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Stock Market Dynamics:

A Psycho-Economic Approach to Speculative Bubbles

von

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Beitrag Nr. 92 Juni 1993

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Abstract

The aim of this paper is to build up a formal model of the stock market in a framework which Shiller (1990) calls a "feedback system (...) with a possibly complicated dynamics." Therefore we attempt to synthesize both aspects, the economic as well as the psychological, in a theoretical model of an artificial stock market. Both elements are here extended into a dynamic context based on ideas of self-organizing systems. In a dynamic analysis we can show how psychological factors interact with the intensity of fluctuations of stock prices and thus endogenously generate over- and undervaluations of the stock prices or even crashes.

1 Introduction

In reality the aggregate behaviour of investors in the stock market has always been determined by psychological as well as economic factors. Theoretical models, however, designed to explain the investors' behaviour almost entirely concentrated on the economic aspect only, based on the conception of a "homo oeconomicus". Even if stochastic elements and the assumption of risk-averse behaviour are included, the "homo oeconomicus" is still acting as a rational individuum driven by self-interest (Frey/Gülker (1988)) and the quest to maximize.

On the other hand, Kindleberger back in 1978 already pointed out that the rationality of speculative markets is more of an a priori assumption than the description of reality. And, startled by the recent stock market crashes economists are more and more looking for new theoretical approaches which allow to break up the strict postulate of rationality.[1]

Contrary to that, contemporary psychology views stock price fluctuations mainly as a phenomenon which reflects the hopes and fears of market participants rather than real economic situations (Maas/Weibler (1988)). Seen from this perspective the stock market should be a rich "playground" for psychologists. But, until now only a few researchers of the psychologic discipline showed a particular interest in that field (Bungard/Schultz-Gambard (1990)). This might be mainly so because the stock market traditionally is a research object of economists and therefore somewhat a taboo for psychology (Wiendieck (1990)). However, a process of rethinking seems to be under way: The works of Maital/Filer/Simon (1986), Schachter/Hood/Andreassen/Gerin (1986), Schachter/Quellette/Whittle/Gerin (1987) and of Andreassen (1987) already try to examine the influences of motivation and the formation of expectations for transactions on the stock market theoretically, empirically and on experimental grounds.

[1] See in this context De Long/Shleifer/Summers/Waldmann (1990) or Topol (1991).

Our contribution aims at a synthesis of economic and psychological ideas at the theoretical and formal level. Thereby, we want to meet recent demands calling for a stronger integration of psychological elements in economics.[2] For this purpose we develop a dynamic model that attributes stock price deviations from their fundamental values, socalled speculative bubbles, primarily to psychological influences like optimism, pessimism and scepticism. By applying results and methods from the theory of self-organizing systems we can show that the dynamic interplay of psychological factors with the level of price deviations can endogenously generate over- and underpricing of stocks and even sudden price falls. Our model therefore reflects formally what Shiller (1990) called a feedback system with complicated dynamics when he examined the American stock market.

We will proceed as follows: In the next section we specify the dynamic model of the stock market and its underlying assumptions. A dynamic analysis of this model in a qualitative way will follow in section three. In the fourth section the model will be numerically specified and supplemented by a number of simulation runs. There, we will also discuss and compare the results of our qualitative and numerical analysis. Section five sums up the main findings and discusses suggestions for further research.

2 The Model

2.1 Basic Principles of the Formation of Expectations

For a riskneutral investor the return of a share holding is ex post calculated as

$$R_t = \frac{p_{t+1} - p_t + d_{t+1}}{p_t}, \tag{1}$$

where p_t, respectively p_{t+1}, denotes the stock price in period t or $t+1$ and d_{t+1} is the payment of dividends in period $t+1$. In an ex ante equation,

[2]See some articles in Hanusch/Recktenwald (1992).

the latter are random variables.

We assume the investor is forming his expectations by using the mode criterion. The mode of a probability distribution belongs to the family of measures of central location like the expected value which is normally used in these kinds of models. For discrete distributions the mode represents the most likely statistical value. It is calculated, in general form, for the random variable X as

$$M(X|I_t) = \max_{\{X\}} \{P(X|I_t)\}, \tag{2}$$

where $P(.)$ denotes a conditional probabilty distribution and $I_t \in I$ gives the information set which is available in period t.

To come to a decision the investor compares his return from stock holdings with the interest rate of the financial market which is assumed to be constant. For him the following condition must hold

$$M(R_t|I_t) \geq r, \ \forall t = 1, 2, \ldots \tag{3}$$

Since r is the minimum return of the asset, the stock price has to fulfill the minimum requirement

$$p_t = \lambda^{-1}\{M(p_{t+1}|I_t) + M(d_{t+1}|I_t)\}, \ \lambda := 1 + r. \tag{4}$$

For the periods $t + i$, $i = 1, 2, \ldots$, we calculate the mode value under the assumption that

$$I_t \subseteq I_{t+i} \ \text{ and } \ M(M(.|I_{t+i})|I_t) = M(.|I_t) \tag{5}$$

holds. Therefore, we get the solution of (4) by backward integrating under the transversality condition $\lim_{n\to\infty} \lambda^{-n} p_{t+n} = 0$ as

$$p_t^M = \sum_{i=1}^{\infty} \lambda^{-i} M(d_{t+i}|I_t). \tag{6}$$

This equation defines the price which an investor is willing to pay for the stock, if he forms his expectations by using the mode criterion.

The mode formation of expectations, just described, represents a procedure on the microeconomic level. That means every investor has its own probability distribution and forms its individual price expectations according to the mode criterion. In order to get from an individual level to the level of the entire stock market one has to solve the socalled aggregation problem.

To do so, we rely on recent psychological studies of the stock market summarized in a survey by Frey/Stahlberg (1990). According to them, the aggregation problem does not play a preeminent role in the formation of stock price expectations, as one would consider reasonable from an economic viewpoint. Those studies show that the expectations on the stock market does not appear as a special micro phenomenon but is rather caused primarily by collective learning, namely through the influence of mass media, through propaganda or by the collective acquisition of expert opinions. This in turn has the effect that large classes of investors have similar expectations, a fact which is reflected on the aggregate level. If we adopt this view of psychologists aggregating individual stock market expectations, then the stock price is determined by equation (6) via self-fulfilling prophecies.

On that basis we can now examine a stock market on which in period t a speculative bubble emerges. This bubble is defined as the deviation y of the stock price p from its fundamental value p^F. For this difference

$$y_t = p_t - p_t^F \tag{7}$$

$y_t \neq 0$ must hold. For a calculation of the stock price's fundamental value p^F we refer to the relevant literature.[3] There, the fundamental value is derived as

$$p_t^F = \sum_{i=1}^{\infty} \lambda^{-i} E(d_{t+i}|I_t)\,, \tag{8}$$

with E(.) as the conditional expected value of future dividend payments.

[3]See for example Blanchard/Watson (1982).

In order to develop the origin of a speculative bubble in more detail, we want to introduce the following additional assumptions:

- To determine analytically the fundamental value from (8) we assume a normal distribution, as it is usually done. For the formation of expectations according to the mode criterion we use the Q-distribution derived in the appendix. In doing so, we move from the mode criterion to the concept of Q-Mode-Expectations (QME).[4]
- The normal distribution is time independent according to the usual assumptions. For the general Q-distribution expression (A.6) in the appendix applies.
- Starting with period τ, the future dividend policy of the considered company shall be stable.[5] This assumption means that in all future periods, i.e. $\tau+1, \tau+2, \ldots$, the same dividend is paid. However, the amount of the dividend payment is still uncertain.

According to these assumptions and using the QME-approach the dividends are given by:

$$M(d_{\tau+i}|I_\tau) = M(d|I_\tau) = \max_{\{d\}} \{P_\tau^Q(d|\alpha,\beta,\gamma,\delta)|I_\tau\} = d_\tau, \tag{9}$$

with $i = 1, 2, \ldots$ We can now transform equation (7) into

$$y_\tau = \xi(d_\tau - \mu), \; \xi := \frac{1}{\lambda - 1}, \tag{10}$$

where $E(d_{\tau+i}|I_\tau) = E(d|I_\tau) = \mu$.

To get a general distribution for the deviation y we do the transformations $\mu = -\beta/4$ and $\xi = \alpha^{1/4}$.

[4] For a detailed description of this approach see v. Natzmer (1985).

[5] We could also introduce a strategy of constant dividend growth.

The new parameters are now location, respectively scale parameters of the general Q-distribution which therefore is topological equivalent to[6]

$$f_\tau^Q(d|\mu,\xi,\gamma,\delta) = exp\{-(\xi^4)(d^4 - 4\mu d^3 + \gamma_\tau d^2 + \delta_\tau d)\}. \qquad (11)$$

Another transformation according to (10) and substituting τ by t leads to the distribution function of the deviation y

$$f_t(y|k,l) = exp\{-(y^4 + k_t y^2 + l_t y)\}, \qquad (12)$$

with $k_t = (\gamma_t - 6\mu^2)\xi^2$ und $l_t = (\delta_t + 2\gamma_t\mu - 8\mu^3)\xi^3$. This function depends only on the two parameters k_t and l_t.

Since in continuous probability distributions the mode criterion implies local maximization of the density function, the speculative bubble y in period t arises from the solution of the maximization problem

$$y_t = \max_{\{y\}} \{f_t(y|k,l)\}, \qquad (13)$$

that is from the first order condition

$$4y^3 + 2k_t y + l_t = 0. \qquad (14)$$

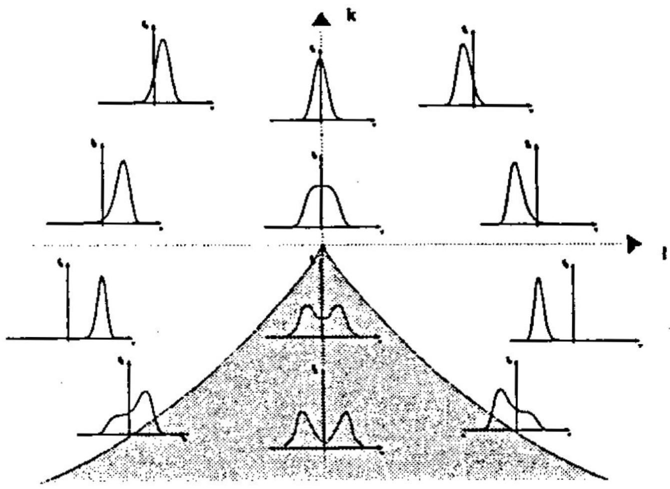
In analogy to the QME-approach the time dependent parameters k and l can now be regarded as psychological influences. Their exact meaning will be specified later on. In the following we want to focus first on how the two parameters influence the shape of the density function $f_t(y|k,l)$. For that let us look at figure 1. There, for a couple of k and l values we can see the corresponding density functions. With this graph we can also demonstrate how the shape of the density function reacts when temporal changes of k and l do occur.

Let us first concentrate on the case where $k \geq 0$. Here, the density is everywhere of an unimodal shape. In the positive intercept of the k-axis,

[6] In the following notations the information condition is neglected and we will no longer distinguish between the distribution function and its topological equivalence.

that means where $l = 0$, it is also symmetrical. This symmetry means that the mode and the expected value of a stock price are identical and, therefore, the stock price equals its fundamental value. Accordingly, a deviation from $y_t = 0$ in our equation (7) maximizes the density function.

Figure 1



Now, if parameter l is positively (negatively) changing over time, the distribution density gets a negative (positive) skewness. Consequently, the deviation y_t becomes negative (positive) and the stock price has to fall below (above) its fundamental value. Outside the hatched area this is also true for the range $k < 0$. So, according to the algebraic sign of l, the stock price is above or below the fundamental value and an over- or undervaluation of the stock takes place by the investors.

Therefore, the parameter l serves as an ideal indicator for optimistic or pessimistic sentiments and tendencies on the stock market.

In the hatched area a switch of the algebraic sign of k causes a change in the number of modes, meaning that the density function takes a bimodal form. To make it quite clear, we assume that the edge of the hatched area is crossed from the left. Then an additional mode emerges, which for $l < 0$ has a lower density value compared to the initial mode. The functional value of the new mode grows with an increasing l. At the negative intercept of the k-axis both modes have the same density value and for $l > 0$ the functional value of the new additional mode lies above the initial one until the latter disappears entirely when crossing the right border.

Coming back to the psychology of the stock market, this can be interpreted in the following way: Beyond the left edge of the hatched area the market is in an optimistic mood ($l < 0$). If this optimism is declining, an additional very likely price movement will arise within the hatched area now laying significantly below its fundamental value. As a consequence the investors start to doubt about their previous optimistic price expectations. They get more and more sceptical about the future price development. Thus, the parameter k, being responsible for this change in orientation, can be used as an indicator for the scepticism on the stock market. Later on we will see that investors within the hatched area may change their expectations even suddenly and very abruptly.

In formal analysis all this has its origin in the solution set of the cubic equation (14), which again is determined by the algebraic sign of the discriminant

$$D = 8k_t^3 + 27l_t^2. \tag{15}$$

For $D \leq 0$ three real solutions exist, in every other case only one. In the case $D = 0$ either two out of the three solutions (if $k_t \neq 0$ and $l_t \neq 0$) or the three solutions altogether (if $k_t = l_t = 0$) coincide. Within the hatched area in figure 1 we get $D < 0$ and on the edge $D = 0$. In the other areas D is positive. Thus, within the hatched area, the density function runs through a transition from an unimodal to a bimodal form .

To complete our discussion here we should mention that in the case of $k > 0$ changes from optimistic to pessimistic expectations are of course also possible. In contrast to the expectation changes discussed above those changes will come forth continuously. Abrupt, discontinuous changes can take place in the hatched area only.

2.2 Elements of Psycho-Dynamics

Our aim now is to conceptualize formally, in a simplified framework, what Shiller (1990) has called "a feedback system (...) with possibly complicated dynamics". When he investigated the American stock market after the crash in 1987 he discovered that this breakdown was not caused by an exogenous trigger in combination with a mass hysteria. Responsible for the crash was, in his opinion, an emotional psychological system with inherent endogenous dynamics. Sharing this opinion means in our context that deviations of prices from the fundamental values, on the one hand, and the indicators for optimism/pessimism and for scepticism, on the other hand, can not be regarded as independent. Rather, there seems to exist an interdependence that should be able to generate a self-dynamic system producing and bursting itself speculative bubbles without regard to the fundamental conditions of the market. To demonstrate such a, in a certain sense, psychological dynamics , we suppose that the events on the stock market are determined by different equations of motion. To build up our formal model we employ the concept of nonlinear dynamics or of selforganizing systems, where the motions regulate themselves via feedbacks.

Since investors form their expectations according to the QME-approach, the following equation guarantees local maximization on the basis of (13):

$$\dot{y} = -\nu(4y^3 + 2ky + l). \tag{16}$$

The term $\nu > 1$ gives the speed of expectation-adjustment to temporal parameter changes of k and l. The dot denotes here and in the following time derivates.

The first order condition from (14) defines for any real values of k and l an equilibrium plane where the corresponding value of y is situated. It can be seen, when condition (14) is met, that $\dot{y} = 0$. As equation (16) assumes a very fast adjustment process, changes of k and l instantaneously are transformed into price deviations. Investors at the stock market, therefore, adjust their expectations without delay to the changed sentiment on the market.

The temporal development of the parameter of scepticism shows the following equation

$$\dot{k} = \frac{1}{\nu}(\varphi(y) + l). \tag{17}$$

We assume that $\varphi(0) = 0$, $\varphi'(y) < 0 \; \forall \; y \neq 0$, $\varphi'(0) = 0$.

From equation (17) it is apparent that the influence of the optimism/pessimism parameter on the temporal development of scepticism is carried out directly. This can reflect a commonly known phenomenon: The higher the degree of optimism ($l < 0$), the more scepticism will grow if the optimistic expectations will come true in the future.

Price deviation y, however, affects the development of the parameter of scepticism only indirectly via function φ. With the help of this function we furthermore assume that in general a positive deviation of the stock price from its fundamental value has a promoting effect for scepticism. While a negative deviation generally should reduce the existing scepticism. This functional coherence is shown in the left part of figure 2.

In addition, we assume that the influence of function φ on the development of the parameter for scepticism is relatively weak within the interval $[y_1, y_2]$. This means, for example, that a slight overvaluation of a share within the limits $0 < y < y_2$ leaves little room for a sceptical attitude concerning the forthgoing positive price movement. But, if the overvaluation increases, and $y > y_2$, the already existing doubts rapidly gain strength.

The equation for the temporal development of the optimism/pessimism parameter looks a bit more complex:

$$\dot{l} = \frac{1}{\nu}(\Lambda - k - \mu l + \phi(l)), \tag{18}$$

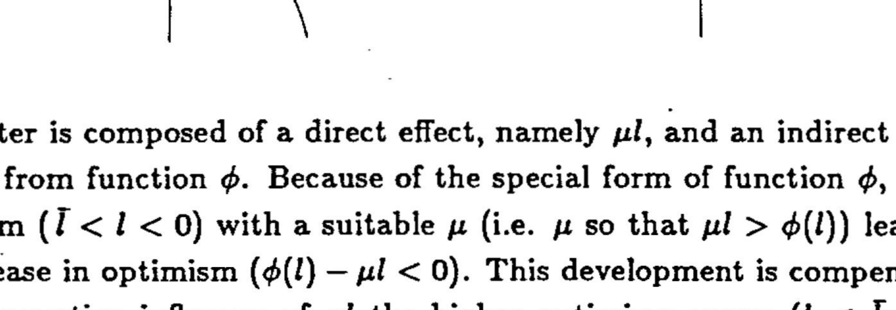
with $\mu > 0,\ \phi(0) = 0,\ \phi'(l) > 0,\ \phi''(0) = 0,\ sign\,\phi''(l) = -sign\,l.$

As before, equation (18) is also determined by direct and indirect influences. The direct influences are:

- an exogenously given parameter $\Lambda \in \mathbb{R}$ which reflects a certain basic sentiment on the stock market;
- the degree of scepticism k,
- and the presently prevailing degree of optimism/pessimism.

While the influence of k on the temporal development of l is unequivocal, meaning that an absence of scepticism ($k > 0$) increases optimism over time ($\dot{l} < 0$), the influence of the respective optimism/pessimism situation (l) is not quite that sure.

Figure 2



The latter is composed of a direct effect, namely μl, and an indirect effect coming from function ϕ. Because of the special form of function ϕ, slight optimism ($\bar{l} < l < 0$) with a suitable μ (i.e. μ so that $\mu l > \phi(l)$) leads to an increase in optimism ($\phi(l) - \mu l < 0$). This development is compensated by the negative influence of μl the higher optimism grows ($l < \bar{l} < 0$). Thus, we can see in the right part of figure 2 that in the $[\bar{l}, 0]$ interval the functional values of ϕ are more negative than the values of μl. This leads to an additional increase in optimism in this area.

Moving from $\bar{l}$ in the negative direction the μl value is more negative than the functional value of ϕ. Thus it causes a reduction of optimism in (18). Therefore the parameter μ is something like a caution parameter of the stock market, because it fixes the intensity of the reverse effect. If the market reacts very cautious, that means if μ is very large, the present optimism will negatively affect the further development of the optimistic sentiment. On the other hand, if the market reacts too careless, the present optimistic sentiment will first increase optimism. Then, if optimism reaches a certain level, namely $\bar{l}$, the prevailing optimism will negatively influence further development.

The speed of adjustment of the two psychological parameters k and l is $1/\nu$. This simply expresses that the adjustment of expectations to sentiments happens faster than the mood is swinging.

3 Dynamic Behaviour of the Model

Formally our model of the stock market consists of the following nonlinear system of differential equations:

$$\begin{aligned} \dot{y} &= -\nu(4y^3 + 2ky + l), \\ \dot{k} &= \tfrac{1}{\nu}(\varphi(y) + l), \\ \dot{l} &= \tfrac{1}{\nu}(\Lambda - k - \mu l + \phi(l)). \end{aligned} \tag{19}$$

In the following we want to examine the dynamic behaviour of this model with the help of a qualitative analysis. Since it is extremely difficult to analyse a 3-dimensional system of nonlinear differential equations qualitatively, we first have to make a simplifying assumption in order to get a 2-dimensional system which is easier to handle formally. For this purpose we assume that the price deviation y of a stock has no influence whatsoever on the development of investors' scepticism, that means $\varphi(y) = \overline{\varphi} = 0$. We are then able to observe changes of k and l, essential for the emergence of a speculative bubble, within the following 2-dimensional system of differential equations:

$$\begin{aligned} \dot{k} &= \tfrac{1}{\nu} l, \\ \dot{l} &= \tfrac{1}{\nu}(\Lambda - k - \mu l + \phi(l)). \end{aligned} \tag{20}$$

The equilibrium of system (20) is given by $(\hat{k}, \hat{l}) = (\Lambda, 0)$. Equilibrium for our stock market means that the stock price only changes with its fundamental value. For $\Lambda \geq 0$ current market price and fundamental value are identical, for $\Lambda < 0$ a constant over-, respectively undervaluation can occur. Later on we will see that this depends on the behaviour of the particular groups of investors. As for $\Lambda = 0$ there do not exist any psychological influences, point $(k^R, l^R) = (0, 0)$ - together with the corresponding price deviation $y^R = 0$ - is equal with the rational expectations' equilibrium. So, we can take this point as starting position for the dynamic development of the model in the following.

The Jacobian matrix evaluated at the equilibrium $(\hat{k}, \hat{l})$ is calculated as

$$J = \frac{1}{\nu} \begin{bmatrix} 0 & 1 \\ -1 & \Pi \end{bmatrix}, \text{ with } \Pi := \phi'(0) - \mu\,. \tag{21}$$

Stability analysis yields that for negative (positive) Π values the equilibrium $(\hat{k}, \hat{l})$ is stable (unstable). The eigenvalues of J are calculated as $\lambda_{1/2} = \frac{1}{2\nu}(\Pi \pm \sqrt{\Pi^2 - 4})$. From this we can see that for $|\Pi| < 2$ the trajectories cyclically converge towards or move away from the equilibrium, depending on the algebraic sign of Π. For $\Pi = \Pi_c = 0$ the eigenvalues become purely complex. An application of Hopf's bifurcation theorem clearly shows the existence of a limit cycle in the environment of Π_c.[7] Further informations concerning this cycle can be gained by using the method of

[7] For the real parts of the eigenvalues denoted as $Re(\lambda)$ the following condition holds: $\partial Re(\lambda)/\partial\Pi|_{\Pi=\Pi_c} = 1/2\nu \neq 0$. For a detailed description of the Hopf theorem see e.g. Lorenz (1989) or Guckenheimer/Holmes (1983).

averaging which is an approximation technique to analyse limit cycles in the plane qualitatively.[8]

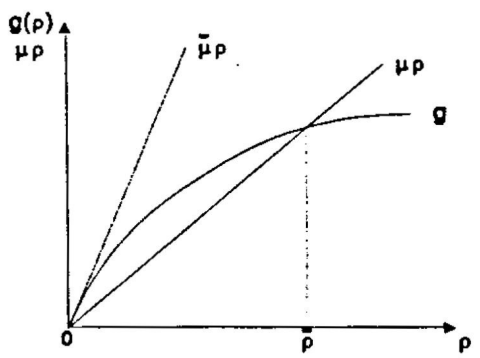
The approximated equation for system (20) is given by

$$\dot{\rho} = \frac{1}{2\nu}(g(\rho) - \mu\rho), \quad \text{with} \quad g(\rho) = \frac{1}{\pi}\int_0^{2\pi} \phi(\rho\sin\theta)\sin\theta\, d\theta. \tag{22}$$

Here ρ approximates the amplitude of the cycle and $\rho\sin\theta$ denotes the variable l in polar coordinates.

The dynamics of this approximated equation is shown in figure 3. Accordingly, system (22) is in equilibrium, if for $\rho \geq 0$ the condition $\mu\rho = g(\rho)$ holds. This condition is graphically represented in figure 3.

Figure 3



The special shape of function $g(\rho)$, which is very important for our analysis, is discussed in the following:

It can easily be seen that $g(0) = 0$ and $g'(\rho) > 0$ holds. Differentiating g once again leads to

$$g''(\rho) = \frac{1}{\pi}\int_0^{2\pi} \phi''(\rho\sin\theta)\sin^3\theta\, d\theta. \tag{23}$$

[8] A good description of this method is given in Chiarella (1990).

The expression above is negative, because g'' is negative over the subintervalls

$$\left[0, \frac{\pi}{2}\right], \left[\frac{\pi}{2}, \pi\right], \left[\pi, \frac{3\pi}{2}\right], \left[\frac{3\pi}{2}, 2\pi\right].$$

Therefore the function $g(\rho)$ has the shape which is drawn in figure 3.

For $\mu > \bar{\mu}$ the differential equation (22) has only one equilibrium point at $\rho = 0$. This equilibrium point corresponds with the stable equilibrium $(\hat{k}, \hat{l})$ from system (20). If, however, the caution parameter μ is decreasing in such a way that $\mu < \bar{\mu}$, equation (22) possesses another equilibrium point at $\bar{\rho}$.

In addition, the first equilibrium point becomes unstable because the variable $\dot{\rho}$ has now a positive value for $0 < \rho < \bar{\rho}$. Consequently, also the equilibrium $(\hat{k}, \hat{l})$ of system (20) becomes unstable. Furthermore, with this method of looking at the approximation $\bar{\rho}$ of the cycle, we can find out whether there exists a limit cycle in the model for $\Pi > 0$ in the neighbourhood of Π_c. The proof for existence, uniqueness and stability of this cycle is given by the following three statements which are fulfilled in this case:

Existence: $\dot{\rho} = 0 \longleftrightarrow \rho = \bar{\rho} \neq 0$,

Uniqueness: $\exists$ only one $\bar{\rho}$ with $\dot{\rho}\,|_{\bar{\rho}} = 0$,

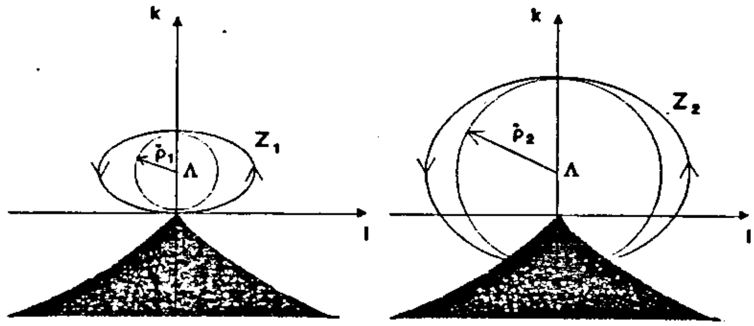
Stability: $\dot{\rho} < (>)0$, if $\rho > (<)\bar{\rho}$.

With the help of (22) we can also carry out a comparative dynamic analysis. Especially the influence of a change in the caution parameter on the amplitude of the limit cycle is of importance in this context. Because of

$$\frac{\partial \bar{\rho}}{\partial \mu} = \frac{\bar{\rho}}{g'(\bar{\rho}) - \mu} < 0, \tag{24}$$

the amplitude of the cycle is decreasing and, consequently, the magnitude of the price deviation must go down the more cautious the market reacts. With a further increase of μ this development will continue until the equilibrium which we discussed before is reached. Figure 4 exemplarily shows the connection between $\bar{\rho}$ and two possible cycles Z_1 and Z_2, for $\mu_2 < \mu_1 < \bar{\mu}$ and a positive Λ.

Figure 4



In the left part of figure 4 a relatively high caution parameter μ_1 corresponds with a relatively low amplitude. In the right part a rather large amplitude exists, because of a lower caution parameter μ_2, so that the cycle runs through the hatched area of the parameter plane. Here, we have the possibility for a discontinuous price correction as mentioned in section 2. As we will explain below, the extent of such a sudden change depends on the strength of the different groups of investors.

Up to now the analysis was purely technical. So let us bring in some economic reasoning in the following.

We first assume that the stock market shall be in the rational expectations equilibrium $(y^R, k^R, l^R) = (0,0,0)$ until period τ. This equilibrium shall then been disturbed by exogenous influences such as good or bad news from the political scene. This disturbance can cause either an optimistic $(\Lambda < 0)$ or a pessimistic $(\Lambda > 0)$ basic sentiment in the market. To present our following considerations more clearly we assume a pessimistic tendency. Then we have to distinguish two cases:

(i) The market reacts very cautiously $(\Pi < 0)$:

The stock price either cyclically or directly returns to its fundamental value after an initial undervaluation. In the new equilibrium we get $(y^{R'}, k^{R'}, l^{R'}) = (0, \Lambda, 0)$.

(ii) The market reacts carelessly ($\Pi \geq 0$):

The stock price either cyclically or directly leaves its fundamental value and approaches a limit cycle. Then again we have to distinguish between two cases, depending on the size of the caution parameter μ as shown in the two pictures of figure 4. Nevertheless, here we want to concentrate on the more interesting case only.

With a relatively low μ, that means if the market reacts extremely careless, the limit cycle, coming from the left handside, runs through the hatched area as can be seen in the right part of figure 4. From section 2 we know that the density function has two modes in this case. So the investors regard two price movements as very likely. Hofstätter (1990), for instance, empirically confirmed that such a behaviour in fact will occur in reality. He observed in the situation of a waning boom a growing contradiction concerning those basic factors which form expectations on the stock market, e.g. the ones published in special media.

Seen strictly mathematically, this insecure behaviour does not exist of course. The dynamics of the system stays at the initial mode until the mode completely vanishes when crossing the right border. Not before then the dynamics carries out a sudden and abrupt transition towards the second mode. This mathematical behaviour, however, should not be simply transferred into the economic sphere. Since investors locally maximize their density function when forming expectations, they can always change their expectations within the hatched area and thereby switch to the other mode.

In order not to overcomplicate the model we divide the investors into different groups. We assume that investors can be distinguished by their typical reactions in the "critical" area mentioned above. In accordance with recent psychological studies, we mentioned before, which ascertain to a large extent a similar investors' behaviour at the aggregate level, we concentrate on three different patterns of behaviour in this area:

(i) normal behaviour,

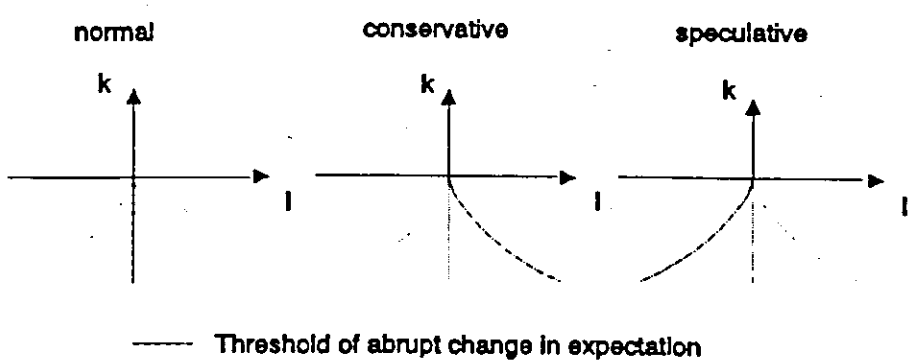
(ii) conservative behaviour,

(iii) speculative behaviour.

With normal behaviour an investor's expectation changes suddenly whenever the new mode reaches a higher density. This is the case on the negative intercept of the k-axis.

Conservative behaviour is characterized by the fact that investors will insist on their kind of expectations once taken in the past. Such an expectation will continue to stay until the initial mode has completely vanished, that means it will finally switch when the right border of the hatched area has been crossed. This behavioural assumption is thus identical with the mathematical behaviour of the system.

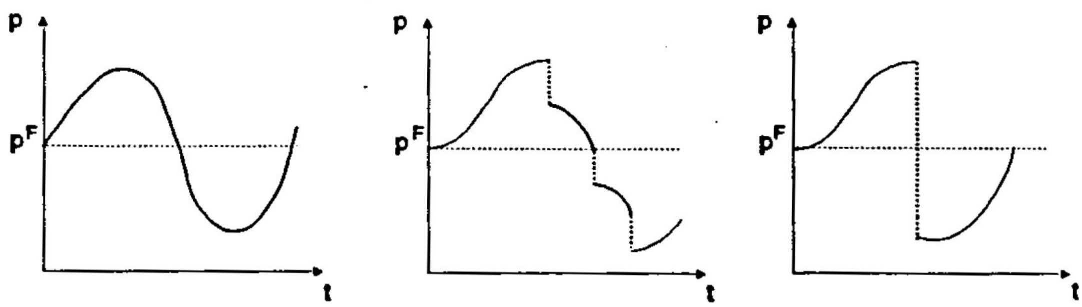
Figure 5



Speculative behaviour can either be pessimistic ($\dot{l} > 0$) or optimistic ($\dot{l} < 0$), depending on the direction of the transition process. Stock markets analysts call that a bear speculation or a bull speculation respectively. The change in expectation happens abruptly, for the pessimistic speculative behaviour at the left border of the hatched area and for the optimistic speculative behaviour at the right border-line. Figure 5 tries to illustrate the thresholds of the particular abrupt expectation changes for the three behavioural patterns. We can see how the different classes of investors react in the "critical" area of the k, l plane. Because of the ambiguous mode, as described above, abrupt jumps in prices occur, the magnitude of which fluctuates with the size of the different groups of investors.

Figure 6 gives an example of three possible price movements. The price movement on the left represents the cycle Z_1 of figure 4. Here the price performance shows a quite smooth and "orderly" pattern, characterized by continuous and constant oscillations around the fundamental value p_t^F. Because of the always unambiguous mode abrupt changes in expectations and the resulting discontinuous stock price jumps do not appear in this case.

Figure 6



The picture in the middle assumes that all three groups of investors are approximately of the same size. In analogy to the thresholds of abrupt expectation changes, as shown in figure 5, discontinuous price changes do now occur, their magnitude varying with the group size.

In the picture on the right investors consist exclusively of speculators. This leads to a single, but very large price jump of the stock. Thus, in our model the dominance of a particular group of investors can lead to a more or less strong swing of the stock price.

We can summarize the price performance of a stock in our model as follows:

In the model changes in investors' sentiments on a artificial stock market are covered by a system of differential equations. Out of these systems a speculative bubble emerges whenever the stable point equilibrium changes into a stable limit cycle as a result of changes in investors' behaviour. The self-dynamics of the cycle is generated by the nonlinear interplay of psychological factors. Its location and amplitude can be influenced by an optimistic or pessimistic basic sentiment and by a cautious or careless behaviour of investors. The latter causes an extension of the cycle's amplitude and leads to abrupt changes in the existing sentiments and consequently to sudden and discontinuous price jumps. Thus, the bursting of a speculative bubble turns out to be an endogenous process. In addition, it seems important to note that the stock price movement occurs while the fundamental value of the stock remains unchanged.

Next we want to examine whether the dynamic behaviour of our simplified system (20), just described, also applies to the initial system (19). So, we now turn back to the initial system. The equilibrium $(y^{R'}, k^{R'}, l^{R'}) = (0, \Lambda, 0)$ of the simplified system is also the equilibrium solution of the initial system. The Jacobian matrix in this point is

$$J = \begin{bmatrix} -2\Lambda k\nu & 0 & -\nu \\ 0 & 0 & \frac{1}{\nu} \\ 0 & -\frac{1}{\nu} & \frac{1}{\nu}\Pi \end{bmatrix}. \tag{25}$$

The appropriate eigenvalues are

$$(\lambda_1, \lambda_2, \lambda_3) = \frac{1}{2\nu}(-4\Lambda\nu^2,\ \Pi + \sqrt{\Pi^2 - 4},\ \Pi - \sqrt{\Pi^2 - 4}). \tag{26}$$

For $\Pi < 0 (> 0)$ the equilibrium is locally stable (unstable) for a positive Λ. In addition it is stable in a saddlepoint sense for a negative Λ. In analogy to the simplified system, for $\Pi = \Pi_c$ the matrix (25) also possesses purely complex eigenvalues. Application of Hopf's bifurcation theorem again indicates the existence of a limit cycle in this situation. This leads to the assumption that the system in (19), at least in the neighbourhood of the local equilibrium point, shows a similar dynamic behaviour as the 2-dimensional system we extensively discussed before. Since appropriate mathematical tools for a far deeper going qualitative analysis of the initial complex 3-dimensional system do not exist, we have to carry out a model simulation in the next section. With the help of this simulation we will examine whether the most important qualitative results of the simplified system are still valid within the much more complex framework of our numerical specification.

4 Some Simulation Runs

So far we examined the psycho-dynamics of our artificial stock market with qualitative means. There, we found out that, with a large probability, the clear results extracted from the analysis of the simplified system (20) are also valid for the initial system (19). However, in order to bring some more light into this subject, we will carry out a simulation for a numerically specified version of the model.

For this purpose we assume the following simulation equations in our model:

$$\begin{aligned} \dot{y} &= -5(4y^3 + 2ky + l), \\ \dot{k} &= 0.2(-0.016y^3 + l), \\ \dot{l} &= 0.2(\Lambda - k + h(l)). \end{aligned} \tag{27}$$

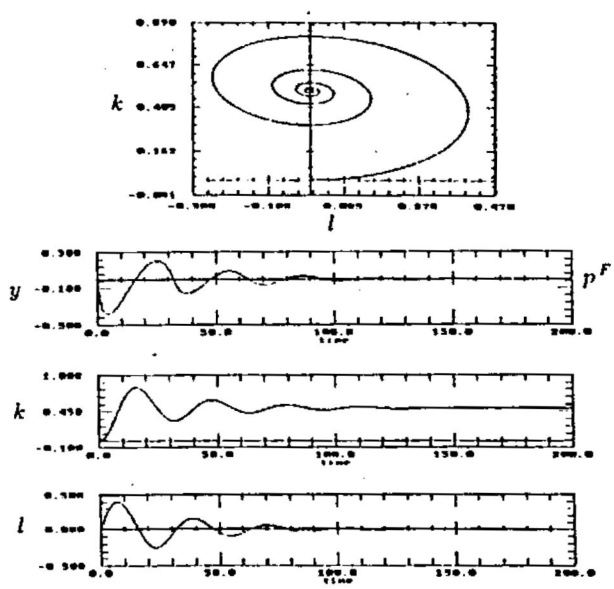
In this case the function $h(l) := -\mu' l - (0.005 l^3 - 5l)$ is topological equivalent to the function $H(l) := -\mu l + \phi(l)$. The equilibrium is at $(0, \Lambda, 0)$ and the corresponding eigenvalues are

$$(\lambda_1, \lambda_2, \lambda_3) = (-10\Lambda,\ 0.1\Pi' + 0.1\sqrt{\Pi'^2 - 4},\ 0.1\Pi' - 0.1\sqrt{\Pi'^2 - 4}), \quad (28)$$

with $\Pi' := 5 - \mu'$. Starting point for our simulation is the equilibrium which results in the case of rational expectations (y^R, k^R, l^R).

To simplify our procedure we exclude the case of saddlepoint stability and assume in the following that Λ is positive. Thus, the equilibrium for $\mu' > 5$ is a point stable one. In figure 7 we have drawn for $\Lambda = 0.5$ and for $\mu' = 5.3$ the cyclical adjustment to the equilibrium in the k, l plane as well as the temporal development of the three variables.

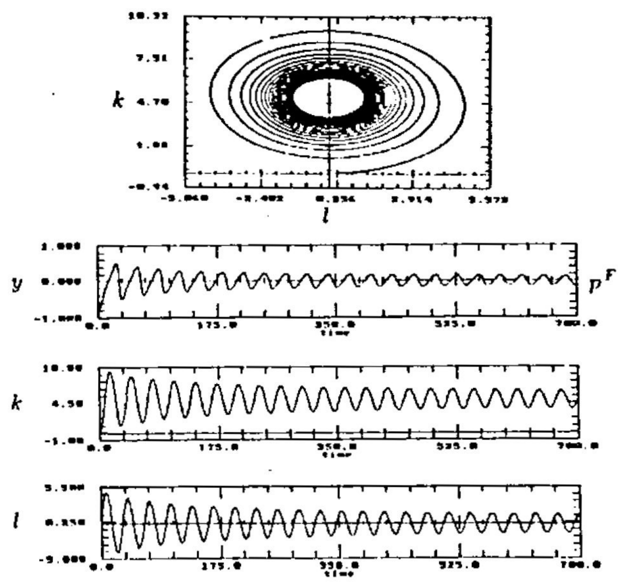
Figure 7



For $\Pi' = 0$ the system (27) undergoes a Hopf bifurcation. This points out the existence of a limit cycle. In figure 8 a cyclical adjustment to the limit cycle is shown which is being reached after approximately 650 time units.

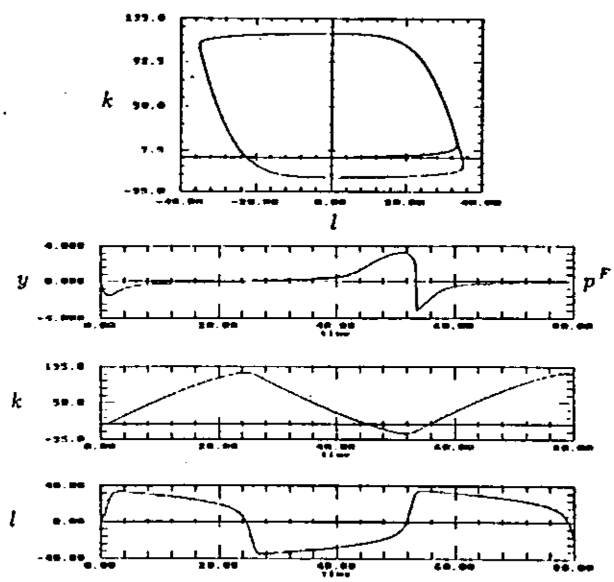
As we can see from figure 8, the cycle runs through the "noncritical" area. The corresponding parameter values are $A = 5$, $\mu' = 4.9998$. Again the adjustment to the cycle can occur cyclical or directly.

Figure 8



We receive even more interesting results, if we again reduce the value of μ'. For example, if we give μ' the value 0.5, the limit cycle runs through the "critical" area of the k, l plane, as it is shown in figure 9. From moment 40 on a speculative bubble emerges which bursts at moment 57. The price deviation there is also much higher than the one in figure 8 respectively. With the help of the above procedure, however, we can only simulate the mathematical development of the system. Our economic interpretation, according to which the behaviour of investors can be classified into three groups, can not be simulated in the same way. Thus, we have to regard the abrupt price adjustment occuring in moment 57 as a result of the stock market situation in which only conservative investors exist. It is worth to mention in this context that the variables k and l do not show jumps of the same kind in their temporal development.

Figure 9



In this section we tried to confirm the results of our qualitative analysis, especially with regard to system (19), by using numerical simulations. To summarize our findings: We can say that cyclical behaviour is not the exceptional case, but rather applies for a large area of all model parameters. Variations of the initial values do also confirm our qualitative results. This has been the case for quite a number of different simulation runs.

5 Summary and Outlook

The aim of our contribution was to show how the working of the stock market can be reproduced in a dynamic model's framework. In his investigations of the behaviour of investors which led to the stock market crash of 1987 Shiller (1990) called the stock market "a feedback system ... with possibly complicated dynamics". In his system mainly psychological factors accounted for the noticed dynamics. By using the QME-approach we succeeded to trespass the traditional approaches built up almost totally on rational behaviour and were able to include certain forms of "non-rational" behaviour of investors. As suggested by Shiller, it were mainly psychological factors that generated the dynamics of our model.

We have shown that an exogenous influence on the stock market may either stimulate the stock price to return to its fundamental value or to oscillate around that value depending on the formulation of the model parameters. Factors like optimistic or pessimistic market sentiments, cautious or careless behaviour of investors and emerging doubts about previous price trends are at the end responsible for these price movements. In addition, our model is able to endogenously create a speculative bubble and let it burst due to its selforganizing structure. Numerical simulations confirm and further clarify the results gained in the qualitative analysis.

In further investigations it would be interesting to endogenously model the process which determines the parameter of cautious or careless behaviour of investors and which is primarily responsible for the dynamics of the model. This would require a microeconomic perspective analysing the quantitative changes in the relations of the different groups of investors in a dynamic context. In such a framework the probability for socalled chaotic behaviour would even still increase.

To empirically check the results of our contribution methodological approaches need to be developed that offer the chance to get better insights into the formation of expectations, an area which is of enormous importance for economics.

Appendix: Derivation of the General Q-Distribution

A probability distribution function which shows due to the choice of the parameter v a different vault is the Exponential Power Distribution

$$\begin{aligned} P^E(X|\mu',\sigma',v') &= (h\sigma')^{-1}\exp\left\{\tfrac{-1}{2}\left|\tfrac{X-\mu'}{\sigma'}\right|^{\frac{2}{1+v}}\right\}, \\ -\infty < X < +\infty,\ h &= 2^{[1+(1+v)/2]}\Gamma\left[1+\tfrac{1+v}{2}\right], \\ \sigma' > 0,\ -\infty < \mu' &< +\infty,\ -1 < v \leq 1. \end{aligned} \qquad (A.1)$$

This density functions are determined by the location parameter μ', the scale parameter σ' and the vault parameter v.[9] The first moments of this distributions can be calculated as

$$E(X) = \mu' \text{ and } E(X-\mu')^2 = \sigma^2 = 2^{(1+v)}\frac{\Gamma\left[\frac{3}{2}(1+v)\right]}{\Gamma\left[\frac{1}{2}(1+v)\right]}\sigma'^2. \qquad (A.2)$$

The function

$$f^E(X|\mu',\sigma',v) = \exp\left\{-\frac{1}{2}\left|\frac{X-\mu'}{\sigma'}\right|^{\frac{2}{1+v}}\right\} \qquad (A.3)$$

is topological equivalent[10] to P^E, because the shape of distribution functions with an exponential part is only determined by that part of the exponent in which the random variable X appears. For $v = 0$ (A.1) turns into the density function of the normal distribution which is topological equivalent to

$$f^N(X|\mu',\sigma) = \exp\left\{-\frac{1}{2}\left|\frac{X-\mu'}{\sigma}\right|^2\right\}. \qquad (A.4)$$

[9] A parameter μ' is denoted as a location parameter if a linear transformation will change the observed values X to $z = rX + a$ and the parameter μ' to $r\mu' + a$. A parameter σ' is called scale parameter, if the multiplication of the observed values like $z = bX$ will transforme the parameter σ' in $|b|\sigma'$.

[10] The topological eqivalence describes the qualitative similarity in the shape of functions.

Another variation to $v = -0.5$ changes f^E into

$$f^E(X|\mu',\sigma') = \exp\left\{-\frac{1}{2}\left|\frac{1}{\sigma'^4}(X^4 - 4\mu'X^3 + 6\mu'^2X^2 - 4\mu'^3X + \mu'^4)\right|\right\}. \qquad (A.5)$$

This function is topological equivalent to a very special case of the general Q-distribution which for its part is again topological equivalent to

$$f_t^Q(X|\alpha,\beta,\gamma,\delta) = \exp\left\{-\alpha_t(X^4 + \beta_tX^3 + \gamma_tX^2 + \delta_tX)\right\}, \qquad (A.6)$$

with $\alpha_t > 0$ for all t. Due to the values of $\alpha,\beta,\gamma,\delta$ - which are modelled in (A.6) in a time dependent way - symmetry, skewness, location and scale parameters of the Q-distribution are changing.

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