DAMPED PERIODICALLY DRIVEN QUANTUM TRANSPORT IN BISTABLE SYSTEMS*

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The quantum dynamics of a quartic double well, subjected to a harmonically oscillating field, is studied in the framework of the Floquet formalism. The modifications of the familiar tunneling process due to driving and dissipation are investigated numerically and explained in terms of the quasienergy spectrum. In absence of dissipation, there is a one-dimensional manifold in the parameter space spanned by amplitude and frequency of the driving force, where tunneling is almost completely suppressed by the coherent driving. The influence of dissipation is described on basis of the reduced density matrix in the Floquet representation. In particular, we consider the effect of weak Ohmic damping. In the classical limit, this system corresponds to a damped bistable Duffing oscillator. The interplay of coherent and incoherent transport processes is studied in terms of the transient time evolution of a temporal autocorrelation function. We find that the coherent suppression of tunneling is *stabilized* by reservoir-induced noise for a suitably chosen temperature. By computing stroboscopic Husimi distributions, we also compare the quantal stationary states with the corresponding classical deterministic attractors.

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1. Introduction

Bistable systems are abundant in physics, from the microscopic to the macroscopic realm. On the macroscopic level, bistability represents a basic concept in nonlinear dynamics. In quantum mechanics, on the other hand, bistable potentials are associated with a paradigmatic coherence effect: tunneling [1]. Accordingly, this class of systems represents a particularly promising field to study the interplay of classical nonlinearity and quantum coherence, and the way this is reflected in phase-space transport.

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In the present work we investigate the influence of periodic driving on the quantal dynamics in a bistable potential. Being equivalent to adding one degree of freedom, external driving is capable of qualitatively altering the dynamics, *e.g.*, in the classical limit, it can render a bistable system chaotic [2, 3]. Here, however, we concentrate on the alterations of the tunneling process, due to the driving, in the deep quantal regime: They take the form of mere quantitative changes of the tunnel splitting, from its complete vanishing up to its augmentation by orders of magnitude, as well as of qualitative changes as a consequence of the admixture of additional levels, beyond the ground-state doublet.

Periodic driving is simple enough to still allow, by way of its discrete time-translational symmetry, for a systematic analytical treatment: The Floquet formalism provides a generalization of the notions of energy eigenvalues and eigenstates to periodically time-dependent systems [4–7]. Since its validity is not restricted to small amplitudes of the driving nor to large characteristic actions, we need not resort to perturbative or semiclassical methods. Furthermore, the Floquet formalism obviously enables us to go beyond the two-state approximation commonly used in the context of tunneling.

In Section 2, we present our working example, a harmonically driven quartic double well, and introduce some analytical concepts for later reference, such as the Floquet operator and the local spectrum. The quantum transport and the local spectrum in absence of dissipation are addressed in Section 3. Section 4 contains our principal results. They form a survey of the friction-modified coherence phenomena that replace driven tunneling in various regimes of the parameter space spanned by amplitude and frequency of the driving force. We summarize our results in Section 5.

This contribution is partially based on results originally published in earlier works by the present authors [8–11].

2. The periodically driven double well

The system we study is a quartic double-well potential driven by a monochromatic force. Its Hamiltonian reads, in dimensionless variables,

$$
\begin{aligned}
H_{\mathrm{DW}}(p,x;t) &\doteq H_0(p,x) + H_1(x;t)\,,\\
H_0(p,x) &= \frac{p^2}{2} - \frac{x^2}{4} + \frac{x^4}{64D}\,,\\
H_1(x;t) &= Sx\cos\omega t\,,
\end{aligned}
\tag{1}
$$

where D denotes the barrier height, and S and ω are the amplitude and frequency of the driving force, respectively.

In systems with a discrete time-translational symmetry, a stroboscopic time evolution is generated by the *Floquet operator* [12–15], the propagator over a single period of the time-dependent force,

$$U_{t_0} = U\left(t_0 + \tfrac{2\pi}{\omega}, t_0\right) = \mathbf{T}\exp\left(-i\int_{t_0}^{t_0+2\pi/\omega} \mathrm{d}t\; H(t)\right), \tag{2}$$

where $\mathbf{T}$ denotes the time-ordering operator. Therefore, U_{t_0} may be called a *quantum map.* According to the Floquet theorem, its eigenstates take the form

$$|\psi_\alpha(t)\rangle = \exp(-i\varepsilon_\alpha t)|\phi_\alpha(t)\rangle\,, \quad \text{with} \quad \left|\psi_\alpha\left(t+\tfrac{2\pi}{\omega}\right)\right\rangle = |\psi_\alpha(t)\rangle\,. \tag{3}$$

The eigenvalues ε_α are called *quasienergies.* In fact, each of them is a representant of an infinite class of eigenvalues $\varepsilon_{\alpha k} = \varepsilon_\alpha + k\omega$, $k = 0, \pm1, \pm2, \ldots$. The $\varepsilon_{\alpha k}$ correspond to solutions equivalent to Eq. (3), as is obvious if one defines $|\phi_{\alpha k}\rangle = \exp(ik\omega t)|\phi_\alpha\rangle$. In other words, the quasienergy spectrum is cyclic, *i.e.*, defined mod ω, similar to the Brillouin-zone structure in the solid-state context.

Another, more special symmetry of the system described by the Hamiltonian (1) goes back to the inversion symmetry: $x \to -x$, $p \to -p$, of phase space for the time-independent system $H_0(p, x)$. This symmetry is destroyed by an arbitrary periodic driving term, but for the harmonic time dependence chosen here, the relation $\cos(\omega t + \pi) = -\cos(\omega t)$ allows for invariance under the operation [8, 16]

$$\mathbf{P}: \quad x \to -x\,, \quad p \to -p\,, \quad t \to t + \tfrac{\pi}{\omega}\,. \tag{4}$$

$\mathbf{P}$ forms a unitary symmetry and may be regarded as a *generalized parity.* As a consequence, the basis formed by the Floquet eigenstates can be divided into an even and an odd subset.

A quantity that provides some condensed information on the transport of probability between the two wells of the bistable potential, and that allows to relate this information directly to the relevant structures in the quasienergy spectrum, is the *probability to return* [17, 18]

$$P^{\Psi(t_0)}(t_n) = \left|\left\langle\Psi\left(t_0 + \tfrac{2\pi n}{\omega}\right)\middle|\Psi(t_0)\right\rangle\right|^2 = |\langle\Psi(t_0)|(U_{t_0})^n|\Psi(t_0)\rangle|^2\,, \tag{5}$$

defined with reference to some initial state $|\Psi(t_0)\rangle$, and with time restricted to a discrete series $t_n = t_0 + 2\pi n/\omega$, $n = 0, \pm1, \pm2, \ldots$. The role of the quasienergies for this time evolution is made explicit by expanding Eq. (5) in the Floquet basis,

$$P^{\Psi(t_0)}(t_n) = \xi^{-1} + \sum_{\alpha \neq \beta} \exp\left(i(\varepsilon_\alpha - \varepsilon_\beta)\left(\tfrac{2\pi n}{\omega}\right)\right) \times |\langle\psi_\alpha(t_0)|\Psi(t_0)\rangle|^2 |\langle\psi_\beta(t_0)|\Psi(t_0)\rangle|^2 . \tag{6}$$

Here, ξ^{-1}, the diagonal part excluded from the double sum in Eq. (6), gives the long-time average of $P^{\Psi(t_0)}(t_n)$. The spectral counterpart of $P^{\Psi(t_0)}(t_n)$ is the two-point cluster function $P_2^{\Psi(t_0)}(\eta)$ of the *local* Floquet spectrum [17–20]. It is related to $P^{\Psi(t_0)}(t_n)$ by Fourier transformation and thus contains all the frequencies involved in the time evolution of $P^{\Psi(t_0)}(t_n)$, weighted according to their relative significance for the specific dynamics starting from $|\Psi(t_0)\rangle$.

3. Driven tunneling in absence of dissipation

In the present Section, we discuss the modifications imposed on the familiar tunneling dynamics, due to periodic driving. That is, we concentrate on the time evolution, under the external force, of a state that is initially prepared as an approximation to a superposition of the two lowest unperturbed eigenstates, $|l(r)\rangle = (|1\rangle + (-)|2\rangle)/\sqrt{2}$, centered in one of the two wells. Accordingly, we trace the quasienergy doublet, corresponding to the unperturbed energies E_1 and E_2, through the parameter space spanned by amplitude S and frequency ω of the driving force. Thereby, we exclude dynamical complexity due to mere preparation effects from our investigation.

There are two regimes in the (S, ω)-plane where tunneling is not qualitatively altered: Both in the limits of slow (adiabatic) and of fast driving, the separation of the time scales of inherent dynamics and external force effectively uncouples these two processes and is reflected in a mere renormalization of the tunneling rate $\Delta(S, \omega)$. Specifically, as both an analytical treatment and numerical experiments show [8], the driving always reduces the effective barrier height and thus augments the tunneling rate in the two limits at issue.

Qualitative changes in the tunneling behavior are expected as soon as the driving frequency becomes comparable with the internal frequencies of the double well, *i.e.*, in particular, the tunnel splitting $\Delta = E_1 - E_2$ and the so-called resonances $E_3 - E_2$, $E_4 - E_1$, $E_5 - E_2, \ldots$. A physical understanding of the temporal complexity in this regime is obtained by relating it to the "landscape" of quasienergy planes $\varepsilon_{\alpha k}(S, \omega)$ in parameter space. Features of particular significance are close encounters of quasienergies: Two

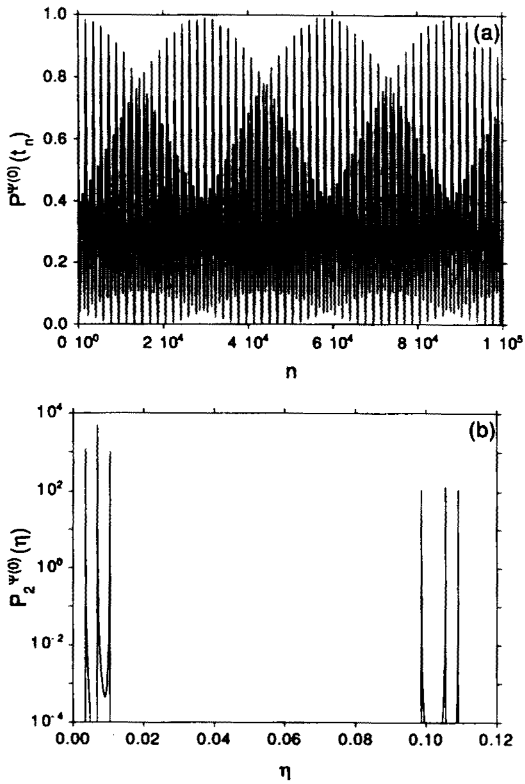
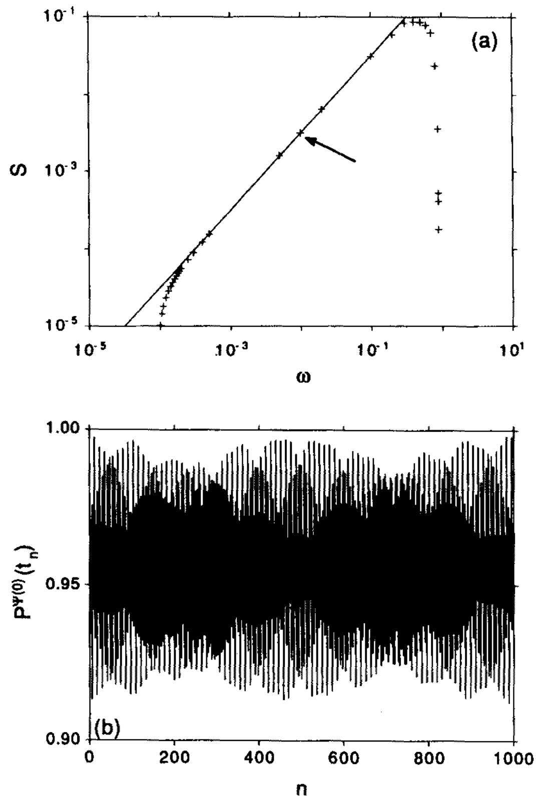


Fig. 1. Driven tunneling at the fundamental resonance $\omega = E_3 - E_2$. (a) Time evolution of $P^{\Psi(t_0)}(t_n)$ over the first 10^5 time steps; (b) local spectral two-point correlation function $P_2^{\Psi(t_0)}(\eta)$ obtained from (a). The parameter values are $D = 2$, $S = 2 \cdot 10^{-3}$, and $\omega = 0.876034$.

quasienergies approaching each other form an exact crossing if they belong to different parity classes, otherwise the crossing will be avoided.

We discuss two specific instances of the quasienergy spectrum with the corresponding tunneling dynamics, one of them featuring an avoided crossing, the other an exact one. The "single-photon transition" at $\omega = E_3 - E_2$ is called the *fundamental resonance.* At $S = 0$, it corresponds to a crossing between the quasienergies $\varepsilon_{2,k}$ and $\varepsilon_{3,k-1}$ and, for $S > 0$, forms an avoided crossing, since the corresponding eigenstates have equal parity. Fig. 1(a) shows the time evolution of $P^{\Psi(t_0)}(t_n)$, at the fundamental resonance ($D = 2$, $S = 10^{-4}$, $\omega = 0.876$), for an initial state prepared as the ground state of a harmonic oscillator approximating one of the wells, *i.e.*, a Gaussian approximation of $|l(r)\rangle$. The monochromatic oscillation of $P^{\Psi(t_0)}(t_n)$, characteristic of unperturbed tunneling, has given way to a more complex beat pattern. The two-point correlation $P_2^{\Psi(t_0)}(\eta)$ of the local spectrum reveals that these beats are mainly composed of two groups of three frequencies each (Fig. 1(b)), which can be identified, in turn, as the quasienergy differences $\varepsilon_{3,-1} - \varepsilon_{2,0}$, $\varepsilon_{2,0} - \varepsilon_{1,0}$, $\varepsilon_{3,-1} - \varepsilon_{1,0}$, and $\varepsilon_{4,-1} - \varepsilon_{3,-1}$, $\varepsilon_{4,-1} - \varepsilon_{2,0}$, $\varepsilon_{4,-1} - \varepsilon_{1,0}$, at the avoided crossing.

In contrast, a two-photon transition that bridges the tunnel splitting Δ is "parity forbidden", and thus the quasienergies $\varepsilon_{1,k+1}$ and $\varepsilon_{2,k-1}$ give rise to an exact crossing. Eq. (6) indicates that a vanishing of the difference $\varepsilon_{2,-1} - \varepsilon_{1,1}$ will have a drastic consequence: For a state prepared as an exact superposition of the corresponding two quasienergy eigenstates only, $P^{\Psi(t_0)}(t_n)$ and all other observables become constants, at least at discrete times $2\pi n/\omega$, and thus it is possible that tunneling comes to a standstill! According to an argument going back to von Neumann and Wigner [21, 22], exact crossings should occur along one-dimensional manifolds in the (S, ω)-plane. Fig. 2(a) shows such a manifold for $\varepsilon_{2,-1} = \varepsilon_{1,1}$, as determined numerically: It is a closed curve, reflection-symmetric with respect to the axis $S = 0$, with an approximately linear frequency dependence for $\Delta \lesssim \omega \lesssim E_3 - E_2$. A typical time evolution of $P^{\Psi(t_0)}(t_n)$ for parameter val-



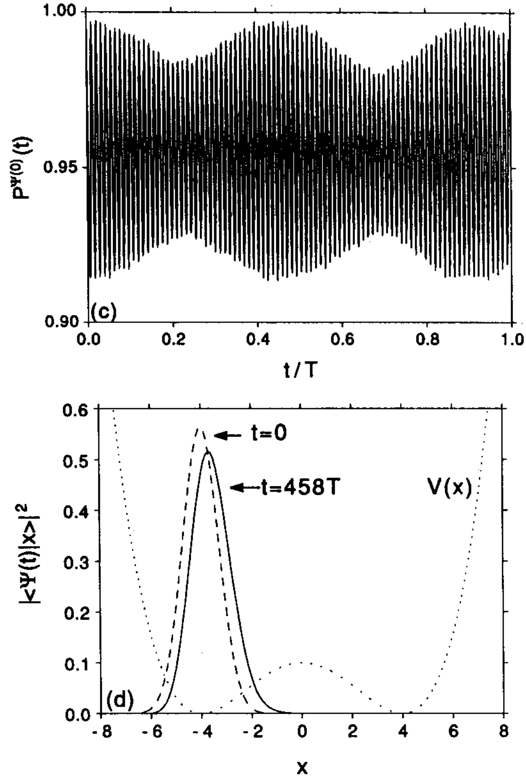


Fig. 2. Coherent suppression of tunneling at an exact crossing $\varepsilon_{2,-1} = \varepsilon_{1,1}$. (a) One of the manifolds in the (S, ω)-plane where this crossing occurs (data obtained by diagonalization of the full Floquet operator for the driven double well are indicated by crosses, the full line has been derived from a two-state approximation, the arrow indicates the parameter point for which the subsequent parts of the figure have been obtained); (b) time evolution of $P^{\Psi(0)}(t_n)$ over the first 10^3 time steps; (c) time evolution of $P^{\Psi(0)}(t)$ within the first period of the driving; (d) $|\langle\Psi(t_n)|x\rangle|^2$ at $n = 458$ (dashed line), compared with the initial state (full line) (the dotted line indicates the unperturbed potential). The parameter values are $D = 3$, $S = 3.171 \cdot 10^{-3}$, and $\omega = 0.01$, that is, $\omega = 52.77\Delta$.

ues on the linear part of that manifold ($D = 2$, $S = 3.171 \cdot 10^{-3}$, $\omega = 0.01$) is presented in Fig. 2(b). It clearly demonstrates the suppression of tunneling. Remaining oscillations of small amplitude can be ascribed to an admixture of higher-lying quasienergy states to the initial state. In addition, the time dependence synchronous with the driving frequency has not completely vanished, as is revealed by the evolution of $P^{\Psi(t_0)}(t)$, resolved within a single period of the driving force (Fig. 2(c)). In Fig. 2(d), we compare $|\langle\Psi(t_n)|x\rangle|^2$, at a time ($n = 458$) where the deviation of $P^{\Psi(t_0)}(t_n)$ from unity is exceptionally large, with the initial state: This confirms that the leakage of probability into the initially empty, opposite well indeed remains extremely small. So the coherent suppression of tunneling truly amounts to a *localization* of the wave packet in one of the wells.

This phenomenon appears to be an elementary quantum-interference effect. In fact, much of it can be understood on basis of a two-state approximation. It is achieved by solving the equations of motion for the expansion coefficients of a localized initial state in the Hilbert space spanned by the unperturbed ground-state doublet $|1\rangle$, $|2\rangle$. The two-state approximation predicts an infinite number of manifolds where localization occurs and yields analytical expressions for them [11, 23].

4. Driven tunneling with dissipation

The approach towards the macroscopic realm comprises, at least, two different aspects: the increase in characteristic phase-space scales allows the use of small-wavelength approximations and lets finer and finer details of the classical phase space flow show up in the quantum dynamics, while the growing role of the ambient degrees of freedom tends to reduce the complexity of the quantum dynamics by degrading coherence effects. We shall here focus on the latter aspect (for semiclassical studies of driven tunneling, see Refs [24, 25]) and present some preliminary results on the influence of dissipation on the quantum dynamics of the driven double well.

We incorporate dissipation by coupling the driven system in (1) to a reservoir, *i.e.*,

$$\begin{aligned} H(t) &= H_{\mathrm{DW}}(t) + H_{\mathrm{I}} + H_{\mathrm{R}}\,, \\ H_{\mathrm{I}} &= x\sum_i (g_i b_i + g^* b_i^\dagger)\,, \\ H_{\mathrm{R}} &= \sum_i \omega_i (b_i^\dagger b_i + \tfrac{1}{2})\,. \end{aligned} \tag{7}$$

Each reservoir oscillator i is described by a frequency ω_i, second-quantization operators b_i, $b_i^\dagger$, and a coupling constant g_i. Starting from the von Neumann equation for the density operator of the full system, we perform the usual procedure [26–28] to trace out the reservoir degrees of freedom under the assumptions *(i)*, that the reservoir is Markovian, *i.e.*, correlation functions for the boson modes decay instantaneously on characteristic time scales of the double-well dynamics, and *(ii)*, that the driven double well is weakly coupled to the reservoir, so that we can neglect the higher order quantum effects in the coupling strength. We assume that the full system starts at time t_0 with a density operator $\rho(t_0) = \sigma(t_0)B_T$, *i.e.*, the driven double well and the reservoir are *initially uncorrelated* and the reservoir is in thermal equilibrium with the canonical statistical operator B_T at temperature T. Here, $\sigma(t_0) := \sigma(t_0, t_0)$ denotes the initial reduced density matrix of the driven double well. Using the interaction picture at t_0 and performing the trace over all reservoir oscillators ($\mathrm{tr_R}$) yields the equation of motion for the reduced density matrix,

$$\dot{\tilde{\sigma}}(t,t_0) = \int\limits_0^{t-t_0} \mathrm{d}t' \mathrm{tr_R} \left[\widetilde{H}_\mathrm{I}(t,t_0), \left[\widetilde{H}_\mathrm{I}(t-t',t_0), B_T\tilde{\sigma}(t,t_0)\right]\right] . \quad (8)$$

In the representation by the Floquet states, the unitary time-translation operator of the undamped system reads

$$U_\mathrm{DW}(t,t_0) = \sum_\alpha \exp(i\varepsilon_\alpha(t-t_0))|\phi_\alpha(t)\rangle\langle\phi_\alpha(t_0)| . \quad (9)$$

This allow us to evaluate the interaction operator $\tilde{x}$ in terms of Fourier components in the Floquet representation [26],

$$\tilde{x}(t,t_0) = U^\dagger_\mathrm{DW}(t,t_0)xU_\mathrm{DW}(t,t_0) = \sum_{\alpha,\beta,k} \exp\left(i\Delta_{\alpha\beta k}(t-t_0)\right) X_{\alpha\beta k}\Gamma_{\alpha\beta}(t_0) . \quad (10)$$

Here, we have introduced the frequencies $\Delta_{\alpha\beta k} = \varepsilon_\alpha - \varepsilon_\beta + k\omega$, the matrix elements of the position operator,

$$X_{\alpha\beta k} = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} \mathrm{d}t \exp(-ik\omega t)\langle\phi_\alpha(t)|x|\phi_\beta(t)\rangle ,$$

and the projectors $\Gamma_{\alpha\beta}(t_0) = |\phi_\alpha(t_0)\rangle\langle\phi_\beta(t_0)|$. In the limit of a dense spectrum of the reservoir oscillators, we introduce the coupling strength

$$J(\omega) = \lim_{\Delta\omega\to 0} \frac{\pi}{\Delta\omega} \sum_{\omega<\omega_l<\omega+\Delta\omega} |g_l|^2 . \quad (11)$$

In this way, Eq. (8) can be rewritten as

$$\begin{aligned}
\dot{\tilde{\sigma}}(t,t_0) = \sum_{\alpha,\beta,k}\sum_{\alpha',\beta',k'} & X_{\alpha\beta k}X^*_{\alpha'\beta'k'} \exp\left((i\Delta_{\alpha\beta k} - \Delta_{\alpha'\beta'k'})(t-t_0)\right) \\
& \times \Big[\Big\{\Theta(\Delta_{\alpha'\beta'k'})S(\Delta_{\alpha'\beta'k'})\left(1+n_\mathrm{th}(\Delta_{\alpha'\beta'k'})\right) \\
& + \tfrac{i}{\pi}\left(P(\Delta_{\alpha'\beta'k'}) + Q_T(\Delta_{\alpha'\beta'k'})\right)\Big\} \\
& \times \left[\Gamma^\dagger_{\alpha'\beta'}(t_0), \tilde{\sigma}(t,t_0)\Gamma_{\alpha\beta}(t_0)\right] \\
& + \left\{\Theta(\Delta_{\alpha'\beta'k'})S(\Delta_{\alpha'\beta'k'})n_\mathrm{th}(\Delta_{\alpha'\beta'k'}) + \tfrac{i}{\pi}Q_T(\Delta_{\alpha'\beta'k'})\right\} \\
& \times \left[\Gamma_{\alpha\beta}(t_0), \tilde{\sigma}(t,t_0)\Gamma^\dagger_{\alpha'\beta'}(t_0)\right]\Big] \quad + \quad \text{h.c.} \quad (12)
\end{aligned}$$

We use the abbreviations $\Theta(x)$ for the unit step function, $n_{\text{th}}(\omega)$ for the thermal occupation of the boson mode with frequency ω, and $P(\omega)$, $Q_T(\omega)$ for the quantities

$$P(\omega) = \mathrm{P}\int_0^\infty \mathrm{d}\omega' \frac{J(\omega')}{\omega - \omega'}, \qquad Q_T(\omega) = \mathrm{P}\int_0^\infty \mathrm{d}\omega' \frac{J(\omega') n_{\text{th}}(\omega')}{\omega - \omega'}, \tag{13}$$

with P indicating the principal value. Specifying the coupling strength as

$$J(\omega) = \frac{\gamma\omega}{\pi(1 + \omega^2/\omega_c^2)}, \tag{14}$$

we obtain, in the classical limit, the Langevin equation

$$\ddot{x} + \gamma\omega_c \int_{-\infty}^{t} \mathrm{d}t' \dot{x}(t') \exp(-\omega_c(t - t')) - \tfrac{x}{2}(1 + 2\gamma\omega_c) + \tfrac{x^3}{16D} + S\cos\omega t = f(t). \tag{15}$$

Here, $f(t)$ is a random force with the autocorrelation function

$$\langle f(t) f(t')\rangle = \gamma k_{\mathrm{B}} T \omega_c \exp(-\omega_c |t - t'|). \tag{16}$$

Eq. (15) describes a bistable Duffing oscillator with Ohmic damping and fluctuations due to the thermal noise.

In order to simplify the equation of motion (12), we introduce the rotating-wave approximation. In the present case, we can distinguish three different aspects of it,

(i) dropping all terms with $\Delta_{\alpha\beta k}\Delta_{\alpha'\beta' k'} < 0$,
(ii) dropping all terms with $(\alpha, \beta) \neq (\alpha', \beta')$,
(iii) dropping all terms with $k \neq k'$.

We start using the full rotating-wave approximation *(i)–(iii)*, but we shall have to partially release it later to allow for the exceptionally small energy scales occurring in the vicinity of quasienergy crossings. In this way, we obtain a quantum-mechanical master equation in the usual form [26–29],

$$\begin{aligned} \dot{\tilde{\sigma}}_{\alpha\alpha}(t, t_0) &= \sum_\nu (W_{\alpha\nu} \tilde{\sigma}_{\nu\nu}(t, t_0) - W_{\nu\alpha}\tilde{\sigma}_{\alpha\alpha}(t, t_0)), \\ \dot{\tilde{\sigma}}_{\alpha\beta}(t, t_0) &= -\tfrac{1}{2}\sum_\nu (W_{\nu\alpha} + W_{\nu\beta})\tilde{\sigma}_{\alpha\beta}(t, t_0), \qquad \alpha \neq \beta, \end{aligned} \tag{17}$$

where

$$\begin{aligned} W_{\alpha\beta} &= \sum_{k=-\infty}^{\infty} \left[\gamma_{\alpha\beta k} + n_{\text{th}}(|\Delta_{\alpha\beta k}|)(\gamma_{\alpha\beta k} + \gamma_{\beta\alpha -k})\right], \\ \gamma_{\alpha\beta k} &= 2\pi\Theta(\Delta_{\alpha\beta k}) S(\Delta_{\alpha\beta k}) |X_{\alpha\beta k}|^2, \\ \tilde{\sigma}_{\alpha\beta}(t) &= \langle \phi_\alpha(t_0)|\tilde{\sigma}(t)|\phi_\beta(t_0)\rangle. \end{aligned}$$

Eq. (17) comprises a closed subset of equations for the approach of the diagonal elements towards a steady state, and another subset describing the decay of the non-diagonal elements. The time-independent coefficients $W_{\alpha\beta}$ contain the specifics of the potential and driving *via* the quasienergy differences $\Delta_{\alpha\beta k}$. Besides the slow dynamics generated by Eq. (17), observables show an additional periodic time dependence, with the period of the driving, which persists even in the steady state. It enters through the time dependence of the quasienergy states $|\phi_\alpha(t)\rangle$ upon calculating expectation values. Eq. (17) amounts, within the approximations made, to a quantization of the driven bistable Duffing oscillator in the limit of weak damping, whose classical chaotic dynamics has been studies in Refs [29, 30].

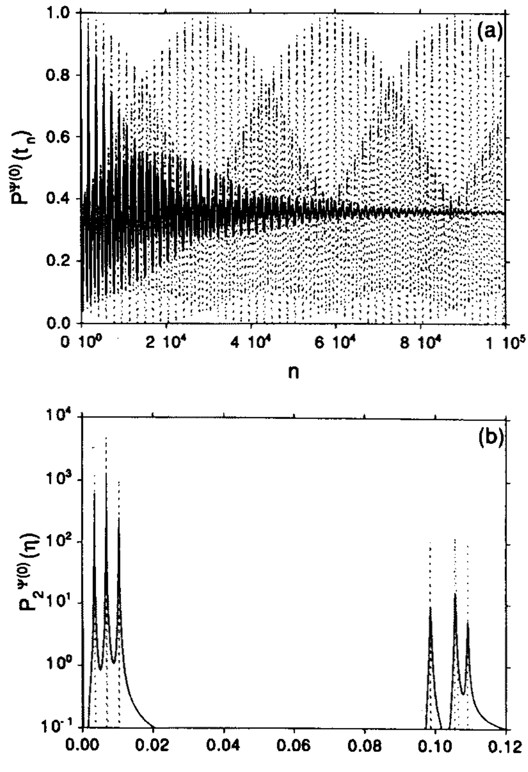
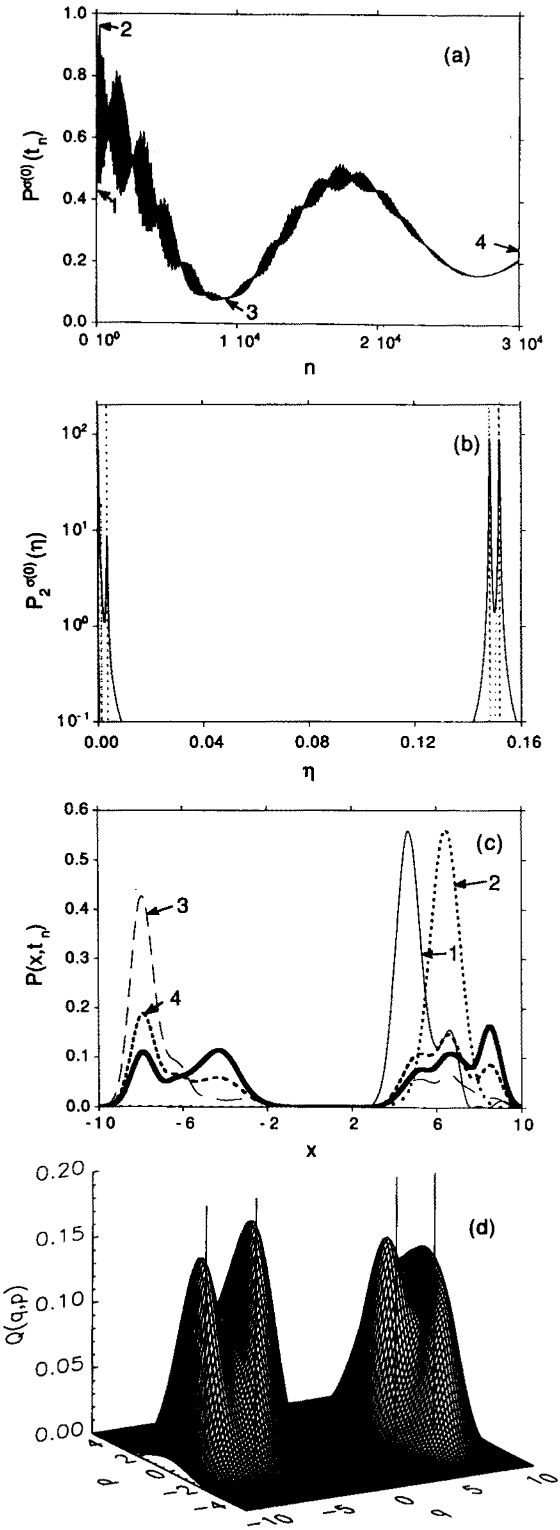


Fig. 3. Driven tunneling with dissipation. (a) Time evolution of $P^{\Psi(0)}(t_n)$ over the first 10^5 time steps; (b) local spectral two-point correlation function $P_2^{\psi(0)}(\eta)$ obtained from (a). The parameter values are as in the corresponding conservative case shown in Fig. 1 (repeated here as dashed lines), but with a finite damping constant, $\gamma = 4 \cdot 10^{-5}$, at $T = 0$.

Fig. 3(a) shows the time evolution of $P^{\sigma(t_0)}(t_n) = \mathrm{tr}(\sigma(t_n, t_0)\sigma(t_0))$,


1.0
0.8
0.6
0.4
0.2
0.0
$P^{\sigma(0)}(t_n)$
(a)
2
1
3
4
0 10^0
1 10^4
2 10^4
3 10^4
n
10^2
10^1
10^0
10^{-1}
$P_2^{\sigma(0)}(\eta)$
(b)
0.00
0.04
0.08
0.12
0.16
η
0.6
0.5
0.4
0.3
0.2
0.1
0.0
$P(x,t_n)$
(c)
-10
-6
-2
2
6
10
x
0.20
0.15
0.10
0.05
0.00
Q(q,p)
(d)
p
q
-10
-5
0
5
10

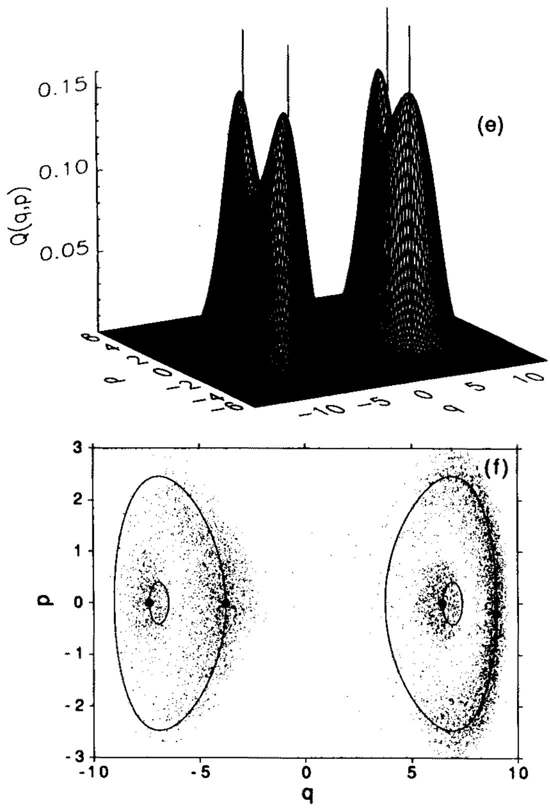
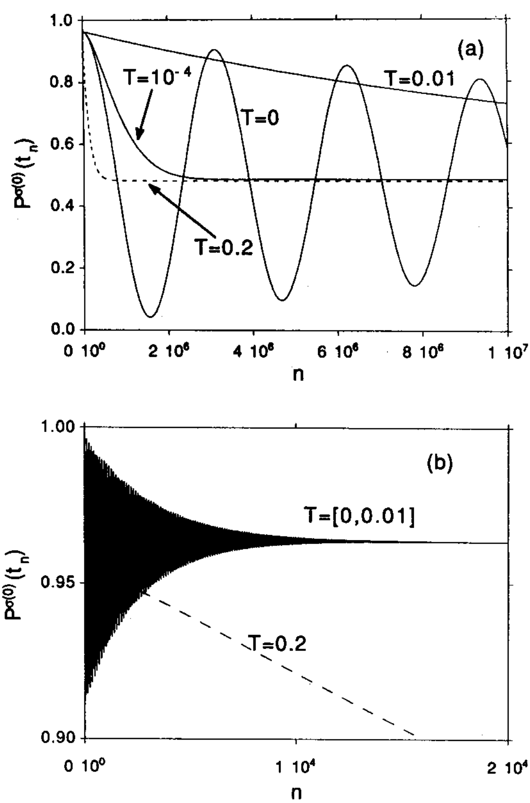


Fig. 4. Tunneling in the periodically driven double well with dissipation, for the parameter values $D = 6$, $S = 0.08485$, $\gamma = 10^{-5}$, and $T = 0$. (a) Time evolution of the autocorrelation function $P^{\sigma(0)}(t_n)$, over the first $3 \cdot 10^4$ time steps, starting from a coherent state centered at one of the classical attractors withing the right well; (b) Fourier transform of (a), corresponding to the local spectral correlation function (dashed: the same function for the undamped system); (c) coordinate distribution at selected times t_n (graph 1: $n = 20$, 2: $n = 40$, 3: $n = 8910$, 4: $n = 5 \cdot 10^4$) as marked in part (a), and stationary state (thick line); (d), (e) Husimi representations of the steady state, compared with the corresponding classical stationary distribution (sharp peaks) at t_n (d) and $t_n + \pi/2\omega$ (e); (f) classical phase-space portrait of the steady state of an ensemble of noisy trajectories (see text), for the same classical parameters and at the same phase as in (d) (dot clouds), compared with the corresponding deterministic point attractors (heavy dots) and the full (*i.e.*, non-strobed) periodic orbits to which they belong (full lines).

with an initial state $\sigma(t_0) = |\Psi(t_0)\rangle\langle\Psi(t_0)|$ and parameters of H_{DW} as in Fig. 1, but with the finite damping constant $\gamma = 4 \cdot 10^{-5}$, at zero temperature. The complex quantum beats, characteristic of the corresponding conservative system (dashed line), die out and give way to a steady state with a finite constant value of $P^{\sigma(t_0)}(t_n)$ (the periodic time dependence of the steady state persisting in a periodically driven system is invisible in a stroboscopic plot like this). The broadening of the quasienergy levels, due to the incoherent transitions described by Eq. (8), can be read off the Fourier transform of $P^{\sigma(t_0)}(t_n)$, Fig. 3(b).

In Fig. 4, we present the quantum dynamics generated by the master equation (17), at a parameter point ($D = 6$, $\omega = 0.9$, and $S = 0.08485$) where the driving frequency ω is close to the fundamental resonance, for $\gamma = 10^{-5}$ and $T = 0$. A pure, coherent state centered at one of the classical attractors within the right well served as the initial state. Fig. 4(a) shows the time evolution of the autocorrelation function at discrete times as in Fig. 3(a). There is a slow oscillation of $P^{\sigma(t_0)}(t_n)$ between 0 and 1 which corresponds, up to an augmented rate, to the familiar tunneling, and there is a superposed fast oscillation of smaller amplitude due to the participation of additional quasienergy states. The corresponding local spectral two-point correlation function is shown in Fig. 4(b): It reflects the primary effect of the incoherent processes induced by the reservoir, a broadening of the quasienergy levels. The broadening is not uniform



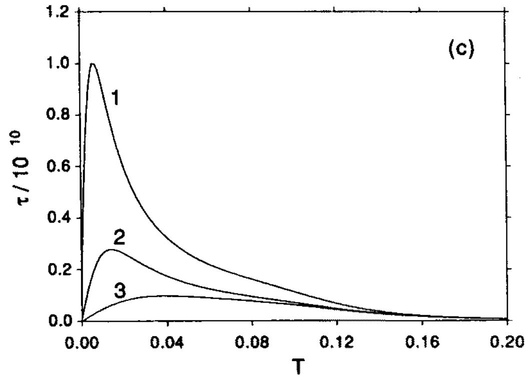


Fig. 5. Coherent suppression of tunneling in the presence of dissipation. (a) Time evolution of the autocorrelation function $P^{\sigma(0)}(t_n)$, over the first 10^7 time steps, at a parameter point ($D = 2$, $S = 3.171 \cdot 10^{-3}$, and $\omega = 0.01$) close to a manifold where the tunnel splitting vanishes, for $\gamma = 10^{-6}$, a cutoff frequency $\omega_c = 1$, and various values of T, starting from a coherent state centered in one of the wells; (b) the first $2 \cdot 10^4$ time steps from (a) on an enlarged time scale); (c) temperature dependence of the decay time τ of $P^{\sigma(0)}(t_n)$ (defined by $P^{\sigma(0)}(t_n) \approx \exp(-n/\tau)$) for three values of the detuning $\Delta\omega = \omega - \omega_{\rm loc}(S)$ from the localization manifold (graph 1: $\Delta\omega = -1.4 \cdot 10^{-7}$, as in part (a), 2: $\Delta\omega = 5.0 \cdot 10^{-7}$ at $S = 3.1712 \cdot 10^{-3}$, 3: $\Delta\omega = 1.4 \cdot 10^{-6}$ at $S = 3.1715 \cdot 10^{-3}$). The other parameters are as in part (a). The data shown do not extend down to $T = 0$, where $\tau(T)$ diverges, but start only with the rising part of this function.

but lets the high-frequency components, contributed by quasienergy pairs separated by a large quasienergy difference, decay faster, as should be expected from the Ohmic reservoir coupling. The spatially resolved states $P(x, t_n) = \langle x|\sigma(t_n, t_0)|x\rangle$ after 20 (graph 1), 40 (2), 8910 (3), and $5 \cdot 10^4$ (4) periods of the driving, respectively, as well as for $t \to \infty$, are presented in Fig. 4(c). While the slow oscillation ((1,2) → 3) corresponds to a flow of probability between the two potential wells, the fast oscillations are associated with transport within the wells. The transport can no longer be attributed to only a few quasienergy states, but it still has the character of a coherent process without close similarity to the classical phase-space flow. The stationary state (thick line in Fig. 4(c)), in turn, does bear the signature of the classical dynamics. Fig. 4(d) shows a phase-space representation of this state in terms of the Husimi function [31]. A comparison with the corresponding stationary state of the deterministic (*i.e.*, noise-free) classical system [29, 30] (sharp peaks in Fig. 4(d)) demonstrates that the occurrence of two pairs of maxima (each pair rotates with the phase of the driving) coincides with the bifurcation of the classical stationary distribution into two separate point attractors (in the stroboscopic dynamics) in each well. Each of these pairs of attractors rotates with the phase of the driving: Fig. 4(e) is analogous to Fig. 4(d), but delayed in phase by $\pi/2$ (due to the symmetry, Eq. (4), the corresponding states at phase π and $3\pi/2$ are related to those shown in Figs 4(d),(e), respectively, by reflections with respect to the origin). The broadening of the quantum-mechanical stationary distributions is essentially an incoherence effect caused by reservoir-induced noise. This is demonstrated in Fig. 4(f): In order to simulate the coupling of the spatial coordinate to the reservoir (see Eq. (7)), we generated noisy classical trajectories by adding uncorrelated Gaussian noise in the x-direction, with arbitrary strength and in time steps much smaller than the driving period. The dot clouds in Fig. 4(f) represent the stationary state of this ensemble corresponding to, and at the same phase as, the state shown in Fig. 4(d). Despite the very crude modeling, we find a good qualitative agreement. The comparison of the noisy ensemble with the full phase-space traces of the associated deterministic classical attractors (continuous lines in Fig. 4(f)) shows in addition that both classical and reservoir-induced quantum-mechanical noise act predominantly in the direction of the stationary deterministic classical flow. (For the present parameter values, there is a fifth classical point attractor which, however, has no discernible counterpart in the quantal stationary state.) Fig. 5 is devoted to the influence of dissipation on the coherent suppression of tunneling. In the vicinity of the manifold where the relevant quasienergy crossing occurs, the conservative time evolution contains very small energy scales and correspondingly large time scales. In order to obtain an adequate description by the master equation for $\sigma(t)$, we start from the full Eq. (12) and restrict the rotating-wave approximation to its part (iii), *i.e.*, drop only the terms with $k \neq k'$. Figs 5(a) and 5(b) show the time evolution as in Fig. 2(b), but for parameters of the driven double well slightly offset from the localization manifold, for $\gamma = 10^{-6}$ and various values of T. For small temperature, $P^{\sigma(t_0)}(t_n)$ exhibits a slowly decaying coherent oscillation with a very long period, due to the small detuning from the quasienergy crossing (the fast decay of superposed oscillations that reflect the admixture of other quasienergy states, which is present also here, is shown in Fig. 5(b)). Asymptotically, the distribution among the wells is completely thermalized. With increasing temperature, the decay time of the coherent oscillation first decreases until this oscillation is suppressed from the beginning (not shown in Fig. 5(c), see below). After going through a minimum, however, the thermalization time increases again. At a characteristic temperature T^*, this time scale reaches a resonance-like *maximum* where the incoherent processes induced by the reservoir *stabilize* the localization of the wave packet in one of the wells and thus compensate for the detuning introduced deliberately. In Fig. 5(c), we present the temperature dependence of the decay time τ (defined by $P^{\sigma(t_0)}(t_n) \approx \exp(-n/\tau)$) for three values of the detuning $\Delta\omega = \omega - \omega_{\rm loc}(S)$: With increasing $\Delta\omega$, the maximum is shifted towards higher temperatures and decreases in height.

5. Summary

The present work is intended to give an overview over various aspects of tunneling in a double well under the influence of periodic driving. (For the effects of periodic driving on the tunneling decay out of a single metastable well, see Ref. [32].) The basic notions to discuss a periodically driven quantal dynamics are provided by Floquet theory, a time-domain analogue of Bloch theory: Quasienergies and quasienergy eigenstates replace the familiar concepts of energy eigenvalues and eigenstates, respectively. Consequently, driven tunneling is adequately analyzed in terms of the quasienergies that contribute to the time evolution of a state initially localized in one of the wells.

In the limits of slow and of fast driving, the familiar tunneling dynamics is merely accelerated. Qualitative modifications occur where the quasienergies corresponding to the groundstate doublet of the unperturbed double well interact, in parameter space, with quasienergies corresponding to higher-lying unperturbed eigenenergies. In particular, avoided crossings can lead to quite complex quantum beats, while at specific exact crossings which form one-dimensional manifolds in parameter space, an almost complete suppression of tunneling occurs. It is essentially a two-quasienergy interference phenomenon, in fact much of it can be understood in terms of a two-state approximation of the double well.

Towards the classical limit, both diffusive transport due to classical chaos and incoherent processes induced by the environment become significant ingredients of the physics of the driven double well. A dissipative dynamics, introduced by coupling the double well to a reservoir, leads to a broadening of the quasienergy lines and to a corresponding decay of the complex quantum beats observed in the conservative case. The steady states approached by this system form the quantal analogues of the attractors of the damped bistable Duffing oscillator. In contrast, the coherent suppression of tunneling shows a less obvious response to incoherent perturbations: At a suitably chosen temperature, it is stabilized rather than destroyed by the coupling to the environment. This surprising result bears some resemblance of the stabilization of classical instable equilibrium states by multiplicative noise [33, 34], but it is not yet understood quantitatively.

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