Superconductivity in restricted geometries

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Abstract

Some aspects of superconductivity in restricted geometries are reviewed. In particular, the suppression of the critical temperature and the order parameter in homogeneously disordered thin films, on the basis of a systematic, self-consistent (one-loop) approximation, is discussed. Furthermore, theoretical results for persistent currents (nonlinear magnetic response in equilibrium) in mesoscopic rings, as well as possible experimental scenarios, are described.

1. Introduction

The interplay between superconductivity and disorder-induced localization is, naturally, a question of fundamental interest. As early as 1959, Anderson [1] explained why nonmagnetic impurities have no considerable effect on superconductivity. But more detailed studies including vertex corrections in the electron–phonon interaction [2] and the Coulomb interaction (see, for example Ref. [3] and references therein), whose effectiveness is enhanced in the diffusive regime, have revealed, for example, that the critical temperature T_c can be strongly shifted. As a rule, it appears that a "low" T_c is enhanced, and a "high" T_c is reduced, with increasing disorder strength [3].

In general, it is useful to distinguish between two idealized classes of superconductors, namely granular and homogeneous systems. It is believed that theoretically, the former (among which I include also Josephson junction arrays) can be described by phase-only models, i.e. a Hamiltonian which includes the Josephson coupling between the grains as well as the charging energy. In this model, phases and charges are canonically conjugate variables. For the (ordered) Josephson junction arrays, it seems to be clear that with decreasing the size of the contacts, i.e. decreasing capacitance and hence increasing charging energy, a superconductor–insulator transition is observed at zero temperature when the charging energy becomes comparable to the Josephson coupling energy (see Ref. [4] and references therein). This result has been confirmed experimentally. Surprisingly, the phase diagram as a function of an external voltage has a very rich structure, including, for example, a "supersolid" phase [5]. It should be mentioned, however, that the mean-field approximations used in this context have problems, especially when the inverse capacitance matrix is of long range [6]. Dynamical properties of vortices in the superconducting phase [7], and close to the transition [8], have been studied in detail (see also Ref. [9]).

In contrast, the situation is less clear for granular films (see, for example, Refs. [10–13]), and whether the transition occurs at a universal value of the sheet resistance (of the order of $h/4e^2$) is a question which remains open.

On the other hand, in homogeneous samples, superconductivity is gradually suppressed when, for example, the thickness d of the film is reduced towards the two-dimensional limit (at fixed composition, especially keeping the mean free path l fixed). It is observed that T_c and the order parameter Δ tend to vanish at a nonuniversal value of the sheet resistance $R_\square$ of the order of a few kΩ [14–16], their ratio being roughly constant [15]. Following the early work by Ovchinnikov [17], the theoretical description of this phenomenon has been discussed by several authors [18–24]. In Section 2, I will discuss in more detail the approach taken by Pelzer and myself [22]. A general overview as well as a discussion of the renormalization group method and of several controversial questions has been given recently by Finkel'stein [20].

When reducing the dimensions even more towards the one-dimensional limit, a further reduction of T_c is observed [25–27] (though the dependence on the width w of the wire is not always clear) as well as a broadening of the resistance versus temperature curves. In particular, the resistive transitions seen experimentally in Pb wires [27] can be reasonably well fitted above T_c with the Aslamasov–Larkin fluctuation contribution to the conductivity, and below T_c with the Langer–Ambegaokar–McCumber–Halperin contribution arising from thermally activated phase slip processes. However, below T_c, there are also clear deviations from the latter which have been interpreted in terms of quantum phase slip processes [28, 29].

In Section 3, I will review a different topic, namely persistent currents in mesoscopic rings. Persistent currents in *normal metal* rings have been observed recently in three beautiful experiments [30–32], and considerable theoretical efforts have been devoted to this problem [33–42]. Below I will describe some of the results Ambegaokar and I [43] obtained recently for the nonlinear (diamagnetic) response of superconducting rings above T_c, and in particular discuss a situation in which a crossover between "persistent" currents above T_c (which can be either diamagnetic or paramagnetic, depending on the impurity configuration) to diamagnetic supercurrents close to T_c should be observable. Experiments on the nonlocal paraconductance of small superconducting rings are discussed by Goldman [44] in these proceedings.

2. From bulk to film

About 20 years ago, Ovchinnikov [17] calculated the shift $\delta T_c = T_c(d^{-1}) - T_c(0)$ of a homogeneous film of thickness d in the regime $l < d < \xi_0$, where $\xi_0 \sim (v_F l/T_c)^{1/2}$ denotes the zero temperature coherence length. Typical values, applicable to the experiments, are $l \sim 5$ Å and $\xi_0 \sim 100$ Å. The main contribution arises from fluctuations of the scalar potential, i.e. the Coulomb interaction, with the result

$$\frac{\delta T_c}{T_c} = -\frac{1}{24\pi}\frac{R_\Box}{R_0}\ln^3\left(\frac{\xi_0^2}{d^2}\right) \tag{1}$$

provided the logarithm is much larger than one. Here $R_\Box = (\sigma d)^{-1}$ is the sheet resistance, $\sigma = 2e^2\mathcal{N}_0 D$ denotes the Drude conductivity and $R_0 = h/4e^2$ the quantum of the resistance. Note that the appearance of d in the logarithm is due to the calculation of difference quantities, T_c^{film} minus T_c^{bulk}, a point discussed in detail in Ref. [22]. Inserting typical parameters as well as putting $d \sim l$ in the logarithm of the above equation [18] corresponds surprisingly well with the data [14]. However, I wish to point out that for d approaching ξ_0, the above asymptotic result as well as general considerations predict a slower than linear initial decrease (see also below) of T_c versus d^{-1}, in particular for $d > \xi_0$, which, however, might be difficult to resolve experimentally.

In order to extend the above result to higher $R_\Box$, Finkel'stein has developed a renormalization group approach which shows that the T_c-degradation slows down with increasing $R_\Box$. His results led to a perfect agreement

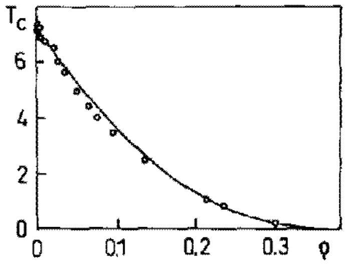

Fig. 1. Suppression of the critical temperature T_c in homogeneous films. The experimental data are from Refs. [14], the solid line is the theoretical fit given by Finkel'stein [19]. Adapted from Refs. [19, 20].

with the experimental data, as is apparent from Fig. 1, with only one fitting parameter, $\ln(1/\tau T_c) \sim 8$, where τ is the elastic scattering time ($l = v_F\tau$).

In order to develop an alternative approach, Pelzer and I [22] formulated the problem in terms of a path integral description of the theory. Starting from a Hamiltonian which includes the pairing as well as the Coulomb interaction, we represented the interactions by fluctuating fields (the complex order parameter field Δ and the scalar potential ϕ) and constructed the appropriate thermodynamic potential through a Legendre transformation. From this potential (which we denoted by Γ), we derived the gap equation by differentiation. In the saddle point approximation for Γ, the BCS expressions are recovered, but fluctuation corrections (one-loop approximation) have also been calculated (with some numerical effort). Thus, our approach is basically similar to that of Ovchinnikov.

We evaluated the resulting expressions close to T_c to calculate the suppression of the critical temperature, and for low temperatures to determine the order parameter suppression. Though the dominant contributions arise from the range $T_c < \hbar\omega < \hbar Dq^2 < \hbar D/d^2$, i.e. from short-wavelength quantum fluctuations, the quantitative analysis, which we have performed self-consistently for various interaction strengths and the Coulomb interaction, shows differences in detail in the behavior of T_c versus Δ. Close to T_c, an important point is that a consistent treatment of phase and potential fluctuations, as in our approach, does not lead to a long-wavelength singularity in the results, a fact which was also discussed in detail recently by Finkel'stein [20]. In other words, though several of the relevant diagrams are divergent for $q \to 0$, this singularity is cancelled out when all diagrams of a certain class are included [23]. This also implies that there is practically no difference between a strong local interaction and the Coulomb interaction $V_q = 2\pi d/q$.

The latter, however, does not apply for the zero-temperature order parameter degradation: There we found a significant contribution from the regime which is dominated by the collective mode, $\omega \sim q^{1/2}$, which leads to a considerably stronger Δ- than T_c-reduction. Some of our

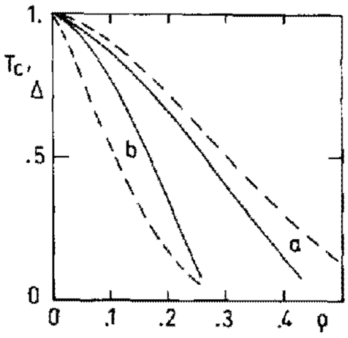

Fig. 2. Critical temperature (continuous lines) and zero-temperature order parameter (dashed lines), normalized to their bulk values, respectively, versus sheet resistance in units of $R_0 = h/4e^2$. (a) Intermediate strength local interaction, $2e^2\mathcal{N}_0 V_0 = 1$; (b) Coulomb interaction, $V_q = 2\pi d/q$. Adapted from Ref. [22].

results are given in Fig. 2, where T_c and Δ, in units of their respective bulk values, are plotted versus $\rho = R_\Box/R_0$. It should be noted that our results do not confirm the slowing down of the T_c-reduction which is apparent in Fig. 1 with increasing ρ, though I suspect that both approaches become questionable close to the point where superconductivity actually disappears.

Our results have been confirmed, close to the critical temperature, in detail [23, 45] with the diagrammatic method. In addition, it has been possible to include a magnetic field [23] and hence to determine the upper critical field, $H_{c2}(T)$. Some results for the perpendicular case are shown in Fig. 3. It has been found that the electron–electron interaction in the Cooper channel mainly affects the slope of $H_{c2}(T)$ versus T near T_c, but in addition the electron–electron interaction in the diffusion channel, for large enough ρ, may lead to a positive curvature (see Fig. 3). From this result it can be interpreted that a magnetic field tends to reduce the magnitude of the T_c-degradation in comparison with the zero-field case.

To the best of my knowledge, a detailed theory of the additional degradation of the critical temperature [25] when

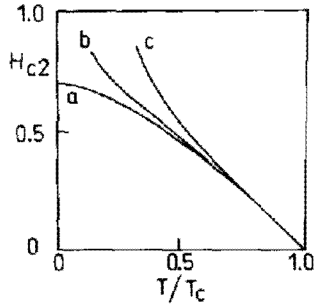

Fig. 3. The caluclated temperature dependences of the upper (perpendicular) critical field of a thin superconducting film, normalized to unit slope near T_c. (a) BCS dirty limit universal curve; (b) $\rho = 0.2$; (c) $\rho = 0.4$ ($\rho = R_\Box/R_0$). Adapted from Ref. [23].

reducing the wire width w, at fixed thickness (experimentally [25] $\delta T_c \sim -w^{-2}$) is not available at present. As discussed above, results based on the long-wavelength singularity of some of the diagrams [21] are erroneous.

3. Mesoscopic rings

In this section, I consider a mesoscopic metal ring in an external magnetic field for the idealized situation where the width is small compared to the perimeter L, so that the ring is quasi-one-dimensional. Without magnetic field penetration, the energy and the appropriate thermodynamic potential depend only on the magnetic flux Φ. As the persistent current $I(\Phi)$ is an equilibrium quantity, it can be calculated as the negative of the flux derivative of the potential. Furthermore, excluding paramagnetic impurities [46], the persistent current has the expansion

$$I = \sum_{n=1}^{\infty} I_n \sin(2\pi n\Phi/\Phi_0) , \qquad (2)$$

where $\Phi_0 = h/e$. As in other mesoscopic phenomena (recall the "universal conductance fluctuations"), the persistent current and its Fourier coefficients $\{I_n\}$ are stochastic quantities, varying from sample to sample. Theoretically, the persistent current is characterized when all correlation functions are known [41, 42]. Here I concentrate on the theoretical results for the average current and the two-point correlator, which should be compared with the experimental observations, Refs. [30] and [31], respectively. Estimates of higher correlation functions are given in Refs. [41, 42].

The characteristic energy scales in the problem are the average level spacing $\delta = (\mathcal{N}_0 \mathcal{V})^{-1}$, the Thouless energy $E_c = \hbar D/L^2$, the average level spacing of an exactly (one transverse channel) one-dimensional ring $\delta_1 = \hbar v_F/L$ (note that $\delta \sim \delta_1/M$, where M is the number of transverse channels) and, for superconducting metals, the critical temperature T_c. Typically $\delta \ll E_c \ll \delta_1$, and the temperature has to be of the order of E_c or lower to make the normal metal persistent currents observable.

3.1. Average persistent current

It is generally agreed that, on average (over samples and hence impurity configurations), the odd harmonics in expansion (2) vanish. Furthermore, for noninteracting electrons, it has been found [36–38] that $\langle I_{2m}\rangle \sim \delta/\Phi_0$, independent of m, which implies at zero temperature the necessity for different techniques [39, 40] or, at least, the phenomenological introduction of a finite pair-breaking parameter (of order δ) in the perturbative expressions of Refs. [36–38] in the regime of small flux. As a result, the zero-temperature maximum persistent current, at $\Phi \sim (\delta/E_c)^{1/2}\Phi_0$, is of the

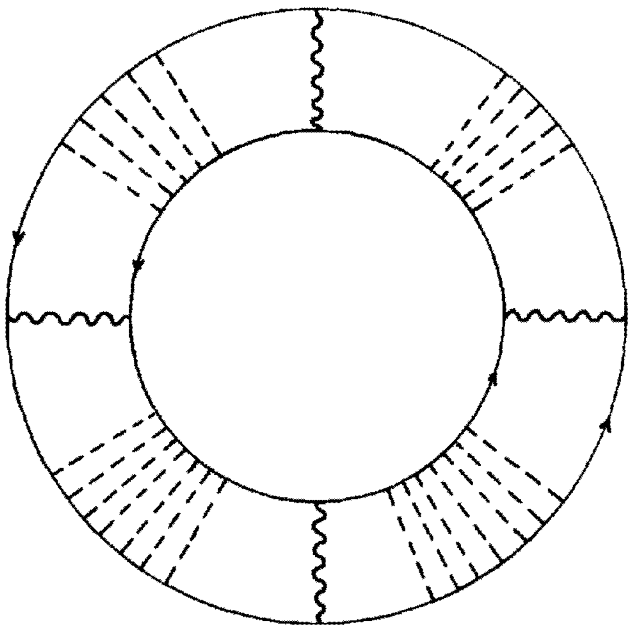

Fig. 4. Typical contribution to the grand potential which, when summing all orders, leads to Eq. (3). The wavy lines denote the attractive interaction, which is chosen to be frequency independent; dashed lines denote standard impurity connections.

order of $(\delta E_c)^{1/2}/\Phi_0$. But at temperatures which can be reasonably achieved experimentally, $T \sim E_c$ ($\sim$ 10 mK), only the $m = 1$ contribution survives [34].

The calculation of the interaction contribution to the average grand potential, and hence the persistent current, is formally identical in normal metals [34, 35] and in superconductors above T_c (see also Refs. [47, 48]). The result for the average potential which, for example, can be found by summing diagrams of the form shown in Fig. 4, or equivalently by the path integral methods briefly described in the previous section, is [43]

$$\langle \Omega \rangle = T \sum_{\omega, q} \ln \mathscr{E}(\omega, q) , \tag{3}$$

where $\mathscr{E}(\omega, q)$ denote the eigenvalues of the pair propagator and ω the Matsubara (Bose) frequencies. The wave vector has to be taken one-dimensional, and in the presence of a magnetic flux is given by

$$q = \frac{2\pi}{L}\left(n - \frac{\Phi}{\Phi_C}\right) \tag{4}$$

with $\Phi_C = h/2e$. Ambegaokar and I [43] evaluated expression (3) for two cases: (A) $T_c \ll T, E_c$ and (B) $E_c \sim T_c \leqslant T$. We found that in case A, apart from the expected opposite sign, the current to a very good approximation is given by our first order result [34], with the dimensionless coupling constant replaced by $\sim [\ln(E_c/T_c)]^{-1}$. Explicit results for the first two (nonvanishing) harmonics are given in Fig. 5.

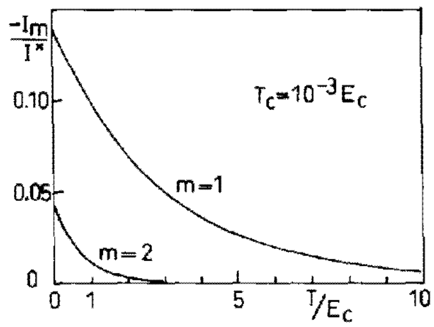

Fig. 5. First and second (nonvanishing) harmonics of the average interaction induced persistent current in units of $I^* \equiv 8E_c/\Phi_C$ for $T_c/E_c = 10^{-3}$ ($\Phi_C = h/2e$, $E_c = \hbar D/L^2$). Adapted from Ref. [43]. Note that $m = 1$ and $m = 2$ correspond to the period Φ_C and $\Phi_C/2$, respectively.

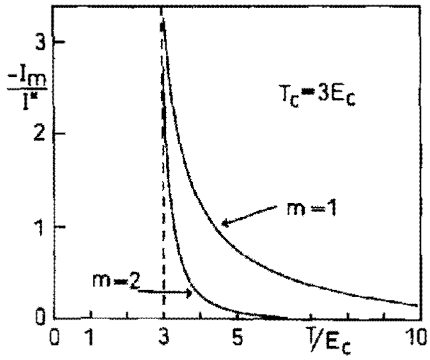

Fig. 6. First and second harmonics of the average interaction induced persistent current for the case $T_c = 3E_c$. Note that both curves reach $3\pi/2 = 4.71$ as $T \to T_c^+$. Adapted from Ref. [43].

Results for case B, on the other hand, are shown in Fig. 6. In particular, close to the critical temperature, the classical (zero frequency) contribution dominates, with the result

$$\langle I_{2m} \rangle = -\frac{4\pi T_c}{\Phi_C} \exp\left(-\frac{mL}{\xi}\right) , \tag{5}$$

where ξ is the temperature dependent coherence length. From this expression, the average persistent current is easily determined in closed form [43]. Note that essentially only those harmonics contribute for which $\xi > mL$.

3.2. Fluctuations of the persistent current

In order to determine the current–current correlation, it is easiest to consider $\langle \Omega(\Phi)\Omega(\Phi') \rangle_c$ and deduce $\langle I(\Phi)I(\Phi') \rangle_c$ by differentiation. A typical diagram is similar to that shown in Fig. 4, except that the wavy lines have to be removed. The corresponding diffuson contribution has to be added. Omitting unimportant terms, the result (see for example, Ref. [42] for a detailed description of the intermediate steps) is

$$\langle \Omega\Omega \rangle_c = -(2T/\pi) \sum_{\omega,q} |\omega| \ln(|\omega| + Dq^2)\,, \qquad (6)$$

where the arguments were suppressed for brevity. Here

$$q = \frac{2\pi}{L}\left(n - \frac{\Phi \pm \Phi'}{\Phi_0}\right) \qquad (7)$$

for the cooperon and the diffuson contribution, respectively. Note that the logarithm in Eq. (6) appears because of the symmetry of the diagrams. Clearly, $\langle \Omega\Omega \rangle_c \sim E_c^2$ for the flux dependent part at low temperatures, and hence $\langle II \rangle_c \sim (E_c/\Phi_0)^2$, i.e.,

$$\langle II \rangle_c^{1/2} \sim ev_F l/L^2\,. \qquad (8)$$

Note, however, that the theoretical results for the average and for the fluctuations are too small to explain the experimental observations, Refs. [30] and [31], respectively.

In order to understand at least the second experiment [31] which indicated that $\langle II \rangle_c^{1/2} \sim ev_F/L$, Schmid and I [42] considered the electron–electron interaction, which we argued should break the symmetry of the relevant diagrams and lead to a result close to experiment. But these arguments were criticized immediately [41], with the conclusion that the Coulomb interaction only gives a renormalization of the mean free path in Eq. (8) (and, in any case, l has to be taken from the experiment). Thus, at present, this problem is open, and for the following discussion, I assume that the result (8) is valid.

3.3. Discussion

Finally, I would like to discuss a situation which would be ideal for seeing both effects, "normal" persistent currents as well as diamagnetic supercurrents.

Consider an experiment on a single, mesoscopic superconducting ring, for which the parameters are such that $T_c \sim E_c$. Given the present microfabrication technology, rings with a perimeter of a few μm, transverse dimensions of a few hundred Å and a mean free path of the same order of magnitude are experimentally achievable. This implies that E_c is a few ten mK. The scale for the temperature dependence of the normal persistent current is also set by E_c, so I may concentrate on the first harmonic. But note that in a single ring experiment, it can be of either sign, depending on the actual impurity configuration. Furthermore, if $T_c \sim E_c$, the diamagnetic current induced by superconducting fluctuations, Eq. (5), is of the same order of magnitude, but of course the flux periodicity is given by $\Phi_C = \Phi_0/2$. Note that the exponent in Eq. (5) can also be written as $L/\xi \sim (T_c/E_c)^{1/2}$ except for the temperature dependence of the coherence length.

I would therefore expect that, for this situation, an experiment should, at $T \sim 2T_c$, show both phenomena, "normal" persistent currents (period Φ_0) as well as diamagnetic supercurrents (period Φ_C), the latter of course becoming more pronounced when the critical temperature is approached. An experiment of this type would be most valuable and definitely be of great help in our understanding of these phenomena. Of course, if the experimental result for single rings [31] can be confirmed, the persistent current will be dominated by the "normal" effect except for temperatures extremely close to the critical temperature.

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References

[1] P.W. Anderson, J. Phys. Chem. Solids 11 (1959) 26.

[2] B. Keck and A. Schmid, J. Low Temp. Phys. 24 (1976) 611.

[3] D. Belitz, Phys. Rev. B 35 (1987) 1636; 35 (1987) 1651; 36 (1987) 47.

[4] For a review of the theoretical and the experimental situation, see for example: R. Fazio, A. van Otterlo, G. Schön, H.S.J. van der Zant and J.E. Mooij, Helv. Phys. Acta 65 (1992) 228, and references therein.

[5] A. van Otterlo and K.-H. Wagenblast, Phys. Rev. Lett., to be published.

[6] J.G. Kissner and U. Eckern, Z. Phys. B 91 (1993) 155.

[7] U. Eckern and A. Schmid, Phys. Rev. B 39 (1989) 6441; U. Eckern and E. B. Sonin, Phys. Rev. B 47 (1993) 505.

[8] R. Fazio, A. van Otterlo and G. Schön, Europhys. Lett. 25 (1994) 453.

[9] A. van Otterlo, Physica B 203 (1994) 504.

[10] H.M. Jaeger, D.B. Haviland, A.M. Goldman and B.G. Orr, Phys. Rev. B 34 (1986) 4920.

[11] A.E. White, R.C. Dynes and J.P. Garno, Phys. Rev. B 33 (1986) 3549.

[12] A.F. Hebard and M.A. Paalanen, Phys. Rev. Lett. 65 (1990) 927.

[13] Y. Liu, D.B. Haviland, B. Nease and A.M. Goldman, Phys. Rev. 47 (1993) 5931.

[14] J.M. Graybeal and M.R. Beasley, Phys. Rev. B 29 (1984) 4167; J.M. Graybeal, M.R. Beasley and R.L. Greene, in: Proc. 17th Int. Conf. Low Temp. Phys. (North-Holland, Amsterdam, 1984) p. 731.

[15] R.C. Dynes, A.E. White, J.M. Graybeal and J.P. Garno, Phys. Rev. Lett. 57 (1986) 2195; J.M. Valles Jr., R.C. Dynes and J.P. Garno, Phys. Rev. B 40 (1989) 6680.

[16] D.B. Haviland, Y. Liu and A.M. Goldman, Phys. Rev. Lett. 62 (1989) 2180.

[17] Yu.N. Ovchinnikov, Zh. Eksp. Teor. Fiz. 64 (1973) 719 [Sov. Phys. JETP 37 (1973) 366].

[18] S. Maekawa and H. Fukuyama, J. Phys. Soc. Japan 51 (1982) 1380; S. Maekawa, H. Ebisawa and H. Fukuyama, J. Phys. Soc. Japan 52 (1983) 1352; H. Takagi and Y. Kuroda, Solid State Commun. 41 (1982) 643.

[19] A.M. Finkel'stein, Pis'ma Zh. Eksp. Teor. Fiz. 45 (1987) 37 [Sov. Phys. JETP Lett. 45 (1987) 46].
[20] A.M. Finkel'stein, Physica B 197 (1994) 636.
[21] H. Ebisawa, H. Fukuyama and S. Maekawa, J. Phys. Soc. Japan 55 (1986) 4408.
[22] U. Eckern and F. Pelzer, J. Low Temp. Phys. 73 (1988) 433.
[23] A.A. Golubov and V.V. Dorin, J. Low Temp. Phys. 78 (1990) 375.
[24] T.R. Kirkpatrick and D. Belitz, Phys. Rev. Lett. 68 (1992) 3232.
[25] J.M. Graybeal, P.M. Mankiewich, R.C. Dynes and M.R. Beasley, Phys. Rev. Lett. 59 (1987) 2697.
[26] N. Giordano and E. Sweetland, Phys. Rev. B 39 (1989) 6455.
[27] F. Sharifi, A.V. Herzog and R.C. Dynes, Phys. Rev. Lett. 71 (1993) 428.
[28] N. Giordano, Phys. Rev. Lett. 61 (1988) 2137.
[29] N. Giordano, Physica B 203 (1994) 460.
[30] L.P. Levy, G. Dolan, J. Dunsmuir and H. Bouchiat, Phys. Rev. Lett. 64 (1990) 2074; L.P. Levy, Physica B 167 (1990) 50.
[31] V. Chandrasekhar, R.A. Webb, M.J. Brady, M.B. Ketchen, W.J. Gallagher and A. Kleinsasser, Phys. Rev. Lett. 67 (1991) 3578.
[32] D. Mailly, C. Chapelier and A. Benoit, Phys. Rev. Lett. 70 (1993) 2020.
[33] H. Bouchiat and G. Montambaux, J. Phys. (Paris) 50 (1989) 2695.
[34] V. Ambegaokar and U. Eckern, Phys. Rev. Lett. 65 (1990) 381; 67 (1991) 3192 (Comment).
[35] U. Eckern, Z. Phys. B 82 (1991) 393.
[36] A. Schmid, Phys. Rev. Lett. 66 (1991) 80.
[37] F. von Oppen and E.K. Riedel, Phys. Rev. Lett. 66 (1991) 84.
[38] B.L. Altshuler, Y. Gefen and Y. Imry, Phys. Rev. Lett. 66 (1991) 88.
[39] K.B. Efetov, Phys. Rev. Lett. 66 (1991) 2794.
[40] A. Altland, S. Iida, A. Müller-Groeling and H.A. Weidenmüller, Europhys. Lett. 20 (1992) 155.
[41] R.A. Smith and V. Ambegaokar, Europhys. Lett. 20 (1992) 161.
[42] U. Eckern and A. Schmid, Europhys. Lett. 18 (1992) 457; Ann. Physik (Leipzig) 2 (1993) 180.
[43] V. Ambegaokar and U. Eckern, Europhys. Lett. 13 (1990) 733; Phys. Rev. B 44 (1991) 10358.
[44] N.E. Israeloff, F. Yu, A.M. Goldman and R. Bojko, Phys. Rev. Lett. 71 (1993) 2130.
[45] R.A. Smith, private communication.
[46] U. Eckern, Physica Scripta T 49 (1993) 338.
[47] B.L. Altshuler, D.E. Khmelnitskii and B.Z. Spivak, Solid State Commun. 48 (1983) 841.
[48] B.L. Altshuler, A.G. Aronov, D.E. Khmelnitskii and A.I. Larkin, in: Quantum Theory of Solids, ed. I.M. Lifshitz (MIR Publishers, Moscow, 1980) p. 130; B.L. Altshuler and A.G. Aronov, in: Electron–Electron Interactions in Disordered Systems, eds. A.L. Efros and M. Pollak (North-Holland, Amsterdam, 1985) p. 1.