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Quadratic Objective Functions from Ordinal Data

Towards More Reliable Representations of Policymakers' Preferences

von

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Summary

An approach recently developed by *Tanguiane* for constructing quadratic objective functions from a minimal set of answers to simple questions about indifference is shown to suffer from a number of deficiencies in practical work. Most importantly, objective functions turn out to be highly non-robust with respect to small errors in the decision maker's answers. For definite objective functions an alternative method is proposed which avoids these problems. In addition a modification is suggested for the case of an indefinite objective function which will at least increase the robustness of the function constructed.

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1. Introduction

Quadratic objective functions are widely used in models of rational decision making. In economics, for example, they have been an essential part of policy analysis based on econometric models since *Theil (1964)* introduced quadratic welfare or loss functions to the theory of economic policy developed earlier by *Tinbergen (1952).* But also purely theoretical analyses as for example in *Barro and Gordon (1983), Alesina and Tabellini (1987)* or, more recently, *Nordhaus (1994)* apply quadratic objective functions in order to describe policy makers' preferences.

As *Theil (1964, p. 4)* pointed out, quadratic objective functions are attractive for a number of reasons. Most importantly, they allow for diminishing marginal rates of substitution between objective variables while at the same time being reasonably simple. In fact, a quadratic objective function can be considered the simplest functional form with this property, given that a linear specification would imply constant marginal rates. Once linear specifications are ruled out quadratic functions also provide the simplest way to allow for convexity or concavity. Finally, it should be noted that a quadratic objective function can be derived from a second-order *Taylor* approximation of a more general preference function, i.e., it can be considered a simplified representation of some more complicated functional form.

Knowing that a decision maker's preferences can be approximated by a quadratic objective function is not very helpful by itself. To analyze optimal decisions we need to know the precise specification of this function which expresses the intensity of the decision maker's preferences towards the objective variables. This is relevant both in empirical and in theoretical work.

Consider again the example of economics. Given an econometric model of an economy, optimal policy can only be derived if the policy maker's preferences for, say, employment and price stability are expressed in the form of coefficients in the objective function. In the field of theory where specifications are often determined by a combination of prior knowledge - central banks dislike inflation more than unemployment etc. - and the desire to keep the analytics simple, it should be noted that the exact form of the objective function can be crucial to the propositions derived. This is particularly true once we consider interactions among independend policy makers such as central banks and governments which control different policy instruments and pursue different objectives. It turns out, for example, that answers to the questions of whether these policy instruments are strategic substitutes or strategic complements and of whether or not policy makers face a collective dilemma situation are very sensitive to the precise specifications of the objective functions.

Given this need to know the coefficients of objective functions, we can then ask how such knowledge can be generated. There might be cases where the logic of revealed preferences can be applied: Observing the values of policy instruments actually chosen by a decision maker and assuming rational behavior, we could infer her preferences from her policies. Notice, however, that this inference can only be made if we observe a sufficiently high number of policy decisions, and restrictions, i.e., the reduced form model connecting instruments and objectives, remain unchanged over the observation period. These preconditions probably prevent us from applying this approach to constructing preference functions for economic policy. Alternatively, we could rely on questionnaires to gather information on preferences. This approach was pursued e.g. by *Husges and Gruber (1991)* in an attempt to derive an objective function for economic policy. In an experimental setup the authors generated a questionnaire of 28 different combinations of four objective variables and asked participants to assign utility levels between 0 and 100 to the alternatives. From the data received they estimated the unknown parameters of a quadratic objective function with ordinary least squares.

Tanguiane (1992) pointed out that the econometric approach pursued by *Husges and Gruber (1991)* assumes cardinal data and therefore violates the fundamental principle that an objective function representing some preference ordering is defined only to within a monotonic transformation. In two papers *Tanguiane (1992, 1993)* developed an alternative concept based on simple questions on indifference which generate only ordinal data. However, closer inspection shows that *Tanguiane's* approach while being theoretically elegant and attractive tends to suffer from a number of deficiencies in practical work. The purpose of the present paper is to show what these deficiencies are and how they can be overcome by either modifying *Tanguiane's* method or by developing an alternative approach.

The plan of the paper is as follows: In section 2 *Tanguiane's (1992, 1993)* idea for constructing a quadratic objective function from ordinal data is briefly outlined (2.1) and a number of weaknesses of this method are pointed out (2.2). Section 3 deals with ways to get around these difficulties. In particular, an alternative method is proposed for definite objective functions (3.1). For the more general case the analysis sticks to *Tanguiane's* approach and suggests a modification in order to make it more robust (3.2). Section 4 briefly sums up.

2. Finding quadratic objective functions - Tanguiane's approach

2.1. Generating minimal data from simple questions on indifference

Consider a decision problem with n objective variables which are collected in a vector $\mathbf{y} = [y_1,\ldots,y_i,\ldots,y_n]'$. For the decision maker's preferences it is assumed that they can be represented by a quadratic objective function $w(\mathbf{y}) = \mathbf{b}'\mathbf{y} + \frac{1}{2}\mathbf{y}'\mathbf{B}\mathbf{y}$ with $\mathbf{B} = \mathbf{B}'$ which is seen as a second-order *Taylor* approximation of some more general function. Notice that an objective function defined in terms of quadratic differences from a vector $\mathbf{y}^*$ of ideal values, i.e., $w(\mathbf{y}) = (\mathbf{y}-\mathbf{y}^*)'\mathbf{B}(\mathbf{y}-\mathbf{y}^*)$, can always be transformed into the specification given before by setting $\mathbf{b} = -\mathbf{B}\mathbf{y}^*$ and ignoring the constant term. Furthermore, in a more general case where the decision maker also holds preferences on her n' instruments $\mathbf{x} = [x_1,\ldots,x_l,\ldots,x_{n'}]'$, these variables can be included in our specification by building a stacked vector of **y** and **x** and extending **b** and **B** appropriately. To keep the notation simple, I stick to **y** and its n components.

Tanguiane (1992, 1993) designed a method of asking a decision maker $(n^2+3n)/2-1$ questions on her indifference among simple alternatives which generate the minimal amount of data necessary to calculate the unknown parameters of **b** and **B**. To understand the logic of this approach consider the case of $n=3$ which can readily be generalized to higher dimensions. Table 1 provides information on the questions asked.

Table 1

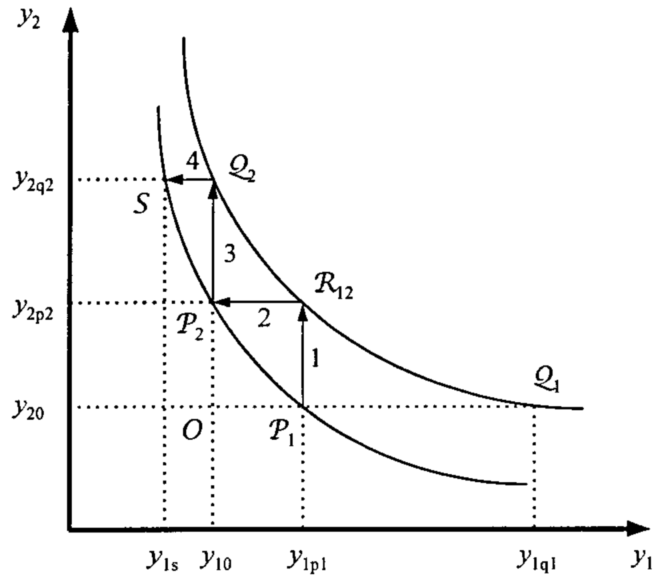
Tanguiane's questionnaire for $n=3$

question	point	y_1	y_2	y_3	welfare index
-	$\mathcal{P}_1$	y_{1p1}	y_{20}	y_{30}	0
-	$\mathcal{Q}_1$	y_{1q1}	y_{20}	y_{30}	1
1	$\mathcal{R}_{12}$	y_{1p1}	$y_{2p2} = ?$	y_{30}	1 $(\sim\mathcal{Q}_1)$
2	$\mathcal{P}_2$	$y_{1p2} = y_{10} = ?$	y_{2p2}	y_{30}	0 $(\sim\mathcal{P}_1)$
3	$\mathcal{Q}_2$	y_{10}	$y_{2q2} = ?$	y_{30}	1 $(\sim\mathcal{Q}_1)$
4	$\mathcal{S}$	$y_{1s} = ?$	y_{2q2}	y_{30}	0 $(\sim\mathcal{P}_1)$
-	$\mathcal{O}$	y_{10}	y_{20}	y_{30}	-
5	$\mathcal{P}_3$	y_{10}	y_{20}	$y_{3p3} = ?$	0 $(\sim\mathcal{P}_1)$
6	$\mathcal{Q}_3$	y_{10}	y_{20}	$y_{3q3} = ?$	1 $(\sim\mathcal{Q}_1)$
7	$\mathcal{R}_{13}$	y_{1p1}	y_{20}	$y_{3r13} = ?$	1 $(\sim\mathcal{Q}_1)$
8	$\mathcal{R}_{23}$	y_{10}	y_{2p2}	$y_{3r23} = ?$	1 $(\sim\mathcal{Q}_2)$

Questioning starts at a vector $(y_{1p1}, y_{20}, y_{30})$ which is called alternative $\mathcal{P}_1$ and is chosen such that it represents the present situation. $(y_{1q1}, y_{20}, y_{30})$ is a second point of reference, denoted by $\mathcal{Q}_1$, which differs from $\mathcal{P}_1$ only in its first component. Assume the difference to be such that the decision maker prefers $\mathcal{Q}_1$ to $\mathcal{P}_1$, i.e., $\mathcal{P}_1 \prec \mathcal{Q}_1$. For later use assign to $\mathcal{P}_1$ a level of 0 and to $\mathcal{Q}_1$ a level of 1 of the welfare index w. A question mark in table 1 indicates an objective variable the value of which has to be given by the decision maker in order to indicate her indifference (~) between this particular alternative and some other alternative in the table. In the first question the decision maker is asked to change the second variable in $\mathcal{P}_1$ such that she is indifferent between $\mathcal{Q}_1$ and the new point $\mathcal{R}_{12}$ she created. Continuing from $\mathcal{R}_{12}$ the first variable is changed in order to achieve indifference with $\mathcal{P}_1$. The new point is denoted by $\mathcal{P}_2$. Substituting the first variable of $\mathcal{P}_2$ into $\mathcal{P}_1$ yields another point denoted by O which is serves as a point of reference in later questions. Altering the second component of $\mathcal{P}_2$ to create indifference with $\mathcal{P}_1$ yields $\mathcal{Q}_2$, from which S is derived by changing the first variable. Notice that the six questions considered so far took the value of y_3 as given at a level of y_{30} and could therefore be represented in (y_1, y_2)-space. Next, points $\mathcal{P}_3$ and $\mathcal{Q}_3$ are generated in (y_1, y_3)-space by starting from O and changing the third variable such that there is indifference to $\mathcal{P}_1$ and $\mathcal{Q}_1$, respectively. Finally, for points $\mathcal{P}_1$ and $\mathcal{P}_2$ the decision maker is asked to name changes in the third variable creating indifference to $\mathcal{Q}_1$ and $\mathcal{Q}_2$, respectively, which yields $\mathcal{R}_{13}$ and $\mathcal{R}_{23}$. All points denoted by $\mathcal{Q}_i$ represent the same welfare level and differ only in component *i* from a corresponding point $\mathcal{P}_i$. There is also indifference among all points $\mathcal{P}_i$ which are on the lower indifference curve. Point $\mathcal{R}_{ij}$ in (y_i, y_j)-space yields the same welfare level as $\mathcal{Q}_i$ and $\mathcal{Q}_j$, and differs from $\mathcal{P}_i$ only in component *j*.

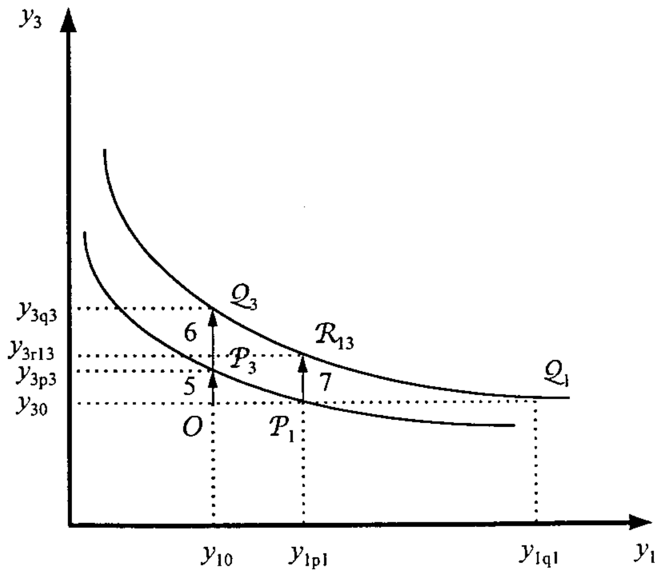
This process of data generation can be depicted in simple diagrams in (y_i, y_j)-space where variables y_k not included in a diagram are fixed at levels y_{k0}. Consider first figure 1 in (y_1, y_2)-space where $\mathcal{P}_1$ and $\mathcal{Q}_1$ are given and all points share a common value y_{30} of the third objective variable. Arrows indicate the four questions asked in this two-dimensional subspace of $\Re^3$. Question 1, for example, is as follows: Starting from $\mathcal{P}_1$, by how much should y_2 be increased in order to yield a point which give the same welfare level as $\mathcal{Q}_1$? Or, to use the coordinates $(y_{1p1}, y_{20}, y_{30})$ of $\mathcal{P}_1$ and $(y_{1q1}, y_{20}, y_{30})$ of $\mathcal{Q}_1$: Which value of y_2 above y_{20} is equivalent to an increase in y_1 from y_{1p1} to y_{1q1}? Once $\mathcal{P}_2$ is known, the unknown coordinate y_{10} of point O which will be needed as a „bridge“ to other two-dimensional spaces can be inferred.

Figure 1
Tanguiane's approach: indifference in (y_1, y_2)-space



Consider next figure 2 in (y_1, y_3)-space with the second variable fixed at y_{20}. Points $\mathcal{P}_1$, $\mathcal{Q}_1$ and O from figure 1 can also be included in this diagram. They serve as reference points for questions 5-7. In the general case of $n > 3$ objective variables equivalent diagrams in (y_1, y_j)-space would have to be drawn for all $j = 3,\ldots,n$, keeping y_k, $k = 2,\ldots,n$, $k \neq j$, at the levels y_{k0}.

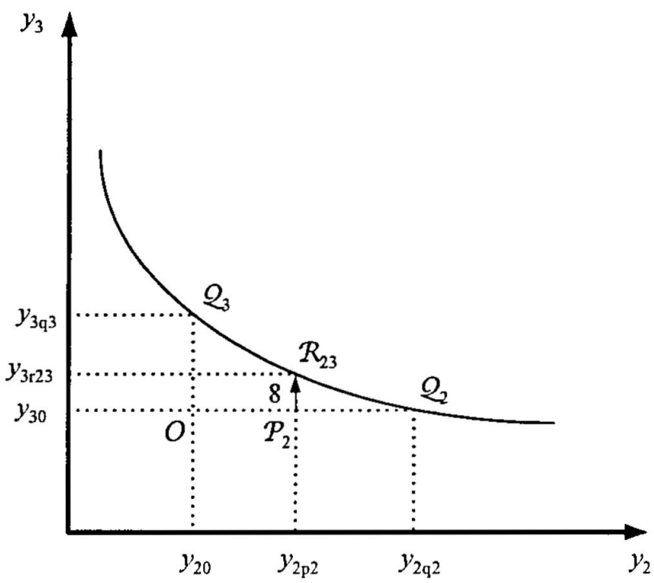
Figure 2
Tanguiane's approach: indifference in (y_1, y_3)-space



For $n=3$ the (final) question 8 can be represented in figure 3 in (y_2, y_3)-space with y_1 fixed at y_{10}. $n>3$ would call for equivalent diagrams in (y_i, y_j)-space for all $j=3,\ldots,n$ and all $i=2,\ldots,j-1$.

Figure 3

Tanguiane's approach: indifference in (y_2, y_3)-space



For a given preference ordering „≺" which by assumption can be represented by a quadratic objective function, *Tanguiane (1992)* showed that $\mathcal{P}_1 \prec \mathcal{Q}_1$ together with the data points S, $\mathcal{P}_i$, $\mathcal{Q}_i$ and $\mathcal{R}_{ij}$, $j=2,3$ and $i=1,\ldots,j-1$, generated in the questionnaire permit unique construction of the objective function w(·) which represents the decision maker's ordinal preferences. A scaling constant d is introduced such that

$$\mathrm{w}(\mathcal{P}_1)+d=0, \qquad \mathrm{w}(\mathcal{Q}_1)+d=1. \tag{1}$$

The way the questionnaire was set up implies

$$\begin{aligned} &\mathrm{w}(S)=\mathrm{w}(\mathcal{P}_1)=\mathrm{w}(\mathcal{P}_2)=\mathrm{w}(\mathcal{P}_3), \\ &\mathrm{w}(\mathcal{Q}_1)=\mathrm{w}(\mathcal{Q}_2)=\mathrm{w}(\mathcal{Q}_3)=\mathrm{w}(\mathcal{R}_{12})=\mathrm{w}(\mathcal{R}_{13})=\mathrm{w}(\mathcal{R}_{23}), \end{aligned} \tag{2}$$

and therefore

$$\begin{aligned} &\mathrm{w}(S)+d=0, && \mathrm{w}(\mathcal{R}_{ij})+d=1, \quad j=2,3, \quad i=1,\ldots,j-1, \\ &\mathrm{w}(\mathcal{P}_j)+d=0, \quad j=1,2,3, && \mathrm{w}(\mathcal{Q}_j)+d=1, \quad j=1,2,3. \end{aligned} \tag{3}$$

For example, the first of these equations is given by

$$\begin{aligned} \mathrm{w}(S)=\mathrm{w}(y_{1s}, y_{2q2}, y_{30}) &= \tfrac{1}{2}B_{11}y_{1s}^2+\tfrac{1}{2}B_{22}y_{2q2}^2+\tfrac{1}{2}B_{33}y_{30}^2+B_{12}y_{1s}y_{2q2} \\ &+B_{13}y_{1s}y_{30}+B_{23}y_{2q2}y_{30}+b_1y_{1s}+b_2y_{2q2}+b_3y_{30}+d=0. \end{aligned} \tag{4}$$

Substituting the values of the three objective variables yields ten such equations which are linear in the constant *d* and in the nine unknown parameters of **b** and **B**. In the general case of n objective variables there are $1+2n+n(n-1)/2=1+(n^2+3n)/2$ equations in *d* and the other $n+n(n+1)/2=(n^2+3n)/2$ parameters of $w(\cdot)$. For systems like (3) a more general notation is useful. Denote by $\mathbf{y}^i \in \Re^n$ a point initially given or generated by *Tanguiane's* approach when there are n objectives. The equation corresponding to point $\mathbf{y}^i$ can be written as

$$\underbrace{[(v\{\mathbf{y}^i(\mathbf{y}^i)' - \tfrac{1}{2}dg(\mathbf{y}^i(\mathbf{y}^i)')\})' \quad (\mathbf{y}^i)' \quad 1]}_{(\breve{\mathbf{y}}^i)'} \cdot \begin{bmatrix} v(\mathbf{B}) \\ \mathbf{b} \\ d \end{bmatrix} = w^i, \; i=1,\ldots,n(n+3)/2+1. \quad (5)$$

The first part of $\breve{\mathbf{y}}^i$ is an $n(n+1)/2$ vector capturing cross- and own-products of all elements of $\mathbf{y}^i$ where the latter are multiplied by a factor 1/2.[1] In total, $\breve{\mathbf{y}}^i$ is a vector of $(n(n+3)/2)+1$ elements which covers all the numerical coefficients of all unknown parameters of **B**, **b** and *d* in an equation like (4). For $n=3$, for example, we have

$$\begin{aligned} &[(v\{\mathbf{y}^i(\mathbf{y}^i)' - \tfrac{1}{2}dg(\mathbf{y}^i(\mathbf{y}^i)')\})' \quad (\mathbf{y}^i)' \quad 1]' = \\ &\qquad [\tfrac{1}{2}y_1^i y_1^i \quad y_1^i y_2^i \quad y_1^i y_3^i \quad \tfrac{1}{2}y_2^i y_2^i \quad y_2^i y_3^i \quad \tfrac{1}{2}y_3^i y_3^i \quad y_1^i \quad y_2^i \quad y_3^i \quad 1] \qquad (6) \\ &[v(\mathbf{B}) \quad \mathbf{b} \quad d]' = [B_{11} \quad B_{12} \quad B_{13} \quad B_{22} \quad B_{23} \quad B_{33} \quad b_1 \quad b_2 \quad b_3 \quad d] \end{aligned}$$

The equations for all points $\mathbf{y}^i$ yield a linear system

$$\begin{bmatrix} (\breve{\mathbf{y}}^1)' \\ \vdots \\ (\breve{\mathbf{y}}^{n(n+3)/2+1})' \end{bmatrix} \cdot \begin{bmatrix} v(\mathbf{B}) \\ \mathbf{b} \\ d \end{bmatrix} = \begin{bmatrix} w^1 \\ \vdots \\ w^{n(n+3)/2+1} \end{bmatrix}, \qquad (7)$$

which after introducing a quadratic matrix $\breve{\mathbf{Y}}$ and vectors $\breve{\mathbf{b}}$ and $\mathbf{w}$ can be written as

$$\breve{\mathbf{Y}} \cdot \breve{\mathbf{b}} = \mathbf{w}. \qquad (8)$$

Solving (8) for $\breve{\mathbf{b}}$ yields *d* and the parameters of the decision maker's objective function.

Whether or not decision makers in general, and economic policy makers in particular, hold well-behaved preferences permitting construction of an objective function from the data generated in *Tanguiane's* questionnaire is an empirical matter. There are, however, a number of conceptual problems inherent to *Tanguiane's* approach which are present even if the decision maker's preferences are well-behaved. The following subsection will address these problems.

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1 $dg(\cdot)$ extracts the main diagonal from a matrix and forms a diagonal matrix. $v(\cdot)$ stacks the columns of a matrix, ignoring all elements above the main diagonal.

2.2. Some weaknesses of Tanguiane's approach

Even for a decision maker with well-behaved preferences it might be difficult to figure out the precise value of an objective variable that creates indifference in the questionnaire. In reality, we can expect answers to deviate slightly from the indifference curves depicted in figures 1-3. Our method of deriving an objective function should therefore be robust against minor mistakes such as, for example, giving a value of 2.2% instead of 2% for an inflation rate in the case of preferences for macroeconomic variables. Unfortunately, the approach developed by *Tanguiane (1992, 1993)* turns out to be very sensitive with respect to small mistakes. Consider a simple example with $n = 2$, where true preferences are assumed to be represented by

$$w(y) = [-6 \quad -4]y + \tfrac{1}{2}y' \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} y . \tag{9}$$

Defining $\mathcal{P}_1$ and $\mathcal{Q}_1$ as (1, 1) and (1.5, 1), respectively, the questioning as indicated in figure 1 yields a system (8). If the decision maker is absolutely precise in her answers, the unique solution to the system is $B_{11} = 1.6$, $B_{12} = B_{21} = 0.8$, $B_{22} = 0.8$, $b_1 = -4.8$ and $b_2 = -3.2$ which under the decision maker's ordinal preferences is equivalent to (9) and leads to the same (y_1, y_2)-combination (2, 2) for a (unique) minimum. Suppose now, the decision maker is slightly mistaken in her answer to the last question. The correct coordinates of point S are given by (0.0450881, 1.90686). If instead she indicates that starting from $\mathcal{Q}_2$ indifference with $\mathcal{P}_1$ or $\mathcal{P}_2$ is given for a value of $y_{1s} = 0{,}0405793$ which is 10% below the true value, the parameters derived will be (approximately) $B_{11} = 1.7$, $B_{12} = B_{21} = 0.9$, $B_{22} = 0.8$, $b_1 = -5.0$ and $b_2 = -3.3$. While the first-order conditions for optimization of this objective function yield (1.93, 1.97) which at first sight looks tolerable since it is not very far away from the values of (2, 2) found for the true function, it turns out that the matrix **B** is now indefinite, i.e., the objective function identified has no optimum value. Things can even get worse if the questionnaire starts out from other initial points $\mathcal{P}_1$ and $\mathcal{Q}_1$. With $\mathcal{P}_1 = (1, 0.5)$ and $\mathcal{Q}_1 = (1.5, 0{,}5)$, for example, a 10% mistake in y_{1s} leads to an objective function with a minimum at (1.66, 1.69); starting from $\mathcal{P}_1 = (1, 0.5)$ and $\mathcal{Q}_1 = (1.25, 0{,}5)$ we end up at a point (0.94, 0.74) which is again neither a minimum nor a maximum. Notice that mistakes at points other than S should have even more severe consequences because they would also affect later questions and answers in the questionnaire.

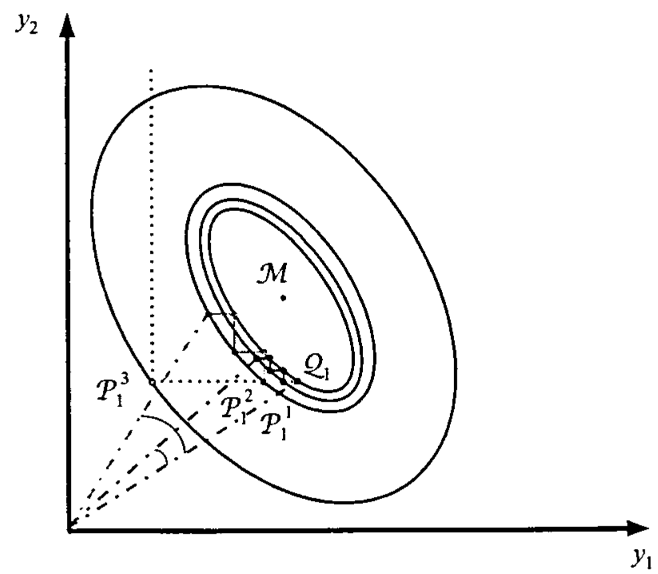
This non-robustness with respect to small errors in the answers is not confined to the example presented here. It can also be found in the three- and four-dimensional examples given by *Tanguiane (1992)* and *Tanguiane and Gruber (1993)*. While the objective functions they assume do not have unconstrained minima or maxima, it can easily be shown that solutions to optimization problems subject to linear restrictions vary wildly in the presence of small errors in the answers.

When looking for the underlying source of this non-robustness, attention is immediately attracted to the numerical properties of the system (8) and the matrix $\breve{\mathbf{Y}}$ in particular. It turns out that the data matrix $\breve{\mathbf{Y}}$ generated by the questionnaire is typically ill-conditioned which means that the solution to (8) is very sensitive to small changes in $\breve{\mathbf{Y}}$. In the terminology of econometrics there is a problem of multicollinearity. The „condition number" $\kappa(\breve{\mathbf{Y}})$ calculated as the ratio of the maximum to the minimum singular value of $\breve{\mathbf{Y}}$ is an indicator for the magnitude of this problem.[2] For the three cases considered in our example we get condition numbers of $7 \cdot 10^2$, $1 \cdot 10^3$ and $3 \cdot 10^5$, respectively, which are way beyond the values of 20-30 normally considered as critical in the literature (see e.g. *Greene, 1993, p. 33*). Results for the examples used by *Tanguiane (1992)* and *Tanguiane and Gruber (1993)* are in a similar range.

For the intuition consider figure 4, where questions 1-4 are represented as in figure 1, but the labels of most data points are omitted to avoid cluttering the diagram.

Figure 4

Tanguiane's approach: the problem of multicollinearity



2 The columns of $\breve{\mathbf{Y}}$ were normalized to 1 (cf. *Belsley et al., 1980, pp. 99, 183*). The condition number could also be calculated from the maximum and minimum eigenvalues of $\breve{\mathbf{Y}}'\breve{\mathbf{Y}}$.

Attention is restricted to the two-dimensional case, but the insights are equally valid for higher dimensions. The greater the angle between the vector (y_{1q1}, y_{20}) belonging to $\mathcal{Q}_1$ and the vector (y_{1s}, y_{2q2}) belonging to S, the smaller the problems of multicollinearity and non-robustness with respect to errors will be. As can be seen from figure 4, initial points $\mathcal{P}_1$ and $\mathcal{Q}_1$ are crucial to the condition of the data matrix generated. For example, using $\mathcal{Q}_1$ and $\mathcal{P}_1^2$ is superior to $\mathcal{Q}_1$ and $\mathcal{P}_1^1$. This suggests an increase in the horizontal distance between $\mathcal{P}_1$ and $\mathcal{Q}_1$ as a solution to the multicollinearity problem. However, figure 4 indicates that there are limits depending on the unknown preferences. If questioning starts out from $\mathcal{Q}_1$ and $\mathcal{P}_1^3$, for example, the decision maker will not be able to answer the first question because there is no (real) value of y_2 creating indifference with $\mathcal{Q}_1$. For a smaller horizontal distance, non-existence of indifference might appear at a later question as can be seen in our numerical example, where question 3 can not be answered if the initial values are (2, 0.5) and (1, 0.5). There is, therefore, a trade-off between making the objective function derived more reliable by reducing multicollinearity and running the risk that at one point the decision maker will not be able to answer a question. Given that the shape of the indifference curves is unknown, there is no way of telling in advance which pair of initial values would perform well in the sense of leading to a well-conditioned data matrix $\breve{\mathbf{Y}}$.[3]

Notice that non-existence of indifference will not appear in cases like the ones presumed in *Tanguiane (1992)* and *Tanguiane and Gruber (1993)*, where only cross-products of the objective variables were included in the underlying objective function which implies hyperbolic indifference curves of the kind depicted in figures 1-3 above. If, however, preferences are truly quadratic in the sense that the squares of the objective variables show up, and if the objective function is (positive or negative) definite, indifference curves have elliptic shapes as in figure 4 (see e.g. *Bronstein and Semendjajew, 1991, pp. 220-221*), implying the possibility of dead ends in the questioning. Such objective functions are indeed the ones considered most relevant e.g. in the analysis of economic policy and in linear-quadratic decision problems in general.[4]

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3 The same holds vis-à-vis the idea that even for a given horizontal distance of $\mathcal{P}_1$ and $\mathcal{Q}_1$ there are - depending on the indifference curves' curvature - pairs of initial values that perform better than others.

4 Indefinite functions like those in *Tanguiane (1992)* and *Tanguiane and Gruber (1993)* carry the disadvantage that existence of an optimal decision depends on the precise form of some relationship $\mathbf{y} = \mathbf{R}\mathbf{x} + \mathbf{s}$ among objectives $\mathbf{y}$, instruments $\mathbf{x}$, and some exogenous variables $\mathbf{s}$. Admittedly, it is an empirical question which type of function prevails.

Elliptic indifference curves cause yet another problem. So far each point seemed to lead to a unique successor. While this was necessarily true in the setup of figures 1-3, it should be noted that questions in figure 4 need not have unique answers. Instead of moving left, the decision maker could also have indicated a point of indifference to the right. Similarly, instead of moving up a little, she could also have moved up more to a second point of indifference in the vertical direction. We could restrict the directions in which the answers are moving and create a unique path by introducing two rules: (1) From $\mathcal{Q}_i$ and $\mathcal{R}_{ij}$ only moves to the left, and from $\mathcal{P}_i$ only upward moves are admitted. (2) Upward moves from $\mathcal{P}_i$ have to be minimal. However, without knowing the center of the indifference curves which represents the minimum or maximum of the objective function, we can not tell in advance whether these rules will increase or decrease the danger of a dead end and whether they will decrease or increase the multicollinearity problem.

Finally, it should be noted that in case the objective function is (negative or positive) definite and the decision maker can name a unique maximum or minimum $\mathbf{y}^*$ in $\Re^n$, the approach developed by *Tanguiane (1992)* is inefficient since it does not make use of all the information available. In this case parameters of **b** need not be calculated because the objective function can be written in the equivalent form $w(\mathbf{y}) = (\mathbf{y} - \mathbf{y}^*)'\mathbf{B}(\mathbf{y} - \mathbf{y}^*)$ and **b** can be derived from $\mathbf{b} = -\mathbf{B}\mathbf{y}^*$. Questioning should therefore start with a question on an ideal vector $\mathbf{y}^*$. If there is one, the number of unknown parameters and the number of further questions to be asked is reduced. If the questionnaire were still minimal, a number of $n(n+1)/2+1$ points $\mathbf{y}^i$ compared to $n(n+1)/2+n+1$ points before would have to be generated, i.e., the number of questions could at most be reduced by $n-1$. Given $\mathbf{y}^*$ the system of equations (8) can be re-written. Using $\mathbf{b} = -\mathbf{B}\mathbf{y}^*$, an equation for observation $\mathbf{y}^i$ is now given by

$$w(\mathbf{y}^i) = -(\mathbf{B}\mathbf{y}^*)'\mathbf{y}^i + \tfrac{1}{2}(\mathbf{y}^i)'\mathbf{B}\mathbf{y}^i + d = (\tfrac{1}{2}\mathbf{y}^i - \mathbf{y}^*)'\mathbf{B}\mathbf{y}^i + d, \tag{10}$$

which we can write in vector notation as

$$\begin{aligned}&[(\mathrm{v}\{\mathbf{y}^i(\mathbf{y}^i)' - \tfrac{1}{2}\mathrm{dg}(\mathbf{y}^i(\mathbf{y}^i)')\} - \mathrm{v}\{\mathbf{y}^i(\mathbf{y}^*)'\} - \mathrm{v}\{(\mathbf{y}^i(\mathbf{y}^*)')' - \mathrm{dg}(\mathbf{y}^i(\mathbf{y}^*)')\})' \quad 1]\\ &\cdot\begin{bmatrix}\mathrm{v}(\mathbf{B})\\ d\end{bmatrix} = \mathrm{w}^i, \quad i = 1,\ldots,n(n+1)/2+1.\end{aligned} \tag{11}$$

Denote the first vector by $\breve{\mathbf{y}}^i$. For $n = 3$ this is

$$\begin{aligned}(\breve{\mathbf{y}}^i)' = [&\tfrac{1}{2}y_1^i y_1^i - y_1^i y_1^* \quad y_1^i y_2^i - y_2^i y_1^* - y_1^i y_2^* \quad y_1^i y_3^i - y_3^i y_1^* - y_1^i y_3^*\\ &\tfrac{1}{2}y_2^i y_2^i - y_2^i y_2^* \quad y_2^i y_3^i - y_3^i y_2^* - y_2^i y_3^* \quad \tfrac{1}{2}y_3^i y_3^i - y_3^i y_3^* \quad 1].\end{aligned} \tag{12}$$

Combining all points initially given or generated from the decision maker's answers leads to a system

$$\begin{bmatrix} (\breve{\mathbf{y}}^1)' \\ \vdots \\ (\breve{\mathbf{y}}^{n(n+1)/2+1})' \end{bmatrix} \cdot \begin{bmatrix} \mathrm{v}(\mathbf{B}) \\ d \end{bmatrix} = \begin{bmatrix} \mathrm{w}^1 \\ \vdots \\ \mathrm{w}^{n(n+1)/2+1} \end{bmatrix}, \tag{13}$$

or with appropriately defined matrix and vectors

$$\breve{\mathbf{Y}} \cdot \breve{\mathbf{b}} = \mathbf{w} . \tag{14}$$

For a definite objective function only data to solve (14) are needed.

To sum up, a number of weaknesses were identified which make *Tanguiane's* approach less attractive in practice compared to its theoretical advantages such as using a minimal set of ordinal data based on very simple questions about indifference. The multicollinearity problem is potentially most damaging because it can make the construction of objective functions very unreliable, if decision makers are a little inaccurate in their answers. As was shown, attempts to „solve“ this problem by choosing initial points skilfully are not very promising and carry the risk of creating dead ends in the questionnaire. One could argue that this could be overcome by a process of trial and error where $\mathcal{P}_1$ and $\mathcal{Q}_1$ are altered until the decision maker is able to answer all questions and the data points generated yield a sufficiently well-conditioned matrix of the linear system (8) or (14). However, this process which lacks much of the theoretical appeal of *Tanguiane's* original idea can be expected to make too many demands on the decision maker's patience. It is therefore worthwhile to examine whether there are improvements to the approach which achieve higher reliability in a more direct fashion.

3. A modified approach

3.1. A questionaire for definite objective functions

For the case of an objective function with a vector $\mathbf{y}^*$ of ideal values a modified approach to constructing the function from ordinal data will be developed which both uses the information on $\mathbf{y}^*$ and reduces the problem of multicollinearity observed before. Consider again $n = 3$ and - more specifically - a minimization problem. In the first question the decision maker is asked to name an ideal combination of the three objective variables which is not inferior to any other combination. If the decision maker is able to provide such a $\mathbf{y}^*$, we proceed according to the questionnaire given in table 2. Otherwise we would return to *Tanguiane's* approach.

Table 2

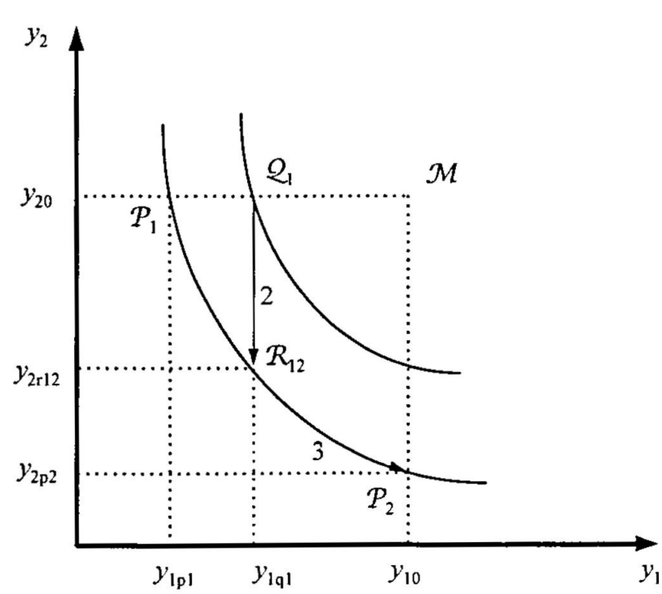
Modified questionnaire for $n = 3$

question	point	y_1	y_2	y_3	welfare index
1	$\mathcal{M}$	y_{10}	y_{20}	y_{30}	-
-	$\mathcal{Q}_1$	y_{1q1}	y_{20}	y_{30}	0
-	$\mathcal{P}_1$	y_{1p1}	y_{20}	y_{30}	1
2	$\mathcal{R}_{12}$	y_{1q1}	$y_{2r12} = ?$	y_{30}	$1\ (\sim\mathcal{P}_1)$
3	$\mathcal{P}_2$	y_{10}	$y_{2p2} = ?$	y_{30}	$1\ (\sim\mathcal{P}_1)$
4	$\mathcal{R}_{13}$	y_{1q1}	y_{20}	$y_{3r13} = ?$	$1\ (\sim\mathcal{P}_1)$
5	$\mathcal{Q}_3$	y_{10}	y_{20}	$y_{3q3} = ?$	$0\ (\sim\mathcal{Q}_1)$
6	S_{23}	y_{10}	$y_{2s23} = ?$	y_{3q3}	$1\ (\sim\mathcal{P}_2)$

Notice a slight change in the notation: While $\mathcal{P}_i \prec \mathcal{Q}_i$ still holds, $\mathcal{P}_i$ and $\mathcal{Q}_i$ are now defined such that they differ from the ideal point $\mathbf{y}^*$ which is denoted by $\mathcal{M}$ only in coordinate *i*. The only difference between $\mathcal{R}_{ij}$ and $\mathcal{Q}_i$ is in coordinate *j*, and the only differenc between S_{ij} and $\mathcal{Q}_j$ is in coordinate *i*. A value of 0 of the welfare index is assigned to all $\mathcal{Q}_i$, whereas all $\mathcal{P}_i$, $\mathcal{R}_{ij}$ and S_{ij} receive a value of 1. Figures 5-7 may be used to clarify the intuition of the questionnaire.

Figure 5

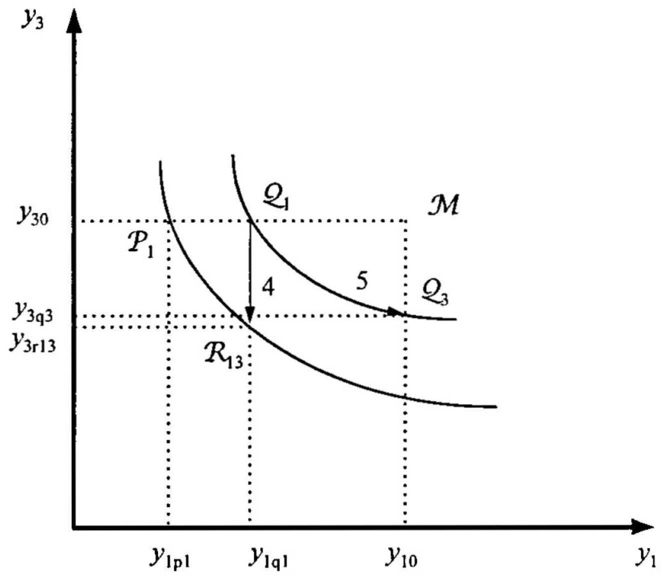
Modified approach: indifference in (y_1, y_2)-space



Given $\mathcal{M}$, $\mathcal{P}_1$ and $\mathcal{Q}_1$ are chosen such that their y_1-values are lower compared to $\mathcal{M}$. Question 2 in figure 5 then determines by how much y_2 can be reduced starting from $\mathcal{Q}_1$ in order to create indifference with $\mathcal{P}_1$. Starting from $\mathcal{M}$, question 3 works analogously.

Figure 6

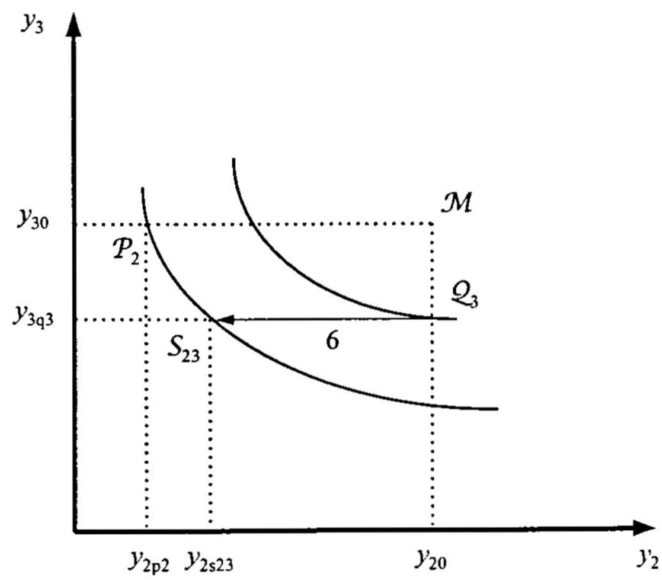
Modified approach: indifference in (y_1, y_3)-space



In figure 6 questions 4 and 5 are asked to determine $\mathcal{R}_{13}$ and $\mathcal{Q}_3$.

Figure 7

Modified approach: indifference in (y_2, y_3)-space



Finally, figure 7 indicates how question 7 is used to find S_{23} starting from $\mathcal{Q}_3$. A total of six questions including the one on $\mathbf{y}^*$ are asked. Together with $\mathcal{P}_1$ and $\mathcal{Q}_1$ this yields eight data points, seven of which can be used in (14) to calculate the seven unknown parameters $B_{11}, B_{12}, B_{13}, B_{22}, B_{23}, B_{33}$ and d.[5]

For the general case of $n \geq 3$ objective variables the following points have to be determined according to the procedure explained in figures 5-7: $\mathcal{Q}_1$ and $\mathcal{Q}_i, i = 3,\ldots,n$, $\mathcal{P}_1$ and $\mathcal{P}_i, i = 2,\ldots,n-1$, $\mathcal{R}_{1i}, i = 2,\ldots,n$, and S_{ij}, where $i = 2,\ldots,n-1$ and $j = i+1,\ldots,n$. This generates a total of $(n-1)(n+4)/2$ points from $(n-1)(n+4)/2-1$ questions including the one on $\mathcal{M}$. Notice that for $n > 3$ the modified approach is not minimal, i.e., it generates more data points than are needed for a solution of (14). Table 3 compares *Tanguiane's* approach to the modified approach with respect to the amount of data processed.

Table 3

Tanguiane's approach versus modified approach

	Tanguiane's approach			modified approach		
n	parameters	points	questions	parameters	points	questions
2	6	6	4	4	4	3
3	10	10	8	7	7	6
4	15	15	13	11	12	11
5	21	21	19	17	18	17
6	28	28	26	22	25	24
⋮	⋮	⋮	⋮	⋮	⋮	⋮
n	$\frac{n^2+3n}{2}+1$	$\frac{n^2+3n}{2}+1$	$\frac{n^2+3n}{2}-1$	$\frac{n^2+n}{2}+1$	$\frac{n^2+3n-4}{2}$	$\frac{n^2+3n-6}{2}$

As can be seen, the modified approach allows the number of questions to be reduced. For $n > 3$, however, this reduction is equal to 2 and constant. At the same time this method generates $n-3$ data points which are superfluous in the sense that (14) can be solved for the unknowns without them. However, since information should not be thrown away, it will make sense to add theses points as additional rows to $\breve{\mathbf{Y}}$, creating

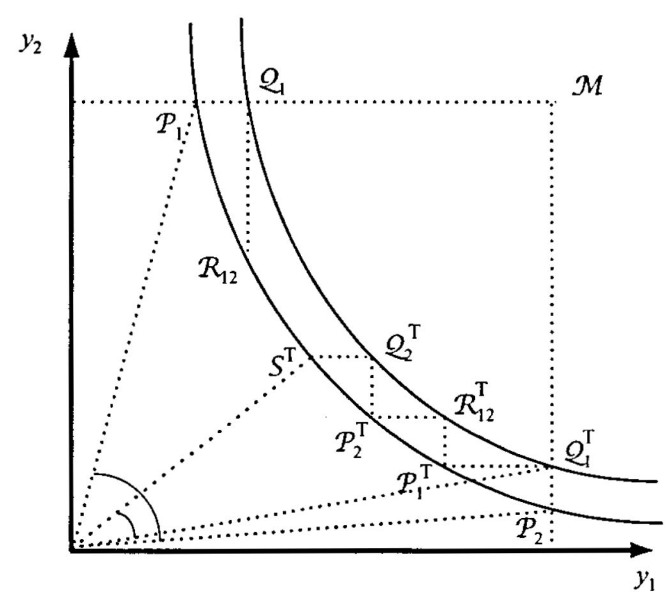
5 Point $\mathcal{M}$ can not be included because it represents neither of the two welfare levels 0 and 1.

an extended data matrix $\breve{\mathbf{Y}}^e$ and an over-determined system $(\breve{\mathbf{Y}}^e)'\breve{\mathbf{Y}}^e\breve{\mathbf{b}} = (\breve{\mathbf{Y}}^e)'\mathbf{w}$. This system can be solved as $\breve{\mathbf{b}} = [(\breve{\mathbf{Y}}^e)'\breve{\mathbf{Y}}^e]^{-1}(\breve{\mathbf{Y}}^e)'\mathbf{w}$ which means that the ordinary least squares estimator is applied to the data. Notice that this use of OLS is not invalidated by the fundamental criticism against employing econometric methods in constructing objective functions. Since there are only two welfare levels 0 and 1 in our data, applying the OLS estimator is not equivalent to implicitly assuming cardinal preferences.

There are other features of the modified approach which appear more important than the number of questions asked. First of all, the potential problem of non-existing indifference is no longer present. As can be seen from figures 5-7, the questions are set up such that there is always a point of indifference.[6] Secondly, and most importantly, as figure 8 indicates the modified approach will lead to a data matrix with a better condition number, making the objective function more robust with respect to mistakes in the decision maker's answers.

Figure 8

Reduction of multicollinearity under the modified approach



This is the result of two facts: (1) In each two-dimensional subspace points horizontally to the left and vertically below $\mathcal{M}$ are created. (2) Since non-existence of indifference

6 Strictly speaking, this statement is only correct if errors in the answers are not too large. If, for example, the y_3-coordinate of Q_3 is so low that Q_3 is erroneously located below the indifference curve belonging to $\mathcal{P}_2$, there will be no real number to answer the question on S_{23}.

can not happen, points $\mathcal{P}_1$ and $\mathcal{Q}_1$ can be moved farther apart which further increases the dispersion of the data points $\mathbf{y}^i$. Consider figure 8 in (y_1, y_2)-space for the intuition. For a given objective function with a unique optimum $\mathcal{M}$ both the points of the modified approach and of *Tanguiane's* approach are included, the latter being marked by a superscript „T". Figure 8 is drawn such that both methods move along the same pair of indifference curves. This is achieved by choosing $\mathcal{Q}_1^{\mathrm{T}}$ such that is is identical to point $\mathcal{Q}_2$ of the modified approach which is not included in the diagram. As can be seen immediately, the data generated by the modified approach span a wider angle than the data created by *Tanguaine's* method. As noted before, this angle could even be increased by moving the pair of initial points farther apart.

Consider a numerical example with $\mathrm{n}=3$ for the effects of this reduction of multicollinearity. Suppose, true preferences can be represented by

$$\mathrm{w}(\mathbf{y}) = [-18 \quad -16 \quad -18]\mathbf{y} + \tfrac{1}{2}\mathbf{y}' \begin{bmatrix} 2 & 3/2 & 2 \\ 3/2 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \mathbf{y}, \tag{15}$$

which is positive definite and has its unique minimum $\mathcal{M}$ in (4, 4, 2). Therefore the function can equivalently be written as

$$\mathrm{w}(\mathbf{y}) = \tfrac{1}{2}[\mathbf{y}-\mathbf{y}^*]' \begin{bmatrix} 2 & 3/2 & 2 \\ 3/2 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} [\mathbf{y}-\mathbf{y}^*], \qquad \text{where} \quad \mathbf{y}^* = [4 \quad 4 \quad 2]'. \tag{16}$$

If *Tanguiane's* approach for given initial points $\mathcal{Q}_1^{\mathrm{T}} =$ (2, 2, 1) and $\mathcal{P}_1^{\mathrm{T}} = (1.5, 2, 1)$ is applied, indifference always exists, and **B** and **b** are found such that the objective function constructed has its unique minimum in (4, 4, 2). However, the condition number of $\check{\mathbf{Y}}$ is about $3 \cdot 10^5$. If, for example, the answer to the final question on point $\mathcal{R}_{23}^{\mathrm{T}}$ is not correct in the sense that a y_3-value 10% below the true value is given, (8) yields totally different parameters **b** and **B** of the objective function. **B** now turns out to be indefinite, and first-order conditions for optimization indicate a saddlepoint at (6.7, 2.0, 0.8).

Initial values for the modified approach are $\mathcal{Q}_1 =$(-0.636809, 4, 2) and $\mathcal{P}_1 =$ (-1.12348, 4, 2). The y_1-values were chosen to force $\mathcal{P}_1$ and $\mathcal{Q}_1$ to be on the same indifference curves as $\mathcal{P}_1^{\mathrm{T}}$ and $\mathcal{Q}_1^{\mathrm{T}}$ in order to make both methods comparable. Under the modified approach the condition number of $\breve{\check{\mathbf{Y}}}$ is about $9 \cdot 10^1$. Given precise answers the method yields the same matrix **B** - and implicitly the same vector **b** - as *Tanguiane's* approach. However, a 10% deviation in the final question, which is a y_2-value of S_{23} 10% below the true value, will only change elements $B_{23} = B_{32}$ of **B** from 0.2105 to 0.1321 - and due to $\mathbf{b} = -\mathbf{B}\mathbf{y}^*$ - implicitly b_2 and b_3 of **b**. 10%-errors in $\mathcal{R}_{12}$ or $\mathcal{R}_{13}$ lead to similar conclusions. Even for errors in points $\mathcal{P}_2$ and $\mathcal{Q}_3$ which are more critical because they influence later questions in the questionnaire, the changes in parameter values are moderate.[7]

A further comparison of the two methods can not be based on their ability to predict the unconstrained optimum point $\mathbf{y}^*$ because under the modified approach $\mathbf{y}^*$ will always be correctly identified by the objective function after the decision maker indicated it as her bliss point. Consider instead a constrained optimization problem where $\mathrm{w}(\mathbf{y})$ is to be minimized subject to $(2,-2,1)\mathbf{y} = 0$. Given the true objective function the solution is (3.3007, 4.4476, 2.2937). Under *Tanguiane's* approach with a 10% error in $\mathcal{R}_{23}^{\mathrm{T}}$ the constrained minimum calculated turns out to be (2.1633, -0.6753, -5.6770) which considerably deviates from the correct solution. Using the modified approach we get a vector (3.2475, 4.4494, 2.4038) which is rather close to the true constrained minimum.

Clearly enough, this numerical example can not prove the superiority of the modified approach in the case of definite objective functions. Additional work is needed in order to more firmly establish the advantages of the modified approach which figure 8 seems to indicate. One way to go would be to simulate the questioning of decision makers according to the two methods discussed here. In such simulations we could allow for the possibility of random error in each answer which would establish a more elaborate and realistic modelling of mistakes.

3.2. Modifications for more general objective functions

The alternative approach outlined above is not applicable if the decision maker can not name a vector $\mathbf{y}^*$ of optimal values. This would be the case in the examples used by *Tanguiane (1992)* and *Tanguiane and Gruber (1993)*. We therefore need a modification within *Tanguiane's* approach in order to reduce the multicollinearity problem.

In figure 9 two auxiliary points in (y_1, y_2)-space are introduced between every pair of data points differing in y_2. Including $\mathcal{H}_i$, $i = 1,\ldots,4$, increases the dispersion of the data and improves the condition number of $\breve{\mathbf{Y}}$ in (8). This could even be strengthened by adding four or six instead of two points.

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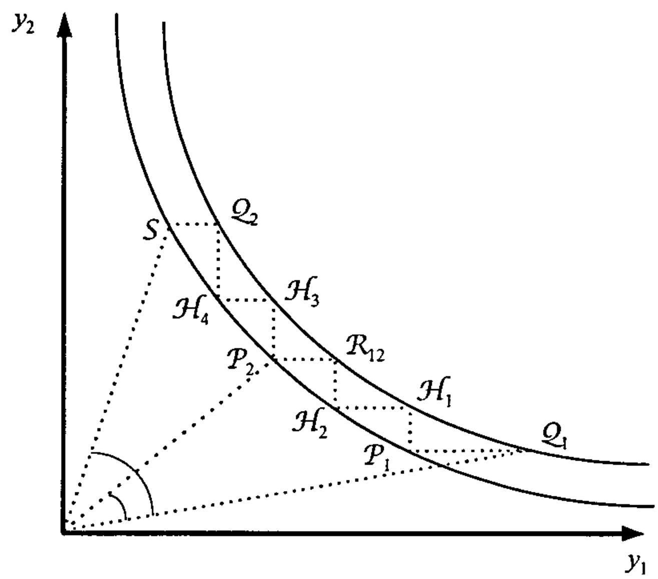
7 In general, *Tanguiane's (1992, 1993)* approach can be expected to be affected more severely by such „chains of errors" because it includes relatively more interdependence among data points. Notice, however, that the relative failure of this method pointed out in the example can not be due to such cumulative errors because a mistake was only made at a point without a successor.

Figure 9

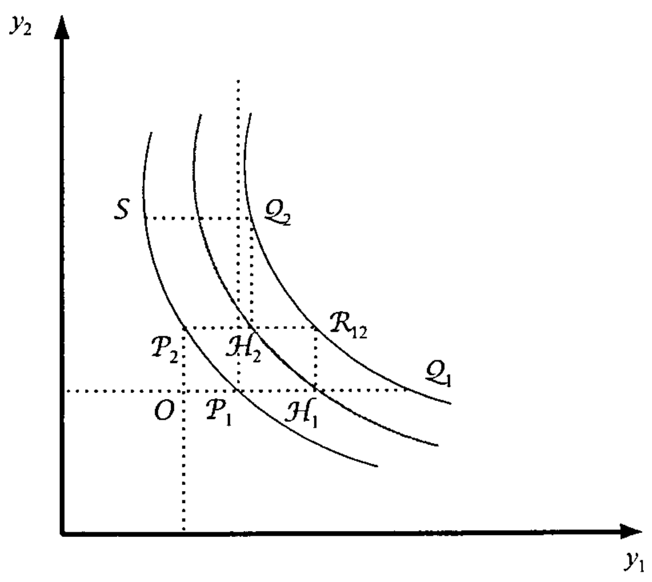
Auxiliary points to reduce multicollinearity



However, the same effect can be achieved by moving the initial points $\mathcal{P}_1$ and $\mathcal{Q}_1$ farther apart. Therefore, auxiliary points as in figure 9 are only of interest, if the information embodied in these data points is also used in calculating the unknown parameters of the objective function. This calls again for an extended data matrix $\breve{\mathbf{Y}}^e$ and a solution $\breve{\mathbf{b}} = [(\breve{\mathbf{Y}}^e)'\breve{\mathbf{Y}}^e]^{-1}(\breve{\mathbf{Y}}^e)'\mathbf{w}$.

Figure 10

Auxiliary welfare levels to reduce multicollinearity



An apparent disadvantage of using auxiliary points is the trade-off between reducing multicollinearity and increasing the risk of non-existence of indifference. Introducing additional welfare levels can be a way to generate more disperse data and at the same time avoid this drawback. Figure 10 shows the intution. Given $\mathcal{P}_1$ and $\mathcal{Q}_1$ which are drawn apart to increase dispersion of the data, there is no answer to the first question in *Tanguiane's* questionnaire. Introduce point $\mathcal{H}_1$ on a third indifference curve between $\mathcal{P}_1$ and $\mathcal{Q}_1$. $\mathcal{H}_1$ can be used to find $\mathcal{R}_{12}$ on the indifference curve of $\mathcal{Q}_1$ which in turn leads to $\mathcal{P}_2$. Since the question on $\mathcal{Q}_2$ can also not be answered from $\mathcal{P}_2$, another point called $\mathcal{H}_2$ is introduced. This can be done - as in figure 10 - by asking a question on indifference with $\mathcal{H}_1$ or by choosing an arbitrary point $\mathcal{H}_2$ between $\mathcal{P}_2$ and $\mathcal{R}_{12}$, thereby introducing yet another welfare level.

Two remarks on this procedure should be noted: Auxiliary points on indifference curves other than those of $\mathcal{P}_1$ and $\mathcal{Q}_1$ must not be included in the system (8). Using them for the calculation of the unknown parameters would amount to implicitly assuming cardinal preferences. Secondly, it should be noticed that as opposed to the original method $\mathcal{Q}_2$ no longer has a y_1-value of y_{10}. This implies that $\mathcal{Q}_2$ can no longer be used in (y_2, y_3)-space (see figure 3). $\mathcal{R}_{23}$ has then to be found by asking for indifference with $\mathcal{Q}_3$. This, however, is only permitted, if $\mathcal{Q}_3$ was found in (y_1, y_3)-space without the use of an auxiliary point.

4. Final remarks

Tanguiane (1992, pp. 2-3) is probably right to call the representation of objectives by scalar-valued objective functions the link in rational decision making based on optimization which is weakest next to finding an appropriate numerical representation of alternatives. In economics, and in the theory of economic policy in particular, this representation of preferences is urgently needed both for empirical work and theoretical analyses. Methods which gather information on policy makers' preferences and permit the construction of objective functions are more than welcome given that currently welfare or loss functions are normally written down in an ad hoc fashion.

This paper pointed out that an approach developed by *Tanguiane (1992, 1993)* which meets this challenge elegantly and with minimum effort still suffers from deficiencies in practical applications. The most important of these is a considerable non-robustness of the function constructed with respect to small variations in the data points. Such variations can arise if answers given by a decision maker are a little inaccurate. For the case of definite objective functions, where a decision maker knows an optimal combination of her objective variables, an alternative method was presented. This method appears to be able to substantially reduce the underlying problem of multicollinearity in the data generated. In addition, modifications to yield more robust specifications for the general case of objective functions without an optimal point were discussed.

In a next step these concepts ought to be put to work. This could be done through questioning of policymakers, in experimental setups, or in simulations. At the current stage, I consider simulation studies the most promising way to go. They would enable us to learn more about the relative merits and drawbacks of different methods proposed for the construction of objective functions. In particular, the consequences of errors in the answers can be examined at low cost in simulation studies. Only such methods should be considered for experiments or questioning of real policymakers which pass a very basic test in these simulations: Given errors in the answers, will a method be able to come up with an estimate of the objective function which is reasonably close to the true objective function?

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