INSTITUT FÜR VOLKSWIRTSCHAFTSLEHRE

der

UNIVERSITÄT AUGSBURG

Multiple Steady States, Indeterminancy and Cycles in a Basic Model of Endogenous Growth

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Beitrag Nr. 140

Oktober 1995

Volkswirtschaftliche Diskussionsreihe

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Abstract

The goal of this paper is to demonstrate that a basic model of endogenous growth with learning by doing may produce a rich array of outcomes. Starting point of our analysis is the Romer (1986a) approach. In contrast to Romer, however, we assume that one unit of investment shows different effects concerning the building up of physical and human capital, so that these variables cannot be merged into one single variable. With this assumption, it can be shown that multiple steady states, indeterminacy of equilibria and persistent cycles may result in our model.

1 Introduction

It is well known that introducing external effects in models of economic growth may lead to sustained per capita growth. Taking for example a learning by doing approach with endogenous savings, as initiated by Arrow (1962) and generalized by Levhari (1966), Romer (1986a) has shown that per capita variables may grow without an upper bound if the spillover effects are sufficiently large. Other examples of endogenous growth models with a perpetual increase of per capita production are those where agents may devote time to increase their stock of human capital which positively impacts per capita output (Lucas, 1988). The vast body of literature on this topic, however, is often only concerned with the analysis of balanced growth paths and aspects concerning the dynamic behaviour of economies with endogenous growth are frequently neglected.

One interesting feature of endogenous growth models may be the existence of a multiplicity of steady states implying that the initial conditions crucially determine the

*Forthcoming in: *Journal of Economics/Zeitschrift für Nationalökonomie*

stationary state to which the system converges in the long run. So, two economies with distinct starting values of the capital stock may reveal completely different steady state values and thus growth rates. This means that one economy always lags behind the other and can never catch up. In this case, we can speak of lock-in effects in the sense of Arthur (1988). Closely related to that point is the question of indeterminacy of equilibria. Here then, two economies with identical initial conditions with respect to the stock of physical and human capital have the same stationary values and growth rates. The transitional dynamics, however, and thus transitory growth rates depend on the starting value of consumption which may be chosen freely. To our knowledge, the only papers which study this question within a model of endogenous growth with externalities are the one by Boldrin and Rustichini (1994), who show that in a discrete-time one-sector model equilibria are unique, and the paper by Benhabib and Farmer (1994), who demonstrate indeterminacy with an aggregate production function with increasing returns and an elastic labour supply. The aspect of indeterminacy as well as the possibility of multiple steady states is also studied by Chamley (1993) in a model where agents can choose between allocating their time to either learning or producing goods. The topic of indeterminacy is also addressed by Benhabib and Perli (1994) for the Lucas model and Benhabib, Perli and Xie (1994) for the Romer (1990) model[1].

Another point of interest concerns the generic dynamic properties of endogenous growth models. Although some papers have recently emerged which study the transitional dynamics on paths to the steady state[2], none of these studies finds an interesting out-of-steady state dynamics for the standard endogenous growth models either the Lucas (1988) or Romer (1990) type. For conventional growth models, i.e. for models with zero per capita growth rates, Benhabib and Nishimura (1979) have shown that in an economy with more than one physical capital good persistent oscillations may result. A similar outcome was observed by Greiner and Hanusch (1994) in a model with learning by doing.

But this question has rarely been studied in continuous time models of endogenous growth with positive per capita growth rates. Benhabib, Perli and Xie (1994) or Benhabib and Perli (1994, p. 124), for example, mention the possibility of persistent cycles in endogenous growth models, but did not go further into the details. Futagami and

[1]See also the papers in the special issue of the Journal of Economic Theory, volume 63, no. 1, which, however, mainly focus on the Lucas (1988) variant of an endogenous growth model.

[2]See King and Rebelo (1993), Mulligan and Sala-i-Martin (1993), Caballe and Santos (1993), Asada, Semmler and Novak (1995) and Koch (1995).

Mino (1995) show for an endogenous growth model with public capital that their model may also produce endogenous cycles. However, these authors only demonstrated the possibility of existence of closed orbits around a steady state by showing that the real parts of two eigenvalues may become zero thus generating a Hopf bifurcation.

The goal of our paper consists in demonstrating that already in a standard one-sector model of endogenous growth with learning by doing, multiple steady states and indeterminacy of equilibria may result. Moreover, we also prove, applying Hopf-Bifurcation theory and numerical examples, that persistent growth cycles may occur in such a model.

The rest of the paper is organized as follows. In section 2, we present our model. Section 3 studies the possibility of multiple steady states and gives conditions for the indeterminacy of equilibria. Moreover, necessary conditions for the emergence of stable limit cycles are derived. Section 4 presents simulations which demonstrate the analytical results. Section 5 concludes the paper.

2 A Model of Endogenous Growth with Learning by Doing

We consider an economy with a macroeconomic production function of the form $Y(t) = b(A(t)L(t))^{\alpha}\bar{K}(t)^{1-\alpha}$, with $Y(t)$ output, $A(t)$ stock of human capital, $L(t)$ labour force and $\bar{K}(t)$ total stock of physical capital. $\alpha \in (0,1)$ is the coefficient in the Cobb-Douglas function determinig the labour share in the production of the output $Y(t)$, and all variables are functions of time.

The labour supply $L(t)$ is assumed to grow at the constant rate n. Macroeconomic output $Y(t)$ may be either consumed or invested, thus increasing the stock of physical capital in the economy. The evolution of the per capita capital stock $K(t)$ is given by the differential equation, $\dot{K}(t) = I(t) - (\delta + n)K(t)$, with $I(t)$ per capita investment and $\delta > 0$ constant depreciation rate. As to the stock of human capital $A(t)$ we assume that it is formed according to the learning by doing approach initiated by Arrow (1962). In contrast to Arrow, however, who uses a vintage approach with fixed coefficients, we assume in our model that technical change is disembodied and the production function is not restricted to fixed coefficients (see Levhari, 1966). Moreover, we suppose that the contribution of gross investment further back in time to the formation of human capital is smaller than recent gross investment. This assumption makes sense economically and can be formalized by defining the stock of human capital as an integral of past gross investment with exponentially declining weights put on investment flows further back in time (cf. Ryder and Heal, 1973, or Feichtinger and Sorger, 1988). $A(t)$ then is given by $A(t) = \rho \int_{-\infty}^{t} e^{\rho(s-t)} I(s) ds$, with $\rho > 0$. The parameter ρ represents the weight given to more recent levels of gross investment. The higher ρ, the larger is the contribution of more recent gross investment to the human capital stock in comparison to flows of investment in the past. There is an alternative interpretation for the parameter ρ. This coefficient can also be considered as the parameter determining the turnover of human capital in the economy. This is seen by differentiating $A(t)$ with respect to t, giving, $\dot{A} = \rho(I - A)$. The higher ρ the larger is the change in human capital induced by one unit of investment and the more of the current stock of human capital depreciates. In this sense we could say that the creation of new knolwedge is subject to the process of creative destruction in the Schumpeterian sense because any new investment creates knowledge but, at the same time, makes a certain part of the existing knowledge obsolete.

It should be mentioned that our approach is equal to the one by Romer (1986a) for $\rho = 1$ and under the assumption that neither the stock of physical nor human capital depreciates. Then, we get $A(t) = \int_{-\infty}^{t} I(s) ds = K(t)$ and our model is the same as the original Romer model[3].

Per capita consumption $C(t)$ in our economy is chosen so as to maximize the welfare functional $\int_0^\infty e^{-rt} u(C(t)) L(t) dt$, subject to the constraint giving the evolution of physical capital. $r > 0$ denotes the constant discount rate and $u(\cdot)$ is a strictly concave utility function, $u'(\cdot) > 0$, $u''(\cdot) < 0$. Moreover, the utility function obeys the Inada conditions $\lim_{C \to 0} u'(C) = \infty$, $\lim_{C \to \infty} u'(C) = 0$, and we suppose that the elasticity of marginal utility $u''(C)C/u'(C) \equiv -\sigma$ is constant.

As usual in this type of growth models with positive external effects, the solution to this optimization problem does not yield the socially optimal outcome. The latter would be achieved by explicitly taking into account an additional differential equation giving the evolution of human capital over time. The question, however, what measures should be taken to achieve the social optimum is beyond the scope of this paper. Instead, we will limit our investigations to the competitive situation, i.e. where only the direct effects of investment on physical capital are taken into account, and study what dynamic behaviour may result in that type of models.

Considering that the labour force grows with the constant rate $n > 0$ and normalizing

[3]We thank a referee for a remark on this topic.

$L(0) \equiv 1$, we can write our optimization problem as

$$\sup_{C(t)} \int_0^\infty e^{-(r-n)t} u(C(t))dt \tag{1}$$

subject to[4]

$$\dot{K} = bA^\alpha K^{(1-\alpha)} - C - (\delta + n)K. \tag{2}$$

The external effect of investment is described by the differential equation,

$$\dot{A} = \rho(bA^\alpha K^{(1-\alpha)} - C - A). \tag{3}$$

Given the fact that the evolution of human capital is exogenous to firms, the solution to this optimization problem is equivalent to the solution of the market economy with competitive firms, which pay wages equal to the marginal product of labour and rent capital with the interest rate equal to the marginal product of capital, and households, which receive labour income and interest payments on their savings.

Before we proceed to use necessary conditions to describe the solution to this optimization problem, we first investigate whether there exists a solution at all. A crucial condition in demonstrating that a solution to our optimization problem exists is to assume that the growth of $A(t)$ and $K(t)$ is bounded. This assumption excludes jumps in these variables and states, in economic terms, that the economy must not grow too fast. The following proposition gives the exact condition.

Proposition 1 *Assuming that $K(t)$ and $A(t)$ are bounded by a function growing with e^{gt}, where $0 < g < r - n$, there exists a unique solution for (1) subject to (2).*

Proof: To prove this assertion, we use the theorem by Romer (1986b). To apply this theorem, we have to write our problem in the form $\max_{\dot{K}} \int_0^\infty u(A, K, \dot{K}) e^{-(r-n)t} dt$. Since (2) is invertible, this is easily done. A solution to the problem exists if $u(\cdot)$ is (i) upper-semicontinuous, (ii) concave in $\dot{K}$ and if, (iii), $u(\cdot) \leq m(t) - |\dot{K}|^p$ holds, with $p > 1$ and $m(t)e^{-(r-n)t}$ integrable. (i) and (ii) are easily seen to be satisfied. To show that (iii) is fulfilled, we first note that $u(C) < u(1) + u'(1)(C-1)$ holds because of the strict concavity of $u(\cdot)$ in C. Stating that $C = bA^\alpha K^{1-\alpha} - (\delta + n)K - \dot{K}$, we may write $u(bA^\alpha K^{1-\alpha} - (\delta + n)K - \dot{K}) < u(1) + u'(1)(bA^\alpha K^{1-\alpha} - (\delta + n)K - \dot{K} - 1) \leq u(1) + u'(1)(bA_0 e^{\alpha gt} K_0 e^{(1-\alpha)gt}) = u(1) + u'(1)(bA_0 K_0 e^{gt})$. Here, it should be noted that we implicitly assume that investment is irreversible, i.e. $I \geq 0$. Now we can find constants

[4] In what follows we will suppress the time argument if no ambiguity arises.

M_1, M_2 and a $p > 1$ so that $u(1) + u'(1)(bA_0K_0e^{gt}) \leq u(1) + u'(1)M_1e^{gt} \leq u(1) + u'(1)M_2e^{gt} - (K_0g)^p(e^{gt})^p \leq u(1) + u'(1)M_2e^{gt} - |\dot{K}|^p$. Setting $u(1) + u'(1)M_2e^{gt} = m(t)$ and knowing that $g < r - n$, it is immediately seen that (iii) is fulfilled. □

With this proposition at hand we can use Pontryagin's maximum principle to describe the optimal solution. The current-value Hamiltonian for our problem is $H(\cdot) = u(C) + \gamma(bA^{\alpha}K^{(1-\alpha)} - C - (\delta + n)K)$.

Maximizing with respect to C yields $u'(C) = \gamma$ for interior solutions. The evolution of γ is given by, $\dot{\gamma} = \gamma(r + \delta) - \gamma(1 - \alpha)b(A/K)^{\alpha}$. Since the Hamiltonian is strictly concave in C and K jointly, the necessary conditions are also sufficient if in addition the transversality condition at infinity $\lim_{t\to\infty} e^{-(r-n)t}\gamma(t)(K(t) - K^{\star}(t)) \geq 0$ is fulfilled with $K^{\star}(t)$ denoting the optimal value. Moreover, strict concavity in C and K also guarantees that the solution is unique (cf. Seierstad and Sydsaeter (1987), pp. 234-235). This holds because for the competitive economy only the evolution of physical capital is taken into account whereas the evolution of human capital is given exogenously.

3 The Dynamic Behaviour

The necessary optimality conditions together with the differential equation describing the exogenously given evolution of human capital determine the dynamics of our economy. It is given by the differential equations

$$\frac{\dot{C}}{C} = \frac{1-\alpha}{\sigma}b\left(\frac{A}{K}\right)^{\alpha} - \frac{r+\delta}{\sigma}, \tag{4}$$

$$\frac{\dot{K}}{K} = b\left(\frac{A}{K}\right)^{\alpha} - \frac{C}{K} - (\delta + n), \tag{5}$$

$$\frac{\dot{A}}{A} = \rho b\left(\frac{A}{K}\right)^{\alpha-1} - \rho\frac{C}{A} - \rho. \tag{6}$$

It is obvious that sustained per capita growth is only feasible if the external effect of investment is strong enough. In fact, if A is constant we have the usual neoclassical growth model with zero per capita growth in the long run. This is well-known and has been pointed out by others before[5]. The novelty of our approach consists in assuming that the contribution of one unit of gross investment shows different effects on the building up of physical capital and human capital. This is certainly reasonable and implies, as a consequence, that these state variables cannot be merged into one single

[5]See e.g. Romer (1986a) or Sala-i-Martin (1990).

variable, as is frequently done. In the following, let us assume that the spillovers in our economy are sufficiently strong so that positive per capita growth is feasible and that the rate of growth is bounded by $0 < g < r - n$.

To explicitly investigate the dynamic behaviour of our economy, we perform a change of variables with $k = K/A$ and $c = C/A$. Differentiating with respect to time gives $\dot{k}/k = \dot{K}/K - \dot{A}/A$ and $\dot{c}/c = \dot{C}/C - \dot{A}/A$. Our new system of differential equations in k and c is then given by

$$\dot{k} = bk^{1-\alpha} - b\rho k^{2-\alpha} - c - k(\delta + n) + \rho k(1 + c), \tag{7}$$

$$\dot{c} = \frac{1-\alpha}{\sigma} bk^{-\alpha} c - \frac{r+\delta}{\sigma} c - b\rho c k^{1-\alpha} + \rho c(1 + c). \tag{8}$$

A rest point of this system corresponds to a balanced growth path of (4) - (6) with $\dot{A}/A = \dot{K}/K = \dot{C}/C = const$. Let us, in a next step, examine whether system (7) - (8) has a steady state. It is immediately seen that $k = 0$ cannot be a steady state value since k is raised to a negative power in (8). This implies that there is no steady state with a zero value for k. Moreover, setting $c = 0$ and k so that $\delta + n - \rho = bk^{-\alpha}(1 - \rho k)$ would yield a stationary point for (7) - (8). This, however, would imply that consumption is zero, a fact which does not make sense from the economic point of view so that we can exclude this rest point a priori, too. Therefore, we can consider the system (7) - (8) in the rates of growth and find its interior stationary points. In Proposition 2, we state conditions which guarantee that the interior stationary state is unique.

Proposition 2 *If $\delta + n \geq (r + \delta)/\sigma \geq \rho$, the existence of an interior steady state for system (7) - (8) implies that it is unique. The growth rate associated with this steady state is an increasing function of the population growth n.*

If the above inequality does not hold, multiple steady states may be possible.

Proof: Setting $\dot{k}/k = 0$ yields $c^\infty = (bk^{1-\alpha} - b\rho k^{2-\alpha} - k(\delta+n) + \rho k)/(1 - \rho k)$. Substituting c^∞ in $\dot{c}/c$ and setting $\dot{c}/c = 0$ gives, $q(k, \cdot) \equiv -\rho bk(1-\alpha)/\sigma - k^\alpha(-\rho + (r+\delta)/\sigma) - \rho k^{\alpha+1}(\delta + n - (r+\delta)/\sigma) + b(1-\alpha)/\sigma = 0$.

For $k = 0$, $q(k, \cdot) = b(1-\alpha)/\sigma > 0$. Moreover, $q'(k) < 0$ if the condition in the Proposition is fulfilled. If this does not hold, multiple solutions to $q(k, \cdot) = 0$ may exist.

Furthermore, the balanced growth rate is given by (4). Differentiating this expression with respect to n and using the fact that $\dot{k} = \dot{c} = 0$ holds at the steady state, then yields $\partial(\dot{C}/C)/\partial n = -\alpha(1-\alpha)b\rho/q'(k)\sigma$, which is positive for $q'(k) < 0$. □

This proposition shows that the technology does not play any role in determining whether multiple steady states exist or not. Only the absolute value of the (constant) marginal elasticity of utility is decisive, besides exogenously given parameters. If the absolute value of the marginal elasticity of utility remains within some boundaries, namely if $(r+\delta)/(\delta+n) \leq \sigma \leq (r+\delta)/\rho$, then the balanced growth path is unique. However, if either $(r+\delta)/(\delta+n) > \sigma$ or $\sigma > (r+\delta)/\rho$, we may observe multiple steady states. This demonstrates that the marginal elasticity of utility plays the decisive role, and the emergence of multiple balanced growth path depends whether its absolute value is lower or larger than a certain numerical value which is determined by the parameter values for r, δ, n and ρ.

The economic meaning of this proposition is that, in case of mulitple steady states, the long run growth rate in an economy crucially depends on the initial conditions of k. Here we can speak of path dependence and lock-in effects in the sense of Arthur (1988) implying that an economy with a lower initial stock of human capital possibly always lags behind another one and never can catch up. This means that two economies may reveal completely different growth rates both transitorily as well as in the long run although they have identical preferences and an identical technology. A feature like this can be used to explain why some less developed countries do not succeed in the process of catching up and continue to show low or even negative per capita growth rates. In this case we may speak of poverty traps in which an economy is caught and no convergence towards high per capita income economies will occur. A similar finding has also been reported in a paper by Futagami and Mino (1993) and an earlier paper by Shell (1967)[6]. These authors, however, could derive their results only for conventional growth models, i.e. for models with a zero per capita growth rate.

A topic closely related to the above is the question of the indeterminacy of equilibria. If the stationary state is completely stable, that is all trajectories satisfying (7) and (8) which start in the neighborhood of this stationary state converge to the rest point, then there exists a continuum of paths $\{k(t), c(t)\}$ all converging to the stationary point. This holds because only the initial condition for physical and human capital, i.e. k_0, is given for an economy, whereas the initial consumption per human capital, c_0, can be chosen freely. Therefore, there exists a continuum of c_0, satisfying the first order conditions, so that we may say that the equilibrium path is indeterminate. What level for c_0 is eventually selected depends on non-economic factors like cultural or institutional ones

[6] See also Azariadis and Drazen (1990) who study this problem with a threshold model.

affecting the transitional paths of the economy until it reaches the long-run balanced growth rate. Thus, the level of the long-run capital stock and of consumption are also determined by C_0, but, of course, not the long-run growth rate. This property might be used to explain why some countries which are similar in structure, like highly developed countries of North America and Western Europe for example, show different growth rates.

It should be mentioned that the above definition refers to local indeterminacy. In contrast to local indeterminacy, there may also exist global indeterminacy in the sense that the initial level of consumption may be crucial in determining to which balanced growth path the economy converges in the long run. These definitions have been first introduced by Benhabib and Perli (1994) and Benhabib, Perli and Xie (1994). In the simulations below we show examples for both local and global indeterminacy.

Another interesting question, besides the number of steady states and the indeterminacy of equilibria, is what generic dynamic behaviour of the growth rates is feasible, especially whether the economy reaches the steady state growth rate at all. This holds all the more because the dynamics of such models are not yet well understood (Caballe and Santos (1993), p. 1043) although the number of papers examining models with endogenous growth has sharply increased during the last few years[7]. For our system, it follows immediately from the Poincáre-Bendixson theorem, that the most complex dynamic behaviour we can expect is a limit cycle. To apply this theorem, we have to find a compact invariant set. Since this is not an easy task to fulfill, we will not apply this theorem, but resort to bifurcation theory to investigate of whether cyclical solutions for our system exist. This, however, gives us only a local result, valid near the steady state under consideration. From an economic point of view, this procedure seems to be justified because from empirical data it is to be expected that real economies are near stationary states.

As to the relevance of growth cycles for real world economies we should like to make two remarks. First, we do not think that neoclassical growth models, where all markets clear instantenously because prices are sufficiently flexible, can explain short run business cycle fluctuations. Instead we should like to maintain a theoretical dichotomy between growth and business cycles. Therefore, second, we believe that the emergence of cycles in this type of models can only be used to explain more medium or long run fluctuations, which we will call growth cycles. As to the empirical relevance of this sort

[7] See the papers cited in the introduction.

of cycles, it is difficult to come to a clear answer. This results from the fact that there is little reliability of the time series to test for long run fluctuations in economic growth. Nevertheless, there do exist studies asserting that long run fluctuations can be observed in the data[8].

Let us go back to our analytical model. In the following we assume that there exists at least one interior rest point for our system (7)-(8). To investigate the local dynamics, we proceed as usual and first calculate the Jacobian matrix. Using the fact that $\dot{k} = \dot{c} = 0$ at a stationary state, the Jacobian at the steady state is given by

$$J = \begin{bmatrix} c/k - \alpha b k^{-\alpha} - (1-\alpha) b \rho k^{1-\alpha} & \rho k - 1 \\ \frac{1-\alpha}{\sigma} bc(-\alpha) k^{-\alpha-1} - \rho c b (1-\alpha) k^{-\alpha} & \rho c \end{bmatrix}.$$

As is well known, the trace of the Jacobian, trJ, and the determinant, $\det J$, determine the local stability properties. If $\det J < 0$, the stationary state is stable in the saddle point sense with a one-dimensional stable manifold leading to the rest point. In this case, the equilibrium is locally unique in the neighborhood of the steady state, i.e. for every k_0 in the neighborhood of k^∞ there exists a unique c_0 in the neighborhood of c^∞ that generates a trajectory converging to $\{k^\infty, c^\infty\}$. If $trJ < 0$ and $\det J > 0$, the real parts of the eigenvalues are negative indicating that the steady state is completely stable. In this case, we have a continuum of equilibria and may speak of indeterminacy. If, however, $trJ = 0$ and $\det J > 0$, the system looses stability and has two purely imaginary eigenvalues. In this case, it may undergoe a Hopf bifurcation leading to stable limit cycles. If this happens, the economy does not converge any longer to a balanced growth path but instead shows cyclical oscillations in the growth rate.

In Proposition 3, we state conditions which determine the dynamic outcome for our analytical model.

Proposition 3 *(i) A necessary and sufficient condition for uniqueness of equilibria is* $(\alpha b(1-\alpha)(1-\rho k) + bk\rho\sigma)/(k^\alpha \rho\sigma) > c$.

(ii) A necessary and sufficient condition for indeterminacy of equilibria is $(\alpha b(1-\alpha)(1-\rho k) + bk\rho\sigma)/(k^\alpha\rho\sigma) < c < (\alpha b k^{-\alpha} + (1-\alpha) b\rho k^{1-\alpha})/(\rho + k^{-1})$.

(iii) A necessary condition for stable limit cycles is $c = (\alpha b k^{-\alpha} + (1-\alpha) b \rho k^{1-\alpha})/(\rho + k^{-1})$ *and* $\alpha + (-u''(\cdot)C/u'(\cdot)) < 1$.

The values for k and c in (i) - (iii) are evaluated at the steady state $\{k^\infty, c^\infty\}$.

[8] See e.g. Bieshaar and Kleinknecht (1984), Rosenberg and Frischtak (1984) or Kleinknecht and Bain (1992).

Proof: The condition for uniqueness of equilibria follows from $\det J < 0$. The first condition for indeterminacy of equilibria follows from $\det J > 0$. The second results from $trJ < 0$. The first condition for stable limit cycles follows from $trJ = 0$. Substituting this c in $\det J$ and knowing that $\det J < 0$ must hold, then yields the second. □

The second necessary condition for growth cycles shows that the higher the labour share in the macroeconomic production function and the higher the absolute value of the elasticity of marginal utility, the less likely is the emergence of endogenous growth cycles. Or, stated in another way, a large capital share and a low absolute value of the elasticity of marginal utility is a necessary condition for persistent growth cycles.

The conditions for uniqueness and indeterminacy, however, cannot be interpreted economically. Therefore, we state the following corollary which gives further insights and states conditions that can be interpreted in economic terms.

Corollary *For $k > \rho^{-1}$ the following turns out to be true:*

$\delta + n - \rho < 0$ and $-\sigma\rho k^{1+\alpha}(\delta + n - \rho) > b\alpha(1-\alpha)(1-\rho k)^2$ are necessary and sufficient for uniqueness of equilibria, while $\delta + n - \rho < 0$ and $-\sigma\rho k^{1+\alpha}(\delta + n - \rho) < b\alpha(1-\alpha)(1-\rho k)^2$ are necessary conditions for indeterminacy of equilibria.

For $k < \rho^{-1}$ the following is true:

$\delta + n - \rho < 0$ and $-\sigma\rho k^{1+\alpha}(\delta + n - \rho) < b\alpha(1-\alpha)(1-\rho k)^2$ are sufficient for uniqueness of equilibria whereas $\delta + n - \rho < 0$ and $-\sigma\rho k^{1+\alpha}(\delta + n - \rho) > b\alpha(1-\alpha)(1-\rho k)^2$ are necessary conditions for indeterminacy of equilibria.

The value for k is evaluated at the steady state $\{k^\infty\}$.

Proof: To proof this corollary we calculate the determinant and the trace of the Jacobian. Taking into account that c at the rest point can be calculated as $c^\infty = (bk^{1-\alpha} - b\rho k^{2-\alpha} - k(\delta + n) + \rho k)/(1-\rho k)$ (from $\dot{k}/k = 0$) and using $\dot{c}/c = 0$ which gives an expression for r at the steady state, the determinant can be computed as $\det J = (-b(1-\rho k) + k^\alpha(\delta + n - \rho)) \cdot (b\alpha(1-\alpha)(1-\rho k)^2 + \sigma\rho k^{1+\alpha}(\delta + n - \rho))/(k^{2\alpha}\sigma(\rho k - 1)^2)$. The sign of this expression is equivalent to $(-b(1-\rho k) + k^\alpha(\delta + n - \rho)) \cdot (b\alpha(1-\alpha)(1-\rho k)^2 + \sigma\rho k^{1+\alpha}(\delta + n - \rho))$. Now, we know that $\dot{k}/k = 0$ implies $-b(1-\rho k) + k^\alpha(\delta + n - \rho) = -c(1-\rho k)k^\alpha$. Combining these two expressions then leads to $(c(1-\rho k)k^\alpha) \cdot (b\alpha(1-\alpha)(1-\rho k)^2 + \sigma\rho k^{1+\alpha}(\delta + n - \rho))$ which determines the sign of $\det J$.

Following the same steps for the trace of the Jacobian gives the expression $b + b\alpha(\rho k - 1) + (\delta + n - \rho)k^\alpha(1 + \rho k)/(\rho k - 1)$ which determines the sign of trJ.

With these expressions giving the sign of $\det J$ and trJ the results in the corollary follow immediately. □

This corollary shows that the ratio of physical to human capital k at the steady state in relation to the parameter ρ, denoting the turnover of human capital, is most decisive. If k is larger than $1/\rho$, i.e. if either the ratio of physical to human capital is large or ρ takes a high value (or both show high values), then the equilibrium path is unique if the absolute value of the elasticity of marginal utility is relatively high, for a given technology and parameter values. On the other hand, a relatively low value (in absolute terms) for the elasticity of marginal utility is a necessary condition for indeterminacy.

If, however, k is smaller than $1/\rho$, the conditions are just reverse. Now, a relatively small elasticity of marginal utility (in absolute terms) is a sufficient (but not necessary) condition for uniqueness of equilibria, while an indeterminate equilibrium path necessarily goes along with a high elasticity.

In contrast to the conditions for limit cycles, which were independent of the steady state value for k and the parameter ρ, the conditions for uniqueness and indeterminacy crucially depend on the sign of $k - \rho^{-1}$ and the elasticity of marginal utility. This demonstrates that it cannot be determined a priori whether the equilibrium path is indeterminate or unique. Instead, the specific conditions of an economy, depending on the parameter values and preferences among others, determine the dynamic outcome.

In order to demonstrate that multiple steady states as well as uniqueness and indeterminacy of equilibria may occur we next present two numerical examples, where we illustrate our analytical results. The simulations are also needed in order to show that endogenously generated cycles in the growth rate may actually occur, since it is rather difficult to analytically check of whether the other conditions leading to stable limit cycles (positive crossing velocity of the eigenvalues and the sign of the coefficient determining the stability of the cycle) are also fulfilled for the model.

4 Numerical Examples

Let us now illustrate our analytical findings with the help of numerical examples.

Example 1. For the utility function, we assume a function with constant relative risk aversion, $u(C) = C^a/a$, $a \leq 1$ (cf. Romer, 1986b, p. 903) and suppose $a = 0.6$. b in the macroeconomic production function is normalized to 1 and the coefficient in the Cobb-Douglas production function is set to $\alpha = 0.5$, which may be seen as a plausible upper bound for the capital share[9]. The depreciation rate is $\delta = 0.19$, the population growth is assumed to be $n = 0.02$ and ρ is set to $\rho = 1.65$. Interpreting one time period as two years then means that the annual depreciation is 9.5 per cent, which is about in the range reported by Maddison (1987 table 7) and the growth rate of the workforce per year is 1 per cent. The value for ρ states that the contribution of investment two years back to the present stock of human capital is $e^{-0.825 \cdot 2} = 0.192$ and the contribution of investment 5 years back is 1.62 per cent. The discount rate r serves as bifurcation parameter. With these parameter values our dynamic system becomes,

$$\begin{aligned}\dot{k} &= k^{0.5} - 1.65k^{1.5} - c - 0.21k + 1.65ck + 1.65k \\ \dot{c} &= 1.25k^{-0.5}c - 2.5(r + 0.19)c - 1.65ck^{0.5} + 1.65c(1 + c),\end{aligned}$$

For $r = 0.041$ figure 1 gives the curve $q(k, \cdot)$, which is the differential equation $\dot{c}/c$ with c taken at the steady state which is computed from $\dot{k}/k$. A point for which $q(k, \cdot) = 0$ holds then yields a balanced growth path for our economy.

Figure 1 about here

It can be seen that there are two interior stationary states which are given by $k_1^\infty = 4.11234$, $c_1^\infty = 1.00431$ and $k_2^\infty = 3.63524$, $c_2^\infty = 0.859293$.

The first point is a stable focus with eigenvalues $\lambda_{1/2} = -0.00911843 \pm 0.151366\sqrt{-1}$, while the second is stable in the saddle point sense. If we vary the discount rate, we see that for a critical value of $r_{crit} = 0.040899$, the eigenvalues become purely imaginary, $\lambda_{1/2} = \pm 0.167918\sqrt{-1}$. The steady state for this value of r shifts to $k_1^\infty = 4.17066$ and $c_1^\infty = 1.02111$. Moreover, we can calculate $\partial Re\lambda(r)/\partial r = -82.536$ for $r = r_{crit}$ so that the crossing velocity is non zero, indicating a Hopf bifurcation. For $r = 0.04075$, stable limit cycles can be observed[10].

Figures 2a and 2b show the situation in the $c - k$ phase diagram with $r = 0.041$ and $r = 0.04075$ respectively[11].

Figure 2a and 2b about here

[9] See King and Rebelo (1993, p. 918).

[10] To do the numerical calculations we used the software Mathematica (see Wolfram Research, 1991).

[11] We show a qualitative representation of the phase diagram because in the phase diagram drawn to scale, the isoclines are nearly identical for $k \in (3, 6)$ such that the intersection points of them cannot be clearly recognized.

This phase diagram gives an idea about the global dynamics of our system. If the initial level of k is smaller than 3.635 the equilibrium is determinate, i.e. convergence to the balanced growth path is only given if it starts on the stable branch of the saddle point. For values of k larger than 3.635 the economy may either converge to the balanced growth path associated with $k_2^\infty = 3.635$ or to the one associated with $k_1^\infty = 4.112$, for $r = 0.041$. This is the case in figure 2a. In this case we may speak of global indeterminacy in the sense that the initial level for c determines whether the economy converges to k_1^∞ or k_2^∞. If $r = 0.04075$, the model produces a stable limit cycle if it converges to k_2^∞ as shown in figure 2b.

In figure 3, we illustrate the case of local indeterminacy and depict two optimal time paths for $k(t)$ for $r = 0.041$ and with the same initial condition for $k(t)$, $k(0) = 4.18$, but different starting values for consumption, namely $c(0) = 1.01$ and $c(0) = 1.02$. It can be observed that the initial conditions for consumption crucially determine the transitional dynamics until the paths converge to the stationary state with a constant total annual growth rate of 1.725 per cent.

Figure 3 about here

Figure 4 shows how the trajectory approaches the limit cycle in the $k(t) - c(t)$ phase diagram for $r = 0.04075$ and inital conditions $k(0) = 4.18$, $c(0) = 1.02$.

Figure 4 about here

It should be noted that the value for $k(t)$ varies about between 3.699 and 4.689 implying that in this case the annual growth rate varies between 2.81 and -0.83 per cent. With these values it cannot be determined whether the functional diverges or not. Nevertheless, if (1) does not converge, the path described by the first order conditions is optimal according to the catching-up criterium (CU-optimal)[12].

The second steady state $k_2^\infty = 3.53563$, $c_2^\infty = 0.827053$ has eigenvalues $\lambda_1 = -0.315808$ and $\lambda_2 = 0.0971835$ for $r = 0.04075$. This rest point, however, implies that the functional (1) diverges.

Example 2. Next, let us present an example with a higher capital share in the production function and set $\alpha = 0.3$. Taking $1 - \alpha = 0.5$ as an upper bound for the capital share, a physical capital share of 0.7 seems to be implausibly high, at first sight.

[12] See Seierstad and Sydsaeter (1987), p. 232-233.

However, in an empirical study Romer (1987) finds that, due to spillovers, the overall capital share in determining long run economic growth falls in the range 0.7 to 1.0, whereas the value for the labour share is in the range 0.1 to 0.3, with values possibly as large as 0.5. Taking these estimates seriously, our choice for α seems to be justified. The coefficient in the utility function is now $a = 0.5$. The depreciation rate is $\delta = 0.09$, the population growth is assumed to be $n = 0.03$, b and ρ are as above.

Analyzing this system with $r = 0.2967$, we again find two interior steady states. The first is given by $c_1^\infty = 1.78019$, $k_1^\infty = 4.47098$, with eigenvalues $\lambda_1 = -0.248538$ and $\lambda_2 = 0.0975281$, indicating that this equilibrium is stable in the saddle point sense. The endogenous per capita growth rate per year is 3.996 per cent.

The second stationary point is $c_2^\infty = 1.93537$, $k_2^\infty = 4.79655$. The eigenvalues associated with this stationary point are $\lambda_{1/2} = -0.025915 \pm 0.157396\sqrt{-1}$, showing that this point is a stable focus. The endogenous per capita growth rate corresponding to this steady state is 3.064 per cent and per year. This analysis demonstrates that the economy only achieves the higher growth rate associated with $\{c_1^\infty, k_1^\infty\}$ if it starts on the stable manifold of the saddle point, depending on the initial conditions of the economy. If the economy is on the unstable branch of the saddle point and near the other steady state $\{c_2^\infty, k_2^\infty\}$, it will converge in the long run to this stationary value resulting in a lower growth rate.

Moreover, if the economy converges to $\{c_2^\infty, k_2^\infty\}$, it will show transitory oscillations until it reaches the stationary value. Furthermore, if we vary the discount rate r, we see that for $r = r_{crit} = 0.296452$, two eigenvalues are purely imaginary. The stationary point for this value of r is shifted to $k^\infty = 4.97537$ and $k^\infty = 2.01863$. Since $\partial Re\lambda(r)/\partial r = -79.3687$ for $r = r_{crit}$ the crossing velocity is non zero indicating a Hopf bifurcation.

For $r = 0.296348$ we can again observe stable limit cycles around the stationary point which changes to $k_2^\infty = 5.02978$ and $c_2^\infty = 2.04371$. The value of $k(t)$ varies between 5.42 and 4.56 implying that the per capita growth rate per year varies between 2.02 per cent and 4.28 per cent. For this value of the discount rate, the other rest point is given by $c_1^\infty = 1.68597$ and $k_1^\infty = 4.27837$ with the eigenvalues $\lambda_1 = -0.362199$ and $\lambda_2 = 0.149095$, indicating that this equilibrium is still stable in the saddle point sense.

Figure 5 shows the time path for $k(t)$ over one cycle, for $r = 0.296348$ with $k(0) = 4.97$ and $c(0) = 2.02$.

Figure 5 about here

As to the analysis in the phase diagram it should be noted that the qualitative picture is analogous to figures 1 and 2. Therefore, we confined ourselves to presenting figure 5 which shows that stable limit cycles may result for this example, too.

5 Conclusion

In this paper we presented a basic model of endogenous growth with learning by doing. In contrast to the usual approach, we supposed that investment in physical capital does not increase the stock of human and physical capital one for one, but shows different effects on the formation of these two state variables. This assumption implies that the stocks of physical and human capital cannot be merged into one single state variable, but instead are treated as two distinct variables, with the evolution of each described by a differential equation.

With this assumption, we could demonstrate that even our extremely simple economy, where we have disregarded the accumulation of human capital (Lucas, 1988) or knowledge capital (Romer, 1990), may show multiple steady states and indeterminacy of equilibrium paths. This result is equivalent to the one detected by Benhabib and Farmer (1994), Benhabib and Perli (1994) and Benhabib, Perli and Xie (1994) who investigated indeterminacy and multiple steady states in a two sector endogenous growth model of Lucas or Romer type. In addition to this result we could demonstrate that the external effect of investment may also generate cyclical growth paths with transitory or, using Hopf-Bifurcation theory, permanent oscillations.

Acknowledgements

Versions of this paper were presented at the Eunetic Conference (Strasbourg, October 1994), at the Second Symposium on Nonlinear Dynamics and Econometrics (New York, March 1995) and at the Tenth Annual Congress of the European Economic Association (Prague, September 1995). We want to thank the participants of these conferences as well as two anonymous referees of the journal for providing detailed comments which helped to improve the paper. Alfred Greiner gratefully acknowledges generous financial support from the Deutsche Forschungsgemeinschaft (DFG).

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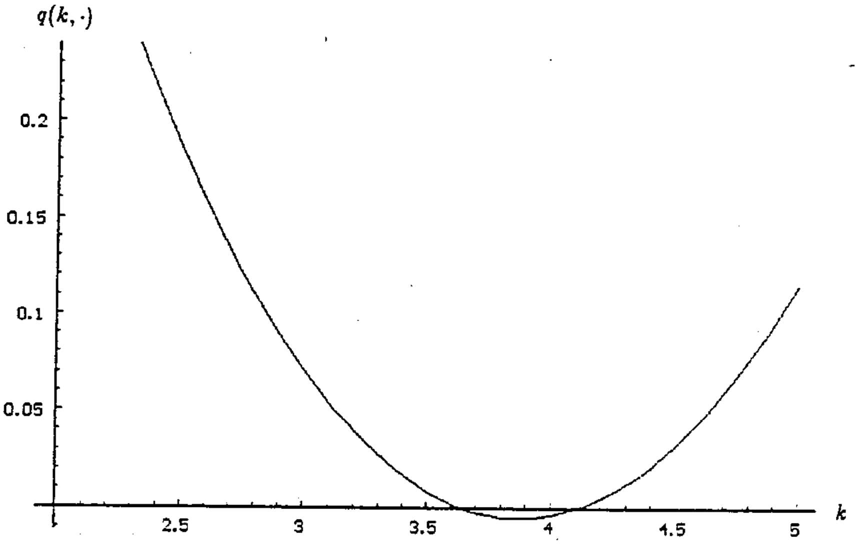
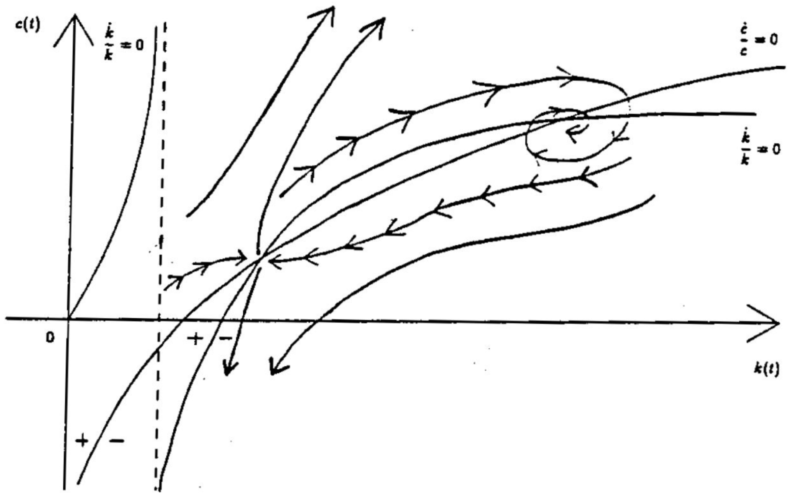


Fig. 1

2a



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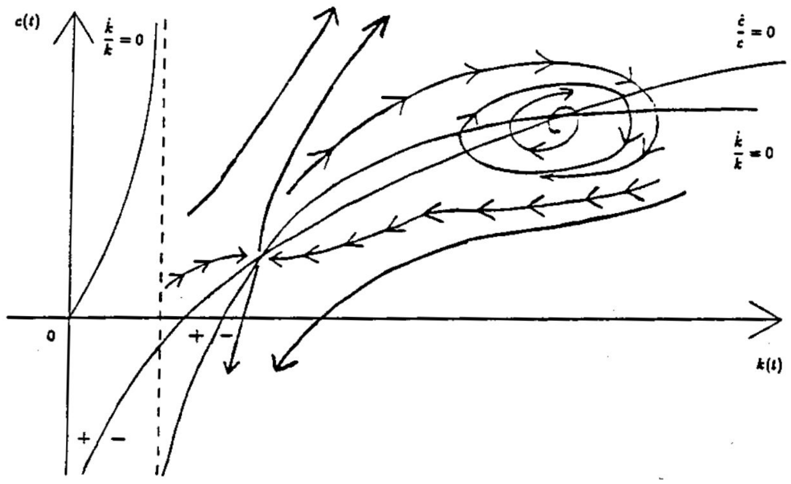

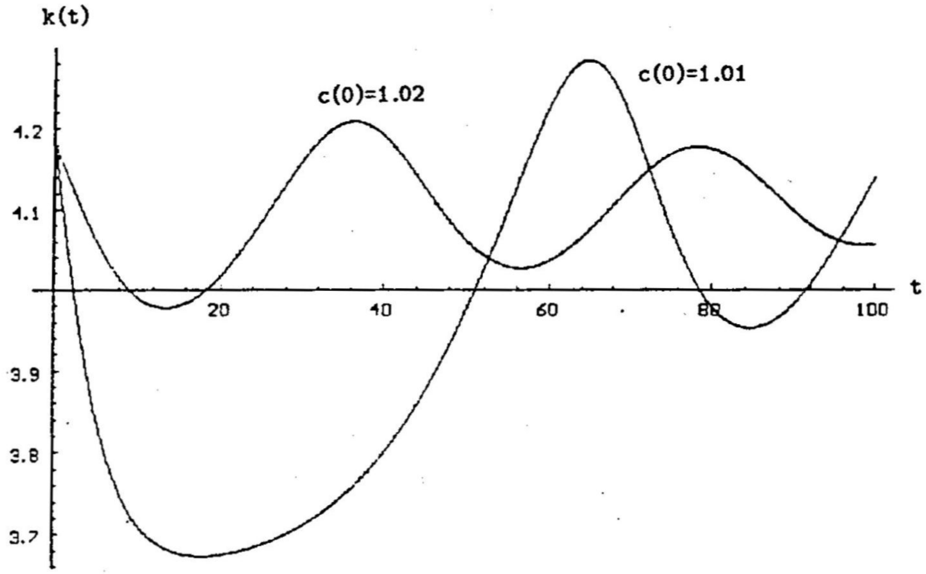

Fig 3

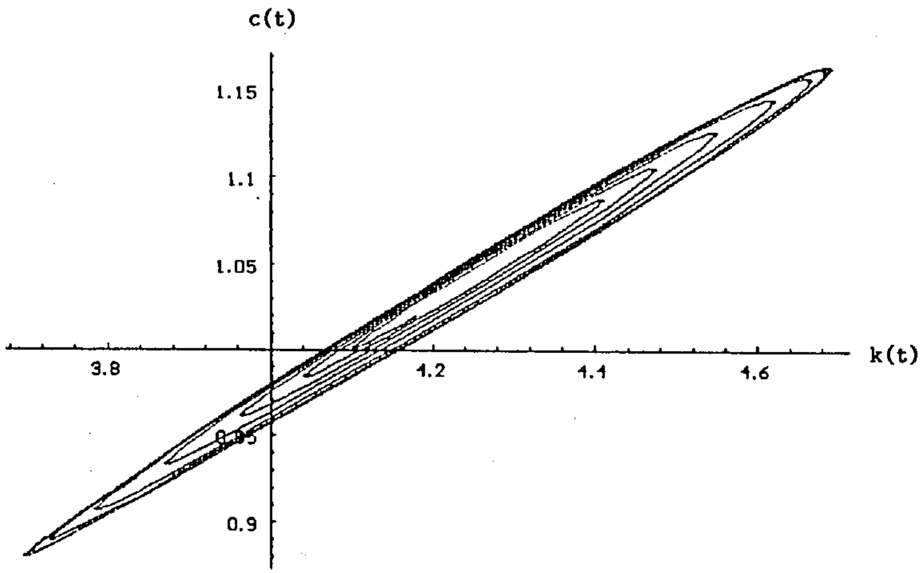

Fig 4

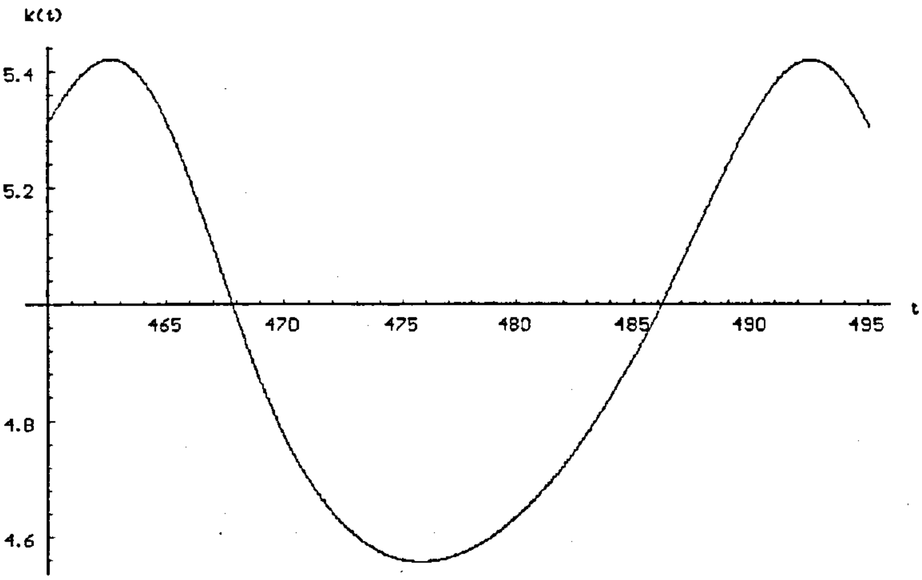

Fig 5

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