice and the use of demand models based on such an assumption will lead to severe distortion of measurement.* So that if the price to be paid for the theoretical consistency of demand models is the necessity of assuming additive preferences, then the price is too high." (Deaton, 1974, p346, emphasis in the original).

Continuing Example

To analyze a situation as outlined above in the original Kőszegi and Rabin (2009) model we have to aggregate those dimensions of utility that are not separable, i.e. c_t^1, c_t^2 and c_t^4. Define $c_t^5 = c_t^1 c_t^2 + c_t^4$, then $m(c_t) = \ln(c_t^5) + \ln(c_t^3)$. We would have to construct a price index of the composite good c_t^5, not an easy task to do. The only relative price change that can be analyzed is that between c_t^3 and the aggregated price index of c_t^5.

Let us for the following sections assume that we can make observations on this compound concept of consumption. Let us also redefine the utility contributing goods related to health as good 1 and the utility contributing goods not related to health as good 2. Then, $m(c_t) = \ln(c_t^1) + \ln(c_t^2)$.

[2] The original proof of Gorman (1968) allows for a generalization where every c_t^k may represent a group of goods.

Although both c_t^1 and c_t^2 can take on the value of each non-negative real number, I will in the following consider situations where both arguments are either 1 or Euler's number e. If both are 1, the pure consumption utility is 0, $m(c_t^1 = 1, c_t^2 = 1) = 0$. If both are e pure consumption utility equals 2, $m(c_t^1 = e, c_t^2 = e) = 2$. If only one of the arguments is 1 and the other is e pure consumption utility is 1, $m(c_t^1 = 1, c_t^2 = e) = m(c_t^1 = e, c_t^2 = 1) = 1$.

2.7 Equality of Models for Single-Attribute Utility

In this section I show that my model is identical to the model of Kőszegi and Rabin (2009) if the consumption vector contains only a single dimension. This is important because all formal propositions in Kőszegi and Rabin (2009) were derived for $K = 1$. Thus, all these results are valid in my model as well.

If I define

$$\begin{aligned} c &\equiv (c_s, \ldots, c_T) \\ \gamma &\equiv (\gamma_{0,T-s}, \ldots, \gamma_{T-s-1,T-s}) \\ \mathcal{F} &\equiv (\mathcal{F}_{s-1,s}, \ldots, \mathcal{F}_{s-1,T}, \mathcal{F}_{s,s+1}, \ldots, \mathcal{F}_{s,T}, \ldots, \ldots, \mathcal{F}_{T-1,T}) \end{aligned}$$

I can state the following proposition.

Proposition 11 *For $K = 1$ my model is equivalent to the one of Kőszegi and Rabin (2009), thus $\bar{U}^s(c, \mathcal{F}; \gamma, m(\cdot), \mu(\cdot)) = \hat{U}^s(c, \mathcal{F}; \gamma, m(\cdot), \mu(\cdot)) \;\; \forall\; s, c, \mathcal{F}, \gamma, m(\cdot), \mu(\cdot)$.*

Proof. [Proposition 11] With $K = 1$ equation (2.12) reduces to

$$\bar{U}^s = \sum_{t=s}^{T} \left\{ m^1(c_t^1) + \sum_{\tau=t}^{T} \left[\gamma_{t,\tau} \int_0^1 \mu \left(m^1(c_{\mathcal{F}^1_{t,\tau}}(p)) - m^1(c_{\mathcal{F}^1_{t-1,\tau}}(p)) \right) dp \right] \right\} \quad (2.17)$$

and equation (2.15) to

$$\hat{U}^s = \sum_{t=s}^{T} \left\{ m(c_t) + \sum_{\tau=t}^{T} \left[\gamma_{t,\tau} \int_0^1 \mu \left(m_{\mathcal{F}_{t,\tau}}(p) - m_{\mathcal{F}_{t-1,\tau}}(p) \right) dp \right] \right\}. \quad (2.18)$$

With $K = 1$ it also holds that

$$\begin{aligned} c_t &= c_t^1 && (2.19) \\ m(\cdot) &= m^1(\cdot) && (2.20) \\ \mathcal{F}_{t,\tau} &= \mathcal{F}^1_{t,\tau} \\ m_{\mathcal{F}_{t,\tau}}(p) &= m^1(c_{\mathcal{F}^1_{t,\tau}}(p)) && (2.21) \end{aligned}$$

where the last equality holds because at each t the function $m(\cdot)$ is increasing in the only argument c_t, as expressed in condition (2.4). Inserting (2.19), (2.20) and (2.21) into (2.18) delivers the same expression as (2.17). ■

Equation (2.21) is probably the only condition that requires more elaboration. I already established that for $K = 1$, $m(\cdot) = m^1(\cdot)$ and $\mathcal{F}_{t,\tau} = \mathcal{F}^1_{t,\tau}$ hold. Therefore, $m^1(c_{\mathcal{F}^1_{t,\tau}}(p)) = m(c_{\mathcal{F}_{t,\tau}}(p))$.

Figure 2.3 illustrates the identity of $m_{\mathcal{F}_{t,\tau}}(p)$ and $m(c_{\mathcal{F}_{t,\tau}}(p))$. In the northeastern quadrant the cumulative distribution function of c_τ according to a belief $\mathcal{F}_{t,\tau}$ is sketched. Every consumption level c_τ is mapped onto the pure consumption utility function $m_\tau = m(c_\tau)$ in the southeastern quadrant. The cumulative distribution function of m_τ according to the belief $\mathcal{F}_{t,\tau}$ is depicted in the southwestern quadrant. For an illustration consider the percentile $p = \tilde{p}$. Whether we take the value of c_τ at the percentile $\tilde{p}$ according to the belief $\mathcal{F}_{t,\tau}$ and evaluate it by the pure consumption utility function $m(\cdot)$ or we directly evaluate m_τ at the percentile $\tilde{p}$ always delivers the same amount of pure consumption utility.

2.8 Inequality of Models for Multi-Attribute Utility

If pure consumption utility depends on several arguments the two compared models will differ in their predictions of choice under risk. This difference persists if pure consumption utility is additively separable in all arguments. The root of this difference is that gain-loss utility is computed at a different level of aggregation. Whereas in the model of Kőszegi and

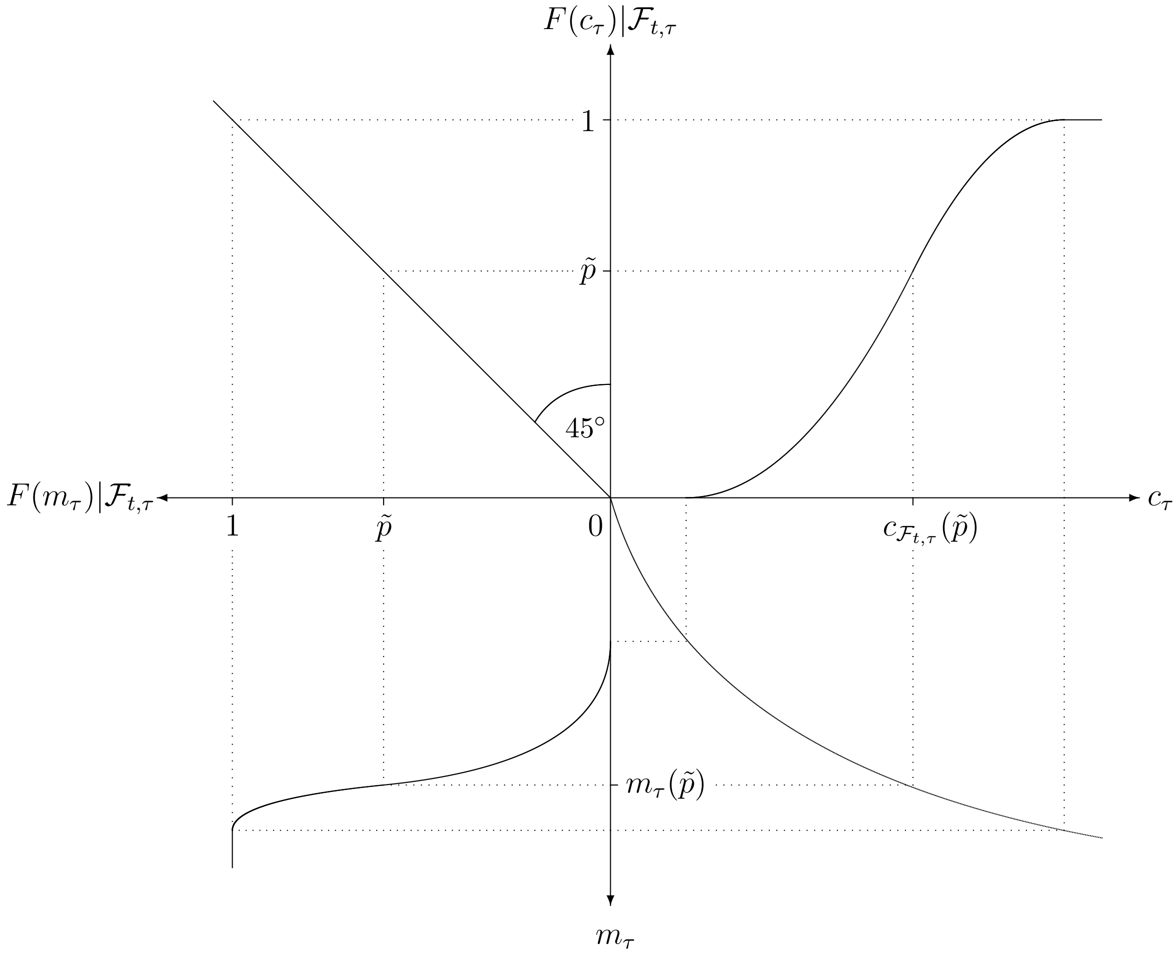

Figure 2.3: Equality of $m_{\mathcal{F}_{t,\tau}}(p)$ and $m(c_{\mathcal{F}_{t,\tau}}(p))$ with $K = 1$

Rabin (2009) it is computed at the level of pure consumption utility of a single dimension in my model it is computed at the level of overall pure consumption utility of all dimensions.

To formally demonstrate the difference with additively separable pure consumption utility a single example would suffice. However, I provide two examples in the following subsections that show that the difference is not a mere technicality but extends to practically relevant situations. At the end of each subsection I continue my example from previous sections.

2.8.1 Substitutability of Changes

In a certain sense in my model the dimensions of consumption are more substitutable than in the model of Kőszegi and Rabin (2009). If a change in the beliefs occurs, individuals assess whether this change would lead to a different probability distribution over the overall pure consumption utility. If it does not, there result no feelings of gain or loss in my model. However, in the model of Kőszegi and Rabin (2009) individuals do not integrate the different dimensions of utility when anticipating the effect of changes. They compare the probability distribution of each dimension of utility separately and enjoy gains or suffer losses along each of these dimensions.

To clarify the difference between the two models I define an event that leads to an update of beliefs in the following way. It leaves the beliefs on overall pure consumption utility unaffected. At the same time it changes the beliefs concerning at least one dimension of utility. If both statements hold simultaneously there have to exist changes in at least two dimensions. Then I show that this event has no effect on aggregate gain-loss utility in my model. Subsequently I show that this event has an unambiguously negative effect on aggregate gain-loss utility in the Kőszegi and Rabin (2009) model.

Proposition 12 *Assume* $m(c_t) = \sum_{k=1}^{K} m^k\left(c_t^k\right)$ *and* $\mu(\cdot)$ *satisfies (A3°). For a fixed combination of* t *and* τ *consider beliefs* $\mathcal{F}_{t,\tau}$ *and* $\mathcal{F}_{t-1,\tau}$ *such that* $m_{\mathcal{F}_{t,\tau}}(p) = m_{\mathcal{F}_{t-1,\tau}}(p)\ \forall\ p$ *and* $\exists\ k, p : c_{\mathcal{F}^k_{t,\tau}}(p) \neq c_{\mathcal{F}^k_{t-1,\tau}}(p)$*. Then* $N(\mathcal{F}_{t,\tau}|\mathcal{F}_{t-1,\tau}) = 0$ *in my model but* $N(\mathcal{F}_{t,\tau}|\mathcal{F}_{t-1,\tau}) < 0$ *in the model of Kőszegi and Rabin (2009).*

Proof. [**Proposition 12**] Is provided in the appendix of section 2.11 ■

The outlined difference in predictions could also be employed in an experiment to distinguish between my own model and the Kőszegi and Rabin (2009) model. For example, one could determine which trade-offs between monetary compensation and effort leaves an individual indifferent in a riskless environment. In such a riskless environment, both models yield identical prediction and decision utility equals pure consumption utility. Then one could test whether in risky environments changes in beliefs over the two dimensions that did not have an effect on pure consumption utility do generate a change in decision utility.

Continuing Example

To illustrate proposition 12 consider the situation where in period $t-1$ the decision maker beliefs that in the future period τ he will be in good health ($c_\tau^1 = e$) with certainty. However, he also thinks that with certainty he will be poor ($c_\tau^2 = 1$). Between period $t-1$ and period t the decision maker is knocked over by a drunken car driver. This has two effects. First, the decision maker knows with certainty that in period τ he will not be able to use his legs normally and thus $c_\tau^1 = 1$. Second, the decision maker receives a financial compensation for non-pecuniary damage from the car driver that allows him to consume $c_\tau^2 = e$ instead of $c_\tau^2 = 1$. The pure consumption utility in period τ, that is $m(c_\tau)$ will not be changed by the accident and equals 1. The assumption that the financial payment does not make the decision maker worse off in terms of pure consumption utility might seem stylized. However, this case could be a reasonable approximation to reality if the legislation has the intention to fully compensate the victim and is very effective in doing so. The outlined event does not have any effect on anticipatory utility in my model since only changes in the anticipated pure consumption utility can generate this. In contrast, it results in an aggregate gain-loss utility of $\mu(1) + \mu(-1) < 0$ in the model of Kőszegi and Rabin (2009).[3]

2.8.2 Changes in Beliefs on Correlation

The discussion of the previous subsection might give the misleading impression that the model of Kőszegi and Rabin (2009) predicts gain-loss utility to arise in unambiguously more situations than in my model. However, we will see that there also exists a class of situations where gain-loss utility can only occur in my model.

To identify this class of situations I define an update of beliefs with the following properties. It does not change any marginal distribution of the single dimensions of consumption. However, the update in beliefs does change the joint probability distribution of at least one

[3] Note that the assumption that deteriorations in health can be fully compensated by other consumption in terms of decision utility is not unique to my model. Only the level at which this compensation is reached differs between the two models. In my model the level is equal to the compensation in riskless situations whereas in the model of Kőszegi and Rabin (2009) it has to be higher.

pair of dimensions of consumption. Then I show that this update in beliefs can never generate any gain-loss utility in the model of Kőszegi and Rabin (2009). In contrast, it generally generates gain-loss utility in my model.

Thus, my model does generate the prediction that the correlation of risks that affect different dimensions of consumption matters for the determination of gain-loss utility. If the individual would for instance learn that losses in two dimensions of consumption are more likely to appear jointly than he thought before, this dismal association can trigger negative feelings of loss in him. In contrast, according to the model of Kőszegi and Rabin (2009) he would not be scared by the insight that two bad news tend to come together.

Proposition 13 *Assume* $m(c_t) = \sum_{k=1}^{K} m^k(c_t^k)$. *For a fixed combination of* t *and* τ *consider beliefs* $\mathcal{F}_{t,\tau}$ *and* $\mathcal{F}_{t-1,\tau}$ *such that* $\mathcal{F}_{t,\tau}^k = \mathcal{F}_{t-1,\tau}^k\ \forall\ k$ *and* $\exists\ k,l : \mathcal{F}_{t,\tau}^{k,l} \neq \mathcal{F}_{t-1,\tau}^{k,l}$. *Then,* $N(\mathcal{F}_{t,\tau}|\mathcal{F}_{t-1,\tau}) = 0$ *in the model of Kőszegi and Rabin (2009). In general,* $N(\mathcal{F}_{t,\tau}|\mathcal{F}_{t-1,\tau}) \neq 0$ *in my model.*

Proof. [**Proposition 13**] Is provided in the appendix of section 2.11 ■

Again, this result could be used to distinguish between the two models experimentally. It could be implemented that the subjects receive information on the correlation of future risks. If the subjects would show not to be indifferent to these news they would hint at a closer resemblance with my model.

Continuing Example

To illustrate proposition 13 consider the following situation. The decision maker thinks in period $t-1$ and in period t that with the probability of $1/2$ he will be healthy in τ. If he is healthy, consumption of good 1 is e, otherwise it is 1. He also thinks in both periods that he will be able to afford the higher level of consumption of good 2 with probability of $1/2$. The higher level of good 2 is e, the lower level is 1. The only difference between the two periods is that in period $t-1$ the decision maker thinks that the levels of the two goods are independently distributed. Think of it as his naive inference from statistical data that states that half of comparable persons are rich and half are healthy.

Between period $t-1$ and t he learns of a number of cases where comparable individuals have suffered from a joint deterioration in both dimensions. This could be in the form of stories on self-induced car accidents. In such accidents the drivers lose the ability to normally use their legs in τ and the damage to the car additionally decreases their consumption in τ. Again, to make the example very clear cut, I assume that the individual naively infers that the injured and poor individuals are always the same and vice versa. Thus, in period t he beliefs that the two dimensions of consumption are perfectly and positively correlated.

In the model of Kőszegi and Rabin (2009) the decision maker will suffer no gain-loss utility. The reason is that only the beliefs on marginal probability distributions of dimensions of consumption are relevant for gain-loss utility and those marginal distributions are unaffected by the update in beliefs. In my model the perceived higher probability of suffering both a low level of health and of material consumption (probability of 1/2 instead of 1/4) induces feelings of loss that outweigh the elation of a higher probability of enjoying both good health and riches (also with a probability of 1/2 instead of 1/4). Formally, in my model the update in beliefs results in aggregate gain-loss utility of $\frac{1}{4}[\mu(1) + \mu(-1)] < 0$.

Although I assumed for illustrational purposes an extreme response to news it is important to point out that also very reasonable and rational individuals update their beliefs as they receive additional information on the correlation of events. The feelings of such individuals that is generated by updates on correlations may be less extreme but nevertheless could change behavior in crucial ways.

2.9 Conclusion

In this chapter I proposed a theoretical model of reference-dependent anticipatory utility that has a broader range of applicability in case of multi-attribute utility than previous models. I provided a thorough comparison to the similar model of Kőszegi and Rabin

(2009). I showed that the difference in predictions of behavior exists only in the multi-attribute utility case. I showed that the difference in the multi-attribute utility case persist even if pure consumption utility is additively separable. I outlined two classes of situations where those differences matter.

Although the ambition is to improve the model of Kőszegi and Rabin (2009) in the respect of wider applicability there is no doubt that my work constitutes only a small step forward compared to the huge improvement the model of Kőszegi and Rabin (2009) achieved in comparison to its predecessors. In this respect I perceive my model to stand in the tradition of studies such as Fishburn (1984), Miyamoto and Wakker (1996), Zank (2001), Bleichrodt and Miyamoto (2003), and Bleichrodt, Schmidt and Zank (2009) which extend other non-EUT models to the multi-attribute case. The latter three studies concern multi-attribute versions of prospect theory. The level at which gain-loss utility is generated can also be employed to categorize these models. While Miyamoto and Wakker (1996) and Zank (2001) assume gain-loss utility to depend on the level of integrated consumption utility, Bleichrodt, Schmidt and Zank (2009) assume that at each dimension of consumption gain-loss utility is generated. In this respect the distinction between these models resembles the distinction between my model and the one of Kőszegi and Rabin (2009). The question of substitutability of changes in different dimensions of consumption arises in those models in a very similar way than we discussed in section 2.8.1 for the two models under consideration. However, the additional issues related to the beliefs in the correlation of dimensions as discussed in section 2.8.2 are novel to anticipatory utility. This is because in the previous non-EUT models changes of expectations did not have any effect.

Possible applications of my model encompass all economically interesting situations where the multi-dimensionality of consumption is known to be important and where the anticipation of events can generate profound impacts on the feelings of individuals. The domain of health comes into mind. It has been argued that health is not integrated with other consumption in a linear way. Also, the anxiety because of potential deterioration in health become extremely important to explain patient behavior. We could, for instance, apply the outlined model to analyze the attitude of patients to medical test that might reveal that the patient is prone to certain illnesses. Especially, the potential event of a false positive test result could explain the reluctance of patients to take such tests. Another application in health economics would be to analyze what benefit insurance can have if it cannot improve health but only raise wealth in the event of an illness. Here, the differences of my model to the one of Kőszegi and Rabin (2009) could also lead to diverging predictions.

2.10 Appendix: Alternative Models

In this section I investigate if there exists an adjustment of the model of Kőszegi and Rabin (2009) that allows pure consumption utility $m(\cdot)$ to be non-additive but at the same time generates gain-loss utility in each dimension k separably. This is only possible if the percentile-wise comparison in each dimension is not conducted over the pure consumption utilities, but instead over the absolute levels of consumption.

In both adjusted models considered in this section the common assumptions of section 2.2 are met. Formally, equations (2.1) to (2.5) hold, thus also equation (2.6) holds.

The features not common to the two models in the main text are

$$m(c_t) \quad : \quad \text{any function satisfying} \quad \frac{\partial m}{\partial c_t^k} > 0 > \frac{\partial^2 m}{\left[\partial c_t^k\right]^2} \quad \forall\, t, k \tag{2.22}$$

$$N(\mathcal{F}_{t,\tau}|\mathcal{F}_{t-1,\tau}) = \sum_{k=1}^{K} N^k(\mathcal{F}_{t,\tau}^k|\mathcal{F}_{t-1,\tau}^k) \tag{2.23}$$

Equation (2.22) distinguishes the adjusted models from the original model of Kőszegi and Rabin (2009), where additionally condition (2.7) has to hold. Equation (2.23) distinguishes the adjusted models from my model.

In addition to the assumptions above, one of the following conditions holds

$$N^k(\mathcal{F}^k_{t,\tau}|\mathcal{F}^k_{t-1,\tau}) = \int_0^1 \mu\left(c_{\mathcal{F}^k_{t,\tau}}(p) - c_{\mathcal{F}^k_{t-1,\tau}}(p)\right) dp \tag{2.24}$$

$$N^k(\mathcal{F}^k_{t,\tau}|\mathcal{F}^k_{t-1,\tau}) = \int_0^1 \mu^k\left(c_{\mathcal{F}^k_{t,\tau}}(p) - c_{\mathcal{F}^k_{t-1,\tau}}(p)\right) dp \tag{2.25}$$

$c_{\mathcal{F}}(p)$ again denotes consumption in percentile p given belief $\mathcal{F}$, and both $\mu(\cdot)$ and $\mu^k(\cdot)$ satisfying assumptions (A0) to (A4) (in the latter case, for all k).

Using specification (2.24), the adjusted Kőszegi and Rabin (2009) model can be written as

$$\dot{U}^s = \sum_{t=s}^{T}\left\{m(c_t) + \sum_{\tau=t}^{T}\left[\gamma_{t,\tau}\sum_{k=1}^{K}\int_0^1 \mu\left(c_{\mathcal{F}^k_{t,\tau}}(p) - c_{\mathcal{F}^k_{t-1,\tau}}(p)\right) dp\right]\right\} \tag{2.26}$$

The drawback of specification (2.24) is that it is not invariant to the scaling of consumption levels. If we would have only a single dimension of utility, hence $K = 1$, we could define $\mu(\cdot)$ as holding for a specific scale of its argument. But since we consider multi-attribute consumption vectors this is no longer possible. Different attributes might not be expressible in the same units to begin with. Even if they are, it seems highly doubtful that a common scale-specific gain-loss utility function $\mu(\cdot)$ would be a reasonable assumption. Both a splendid bottle of wine and a package of shampoo can be expressed in milliliter or gallons. However, it seems far fetched that a loss of the same quantity would result in equal feelings of loss.

This discussion also demonstrates why in both models discussed in the main text disaggregate gain-loss utility is based on the differences in pure consumption utilities. The pure consumption utility operates as an equalizer of all dimensions of consumption. It translates the attribute-specific units into a single measure that can be interpreted across dimensions.

Using specification (2.25), the adjusted Kőszegi and Rabin (2009) model can be written as

$$\ddot{U}^s = \sum_{t=s}^{T} \left\{ m(c_t) + \sum_{\tau=t}^{T} \left[\gamma_{t,\tau} \sum_{k=1}^{K} \int_0^1 \mu^k \left(c_{\mathcal{F}^k_{t,\tau}}(p) - c_{\mathcal{F}^k_{t-1,\tau}}(p) \right) dp \right] \right\} \tag{2.27}$$

The drawback of specification (2.25) is that one has to define K different gain-loss functions $\mu^1(\cdot), \ldots, \mu^K(\cdot)$. These additional degrees of freedom run counter to the stated objective to find a common theory for many phenomena.[4] It introduces $K-1$ additional degrees of freedom into the model. One consequence would be that no elicitation of gain-loss utility over one dimension of utility can be used to generate predictions on behavior concerning gain-loss utility of a different dimension. It diminishes the accomplishment of Kőszegi and Rabin (2006) and subsequent models to reduce arbitrary and non-verifiable assumptions.

2.11 Appendix: Proofs

Proof. [Lemma 2] Firstly, I show that the equality in the lemma holds

$$\begin{aligned}
\text{By assumption:} \quad & \omega \text{ is symmetrically distributed around } 0 \\
\Rightarrow \quad & \sum_{k=1}^{K} \left[\omega^k\right] = 0 \\
\text{using (A0)} \Rightarrow \quad & \mu\left(\sum_{k=1}^{K} \left[\omega^k\right]\right) = 0.
\end{aligned}$$

[4]On this goal see Kőszegi and Rabin (2006, p1136-1137):“[...] our general approach has an attractive methodological feature: [...] it moves us closer to a universally applicable, zero-degrees-of-freedom way to translate any existing reference-independent model into the corresponding reference-dependent one.”

Secondly, I show that the inequality in the lemma holds. In a first step I rewrite the sum as

$$
\begin{array}{rrcl}
 & \sum_{k=1}^{K}\left[\mu\left(\omega^k\right)\right] & = & \sum_{k:\omega^k=0}\left[\mu\left(\omega^k\right)\right] + \sum_{k:\omega^k\neq 0}\left[\mu\left(\omega^k\right)\right] \\
\Leftrightarrow & \sum_{k=1}^{K}\left[\mu\left(\omega^k\right)\right] & = & \sum_{k:\omega^k=0}\left[\mu\left(0\right)\right] + \sum_{k:\omega^k\neq 0}\left[\mu\left(\omega^k\right)\right] \\
\text{using (A0)} \Rightarrow & \sum_{k=1}^{K}\left[\mu\left(\omega^k\right)\right] & = & \sum_{k:\omega^k\neq 0}\left[\mu\left(\omega^k\right)\right] \\
\multicolumn{4}{l}{\text{using that } \omega \text{ is symmetrically distributed around } 0} \\
\Rightarrow & \sum_{k=1}^{K}\left[\mu\left(\omega^k\right)\right] & = & \sum_{k:\omega^k>0}\left[\mu\left(\omega^k\right) - \mu\left(-\omega^k\right)\right].
\end{array}
$$

Now I show that this term is negative as long as there exists at least one k for which $\omega^k > 0$

$$
\begin{array}{rrcll}
\text{using (A2)}: & \mu(y)+\mu(-y) & < & \mu(x)+\mu(-x) & \forall \;\; y > x \geq 0 \\
\Rightarrow & \mu(y)+\mu(-y) & < & 2\mu(0) & \forall \;\; y > 0 \\
\text{using (A0)} \Rightarrow & \mu(y)+\mu(-y) & < & 0 & \forall \;\; y > 0 \\
\Leftrightarrow & \mu\left(\omega^k\right)+\mu\left(-\omega^k\right) & < & 0 & \forall \;\; \omega^k > 0 \\
\Rightarrow & \sum_{k:\omega^k>0}\left[\mu\left(\omega^k\right)+\mu\left(-\omega^k\right)\right] & < & 0. &
\end{array}
$$

The last step of the proof is to show that there exists always a k for which $\omega^k > 0$

$$
\begin{array}{rlcl}
\text{By assumption:} & \multicolumn{3}{l}{\omega \text{ is non-degenerate}} \\
\Leftrightarrow & \exists\, k: \;\; \omega^k & \neq & 0 \\
\text{using that } \omega \text{ is symmetrically distributed around } 0 \Rightarrow & \exists\, k: \;\; \omega^k & > & 0.
\end{array}
$$

■

Proof. [Lemma 3] First I show that the equality in the lemma holds

$$
\begin{array}{rrcl}
\text{By assumption:} & \sum_{k=1}^{K}\left[\omega^k\right] & = & 0 \\
\text{using (A0)} \Rightarrow & \mu\left(\sum_{k=1}^{K}\left[\omega^k\right]\right) & = & 0.
\end{array}
$$

Now I show that the inequality in the lemma holds.

$$\begin{array}{rrcl}
\text{By assumption:} & \sum_{k=1}^{K}\left[\omega^k\right] & = & 0 \\
\Leftrightarrow & \sum_{k:\omega^k>0}\left[\omega^k\right]+\sum_{k:\omega^k=0}\left[\omega^k\right]+\sum_{k:\omega^k<0}\left[\omega^k\right] & = & 0 \\
\Leftrightarrow & \sum_{k:\omega^k>0}\left[\omega^k\right]+\sum_{k:\omega^k<0}\left[\omega^k\right] & = & 0 \\
\Leftrightarrow & \sum_{k:\omega^k>0}\left[\omega^k\right] & = & -\sum_{k:\omega^k<0}\left[\omega^k\right].
\end{array}$$

I can also derive

$$\begin{array}{rrcll}
\text{by assumption (A3°):} & \lim_{x\to-\infty}\mu'(x) & > & \lim_{x\to 0}\mu'(|x|) & \\
\text{using (A0) \& (A2)} \Rightarrow & \mu'(x) & > & \mu'(y) & \forall \;\; x<0<y \\
\Leftrightarrow & \mu'\left(\omega^k\right) & < & \mu'\left(\omega^l\right) & \forall \;\; \omega^l<0<\omega^k.
\end{array}$$

Using this result and assumption (A1), I can state that

$$\begin{array}{rrcl}
\text{from:} & \omega^k>0>\omega^l \;\; \forall\, k,l \quad \text{and} & & \sum_k\left[\omega^k\right]=\sum_l\left[\omega^l\right] \\
\text{it follows that:} & \sum_k\left[\mu(\omega^k)-\mu(0)\right] & < & \sum_l\left[\mu(0)-\mu(\omega^l)\right] \\
\text{using (A0)} \Rightarrow & \sum_k\left[\mu(\omega^k)\right] & < & -\sum_l\left[\mu(\omega^l)\right].
\end{array}$$

Therefore,

$$\begin{array}{rrcl}
 & \sum_{k:\omega^k>0}\left[\mu(\omega^k)\right] & < & -\sum_{k:\omega^k<0}\left[\mu(\omega^k)\right] \\
\Leftrightarrow & \sum_{k:\omega^k>0}\left[\mu(\omega^k)\right]+\sum_{k:\omega^k<0}\left[\mu(\omega^k)\right] & < & 0 \\
\text{using (A0)} \Rightarrow & \sum_{k:\omega^k>0}\left[\mu(\omega^k)\right]+\sum_{k:\omega^k=0}\left[\mu(\omega^k)\right]+\sum_{k:\omega^k<0}\left[\mu(\omega^k)\right] & < & 0 \\
\Leftrightarrow & \sum_{k=1}^{K}\left[\mu(\omega^k)\right] & < & 0.
\end{array}$$

■

Proof. [Proposition 12] I first show that in my model the change in beliefs results in no gain-loss utility.

$$
\begin{array}{rrcll}
\text{By assumption:} & m_{\mathcal{F}_{t,\tau}}(p) & = & m_{\mathcal{F}_{t-1,\tau}}(p) & \forall \;\; p \\
\Leftrightarrow & m_{\mathcal{F}_{t,\tau}}(p) - m_{\mathcal{F}_{t-1,\tau}}(p) & = & 0 & \forall \;\; p \\
\text{using (A0)} \Rightarrow & \mu\left(m_{\mathcal{F}_{t,\tau}}(p) - m_{\mathcal{F}_{t-1,\tau}}(p)\right) & = & 0 & \forall \;\; p \\
\Rightarrow & \int_0^1 \mu\left(m_{\mathcal{F}_{t,\tau}}(p) - m_{\mathcal{F}_{t-1,\tau}}(p)\right) dp & = & 0 & \\
\text{using (2.14)} \Rightarrow & N\left(\mathcal{F}_{t,\tau}|\mathcal{F}_{t-1,\tau}\right) & = & 0 &
\end{array}
$$

I now show that in the model of Kőszegi and Rabin (2009) there results a negative gain-loss utility. For this purpose consider the variable $\omega(p)$ whose K outcomes are defined by $\omega^k(p) = m^k(c_{\mathcal{F}^k_{t,\tau}}(p)) - m^k(c_{\mathcal{F}^k_{t-1,\tau}}(p))$ for $k = 1, \ldots, K$. Then,

$$
\begin{array}{rrcll}
\text{by assumption:} & m_{\mathcal{F}_{t,\tau}}(p) & = & m_{\mathcal{F}_{t-1,\tau}}(p) & \forall \;\; p \\
\text{using (2.7)} \Rightarrow & \sum_{k=1}^{K} m^k(c_{\mathcal{F}^k_{t,\tau}}(p)) & = & \sum_{k=1}^{K} m^k(c_{\mathcal{F}^k_{t-1,\tau}}(p)) & \forall \;\; p \\
\Leftrightarrow & \sum_{k=1}^{K} \left[m^k(c_{\mathcal{F}^k_{t,\tau}}(p)) - m^k(c_{\mathcal{F}^k_{t-1,\tau}}(p))\right] & = & 0 & \forall \;\; p,
\end{array}
$$

therefore the sum over $\omega^k(p)$ is zero for all p. Also,

$$
\begin{array}{rlrcl}
\text{by assumption:} & \exists \; k, p : & c_{\mathcal{F}^k_{t,\tau}}(p) & \neq & c_{\mathcal{F}^k_{t-1,\tau}}(p) \\
\text{using (2.7)} \Rightarrow & \exists \; k, p : & m^k(c_{\mathcal{F}^k_{t,\tau}}(p)) & \neq & m^k(c_{\mathcal{F}^k_{t-1,\tau}}(p)) \\
\Leftrightarrow & \exists \; k, p : & m^k(c_{\mathcal{F}^k_{t,\tau}}(p)) - m^k(c_{\mathcal{F}^k_{t-1,\tau}}(p)) & \neq & 0,
\end{array}
$$

therefore $\omega(p)$ is non-degenerate for some p. By applying lemma 3 I receive

$$\begin{array}{rrcl}
 & \sum_{k=1}^{K}\left[\mu\left(\omega^k(p)\right)\right] & < & 0 \quad \forall \quad p \\
\Leftrightarrow & \sum_{k=1}^{K}\left[\mu\left(m^k(c_{\mathcal{F}^k_{t,\tau}}(p)) - m^k(c_{\mathcal{F}^k_{t-1,\tau}}(p))\right)\right] & < & 0 \quad \forall \quad p \\
\Rightarrow & \int_0^1 \sum_{k=1}^{K}\left[\mu\left(m^k(c_{\mathcal{F}^k_{t,\tau}}(p)) - m^k(c_{\mathcal{F}^k_{t-1,\tau}}(p))\right)\right] dp & < & 0 \\
\Leftrightarrow & \sum_{k=1}^{K}\int_0^1 \left[\mu\left(m^k(c_{\mathcal{F}^k_{t,\tau}}(p)) - m^k(c_{\mathcal{F}^k_{t-1,\tau}}(p))\right)\right] dp & < & 0 \\
\text{using (2.11)} \Rightarrow & \sum_{k=1}^{K} N^k\left(\mathcal{F}^k_{t,\tau}|\mathcal{F}^k_{t-1,\tau}\right) & < & 0 \\
\text{using (2.10)} \Rightarrow & N\left(\mathcal{F}_{t,\tau}|\mathcal{F}_{t-1,\tau}\right) & < & 0.
\end{array}$$

■

Proof. [**Proposition 13**] I first show that in the model of Kőszegi and Rabin (2009) the change in beliefs results in no gain-loss utility.

$$\begin{array}{rrcll}
\text{By assumption:} & \mathcal{F}^k_{t,\tau} & = & \mathcal{F}^k_{t-1,\tau} & \forall \quad k \\
\Leftrightarrow & c_{\mathcal{F}^k_{t,\tau}}(p) & = & c_{\mathcal{F}^k_{t-1,\tau}}(p) & \forall \quad k,p \\
\text{using (2.8)} \Rightarrow & m^k(c_{\mathcal{F}^k_{t,\tau}}(p)) & = & m^k(c_{\mathcal{F}^k_{t-1,\tau}}(p)) & \forall \quad k,p \\
\Leftrightarrow & m^k(c_{\mathcal{F}^k_{t,\tau}}(p)) - m^k(c_{\mathcal{F}^k_{t-1,\tau}}(p)) & = & 0 & \forall \quad k,p \\
\text{using (A0)} \Rightarrow & \mu\left(m^k(c_{\mathcal{F}^k_{t,\tau}}(p)) - m^k(c_{\mathcal{F}^k_{t-1,\tau}}(p))\right) & = & 0 & \forall \quad k,p \\
\Rightarrow & \int_0^1 \mu\left(m^k(c_{\mathcal{F}^k_{t,\tau}}(p)) - m^k(c_{\mathcal{F}^k_{t-1,\tau}}(p))\right) dp & = & 0 & \forall \quad k \\
\text{using (2.11)} \Rightarrow & N^k\left(\mathcal{F}^k_{t,\tau}|\mathcal{F}^k_{t-1,\tau}\right) & = & 0 & \forall \quad k \\
\text{using (2.10)} \Rightarrow & N\left(\mathcal{F}_{t,\tau}|\mathcal{F}_{t-1,\tau}\right) & = & 0 &
\end{array}$$

I now show that in my model generally $N(\mathcal{F}_{t,\tau}|\mathcal{F}_{t-1,\tau}) \neq 0$ holds. To show that the strict inequality in the proposition can hold, it suffices to give a single example. Suppose that $K = 2$, $m\left(c^1_\tau, c^2_\tau\right) = \ln\left(c^1_\tau\right) + \ln\left(c^2_\tau\right)$, and $c^1_\tau, c^2_\tau \in \{1, e\}$, where e denotes Euler's number. For the considered t, τ, the beliefs over the joint distribution of c_τ^1 and c_τ^2 are as follows

$$\begin{aligned}
\mathcal{F}_{t-1,\tau} &= \mathcal{F}_{t-1,\tau}^{1,2} &: \ \Pr\left(c_\tau^1 = 1, c_\tau^2 = e\right) = \Pr\left(c_\tau^1 = e, c_\tau^2 = 1\right) = 1/2 \\
\mathcal{F}_{t,\tau} &= \mathcal{F}_{t,\tau}^{1,2} &: \ \Pr\left(c_\tau^1 = 1, c_\tau^2 = 1\right) = \Pr\left(c_\tau^1 = e, c_\tau^2 = e\right) = 1/2
\end{aligned}$$

The marginal probabilities do not change, therefore $\mathcal{F}_{t-1,\tau}^k = \mathcal{F}_{t,\tau}^k$ for $k = 1, 2$. My model relies only on the beliefs over the full pure consumption utility. Thus, these beliefs can equivalently be stated as

$$\begin{aligned}
\mathcal{F}_{t-1,\tau} &: \ \Pr\left(m\left(c_\tau^1, c_\tau^2\right) = 1\right) = 1 \\
\mathcal{F}_{t,\tau} &: \ \Pr\left(m\left(c_\tau^1, c_\tau^2\right) = 0\right) = \Pr\left(m\left(c_\tau^1, c_\tau^2\right) = 2\right) = 1/2
\end{aligned}$$

Then I can state that in my example

$$N(\mathcal{F}_{t,\tau}|\mathcal{F}_{t-1,\tau}) = \tfrac{1}{2}\left[\mu(-1) + \mu(1)\right]$$

This expression is strictly negative according to assumption (A2).

I now show that the inequality in the proposition holds only generally and there exist exemptions. In those exemptions no gain-loss utility results in my model. One exemption is the following. Suppose that $K = 3$, $m\left(c_\tau^1, c_\tau^2, c_\tau^3\right) = \ln\left(c_\tau^1\right) + \ln\left(c_\tau^2\right) + \ln\left(c_\tau^3\right)$, and $c_\tau^1, c_\tau^2, c_\tau^3 \in \{1, e\}$. For the considered t and τ, the beliefs over the joint distribution of c_τ^1, c_τ^2 and c_τ^3 can be represented as follows

$$\begin{aligned}
\mathcal{F}_{t-1,\tau} = \mathcal{F}_{t-1,\tau}^{1,2,3} \ : \quad & \Pr\left(c_\tau^1 = 1, c_\tau^2 = 1, c_\tau^3 = 1\right) = \Pr\left(c_\tau^1 = 1, c_\tau^2 = 1, c_\tau^3 = e\right) \\
= \ & \Pr\left(c_\tau^1 = e, c_\tau^2 = e, c_\tau^3 = 1\right) = \Pr\left(c_\tau^1 = e, c_\tau^2 = e, c_\tau^3 = e\right) \quad = 1/4 \\
\mathcal{F}_{t,\tau} = \mathcal{F}_{t,\tau}^{1,2,3} \ : \quad & \Pr\left(c_\tau^1 = 1, c_\tau^2 = 1, c_\tau^3 = 1\right) = \Pr\left(c_\tau^1 = e, c_\tau^2 = 1, c_\tau^3 = 1\right) \\
= \ & \Pr\left(c_\tau^1 = 1, c_\tau^2 = e, c_\tau^3 = e\right) = \Pr\left(c_\tau^1 = e, c_\tau^2 = e, c_\tau^3 = e\right) \quad = 1/4
\end{aligned}$$

Again, the marginal probabilities do not change, and therefore $\mathcal{F}_{t-1,\tau}^k = \mathcal{F}_{t,\tau}^k$ for $k = 1, 2, 3$. However, $\mathcal{F}_{t-1,\tau}^{k,l} \neq \mathcal{F}_{t,\tau}^{k,l}$ holds for both $(k, l) = (1, 2)$ and $(k, l) = (2, 3)$. Thus, the conditions in the proposition are met in this example.

As again my model relies only on the beliefs over pure consumption utility I can state the full set of beliefs equivalently as

$$\begin{aligned}\mathcal{F}_{t-1,\tau}, \mathcal{F}_{t,\tau} \quad : \quad & \Pr\left(m\left(c_\tau^1, c_\tau^2, c_\tau^3\right) = 0\right) = \Pr\left(m\left(c_\tau^1, c_\tau^2, c_\tau^3\right) = 1\right) \\ = \quad & \Pr\left(m\left(c_\tau^1, c_\tau^2, c_\tau^3\right) = 2\right) = \Pr\left(m\left(c_\tau^1, c_\tau^2, c_\tau^3\right) = 3\right) \quad = \quad 1/4\end{aligned}$$

The beliefs over the pure consumption utility did not change between period $t-1$ and t. Therefore, in my second example $N(\mathcal{F}_{t,\tau}|\mathcal{F}_{t-1,\tau}) = 0$ holds in my model also. ■

Part III

Model-Independent Experimental Approaches

Chapter 3

Intensity of Risk Aversion*

3.1 Introduction

In order to measure individuals' risk attitudes, the multiple price-list method of Holt and Laury (2002) has become the industry standard in experimental economics. Major advantages that led to the popularity of the Holt and Laury (HL) tables include its transparency to subjects (easy to explain and implement), its incentivized elicitation, and that it can be easily attached to other experiments where risk aversion may have an influence. Nevertheless, the HL method has also its drawbacks. The major disadvantage is that it requires a specific utility framework such as expected utility theory (EUT) in order to classify subjects as more or less risk-averse.[1] If individuals' risk preferences are heterogeneous in the way that some act according to EUT while others rather act according to non-EUT, it becomes problematic to use the HL tables in order to classify subjects' risk attitudes. The reason is that the HL method is not based on a general notion of increasing risk which is satisfied by EUT *and* non-EUT models.

To account for this disadvantage, we propose a modification of the HL tables. This new method is based on the well-known increasing risk definitions of Rothschild and Stiglitz

*This chapter is based on joint work with Johannes Maier. Both authors contributed equally to this work.

[1] Holt and Laury (2002) use specific parametric forms of EUT in order to classify subjects.

(1970). These definitions are solely in terms of attributes of the distribution function and are therefore independent of a utility framework. Moreover, they have been used to characterize risk aversion in both EUT and non-EUT models (e.g. Machina, 1982; or Chew, Karni and Safra, 1987). By just imposing 'duality' asserting that less risk-averse individuals accept riskier gambles (see Diamond and Stiglitz, 1974), our method enables us to classify subjects as more or less risk-averse without assuming a specific utility framework. It is therefore applicable with heterogeneous risk preferences. Furthermore, our approach is more robust toward probability weighting, since it uses variations in outcomes (i.e. mean preserving spreads) and holds probabilities of outcomes constant at 50 %.

In a laboratory experiment we directly compare the HL method and our method using low and high stakes. We find that both methods yield the same classification of individuals concerning the *direction* of risk attitudes (i.e. risk-averse, risk-neutral, or risk-seeking). However, we also find that both methods yield diverging results concerning the *intensity* of risk attitude. The classification of individuals as being more or less risk-averse than others is quite different between both methods. Moreover, we find that our method yields higher levels of risk aversion intensity that are much closer to what is observed in the field. These estimates of risk aversion intensity are robust toward multiplying the stakes by five only when our method is used. For the HL method we can confirm the result of Holt and Laury (2002) that increasing the stakes increases risk aversion. It is also shown that these results are robust toward possible confounds like certain switching preferences or order effects.

The chapter is structured as follows. In section 3.2 we review the HL tables and note further advantages and disadvantages. We then propose a new method that shares the advantages but not the disadvantages of the HL tables in section 3.3. The experiment used to directly compare both methods is explained in section 3.4. The results of our experiment as well as their robustness are discussed in section 3.5 and the conclusion can be found in section 3.6. In the appendix of section 3.7 we generalize our new method so that it can be used for different purposes of measuring the intensity of risk attitude.

3.2 The Holt and Laury Method

Measuring the intensity of risk preferences is very important for theoretical predictions. Also, in experiments individuals' decisions are often (partly) driven by their risk preferences. In order to control for that, the multiple price-list method of Holt and Laury (2002) is commonly used in experiments nowadays. Table 3.1 presents the original HL design.

Table 3.1: The Holt and Laury method

	Option A		Option B		RRA if row was		
Row No.	Outcome A1 = \$2.00	Outcome A2 = \$1.60	Outcome B1 = \$3.85	Outcome B2 = \$0.10	last choice of A and below all B	EV[A]$-$ EV[B]	Var[A]$-$ Var[B]
1	Prob. 1/10	Prob. 9/10	Prob. 1/10	Prob. 9/10	[-1.71 ; -0.95]	1.17	-1.25
2	Prob. 2/10	Prob. 8/10	Prob. 2/10	Prob. 8/10	[-0.95 ; -0.49]	0.83	-2.22
3	Prob. 3/10	Prob. 7/10	Prob. 3/10	Prob. 7/10	[-0.49 ; -0.14]	0.50	-2.92
4	Prob. 4/10	Prob. 6/10	Prob. 4/10	Prob. 6/10	[-0.14 ; 0.15]	0.16	-3.34
5	Prob. 5/10	Prob. 5/10	Prob. 5/10	Prob. 5/10	[0.15 ; 0.41]	-0.18	-3.84
6	Prob. 6/10	Prob. 4/10	Prob. 6/10	Prob. 4/10	[0.41 ; 0.68]	-0.51	-3.34
7	Prob. 7/10	Prob. 3/10	Prob. 7/10	Prob. 3/10	[0.68 ; 0.97]	-0.85	-2.92
8	Prob. 8/10	Prob. 2/10	Prob. 8/10	Prob. 2/10	[0.97 ; 1.37]	-1.18	-2.22
9	Prob. 9/10	Prob. 1/10	Prob. 9/10	Prob. 1/10	[1.37 ; ∞)	-1.52	-1.25
10	Prob. 10/10	Prob. 0/10	Prob. 10/10	Prob. 0/10	non-monotone	-1.85	0.00

An individual makes a decision between option A and option B in each of the ten rows. Option A as well as option B can have two different outcomes (A1 or A2 and B1 or B2) with varying probabilities over the ten rows. The expected outcome of option A is higher for the first four rows and lower for the last six rows (as indicated by the second-to-last column in Table 3.1). So, a risk-neutral subject should choose option A in row 1 to 4 and then switch over and choose option B in row 5 to 10. However, as option B has a higher variance (indicated by the last column in Table 3.1), there is a trade-off when to switch to option B. Clearly, by row 10 everybody should have switched to option B as it yields the higher outcome with certainty. An individual who switches to option B between row 6 and row 10 is classified as being risk-averse and the more risk-averse individual will switch later as she needs a higher expected value to choose the more variable option. Someone who switches earlier to option B (between row 1 and row 4) is classified as risk-seeking by similar arguments. In column 6 of Table 3.1 we report the risk preference intensity measured by the amount of relative risk aversion (RRA) that is induced from the switching behavior if we assume the class of constant relative risk-averse (CRRA) utility functions.[2]

The advantages of the HL method are due to its design. It is very easy to explain to subjects since they only have to choose between option A and option B in each row.[3] It is incentivized and usually one of the ten rows is randomly selected and paid out for real. And because it is so easy to implement, the HL table can be attached to other experiments where risk aversion may play a role.

Nevertheless, the HL method also has its disadvantages. One disadvantage is that there is no flexibility in adjusting the ranges of RRA without affecting the round-numbered probabilities. So, for instance, if one would want to decrease the ranges of the RRA intervals in row 4 to 6 in order to better classify most subjects' risk attitudes (according to Holt and Laury (2002) 75 % of the subjects fall into this category), one would have to give up the round-numbered probabilities in Table 3.1. One way to circumvent this problem is proposed by Andersen et al. (2006) using a complex more-stage procedure and thereby loosing the advantages of the simple HL design mentioned above.

The use of variations in probabilities (whereas outcomes are held constant) makes the HL tables sensitive to probability weighting. For instance, by using the standard parametric Prospect Theory assumptions (Tversky and Kahneman, 1992) on the probability weighting function, we obtain the result that a subject with a linear utility function should choose A only for the first three and not for the first four rows. Such an individual would be classified as risk-seeking in HL. Of course, this makes it difficult to draw conclusions about the shape of the utility function.

The major disadvantage of the HL tables is, however, that they are not based on a general notion of increasing risk. They need a specific utility framework, namely EUT, in order to

[2]Note that the bound in rows 3 and 4 of $r = -0.15$ as reported in the original article of Holt and Laury (2002) is in fact according to our calculation $r = -0.14$. Also, if the subject chooses always option B, his relative risk aversion is $r \in (-\infty; -1.71]$.

[3]A variant of the design is to induce a single switching point, i.e. to ask for the row where the subjects would want to switch from A to B.

classify subjects as more or less risk-averse.[4] However, evidence rather suggests that risk preferences are heterogeneous and subjects follow different models of risky choice.[5] It is then not only problematic to impose EUT in order to classify subjects, but also to assume the very same choice model over all subjects. Hence, a more *general* measure of risk aversion intensity is needed that allows for a classification across different underlying models. In the next section, we therefore propose a modification of the HL method that is based on a general 'behavioral' notion of risk aversion, namely an aversion to mean preserving spreads.

3.3 A Model-Independent Method

In this section we propose a new method that shares the advantages of the HL table but not its disadvantages as mentioned above. Table 3.2 presents our new approach.

Table 3.2: Our elicitation method

Row No.	Option A Prob. 1/2 Outcome A1	Option A Prob. 1/2 Outcome A2	Option B Prob. 1/2 Outcome B1	Option B Prob. 1/2 Outcome B2	RRA if row was first choice of A and above all B	RRA if row was last choice of A and below all B
1	0.05	4.95	2.65	2.75		[−0.51 ; −0.13]
2	1.10	3.90	2.65	2.75		(−∞ ; −0.51]
3	**2.40**	**2.60**	**2.65**	**2.75**		non-monotone
4	2.40	2.60	2.00	3.40	[2.27 ; ∞)	
5	2.40	2.60	1.90	3.50	[1.70 ; 2.27]	
6	2.40	2.60	1.75	3.65	[1.18 ; 1.70]	
7	2.40	2.60	1.60	3.80	[0.86 ; 1.18]	
8	2.40	2.60	1.45	3.95	[0.65 ; 0.86]	
9	2.40	2.60	1.05	4.35	[0.36 ; 0.65]	
10	2.40	2.60	0.20	5.20	[0.13 ; 0.36]	

Again, subjects choose in each row between option A and option B. As in the HL table, option A as well as option B has two possible outcomes. However, instead of varying the

[4]In order to discriminate between intensities of risk aversion, HL use EUT and the specific class of CRRA functions. Using the HL method and assuming for instance Tversky and Kahneman's (1992) Prospect Theory would require a 'trade-off' between the curvatures of the utility function and the probability weighting function since both simultaneously influence the level of risk aversion. However, even if such a 'trade-off' could be made, there is no way to compare subjects in case they follow different models of risky choice.

[5]The evidence that many individuals behave according to non-EUT models is vast. For instance, a recent study by Harrison, Humphrey and Verschoor (2010) finds a 50/50 share of EUT and Prospect Theory.

probabilities and keeping the outcomes constant over all rows as in the HL table, we rather vary the outcomes and keep the probabilities constant (i.e. all probabilities are equal, namely 50 %). First note that an individual with monotone preferences will always prefer option B over option A in row 3 of Table 3.2 (this is similar to row 10 in Table 3.1) as here option B first-order stochastically dominates (or more specifically, state-wise dominates) option A.

We now compare options in row 4 to those in row 3. While option A is identical to the one in row 3, option B in row 4 is a mean preserving spread of the one in row 3. We can therefore say that option B becomes more risky in the sense of the very general increasing risk definition of Rothschild and Stiglitz (1970), while option A stays the same. In row 5 option A is again unaltered whereas option B is a further mean preserving spread of the one in row 4 and thus a further increase in risk. This continues until row 10. By just imposing 'duality' stating that less risk-averse individuals should take riskier gambles, we can say that someone (call her j) who preferred option B in the first four rows and option A in the last six rows is more risk-averse than someone (call her i) who preferred option B in the first five rows and option A in the last five rows. Such a statement can be made without referring to any particular utility framework. The property of 'duality' is based on the concept of mean utility preserving spreads first proposed by Diamond and Stiglitz (1974).[6] To see how, note that there exists a hypothetical gamble that is a mean preserving spread of option B in row 4, a mean preserving contraction of B in row 5, and leaves j just indifferent between A and B. This hypothetical gamble then is a mean utility preserving spread of A for j. At this point i still prefers B, so her hypothetical gamble representing a mean preserving spread of B in row 5, a mean preserving contraction of B in row 6, and leaving i just indifferent between A and B is a mean preserving spread of j's hypothetical gamble. It follows that j is more risk-averse than i. To illustrate how our table relates to the one of Holt and Laury (2002), we state in the last two columns of Table 3.2 how our method would elicit measures of RRA if we would also assume CRRA.

[6]Diamond and Stiglitz (1974) study this concept solely within EUT. By referring to simple compensated spreads, Machina (1982) and Chew, Karni, and Safra (1987) used it to analyze risk aversion in non-EUT models.

Risk seeking is identified through switches of choices in the first two rows of Table 3.2. Consider again the options in row 3, but now compare them to those in row 2. Now the 'less attractive' option A is altered by a mean preserving spread when going from row 3 to row 2, while option B stays the same. Only a very risk-seeking individual would like this spread so much that she would now prefer option A in row 2. In row 1 option A is a further mean preserving spread. Now, also less extreme risk seekers, who in row 2 were still choosing option B, are lured by the further increase in risk toward choosing option A in row 1. An individual who is risk-neutral, or is very close to being risk-neutral, will always choose option B in Table 3.2 since its expected value is higher than the one of option A in all rows.

Both options in Table 3.2, option A and option B, are always risky. This avoids the 'certainty effect', a well-known problem of any elicitation method using certainty equivalents.[7] The concept of riskiness we use in our table follows the established theoretical literature. "Clearly riskiness is related to dispersion, so a good riskiness measure should be monotonic with respect to second-order stochastic dominance. Less well understood, perhaps, is that riskiness should also relate to location and thus be monotonic with respect to first-order stochastic dominance, in particular, that a gamble that is sure to yield more than another should be considered less risky. Both stochastic dominance criteria are uncontroversial [...]" (Aumann and Serrano, 2008, p811)

In Table 3.2 we use both criteria. Option B first-order stochastically dominates option A in row 3 and can therefore be considered less risky. Going downward from row 3 option A stays unaltered whereas option B gets worse in terms of second-order stochastic dominance. Going upward from row 3 option A gets worse in terms of second-order stochastic dominance whereas option B stays the same. Individuals who switch from option B to A after row 3 are risk-averse (the earlier the more risk-averse). And individuals who switch from option A to option B before row 3 are risk-seeking (the later the more risk-seeking).[8]

[7] "The certainty effect introduces systematic errors into any method based on certainty equivalents." (McCord and de Neufville, 1986, p57). It describes the widely observed phenomenon that certain alternatives are perceived in a fundamentally different way than risky alternatives, even if the risk is negligible.

[8] Note that mean preserving contractions of option A (option B) could be applied in addition to the mean preserving spreads of option B (option A) when going downward (upward) from row 3. This variant of the

Although the two concepts we use for our elicitation method, mean preserving spreads and mean utility preserving spreads, were originally analyzed within EUT, it has become common in other models as well to understand risk aversion in terms of this more behavioral definition, namely as an aversion to mean preserving spreads. Subsequently, it is this definition that is used when risk aversion is analyzed in non-EUT models (see e.g. Machina, 1982; Chew, Karni and Safra, 1987; or Schmidt and Zank, 2008). And as Machina (2008, p80) notes, most non-EUT models "are capable of exhibiting first-order stochastic dominance preference, [and] risk aversion [. . .]." This shows that our method can classify subjects as more or less risk-averse across various models of risky choice.

Using variations of outcomes (i.e. mean preserving spreads) not only makes it easy for subjects to compute expected values but also allows us quite some flexibility in designing the range of the intervals to elicit estimates of relative risk aversion if we adopt the CRRA framework of Holt and Laury (2002). In principle, this could also be achieved in the HL table, but only at the price of stating odd probabilities. By contrast, in our table probabilities stay always at 50 % and only outcomes vary. We believe that subjects are more experienced in dealing with odd outcomes (such as price tags) than with odd probabilities.[9] More importantly, constant probabilities of 1/2 are much less sensitive to probability weighting. Already Quiggin (1982) uses 1/2 as plausible fixed point in his theory. "The claim that the probabilities of 50-50 bets will not be subjectively distorted seems reasonable, and [. . .] has proved a satisfactory basis for practical work [. . .]." (Quiggin, 1982, p328). This becomes important when decisions taken in the table are interpreted solely in terms of the curvature of a utility function.

design could be useful if one wanted to induce similar changes in both options over all rows of our table. However, since it may come at the cost of increased complexity for the subjects, we chose not to do so.

[9] In the appendix in section 3.7 we provide a more general treatment of our method. This shows how our method can be easily modified to meet different requirements on the elicitation ranges.

3.4 The Experiment

The experiment was computer-based and was conducted in November 2009 at the experimental laboratory MELESSA of the University of Munich. It used the experimental software z-Tree (Fischbacher, 2007) and the organizational software Orsee (Greiner, 2004). 232 subjects (graduate students were excluded) participated in 10 sessions and earned 11 euros (including 4 euros show-up fee) on average (with a maximum (minimum) of 30 (4.10) euros) for a duration of approximately one hour.

In the beginning of the experiment subjects received written instructions that were read privately by them. At the end of these instructions they had to answer test questions that showed whether everything was understood. There was no time limit for the instructions and subjects had the opportunity to ask questions in private. The experiment started on the computer screen only after everybody answered the test questions correctly and there were no further questions.

The further procedure of the experiment was the following. Each subject made decisions in four tables.[10] Again, they could take as much time as they wanted in order to make their decisions. After all subjects had made their decisions, an experimental instructor came to each subject to let them randomly determine their payoff from the tables.[11] Before they saw what their payoff from the experiment was they could again see how they actually decided in the randomly determined relevant table. At the end of the experiment all subjects further answered a questionnaire about their socio-economic characteristics. As soon as everybody had answered the questionnaire they were payed in private (not by the experimenter) and could leave.

As mentioned above, each subject made decisions in four tables. Two of the tables were low-stakes tables and two of them were high-stakes tables, where all low-stake outcomes were

[10]Eight treatments varied which tables in which order a subject received. The treatments are further explained below.

[11]Each subject had to role four dice. First, a four-sided die determined which of the four tables was payoff-relevant. Second, a ten-sided die determined which row in the payoff-relevant table was selected. And lastly, two ten-sided dice determined whether the amount A1 or A2 (if A was chosen in the relevant table and row) or whether the amount B1 or B2 (if B was chosen in the relevant table and row) was payed out to them (in addition to the show-up fee of 4 euros).

Table 3.3: Our adjusted elicitation method

Row No.	Option A Prob. 1/2 Outcome A1	Option A Prob. 1/2 Outcome A2	Option B Prob. 1/2 Outcome B1	Option B Prob. 1/2 Outcome B2	RRA if row was first choice of A and above all B	RRA if row was last choice of A and below all B
1	0.03	4.89	2.62	2.72		[−0.49 ; −0.14]
2	1.01	3.91	2.62	2.72		[−0.96 ; −0.49]
3	1.40	3.52	2.62	2.72		[−1.70 ; −0.96]
4	1.65	3.27	2.62	2.72		(−∞ ; −1.70]
5	**2.36**	**2.56**	**2.62**	**2.72**		non-monotone
6	2.36	2.56	1.77	3.57	[1.37 ; ∞)	
7	2.36	2.56	1.61	3.73	[0.97 ; 1.37]	
8	2.36	2.56	1.42	3.92	[0.68 ; 0.97]	
9	2.36	2.56	1.09	4.25	[0.41 ; 0.68]	
10	2.36	2.56	0.26	5.08	[0.15 ; 0.41]	

multiplied by five. In total, we used eight different tables. One of them was the original HL table (HLol) as outlined in section 3.2 (Table 3.1) and another was the original HL table but with all outcomes multiplied by five (HLoh). In order to being able to directly compare the HL method and our method, we adjusted our tables to the exact same ranges of RRA that were used by Holt and Laury (2002). A third table therefore used our method but adjusted to the original low-stake outcomes of the HL table (MRal) as outlined in Table 3.3. And a fourth table used our method adjusted to the high-stake version of the original HL table (MRah), where all outcomes in Table 3.3 are multiplied by five.

Subjects further received our table (Table 3.2) from section 3.3 (MRol). In designing this table we employed criteria mentioned by Holt and Laury (2002). There is an approximately symmetric range of RRA around 0, 0.5, 1, and 2. Based on the experimental results of Holt and Laury (2002), our table has only two risk seeking ranges and therefore more ranges for reasonable degrees of risk aversion. There was also a high-stakes version of this table where all outcomes are multiplied by five (MRoh). Again, in order to directly compare both methods, we also adjusted the HL tables to the exact same ranges of RRA that were used in our tables. Table 3.4 shows the adjusted HL table for low stakes (HLal). Again, the high-stakes version of Table 3.4 (HLah) multiplied all outcomes by five.

Each of all eight different tables was received by 116 subjects and all 232 subjects were

Table 3.4: The adjusted Holt and Laury method

Row No.	Option A: Outcome A1 = $2.00	Option A: Outcome A2 = $1.60	Option B: Outcome B1 = $3.85	Option B: Outcome B2 = $0.10	RRA if row was last choice of A and below all B
1	Prob. 29/100	Prob. 71/100	Prob. 29/100	Prob. 71/100	[−0.53 ; −0.14]
2	Prob. 40/100	Prob. 60/100	Prob. 40/100	Prob. 60/100	[−0.14 ; 0.12]
3	Prob. 49/100	Prob. 51/100	Prob. 49/100	Prob. 51/100	[0.12 ; 0.36]
4	Prob. 58/100	Prob. 42/100	Prob. 58/100	Prob. 42/100	[0.36 ; 0.65]
5	Prob. 69/100	Prob. 31/100	Prob. 69/100	Prob. 31/100	[0.65 ; 0.85]
6	Prob. 76/100	Prob. 24/100	Prob. 76/100	Prob. 24/100	[0.85 ; 1.19]
7	Prob. 86/100	Prob. 14/100	Prob. 86/100	Prob. 14/100	[1.19 ; 1.70]
8	Prob. 95/100	Prob. 5/100	Prob. 95/100	Prob. 5/100	[1.70 ; 2.37]
9	Prob. 99/100	Prob. 1/100	Prob. 99/100	Prob. 1/100	[2.37 ; ∞)
10	Prob. 100/100	Prob. 0/100	Prob. 100/100	Prob. 0/100	non-monotone

in either of eight different treatments. The treatments were designed to control for order effects, not only whether subjects answered low- or high-stakes tables first, but also whether HL tables or our tables (adjusted and original) were answered first. The eight treatments ensured that every subject had the same ex-ante expected income.[12]

In the comparison of the HL method and our method we will ask several questions. Firstly, whether both methods yield the same classification of individuals concerning both the direction and the intensity of risk attitude. Secondly, whether both methods yield similar levels of risk aversion intensity. Thirdly, what the effect is of increasing the stakes (i.e. multiplying all outcomes by five) on these RRA estimates. And lastly, how robust our results are.

3.5 Results

3.5.1 Directions of Risk Attitudes

Before analyzing the intensities of risk attitudes, we can ask how many of the subjects that can be classified are risk-averse, risk-neutral, and risk-seeking. Under low stakes, we

[12]The eight treatments were: 1. HLol, MRal, HLoh, MRah; 2. MRol, HLal, MRoh, HLah; 3. MRal, HLol, MRah, HLoh; 4. HLal, MRol, HLah, MRoh; 5. HLoh, MRah, HLol, MRal; 6. MRoh, HLah, MRol, HLal; 7. MRah, HLoh, MRal, HLol; 8. HLah, MRoh, HLal, MRol.

find that 79 % are risk-averse, 11 % are risk-neutral, and 10 % are risk-seeking in the HL tables. The respective numbers for our tables are 81 %, 9 %, and 10 %. Under high stakes, 88 % are risk-averse, 7 % are risk-neutral, and 5 % are risk-seeking in the HL tables whereas the respective numbers are 88 %, 6 %, and 6 % in our tables. This suggests that both methods yield identical classifications of subjects concerning the *direction* of risk attitude.

This finding is confirmed when we look at within subject classifications. Among the subjects that can be classified, we find that 84 % have the same direction of risk attitude across both methods under low stakes, where 76 % are risk-averse, 4 % are risk-neutral, and 4 % are risk-seeking in both methods. Under high stakes, 90 % have the same direction of risk attitude in both methods with 84 % being risk-averse, 4 % risk-neutral, and 2 % risk-seeking.

For each relevant comparison of both methods (i.e. HLol vs. MRal, HLoh vs. MRah, HLal vs. MRol, and HLah vs. MRoh) we perform sign tests in order to see whether the HL tables yield a different classification on the direction of risk attitude than our tables. We find that none of the comparisons is significant.[13] We can now state our first result.

Result 1 *The HL method and our method do not yield a different classification of individuals concerning the direction of risk attitude (risk-averse, risk-neutral, or risk-seeking).*

3.5.2 Intensities of Risk Attitudes

While Result 1 shows that both methods yield the same classification concerning the *direction* of risk attitude, another question is whether both methods also yield the same classification concerning the *intensity* of risk attitude. Are individuals similarly classified as more or less risk-averse across both methods? This is an important question as it answers whether the methodological problem of using the HL method (due to the required assumption of EUT) is indeed an empirically relevant one. If both methods yielded the same classification

[13] The results (p-values, number of observations N) of two-sided Sign tests are: HLol vs. MRal ($p = 1.0000$, $N = 76$), HLoh vs. MRah ($p = 1.0000$, $N = 71$), HLal vs. MRol ($p = 0.1094$, $N = 66$), and HLah vs. MRoh ($p = 1.0000$, $N = 71$).

of subjects concerning the intensity of risk attitude, results of experimental studies that used the HL method (and thereby assumed EUT) in order to classify subjects as more or less risk-averse would not be flawed. If, however, subjects are differently classified as more or less risk-averse in both methods, the methodological problem of assuming EUT for the classification in the HL method is in fact empirically relevant.

In order to answer this question we take the following approach. We first take a subject in a HL table and determine whether she is classified as more, less, or equally risk-averse than any other subject. We then take the same subject in our table and determine as well whether she is classified as more, less, or equally risk-averse than each of the other subjects. When we now compare this subjects pairwise comparisons (to all other subjects) in both tables, we can identify for this subject whether she is classified the same or differently (against each of the other subjects) across both tables. Thus, we know against how many of the other subjects this subject changes her risk aversion ranking across both tables. Since we perform this task not only for one specific but for all subjects, we can identify against how many of the other subjects a subject changes her ranking across methods on average. When we compare the HL tables to our tables we find that on average subjects change their ranking to 49 % of the other subjects.[14] So, the average subject has a different ranking toward almost half of the other subjects across methods.[15]

Result 2 *The HL method and our method yield a different classification of individuals concerning the intensity of risk aversion relative to other individuals.*

[14] The comparisons we make yield the following results. The average subject changes her ranking (of being more, less, or equally risk-averse) to 54 %, 52 %, 47 %, and 42 % of the other subjects in the respective comparisons of HLol vs. MRal, HLoh vs. MRah, HLal vs. MRol, and HLah vs. MRoh.

[15] Note that subjects also change their ranking to other subjects across stakes (but within methods). However, we can test whether subjects change their ranking across methods more than across stakes. Holding the stakes effect constant, we find that subjects increase their ranking changes to other subjects on average by 36 % when the method in addition to the stakes changes. When performing two-sided Wilcoxon signed-rank tests we find that subjects change their ranking significantly more across methods than across stakes (HLol vs. HLoh against HLol vs. MRah ($z = -5.262$, $p = 0.0000$, $N = 68$), HLal vs. HLah against HLal vs. MRoh ($z = -6.366$, $p = 0.0000$, $N = 67$), MRol vs. MRoh against MRol vs. HLah ($z = -2.431$, $p = 0.0151$, $N = 63$), and MRal vs. MRah against MRal vs. HLoh ($z = -3.169$, $p = 0.0015$, $N = 57$)). Results do not change when using two-sided Mann Whitney U tests instead (HLol vs. HLoh against HLol vs. MRah ($p = 0.0000$, $N = 170$), HLal vs. HLah against HLal vs. MRoh ($p = 0.0000$, $N = 157$), MRol vs. MRoh against MRol vs. HLah ($p = 0.0273$, $N = 135$), and MRal vs. MRah against MRal vs. HLoh ($p = 0.0000$, $N = 133$)).

Results 1 and 2 are of great interest for other experimental studies that use the HL method to control for risk aversion in observed behavior but which are not specifically interested in the absolute level of risk aversion intensity. However, there are other studies where the absolute level of risk aversion is important in order to derive quantitative predictions of a theoretical model. In the following we therefore investigate what the levels of risk aversion intensity are in both methods and how robust these RRA estimates are toward increasing the stakes.

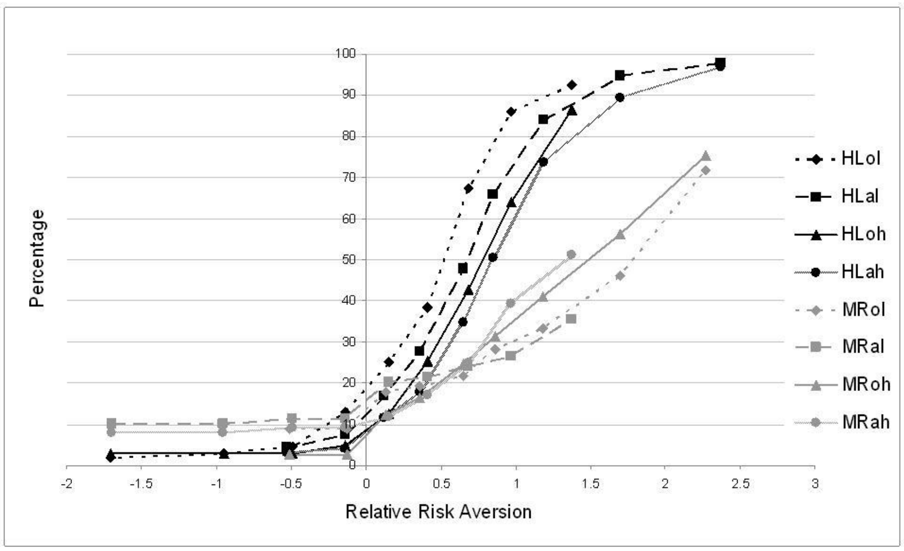

Figure 3.1: Cumulative distributions of RRA for all tables

Concerning the level of the intensity of risk attitude we again find systematic differences between both methods. Figure 3.1 shows the cumulative distributions of RRA for all eight different elicitation tables.[16] The cumulative distributions of relative risk aversion of all four HL tables lie above those of our four tables. While almost none of the subjects lies in the highest RRA range in the HL tables, many subjects fall into the highest RRA range when our method is used.[17] The medians of RRA using the HL method are all below the medians when our method is used. In the HL tables the medians lie in RRA ranges below one (HLol:

[16]We used uniform distributions within the RRA ranges.

[17]Note that as the highest range goes to infinity, the cumulative distribution functions do not end at 100 for the displayed values of RRA. Similarly, as the lowest range goes to minus infinity, the cumulative distribution functions do not start at 0 for the reported RRA values. This is also the reason why we cannot investigate the means of RRA but only the medians.

[0.41; 0.68]; HLal: [0.65; 0.85]; HLoh: [0.68; 0.97]; HLah: [0.65; 0.85]) but they lie in RRA ranges above one with our tables (MRol: [1.70; 2.27]; MRal: [1.37; ∞); MRoh: [1.18; 1.70]; and MRah: [0.97; 1.37]). While Figure 3.1 shows the overall picture, Figures 3.2 and 3.3 show the specific distributions that need to be compared in analyzing our data.[18]

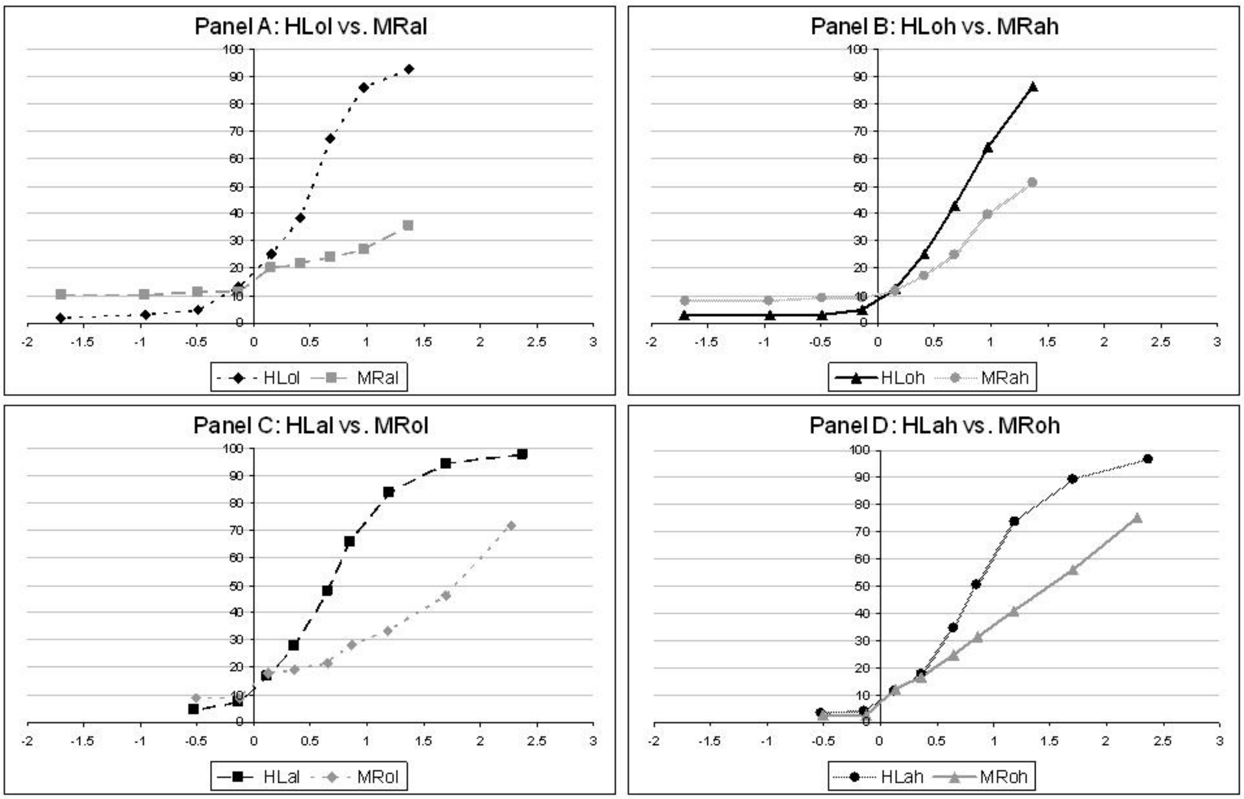

Figure 3.2: Cumulative distributions of RRA: Comparing methods

Figure 3.2 compares the HL method and our method. For low and high stakes we can compare the original to the adjusted tables since the adjustment was such that the ranges of RRA were identical in both tables. In all four panels the HL method yields clearly lower measures of RRA. This is the case no matter whether the adjustment took place for the HL method (Panels 2C and 2D) or for our method (Panels 2A and 2B) or whether we look at low (Panels 2A and 2C) or high stakes (Panels 2B and 2D).

Figure 3.3 shows the effect of increasing the stakes in both methods. Since the cumulative

[18]In Figures 3.2 and 3.3 we omitted naming the axis, but since all distribution lines are taken from Figure 3.1 and just considered in isolation, the notation of Figures 3.2 and 3.3 is of course the same as in Figure 3.1.

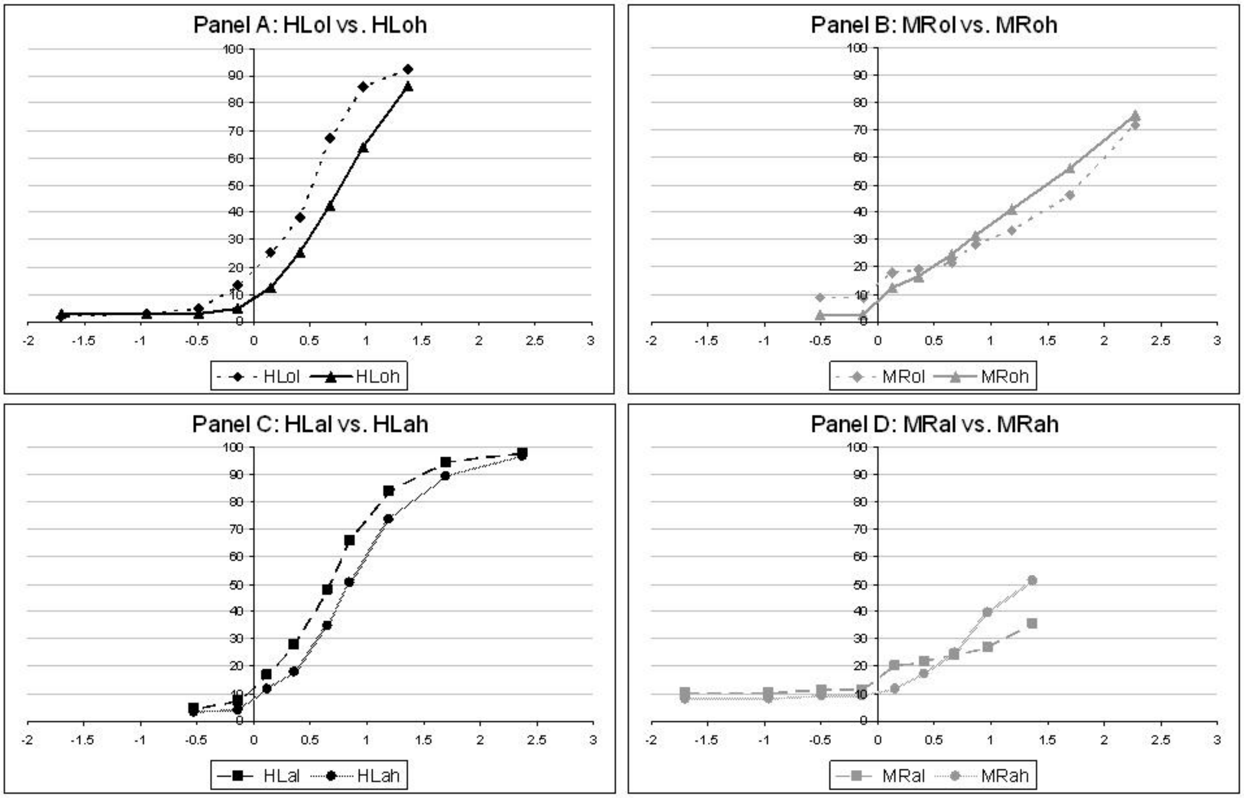

Figure 3.3: Cumulative distributions of RRA: Comparing stakes

distributions of the high-stakes HL tables lie below those of the low-stakes HL tables (Panels 3A and 3C), increasing the stakes seems to increase relative risk aversion. This is in contrast to our method where increasing the stakes does not cause risk aversion to increase. The cumulative distributions of our high-stakes tables rather cross those of our low-stakes tables (Panels 3B and 3D). This picture seems not to be affected by the fact whether we compare original (Panels 3A and 3B) or adjusted tables (Panels 3C and 3D).

Since each subject made decisions in four tables, we also test these differences using matched pairs. Table 3.5 relates the original HL tables to our adjusted tables, such that RRA ranges are identical and can be directly compared. And Table 3.6 relates the adjusted HL tables to our original tables such that all comparisons have identical RRA intervals. Reported are the number of observations (N), the z-values, and the p-values of two-sided Wilcoxon signed-rank tests.[19] In both tables, Table 3.5 and Table 3.6, in each cell it is tested

[19] Our results do not change when using two-sided Kolmogorov-Smirnov tests instead. The p-values are all $p = 0.000$ for the comparisons of Figure 3.2 (with $N = 186, 179, 172, 168$ respectively for Panels 2A, 2B,

if the elicitation method to the left yields different measures of RRA than the elicitation method above. Consider an example from Table 3.5. If HLol is compared to MRal, then a z-value of $z = -4.922$ indicates that the left table (HLol) yields lower measures of RRA than the right table (MRal).

Table 3.5: Wilcoxon signed-rank tests (HLo and MRa)

Cell format: (N) / **z-value** / **p-value**

	HLoh	**MRal**	**MRah**
MRal			(61) 0.488 0.6257
HLoh		(72) -4.000 0.0001	(71) -2.580 0.0099
HLol	(99) -5.081 0.0000	(76) -4.922 0.0000	(71) -4.815 0.0000

Table 3.6: Wilcoxon signed-rank tests (HLa and MRo)

Cell format: (N) / **z-value** / **p-value**

	HLah	**MRol**	**MRoh**
MRol			(65) -0.270 0.7875
HLah		(70) -3.215 0.0013	(71) -4.293 0.0000
HLal	(90) -4.957 0.0000	(66) -3.756 0.0002	(67) -4.654 0.0000

Comparing first the HL method and our method, we observe significantly higher measures of RRA when our method is used (HLol vs. MRal and HLoh vs. MRah in Table 3.5; and HLal vs. MRol and HLah vs. MRoh in Table 3.6). This holds at the 1 %-level for all four comparisons of Figure 3.2. We can therefore state the following result.

2C, 2D). Thus, compared to the HL method our method yields significantly higher measures of RRA. For the comparisons of Figure 3.3, we get $p = 0.798$ ($N = 151$) and $p = 0.227$ ($N = 155$) for Panels 3B and 3D, respectively. And we get $p = 0.002$ ($N = 210$) and $p = 0.167$ ($N = 189$) for the respective Panels 3A and 3B. Hence, while measures of RRA are not significantly different between low and high stakes when our method is used, this is not the case with the HL method. Here, increasing the stakes significantly increases measures of RRA in the original (two-sided at 1 %-level) as well as in the adjusted (one-sided at 10 %-level) HL tables.

Result 3 *Our method yields a higher intensity of risk aversion than the HL method.*

Looking at the effect of increasing the stakes, we see that there is a significantly positive effect on the RRA measure in the HL tables (HLol vs. HLoh in Table 3.5; and HLal vs. HLah in Table 3.6). In contrast, there is no such effect observed when our method is used (MRal vs. MRah in Table 3.5; and MRol vs. MRoh in Table 3.6). So, for the comparisons of Figure 3.3, we see that increasing the stakes by a factor of five increases risk aversion significantly at the 1 %-level only when the HL method is used. With our method, there is no significant effect of increasing stakes.[20]

Result 4 *While increasing the stakes by a factor of five increases the intensity of risk aversion with the HL method, increasing the stakes has no effect on the intensity of risk aversion with our method.*

Results 3 and 4 show that our method not only yields higher risk aversion estimates than the HL method, but also that our estimates are robust toward multiplying all outcomes by five (thereby indicating CRRA). This is not the case for the HL estimates. Here, we find increasing relative risk aversion (IRRA). Our findings for the HL method are completely in line with the findings of Holt and Laury (2002). Nevertheless, the results for our method are much closer to what is observed in the empirical literature. Several empirical studies indicate a measure of RRA roughly between 1 and 2 (e.g. Tobin and Dolde, 1971; Friend and Blume, 1975; Kydland and Prescott, 1982; Hildreth and Knowles, 1982; Szpiro, 1986; Chetty, 2006; or Bombardini and Trebbi, 2010) and Mehra (2003, p59) notes that "most studies indicate a value for α that is close to 2."[21] An experimental study by Levy (1994) rejects the existence of IRRA. Other empirical studies (e.g. by Szpiro, 1986; Friend and Blume, 1975; Brunnermeier and Nagel, 2008; or Calvet and Sodini, 2010) find supportive evidence for CRRA. Fehr-Duda et al. (2010) show in their experimental study that IRRA is entirely driven by transformations of the probability weighting function as stakes increase.

[20] As can be seen from Tables 3.5 and 3.6, even the sign of the z-value is positive (adjusted tables) *and* negative (original tables).

[21] Here, α is the measure of RRA.

In contrast to the HL method, our method is invariant to probability weighting. Hence, this may be the reason why there is a stakes effect only with the HL method.

Another possible explanation for the results of Holt and Laury (2002) and ours is that subjects need higher incentives when they have to exert more cognitive effort. In the HL tables, subjects need to exert more cognitive effort than in our tables since the varying probabilities are difficult to handle. By contrast, in our tables all probabilities are one half and 50/50 odds seem easy to work with. When subjects have too little incentives to exert the cognitive effort that is required to reveal their true level of risk aversion, it seems reasonable that they anchor their decision on the 50/50 choice (i.e. row 5 in the original HL tables).[22] Our results and the results of Holt and Laury (2002) suggest that increasing the stakes increases risk aversion with the HL method. So, as incentives increase subjects are more willing to exert such effort and thereby show their true level of risk aversion.[23] This might be the reason that Holt and Laury (2002) do not observe such a stake effect with hypothetical payoffs. With our method, subjects do not need much cognitive effort to handle 50/50 chances. Already our low-stakes tables give sufficient incentives to show subjects' true level of risk aversion. Increasing the stakes has therefore no effect on risk aversion with our method. It is interesting to note that increasing the stakes by a factor of five seemed not to be sufficient to show subjects' true level of risk aversion in the HL tables since risk aversion was still lower than in our tables. This is, however, consistent with the findings of Holt and Laury (2002) who increased stakes by factors of 20, 50 and 90. They showed that risk aversion increased with these higher stakes and that the RRA of one third of subjects was in the highest RRA range when stakes are increased by a factor of 90. Harrison at al. (2005) use a design similar to Holt and Laury (2002) but control for order effects. They find that stakes effects are only significant between a factor of 20 and 50 (or 90), but not between 50 and 90. Of course, if incentives increase even further beyond a factor of 50, only few subjects need such high incentives to exert their required cognitive effort and we

[22]Note that this is also the modal switching point under low stakes in Holt and Laury (2002).

[23]Note that in Holt and Laury (2002) there are also less inconsistent subjects under high stakes and more inconsistencies under comparable hypothetical payoffs.

should expect that the increase in risk aversion is not significant anymore. Although this cognitive effort explanation may explain the results of our experiment, we did not design the experiment to test this explanation. Further research should therefore investigate this potential explanation more closely in order to answer the question what the true level of risk aversion is.

3.5.3 Robustness

One may be concerned that the differences in the RRA estimates between the HL method and our method may be due to a preference for switching in the same row across tables. For instance, if a subject always switches after row 5 (from A to B in the HL tables and from B to A in our tables) we would measure a higher RRA in our tables. When comparing the HL tables to our tables, there are always the same number of rows where identical switching behavior would induce a higher RRA with our method as there are where identical switching leads to a higher RRA with the HL method.[24] Nevertheless, if a majority of subjects switched in exactly those rows where we would measure a higher RRA for our tables, Result 3 may simply be explained by a preference for identical switching behavior.

For all four comparisons of Figure 3.2, we find that only a minority of subjects switches in the same row in both tables. The respective numbers are 19.7 %, 9.9 %, 10.6 %, and 12.7 % for Panels 2A, 2B, 2C, and 2D.[25] Nevertheless, of those subjects that switch in the same row a majority switches in those rows that induce a higher RRA with our method. For the respective Panels 2A, 2B, 2C, and 2D 80 %, 85.7 %, 71.4 %, and 55.6 % of the 'same row switchers' switch in rows where our tables yield a higher RRA, and only 13.3 %, 0 %, 28.6

[24] When comparing original HL tables to our adjusted tables (HLol vs. MRal and HLoh vs. MRah), switching after row 1, 5, and 6 yields a higher RRA in our tables, switching after row 3, 4, 8, and 9 yields a higher RRA in the HL tables, and switching after row 2 and 7 yields the same RRA in both tables. Never switching (choosing always B in both tables) also leads to a higher RRA in our tables. When comparing our original to the adjusted HL tables (HLal vs. MRol and HLah vs. MRoh), switching after row 3, 4, and 5 yields a higher RRA in our tables, switching after row 2, 7, 8, and 9 yields a higher RRA in the HL tables, and switching after row 1 and 6 leads to the same RRA in both tables. Again, never switching and choosing always B yields a higher RRA in our tables.

[25] Note that all subjects answered not only two but four tables. And there are only two subjects that switch in the same row in all four tables.

%, and 11.1 % switch in rows where the HL tables yield a higher RRA. It may therefore be possible that a preference for identical switching behavior drives Result 3.

However, we can test whether Result 3 still holds for those subjects that switch in different rows. If we exclude the 'same row switchers', we can test whether Result 3 is driven by a preference for switching in the same row across both types of tables. For the comparisons of Figure 3.2, we again perform two-sided Wilcoxon signed-rank tests where we exclude 'same row switchers'.[26] For all comparisons we observe that differences are still significant. For the comparison of HLol vs. MRal (Panel 2A) we get $z = -4.540$, $p = 0.0000$, $N = 61$. For the comparison of HLoh vs. MRah (Panel 2B) we get $z = -1.787$, $p = 0.0740$, $N = 64$. The comparison of HLal vs. MRol (Panel 2C) yields $z = -3.908$, $p = 0.0001$, $N = 59$, and the comparison of HLah vs. MRoh (Panel 2D) yields $z = -3.982$, $p = 0.0001$, $N = 62$. Thus, the differences of Panels 2A, 2C, and 2D are still significant at the 1 %-level, and the difference of Panel 2B is still significant at the 10 %-level. We can now state the following result.

Result 5 *Result 3 is not due to a preference for switching in the same row across both methods.*

Since we had eight different treatments in our experiment, we can also test whether the order in which the tables were presented to the subjects had an effect on the elicited intensity of risk aversion. This could be the case if subjects are primed to think about all tables in terms of the first table they completed. To control for such order effects, we first test for each of the comparisons of Figure 3.2 whether it made a difference if a specific table was completed *before* or *after* its corresponding 'other-type' table. As an example, consider the HLol table. Here, we test whether the RRA distribution of HLol in treatment 1 and 5 (where it is completed before MRal) is different to the RRA distribution of Hlol in treatment 3 and 7 (where it is completed after MRal). For each of the eight different tables we perform a two-sided Kolmogorov-Smirnov test on the equality of RRA distributions and find no significant

[26]When performing two-sided Kolmogorov-Smirnov tests instead, we observe similar results. All comparisons of Figure 3.2 stay significant at the 1 %-level ($p = 0.000$, $N = 122$ for Panel 2A; $p = 0.008$, $N = 128$ for Panel 2B; $p = 0.000$, $N = 118$ for Panel 2C; and $p = 0.000$, $N = 124$ for Panel 2D).

difference in either table.[27]

To control for order effects in the comparisons of Figure 3.3, we also test for each table whether it made a difference if it was completed *before* or *after* its corresponding 'other-stake' table. Consider again as an example the HLol table. Here, we test whether the RRA distribution of HLol in treatment 1 and 3 (where it is completed before HLoh) is different to the RRA distribution of HLol in treatment 5 and 7 (where it is completed after HLoh). Again, none of the two-sided Kolmogorov-Smirnov tests shows a significant difference.[28] We are now able to summarize these results on order effects. The order effects that are captured by Result 6 test whether high-stakes tables are completed before or after corresponding low-stakes tables (and vice versa) as well as whether original (HL and MR) tables are completed before or after corresponding adjusted (MR and HL) tables (and vice versa).

Result 6 *The order in which tables were completed by subjects does not influence the elicited intensities of risk aversion.*

The analysis of sections 3.5.1 and 3.5.2 excluded subjects that switched multiple times from one option to another, or chose the first-order dominated option, and thus were inconsistent in completing a table. Over all eight different tables, we find that on average 24 % of subjects are inconsistent which is in line with other studies.[29] However, Holt and Laury

[27] The results of the two-sided Kolmogorov-Smirnov tests are: HLol ($p = 0.199$, $N = 107$), HLal ($p = 0.822$, $N = 94$), HLoh ($p = 1.000$, $N = 103$), HLah ($p = 0.949$, $N = 95$), MRol ($p = 0.424$, $N = 78$), MRal ($p = 0.896$, $N = 79$), MRoh ($p = 0.545$, $N = 73$), and MRah ($p = 0.763$, $N = 76$).

[28] The results of the two-sided Kolmogorov-Smirnov tests are: HLol ($p = 0.188$, $N = 107$), HLal ($p = 0.949$, $N = 94$), HLoh ($p = 0.990$, $N = 103$), HLah ($p = 0.940$, $N = 95$), MRol ($p = 0.796$, $N = 78$), MRal ($p = 0.979$, $N = 79$), MRoh ($p = 0.775$, $N = 73$), and MRah ($p = 0.957$, $N = 76$).

[29] Of those 24 %, 18 % switched multiple times and 6 % chose the first-order dominated option. Bruner, McKee and Santore (2008), for instance, find 30 % inconsistent subjects where 25 % are multiple switchers and 5 % have non-increasing utility. As noted by Andersen et al. (2006, p386) "it is quite possible that [multiple] switching behavior is the result of the subject being indifferent between the options. The implication here is that one simply use a "fatter" interval to represent this subject in the data analysis, defined by the first row that the subject switched at and the last row that the subject switched at." When allowing for indifference, Andersen et al. (2006) find that multiple switching is only 5.8 % and 24.3 % choose the indifference option. When imposing a single switching point, choosing indifference increases to 30 %, which again indicates that multiple switching behavior is caused by 'thicker' indifference curves. In contrast, Bruner (2007) suggests an instructional variation emphasizing that only *one* decision will determine earnings (and further emphasizing incentive compatibility of the payment rule) and finds multiple switching reduced, from 25.8 % to 6.7 % and from 13.3 % to 2.3 % in two different elicitation formats. This would rather suggest that multiple switching is caused by errors.

(2002) treated those inconsistent subjects differently than we did. They simply counted how often subjects chose each option (even if the option was not only chosen in subsequent rows) and assumed the inconsistency away. If we treat our inconsistent subjects in the same way, we can test whether the inconsistent subjects show a different pattern of risk aversion than the consistent subjects. When performing Kolmogorov-Smirnov tests, we do not find any significant difference in the distributions of consistent and inconsistent subjects.[30]

3.6 Conclusion

The multiple price-list method of HL has become *the* standard way to measure the intensity of individuals' risk attitudes in experiments. Several other methods which also involve choices between lotteries have been proposed to accomplish a similar task. Binswanger (1980, 1981) asks subjects to choose between pairs of different 50/50 gambles, where higher expected values come at the cost of higher standard deviations. Eckel and Grossman (2008) use a gamble design similar to Binswanger (1980, 1981) but subjects choose one out of five different 50/50 gambles. Hey and Orme (1994) ask a battery of 100 choices with more complex probabilities. None of these approaches uses mean preserving spreads *among* different choice situations in a way similar to our approach. Since it is exactly this feature that allows us to classify subjects as more or less risk-averse independently of a specific model, other elicitation methods need to impose the same utility structure on all individuals in order to classify them. None of the methods is model-independent in that it can rank individuals as more or less risk-averse when they have heterogeneous risk preferences, such as EUT *and* non-EUT. This seems especially problematic in light of existing evidence.

After modifying the HL method and thereby proposing a new model-independent multiple price-list method to elicit the intensity of individuals' risk attitudes, we further compared our proposed method to the HL method in a lab experiment. Our results for the HL method

[30]The results of the two-sided Kolmogorov-Smirnov tests are: HLol (p = 0.916, N = 116), HLal (p = 0.390, N = 113), HLoh (p = 0.328, N = 116), and HLah (p = 0.180, N = 116). Subjects were only excluded in case they never chose option B (3 subjects in HLal), since then even reshuffling their choices cannot make them consistent.

replicate the findings of Holt and Laury (2002). Furthermore, concerning the direction of risk attitude (i.e. risk-averse, risk-neutral, or risk-seeking) we find that individuals are classified the same across both methods.

However, with our method we found systematic differences concerning the intensity of risk attitude. The classification of individuals as being more or less risk-averse than other individuals is quite different across methods. This is important for experimental studies that use the HL method (and thereby assume EUT for the classification) in order to control for risk aversion in observed behavior. Concerning the level of individuals' intensity of risk aversion we found that our method yields higher measures of risk aversion that are much closer to what is observed in the field. Furthermore, while increasing the stakes increases risk aversion with the HL method, our method is robust toward such stakes effects. We also offered a cognitive effort explanation of our results that needs to be tested in future research. This may help to answer the important question of what subjects' true level of risk aversion is after all.

3.7 The General Model-Independent Method

Define row i as consisting of two alternatives A_i and B_i. Each alternative consists of two possible outcomes, $A1_i$ and $A2_i$ of alternative A_i and $B1_i$ and $B2_i$ of alternative B_i. Given row i is the row played (if only one row of the table is randomly selected to be played), each outcome of each alternative is realized with probability 1/2. Choose a row $i = m$ and define $A1_m \equiv a, A2_m \equiv b, B1_m \equiv c, B2_m \equiv d$, where $a, b, c, d \in \mathbb{R}$. Without loss of generality, choose a and b such that $a < b$, and c and d such that $c < d$.[31] Our method of elicitation requires that $a \leq c$, $b \leq d$, and at least one relation holds strictly. It follows that

$$A_m \prec_{FSD} B_m. \tag{3.1}$$

[31] We chose to make these inequalities strict in order to avoid 'certainty effect' issues.

(3.1) means that B_m first-order stochastically dominates (FSD) A_m.[32] Every individual with strictly increasing utility prefers B_m over A_m.

Then define values of row $m+1$ as follows: $A1_{m+1} \equiv a, A2_{m+1} \equiv b, B1_{m+1} \equiv c - k_1, B2_{m+1} \equiv d + k_1$. Further define values of row $m+2$ in the following way: $A1_{m+2} \equiv a, A2_{m+2} \equiv b, B1_{m+2} \equiv c - k_1 - k_2, B2_{m+2} \equiv d + k_1 + k_2$. Continue these mean preserving spreads (MPS's) of option B until the last row is reached. Generally, for $\check{n} > 0$ we can define $A1_{m+\check{n}} \equiv a, A2_{m+\check{n}} \equiv b, B1_{m+\check{n}} \equiv c - k_1 - k_2 - \ldots - k_{\check{n}}, B2_{m+\check{n}} \equiv d + k_1 + k_2 + \ldots + k_{\check{n}}$, where $k_1, k_2, \ldots \in \mathbb{R}^+$.

In a similar way, we can define values of row $m-1$ as follows: $A1_{m-1} \equiv a - k^1, A2_{m-1} \equiv b + k^1, B1_{m-1} \equiv c, B2_{m-1} \equiv d$. Now define values of row $m-2$ as follows: $A1_{m-2} \equiv a - k^1 - k^2, A2_{m-2} \equiv b + k^1 + k^2, B1_{m-2} \equiv c, B2_{m-2} \equiv d$. Again, these MPS's of option A can be continued until the first row is reached. Generally, for $\hat{n} > 0$ define $A1_{m-\hat{n}} \equiv a - k^1 - k^2 - \ldots - k^{\hat{n}}, A2_{m-\hat{n}} \equiv b + k^1 + k^2 + \ldots + k^{\hat{n}}, B1_{m-\hat{n}} \equiv c, B2_{m-\hat{n}} \equiv d$, where $k^1, k^2, \ldots \in \mathbb{R}^+$.

With these definitions it follows that for all i the expected values are $\mathbb{E}[A_i] = \frac{a+b}{2}$ and $\mathbb{E}[B_i] = \frac{c+d}{2}$. Table 3.7 illustrates our generalized elicitation method.

Table 3.7: Our generalized elicitation method

Row	Option A_i		Option B_i	
No.	Prob. 1/2	Prob. 1/2	Prob. 1/2	Prob. 1/2
i	Outcome $A1_i$	Outcome $A2_i$	Outcome $B1_i$	Outcome $B2_i$
...	...	...	...	...
$m-\hat{n}$	$a - k^1 - k^2 - \ldots - k^{\hat{n}}$	$b + k^1 + k^2 + \ldots + k^{\hat{n}}$	c	d
...	...	...	...	...
$m-2$	$a - k^1 - k^2$	$b + k^1 + k^2$	c	d
$m-1$	$a - k^1$	$b + k^1$	c	d
m	a	b	c	d
$m+1$	a	b	$c - k_1$	$d + k_1$
$m+2$	a	b	$c - k_1 - k_2$	$d + k_1 + k_2$
...	...	...	...	...
$m+\check{n}$	a	b	$c - k_1 - k_2 - \ldots - k_{\check{n}}$	$d + k_1 + k_2 + \ldots + k_{\check{n}}$
...	...	...	...	...

[32] In order to make this more salient, we even used state-wise dominance as a special case of FSD in the experiment.

From $k_1, k_2, \ldots > 0$ and $k^1, k^2, \ldots > 0$ it follows that for all $i \geq m$ we can state $B1_i > B1_{i+1}$ and $B2_i < B2_{i+1}$. Also, for all $i \leq m$ it holds that $A1_i > A1_{i-1}$ and $A2_i < A2_{i-1}$.

Let us first consider all rows with $i \geq m$. Since all $A1_i$ and $A2_i$ are identical, it follows that

$$\begin{aligned} & A_m = A_{m+1} = \ldots = A_{m+\check{n}} \\ \Rightarrow \quad & A_m \sim A_{m+1} \sim \ldots \sim A_{m+\check{n}}. \end{aligned} \tag{3.2}$$

Since $B1_{m+\check{n}} = B1_{m+\check{n}-1} - k_{\check{n}}$ and $B2_{m+\check{n}} = B2_{m+\check{n}-1} + k_{\check{n}}$ it follows that $B_{m+\check{n}}$ is a MPS of $B_{m+\check{n}-1}$.[33] We use the standard (behavioral) definition of risk aversion and say that an individual is risk-averse if she dislikes increases in risk, i.e. if she dislikes MPS's. We thus employ the increasing risk definitions of Rothschild and Stiglitz (1970). For every risk-averse individual

$$\begin{aligned} & B_m \prec_{MPS} B_{m+1} \prec_{MPS} \ldots \prec_{MPS} B_{m+\check{n}} \\ \Rightarrow \quad & B_m \succ B_{m+1} \succ \ldots \succ B_{m+\check{n}}. \end{aligned} \tag{3.3}$$

Combining (3.1), (3.2), and (3.3) we can derive that the choice between $A_{m+\check{n}}$ and $B_{m+\check{n}}$ for every $\check{n} > 0$ is a trade-off between the advantage of the FSD-improvement from B_m over A_m ($= A_{m+\check{n}}$) and the disadvantage of the increase(s) in risk (MPS('s)) when going from B_m to $B_{m+\check{n}}$.[34]

Suppose an individual chooses $A_{m+\check{n}}$ over $B_{m+\check{n}}$, but chooses $B_{m+\check{n}-1}$ over $A_{m+\check{n}-1}$. Then, the MPS's from row m until row $m + \check{n} - 1$ were not enough to distract him from the FSD-

[33] In going from $B_{m+\check{n}-1}$ to $B_{m+\check{n}}$, general MPS's could be applied. However, since we defined each option as having two possible outcomes only (where each occurs with probability 1/2) a MPS can only be attained by adding and subtracting k_n. In giving up that all outcomes of the table are equally likely, one would not only require that subjects understand situations other than certainty and equally likely, but one would also introduce complications such as probability weighting.

[34] Note that one could also use mean preserving contractions (i.e. decreases in risk) when going from A_m to $A_{m+\check{n}}$, either instead or in addition to the MPS's that are used when going from B_m to $B_{m+\check{n}}$. This would be useful if one wanted to induce similar changes between the two alternatives over the rows of the table.

improvement of B_m over A_m. In contrast, the MPS's from row m until row $m+\check{n}$ were enough to outweigh the FSD-improvement of B_m over A_m. So, we can define an 'intermediate' hypothetical option $\bar{B}_{m+\check{n}}$ with $\bar{B}1_{m+\check{n}} = B1_{m+\check{n}-1} - \kappa k_{\check{n}}$ and $\bar{B}2_{m+\check{n}} = B2_{m+\check{n}-1} + \kappa k_{\check{n}}$. The $\kappa \in [0;1]$ is chosen such that $A_{m+\check{n}} \sim \bar{B}_{m+\check{n}}$. Put differently, κ is the fraction of $k_{\check{n}}$ that would make the individual indifferent between $A_{m+\check{n}}$ and $B_{m+\check{n}}$ if $\kappa k_{\check{n}}$ instead of $k_{\check{n}}$ would have been used. For this individual $\bar{B}_{m+\check{n}}$ is then a mean utility preserving spread of $A_{m+\check{n}}$ in the sense of Diamond and Stiglitz (1974).

Now, suppose there is another individual who is less risk-averse. This individual would then still strictly prefer $\bar{B}_{m+\check{n}}$ over $A_{m+\check{n}}$. It follows, that this individual would choose at least as many times B over A as the more risk-averse individual.[35] The smaller the k_i's, the finer are the differences in risk aversion that can be observed. Any observed differences in behavior must be due to (sufficiently strong) differences in risk aversion.

In order to distinguish risk-seeking individuals, consider all rows with $i \leq m$. Since all $B1_i$ and $B2_i$ are identical, it follows that

$$\begin{aligned} & B_m = B_{m-1} = \ldots = B_{m-\hat{n}} \\ \Rightarrow \quad & B_m \sim B_{m-1} \sim \ldots \sim B_{m-\hat{n}}. \end{aligned} \tag{3.4}$$

Since $A1_{m-\hat{n}} = A1_{m-\hat{n}+1} - k^{\hat{n}}$ and $A2_{m-\hat{n}} = A2_{m-\hat{n}+1} + k^{\hat{n}}$ it follows that $A_{m-\hat{n}}$ is a MPS of $A_{m-\hat{n}+1}$. Of course, every risk-seeking individual likes MPS's and thus

$$\begin{aligned} & A_m \prec_{MPS} A_{m+1} \prec_{MPS} \ldots \prec_{MPS} A_{m-\hat{n}} \\ \Rightarrow \quad & A_m \prec A_{m-1} \prec \ldots \prec A_{m-\hat{n}}. \end{aligned} \tag{3.5}$$

Combining (3.1), (3.4) and (3.5) we can derive that the choice between $A_{m-\hat{n}}$ and $B_{m-\hat{n}}$ for every $\hat{n} > 0$ is a trade-off between the advantage of the FSD-improvement from B_m ($= B_{m-\hat{n}}$) over A_m and the advantage of the increase(s) in risk when going from A_m to

[35] Strictly speaking, an individual who chooses as many times B over A as another individual could still be slightly more or slightly less risk-averse. In this case her κ would be lower or higher, respectively.

$A_{m-\hat{n}}$. By similar arguments as before, it follows that a less risk-seeking individual would choose at least as many times B over A as the more risk-seeking individual.

As in the HL tables, for risk-averse as well as for risk-seeking individuals it is optimal to switch options only once. Risk-averse individuals switch from B to A after row m (the earlier the more risk-averse) and risk-seeking individuals switch from A to B before row m (the earlier the less risk-seeking).[36] For risk-neutral individuals it is optimal to choose option B throughout since it offers the higher expected payoff.

[36] Of course, if one wanted to induce switching from A to B for most (i.e. risk-averse) subjects as in the HL tables, one could achieve this by just exchanging alternatives A and B.

Chapter 4

Measuring Higher-Order Preferences*

4.1 Introduction

In this chapter I present an experimental test of higher-order risk preferences. We evaluate the subjects level of consistency with the concepts of risk aversion (second-order preferences), prudence (third-order preferences, sometimes also called downside risk aversion), and temperance (fourth-order preferences, sometimes also called outer risk aversion). Many theories predict a substantial difference in risk preferences concerning gains and losses. Therefore, a focus of our experimental design was to inflict real monetary losses on subjects.

While second-order risk preferences have been extensively studied in the experimental literature, the experimental evidence on higher orders remains relatively scarce. Some attempts have been made to measure behavior *associated* with higher-order risk preferences. Ballinger, Palumbo and Wilcox (2003) showed in an experiment simulating life-cycle consumption and saving that individuals make precautionary savings, but to a much lesser extend than predicted by theory. Harrison, List and Towe (2007) found in a field experiment that the risk aversion of coin traders is significantly higher when they faced an additional background risk. This supports the notion that temperance is present.

Very recently, there has emerged also some *direct* experimental evidence on higher-

*This chapter is based on joint work with Johannes Maier. Both authors contributed equally to this work.

order preferences. Based on the model-independent gamble definitions of Eeckhoudt and Schlesinger (2006), Deck and Schlesinger (2010) tested whether subjects apportion risks consistent with prudence and temperance. They found evidence for prudence but also against temperance (i.e. intemperance). From this, they conclude that common specifications of expected utility theory (EUT), such as CRRA and CARA functions, are not reconcilable with the combination of prudence and intemperance. However, popular specifications of cumulative prospect theory (of Tversky and Kahneman, 1992) are consistent with both prudence and intemperance. Hence, they interpret their findings as support of cumulative prospect theory (CPT). A similarly designed experiment is Ebert and Wiesen (2010). However, it is confined to third-order risk preferences. They find supportive evidence for prudence.

Compared to EUT, one crucial difference of reference-dependent models, such as prospect theory, is the distinct evaluation of gains and losses. In experiments, it is usually difficult to impose losses on subjects. Most experiments endow subjects with enough money right before they impose 'losses' on them. Since subjects in these experiments cannot make real losses, it is hard to believe that they behave as if they would. The problem of house money effects in economic experiments of risky choice is well-established since Thaler and Johnson (1990). The house money effect describes the behavior of subjects to treat all money earned in the experiment as money to play with. Consequently, they are willing to take risks they normally would not accept. This problem arises in any design where subjects are endowed with a participation fee or previous earnings and can subsequently only lose this amount of money. Bosch-Domènech and Silvestre (2006) instead introduced a design where subjects have to attend two dates of the experiment. This design enables them to impose real losses on the second date. However, they only consider risk aversion. To our knowledge, no experiment has been conducted so far that tests higher-order risk preferences in the domain of real monetary losses.

In our experiment, we investigate second- to fourth-order risk preferences over gains *and* losses. Subjects attended two dates of the experiment. The purpose of the first date was to let subjects earn enough money which they possibly could lose at the second date, several weeks later. At the second date subjects made 84 binary choices that are also based on the gamble definitions of Eeckhoudt and Schlesinger (2006). However, while Deck and Schlesinger (2010) and Ebert and Wiesen (2010) asked subjects where they wanted to add 'harms', we rather presented the gambles in final outcomes. We chose this design, as it is well established in the literature[1] that in experiments subjects have major difficulties in solving compound lotteries.

Results on risk aversion and on prudence closely resemble the results of previous experiments. However, our results on temperance contrast sharply with those of the only previously conducted experiment by Deck and Schlesinger (2010). Since we elicit temperance using 28 choices by each subject[2] we are confident that our results are robust. As a consequence, we do not reject the possibility that common specifications of EUT can be consistent with the data. We also find that risk preferences of different order are empirically strongly related. Probably surprisingly, hardly any variation in behavior was observed between gambles in gains and in losses.

Section 4.2 introduces same theoretical concepts that are underlying the experimental design. The specific design of our experiment is described in section 4.3. Results and their robustness are discussed in section 4.4. Section 2.9 concludes.

4.2 Theoretical Prerequisites

In this section we outline some basic concepts and definitions that clarify what is precisely measured in the experiment. It illustrates what we mean when stating that somebody exhibits risk preferences of a certain kind.

Let $y \in \mathbb{R}$ be the level of wealth. Let $k \in \mathbb{R}$, with $k > 0$, be a sure reduction in wealth. $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ are random variables which are non-degenerate, independent of all other random variables that affect wealth, and with $\mathbb{E}[\tilde{\varepsilon}_1] = \mathbb{E}[\tilde{\varepsilon}_2] = 0$. Now, we define the following

[1] See e.g. Kahneman and Tversky (1979) in their discussion of the isolation effect, Bernasconi (1994), and Friedman (2005).

[2] Deck and Schlesinger (2010) use four choices by each subject to elicit temperance.

standard gambles, where each element denotes an outcome and each outcome of a specific gamble is realized with equal probability.[3] Such defined outcomes can be either deterministic or stochastic themselves.

$$\begin{aligned} B_2 &\equiv [y] & A_2 &\equiv [y+\tilde{\varepsilon}_1] \\ B_3 &\equiv [y-k, y+\tilde{\varepsilon}_1] & A_3 &\equiv [y, y-k+\tilde{\varepsilon}_1] \\ B_4 &\equiv [y+\tilde{\varepsilon}_1, y+\tilde{\varepsilon}_2] & A_4 &\equiv [y, y+\tilde{\varepsilon}_1+\tilde{\varepsilon}_2]. \end{aligned}$$

The definitions used in the theoretical literature are the following

Definition 4 *Eeckhoudt and Schlesinger (2006)*

$$\begin{array}{llllll} i \text{ is risk-averse} & \Leftrightarrow & B_2 \succsim^i A_2 & \forall\, y, \tilde{\varepsilon}_1 & (\Leftrightarrow u_i''(x) & \leq 0 \text{ in EU models}), \\ i \text{ is prudent} & \Leftrightarrow & B_3 \succsim^i A_3 & \forall\, y, \tilde{\varepsilon}_1, k & (\Leftrightarrow u_i'''(x) & \geq 0 \text{ in EU models}), \\ i \text{ is temperate} & \Leftrightarrow & B_4 \succsim^i A_4 & \forall\, y, \tilde{\varepsilon}_1, \tilde{\varepsilon}_2 & (\Leftrightarrow u_i''''(x) & \leq 0 \text{ in EU models}). \end{array}$$

Note that these definitions are deterministic. They define preferences, which are underlying characteristics of the subjects. There are several approaches to relate these preferences to observations. One way to proceed would be to state that preferences are to be called the respective name only if all observed actions are in accordance with this definition. However, then a single observation contradicting one of the definitions would lead to a rejection of the concept. For instance, somebody who is choosing B_3 over A_3 in 27 out of 28 decisions, but chooses A_3 once, would be termed neither prudent nor imprudent. It becomes clear, both from our experimental results as well as from those of Deck and Schlesinger (2010) and Ebert and Wiesen (2010), that with such a strict application of the definitions, almost no subject could be classified as having a certain risk preference. However, we think that the concepts of risk aversion, prudence and temperance still are valuable to characterize individuals who often but not always act according to the definitions above.

This leads us to a stochastic definition of higher-order risk attitudes, replacing the deter-

[3] We defined all gambles having a mean of y if $k = 0$ instead of 0 as in Eeckhoudt and Schlesinger (2006). We do so because only then complete gambles can lie in the domain of gains or losses.

ministic definition. Both Deck and Schlesinger (2010) and Ebert and Wiesen (2010) do so implicitly when making inferences from their results. We state these stochastic definitions explicitly in order to clarify the concepts used to analyze the data. The notion that deterministic theories only represent the structural part of preferences whereas the entire system of preferences incorporates a random or stochastic part is widespread among researchers who try to closely match theories and empirical observations. The fact that subjects of an experiment sometimes make different choices on the very same decision problem and under the same conditions is only reconcilable with stochastic theories.[4] For a comprehensive survey on the different variants of stochastic utility models consult Wilcox (2007).

Let P_n^i be the probability that individual i prefers B_n over A_n for a given set of parameters. Then it follows that

$$\begin{array}{llllllll} P_2^i & \text{is the probability that} & i & \text{chooses} & B_2 & \text{over} & A_2 & \text{for given } y, \tilde{\varepsilon}_1, \\ P_3^i & \text{is the probability that} & i & \text{chooses} & B_3 & \text{over} & A_3 & \text{for given } y, \tilde{\varepsilon}_1, k, \text{and} \\ P_4^i & \text{is the probability that} & i & \text{chooses} & B_4 & \text{over} & A_4 & \text{for given } y, \tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \end{array}$$

and we can state definitions for stochastically specified risk preferences.

Definition 5 *Second-order stochastic risk preferences are defined by the relations*

$$\begin{array}{llcccc} i & \textit{is stochastically risk averse} & \Leftrightarrow & P_2^i & > & 1/2, \\ i & \textit{is stochastically risk neutral} & \Leftrightarrow & P_2^i & = & 1/2, \\ i & \textit{is stochastically risk seeking} & \Leftrightarrow & P_2^i & < & 1/2. \end{array}$$

Third-order stochastic risk preferences are defined by the relations

$$\begin{array}{llcccc} i & \textit{is stochastically prudent} & \Leftrightarrow & P_3^i & > & 1/2, \\ i & \textit{is stochastically prudence neutral} & \Leftrightarrow & P_3^i & = & 1/2, \\ i & \textit{is stochastically imprudent} & \Leftrightarrow & P_3^i & < & 1/2. \end{array}$$

[4] On experimental evidence on this see e.g. Camerer (1989), Starmer and Sugden (1989), Ballinger and Wilcox (1997), and Loomes and Sugden (1998).

Fourth-order stochastic risk preferences are defined by the relations

$$\begin{array}{llcl} i & \textit{is stochastically temperate} & \Leftrightarrow & P_4^i > 1/2, \\ i & \textit{is stochastically temperance neutral} & \Leftrightarrow & P_4^i = 1/2, \\ i & \textit{is stochastically intemperate} & \Leftrightarrow & P_4^i < 1/2. \end{array}$$

These definitions will allow us to make statements on the risk attitudes of a single individual. Sometimes, however, we will make interpersonal comparisons. For such comparisons we need definitions that let us order subjects in terms of the level of consistency with a certain risk attitude. We will say that

Definition 6

$$\begin{array}{llllcl} i & \textit{is stochastically more risk-averse} & \textit{than} & j & \Leftrightarrow & P_2^i > P_2^j \quad \forall\, y, \tilde{\varepsilon}_1, \\ i & \textit{is stochastically more prudent} & \textit{than} & j & \Leftrightarrow & P_3^i > P_3^j \quad \forall\, y, \tilde{\varepsilon}_1, k, \\ i & \textit{is stochastically more temperate} & \textit{than} & j & \Leftrightarrow & P_4^i > P_4^j \quad \forall\, y, \tilde{\varepsilon}_1, \tilde{\varepsilon}_2. \end{array}$$

The definition of stochastically more risk averse was formally stated first by Hilton (1989). Recently, Wilcox (forthcoming) extended the definition for stochastic y. The definitions of stochastically more prudent and stochastically more temperate were not stated explicitly before but are a straightforward extension of the stochastically more risk averse definition to higher orders.

It should be emphasized that the concepts of definition 6 are not to be confounded with the definitions of higher risk aversion (or parallel concepts of higher downside risk aversion etc.) as they were used in the previous chapter. Here, we define the level of consistency with a qualitative concept whereas in the previous experiment we were concerned with the quantitative strength of a risk preference. Only under very specific conditions these concepts relate to each other in a one to one fashion. Observable actions can always be described as being the result of stochastic preferences that are composed of a systematic and a non-systematic part. If we allow the non-systematic part of preferences to differ between subjects (e.g. some subjects make more errors than others) the measures of consistency and of intensity may yield different orderings of subjects. For instance, it is well conceivable that a subject is very confident about rejecting a mean-preserving spread but is not willing to give up a lot in order to avoid one. Another subject might be ready to pay a lot to avoid such a risk in principle but has not thought about such decisions much and is therefore prone to make many mistakes.

Definitions 5 and 6 are still in terms of underlying characteristics of an individual. However, using our observations we can make statistical inferences on these unknown characteristics of the subjects. When labeling a subject as risk averse, prudent or temperate in the subsequent experiment we imply these stochastic definitions.

4.3 The Experiment

The experiment was divided into two distinct dates for each subject. This was necessary because the focus was to induce real monetary losses. Institutional restraints ruled out that on a single-date experiment subjects could make real monetary losses not compensated by any fixed payments such as the participation fee. However, by dividing the experiment into two dates we were able to induce real monetary losses on the second date.

On the first date everybody made real monetary gains. These gains were the lower bound of losses we could inflict at the second date. By letting the second date be separated by several weeks from the second date, it seems likely that the gains of the first date were internalized by the subjects. Furthermore, the first date demanded the completion of real tasks by the subjects and payments were received in cash. The real gains of the first task thus were likely perceived as rightfully earned and not as house money to gamble with at the second date. The subjects also received the information that gains and losses were equally likely outcomes of the second date. It is therefore reasonable to assume that the status quo prior to the second date is the relevant reference point for decisions made during the second date.

A similar approach to inflict real monetary losses was employed by Bosch-Domènech and

Silvestre (2006). They restricted their experiment to the elicitation of risk aversion. Our results for risk aversion resemble their results closely, but our elicitation of higher-order risk preferences in the domain of real monetary losses has, to our knowledge, not been investigated before.

Both dates of the experiment took place in the MELESSA laboratory of the University of Munich. Subjects were recruited using the software ORSEE by Greiner (2004). At both dates subjects first received written instructions. Then they answered control questions ensuring comprehension of the instructions. Finally, participants made their choices through a computer interface that displayed the options. The interfaces were programmed using the z-Tree software of Fischbacher (2007). All random processes that determined the monetary outcomes were realized through the drawings of numbered balls from urns by the subjects themselves.[5]

4.3.1 First Date

72 subjects participated in the first date of the experiment, which took place on November 25, 2008. In each of the three sessions 24 subjects participated. They were randomly assigned to seats in the laboratory. The first task the subjects conducted was to fill out tables that measure intensities of risk preferences. These tables are not a topic of this thesis. However, it is of importance that the subjects carried out a meaningful task, so that the payment at the end of the first date can be perceived as rightfully earned by the subjects. After filling out the tables, the subjects answered a detailed questionnaire on personal characteristics.

The most important feature of the first date for our present analysis is that subjects received payments of 23.30 to 29.20 euros 31.96 to 40.05 U.S. dollars. These were payed out in cash, in private and not by the experimenters. The payments at the first date include both the participation fee for the first and the second date of the experiment. Answering the

[5]We chose not to let the randomization be done by a computer because we agree with Harrison and Rutsröm (2008): "In our experience subjects are suspicious of randomization generated by computers. Given the propensity of many experimenters in other disciplines to engage in deception, we avoid computer randomization whenever feasible."(p.133).

task and the questionnaire, drawing balls from urns and paying participants took around 60 minutes.

4.3.2 Second Date

The second date took place exactly three weeks after the first date, on December 16, 2008, in the same room as the first date. Again, subjects were randomly assigned to seats. 67 participants (93 %) showed up at the second date. The five subjects who did not show up at the second date could in principle have decided not to show up because of fundamentally different risk preferences compared with the other participants. However, we have detailed information on the subjects to check whether this is the case. First, we have the detailed answers to the questionnaires, and second, we have the responses to the tables eliciting the intensities of risk preferences. We found no systematic differences in the answers of those subjects participating only on the first date.

Each subjects made 84 choices on the second date of the experiment. Each choice was a binary decision between two options, called option A and option B. The framing was always in final outcomes. That is, each option was represented only by the final outcomes that could result if the option was played out. For a given decision, all stated outcomes of a decision were realized with equal probability. Thus, a gamble can be fully represented by the outcomes without stating probabilities separately. In the instructions the options were described in a way that even subjects who are not familiar with the basic notions of probabilities could understand how the outcome was determined. Only the comprehension that each ball was equally likely to be drawn from an urn was needed. To assist those subjects who are more familiar with representations in terms of probabilities we provided additionally an alternative explanation. The order of appearance of the 84 decisions was randomly chosen for each of the three sessions, hence we can check whether the order of appearance influences results.

A key feature of the decisions was that their outcomes lie either in the domain of gains, in the domain of losses or in a mixed domain including both. We classify a decision as being a decision in gains if all final outcomes that can potentially occur regardless of the actual choice are positive values. Likewise, we define decisions as being in losses if all outcomes are negative. We define a decision as being over a mixed domain if both positive and negative values can result.

Table 4.1: Decisions eliciting risk aversion

dec-ision no	para-meters y	ε_1	displayed outcomes Option A A1	A2	Option B B1	B2	in do-main	appearance in session 1	2	3	% of B_2
01	45	30	15	75	45	45	G	29	12	75	58
02	-5	6	-11	1	-5	-5	M	1	39	76	52
03	10	2	8	12	10	10	G	2	40	77	51
04	5	3	2	8	5	5	G	3	64	19	69
05	0	12	-12	12	0	0	M	57	41	49	55
06	57	5	52	62	57	57	G	30	65	50	51
07	28	20	8	48	28	28	G	4	66	20	51
08	0	9	-9	9	0	0	M	58	67	78	55
09	62	17	45	79	62	62	G	59	68	21	57
10	-9	3	-12	-6	-9	-9	L	31	13	79	51
11	-5	3	-8	-2	-5	-5	L	32	42	22	67
12	6	5	1	11	6	6	G	5	43	80	45
13	46	32	14	78	46	46	G	33	69	51	57
14	3	2	1	5	3	3	G	60	14	81	58
15	15	11	4	26	15	15	G	34	44	52	57
16	-6	5	-11	-1	-6	-6	L	61	45	23	51
17	0	2	-2	2	0	0	M	62	70	82	52
18	0	5	-5	5	0	0	M	6	15	53	64
19	-3	2	-5	-1	-3	-3	L	7	46	54	66
20	-7	4	-11	-3	-7	-7	L	35	47	24	66
21	5	6	-1	11	5	5	M	8	71	25	60
22	-1	10	-11	9	-1	-1	M	63	72	26	48
23	9	3	6	12	9	9	G	9	73	83	55
24	-10	2	-12	-8	-10	-10	L	36	16	55	49
25	-7	5	-12	-2	-7	-7	L	10	17	84	52
26	1	10	-9	11	1	1	M	11	18	27	70
27	7	4	3	11	7	7	G	37	74	56	55
28	7	5	2	12	7	7	G	38	48	28	54

28 of the decisions were designed to elicit risk aversion. With $\tilde{\varepsilon}_1 = [-\varepsilon_1, \varepsilon_1]$ the definition

of risk aversion becomes

$$B_2 = [y] = [y, y] \succsim [y - \varepsilon_1, y + \varepsilon_1] = [y + \tilde{\varepsilon}_1] = A_2.$$

In order to make both B_2 and A_2 more comparable in terms of complexity we display B_2 also as a two-outcome of equal probability gamble, hence we chose the representation $B_2 = [y, y]$. Table 4.1 describes the 28 gambles to elicit risk aversion both in terms of parameters (y and ε_1) and in terms of final outcomes displayed (A1, A2, B1, and B2). The values of y were drawn from the range of -10 to 62, y also equals the expected value of both B_2 and A_2. The values of ε_1 were drawn from the range of 2 to 32, ε_1 is also the standard deviation of A_2. This leads to final outcomes in the range of -12 to 79 euros. Column 8 of Table 4.1 states whether the decision lies in the domain of gains, losses or in mixed domains (indicated by a G, L and M, respectively). A decision eliciting risk aversion is in gains if $y - \varepsilon_1 > 0$, in losses if $y + \varepsilon_1 < 0$ and in mixed domains if neither is the case. Of the 28 decisions to elicit risk aversion, 13 were in gains, seven in losses and eight in mixed domains. Displayed in columns 9, 10 and 11 are the orders of appearance of the respective decision in the three sessions.

28 of the decisions were designed to elicit prudence. With $\tilde{\varepsilon}_1 = [-\varepsilon_1, \varepsilon_1]$ the definition of prudence can be stated as

$$\begin{aligned} B_3 &= [y - k, y + \tilde{\varepsilon}_1] = [y - k, y - k, y - \varepsilon_1, y + \varepsilon_1] \\ &\succsim [y, y, y - k - \varepsilon_1, y - k + \varepsilon_1] = [y, y - k + \tilde{\varepsilon}_1] = A_3. \end{aligned}$$

Table 4.2 describes the 28 gambles to elicit prudence both in terms of parameters (y, k and ε_1) and in terms of final outcomes (A1 - A4 and B1 - B4), which were actually displayed to the subjects. y is drawn from the range of -8 to 67, k from 2 to 15 and ε_1 from 2 to 21. The expected value of both B_3 and A_3 is $y - k/2$, and it ranges from -10 to 61.5. The standard deviation of both B_3 and A_3 is $\varepsilon_1/2$ and ranges from 1 to 10.5. This leads to final outcomes in the range of -12 to 72. Column 13 of Table 4.2 states whether the decision lies in the domain of gains, losses or in mixed domains. A decision eliciting prudence is in gains

Table 4.2: Decisions eliciting prudence

Dec-ision no	para-meters y	k	ε_1	displayed outcomes Option A A1	A2	A3	A4	Option B B1	B2	B3	B4	in do-main	appearance in session 1	2	3	% of B_3
29	-4	3	3	-10	-4	-4	-4	-7	-7	-7	-1	L	12	75	29	58
30	-5	2	3	-10	-5	-5	-4	-8	-7	-7	-2	L	39	76	1	49
31	10	2	2	6	10	10	10	8	8	8	12	G	40	77	2	49
32	-6	4	2	-12	-6	-6	-8	-8	-10	-10	-4	L	64	19	3	58
33	9	6	2	1	9	9	5	7	3	3	11	G	41	49	57	60
34	-6	2	4	-12	-6	-6	-4	-10	-8	-8	-2	L	65	50	30	49
35	57	5	20	32	57	57	72	37	52	52	77	G	66	20	4	54
36	0	6	4	-10	0	0	-2	-4	-6	-6	4	M	67	78	58	54
37	10	4	2	4	10	10	8	8	6	6	12	G	68	21	59	57
38	62	15	21	26	62	62	68	41	47	47	83	G	13	79	31	70
39	0	2	2	-4	0	0	0	-2	-2	-2	2	M	42	22	32	58
40	0	4	6	-10	0	0	2	-6	-4	-4	6	M	43	80	5	49
41	8	2	4	2	8	8	10	4	6	6	12	G	69	51	33	57
42	51	12	5	34	51	51	44	46	39	39	56	G	14	81	60	61
43	0	10	2	-12	0	0	-8	-2	-10	-10	2	M	44	52	34	55
44	67	11	11	45	67	67	67	56	56	56	78	G	45	23	61	51
45	-3	2	2	-7	-3	-3	-3	-5	-5	-5	-1	L	70	82	62	63
46	-3	6	2	-11	-3	-3	-7	-5	-9	-9	-1	L	15	53	6	52
47	-3	2	6	-11	-3	-3	1	-9	-5	-5	3	M	46	54	7	54
48	5	2	2	1	5	5	5	3	3	3	7	G	47	24	35	67
49	-4	2	5	-11	-4	-4	-1	-9	-6	-6	1	M	71	25	8	55
50	4	2	5	-3	4	4	7	-1	2	2	9	M	72	26	63	51
51	42	8	16	18	42	42	50	26	34	34	58	G	73	83	9	70
52	10	7	5	-2	10	10	8	5	3	3	15	M	16	55	36	52
53	0	6	6	-12	0	0	0	-6	-6	-6	6	M	17	84	10	58
54	-8	2	2	-12	-8	-8	-8	-10	-10	-10	-6	L	18	27	11	58
55	0	7	3	-10	0	0	-4	-3	-7	-7	3	M	74	56	37	48
56	0	2	10	-12	0	0	8	-10	-2	-2	10	M	48	28	38	42

if $y - k - \varepsilon_1 > 0$, in losses if $y + \varepsilon_1 < 0$ and in mixed domains if neither is the case. Of the 28 decisions to elicit prudence ten are in gains, seven in losses and eleven in mixed domains. Columns 14, 15 and 16 show the orders of appearance of the respective decision in the three sessions.

Lastly, 28 of the decisions were designed to elicit temperance. With $\tilde{\varepsilon}_1 = [-\varepsilon_1, \varepsilon_1]$ and $\tilde{\varepsilon}_2 = [-\varepsilon_2, \varepsilon_2]$ the definition of temperance can be stated as

$$\begin{aligned} B_4 &= [y + \tilde{\varepsilon}_1, y + \tilde{\varepsilon}_2] = [y - \varepsilon_1, y + \varepsilon_1, y - \varepsilon_2, y + \varepsilon_2] \\ &= [y - \varepsilon_1, y - \varepsilon_1, y + \varepsilon_1, y + \varepsilon_1, y - \varepsilon_2, y - \varepsilon_2, y + \varepsilon_2, y + \varepsilon_2] \\ &\succsim [y, y, y, y, y - \varepsilon_1 - \varepsilon_2, y - \varepsilon_1 + \varepsilon_2, y + \varepsilon_1 - \varepsilon_2, y + \varepsilon_1 + \varepsilon_2] \\ &= [y, y + \tilde{\varepsilon}_1 + \tilde{\varepsilon}_2] = A_4. \end{aligned}$$

We used the representation of $[y-\varepsilon_1, y-\varepsilon_1, y+\varepsilon_1, y+\varepsilon_1, y-\varepsilon_2, y-\varepsilon_2, y+\varepsilon_2, y+\varepsilon_2]$ for B_4 in order to make B_4 and A_4 gambles with an equal number of outcomes and thus to make them similarly complex. Table 4.3 describes the 28 gambles to elicit temperance both in terms of parameters (y, ε_1 and ε_2) and in terms of final outcomes (A1 - A8 and B1 - B8) which were actually displayed. y was drawn from the range of -8 to 62, it also equals the expected value of both B_4 and A_4. The values for ε_1 were drawn from the range of 2 to 22, those of ε_2 from the range of 2 to 12. Without loss of generality we chose $\varepsilon_1 \geq \varepsilon_2$. The standard deviation of both B_4 and A_4 is $\sqrt{(\varepsilon_1^2 + \varepsilon_2^2)/2}$ and ranges from 2 to $\sqrt{246.5} \approx 15.7$. The final outcomes range from -12 to 84 euros. Again, the next column states whether the decision lies in the domain of gains, losses or in mixed domains. A decision eliciting temperance is in gains if $y - \varepsilon_1 - \varepsilon_2 > 0$, in losses if $y + \varepsilon_1 + \varepsilon_2 < 0$ and in mixed domains if neither is the case. Of the 28 decisions to elicit temperance ten are in gains, seven in losses and eleven in mixed domains. Also displayed are the orders of appearance of the respective decision in the three sessions.

Table 4.3: Decisions eliciting temperance

dec-ision	para-meters			displayed outcomes																in do-main	appearance in session			% of
				Option A								Option B												
no	y	ε_1	ε_2	A1	A2	A3	A4	A5	A6	A7	A8	B1	B2	B3	B4	B1	B2	B3	B4	main	1	2	3	B_4
57	0	4	3	-7	-1	0	0	0	0	1	7	-4	-4	-3	-3	3	3	4	4	M	75	29	12	55
58	52	18	5	29	39	52	52	52	52	65	75	34	34	47	47	57	57	70	70	G	76	1	39	63
59	-4	3	2	-9	-5	-4	-4	-4	-4	-3	1	-7	-7	-6	-6	-2	-2	-1	-1	M	77	2	40	69
60	6	4	2	0	4	6	-6	-6	-6	8	12	2	2	4	4	8	8	10	10	G	19	3	64	55
61	-8	2	2	-12	-8	-8	-8	-8	-8	-8	-4	-10	-10	-10	-10	-6	-6	-6	-6	L	49	57	41	60
62	48	12	12	24	48	48	48	48	48	48	72	36	36	36	36	60	60	60	60	G	50	30	65	57
63	-4	2	1	-7	-5	-4	-4	-4	-4	-3	-1	-6	-6	-5	-5	-3	-3	-2	-2	L	20	4	66	52
64	0	8	4	-12	-4	0	0	0	0	4	12	-8	-8	-4	-4	4	4	8	8	M	78	58	67	57
65	0	6	6	-12	0	0	0	0	0	0	12	-6	-6	-6	-6	6	6	6	6	M	21	59	68	45
66	-1	6	2	-9	3	-1	-1	-1	-1	-5	7	-3	-3	-7	-7	5	5	1	1	M	79	31	13	76
67	-6	3	2	-11	-7	-6	-6	-6	-6	-5	-1	-9	-9	-8	-8	-4	-4	-3	-3	L	22	32	42	69
68	35	22	4	9	17	35	35	35	35	53	61	13	13	31	31	39	39	57	57	G	80	5	43	55
69	-5	2	2	-9	-5	-5	-5	-5	-5	-5	-1	-7	-7	-7	-7	-3	-3	-3	-3	L	51	33	69	60
70	7	4	1	2	4	7	7	7	7	10	12	3	3	6	6	8	8	11	11	G	81	60	14	57
71	0	7	5	-12	-2	0	0	0	0	2	12	-7	-7	-5	-5	5	5	7	7	M	52	34	44	51
72	0	9	2	-11	-7	0	0	0	0	7	11	-9	-9	-2	-2	2	2	9	9	M	23	61	45	60
73	0	10	2	-12	-8	0	0	0	0	8	12	-10	-10	-2	-2	2	2	10	10	M	82	62	70	58
74	62	13	9	40	58	62	62	62	62	66	84	49	49	53	53	71	71	75	75	G	53	6	15	43
75	10	7	5	-2	12	10	10	10	10	8	22	5	5	3	3	17	17	15	15	M	54	7	46	61
76	-7	4	1	-12	-10	-7	-7	-7	-7	-4	-2	-11	-11	-8	-8	-6	-6	-3	-3	L	24	35	47	43
77	1	6	2	-7	5	1	1	1	1	-3	9	-1	-1	-5	-5	7	7	3	3	M	25	8	71	37
78	18	9	6	3	15	18	18	18	18	21	33	9	9	12	12	24	24	27	27	G	26	63	72	73
79	45	6	6	33	45	45	45	45	45	45	57	39	39	39	39	51	51	51	51	G	83	9	73	57
80	0	2	2	-4	0	0	0	0	0	0	4	-2	-2	-2	-2	2	2	2	2	M	55	36	16	45
81	8	2	2	4	8	8	8	8	8	8	12	6	6	6	6	10	10	10	10	G	84	10	17	69
82	7	3	2	2	6	7	7	7	7	8	12	4	4	5	5	9	9	10	10	G	27	11	18	52
83	-6	4	2	-12	-8	-6	-6	-6	-6	-4	0	-10	-10	-8	-8	-4	-4	-2	-2	L	56	37	74	51
84	-7	3	2	-12	-8	-7	-7	-7	-7	-6	-2	-10	-10	-9	-9	-5	-5	-4	-4	L	28	38	48	40

4.4 Results

We observed 5628 binary choices that we can employ for our analysis, 84 choices for each of the 67 subjects. A first analysis in subsection 4.4.1 treats all choices as independent observations. This provides a first overview of the results. However, it neglects the fact that subsets of 84 choices each are the actions of a single individual. Therefore, we focus in the following subsections 4.4.2 to 4.4.4 on a within-subject analysis. Deck and Schlesinger (2010) let subjects make only ten decisions (six eliciting prudence and four eliciting temperance) and therefore have to refrain from an actual within-subject analysis.

4.4.1 Pooling Subjects

Table 4.4: Descriptive results pooling subjects

choices eliciting	across domains	within gains	within losses	in mixed gambles
Risk-Aversion	56 %	55 %	57 %	57 %
(N)	(1876)	(871)	(469)	(536)
Prudence	56 %	60 %	55 %	52 %
(N)	(1876)	(670)	(469)	(737)
Temperance	56 %	58 %	54 %	56 %
(N)	(1876)	(670)	(469)	(737)

Table 4.4 outlines our results if we pool all subjects. All results are rounded to the full percentage point. The second column states in how many decisions B_n was chosen over A_n for each $n = 2, 3, 4$. In this column no distinction between the decisions were made other than whether they were designed to elicit risk aversion, prudence or temperance. The hypothesis that the probability of making a risk averse choice in a second order decision, a prudent choice in a third-order decision, or a temperate choice in a fourth-order decision is equal or lower than 1/2 is rejected by the three binomial tests with p-values rounded to 0.000000.

In decisions eliciting third-order preferences, 56 % of the choices were in favor of the prudent choice, that is B_3 was preferred over A_3. Both Deck and Schlesinger (2010) and Ebert and Wiesen (2010) also observed a majority of prudent choices. At first glance their results indicate a higher consistency with the concept of prudence since they found 61 % (in Deck and Schlesinger, 2010) and 65 % (in Ebert and Wiesen, 2010) of choices in favor of the prudent alternative. Note, however, that in their experiments subjects could only make real monetary gains. The result which is comparable to their analysis best is reported in the third column of Table 4.4. Here, we find that 60 % of choices in gains were in favor of the prudent choice, quite close to the results of the other two experiments.

In decisions eliciting fourth-order preferences, 56 % of the choices were in favor of the temperate choice. This is in sharp contrast to the results of Deck and Schlesinger (2010), the only other experiment on temperance so far. They report that only 38 % of the choices favor the temperate choice and thus individuals are on average intemperate. However, we are confident that our results are robust since our design was in final outcomes only and thus could be very easily understood by subjects and our analysis of temperance rests on 1876 observations (Deck and Schlesinger, 2010, used 396 observations to elicit temperance). This differing result could have strong consequences since the results of Deck and Schlesinger (2010) on temperance lead them to conclude that cumulative prospect theory better explains data on higher order risk attitudes than EUT with CRRA or CARA functions. If we chose only temperate choices in gains as the appropriate comparison to the results of Deck and Schlesinger (2010) the gap in the results even broadens by two percentage points.

A striking feature of the results in Table 4.4 is the fact that the percentage of risk averse, of prudent, and of temperate choices are so close that they all are rounded to 56 % (the more exact percentages are 56.24 % risk averse, 55.70 % prudent, and 56.02 % temperate choices).[6] In subsection 4.4.3 we will explore whether this stems from the fact that the same individuals who are risk averse, are also prudent and temperate.

[6] If we compute the 90 % confidence intervals of the underlying population percentages we receive 54.14-58.14, 53.78-57.61, and 54.11-57.93, respectively. The fact that these intervals widely overlap again highlights the possibility that the underlying percentage of choosing the 'averse' option may well be equal or very close for different orders.

Based on these first results we can summarize the following. If we would accept the assumptions that individuals are homogeneous and observations are independent, we could state with a high degree of confidence that the behavior of the representative subject exhibits stochastic risk aversion, stochastic prudence, and stochastic temperance.

Column 3 to 5 of Table 4.4 state for each domain separately in how many decisions B_n was chosen over A_n (for each $n = 2, 3, 4$). Choices eliciting risk aversion were very homogeneous across domains. Also, choices eliciting temperance did not show much variation. Only choices eliciting prudence seem to slightly depend on the domain they lie in. To make more precise statements we refer to subsection 4.4.4, which explores the relations of preferences in different domains in more detail. For each cell of Table 4.4 we conducted a binomial test to determine at which level of confidence we can reject the hypothesis that the underlying probability of choosing B_n is equal or below 50 %. All tests yield a p-value below 0.05 except for choices on temperate decisions in losses where the p-value equals 0.069713.

4.4.2 Comparing Subjects

In this and the next two subsections we use the data to characterize individual subjects. Here, we analyze risk preferences of different orders separately. In the following subsection we investigate the *relationship* between risk preferences of different orders. These two subsections have in common that we do not distinguish between decisions in gains, in losses, and in mixed domains. Finally, in subsection 4.4.4 we check if risk preferences depend on the domain the decisions are defined over.

Figures 4.1, 4.2, and 4.4 present our results on the level of the individual subject. We count how often each individual chooses B_n over A_n for each n and show the relative frequencies of subjects with a certain number of risk averse, prudent, or temperate choices. The red distribution in Figure 4.1 shows the empirical frequency distribution of the number of risk averse choices made by the subjects. The blue distribution shows the frequency distribution that had occurred if every subject would have chosen randomly between B_2 and A_2. This would also have been the distribution if every subject would have been stochastically risk

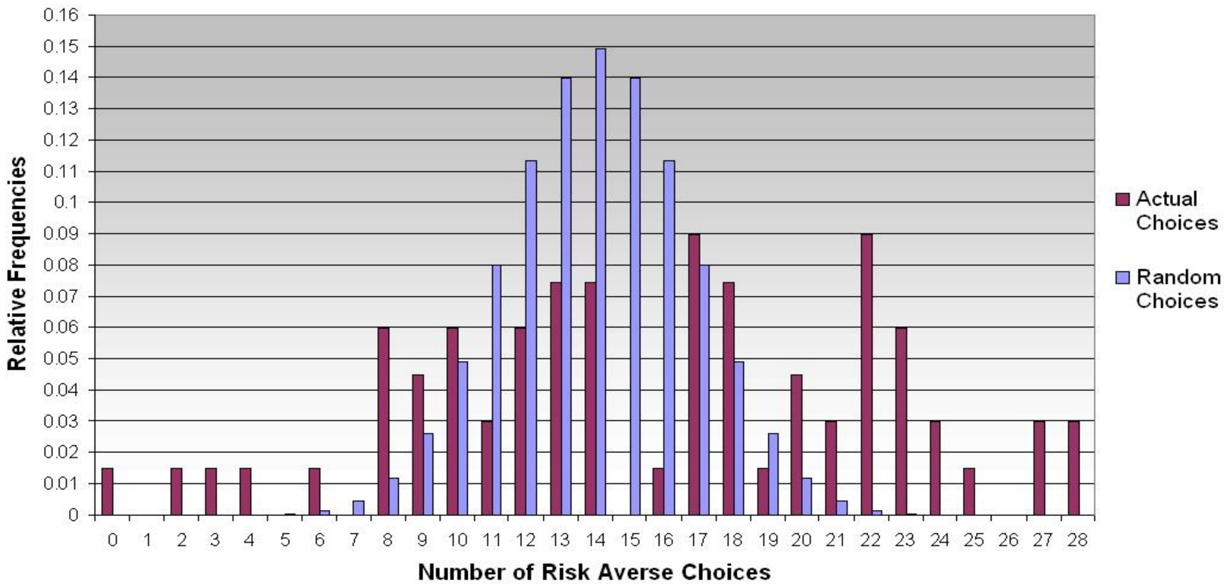

Figure 4.1: Characterizing subjects: Risk aversion

neutral, since this is equivalent to the behavior of choosing B_2 with a probability of 1/2. The empirical distribution clearly does not follow random behavior. The median of the empirical distribution (17 risk averse choices) is above the median of the random distribution (14 risk averse choices). This indicates risk aversion. The empirical distribution also has fatter tails. Hence, there exist both risk seekers and risk averters. However, more subjects are located in the right tail than in the left. Therefore, the number of subjects who are clearly risk averse is higher than the number of clear risk seekers.

In Figure 4.2 the red distribution shows the empirical frequency distribution of the number of prudent choices. The blue distribution again shows the frequency distribution if every subject would have chosen randomly. As before, the two distributions are distinct. Again, the median of the empirical distribution is higher in the actual distribution (15 prudent choices), the tails are fatter, and the right tail is fatter than the left tail. This means subjects tend to be prudent, but also that there is quite a significant level of heterogeneity. Another large part of the subject pool is not distinguishable from being prudence neutral. Also, a small group of subjects clearly seems to be imprudent. Compared with the empirical distribution of choices eliciting risk aversion, the tails are less fat. Additionally, the dominance of the right tail against the left tail is less pronounced than in the case of risk aversion.

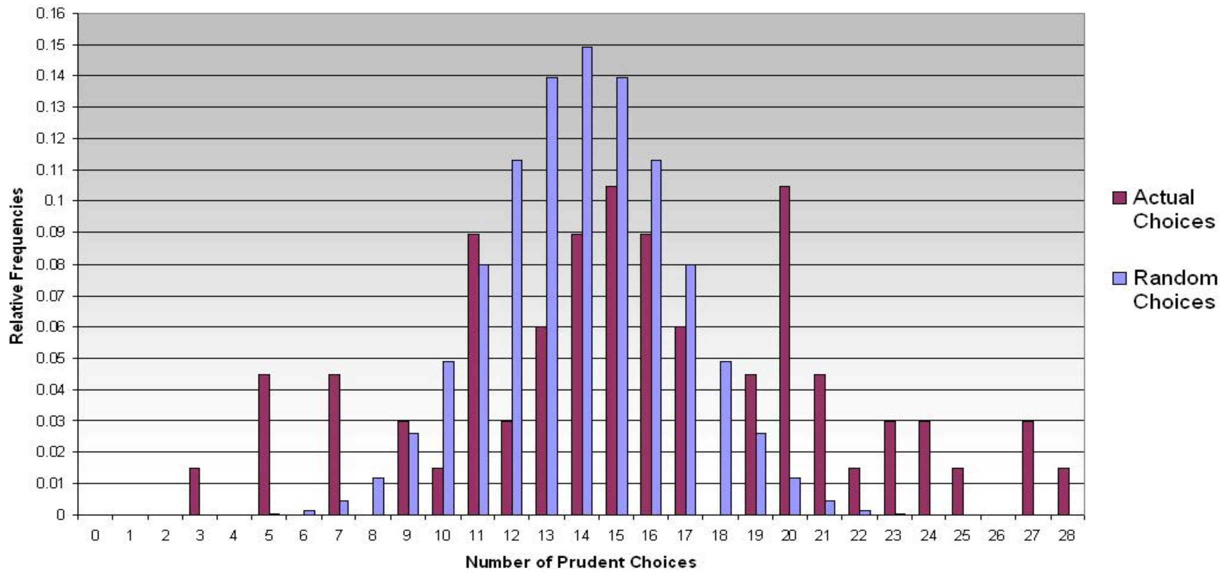

Figure 4.2: Characterizing subjects: Prudence

Figure 4.3: Deck and Schlesinger (2010, Figure 4): Prudence

We can directly compare our results with those of Deck and Schlesinger (2010). In Figure 4.3 we replicate Deck and Schlesinger (2010, Figure 4). It shares the central features of our empirical distribution, namely that the empirical median is above the median of the random distribution (four versus three prudent choices) and significantly more mass is on the right tail than in the random distribution.

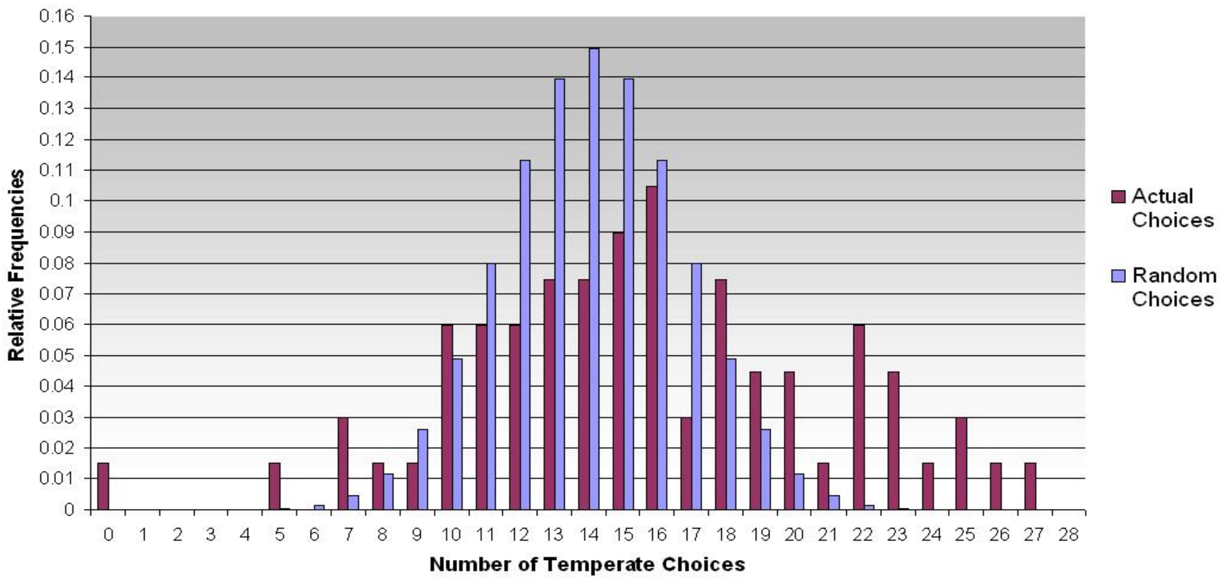

Figure 4.4: Characterizing subjects: Temperance

Finally, Figure 4.4 shows our results of temperate choices against the distributions that would occur under random behavior. It shows that individuals tend to be temperate, the median individual chooses 15 times the temperate option. The left tail has only slightly more mass than would be predicted under random behavior, indicating that the share of subjects who are clearly intemperate is very low. In contrast, the right tail of the distribution has significantly more mass than under the random distribution. While this distribution might not look surprising when compared with our empirical distributions of risk aversion and prudence, it is very different from the results of the only previous experimental study on temperance. In Figure 4.5 we replicate the results of Deck and Schlesinger (2010, Figure 3). The empirical distribution of Deck and Schlesinger (2010) is skewed to the right, while ours is skewed to the left. Deck and Schlesinger (2010) conclude that subjects must be

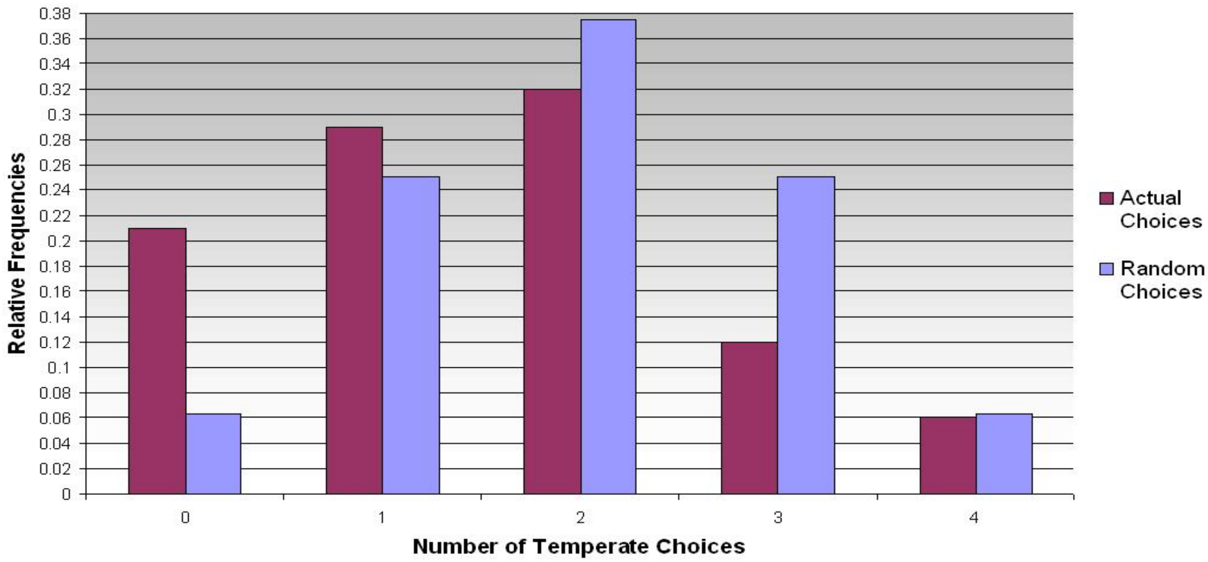

Figure 4.5: Deck and Schlesinger (2010, Figure 3): Temperance

intemperate.[7] They deduce that conventional functional forms of EUT are not reconcilable with individuals being intemperate. Based on our data on temperance we would come to a different conclusion.

4.4.3 Relating Risk Preferences of Different Order

The observation that aggregate accounts of different order preferences closely resemble each other (see Table 4.4), leads to the question of how tightly the risk preferences of different orders are related. We can conduct a within subject analysis of this question, since we can link the decisions of a single subject on different orders. The relation of risk preferences of different orders is of interest because it indicates whether the elicited risk preferences of one order can be a good predictor for the risk preferences of a another order. This has been a standard procedure in many empirical studies on precautionary saving. In these articles, measures of risk aversion were used to predict the amount of prudence and subsequently the presumable optimal levels of precautionary savings.[8]

Figure 4.6 illustrates whether the same subjects that are risk averse also are prudent. We

[7]Whether the exact median of their observed data equals one or two temperate choices (versus a median of two choices under the random distribution) is not inferrable from their reported data. They report that 50 % of subjects made either no or a single temperate choice. Since they analyze 99 subjects the median

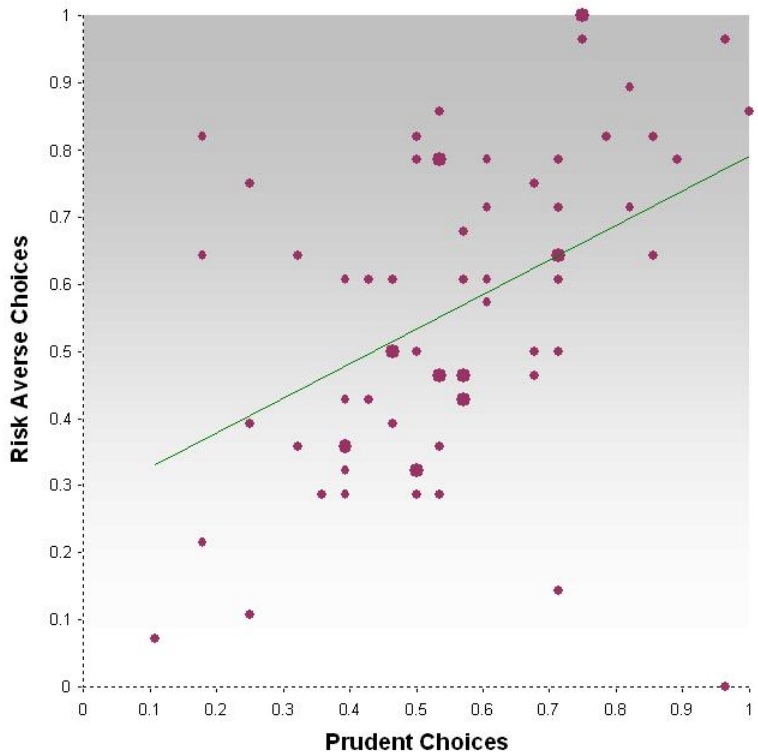

Figure 4.6: Relation of risk aversion and prudence

map each subjects relative frequency of risk averse and prudent choices. Each small point represents a single individual and each larger point represents two individuals who share exactly the same pattern. There clearly is a positive relation between the frequencies of risk averse and prudent choices. The straight line represents a least squares regression line. It has the functional form $Y = 0.2758 + 0.5144\,X$, where X represents the relative frequency of prudent choices and Y the relative frequency of risk averse choices. The coefficient of determination of the regression line is $R^2 = 0.193$ and the Pearson product-moment correlation coefficient is thus $r = 0.4393$. However, since we deal with data measured on an ordinal scale, for a quantification of the association, the Spearman rank-order correlation coefficient r_S is more appropriate. The Spearman correlation is $r_S = 0.4948$, and if we test whether r_S is different from 0 we can confidently reject this with a p-value of 0.0000. We infer that there exists a considerable positive relation between risk aversion and prudence.

has to be either one or two choices.

[8]For a survey article discussing this approach and its results, see Carroll and Kimball (2008).

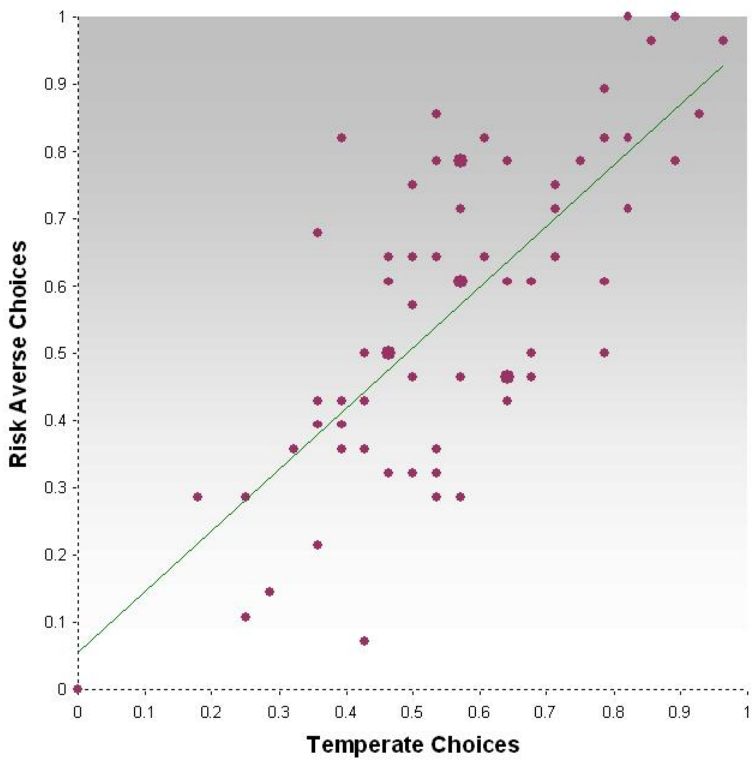

Figure 4.7: Relation of risk aversion and temperance

Figure 4.7 illustrates the relationship between risk aversion and temperance. Again, each small point represents a single individual, each larger point represents two individuals. The positive relation between the frequency of risk aversion and prudence is even more eminent. The least squares regression line is $Y = 0.0547 + 0.9062\,X$, where X represents the relative frequency of temperate choices and Y the relative frequency of risk averse choices (with $R^2 = 0.5391$ and $r = 0.7342$). The Spearman rank-order correlation coefficient is $r_S = 0.6853$, and if we test whether r_S is different from 0 we can confidently reject this with a p-value of 0.0000. This illustrates, that there is a very strong relation between the degree of risk aversion and temperance of an individual, according to our data.

Finally, Figure 4.8 relates prudence and temperance. In this diagram we use three sizes of data points. Each small point is the representation of a single individual, each medium-sized point that of two, and each large-sized point that of three individuals. The least squares regression line is $Y = 0.2221 + 0.5978\,X$, where X represents the relative frequency

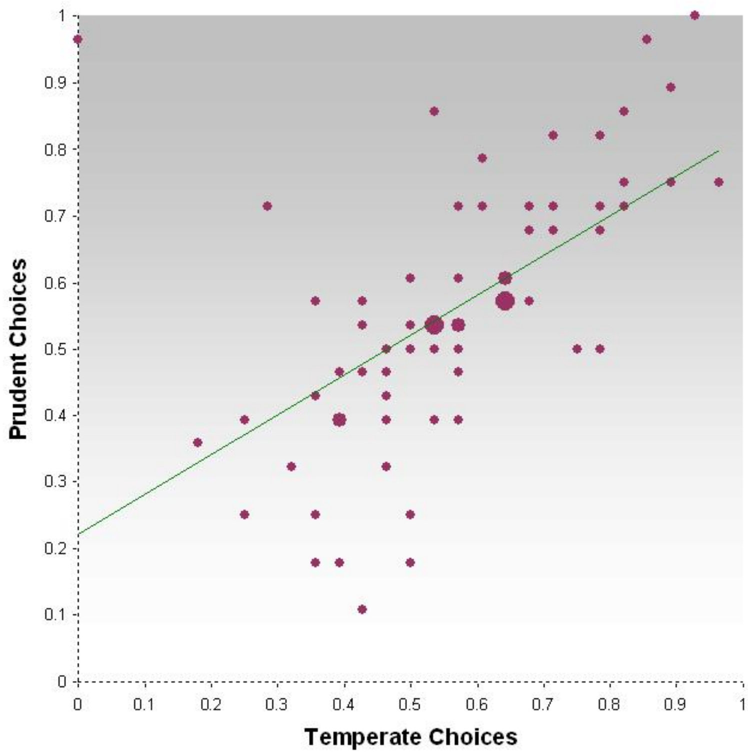

Figure 4.8: Relation of prudence and temperance

of temperate choices and Y the relative frequency of prudent choices (with $R^2 = 0.3216$ and $r = 0.5671$). The Spearman rank-order correlation coefficient is $r_S = 0.6795$, and if we test whether r_S is different from 0 we can confidently reject this with a p-value of 0.0000. The relationship between prudence and temperance is almost as strong as the one between risk aversion and temperance, and stronger than the one between risk aversion and prudence.

The empirical distributions of the three orders of risk preferences seem very similar, judging by the descriptive diagrams in Figures 4.1, 4.2, and 4.4. We also found a high degree of correlation between the risk preferences of different order. The question arises, whether the risk preferences of different orders might be drawn from the same distribution. Therefore, we now analyze whether they are statistically distinct. Since we have a frequency of risk averse, prudent, and temperate choices for every single individual we can employ a two-sided Wilcoxon signed-rank test to compare a pair of two risk preferences. We report the results in Table 4.5. The null hypothesis is that the underlying distribution of the risk preference to the left of a cell and the underlying distribution of the risk preference above a cell are identical. All p-values are very far from any customary significance level. Closest comes the

Table 4.5: Wilcoxon signed-rank tests: differences across risk preferences?

p-value z-statistic (N)	Prudence	Temperance
Prudence		0.3600 -0.915 (67)
Risk Aversion	0.9253 0.094 (67)	0.9054 0.119 (67)

examination on whether prudence and temperance are different, but this is still quite far from being statistically significant. Hence, we clearly cannot reject the null hypothesis that the distributions of risk preferences of different orders are identical.

4.4.4 Comparing Domains

In this subsection we examine the question whether risk preferences differ according to the domain the gambles are defined over: gains, losses, or mixed domains. This is of interest because domain-specific risk preferences are a key feature of many non-EUT models, including cumulative prospect theory (CPT).

We begin with the analysis of risk aversion. Since every subject answered all 28 questions eliciting risk aversion we can again use the two-sided Wilcoxon signed-rank test. We report results in Table 4.6. The null hypothesis is that the underlying distribution of risk aversion is the same in the domain to the left of the cell and in the domain above the cell. We find no pairwise comparison of domains where we could reject the null at the significance level of 10 %. The classical literature on prospect theory found in numerous experiments that individuals are risk-seeking in losses. However, these conclusions were predominantly based on experiments with hypothetical choices. In contrast, the experiment of Bosch-Domènech

Table 4.6: Wilcoxon signed-rank tests: does risk aversion depend on domains?

p-value z-statistic (N)	Losses	Mixed
Losses		0.6482 0.456 (67)
Gains	0.1822 -1.334 (67)	0.4095 -0.825 (67)

and Silvestre (2006) finds a similar pattern as we do, subjects are risk averse in losses. Since they also inflict real monetary losses on subjects, this feature seems to be driving the differing results.

Table 4.7: Wilcoxon signed-rank tests: does prudence depend on domains?

p-value z-statistic (N)	Losses	Mixed
Losses		0.4860 0.697 (67)
Gains	0.1981 1.287 (67)	0.0176 2.374 (67)

Table 4.7 reports the results of Wilcoxon signed-rank tests for the distribution of prudence in different domains. The only statistically significant effect of domains on any measure of risk preferences is found for prudence, between the distribution of prudence in gains and in mixed domains. This confirms our conjecture from the descriptive results of Table 4.4. We reject the null hypothesis that the distribution of prudence is identical in the domain of gains and in the mixed domain with a significance level of 5 %. In contrast, it cannot be rejected that the distributions of prudence in gains and in losses are identical. Neither can the hypothesis be rejected that the distributions in losses and in mixed domains are

identical.

Table 4.8: Wilcoxon signed-rank tests: does temperance depend on domains?

p-value z-statistic (N)	Losses	Mixed
Losses		0.3834 -0.872 (67)
Gains	0.1507 1.437 (67)	0.6260 0.487 (67)

Table 4.8 displays the results of Wilcoxon signed-rank tests for the distribution of temperance across domains. Similarly to the analysis of risk aversion we cannot reject the hypothesis that the underlying distributions of risk preferences of different domains are identical in any of the possible cases.

To conclude, we do not find that risk preferences over decisions in losses are fundamentally different than preferences over decisions in gains. Also, preferences over decisions with a mixed domain do not substantially differ from decision which lie entirely in one of the two domains. Note however, that this result should by no means be interpreted as a rejection of the general concept of a reference point. The concept of loss aversion is not systematically testable in our experimental design.[9] Also, we are not in a position to test whether the strength of risk preferences depend on the distance of outcomes to a reference point or not. What we can conclude is that the direction of the qualitative concepts of risk aversion, prudence and temperance seems not to depend on the domain of the decision. Therefore, the functional form of the value function seems not to change radically at the reference point.

[9] If we examine the second-order decisions over mixed domains, we find some indications that loss aversion is present. Subjects choose B_2 noticeably more often (65 % versus 50 %) if this choice provides a sure gain (decisions 21 and 26) than if this choice leads to a sure loss (decisions 2 and 22). However, the number of decisions is to small to allow for a detailed statistical analysis.

4.4.5 Cumulative Prospect Theory

In this section we examine whether cumulative prospect theory (CPT) can be a better predictor of higher-order risk preferences than EUT with a commonly used utility function.[10] This is a key conclusion Deck and Schlesinger (2010) draw from their experiment. They use the parametrization of Tversky and Kahneman (1992) and assume that the reference point is equal to the mean of the alternatives in a specific decision. With these assumptions, the predictions of CPT over the decisions of their experiment is that subjects will always prefer the prudent choice and will always prefer the intemperate choice. EUT with a commonly used utility function predicts that subjects choose always the prudent and the temperate choice. In all six decisions eliciting third-order preferences, more subjects preferred the prudent choice and in all four decisions eliciting fourth-order preferences more subjects preferred the intemperate choice. Therefore, while both CPT and EUT predict prudence correctly, the finding of temperance is only predicted by CPT.

We carefully designed our experiment to implement the status quo prior to the second date as the reference point. If we adopt the parametric assumptions of Tversky and Kahneman (1992), we find that there is a variety of predictions for each order. CPT predicts in seven of our decisions eliciting second-order preferences that the risk seeking choice will be taken and in 21 decisions it predicts that the risk averse choice will be preferred. Out of the 28 decisions eliciting third-order preferences it predicts only one imprudent choice. In the decisions eliciting fourth-order preferences it predicts twelve temperate choices versus 14 intemperate choices.[11] We can compute in how many decisions CPT correctly predicted the majority of choices. In 59 out of 84 decisions the prediction was correct. To receive an intuition on the performance of this as a predictive theory, we compare it to the result under EUT with standard utility functions. Here, 69 out of the 84 decisions were predicted correctly.[12] To summarize, based on our experiment, we would conclude that the parametric

[10] As commonly used utility functions we denote all functional forms that have derivatives with alternating signs and positive marginal utility, such as CRRA or CARA utility functions.

[11] EUT is independent of domains. If we additionally assume a commonly used utility function, it therefore predicts that subjects always make the risk averse, the prudent and the temperate choice.

[12] Choices in decisions 10, 11, 16, 19, 20, 25, 57, 61, 63, 64, 67, 68, 69, 71, 72, 73 and 83 were correctly

version of CPT most commonly used is performing worse as a predictive theory than EUT with any standard utility function.

If we would adopt our definition of the reference point to the experiment of Deck and Schlesinger (2010), the two compared theories would predict exactly the same behavior. We defined the reference point to be the status quo prior to the experiment. According to this definition, all decisions in Deck and Schlesinger (2010) lie in the domain of gains. In gains, the value function of CPT is identical to a specific CRRA function. In the decision problems of Deck and Schlesinger (2010), the nonlinear influence of probabilities does not change these predictions. Hence, both theories yield identical predictions.

Although we carefully designed our experiment to induce the status quo prior to the second date as the reference point, one could argue that the precise nature of the reference point is never verifiable. Maybe the predictive performance of CPT increases if other reference points are assumed. If we adopt the definition of the reference point from Deck and Schlesinger (2010) to our experiment we find that in this case CPT predicts that subjects always choose the risk averse, the prudent and the intemperate alternative. Only in 53 out of 84 decisions the predictions of CPT would be correct. Therefore, if the mean of the alternatives in any decision problem is assumed to constitute the reference point, CPT performs even worse than under our preferred assumption.

4.4.6 Robustness

In this subsection we investigate whether the sequence in which the decision problems were displayed and answered by the subjects had any effect on the answers. In the three sessions we conducted, the decisions were displayed in different sequences (all determined by a random process). Therefore, we approach this question by observing whether the pattern of answers differs between the three sessions.

In Table 4.9 we analyze if the probability of choosing the risk averse choice differs between

predicted by EUT but not by CPT. Choices in decisions 24, 43, 65, 76, 77, 80 and 84 were correctly predicted by CPT but not by EUT. Choices in decisions 12, 22, 30, 31, 34, 40, 55 and 74 were wrongly predicted by both theories. Choices in the 52 remaining decisions were correctly predicted by both theories.

Table 4.9: Does the elicited risk aversion depend on the ordering of decisions?

	% of	K-S test vs. session 2			K-S test vs. session 3		
	B_2	p-value	D-statistic	(N)	p-value	D-statistic	(N)
session 1	61	0.061	0.3755	(45)	0.387	0.2727	(44)
session 2	52				0.251	0.2866	(45)
session 3	55						

K-S test: Kolmogorov-Smirnov tests for equality of distribution functions

the three sessions. In the second column we state the share of risk averse choices in the three sessions as a first descriptive analysis. In all three sessions a majority of choices were an favor of the risk averse option. In session 1 the share of risk averse choices was highest, in session 2 it was lowest. In the following columns we report the results of three Kolmogorov-Smirnov tests for equality of distribution functions. The null hypothesis is that the underlying probability of choosing the risk averse choice is identical in two sessions. We cannot reject this hypothesis in any of the three tests at a level of significance of 5 %. At a significance level of 10 % we would reject the hypothesis that the probability of choosing B_2 is identical in session 1 and in session 2. In fact, this is the only rejected hypothesis of equality at the 10 %-level in the whole subsection. Since we perform nine tests in total it is not surprising that one indicates a significance at the 10 %-level. We therefore do not conclude that the sequence of questions alters behavior in our design. However, we recognize that if any sequence effect exists, it is most likely to be present in the decisions eliciting risk aversion.

Table 4.10 displays the share of prudent choices in the three sessions as well as the results of three Kolmogorov-Smirnov tests. The shares of prudent choices differ by six percentage points at most. Also, the tests yield p-values that are far from any customary significance level. We therefore clearly cannot reject the hypothesis that the distribution of the underlying probability to choose a prudent option is identical in the three sessions.

Finally, in Table 4.11 we check for the robustness of temperance elicitation to sequence

Table 4.10: Does the elicited prudence depend on the ordering of decisions?

	% of	K-S test vs. session 2			K-S test vs. session 3		
	B_3	p-value	D-statistic	(N)	p-value	D-statistic	(N)
session 1	54	0.313	0.2727	(45)	0.842	0.1818	(44)
session 2	60				0.954	0.1364	(45)
session 3	54						

K-S test: Kolmogorov-Smirnov tests for equality of distribution functions

Table 4.11: Does the elicited temperance depend on the ordering of decisions?

	% of	K-S test vs. session 2			K-S test vs. session 3		
	B_4	p-value	D-statistic	(N)	p-value	D-statistic	(N)
session 1	56	0.854	0.1621	(45)	0.632	0.1818	(44)
session 2	58				0.194	0.2984	(45)
session 3	54						

K-S test: Kolmogorov-Smirnov tests for equality of distribution functions effects. The maximum difference in the share of temperate choices is four percentage points. Neither of the three Kolmogorov-Smirnov tests indicates that the hypothesis of identical distributions can be rejected.[13]

4.5 Conclusion

We analyzed higher-order risk preferences, namely risk aversion, prudence and temperance in a laboratory experiment. Each analyzed subject participated in two distinct dates of the experiment, separated by several weeks. This allowed us to inflict real monetary losses on subjects on the second date if decisions and chance determined that the pay-offs were negative. Thus, we designed the experiment in order to clearly implement the status quo prior to the second date experiment as the reference point.

Our findings for risk aversion are in line with the most recent results from other experiments that use monetary incentives. Especially, we find no hint at risk-seeking behavior in losses, a behavior that has been found in the early experimental literature relying on hypothetical questions only.

Our results on prudence resemble those of the other two experimental studies that have examined this matter. Prudence is the only risk preference analyzed that relies on the domain the decision lies in, although the effect seems not to be very strong. Decisions in the mixed domain reveal a statistically significant lower level of consistency with prudence than those in gains. Note, however that this does not change the qualitative result that in all domains, subjects prefer the prudent more often than the imprudent alternative.

The results of our experiment for temperance are clearly the most notable. There exists only one previous experimental study on such behavior (by Deck and Schlesinger, 2010) and our experiment yields almost opposite results. Since our set of data included more than four

[13] One may be puzzled by the results of the Kolmogorov-Smirnov tests between session 1 and 3 for third- and for fourth-order preferences (in Tables 4.10 and 4.11, respectively). They yield the same D-statistic and are based on the same sample size, yet they exhibit different p-values. The reason is that in Tables 4.9, 4.10 and 4.11 we report the exact p-values of the tests while the p-values directly based on the D-statistics are approximations in case of ties (and are likewise both 0.860 for the two tests under consideration).

times as many observations than that of Deck and Schlesinger (2010) we are confident that our results are robust. This is of importance because the previous result of intemperance was used to reject common specifications of EUT and to support cumulative prospect theory instead. Our results, however, are better reconcilable with standard assumptions in EUT, such as CRRA or CARA, than with their favored variant of cumulative prospect theory. Also, we found very strong relationships between risk preferences of different orders. Standard preferences, such as CRRA or CARA, predict that the higher the degree of risk aversion, the higher the degrees of prudence and temperance.

So, the shape of gain-loss utility does not seem to be of a fundamentally different nature for gains and for losses. As a qualification of our results, we have to state that the stakes in our experiment are small compared to some decisions individuals face outside the laboratory. If individuals are threatened by huge losses, they may be tempted to speculate on a series of favorable outcomes of events and take risks they otherwise would shun. They might be willing to aggregate risks in order to leave at least one possible outcome unaffected. This unaffected outcome then could function as a glimmer of hope for the subjects. However, behavior in reaction to monetary losses that do not stir such existential fears seems more accessible to economic analysis. Although we cannot infer the functional shape of the value function for all levels of stakes from our results, we can only dismiss the notion that at the reference point the function radically changes its characteristics.

Part IV

Conclusion and Outlook

The general approach I followed in this thesis in order to contribute to the existing body of literature is twofold. The first strategy was to extend established theoretical models that represent the current frontier of behavioral economics. The second strategy was to develop model-independent methods that allow experimental preference elicitations which can be interpreted in a wide range of plausible models. Both approaches have their unique benefits. Moreover, rather than being substitutes, they can complement each other.

In the attempt to extend existing theoretical models, the focus was on descriptive models that are based on the consistent maximization of an objective function. This ensures that the specified theories can be employed in a wide range of situations and are not limited to particular contexts. Furthermore, the extensions concentrated on recently proposed models of reference-dependent preferences because of two reasons. On the one hand, models of reference dependence are rapidly gaining acceptance among researchers in this field. This indicates that the general importance of reference points for decisions under risk is becoming an element of a consensus among scholars. On the other hand, theoretical models of endogenous reference points are a relatively recent development. Therefore, for many well-established strands of literature inside expected utility theory (EUT) it has not yet been analyzed how equivalent concepts in models of reference-dependent preferences apply. However, the question whether these concepts can be reasonably applied in the new models is interesting since the new models aim to achieve a level of universality comparable with that of EUT.

In chapter 1, an analysis of the properties of higher-order risk preferences, such as prudence and temperance, in reference-dependent models was presented. Concepts of risk preferences beyond risk aversion have not been systematically analyzed before in models of reference dependence. It was shown that the general pattern of higher-order risk preferences is fundamentally different compared to commonly assumed models in EUT. Also, assuming different types of reference points influences the results significantly. The assumption of recent expectations as the reference point, as it has been proposed in the popular models by Kőszegi and Rabin (2006, 2007), explains two puzzles in the literature concerning higher-order risk preferences. First, the coexistence of a considerable amount of risk aversion and modest levels of precautionary saving are consistent with this theory. Second, the demand for costly insurance is predicted by this theory, even in the presence of buffer stock savings. Other concepts of reference points were not able to explain these widely-observed phenomena. This illustrates that the assumption on what shapes the reference point is essential for predictions of the reference-dependent model. In contrast, the precise functional form evaluating gains and losses did not play a major role. The central conclusions in chapter 1 were derived for very general classes of gain-loss utility. We concluded that expectation-based reference points can constructively contribute to explain economic behavior which is based on higher-order risk preferences.

Kőszegi and Rabin (2009) proposed an extension of reference-dependent preferences that allows the inclusion of anticipatory utility into the general framework. In chapter 2, I presented an alternative model that follows the same intuitive notions but allows for more flexibility in the case of multi-attribute utility. This broadens the range of applicability of the model beyond additively-separable utility functions. It was further shown that the two models differ in predictions, even if risk-less preferences are such that both models are applicable. This shows that the alternative model is not simply an extension of the previous model. Two scenarios were defined in which predictions of the two models differ. First, in my model, different attributes of utility are more substitutable than in the previous model. In particular, all changes that do not affect pure consumption utility have no influence on decision utility in my model. Second, changes in beliefs concerning the correlation of risks that influence different attributes have an effect on decision utility only in my model. The aim of this analysis was to adjust the existing model in a way that preserves its numerous advantageous features but allows for a generality of pure consumption utility functions that is comparable to those in EUT.

The two contributions to the theoretical literature have in common that they analyze concepts which are well-established in EUT, in models of reference-dependence. These concepts were higher-order risk preferences and multi-attribute utility, respectively. The theoretical extensions have not necessarily to be confined to a single variant of a reference-dependent model. For instance, we analyzed risk preferences of higher orders with many different reference points in the first chapter. Also, alternative possibilities to achieve a wider range of applicability were explored in the second chapter. If we derive results in different theoretical models, we can isolate the predictions that the models have in common and the predictions that distinguish them. For instance, based on the analysis of chapter 1 it can be concluded that for higher-order risk preferences it makes a substantial difference whether reference points are formed by expectations or by feelings of regret. In contrast, only very general assumptions concerning the functional form of the value function were made to arrive at the conclusions.

The second strategy followed in this thesis was to employ features which many popular models share, to design methods of empirical evaluation. These methods then can be used in experimental studies and the interpretation of its results are not confined to narrowly defined theories. Both proposed methods were implemented in incentivized experiments to assess their empirical implications.

The first model-independent method proposed an experimental procedure to elicit the intensity of risk aversion. Its design is based on the canonical definition of risk increases. The concept of duality of risk aversion and risk was used. Namely, an individual is considered as having a lower level of risk aversion if it is ready to accept more increases in risk in exchange for a compensation. The used notions of risk increases and of duality have been applied in a wide range of theoretical models, both inside and outside of EUT. Therefore, these concepts can be labeled as model-independent. We experimentally implemented our proposed method jointly with the most popular risk aversion elicitation technique. This allows us to assess if our method yields different results, and hence, if the methodological issues are of empirical relevance. It was found that the ordering of subjects according to their intensity of risk aversion is fundamentally different for both methods. Also, if additional assumptions are made to receive a quantitative measure of risk aversion, our method results in substantially higher estimates.

The second model-independent method outlined in this thesis is a design that allows to investigate the higher-order risk preferences of individuals. Since this has only very recently be implemented in laboratory experiments, there exist no established procedures yet. This method was also applied in a laboratory experiment. We find that subjects generally tend to be risk averse, prudent, and temperate. However, we also observed a considerable amount of heterogeneity. A focus in the experiment was to induce real monetary losses for subjects. Many theoretical models that deviate from EUT are based on distinctive evaluations of gains and losses. In our experiment we were able to observe whether risk preferences differ drastically between decisions in gains and in losses. This would be predicted if the value function follows fundamentally different functional forms in losses and in gains. However, we find no substantial differences in the qualitative direction of risk preferences in gains and in losses.

The results of the experiment outlined in chapter 4 indicates that inside the domains of gains and losses, the local properties of the value function seem not to differ in fundamental ways. However, the value function could well have a kink at the reference point, representing loss aversion. More substantially, the analysis of chapter 1 has shown that independently of the functional form of the value function, reference dependence can influence decisions. In contrast, the precise nature of the reference point has emerged as a primary factor that is driving behavior.

Prospects for future research include applications of the two theoretical models. Numerous examples of recent applications of the basic model of Kőszegi and Rabin (2006) were provided in section 1.2.2 of chapter 1. However, most of these applications do not employ the subtle differences of the model, such as the separation along different dimensions of utility or the different concepts of personal equilibria. Remaining theoretical considerations that go beyond mere applications are the precise timing of the adjustment of the reference point to expectations or the principles according to which dimensions of utility are defined.

Even more promising seem descriptive theories of behavior that leave the concept of risk behind. For a long time, such theories have been confined to very abstract and theoretical deliberations. But, as more ways are explored to derive testable predictions, these general concepts of uncertainty increasingly allude to their considerable potential. These theories have in common that they acknowledge the immense difficulties individuals have to capture the multiple contingencies of the future in terms of risk. Examples are theories of ambiguity and of case-based decision making. In theories of ambiguity, it is not assumed that individuals form a single probability distribution representing their subjective beliefs. Instead, a whole set of probability distributions is defined. The uncertainty over the right probability distribution is additionally influencing the decision maker.[14] In models of case-based decision making, observed situations of the past are compared to the present decision problem in terms of similarity. Then, conclusions from the outcomes in these past cases are used by the individuals as guidance for a decision in the problem at hand.[15]

Applying the tool kit developed and refined in the analysis of risk preferences for decades to these theories, has the potential to yield many fruitful insights. Until now, these theories have been characterized by a multitude of sets of axioms. In order to derive easily exercisable versions, based on tractable functional forms, both theoretical and experimental methods of risk preference elicitation could be adopted.

[14]For a recent comprehensive survey of the literature consult Gilboa and Marinacci (2011).

[15]The decision-theoretic analysis of case-based decision making originates in Gilboa and Schmeidler (1995).

Part V

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Tabellarischer Lebenslauf

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