INSTITUT FÜR VOLKSWIRTSCHAFTSLEHRE

der

UNIVERSITÄT AUGSBURG

Schumpeter´s Circular Flow, Learning by Doing and Cyclical Growth

von

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Beitrag Nr. 104 September 1993

olkswirtschaftliche Diskussionsreihe

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Abstract

Our goal with this paper is to show that learning may be sufficient to break up the circular flow as described by Schumpeter. Starting point of our analysis is a two-sector competitive economy without innovations which serves as representation of the circular flow. A brief analysis of that model shows that the dynamic behaviour of the variables is always a monotonic one. In a next step we extend this model by assuming that agents can learn. As to the learning mechanism we suppose a learning by doing framework. Analyzing this model shows that we may now observe oscillations of the economic variables. So there may be transitory oscillations with declining amplitude. On the other hand, application of the Hopf Bifurcation theorem demonstrates that even persistent oscillations may be the outcome.

*Paper presented at the Euro-Conference *Evolutionary and Neoclassical Perspectives on Market Structure and Economic Growth*.

1 Introduction

According to Goodwin (1965), one of Schumpeter's most important contributions to economic theory consists in assuming that economic growth and progress occur in spurts and thus fusing together business cycle theory and growth theory. The cyclicity of economic data for its part, according to Schumpeter, is due to the occurence of revolutionary advances in technique. An important aspect of that theory lies in the fact that the latter are caused endogenously, that is from within the economic system. So if the system is ever to reach a point of equilibrium it is an innovation brought about by an entrepreneur which causes its destruction and makes the economy evolve. Schumpeter uses the notion of "creative destruction" in order to describe this phenomenon (cf. Schumpeter (1947), p. 83).

And it is exactly that aspect which makes an evolutionary economy differ from a static economy in which no innovational changes occur and, as a consequence, production and consumption are completely in accordance. This last situation is also refered to as the circular flow by Schumpeter.

Although these ideas have gained increasing support in the last few decades, little effort seems to be undertaken to integrate Schumpeterian elements into growth theory which, for its part, has also received growing attention with the publications by Romer (1986, 1990) and Lucas (1988). These papers indeed presented new ideas giving rise to the so-called new growth theory in which persistent per-capita growth may be sustained. It is that property which makes the new growth theory differ from the conventional (neoclassical) growth theory where a zero per-capita growth rate represents the final outcome. The exogenously given growth rate of the labour force then plays the decisive role giving the balanced growth path. The papers by Romer and Lucas triggered off a good deal of research, all concerned with endogenously determined growth rates (see for example Barro (1990), Rebelo (1991), Schmitz (1992), Shaw (1992) or for a survey Sala-i-Martin (1990)).

All what those models have in common, however, is that they only consider monotonic growth rates. The important aspect of cyclical growth in the Schumpeterian sense is not taken account of, neither in the traditional growth theory nor in the new growth theory[1].

Our goal with this paper now is to analyze the circular flow in the sense of Schumpeter and to work out the effects if human capital is allowed for in that model. As to the formation of human capital, however, we do not suppose that firms intentionally invest in the creation of it but that it is formed as a by-product of gross investment, i.e. we adopt a learning by doing approach (see Arrow (1962)). We can demonstrate that then the assumption of a steady state growth rate for a conventional two-sector neoclassical growth model does not necessarily hold any longer. Thus we are able to show that learning mechanisms may be sufficient to break up the circular flow.

The rest of the paper is organized as follows. In section 2 we present the model of a competitive two-sector economy and briefly recapitulate its results. This model of the circular flow is extended by assuming that the economic agents learn. The effects of that assumption are then studied in section 3. In section 4, we present a simulation run in order to demonstrate our analytical results and section 5 finally concludes the paper.

2 The Circular Flow

As mentioned above, the circular flow, according to Schumpeter, can be characterized as a situation in which no innovations occur and demand and supply correspond. Such a situation is formally best described by a competitive economy. We will suppose a two-sector economy with a con-

[1]There are growth models which emphasize the posibility of cyclical growth. But these only consider conventional input factors and neglect intagible ones, such as human capital e.g. (see Benhabib and Nishimura (1979) or Nishimura and Takashi (1992)).

sumption good sector and an investment good sector. The two industries which can be imagined as being composed of a large number of identical firms, are distinguished because of different technologies available and, of course, because of the different outputs, namely a consumption good and an investment good. The production functions are given by $C(t) = g^0(L_0, K_0)$ and $I(t) = g^1(L_1, K_1)$. L_0 and L_1 denote labour effort to produce one unit of the consumption good and one unit of the investment good, respectively, with $L_0 + L_1 = L \equiv 1$, i.e. L which is normalized to one is the total amount of labour available to the economy. K_0 and K_1 represent the amounts of capital devoted to the production of consumption and investment, respectively, and $K_0 + K_1 = K$ denotes the total amount of capital in our economy. The functions $g^0(\cdot)$ and $g^1(\cdot)$ are assumed to be C^2 and concave jointly in L_j, K_j, $j = 0, 1$. Moreover, they are strictly concave in each separate factor and linear homogenous in L_j, K_j, $j = 0, 1$. Furthermore, they are increasing in both factors.

Our firms take the price sequence $\{w(t), i(t), q(t)\}$ as given, where $w(t)$ denotes the labour wage rate, $i(t)$ is the gross capital rental and $q(t)$ represents the price of capital (of the investment good) in terms of the consumption good which serves as numeraire. The decision problem of both the firm producing the consumption good and the firm producing the investment good then consists in choosing the optimal input levels of capital K_j and labour L_j, $j = 0, 1$, in order to maximize profits.

Therefore, the consumption good sector solves

$$\max C(t) - i(t)K_0(t) - w(t)L_0(t), \ s.t.\ C(t) = g^0(L_0(t), K_0(t)),$$

for all t and the capital good sector solves

$$\max q(t)I(t) - i(t)K_1(t) - w(t)L_1(t), \ s.t.\ I(t) = g^1(L_1(t), K_1(t)),$$

for all t.

As to the household sector we suppose one single representative agent who lives forever and who faces a budget constraint of the form, $C(t) +$

$q(t)(\dot{K}(t) - \delta K(t)) = i(t)K(t) + w(t)$, where δ denotes the depreciation rate of capital. This budget constraint implies that our individual is endowed with one unit of labour he supplies inelastically to the productive sectors. The goal of the individual consists in maximizing the discounted stream of his lifetime utility arising from consumption, such that his optimization problem reads

$$\max \int_0^\infty e^{-rt} u(C(t))dt,$$

subject to his budget constrained mentioned above. r denotes the discount rate and the utility function $u(\cdot)$ is, as usual, assumed to be an increasing, concave function from $[0,\infty)$ into $[0,\infty)$ and C^2 on $[0,\infty)$.

The circular flow can now be characterized by an intertemporal competitive equilibrium which may be defined as a situation in which, for a given price sequence, all economic agents can fulfill their desired transactions and none of them is rationed. Here, we should mention that this model exactly represents the continuous-time version of a discrete-time model presented in a paper by Boldrin (1989). Therefore, we do not go too much into the details of this model but only refer to the Boldrin paper for a more thorough treatment and an exacter definition of the intertemporal equilibrium. Nor don't we investigate the conditions under which there exists a unique equilibrium since this question can also be solved by standard arguments.

It turns out that the circular flow, i.e. the intertemporal competitive equilibrium is equivalent to the solution of the intertemporal optimization problem

$$\max \int_0^\infty e^{-rt} u(C(t))dt,$$

subject to $C(t) = T(K(t), I(t))$, $\dot{K}(t) = I(t) - \delta K(t)$, $K(0) = K_0 > 0$. $T(K(t), I(t))$ denotes the production possibility frontier (PPF) and is obtained as $T(K(t), I(t)) = \max C(t) = g^0(L_0(t), K_0(t))$ subject to $I(t) = g^1(L_1(t), K_1(t))$, $K(t) = K_0(t) + K_1(t)$, $1 = L_0(t) + L_1(t)$, $L_j(t), K_j(t) \geq 0$, $j = 0, 1$.

From the economic point of view the equivalence between the competitive economy and the intertemporal optimization problem follows from the first and second welfare theorem stating that, loosely speaking, a competitive economy yields a Pareto optimal solution and vice versa. A formal mathematical proof can be obtained by adopting the arguments in the paper by Becker (1981). Concerning the properties of the production possibility frontier, $T(K, I)$, it turns out to be concave and, under some weak technical conditions, twice continuously differentiable. Moreover, it is increasing in K and decreasing in I. For our subsequent analysis, we will assume that it is in addition strictly concave in I. For a survey of these results we again refer to Boldrin (1989). A more thorough treatment can be found in Kuga (1971), Hirota and Kuga (1972) and in Benhabib and Nishimura (1979). With these assumptions, we can investigate the solution of our dynamic optimization problem, which may also be given the interpretation of a single representative individual which has to decide what amount to consume and what amount to invest in order to increase the production possibilities in the future. To find out the optimum, we formulate the current-value Hamiltonian as $H(\cdot) = \gamma_0 T(K, I) + \gamma_1 (I - \delta K)$, where, for simplicity, we assumed the utility function to be linear. This, however, does not change any of our results derived below. Maximizing with respect to I gives $T_I(\cdot) + \gamma_1 = 0$, for $\gamma_0 = 1$,[2] implicitely defining investment as a function of K and γ_1, with $I_K = -\frac{T_{IK}}{T_{II}}$ and $I_{\gamma_1} = -\frac{1}{T_{II}} > 0$. Inserting this relationship in the other necessary conditions giving the evolution of the capital stock and its shadow price γ_1, which are also sufficient for this model, provides us with the modified Hamiltonian system, namely $\dot{K}(t) = I(K(t), \gamma_1(t)) - \delta K(t)$ and $\dot{\gamma}_1(t) = (r + \delta)\gamma_1(t) - T_K(K(t), I(K(t), \gamma_1(t))$. The transversality conditions which are necessary and sufficient in this case are given by $\lim_{t \to \infty} e^{-rt}\gamma_1(t) = 0$. Analyzing this system, it can easily be

[2]The fact that this problem is a normal one can easily be seen for interior solutions which for its part may be justified by imposing Inada-type conditions on $T_I(\cdot)$.

checked that it is stable in the saddle point sense with the stable manifold representing the optimal solution if $T_{IK} \leq 0$.[3] If, however, $T_{IK} > 0$ the canonical system of differential equations may also be unstable since then the determinant of the Jacobian may become positive. We will not investigate how the optimal solution will look like in that case but only refer to Feichtinger and Hartl (1986), chapter 4.5.2., where it is shown in general how the optimal solution looks like in such a case. For the stability of two-sector growth models see also Uzawa (1964) and Inada (1964) or Intriligator (1971).

A second and from the economic point of view more interesting aspect is that the dynamic behaviour of the economic variables is a monotonic one. This results from the fact that only one capital good is produced (see Hartl (1987)). In fact, Benhabib and Nishimura (1979) have shown that persistent oscillations may occur if more than one capital good is produced. From the economic point of view it may thus be argued that in extremely simple economies with only one sort of capital good, Schumpeter's idea of the circular flow with constant growth rates can be represented. If more than one capital good is produced, a monotonic growth rate does indeed not have to hold true any longer. We with this paper, however, intend to demonstrate that even with one capital good a monotonic growth rate does not necessarily represent the outcome if learning by doing is taken into account. Then, we may observe a permanent cyclical behaviour of the variables or transitory oscillations.

[3]The existence of a unique steady state can easily be checked given the strict concavity of $T(\cdot)$ in I and assuming Inada-type conditions for $T_K(\cdot)$.

3 The Effects of Learning

As mentioned in the last section we will now investigate the effects of learning on the circular flow. As to the learning mechanism, we assume that the stock of human capital which reflects acquired knowledge is built according to Arrow's learning by doing. That approach asserts that human capital is formed as a by-product of gross investment and therefore is determined by accumulated past gross investment. We will slightly depart from this formulation and suppose that investment at different dates is weighted differently. The reason for that lies in the economically reasonable assumption that investment further back in time contributes less to the stock of human capital than more recent investment. Therefore we define the stock of human capital, denoted as $A(t)$, as a function of an integral of past gross investment with exponentially declining weights put on investment flows further back in time, i.e. $A(t) = F(\rho \int_{-\infty}^{t} e^{\rho(s-t)} I(s)ds)$, with $F'(\cdot) > 0$. ρ gives the weight attributed to more recent gross investment. The larger ρ, the higher is the contribution of more recent investment compared to flows of investment dating back further in time. In what follows we will assume that $F(\cdot)$ is a linear function such that we may write $A(t) = \rho \int_{-\infty}^{t} e^{\rho(s-t)} I(s)ds$. Note that from the economic point of view this does not restrict the economic content and our results derived below will be independent of that assumption.

The effect of the stock of human capital consists in acting as an efficiency index positively influencing the amount of labour employed in the production of the investment good as well as in the production of the consumption good. The production functions then are given by $g^j(AL_j, K_j)$, $j = 0, 1$. It should be noted that the positive external effect of investment on the creation of human capital which may be termed a spillover effect is not taken into acount in the decision problem of our investment good sector. That is, the stock of human capital is formed as a by-product of investment and positively affects both production processes, leading to increasing returns (recall that the production functions are linear homogenous in both capital and labour). The reason why firms of the investment good sector do not take account of these positive spillovers simply lies in the fact that they do not get any remuneration for it, i.e. the investment good is still only paid its competitive price $q(t)$. For the social optimum, this deficiency could be remedied by paying a subsidy for any unit of investment good produced which could be financed by imposing a tax on the consumption good. But that question is beyond the scope of our paper here, instead we will investigate the dynamic behaviour of the variables of our competitive economy. Despite these positive externalities the existence of an intertemporal competitive economy can nevertheless be maintained since the positive spillovers bringing about those scale effects are external to the single optimizing firm.

As to the intertemporal optimization problem, we now have to take account of the stock of human capital and the production possibility frontier is thus given by $T(A(t), K(t), I(t))$. The optimization problem, which can again be considered as the problem of a representative individual which has to decide what amount to invest and what amount amount to consume, then is given by

$$\max \int_0^\infty e^{-rt} u(C(t))dt,$$

subject to $C(t) = T(A(t), K(t), I(t))$, $\dot{K}(t) = I(t) - \delta K(t)$, $K(0) = K_0 > 0$.

The PPF is still concave in K and I for a given value of A, but not necessarily when varying all three arguments at the same time. Nevertheless the existence of a solution to our maximization problem is guaranteed if upper bounds for the marginal products of K and A are assumed. This follows from a standard result in control theory (cf. Seierstad and Sydsaeter (1987), p. 237, the proof in detail is available on request).

At that point it should also be noted that the stock of human capital which positively influences the production possibilities but what is not intentionally taken account of, does not stay constant over time. From our definition of $A(t)$ it becomes clear that human capital evolves over time as long as positive gross investments are undertaken contributing to its formation. On the other hand, this stock is also subject to depreciation as time goes by, since investment further back in time becomes less and less important for the workers' actual skill. Mathematically, this can be seen by differentiating $A(t)$ with respect to time yielding, $\dot{A}(t) = \rho(I(t) - A(t))$.[4]

In order to derive the equations of motion giving the dynamic behaviour of our economic variables we first formulate the current-value Hamiltonian which is equivalent to the one in the situation without learning. The reason for that lies in the fact that our individual does not take account of the positive spillovers of investment and treats the evolution of the stock of human capital as exogenously given. For a linear utility function, the Hamiltonian is thus again $H(\cdot) = \gamma_0 T(A,K,I) + \gamma_1(I - \delta K)$. Maximizing with respect to investment gives this control variable again as an implicitely defined function which now also depends on the stock of human capital. The derivatives are seen to be $I_K = -\frac{T_{IK}}{T_{II}}$, $I_{\gamma_1} = -\frac{1}{T_{II}} > 0$ and $I_A = -\frac{T_{IA}}{T_{II}}$. The other necessary conditions are again given by the differential equations giving the evolution of the capital stock and the shadow price. But now we also have to take account of the evolution of the stock of human capital which evolves over time as a by-product of investment and influences the evolution of the capital stock and its shadow price, although this effect is not explicitely taken into account. Our system of differential equations can therefore be written as

$$\dot{K}(t) = I(A(t), K(t), \gamma_1(t)) - \delta K(t), \tag{1}$$

$$\dot{\gamma}_1(t) = (r + \delta)\gamma_1(t) - T_K(\cdot), \tag{2}$$

$$\dot{A}(t) = \rho I(A(t), K(t), \gamma_1(t)) - \rho A(t). \tag{3}$$

The transversality conditions which are again necessary (demonstrated in the appendix available on request) are $\lim_{t\to\infty} e^{-rt}\gamma_1(t) = 0$.

To investigate the dynamic (local) behaviour of this system let us first

[4]For this derivative see Seierstad (1981), p. 175 or Ryder and Heal (1973).

state that there is a unique steady state. This can easiliy be seen once it is recognized that there is a unique rate of investment which follows from the strict concavity of the Hamiltonian in $I(t)$. This result then gives a unique capital stock and a unique stock of human capital. The uniqueness of the shadow price can be assured if Inada-type conditions are imposed on $T_K(\cdot)$.

In order to determine the dynamic behaviour of our system (1)-(3) we proceed as ususal, calculate the Jacobian and then determine the eigenvalues of that matrix. The Jacobian can easily be seen to have the following form,

$$J = \begin{bmatrix} -\frac{T_{IK}}{T_{II}} - \delta & -1/T_{II} & -\frac{T_{IA}}{T_{II}} \\ -T_{KK} + \frac{T_{IK}^2}{T_{II}} & \frac{T_{IK}}{T_{II}} + r + \delta & -T_{KA} + \frac{T_{IK}T_{IA}}{T_{II}} \\ -\rho\frac{T_{IK}}{T_{II}} & -\rho/T_{II} & -\rho - \rho\frac{T_{IA}}{T_{II}} \end{bmatrix}.$$

The characteristic equation then is obtained as

$$\lambda^3 + (-\text{trace}\, J)\lambda^2 + K_2\lambda + (-\det J) = 0, \tag{4}$$

with K_2 being defined as

$$K_2 = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix},$$

with a_{ij} element of the i-th row and j-th column of J.

The dynamic behaviour of our economic variables may be quite different depending on the eigenvalues of our characteristic equation. In particular we are interested in cyclical solutions of the variables which may result when our system undergoes a Hopf bifurcation[5]. The technical prerequisite for that occurence lies in two eigenvalues crossing the imaginary axis. In order to work out the particular conditions for our model to give rise to

[5]For a description of that phenomenon see Guckenheimer and Holmes (1983), chapter 3, especially p. 151/152.

oscillating solutions we use a lemma which can be found in the paper by Asada and Semmler (1992). According to that lemma, two pure imaginary roots for the characteristic equation (4) occur if and only if $K_2 > 0$ and $K_2 \cdot (-\text{trace}\, J) + \det J = 0$ holds. Applying this result to our problem we have to find out that the second condition becomes extremely complicated and cannot be interpreted economically. Therefore we will only focus on the first one thus gaining solely necessary conditions for cyclical solutions. This shortcoming, however, is partly offset in our next section where we will present a numerical example in order to illustrate our results.

Calculating the constant K_2, we see that it can be written as $K_2 = -a+A_1+c+d$, with $a = \delta(r+\delta) > 0$, $A_1 = -(1/T_{II})[(r+2\delta)T_{IK}+T_{KK}]$, $c = -(1/T_{II})r\rho(T_{IA}+T_{II})$, $d = -\rho T_{KA}/T_{II}$. As to the elements of K_2 we see that all may have positive or negative signs except a which is always positive.

Concerning the economic mechanisms we see that A_1 can serve as a measure for complementarity over time.

If $A_1 > 0$ we can speak of adjacent complementarity with respect to the capital stock K. That means increasing investment at time t_3 implies a reallocation of resources from distant dates t_1 to nearby dates t_2. Correspondingly, if $A_1 < 0$ we speak of distant complementarity with respect to the capital stock, meaning that an increase in investment at time t_3 leads to a reallocation of resources from nearby dates t_2 to distant ones t_1[6].

Given these notions, we immediately see that adjacent complementarity is favourable for the emergence of persistent oscillations. In fact, if the cross derivatives T_{IA}, T_{KA} are non-positive, stable limit cycles are only possible if adjacent complementarity prevails.

If, however, positive cross derivatives can be observed, then persistent oscillations may occur even for distant complementarity with respect to the capital stock. The economic reason for that result seems to be clear:

[6]For a derivation of these concepts we refer to Dockner and Feichtinger (1991). See also Wan (1970) and Ryder and Heal (1973).

cyclical time paths require a sort of acceleration effect between the two stocks. If the cross derivative is non-positive, however, it can be argued that an additional unit of human capital has a satiating effect on capital, implying convergence to the steady state. Moreover, it seems that a positive cross effect represents the more plausible case, since an additional unit of capital yields a higher return if the workers operating the machines dispose of higher skills, i.e. if the stock of human capital is higher.

The same holds if a higher stock of human capital lowers the opportunity cost of investment. In this case, a higher stock of human capital implies that less investment has to be given up in order to produce an additional unit of the consumption good.

To summarize our results, we can state that adjacent complementarity with respect to the capital stock or positive cross effects between either the stock of human capital and physical capital or between human capital and investment are necessary for persistent oscillations of our economic variables[7]. We would also like to point to the fact that assuming a two-sector economy is not crucial for our results. If we had supposed a one-sector economy and a strictly concave utility function $u(c(t))$, it can easily be seen that all of our results remain valid if $T(A, K, I)$ is replaced by $u(F(A, K) - I)$, with $F(A, K)$ denoting the macroeconomic production function. This shows that our result is independent of the assumption of a two-sector economy.

As mentioned above, we will in the next section present a numerical example in order to illustrate our analytical results derived in this section. But before let us briefly summarize what we have done up to now. We saw that in our simple economy characterized by the circular flow without any innovation the variables always display a monotonic behaviour. If, however, we allow for learning in that economy this result changes drastically. Then,

[7]Note that the intertemporal substitution effect also only works if there is a positive cross effect between investment and physical capital.

we may observe that the variables do not necessarily reveal monotonic time paths any longer. Instead, persistent oscillations may be the outcome. Even if convergence still holds, the path to the steady state may show transitory oscillations which can also matter since the solution property of this system of differential equations is an asymptotic one meaning that the steady state is not reached within finite time. Thus we were able to show that only taking into account learning mechanisms may be sufficient to destroy the circular flow. Then cyclically oscillating investment rates may turn out to be optimal and bring about cyclical time paths of human capital and other economic variables, too.

4 A Simulation Run

Let us now present a numerical example to illustrate our results from the last section. For the PPF we assume that it is given by $T(A,K,I) = a_1K + a_2A - aI^2/(A+K) + b_1IK + b_2IA$ for a similar relationship see Kuga (1972), p. 735). The evolution of the capital stock is given by $\dot{K}(t) = I(t) - \delta K(t)$. Forming the current-value Hamiltonian, maximizing with respect to $I(t)$ gives $I(t) = (b_1K(t) + b_2A(t) + \gamma_1(t))(A(t) + K(t))/2a$. Substituting this value in the differential equations giving the dynamic behaviour of the capital stock and the co-state variable (shadow price) $\gamma_1(t)$ as well as in the equation determining the evolution of the stock of human capital, $\dot{A}(t) = \rho(I(t) - A(t))$, then gives our system of differential equations as

$$\begin{aligned}
\dot{K}(t) &= (b_1K(t) + b_2A(t) + \gamma_1(t))(A(t) + K(t))/2a - \delta K(t),\\
\dot{\gamma}_1(t) &= (r+\delta)\gamma_1(t) - \frac{1}{4a}(b_1K(t) + b_2A(t) + \gamma_1(t))^2\\
&- b_1(b_1K(t) + b_2A(t) + \gamma_1(t))(A(t) + K(t))/2a - a_1,\\
\dot{A}(t) &= \rho(b_1K(t) + b_2A(t) + \gamma_1(t))(A(t) + K(t))/2a - \rho A(t).
\end{aligned}$$

Using the parameter values $a = 1$, $a_1 = 0.35$, $a_2 = 1$, $b_2 = -0.25$, $r = 0.25$ and $\delta = 0.75$, $\rho = 0.5$ and taking b_1 as bifurcation parameter we see that for $b_{1,crit} = 0.16541$ two eigenvalues of the corresponding Jacobian matrix are pure imaginary. The steady states for this value of b_1 are given by $K^\infty = 3.17228$, $\gamma_1^\infty = 0.9272186$, $A^\infty = 2.37921$, $I^\infty = 2.37921$, $C^\infty = 2.3031218$, $GNP^\infty = 4.5091696$. $GNP(t)$ denotes gross national product and is given by $GNP(t) = C(t) + \gamma_1(t)I(t)$. Note that $\gamma_1(t)$ denotes the price of investment in terms of the consumption good which is used as numeraire and is equal to $q(t)$ (see Boldrin (1989), p. 235 or Otani and El-Hodiri (1987), p. 226). The derivative of the real part with respect to the bifurcation parameter of the pure imaginary eigenvalues at $b_1 = b_{1,crit}$, is $Re\,\lambda_1'(b_{1,crit}) = 4.8916$ indicating the emergence of a Hopf bifurcation[8] possibly leading to stable limit cycles.

As to the degree of adjacent complementarity we calculate for A_1, $A_1 = 1.36981$ whereas both c and d are negative, for $b_1 = b_{1,crit}$. By varying b_1 we determine the sign of A_1 giving the degree of complementarity of the capital stock with respect to time.

Taking $b_1 = 0.16$ a little smaller than $b_{1,crit}$ we calculate the eigenvalues of the Jacobian as $\lambda_{1,2} = -0.0291483 \pm 0.626392\sqrt{-1}$, $\lambda_3 = -0.359898$ indicating that for this case the dynamic behaviour of the variables is characterized by a stable focus.

In figure 1 the time path for consumption is depicted. It can be seen that the path shows cyclical oscillations with declining amplitude.

Figure 1 about here

In figure 2 the stable focus in the $A(t) - K(t)$ phase diagram is depicted. It can be seen how the path approaches the steady state in the long run.

Figure 2 about here

Figure 3 finally shows a three-dimensional projection of the stable focus in

[8]For the numerical computations and the solution of the differential equations we used the computer software Mathematica (see Wolfram Research (1991)).

the $I(t) - K(t) - GNP(t)$ diagram, with GNP(t) denoted as BSP(t).

Figure 3 about here

As mentioned above, our dynamical system undergoes a Hopf bifurcation for $b_1 = 0.16541$. For values smaller than this critical value, the system is stable. If we take b_1 a little larger than 0.16541 and take $b_1 = 0.16542$ we can observe that we now have stable limit cycles. In figure 4 again the time path for consumption is depicted. We see that the amplitude of the oscillations now remains constant.

Figure 4 about here

Figure 5 shows the time path for technical knowledge and investment in the $A(t) - I(t)$ phase diagram which is a stable limit cycle. The orientation is counter-clockwise.

Figure 5 about here

In figure 6 the limit cycle is depicted in the $I(t) - K(t) - C(t)$ phase diagram. It can be seen how the path approaches the limit cycle which is an attractor for our system.

Figure 6 about here

5 Conclusion

In this paper we demonstrated how the introduction of learning in a conventional two-sector growth model may lead to transitory or persistent oscillations. In the introduction we refered to Schumpeter as being one of the first to emphasize the endogenity of growth cycles in capiatlist economies. Therefore one could be tempted to call our model Schumpeterian. But on the other hand it must be clearly underlined that our model is too conventional to be termed Schumpeterian and we do not want to pretend aspects this model does not contain. So it must be clear that we still have competitive markets with equilibria in it. To be Schumpeterian we would have to give up that assumption and use a different market form. But that choice also resulted from our assumption that human capital is built as a by-product of investment. If we had supposed that firms intentionally invest in the creation of new technologies the assumption of competitive markets could not have been maintained if positive spillovers were allowed for. This point has also been recognized by Schumpeter himself and was repeated by a good deal of economists later on (Schumpeter (1947), Sheshinski (1967), Shell (1967), Romer (1990)). But on the other hand it was our intention to show that the presence of learning alone may be sufficient to generate cyclical behaviour of economic variables.

As to future works, we believe that more realistic models should be constructed in which the fact that firms intentionally invest in the creation of new technologies or their workers' skill is taken into consideration. Then persistent cyclical per-capita growth rates of the economic variables may be the outcome, too.

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Captions of the Figures

Figure 1: Optimal path of consumption (stable focus).
Figure 2: Stable focus in the human capital - physical capital plane.
Figure 3: Stabel focus in the investment - capital - GNP space.
Figure 4: Optimal path of consumption (stable limit cycle).
Figure 5: Limit cycle in the human capital - investment plane.
Figure 6: Limit cycle in the investment - capital - consumption space.

Figure 1

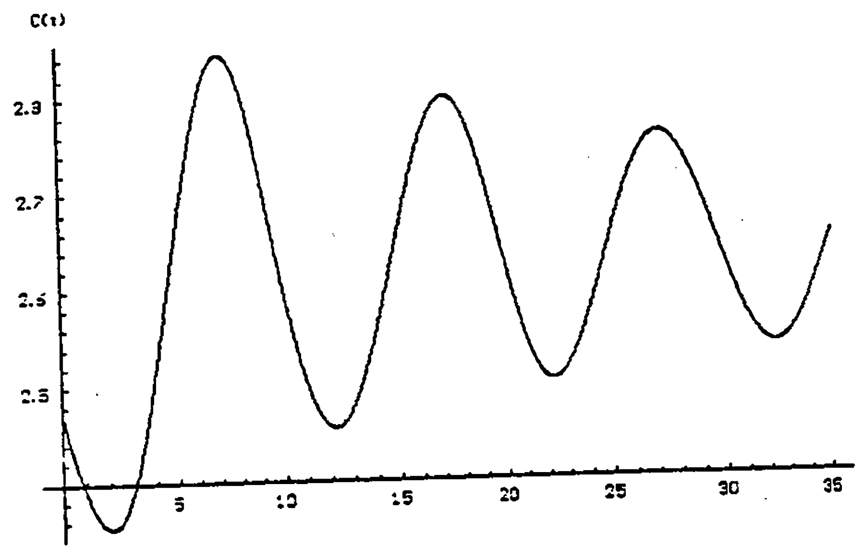


Figure 2

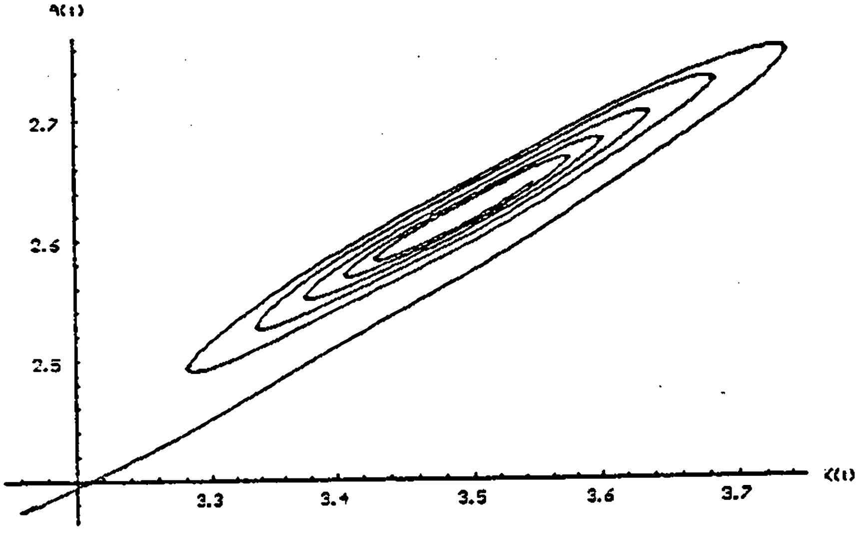


Figure 3

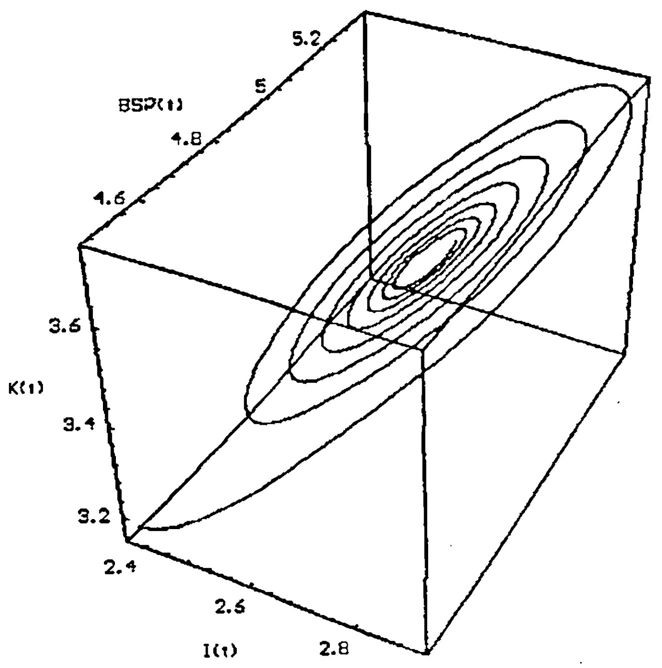


Figure 4

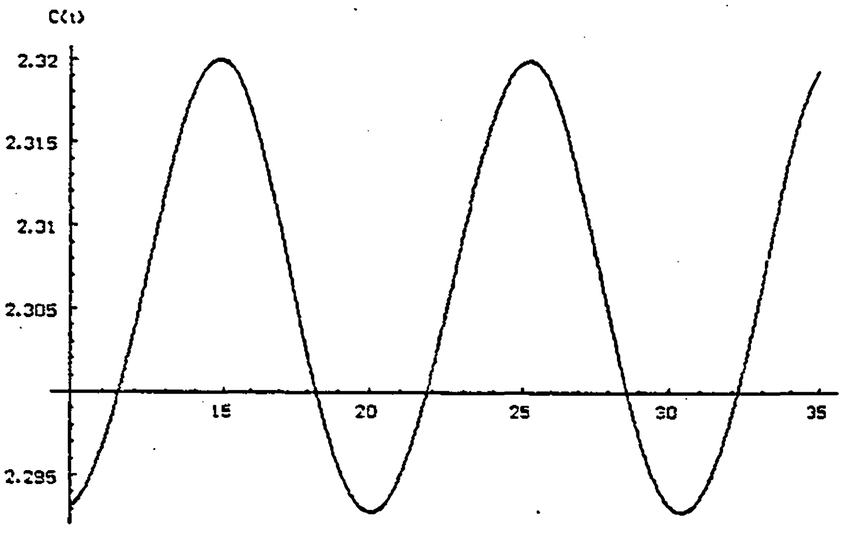


Figure 5

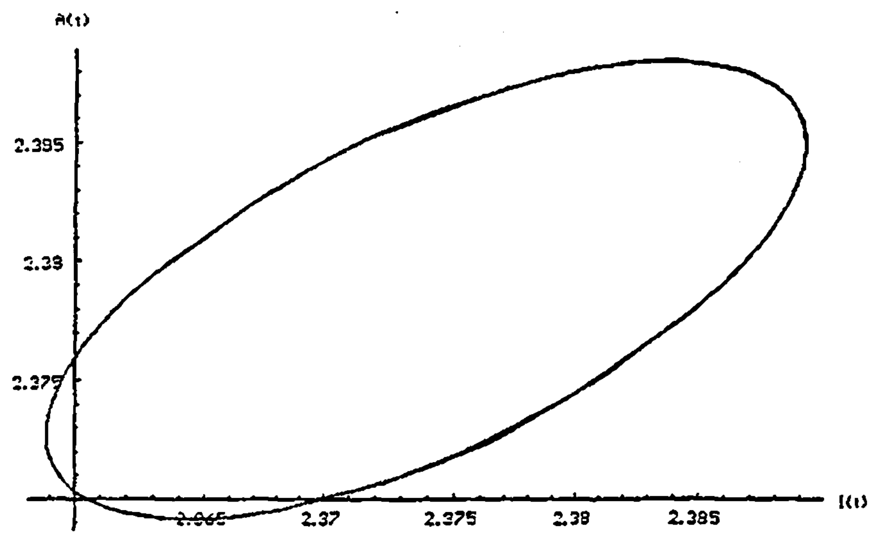
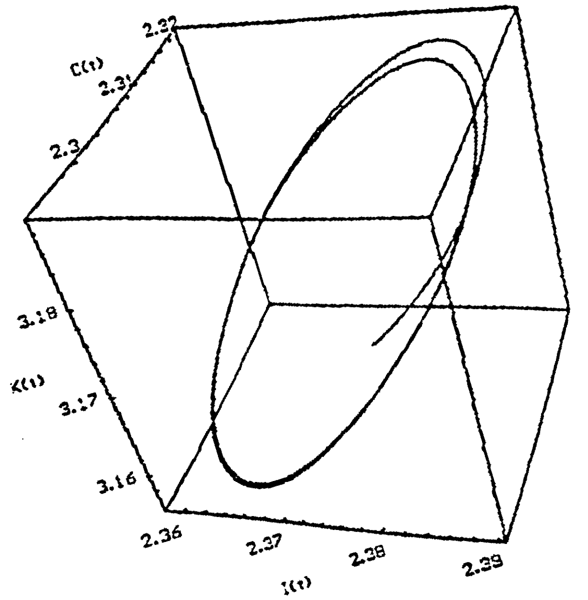


Figure 6



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