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Peter Schwab, Ulrich Eckern

Angaben zur Veröffentlichung / Publication details:

Schwab, Peter, and Ulrich Eckern. 1996. "Impurity spin dynamics and quantum coherence in mesoscopic rings." *Annalen der Physik* 508 (1): 57–87. https://doi.org/10.1002/andp.2065080106.

Impurity spin dynamics and quantum coherence in mesoscopic rings

P. Schwab and U. Eckern

Institut für Physik, Universität Augsburg, Memminger Str. 6, D-86135 Augsburg, Germany

Abstract We investigate the effects of impurity spin dynamics on quantum coherence in mesoscopic metallic rings. Spin relaxation induces temporal current fluctuations which are closely related to persistent currents in such systems. The persistent current is suppressed in the presence of magnetic impurities. We discuss spin-polarisation effects, spin-glass ordering, and the Kondo effect which are shown to restore the current even in the presence of magnetic scattering.

Keywords: Mesoscopic quantum systems; persistent current; magnetic impurities.

1 Introduction

Recent experiments on metallic [1, 2] and semiconducting [3, 4] mesoscopic rings demonstrated the existence of persistent currents at low temperatures. In the first experiment the persistent current in an ensemble of 10^7 copper rings [1] was measured. The current was found to be periodic in the magnetic flux, ϕ, with the periodicity given by half a flux quantum, $\phi_0/2 = h/2e$, and with an amplitude of the order $I \sim ev_F l/L^2$ (v_F is the Fermi velocity, l the mean free path, and L the circumference of the ring). Another group measured the current in single gold rings [2]. In that experiment the periodicity was one flux quantum, the amplitude was of the order $I \sim ev_F/L$. Qualitatively these experimental findings can be explained assuming strong fluctuations of the current from one sample to the other. If these fluctuations allow for different signs of the current in different samples, the typical current measured in a single ring is much larger than the mean current in an ensemble. We can also explain the different periodicities: Expanding the current in each ring in a Fourier series,

$$I(\phi) = I_1 \sin(2\pi\phi/\phi_0) + I_2 \sin(4\pi\phi/\phi_0) + \cdots,$$

the experimental observation is consistent with the theoretical result that the average of I_1 vanishes.

The amplitudes, however, of both the mean current and the current found in the single ring experiment are larger by about two orders than predicted by theory. Many efforts have been devoted to an understanding of this discrepancy. Early calculations for noninteracting electrons predicted an exponentially small current in a grand cano-

nical ensemble. An important step was the realisation that a much larger current is found in a canonical theory, i.e. the particle number is fixed instead of the chemical potential. Unfortunately the result is still two orders of magnitude too small. On the other hand, it was found [5] that the Coulomb interaction gives an important contribution to the mean current, namely $I \sim \lambda_c e v_F l/L^2$, where λ_c is a dimensionless constant characterising the strength of the interaction. Parametrically this is the same as the observed current, but the estimated value of λ_c is again far too small. It is remarkable that the temperature dependence of the mean current is described quite accurately by this theory [5].

The situation is even less clear for the single ring experiment. Analytical calculations for non-interacting electrons determined the root mean square current to be $\langle I^2 \rangle^{1/2} \sim e v_F l/L^2$, much too small, since $l/L \sim 10^{-2}$. The role of the Coulomb interaction for the persistent current fluctuations is still unclear: Analytic calculations which predicted an enhancement of the current were not convincing [6–8]. Numerical investigations are restricted to small systems. In weakly disordered, strictly one dimensional systems it was found that the Coulomb interaction suppresses the current [9, 10], while in systems with many transverse channels an enhancement of the current has been reported [11]. However, compared with the experiment it is still too small.

In the present article we investigate the effects of magnetic scattering on quantum coherence in mesoscopic rings. In Chapter 2 we briefly review aspects of the theory of persistent currents. Using the methods of perturbation theory, we consider both interacting and non-interacting electrons.

In Chapter 3 we extend these calculations to include magnetic scattering, i.e. spin-flip scattering, spin-orbit scattering, and Zeeman effects. We first discuss paramagnetic impurities above the Kondo temperature, and then apply the results to the spin-glass case. At the end of this chapter we take the Kondo effect into account. We must point out that we are not able to explain the large persistent currents observed experimentally, as we do not find a scenario where the persistent current in the presence of magnetic impurities is enhanced over the value in the "clean" limit, i.e. without magnetic impurities. On the other hand, our results for the current as a function of parameters like the impurity concentration, magnetic field and so on, may serve as a test for the applicability of the theoretical concepts.

In Chapter 4 we consider a related phenomenon, i.e. temporal current fluctuations which are induced by impurity spin dynamics. Our results are summarised in Chapter 5.

2 Persistent currents

In the idealised situation, where the effects of magnetic field penetration can be neglected, the persistent current in a ring penetrated by a magnetic flux ϕ is given by the derivative of the thermodynamic potential,

$$I(\phi) = -\frac{\partial}{\partial \phi} K(\phi). \tag{1}$$

In a system with fixed particle number $K(\phi) = F(N, \phi)$, where F is the canonical free energy, while for a system coupled to a particle reservoir, $K(\phi) = \Omega(\mu, \phi)$, where Ω is the grand canonical potential. Clearly $K(\phi)$ is periodic in the flux, ϕ,

with the period given by the flux quantum, $\phi_0 = h/e$. The thermodynamic potentials and the current can be represented as a Fourier series,

$$K(\phi) = K_0 + \sum_{m=1}^{\infty} [K_m \cos(2\pi m\phi/\phi_0) + \tilde{K}_m \sin(2\pi m\phi/\phi_0)] \tag{2}$$

and

$$I(\phi) = \sum_{m=1}^{\infty} [I_m \sin(2\pi m\phi/\phi_0) + \tilde{I}_m \cos(2\pi m\phi/\phi_0)]. \tag{3}$$

In the case of time reversal symmetry, $K(\phi)$ is an even function in ϕ, and $I(\phi)$ is odd, i.e. $\tilde{K}_m = \tilde{I}_m = 0$ for all m. In a disordered system the impurity-averaged quantities $\langle K \rangle$, $\mathcal{M}_K = \langle KK \rangle_c = \langle KK \rangle - \langle K \rangle \langle K \rangle$, and the higher correlation functions are the central objects to study. In the metallic regime (diffusive electron motion) the following results are generally accepted:

$$\langle I_{2m+1} \rangle = 0 \tag{4}$$

$$\langle I_m I_{m'} \rangle_c = 0 \quad \text{if} \quad m \neq m' \tag{5}$$

Higher connected correlation functions are small, provided the number of electrons is large [6–8].

We consider a weakly disordered metal in the diffusive regime. In a typical sample, we may assume the following inequalities between relevant length scales: $\lambda_F \ll l \ll L \ll \xi \sim Ml$. Here λ_F is the Fermi wavelength, l the elastic mean free path, L the perimeter of the ring, and ξ is the localisation length; the latter is in quasi-onedimensional systems proportional to the number of transverse channels $M = k_F^2 \mathcal{A}/4\pi$, where $\mathcal{A}$ is the cross section of the ring. Alternatively, we may consider the relevant energy scales, which are in descending order: ε_F, the Fermi energy, $\hbar/\tau = \hbar v_F/l$, the elastic scattering rate, $\hbar v_F/L$, the energy to localise a particle in one half of the ring, $E_c = \hbar D/L^2$, the Thouless energy ($D = v_F l/3$ is the diffusion constant), $\Delta = 1/2\mathcal{N}_0\mathcal{V}$, the mean level spacing at the Fermi level ($\mathcal{N}_0$ is the density of states, $\mathcal{V}$ the volume). For a general orientation, we give the values which are applicable for the experiment on single gold rings (in units of Kelvin): $\varepsilon_F \sim 10^5$ K, $\hbar/\tau \sim 10^2$ K, $\hbar v_F/L \sim 1$ K, $E_c \sim 4$ mK, and $\Delta \sim 0.02$ mK. Clearly, all these energy scales are well separated.

2.1 Non-interacting electrons

Since the impurity-averaged density of states is insensitive to a magnetic flux, it follows immediately that the mean persistent current in a grand canonical ensemble is practically zero, i.e. exponentially small. Considering then a system with fixed particle number, the persistent current is given by [12–14], $I(\phi) = -\partial_\phi F(N, \phi)$. The free energy, F, and the grand potential Ω are related via a Legendre transformation, $F(N, \phi) = \Omega(\mu(N, \phi), \phi) + \mu(N, \phi)N$. Hence we conclude that $\partial_\phi F(N, \phi) = \partial_\phi \Omega(\mu, \phi)|_{\mu=\mu(N,\phi)}$, where the chemical potential, $\mu(N, \phi) = \partial_N F(N, \phi)$, varies as a function of the flux ϕ such that the particle number is fixed. Writing $\mu(N, \phi) = \mu_0 + \delta\mu(\phi)$ and expanding the grand potential, the result is

$$I(\phi) = -\frac{\partial}{\partial\phi}\Omega(\mu,\phi)|_{\mu=\mu_0} - \delta\mu(\phi)\frac{\partial^2}{\partial\mu\partial\phi}\Omega(\mu,\phi)|_{\mu=\mu_0}. \tag{6}$$

At fixed chemical potential the particle number varies due to fluctuations of the density of states, i.e. $N(\mu_0) = -\partial_\mu\Omega(\mu,\phi)|_{\mu=\mu_0} = N + \delta N(\phi)$. On the other hand,

$$N = -\frac{\partial}{\partial\mu}\Omega(\mu,\phi)|_{\mu=\mu_0+\delta\mu} = -\frac{\partial}{\partial\mu}\Omega(\mu,\phi)|_{\mu=\mu_0} - \delta\mu(\phi)\frac{\partial^2}{\partial\mu^2}\Omega(\mu,\phi)|_{\mu=\mu_0} \tag{7}$$

from which we can relate $\delta N(\phi)$ and $\delta\mu(\phi)$: $\delta N(\phi) = \delta\mu(\phi)\partial_\mu^2\Omega|_{\mu=\mu_0}$. The mean level spacing, Δ, is given by $\Delta = -(\partial_\mu^2\Omega|_{\mu=\mu_0})^{-1} \simeq (2\mathcal{N}_0\mathcal{V})^{-1}$. Collecting these relations, we express the average persistent current at fixed particle number as follows:

$$\langle I\rangle = -\frac{\partial}{\partial\phi}\langle F(N,\phi)\rangle \simeq -\frac{\Delta}{2}\frac{\partial}{\partial\phi}\langle(\delta N)^2\rangle, \tag{8}$$

where the right hand side of this equation depends on expressions at the chemical potential $\mu = \mu_0$ only.

The last expression in Eq. (8), which assumes for example that the average level spacing is independent of the magnetic flux, is certainly valid when neglecting terms higher than quadratic in $\delta\mu$. However, it has been argued [15] that Eq. (8) is exact when one considers a large ensemble of rings with varying particle number. It should be noted that for strong spin-orbit scattering and near the Kramers degeneracy (zero flux) at very low temperature, there are subtle even-odd effects which are lost when averaging over the particle number [16].

Below we will *evaluate* Eq. (8) perturbatively within the cooperon-diffuson expansion; for such an analysis to be valid, one has to require that either the temperature or the phase-breaking rate is large compared to Δ.

The current fluctuations, $\langle I(\phi)I(\phi')\rangle_c$, can be found from the fluctuations of the thermodynamic potential, $\langle I(\phi)I(\phi')\rangle_c = \partial_\phi\partial_{\phi'}\langle K(\phi)K(\phi')\rangle_c$. For this quantity, the condition of fixed particle number seems not to be significant. We mention also that the particle number correlator can be found from the correlations of the grand potential, $\mathcal{M}_\Omega = \langle\Omega(\mu,\phi)\Omega(\mu',\phi')\rangle_c$, by differentiation with respect to μ and μ'.

Alternatively, we can directly calculate the expectation value of the current operator. Using the standard Green's functions, the (thermal average of the) current is

$$I(\phi) = \int\frac{d\varepsilon}{2\pi i}\sum_{\mathbf{k}\sigma} I_x n_F(\varepsilon)\left[G^A_{\mathbf{k}\sigma,\mathbf{k}\sigma}(\varepsilon) - G^R_{\mathbf{k}\sigma,\mathbf{k}\sigma}(\varepsilon)\right]. \tag{9}$$

$n_F(\varepsilon)$ is the Fermi function, and the current vertex is given by $I_x = (-e)\hbar k_x/mL$. In the ring geometry under consideration k_x assumes the values $k_x = 2\pi/L\cdot(n+\phi/\phi_0)$. Furthermore, $G^R(G^A)$ is the retarded (advanced) Green's function, $G^{R(A)}_{\mathbf{k}\mathbf{k}'}(\varepsilon)$ denotes the exact Green's function, before impurity averaging. For the Green's functions averaged over impurity configurations, which are diagonal in momentum space, we use the notation $G^{R(A)}(\varepsilon,\mathbf{k})$. For example, for noninteracting electrons and within the Born approximation, the retarded Green's function is $G^R(\varepsilon,\mathbf{k}) = (\varepsilon+\mu-\varepsilon_{\mathbf{k}}+i\hbar/2\tau)^{-1}$. Note that in many cases the expressions for $\Omega(\mu,\phi)$ are more compact than their derivatives, as we will see below.

To determine the mean current at fixed particle number within the Green's function approach, we follow the procedure described above: The chemical potential is set equal to $\mu = \mu_0 + \delta\mu$, and we expand Eq. (9) to first order in $\delta\mu$:

$$\langle I(\phi)\rangle = -2\int \frac{d\varepsilon}{2\pi}\sum_{\mathbf{k},\sigma} I_x n_F(\varepsilon)\mathrm{Im}\left\{G^R(\varepsilon,\mathbf{k})|_{\mu=\mu_0} - \delta\mu\left[G^R(\varepsilon,\mathbf{k})|_{\mu=\mu_0}\right]^2\right\}. \quad (10)$$

The first term gives the mean current at a fixed chemical potential μ_0 and can be neglected. We rewrite the second term using $I_x[G^R(\varepsilon,\mathbf{k})]^2|_{\mu=\mu_0} = -\partial_\phi G^R(\varepsilon,\mathbf{k})$ and find the expression

$$\langle I(\phi)\rangle = -2\delta\mu(\phi)\frac{\partial}{\partial\phi}\sum_{\mathbf{k}\sigma}\int\frac{d\varepsilon}{2\pi}n_F(\varepsilon)\mathrm{Im}G^R(\varepsilon,\mathbf{k})|_{\mu=\mu_0} \quad (11)$$

$$= +\delta\mu(\phi)\frac{\partial}{\partial\phi}N(\mu_0,\phi) \quad (12)$$

which agrees with Eq. (6).

The calculation of the current correlator involves averages of products of retarded and advanced Green's functions. The average of two retarded or two advanced Green's functions factorises, $\langle G^R_{\mathbf{k}\mathbf{k}}G^R_{\mathbf{k}'\mathbf{k}'}\rangle \approx \langle G^R_{\mathbf{k}\mathbf{k}}\rangle\langle G^R_{\mathbf{k}'\mathbf{k}'}\rangle$, while products of a retarded and an advanced Green's function give a connected, flux-sensitive contribution: $\langle G^R_{\mathbf{k}\mathbf{k}}G^A_{\mathbf{k}'\mathbf{k}'}\rangle = \langle G^R_{\mathbf{k}\mathbf{k}}G^A_{\mathbf{k}'\mathbf{k}'}\rangle_c + \langle G^R_{\mathbf{k}\mathbf{k}}\rangle\langle G^A_{\mathbf{k}'\mathbf{k}'}\rangle$. Using Eq. (9) we find

$$\langle I(\phi)I(\phi')\rangle_c = -8\left(\frac{e\hbar}{mL}\right)^2\int\frac{d\varepsilon_-}{2\pi}\frac{d\varepsilon_+}{2\pi}n_F(\varepsilon_-)[1-n_F(\varepsilon_+)]$$

$$\times\sum_{\mathbf{k},\mathbf{k}'}k_x k'_x\mathrm{Re}\langle G^A_{\mathbf{k}'\mathbf{k}'}(\varepsilon_-)G^R_{\mathbf{k}\mathbf{k}}(\varepsilon_+)\rangle_c, \quad (13)$$

where a spin factor of four has been included.

We consider now explicitly the expression for $\langle I(\phi)I(\phi')\rangle_c$ and relate it to the expression for $\mathcal{M}_\Omega$, within the usual perturbative treatment for the average $\langle G^A_{\mathbf{k}'\mathbf{k}'}(\varepsilon_-)G^R_{\mathbf{k}\mathbf{k}}(\varepsilon_+)\rangle_c$. Note that this approach has some limitations [15, 17]. The leading diagrams are shown in Fig. 1. The solid lines represent Green's functions, averaged over

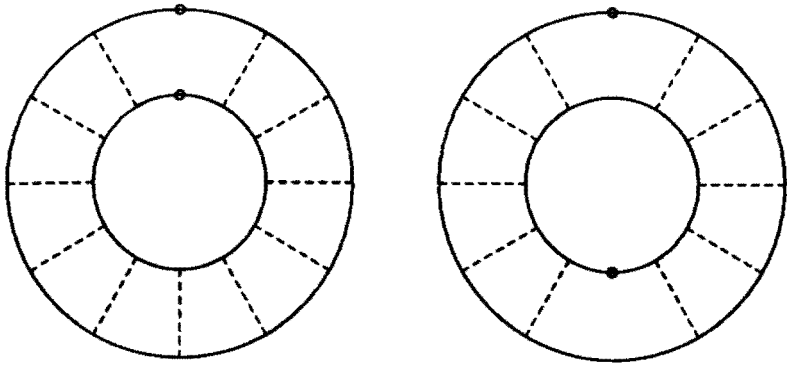

Fig. 1 Diagrams used for the calculation of the current correlator.

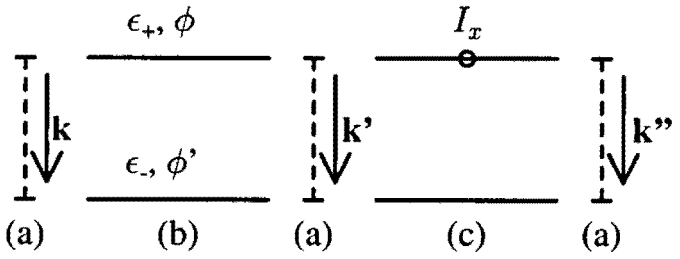

Fig. 2 The elements building up the diagrams in Fig. 1. The broken lines are impurity lines, the continuous lines represent Green's functions. The small circle is a current vertex.

impurity configurations. The broken lines are impurity lines, and current vertices are represented by small circles. The leading diagrams are those which do not have crossed impurity lines [18]. We did not explicitly assign arrows to the Green's functions, since there are always two possibilities (parallel or antiparallel) which have to be added. The diagrams are built up by the following elements, compare Fig. 2:

(a) Impurity lines, associated with a factor $X_0 = \hbar/2\pi\mathcal{N}_0\tau$. These lines carry the momentum $\hbar\mathbf{k}$ (but no energy).

(b) Products of an advanced and a retarded Green's function, integrated over the momentum:

$$\Pi^{C,D}(\mathbf{q}, \varepsilon_+ - \varepsilon_-) = \int \frac{d^3k}{(2\pi)^3} G^R(\varepsilon_+, \mathbf{k}) G^A(\varepsilon_-, \mp\mathbf{k} \pm \mathbf{q}). \tag{14}$$

C and D denote the cooperon and the diffuson. If we include the arrows in Fig. 1, the diagrams with the parallel arrows correspond to cooperon contributions, and $\mathbf{q}$ is the sum of the momenta of the two Green's functions. For antiparallel arrows we have a diffuson contribution, and $\mathbf{q}$ is the difference of the momenta.

(c) Blocks build up of three or four Green's function plus one or two current vertices. Since $I_x[G^{R,A}(\varepsilon, \mathbf{k})]^2 = -\partial_\phi G^{R,A}(\varepsilon, \mathbf{k})$, these blocks can be related to derivatives of $\Pi^{C,D}$.

In three dimensions and for $q \ll k_F$; $\varepsilon_+, \varepsilon_- \ll \varepsilon_F$, we find

$$\Pi^{C,D}(q, \varepsilon_+ - \varepsilon_-) = \frac{\pi\mathcal{N}_0\tau}{\hbar} \frac{i}{ql} \ln \frac{ql + (\varepsilon_+ - \varepsilon_-)\tau/\hbar + i}{-ql + (\varepsilon_+ - \varepsilon_-)\tau/\hbar + i}, \tag{15}$$

where $l = v_F\tau$ is the elastic mean free path. In the limit $ql \ll 1$, $(\varepsilon_+ - \varepsilon_-)\tau/\hbar \ll 1$, $\Pi^{C,D}(q, \varepsilon_+ - \varepsilon_-) \simeq [1 + i(\varepsilon_+ - \varepsilon_-)\tau/\hbar - D\tau q^2]2\pi\mathcal{N}_0\tau/\hbar$, with $D = v_F l/3$ the diffusion constant. Summing up an impurity ladder leads to the following result:

$$X(q, \varepsilon_+ - \varepsilon_-) = X_0 \sum_{M=0}^{\infty} (\Pi^{C,D} X_0)^M \tag{16}$$

$$= X_0 \frac{1}{1 - \Pi^{C,D} X_0} = \frac{\hbar^2}{2\pi\mathcal{N}_0\tau^2} \frac{1}{-i(\varepsilon_+ - \varepsilon_-) + \hbar D q_\pm^2} \tag{17}$$

where $X = C, D$ is the cooperon or diffuson propagator. The notation $q_\pm$ indicates that q is the sum or difference of two momenta.

Now we can transform the diagrams in Fig. 1 into an analytic expression. The contribution to the current fluctuation from the left diagram of Fig. 1, for example, is

$$\langle I(\phi)I(\phi')\rangle_c = -8\int \frac{d\varepsilon_-}{2\pi}\frac{d\varepsilon_+}{2\pi} n_F(\varepsilon_-)[1-n_F(\varepsilon_+)]$$

$$\times \mathrm{Re}\left[\sum_q (\partial_\phi \partial_{\phi'}\Pi^{C,D})\frac{X_0}{1-\Pi^{C,D}X_0}\right]_{C+D}. \tag{18}$$

While in general, $\mathbf{q}$ is a threedimensional vector, for the ring geometry q can be considered onedimensional. The subscript $C+D$ indicates that we have to sum over both (cooperon and diffuson) contributions.

After a change of variables to $y = \varepsilon_+ - \varepsilon_-$ we can integrate over ε_-:

$$\int d\varepsilon_- n_F(\varepsilon_-)[1-n_F(y+\varepsilon_-)] = \frac{y e^{y/2k_BT}}{e^{y/2k_BT}-e^{-y/2k_BT}}. \tag{19}$$

The combined contribution of both diagrams in Fig. 1 is of the form

$$(\partial_\phi\partial_{\phi'}\Pi^{C,D})\frac{X_0}{1-\Pi^{C,D}X_0} + (\partial_\phi\Pi^{C,D})(\partial_{\phi'}\Pi^{C,D})\left(\frac{X_0}{1-\Pi^{C,D}X_0}\right)^2$$

$$= -\partial_\phi\partial_{\phi'}\ln(1-\Pi^{C,D}X_0). \tag{20}$$

Note that only $\Pi^{C,D}$ depends on the fluxes. Here we realise that it is more convenient to consider the (flux-dependent part of the) correlations of the thermodynamic potential,

$$\langle\Omega(\phi)\Omega(\phi')\rangle_c = \frac{2}{\pi^2}\int_0^\infty dy y \coth\frac{y}{2k_BT}\mathrm{Re}\left[\sum_q \ln(-iy+\hbar D q_\pm^2)\right]_{C+D}. \tag{21}$$

The diagrammatic representation of this expression is shown in Fig. 3: Graphically one only has to remove the current vertices from the diagrams for the current fluctuations. The remaining diagrams are vacuum diagrams, as is normal for the thermodynamic potential. The summation is over momenta $q_\pm = 2\pi[n+(\phi\pm\phi')/\phi_0]/L$, with integer n and plus or minus for the cooperon or diffuson contributions. Problems of convergence can be overcome by subtracting, in Eq. (21), a flux independent constant.

In the next step we determine the Fourier coefficients, compare Eqs. (2) and (3). Writing the summation over q in the form

$$\mathcal{M}_\Omega = \sum_{n=-\infty}^{n=+\infty}\left[G\left(4\pi^2 E_c(n+x)^2\right)\right]_{C+D} \tag{22}$$

with $x = (\phi\pm\phi')/\phi_0$ and a suitable function G, we obtain

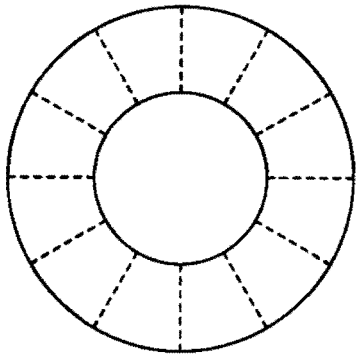

Fig. 3 Diagram used for the calculation of the fluctuations of the grand potential $\mathcal{M}_\Omega$.

$$\mathcal{M}_\Omega = \sum_{m=-\infty}^{m=+\infty} \left[e^{2\pi i m x} G_m\right]_{C+D} \tag{23}$$

where

$$G_m = \int_{-1/2}^{1/2} dx e^{-2\pi i m x} \sum_{n=-\infty}^{n=+\infty} G\left(4\pi^2 E_c (n+x)^2\right) = \int_{-\infty}^{+\infty} \frac{dx}{2\pi} e^{-imx} G(E_c x^2). \tag{24}$$

Since $G_m = G_{-m}$ only terms $\propto \cos(2\pi m x)$ survive in Eq. (23). In the present example, Eq. (21), the integration over x is easily achieved after integrating by parts:

$$\int_{-\infty}^{\infty} \frac{dx}{2\pi} e^{-imx} \mathrm{Re}\ln(-iy + E_c x^2) = \int_{-\infty}^{\infty} \frac{dx}{2\pi i m} e^{-imx} \left(\frac{E_c x}{-iy + E_c x^2} + \frac{E_c x}{iy + E_c x^2}\right)$$

$$= -\frac{1}{m} e^{-\sqrt{ym^2/2E_c}} \cos\sqrt{ym^2/2E_c}. \tag{25}$$

The combination of cooperon and diffuson terms is of the form

$$2\sum_{m=1}^{\infty} G_m \left[\cos\left(\frac{\phi + \phi'}{\phi_0} 2\pi m\right) + \cos\left(\frac{\phi - \phi'}{\phi_0} 2\pi m\right)\right]$$

$$= 4\sum_{m=1}^{\infty} G_m \cos\left(\frac{2\pi m\phi}{\phi_0}\right) \cos\left(\frac{2\pi m\phi'}{\phi_0}\right). \tag{26}$$

The explicit result at zero temperature is the following[1]:

$$\mathcal{M}_\Omega = \frac{96E_c^2}{\pi^2} \sum_{m=1}^{\infty} \frac{1}{m^5} \cos\frac{2\pi m\phi}{\phi_0} \cos\frac{2\pi m\phi'}{\phi_0}, \tag{27}$$

from which we find $\langle I_m I_{m'}\rangle_c = \delta_{mm'}(eE_c)^2 96/(\hbar^2\pi^2 m^3)$.

[1] This quantity was first estimated in [19].

The calculation of the mean current is very similar. First we determine the potential correlator at different chemical potentials μ and μ'. The expression is the same as in Eq. (21), except for the replacement $\ln(-iy+\hbar Dq^2) \to \ln[-i(y+\mu-\mu')+\hbar Dq^2]$. The potential correlator has to be taken at equal fluxes $\phi = \phi'$, i.e. we need the cooperon contribution only. The explicit result for the current at zero temperature is $\langle I_{2m}\rangle_{CE} = 4e\Delta/\hbar\pi^2$ (CE means canonical ensemble).

2.2 *Interacting electrons*

As pointed out in [12], the flux-dependent contribution to the canonical potential given in Eq. (7) can be explained by the absence of global charge fluctuations. This becomes apparent by considering a grand canonical ensemble, and adding a capacitive energy [20,21], i.e.

$$H_C = \frac{e^2}{2C}(\delta N)^2. \tag{28}$$

If we take this expression – which is similar to (8) except that the charging energy, $e^2/2C$, replaces the level spacing – directly, we find a persistent current which is comparable in size to the result for onedimensional non-interacting electrons on a perfect ring (since $C \sim L$, and $e^2 \sim \hbar v_F$ for metallic densities, implying $e^2/2C \sim \hbar v_F/L$). However the standard RPA approximation leads to the replacement

$$\frac{e^2}{C} \to \frac{e^2/C}{1+\Delta^{-1}e^2/C} \simeq \Delta \tag{29}$$

since $\Delta \ll e^2/C$, and we are back to (8), i.e.

$$\langle\delta\Omega\rangle_C = \frac{1}{2}\Delta\langle(\delta N)^2\rangle. \tag{30}$$

From this argument, it appears reasonable that an interaction, which suppresses the charge fluctuations locally and hence is a stronger constraint, leads to a larger persistent current. For example, consider the Hartree contribution to the grand potential,

$$\langle\delta\Omega\rangle_{EEI} = \frac{1}{2}\int d^3r d^3r' \tilde{v}(\mathbf{r}-\mathbf{r}')\langle\delta n(\mathbf{r})\delta n(\mathbf{r}')\rangle \tag{31}$$

where $\tilde{v}$ denotes the screened Coulomb interaction. The subscripts C and EEI denote capacitance and electron-electron interaction. Both parts of $\langle\delta\Omega\rangle$ are represented graphically in Fig. 4. By definition, the momentum transfer of an interaction line in $\langle\delta\Omega\rangle_C$ is $\mathbf{k}=0$, i.e. the leading diagram consists of two cooperons. The momentum transfer of the local interaction line in $\langle\delta\Omega\rangle_{EEI}$ is of the order $k \sim k_F$, i.e. the leading diagram consists of only one cooperon. The latter calculation is more involved since Hartree and exchange (Fock) contributions are of the same order, and also higher orders in the interaction $\tilde{v}$ have to be considered. We approximate $\tilde{v}$ by a local interaction, $\tilde{v}(\mathbf{x}) \approx \tilde{w}\delta(\mathbf{x})$, and introduce the dimensionless coupling constant, $\lambda_0 = \tilde{v}\mathcal{N}_0$.

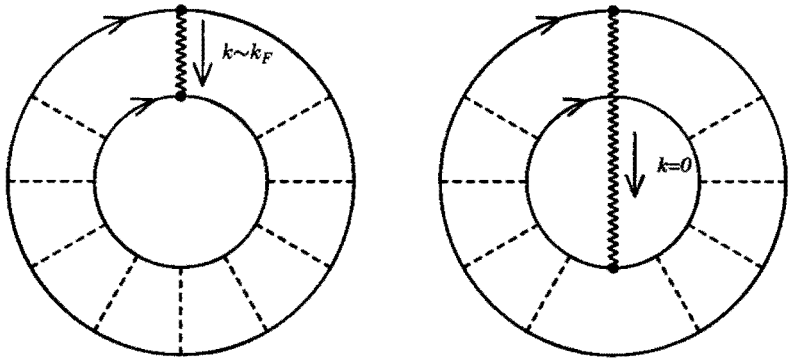

Fig. 4 Diagrams representing the flux-sensitive part of the grand potential Ω (Hartree contribution). The wavy line represents the Coulomb interaction. In the diagram on the left, the Coulomb interaction couples to local charge fluctuations, i.e. the interaction line carries a momentum of typically $k \sim k_F$. In the diagram on the right, the Coulomb interaction couples to global charge fluctuations, i.e. the interaction line carries zero momentum. The latter corresponds to the capacitance model.

Higher order corrections can be absorbed in a renormalisation of the coupling constant, $\lambda_0 \to \lambda_c \ll \lambda_0$. While the bare coupling constant can be reliably determined (copper: $\lambda_0 \sim 0.3$), the estimate $\lambda_c \sim \lambda_0/10$ is unfortunately very rough. Including Hartree and Fock contribution [5], the electron-electron interaction contribution to the grand potential is the following:

$$\langle \delta\Omega \rangle_{EEI} = -2\lambda_c \sum_q \int_0^\infty \frac{dy}{2\pi} y \coth \frac{y}{2k_B T} \mathrm{Re} \frac{1}{-iy + \hbar D q^2}, \tag{32}$$

where at present $q = 2\pi(n + 2\phi/\phi_0)/L$. The final result for the Fourier coefficients [5] of the persistent current at $T = 0$ is $\langle I_{2m} \rangle_{EEI} = 8\lambda_c e E_c/\hbar\pi m^2 \sim \lambda_c (e v_F l/L^2) m^{-2}$.

3 Magnetic impurities

The sensitivity of quantum coherence with respect to magnetic scattering is well known. It has been discussed in detail for weak localisation [22–24] and conductance fluctuations [25–27]. Persistent currents in the presence of magnetic impurities have been discussed in [28–30], including effects of spin-flip scattering, spin-orbit scattering and Zeeman splitting. In these papers the spin configuration was considered static, leaving the role of impurity spin dynamics an open problem.

In order to include impurity spins, we start from the usual Hamiltonian where the local magnetic moments are coupled to the conduction band electrons by a local exchange coupling,

$$H_{sd} = -\frac{J}{\mathcal{V}} \sum_{\mathbf{R}} \sum_{\mathbf{k}\mathbf{k}'\sigma\sigma'} \exp[-i(\mathbf{k} - \mathbf{k}')\mathbf{R}] c^+_{\mathbf{k}\sigma} c_{\mathbf{k}'\sigma'} \vec{\sigma}_{\sigma\sigma'} \vec{S}_{\mathbf{R}}. \tag{33}$$

J is the coupling constant, $\mathbf{R}$ are the impurity sites, $c^+_{\mathbf{k}\sigma}$ and $c_{\mathbf{k}\sigma}$ the Fermi operators, $\vec{\sigma}$ are the Pauli matrices and $\vec{S}_{\mathbf{R}}$ denotes the spin operator for a spin $S = 1/2$ impurity at site $\mathbf{R}$.

This model contains a number of "magnetic" energy scales which are characteristic for the system: The spin-flip scattering rate, $\hbar/\tau_s = n_s S(S+1)J^2 2\pi\mathcal{N}_0$ ($n_s = N_s/\mathcal{V}$ is the density of magnetic impurities), the Korringa rate for spin relaxation, $\hbar/\tau_K \sim k_B T(J\mathcal{N}_0)^2$, the Kondo temperature, $k_B T_K \sim \varepsilon_F \exp(-1/2|J|\mathcal{N}_0)$, and the spin glass temperature, $k_B T_{SG} \sim n_s J^2 \mathcal{N}_0 \sim \hbar/\tau_s$.

3.1 Spin polarisation in magnetic fields

Here we discuss magnetic scattering perturbatively in J, i.e. we exclude the Kondo effect. We include both the Zeeman splitting of the conduction band and the effect of Zeeman splitting on exchange scattering. In a magnetic field, the impurity spins are polarised, $\langle\vec{S}\rangle \neq 0$. Assuming that the magnetic field is in z-direction, we have $\langle S^x\rangle = \langle S^y\rangle = 0$, and $\langle S^z\rangle = \tanh(\omega_s/2T)/2$. $\langle\cdots\rangle$ denotes the thermal average[2]. The corresponding first order contribution to the electron self energy is

$$\Sigma^{(1)}_{\alpha\beta} = -n_s J\vec{\sigma}_{\alpha\beta}\langle\vec{S}\rangle = -n_s J\sigma^z_{\alpha\beta}\frac{1}{2}\tanh\frac{\omega_s}{2T}. \tag{34}$$

In second order in J, there is an effective electron-electron interaction, V, due to the coupling of conduction band electrons and magnetic impurities. In imaginary time representation it is given by

$$V_{\alpha\beta\gamma\delta}(i\omega_n) = -\left(\frac{J}{\mathcal{V}}\right)^2 \int_0^\beta d\tau e^{i\omega_n\tau} \sum_{a,b=x,y,z} \sigma^a_{\alpha\gamma}\sigma^b_{\beta\delta}\langle T_\tau S^a(\tau)S^b(0)\rangle. \tag{35}$$

T_τ is the time ordering. The result is

$$\begin{aligned} V_{\alpha\beta\gamma\delta} &= -\left(\frac{J}{\mathcal{V}}\right)^2 \\ &\times \left(\frac{1}{T}\frac{1}{4}\delta_{0\omega_n}\sigma^z_{\alpha\gamma}\sigma^z_{\beta\delta} + \tanh\frac{\omega_s}{2T}\frac{1}{i\omega_n+\omega_s}\sigma^+_{\alpha\gamma}\sigma^-_{\beta\delta} - \tanh\frac{\omega_s}{2T}\frac{1}{i\omega_n-\omega_s}\sigma^-_{\alpha\gamma}\sigma^+_{\beta\delta}\right) \end{aligned} \tag{36}$$

where $\omega_n = 2\pi nT$ is a bosonic Matsubara frequency. There is no energy transfer if no spins are flipped, i.e. in the $\sigma^z_{\alpha\gamma}\sigma^z_{\beta\delta}$ term. For weak magnetic fields, $\omega_s/T \to 0$, all contributions to $V_{\alpha\beta\gamma\delta}$ become static,

$$V_{\alpha\beta\gamma\delta} = -\frac{1}{T}\frac{1}{4}\left(\frac{J}{\mathcal{V}}\right)^2 \vec{\sigma}_{\alpha\gamma}\vec{\sigma}_{\beta\delta}\delta_{0\omega_n}. \tag{37}$$

The electron self energy second order in J is

$$\Sigma^{(2)}_\gamma(i\varepsilon_n) = N_s(-T)\sum_{\omega_l}\sum_{\mathbf{k}\alpha} V_{\gamma\alpha\alpha\gamma}(i\omega_l)G(i\varepsilon_n - i\omega_l, \mathbf{k}). \tag{38}$$

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[2] In the following we use units where $\hbar = k_B = 1$.

Since $\Sigma^{(2)}$ (and $\Sigma^{(1)}$) is diagonal in the spin, here and below we use a simplified notation, $\Sigma_{++} \to \Sigma_+$, $\Sigma_{--} \to \Sigma_-$. Details are given in App. A. After analytical continuation, $i\varepsilon_n \to \varepsilon + i0$, the retarded self energy in second order is found as [31]

$$\mathrm{Re}\,\Sigma_\gamma^{R(2)}(\varepsilon,\omega_s) = n_s J^2 \mathcal{N}_0 \tanh\frac{\gamma\omega_s}{2T}\left[\ln\frac{\varepsilon_F}{2\pi T} - \mathrm{Re}\,\Psi\left(\frac{1}{2} - i\frac{\varepsilon+\gamma\omega_s}{2\pi T}\right)\right] \tag{39}$$

$$\mathrm{Im}\,\Sigma_\gamma^{R(2)}(\varepsilon,\omega_s) = -n_s\pi\mathcal{N}_0 J^2 \tanh\frac{\gamma\omega_s}{2T}\left[n_B(\gamma\omega_s) + n_F(\gamma\omega_s+\varepsilon)\right] - \frac{1}{4}\pi\mathcal{N}_0 n_s J^2. \tag{40}$$

n_s is the density of the magnetic impurities, n_B and n_F the Bose and Fermi functions, $\gamma = \pm 1$, and Ψ the digamma function. The term first order in J, Eq. (34), renormalises the electron Zeeman energy, $\omega_s \to \omega_s + n_s J \tanh(\omega_s/2T)$, compare Eq. (41) below. Above the Kondo temperature, the second order correction to the real part of Σ is small compared to the first order, nevertheless it has to be included since it is of the same order as $1/\tau_s$.

For $\omega_s = 0$, only the second order contribution to the self energy survives, which is purely imaginary, $-\mathrm{Im}\Sigma_\gamma^R(\varepsilon) = n_s 3\pi\mathcal{N}_0 J^2/4 = 1/2\tau_s$. In the presence of magnetic fields, $\mathrm{Im}\Sigma_\gamma^R(\varepsilon)$ is energy and spin dependent. The first of the two terms on the right hand side of Eq. (40) describes spin-flip processes; for $|\omega_s| \gg T$ and $|\varepsilon| < \omega_s$, spin-flip is forbidden and this term is negligible.

The impurity averaged retarded Green's function is given by

$$G_\gamma^R(\varepsilon_+,\mathbf{k}) = \frac{1}{\varepsilon_+ - \mathbf{k}^2/2m + \mu + \gamma\omega_s/2 + i/2\tau + i/2\tau_{so} - \Sigma_\gamma^R(\varepsilon_+)}, \tag{41}$$

where $1/\tau$ and $1/\tau_{so}$ are the nonmagnetic and the spin-orbit scattering rates, and $\Sigma_\gamma^R(\varepsilon_+)$ is the spin- and energy-dependent contribution of the paramagnetic impurities to the self energy as given in Eqs. (34), (39) and (40).

3.1.1 Noninteracting electrons

For noninteracting electrons the main task is to determine the potential correlator $\langle\Omega(\phi)\Omega(\phi')\rangle_c$, from which both the mean current and the current fluctuations can be found. The quantity $\Pi_{\gamma\delta}^{C,D}$, defined now by (compare Eq. (14))

$$\Pi_{\gamma\delta}^{C,D} = \int \frac{d^3k}{(2\pi)^3} G_\gamma^R(\varepsilon_+,\mathbf{k}) G_\delta^A(\varepsilon_-, \mp\mathbf{k} \pm \mathbf{q}) \tag{42}$$

is given by

$$\Pi_{\gamma\delta}^{C,D}/2\pi\mathcal{N}_0\tau = 1 - \tau\left[-i(\varepsilon_+ - \varepsilon_-) + Dq_\pm^2 - i\frac{\gamma\omega_s - \delta\omega_s'}{2} + \gamma_{\gamma\delta} + \frac{1}{\tau_{so}}\right] \tag{43}$$

where $\gamma_{\gamma\delta} = i(\Sigma_\gamma^R - \Sigma_\delta^A)$. We omitted the dependence of $\Pi_{\gamma\delta}^{C,D}$ on the difference of chemical potentials. The spin-flip scattering rate, the spin-orbit scattering rate, and

the Zeeman energy are assumed to be small compared to the non-magnetic elastic scattering rate, $1/\tau$.

Considering correlations between two different systems, the bare impurity vertices $X^0_{\alpha\beta\gamma\delta}$ are given by

$$2\pi\mathcal{N}_0 D^0_{\alpha\beta\gamma\delta} = \frac{1}{\tau}\delta_{\alpha\gamma}\delta_{\delta\beta} + \frac{1}{3\tau_{so}}\vec{\sigma}_{\alpha\gamma}\vec{\sigma}_{\delta\beta} + \gamma_z\sigma^z_{\alpha\gamma}\sigma^z_{\delta\beta} \tag{44}$$

and

$$2\pi\mathcal{N}_0 C^0_{\alpha\beta\gamma\delta} = \frac{1}{\tau}\delta_{\alpha\gamma}\delta_{\beta\delta} - \frac{1}{3\tau_{so}}\vec{\sigma}_{\alpha\gamma}\vec{\sigma}_{\beta\delta} + \gamma_z\sigma^z_{\alpha\gamma}\sigma^z_{\beta\delta} \tag{45}$$

where

$$\gamma_z = 2\pi\mathcal{N}_0 n_s J^2 \langle S^z\rangle\langle S^z\rangle' = n_s\frac{1}{2}\pi\mathcal{N}_0 J^2 \tanh\left(\frac{\omega_s}{2T}\right)\tanh\left(\frac{\omega_s'}{2T}\right). \tag{46}$$

In order to determine the potential correlator $\mathcal{M}_\Omega$, we have to compute the eigenvalues λ_j^X of the 4×4 matrix $X^0_{\alpha\beta\gamma\delta}\Pi^X_{\gamma\delta}$,

$$\langle\Omega(\phi)\Omega(\phi')\rangle_c : \quad \sum_{X=C,D}\sum_{m=1}^{\infty}\frac{1}{m}\mathrm{Tr}(X^0\Pi^X)^m = -\sum_{X=C,D}\sum_{j=1}^{4}\ln(1-\lambda_j^X). \tag{47}$$

We express the cooperon parts using the combinations

$$N_0 = -i(\varepsilon_+ - \varepsilon_-) + Dq_+^2 + \frac{\gamma_{+-}+\gamma_{-+}}{2} + \gamma_z \tag{48}$$

$$N_1 = -i(\varepsilon_+ - \varepsilon_-) + Dq_+^2 + \frac{4}{3\tau_{so}} + \frac{\gamma_{+-}+\gamma_{-+}}{2} + \gamma_z \tag{49}$$

$$N_2 = -i(\varepsilon_+ - \varepsilon_-) + Dq_+^2 + \frac{4}{3\tau_{so}} + \frac{\gamma_{+-}+\gamma_{-+}}{2} - \gamma_z, \tag{50}$$

and the diffuson contributions using the combinations

$$M_0 = -i(\varepsilon_+ - \varepsilon_-) + Dq_-^2 + \frac{\gamma_{++}+\gamma_{--}}{2} - \gamma_z \tag{51}$$

$$M_1 = -i(\varepsilon_+ - \varepsilon_-) + Dq_-^2 + \frac{4}{3\tau_{so}} + \frac{\gamma_{++}+\gamma_{--}}{2} - \gamma_z \tag{52}$$

$$M_2 = -i(\varepsilon_+ - \varepsilon_-) + Dq_-^2 + \frac{4}{3\tau_{so}} + \frac{\gamma_{++}+\gamma_{--}}{2} + \gamma_z. \tag{53}$$

The notation $N_0 \ldots M_2$ is chosen such that N_0 and M_0 correspond to singlet, and $N_{1,2}$ and $M_{1,2}$ to triplet parts. The explicit result for the potential correlator is

$$\langle\Omega(\phi)\Omega(\phi')\rangle_c = 2\int \frac{d\varepsilon_-}{2\pi}\frac{d\varepsilon_+}{2\pi} n_F(\varepsilon_-)[1-n_F(\varepsilon_+)]\mathrm{Re}\sum_q \ln$$
$$\times\left\{\left[N_2N_2-\left(\frac{\gamma_{++}-\gamma_{--}}{2}-i\frac{\omega_s-\omega_s'}{2}\right)^2\right]\right.$$
$$\times\left[N_0N_1-\left(\frac{\gamma_{-+}-\gamma_{+-}}{2}+i\frac{\omega_s+\omega_s'}{2}\right)^2\right]$$
$$\times\left[M_2M_2-\left(\frac{\gamma_{-+}-\gamma_{+-}}{2}+i\frac{\omega_s+\omega_s'}{2}\right)^2\right]$$
$$\left.\times\left[M_0M_1-\left(\frac{\gamma_{++}-\gamma_{--}}{2}-i\frac{\omega_s-\omega_s'}{2}\right)^2\right]\right\}. \tag{54}$$

In the absence of spin effects, $N_0 = \ldots = M_0 = \ldots = -i(\varepsilon_+ - \varepsilon_-) + Dq_\pm^2$, $\gamma_{\mu\nu} = \omega_s = 0$, i.e. the terms in the logarithm depend only on the difference of energies, $y = \varepsilon_+ - \varepsilon_-$. One of the two energy integrations is feasible and, finally, we recover Eq. (21).

If we reverse the sign of the magnetic field in one of the thermodynamic potentials, ϕ, $\omega_s \to -\phi, -\omega_s$, we have to replace $\gamma_z \to -\gamma_z$, $\gamma_{++} \to \gamma_{-+}$, and so on. In that way diffuson contributions transform into cooperon contributions, and vice versa. As a result, we have shown explicitly that $\mathcal{M}_\Omega(-\omega_s, -\phi, \omega_s', \phi') = \mathcal{M}_\Omega(\omega_s, \phi, \omega_s', \phi')$, as implied by time reversal symmetry.

For the following discussion, we consider ω_s and ϕ as independent variables. The persistent current is a periodic function in ϕ, but the Fourier components depend on the Zeeman energy ω_s:

$$I = \sum_{m=1}^{\infty}\left[I_m(\omega_s)\sin(2\pi m\phi/\phi_0) + \tilde{I}_m(\omega_s)\cos(2\pi m\phi/\phi_0)\right]. \tag{55}$$

Due to time reversal symmetry $I_m(\omega_s)$ and $\tilde{I}_m(\omega_s)$ are even and odd, respectively; in particular, $\tilde{I}_m(\omega_s = 0) = 0$.

In the ideal case, i.e. $T = 0$, and $1/\tau_s = 1/\tau_{so} = \omega_s = 0$, one finds for the Fourier components of the current correlator

$$\langle I_m I_{m'}\rangle_{id} = \delta_{mm'}\frac{96}{\pi^2}e^2E_c^2\frac{1}{m^3}. \tag{56}$$

In general, we define h_m through the relation $\langle I_m(\omega_s)I_m(\omega_s')\rangle_c = h_m\langle I_m^2\rangle_{id}$, i.e. h_m is a function of the T, $1/\tau_s$, $1/\tau_{so}$, ω_s, ω_s'. For example, the temperature suppression is given by

$$h_m(\Gamma) = -\frac{2}{3}\int_0^\infty ds s^3\coth(s^2/\Gamma)e^{-s}\cos(s) \tag{57}$$

with $\Gamma = T/T_m$, $T_m = D/(mL)^2$. The asymptotic results for low and high temperature are $h_m(0) = 1$, and $h_m(\Gamma \gg 1) \simeq (\pi^2\Gamma^2/3)\exp(-\sqrt{2\pi\Gamma})$. The temperature scale for the decay of the m^{th} Fourier component is set by T_m, so at a given temperature T the higher components are suppressed stronger than the lower Fourier components. Considering the suppression of the current due to exchange scattering, we find at zero temperature, and for $1/\tau_{so} = \omega_s = \omega_s' = 0$:

$$h_m(\Gamma) = e^{-\sqrt{\Gamma}}(1 + \sqrt{\Gamma} + \frac{1}{3}\Gamma), \tag{58}$$

with $\Gamma = 1/\tau_s T_m$. If Zeeman splitting is the only mechanism destroying the coherence, i.e. $T = 0$, $1/\tau_s = 1/\tau_{so} = 0$, and for $\omega_s = \omega_s'$, we find

$$h_m(\Gamma) = \frac{1}{2} + \frac{1}{2}e^{-\sqrt{\Gamma/2}} \times \left[\cos\left(\sqrt{\Gamma/2}\right)\left(1 + \sqrt{\Gamma/2}\right) + \sin\left(\sqrt{\Gamma/2}\right)\left(\sqrt{\Gamma/2} + \frac{\Gamma}{3}\right)\right]; \tag{59}$$

here $\Gamma = \omega_s/T_m$. Equation (59) can be obtained from Eq. (58), taking into account that only two of the four diffuson/cooperon channels are suppressed, and $\Gamma \to \pm i\Gamma$ in the remaining channels. Fig. 5 depicts h_1 as a function of the Zeeman energy in the two limiting cases, $1/\tau_s = 0$, compare Eq. (59), and for strong spin-flip scattering, $1/\tau_s \gg E_c$. Remarkably, h_1 is a non-monotonic function in ω_s/E_c. The asymptotic value $h_1 \to 1/2$ is only reached for rather high values of the magnetic field. The broken line is the curve for strong spin scattering; we have chosen $1/\tau_s = 40E_c$, and $T = 0.2E_c$. We took into account the second order self energy, $\Sigma_{\pm}^{(2)}(\varepsilon)$, only, since the first order term $\Sigma_{\pm}^{(1)}$ is irrelevant: For $1/\tau_s \gg E_c$ and $\omega_s = \omega_s'$, the relevant

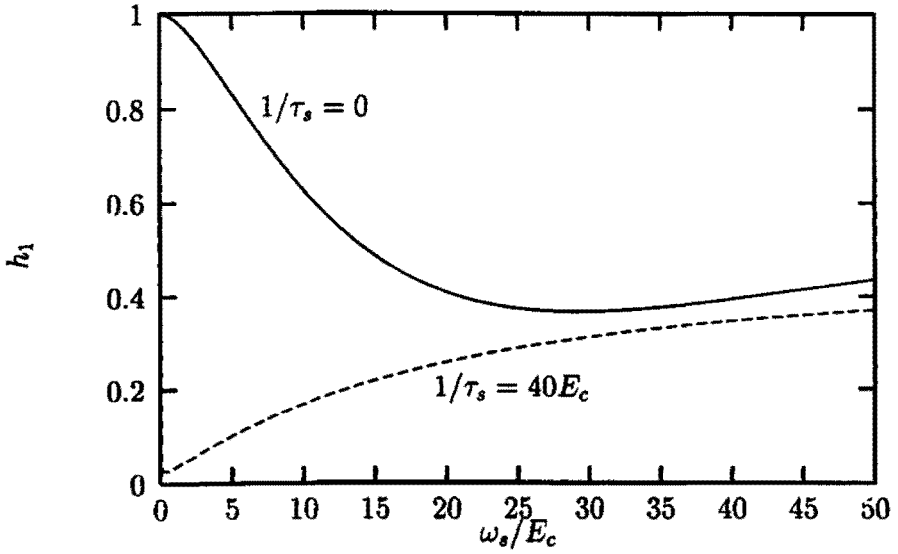

Fig. 5 Normalised amplitude of the first harmonic of the current, $\langle I_1^2\rangle_c = h_1\langle I_1^2\rangle_{id}$ as a function of ω_s/E_c. The continuous line shows the result in the absence of magnetic impurities, where the current is reduced due to Zeeman splitting. The dashed line shows the result in the presence of strong spin-flip scattering. The temperature is $T = 0.2E_c$.

parts for the potential correlator are the N_2 contributions for the cooperon, and M_0, M_1 for the diffuson. $\Sigma_{\pm}^{(1)}$ cancels in γ_{++} and γ_{--}, i.e. the relevant parts are independent of $\Sigma_{\pm}^{(1)}$.

The limit $\omega_s \ll T$ has been discussed before (compare Eq. (58)) and explains the limiting value of h_1 for low ω_s. Note that the condition $\omega_s \gg T$ is not at all sufficient to find a "large" current: For $T \ll \omega_s \ll T_m$, spin-flip processes are still allowed since, for calculating $\langle I_m^2 \rangle_c$, typical energies $\varepsilon_{\pm}$ in the integrations are of the order $\varepsilon_{\pm} \sim T_m$ or even larger. Further increasing the strength of the magnetic field, spin-flip processes will be forbidden for $\omega_s \gg T_m$, i.e. $\mathrm{Im}\Sigma_{\gamma}^{R} = -1/6\tau_s$ in the energy range of interest. Using the relation $\partial_{\varepsilon}\Sigma_{\pm}^{R}\big|_{\varepsilon=0} = -2/(3\pi\tau_s\omega_s)$, we find an algebraic reduction of the current:

$$h_m = \frac{1}{2}\frac{1}{(1+2/3\pi\tau_s\omega_s)^2}. \tag{60}$$

This function fits the dashed curve in Fig. 5 for high values of ω_s/E_c.

3.1.2 Interacting electrons (mean current)

In this case the two Green's functions which determine the cooperon correspond to the same thermodynamic potential. Because of this the bare impurity vertex $C^0_{\alpha\beta\gamma\delta}$ is different to that in Eq. (45): Instead of inserting the term $\gamma_z \sigma^z_{\alpha\gamma}\sigma^z_{\beta\delta}$, the Green's functions are connected by the interaction $V_{\alpha\beta\gamma\delta}$ as defined in Eq. (36). We restrict ourselves to two cases, $\omega_s \ll T$, and $\omega_s \gg T, E_c$. For a weak magnetic field $V_{\alpha\beta\gamma\delta}$ is dominated by its static part from which we find

$$2\pi\mathcal{N}_0 C^0_{\alpha\beta\gamma\delta} = \frac{1}{\tau}\delta_{\alpha\gamma}\delta_{\beta\delta} + \left(\frac{1}{3\tau_s} - \frac{1}{3\tau_{so}}\right)\vec{\sigma}_{\alpha\gamma}\vec{\sigma}_{\beta\delta} \tag{61}$$

for the bare cooperon. It is interesting that in this case the current, related to global charge fluctuations, $\langle I(\phi)\rangle = -\partial_{\phi}\langle\delta\Omega\rangle_C$, is *not* equal to the mean current found for non-interacting electrons. Although the current is reduced in the presence of magnetic impurities in both cases, the decay of the cooperons is not identical. $\langle I\rangle_{EEI} = -\partial_{\phi}\langle\delta\Omega\rangle_{EEI}$ has been discussed in detail in [28], where expressions similar to Eqs. (57)-(59) have been given. The flux-dependent part of the grand potential was found as

$$\langle\delta\Omega\rangle_{EEI} = -\lambda_c 2\pi\mathcal{N}_0\tau^2 \sum_{q,\alpha,\beta}\int_0^{\infty}\frac{dy}{2\pi} y \coth\frac{y}{2T}\mathrm{Re}\left(C_{\alpha\beta\alpha\beta} - C_{\alpha\beta\beta\alpha}\right). \tag{62}$$

We assumed λ_c, the effective interaction constant in the Cooper channel, to be the same for Hartree and exchange contributions. The relevant combinations of the spin-dependent cooperon can be expressed in terms of singlet and triplet components, $\sum_{\alpha\beta} C_{\alpha\beta\alpha\beta} = 3C_1 + C_0$, and $\sum_{\alpha\beta} C_{\alpha\beta\beta\alpha} = 3C_1 - C_0$, such that only the singlet component is relevant for $\langle\delta\Omega\rangle_{EEI}$. In particular, $C_0(q,\omega) = (1/2\mathcal{N}_0\tau^2)(-i\omega + Dq^2 + 2/\tau_s)^{-1}$. Note that C_0 is independent of spin-orbit scattering.

In the opposite limit, $\omega_s \gg E_c, T$, the inelastic parts of $V_{\alpha\beta\gamma\delta}$, that is the spin-flip contributions, can be neglected. As a consequence $C^0_{\alpha\beta\gamma\delta}$ is given by

$$2\pi\mathcal{N}_0 C^0_{\alpha\beta\gamma\delta} = \frac{1}{\tau}\delta_{\alpha\gamma}\delta_{\beta\delta} - \frac{1}{3\tau_{so}}\vec{\sigma}_{\alpha\gamma}\vec{\sigma}_{\beta\delta} + \gamma_z\sigma^z_{\alpha\gamma}\sigma^z_{\beta\delta} \tag{63}$$

which is identical to $C^0_{\alpha\beta\gamma\delta}$ given in Eq. (45). For the thermodynamic potential we can use the previously defined quantities N_0 and N_1 as well as the expressions for the cooperon $C_{\alpha\beta\gamma\delta}$ given in App. B. We find

$$\langle\delta\Omega\rangle_{EEI} = -2\lambda_c \sum_q \int_0^\infty \frac{dy}{2\pi} y \coth\frac{y}{2T} \mathrm{Re}\frac{N_1}{N_0N_1 - [(\gamma_{-+} - \gamma_{+-})/2 + i\omega_s]^2}. \tag{64}$$

Note that N_0, N_1 are functions of $q = 2\pi(n + 2\phi/\phi_0)/L$ and $y = \varepsilon_+ - \varepsilon_-$.

Some results are collected in Table 1 and 2. We consider the zero temperature limit and the asymptotic results only. We give both the results for the current fluctuation, $\langle I^2_m\rangle_c$, and for the mean current, i.e. the free electron contribution in a canonical ensemble, $\langle I_{2m}\rangle_{CE}$, and the collective contribution due to electron-electron interaction, $\langle I_{2m}\rangle_{EEI}$. All the numbers in the tables are normalized to the values without spin effects (see second row in Table 1). Table 1 shows the results in the absence of spin-orbit scattering. All contributions are strongly suppressed if $1/\tau_s \gg T_m$ and for weak magnetic fields. In the presence of strong magnetic fields, $\omega_s \gg T_m$, the collective contribution remains suppressed, the free electron contributions are only suppressed algebraically (Eq. 60) and can reach one half of their ideal values.

In the presence of strong spin-orbit scattering, the results are somewhat different (see Table 2). In this case, only the singlet, i.e. the M_0 and N_0 components, contribute to the current. If there are no exchange scatterers, the current is only weakly dependent on Zeeman splitting, in the sense that the typical energy scale is set by

Table 1 Relative magnitude of the typical current, the free electron contribution, and the collective contribution to the mean current as a function of the exchange scattering rate $1/\tau_s$ and the Zeeman energy ω_s. In the regime $T_m < \omega_s < 1/\tau_s$ the current is suppressed algebraically, compare Eq. (60). The spin-orbit scattering rate $1/\tau_{so}$ is set equal to zero.

	$1/\tau_s \ll T_m$		$1/\tau_s \gg T_m$	
	$\omega_s \ll T_m$	$\omega_s \gg T_m$	$\omega_s \ll T_m$	$\omega_s \gg T_m, 1/\tau_s$
$\langle I^2_m\rangle_c$:	1	1/2	0	1/2
$\langle I_{2m}\rangle_{CE}$:	1	1/2	0	1/2
$\langle I_{2m}\rangle_{EEI}$:	1	0	0	0

Table 2 Relative magnitude of the current in the presence of strong spin-orbit scattering. Treating ω_s and ϕ as independent variables, we find odd and even contributions to the current, compare Eq. (55).

	$1/\tau_s \ll T_m$		$1/\tau_s \gg T_m$	
	$\omega_s(\omega_s\tau_{so}) \ll T_m$	$\omega_s(\omega_s\tau_{so}) \gg T_m$	$\omega_s \ll T_m$	$\omega_s \gg T_m, 1/\tau_s$
$\langle I^2_m\rangle_c$:	1/4	1/8	0	1/8
$\langle I^2_m\rangle_c$:	0	1/8	0	1/8
$\langle I_{2m}\rangle_{CE}$:	1/4	0	0	0
$\langle I_{2m}\rangle_{EEI}$:	1	0	0	0

$\omega_s \cdot \omega_s \tau_{so}$ n, instead of $\omega_s \sim T_m$ as before. Note that the collective contribution is not reduced by spin-orbit scattering. For very high Zeeman energies only the M_0 component contributes to the current, so $\langle I_{2m} \rangle_{CE} = 0$ and $\langle I_m^2 \rangle_c$ is reduced by eight. Since the symmetry of diffuson and cooperon contributions is broken, there are also even harmonics, $\langle \tilde{I}_m^2 \rangle_c = \langle I_m^2 \rangle_c$.

3.2 Spin glasses

An interaction of the type given in Eq. (33) induces an effective interaction among the localised spins, which is called the RKKY interaction,

$$H_{eff} = -\sum_{i,j} J_{RKKY}(\mathbf{R}_i, \mathbf{R}_j) \vec{S}_{\mathbf{R}_i} \vec{S}_{\mathbf{R}_j}. \tag{65}$$

Below the spin glass temperature, T_{SG}, the spins are in an ordered phase, and within a mean field description each impurity spin can be considered in an effective magnetic field H_{SG} due to the other spins. This magnetic field is of the order $\mu_B H_{SG} \sim T_{SG}$. Generally, the electron self energy $\Sigma_{\alpha\beta}(\varepsilon)$ is the sum of a scalar part, Σ, and a vector part, $\vec{\Sigma}$: $\Sigma_{\alpha\beta}(\varepsilon) = \Sigma(\varepsilon)\delta_{\alpha\beta} + \vec{\Sigma}(\varepsilon)\vec{\sigma}_{\alpha\beta}$. If the internal magnetic fields are distributed isotropically and no external magnetic field is present the vector part of the self energy is zero. The scalar part of the self energy can be found from Eqs. (39) and (40), $\Sigma = (\Sigma_+ + \Sigma_-)/2$. The result is:

$$\begin{aligned} \mathrm{Re}\Sigma^R(\varepsilon) = &-\frac{1}{2\mathcal{V}} J^2 \mathcal{N}_0 \sum_{\mathbf{R}} \tanh \frac{\omega_{\mathbf{R}}}{2T} \\ &\times \left[\mathrm{Re}\Psi\left(\frac{1}{2} - i\frac{\varepsilon + \omega_{\mathbf{R}}}{2\pi T}\right) - \mathrm{Re}\Psi\left(\frac{1}{2} - i\frac{\varepsilon - \omega_{\mathbf{R}}}{2\pi T}\right) \right] \end{aligned} \tag{66}$$

$$\begin{aligned} \mathrm{Im}\Sigma^R(\varepsilon) = &-\frac{1}{4}\pi \mathcal{N}_0 n_s J^2 - \frac{1}{2\mathcal{V}} \pi \mathcal{N}_0 J^2 \sum_{\mathbf{R}} \tanh \frac{\omega_{\mathbf{R}}}{2T} \\ &\times [n_B(\omega_{\mathbf{R}}) + n_F(\omega_{\mathbf{R}} + \varepsilon) - (\omega_{\mathbf{R}} \leftrightarrow -\omega_{\mathbf{R}})]. \end{aligned} \tag{67}$$

Here $\omega_{\mathbf{R}} > 0$ is the Zeeman energy associated with the internal magnetic field H_{SG} at the impurity site $\mathbf{R}$. In the fully polarised regime, $T \ll T_{SG}$, the spin-flip scattering rate is now given by $1/\tau_s = 2\pi \mathcal{N}_0 n_s J^2 S^2 = \pi \mathcal{N}_0 n_s J^2/2$, which is by a factor three smaller than the scattering rate of paramagnetic impurities. Another consequence of the random distribution of the directions of internal fields is that $\sum_{\mathbf{R}} \langle S_{\mathbf{R}}^a \rangle \langle S_{\mathbf{R}}^b \rangle = \delta_{ab} \sum_{\mathbf{R}} (\langle \vec{S}_{\mathbf{R}} \rangle)^2/3$. For $\omega_{\mathbf{R}} \gg T$ the bare impurity vertices are

$$2\pi \mathcal{N}_0 D^0_{\alpha\beta\gamma\delta} = \frac{1}{\tau}\delta_{\alpha\gamma}\delta_{\delta\beta} + \left(\frac{1}{3\tau_s} + \frac{1}{3\tau_{so}}\right)\vec{\sigma}_{\alpha\gamma}\vec{\sigma}_{\delta\beta} \tag{68}$$

$$2\pi \mathcal{N}_0 C^0_{\alpha\beta\gamma\delta} = \frac{1}{\tau}\delta_{\alpha\gamma}\delta_{\beta\delta} + \left(\frac{1}{3\tau_s} - \frac{1}{3\tau_{so}}\right)\vec{\sigma}_{\alpha\gamma}\vec{\sigma}_{\beta\delta} \tag{69}$$

A scattering of this type has been considered in [29] and [30]. In the present work we consider the full energy-dependent electron self energy given in Eqs. (66) and (67), instead of the simple approximation $\Sigma^R = -i/2\tau_s$, which is the first term in Eq. (67). For $1/\tau_s > E_c$ only the diffuson singlet component contributes to the potential correlator, which is of the form

$$\langle \Omega(\phi)\Omega(\phi')\rangle_c : \mathrm{Re}\sum_q \ln\left[-i(\varepsilon_+ - \varepsilon_-) + Dq^2 + i(\Sigma^R(\varepsilon_+) - \Sigma^A(\varepsilon_-)) - 1/\tau_s\right]. \quad (70)$$

The internal magnetic fields break time reversal invariance and Ω is not an even function in ϕ, equivalently $I(\phi)$ has odd and even parts, I_m and $\tilde{I}_m$. They are of the same order, $\langle I_m^2\rangle_c = \langle \tilde{I}_m^2\rangle_c$. Considering the amplitude of the current we can use the results from the previous section. Spin polarisation has the tendency to restore the current, but as long as $\omega_R < E_c$ the current remains small, i.e. the current is strongly reduced if $T_{SG} \ll E_c \ll 1/\tau_s$. For $T_{SG} \gg E_c$ there is no exponential suppression of the current, but there remains a suppression of the Fourier components $\langle I_m^2\rangle_c$ given by $(1/8)(1 + 1/\pi\tau_s\mu_B H_{SG})^{-2}$, compare Eq. (60). The factor eight takes into account that only one diffuson channel, instead of four diffuson and four cooperon channels, contributes to the current. In the presence of an external magnetic field there is a crossover from the regime of random spin directions to the regime of aligned spins at $\omega_s \sim \mu_B H_{SG}$. The regime of aligned spins has been discussed in the previous chapter. For weak spin-orbit scattering the current increases, since more diffuson and cooperon channels contribute.

3.3 Kondo effect

For temperatures lower than the Kondo temperature, $T_K \sim \varepsilon_F \exp(-1/2|J|\mathcal{N}_0)$, perturbation theory in J breaks down, but in principle we can calculate the potential correlator as before. We express the self energies using the T-matrix, $\Sigma_\gamma^{R,A}(\varepsilon) = n_s T_\gamma^{R,A}(\varepsilon)$. In addition, we replace in Eqs. (44) and (45) $\gamma_z \sigma^z_{\alpha\gamma}\sigma^z_{\beta\delta}$ by $n_s 2\pi\mathcal{N}_0 T_\gamma^R(\varepsilon_+)T_\delta^A(\varepsilon_-)\delta_{\alpha\gamma}\delta_{\beta\delta}$. In the weak coupling limit $T_\gamma^R = \gamma(-J)/2 \cdot \tanh(\omega_s/2T)$, and we recover the results given above (Eqs. (39) and (46)).

Neglecting Zeeman effects and spin-orbit scattering the potential correlator can be determined from

$$\langle \Omega(\phi)\Omega(\phi')\rangle_c : \quad \mathrm{Re}\sum_q \ln\left[-i(\varepsilon_+ - \varepsilon_-) + Dq_\pm^2 + 1/\tau_\varphi\right] \quad (71)$$

where $1/\tau_\varphi = n_s[iT^R(\varepsilon_+) - iT^A(\varepsilon_-) - 2\pi\mathcal{N}_0 T^R(\varepsilon_+)T^A(\varepsilon_-)]$. Although many exact results for the Kondo model exist, an analytic expression for the T-matrix is not known. As an approximation for the T-matrix we use (for $E_c > T_K$) the expression given by Haman [32]

$$T_{Ha}^R(\varepsilon) = \frac{-i}{2\pi\mathcal{N}_0}\left[1 - x\left(x^2 + 3\pi^2/4\right)^{-1/2}\right] \quad (72)$$

with $x = \ln[(\varepsilon + iT)/iT_K]$. This approximation gives the correct behaviour for high energy or temperature. It interpolates into the low temperature regime such that

$T^R(\varepsilon = 0, T = 0)$ is correct, i.e. the so called unitarity limit is reached, $T_0^R = 1/(i\pi\mathcal{N}_0)$. If $T^R(\varepsilon, T)$ is replaced by this constant value, there is no suppression of the persistent current since $i(T_0^R - T_0^A) - 2\pi\mathcal{N}_0 T_0^R T_0^A = 0$. However Eq. (72) is not sufficiently accurate in the low energy regime. Exact results for low temperature can be found from Fermi liquid theory. A systematic expansion around the Fermi liquid fixed point has been formulated by Hewson [33]. For low energy the T matrix is given by [34]

$$T_{FL}^R(\varepsilon) = \frac{1}{\pi\mathcal{N}_0}\frac{4T_K}{\pi w}\frac{1}{\varepsilon + 4iT_K/\pi w}. \tag{73}$$

$w \approx 0.41$ is the Wilson number. Higher corrections are of the order $(T/T_K)^2$ or $(\varepsilon/T_K)^2$. Fig. 6 shows the normalised current as a function of E_c/T_K. Fixing E_c/T_K one parameter remains undetermined, namely the impurity concentration n_s. Here we choose $n_s = 50 \cdot 2\pi\mathcal{N}_0 E_c$. For the solid line we used T_{Ha}^R as an approximation for the T-matrix, and for the dashed line we used T_{FL}^R. There is a maximum suppression of the current for $E_c \sim T_K$. The physical reason is that the spin-flip scattering rate has a maximum at $T \sim T_K$ (or equivalently $\varepsilon \sim T_K$). For $E_c \ll T_K$, i.e. when the impurity spins are well screened, we find from Eqs. (71) and (73) for the relative magnitude of the current fluctuation

$$h_m = 1/(1 + n_s w/4\mathcal{N}_0 T_K)^2. \tag{74}$$

Note that Eq. (73) for the T-matrix is exact for $\varepsilon \ll T_K$ and for the single impurity problem. For the present situation with a finite concentration of magnetic impurities, Eq. (73) is thus only appropriate if the concentration of magnetic impurities is low. Recent experiments have shown a size dependence of the Kondo effect in systems of reduced dimensionality [35]. These experimental results are still controversial since

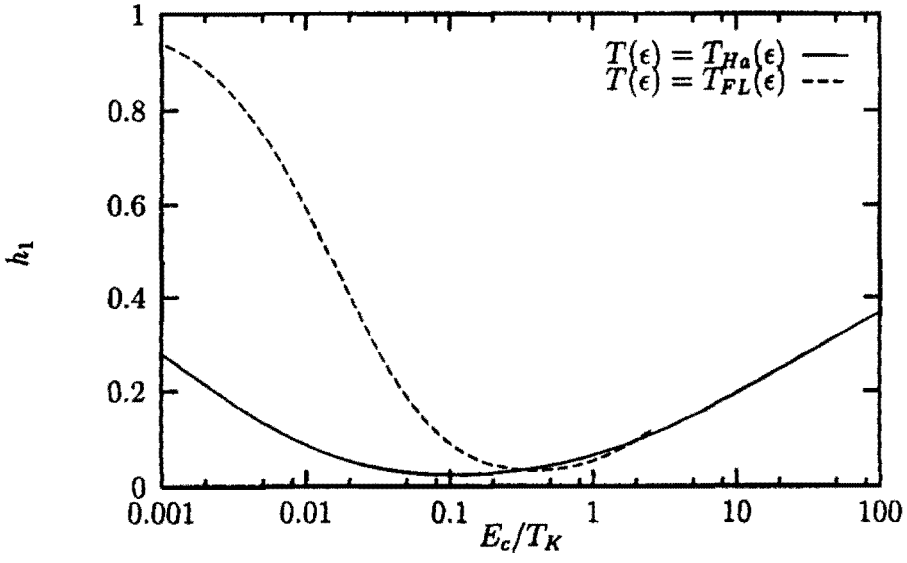

Fig. 6 Normalised amplitude of the first harmonic of the current, $\langle I_1^2\rangle_c = h_1\langle I_1^2\rangle_{id}$, as a function of E_c/T_K. We use two different approximations for the T-matrix, as explained in the text. The impurity concentration is chosen to be $n_s = 50 \cdot 2\pi\mathcal{N}_0 E_c$. If we estimate the ratio $E_c/\Delta = 2\mathcal{N}_0 \mathcal{V} E_c \sim 10^2$, and the number of electrons in the sample to be $N_{el} \sim 10^{10}$, this corresponds to $N_s/N_{el} \sim 1.5 \cdot 10^{-6}$.

they could not be confirmed [36]. We would point out, that the limits of applicability of the single impurity results are far from clear. A rough estimate for the maximum number of magnetic impurities in systems with Kondo screening is given by comparing the ratio of the number of magnetic impurities, N_s, and the number of electrons, N_{el}, with the ratio of the Kondo temperature and the Fermi energy, i.e. $N_s/N_{el} \sim T_K/\varepsilon_F$. Since $N_s/\mathcal{V} = n_s$ and $\varepsilon_F/N_{el} \sim \Delta = 1/2\mathcal{N}_0\mathcal{V}$ we can rewrite this condition as $n_s/2\mathcal{N}_0 T_K \sim 1$. As a consequence we expect that the result given in Eq. (74) is not accurate in systems with Kondo screening, when h_m becomes much smaller than one.

4 Temporal current fluctuations

In metals there are current fluctuations on many different time scales and due to different mechanisms. Here we concentrate on one aspect, i.e. current fluctuations which are induced by the dynamics of magnetic impurities. We follow the ideas of [26, 27] where the effects of spin relaxation on conductance fluctuations and on weak localisation were studied.

For this calculation we use the real time Green's function formalism, originally formulated by Keldysh [37]. Our notation follows the review by Rammer and Smith [38]. In this formalism, Green's functions are matrices,

$$G = \begin{pmatrix} G^R & G^K \\ 0 & G^A \end{pmatrix} \tag{75}$$

where G^R and G^A are the usual retarded and advanced Green's functions. G^K is called the Keldysh component; in equilibrium it is related to the retarded and advanced function by the relation $G^K(\varepsilon) = \tanh(\varepsilon/2T)[G^R(\varepsilon) - G^A(\varepsilon)]$. The current can be determined from the Keldysh component, according to

$$I(t) = \frac{-i}{2}\sum_{\mathbf{k}\sigma} I_x G^K_{\mathbf{k}\sigma\mathbf{k}\sigma}(t,t). \tag{76}$$

In equilibrium this is, by definition, the persistent current (compare Eq. (9)). In the following we determine the current correlations including correlations of the impurity spins for different times, assuming the spin dynamics to be given as described below.

4.1 The effective electron-electron interaction

Interactions are also represented by matrices. The effective electron-electron interaction due to magnetic impurities, which we presented in Eq. (36) in Matsubara representation, is now given by

$$V^R_{\alpha\beta\gamma\delta}(t-t') = -i\Theta(t-t')\left(\frac{J}{\mathcal{V}}\right)^2 \sum_{a,b=x,y,z} \sigma^a_{\alpha\gamma}\sigma^b_{\beta\delta}\langle[S^a(t), S^b(t')]\rangle \tag{77}$$

$$V^A_{\alpha\beta\gamma\delta}(t-t') = i\Theta(t'-t)\left(\frac{J}{\mathcal{V}}\right)^2 \sum_{a,b=x,y,z} \sigma^a_{\alpha\gamma}\sigma^b_{\beta\delta}\langle[S^a(t), S^b(t')]\rangle \tag{78}$$

$$V^K_{\alpha\beta\gamma\delta}(t-t') = -i\left(\frac{J}{\mathcal{V}}\right)^2 \sum_{a,b=x,y,z} \sigma^a_{\alpha\gamma}\sigma^b_{\beta\delta}\langle\{S^a(t), S^b(t')\}\rangle. \tag{79}$$

Here $[,]$ is the commutator, and $\{,\}$ the anticommutator. For fixed impurities (time independent), $\langle S^a(t)S^b(0)\rangle = \delta_{ab}S(S+1)/3$, and the retarded and advanced components of V are exactly zero. The impurity spin dynamics is induced by a magnetic field or due to coupling to the conduction band. The retarded (advanced) component of V is related to the dynamical response functions, which have been studied in detail in the context of spin resonance experiments. Using phenomenological arguments it has been found that for small magnetic fields, $\omega_s \ll T$,

$$i\int_0^\infty dt\langle[S^z(t), S^z(0)]\rangle e^{i\omega t} \simeq \frac{i\Gamma_1}{\omega + i\Gamma_1}\chi_0 \tag{80}$$

$$\frac{1}{2}i\int_0^\infty \langle[S^-(t), S^+(0)]\rangle e^{i\omega t} \simeq \frac{-\omega_R + i\Gamma_2}{\omega - \omega_R + i\Gamma_2}\chi_0, \tag{81}$$

where $\chi_0 = 1/4T$ is proportional to the static spin susceptibility. In a microscopic calculation these quantities have been determined under the conditions $|\omega - \omega_R| \ll T$, and Γ_1, $\Gamma_2 \ll T$ (see e.g. [39]). The resonance frequency ω_R is shifted by the coupling to the conduction band, $\omega_R = \omega_s(1 + J\mathcal{N}_0 + \cdots)$, and the relaxation rates are given by $\Gamma_1 = \Gamma_2 = 4\pi(J\mathcal{N}_0)^2 T = 1/\tau_K$.

The Keldysh component is found from the fluctuation dissipation theorem, $V^K(\omega) = \coth(\omega/2T)[V^R(\omega) - V^A(\omega)] \approx (2T/\omega)[V^R(\omega) - V^A(\omega)]$. For the frequencies we consider, $\omega \ll T$, the retarded and advanced component of the interaction are much smaller than the Keldysh component $|V^{R,A}(\omega)| \ll |V^K(\omega)|$, and will therefore be neglected in the following. In the absence of magnetic fields this is equivalent to the ansatz in [24, 26, 27] where the impurity spin dynamics was assumed to be $\langle S^a(t)S^b(0)\rangle = \delta_{ab}S(S+1)e^{-|t|/\tau_K}/3$. This expression is symmetric in S^a and S^b such that the commutators of two spin operators and therefore the retarded and advanced components of the effective interaction are neglected from the beginning.

In the Keldysh formalism an interaction generally has a rather complicated structure. We already mentioned that Green's functions and interactions are matrices. Vertices are tensors of third rank, and absorption γ^k_{ij} and emission vertices $\tilde{\gamma}^k_{ij}$ are distinct. An interaction, $V_{kk'}$, where k and k' are these matrix indices, is to be translated into $i\sum_{kk'}\gamma^k_{ii'}V_{kk'}\tilde{\gamma}^{k'}_{jj'}$. Within the notation used in [38], $\tilde{\gamma}^1_{ij} = \gamma^2_{ij} = \sigma^x_{ij}/\sqrt{2}$. If we only consider the Keldysh component ($V^K = V_{12}$) of the interaction, this structure simplifies considerably according to $i\sum_{kk'}\gamma^k_{ii'}V_{kk'}\tilde{\gamma}^{k'}_{jj'} \to i\delta_{ii'}\delta_{jj'}V^K$. Within this approximation, the structure of the interaction is identical in form to a time-dependent external field [38].

4.2 Diffuson and cooperon

We determine the diffuson and cooperon in the absence of magnetic fields, $\omega_s = 0$. In the absence of magnetic impurities we found the following result, see Eq. (16):

$$C(\varepsilon_+ - \varepsilon_-) = \frac{1}{2\pi\mathcal{N}_0\tau^2} \frac{1}{-i(\varepsilon_+ - \varepsilon_-) + Dq_+^2}. \tag{82}$$

In the presence of spin effects the cooperon is not simply diagonal in spin space, as in the equation above, but it can be represented conveniently in the form of singlets and triplets. As shown above, spin-flip scattering is formally identical to a time-dependent external field, and we can make use of the methods in [40, 41], where the cooperon in the presence of a time-dependent electromagnetic field was discussed.

The cooperon including spin-flip scattering is expressed as a function of the "unperturbed" cooperon as

$$C_j(\varepsilon_+, \varepsilon_-; \varepsilon'_+, \varepsilon'_-) = C(\varepsilon_+ - \varepsilon_-)\left[(2\pi)^2\delta(\varepsilon_+ - \varepsilon'_+)\delta(\varepsilon_- - \varepsilon'_-) + \hat{\zeta}_j C_j(\varepsilon_+, \varepsilon_-; \varepsilon'_+, \varepsilon'_-)\right], \tag{83}$$

where C denotes the cooperon without spin effects. The δ-functions reflect that there is no energy transfer between the two Green's functions defining C. C_j is the singlet ($j = 0$) or triplet ($j = 1$) component of the full propagator. Spin-flip scattering is incorporated using the operator

$$\hat{\zeta}_j = -2\pi\mathcal{N}_0\tau^2\left[\frac{1}{\tau_s} + c_j\frac{1}{\tau_s}\int\frac{d\omega}{2\pi}\frac{2\tau_K}{(\omega\tau_K)^2 + 1}e^{-\omega\partial_{\varepsilon_-} + \omega\partial_{\varepsilon_+}}\right]. \tag{84}$$

A graphical representation is shown in Fig. 7. The first term appears as a result of the two self energy diagrams, the second term corresponds to the diagram where both sides are connected by an interaction line. The interaction transmits an energy ω, so the cooperon energies are shifted by $\pm\omega$. ($c_0 = 1$ and $c_1 = -2/3$ for the singlet and triplet channel, respectively.)

We multiply Eq. (83) with $[C(\varepsilon_+ - \varepsilon_-)]^{-1}$ and Fourier transform into the time representation, $\varepsilon_+ \to t_+$, and so on. After a change of variables, defining $t = (t_+ + t_-)/2$ and $\eta = t_+ - t_-$, we find, for the cooperon, an expression that depends on four time variables, but only two are relevant: It has been shown [40] that the cooperon can always be represented as $C(t, t', \eta, \eta') = \delta(t - t')C(t, \eta, \eta')$. In our special case $C(t, \eta, \eta')$ does not depend on t, i.e. we have to consider the function $C(\eta, \eta')$ only. It is related to the cooperon in frequency representation by

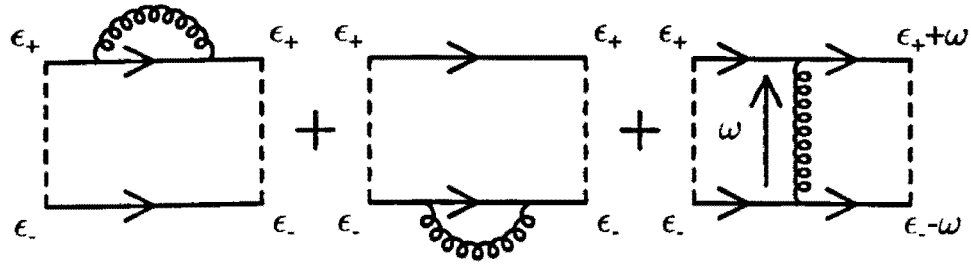

Fig. 7 Graphical representation of the operator $\hat{\zeta}$. The curly line is the effective interaction due to spin-spin correlations.

$$C_j(\varepsilon_+,\varepsilon_-;\varepsilon'_+,\varepsilon'_-) = 2\pi\delta(\varepsilon'_+ + \varepsilon'_- - \varepsilon_+ - \varepsilon_-)\int d\eta d\eta' C_j(\eta,\eta') \times \exp\left\{i\frac{\eta}{2}(\varepsilon_+ - \varepsilon_-) - i\frac{\eta'}{2}(\varepsilon'_+ - \varepsilon'_-)\right\}. \quad (85)$$

The cooperon obeys the following differential equation:

$$\left[2\partial_\eta + Dq_+^2 + \frac{1}{\tau_s}\left(1 + c_j e^{-|\eta|/\tau_K}\right)\right] C_j(\eta,\eta') = \frac{1}{2\pi\mathcal{N}_0\tau^2}\delta(\eta - \eta'). \quad (86)$$

The diffuson can be determined analogously. Again we use a notation with two time variables, $D(t,t',\eta,\eta') = \delta(\eta - \eta')D(t - t',\eta)$. The Fourier transformation from time to frequency representation is

$$D_j(\varepsilon_+,\varepsilon_-;\varepsilon'_+,\varepsilon'_-) = 2\pi\delta(\varepsilon_+ - \varepsilon_- - \varepsilon'_+ + \varepsilon'_-) \times \int dt d\eta D_j(t,\eta)\exp\left\{it(\varepsilon_+ - \varepsilon_-) + i\frac{\eta}{2}(\varepsilon_+ + \varepsilon_- - \varepsilon'_+ - \varepsilon'_-)\right\}. \quad (87)$$

The differential equation for the diffuson is

$$\left[\partial_t + Dq_-^2 + \frac{1}{\tau_s}\left(1 + d_j e^{-|\eta|/\tau_K}\right)\right] D_j(t,\eta) = \frac{1}{2\pi\mathcal{N}_0\tau^2}\delta(t) \quad (88)$$

with $d_0 = -1$ and $d_1 = 2/3$ for the singlet and triplet, respectively.

4.3 Temporal current fluctuations

Again, we use the approximation, that spin-flip scattering is in form identical to a time-dependent external field. Explicitly we consider

$$\langle I(t)I(0)\rangle_c = -\frac{1}{4}\langle\sum_{\mathbf{k}\sigma} I_x G^K_{\mathbf{k}\sigma\mathbf{k}\sigma}(t,t)\sum_{\mathbf{k}'\sigma'} I_x G^K_{\mathbf{k}'\sigma'\mathbf{k}'\sigma'}(0,0)\rangle_c, \quad (89)$$

compare Eq. (76). The relevant diagrams are given in Fig. 1, showing that they are of the same structure as the diagrams for the persistent current fluctuations. Consider for example the one-cooperon diagram. Concentrating on the frequency dependencies, it is of the form

$$\int\frac{d\omega}{2\pi}\int\frac{d\varepsilon_+}{2\pi}\frac{d\varepsilon_-}{2\pi} n_F(\varepsilon_-)[1 - n_F(\varepsilon_+)]\mathrm{Re}\left[e^{-i\omega t}C_j(\varepsilon_+,\varepsilon_- + \omega;\varepsilon_+ + \omega,\varepsilon_-)\right] \quad (90)$$

$$= \int\frac{d\varepsilon_+}{2\pi}\frac{d\varepsilon_-}{2\pi} n_F(\varepsilon_-)[1 - n_F(\varepsilon_+)] \times \mathrm{Re}\int\frac{d\omega}{2\pi}\int dt_1 dt_2 e^{-i\omega(t + t_1/2 + t_2/2)}e^{i(\varepsilon_+ - \varepsilon_-)(t_1 - t_2)/2}C_j(t_1,t_2) \quad (91)$$

Some of the integrations can be achieved rather easily, e.g. from the ω-integration we find the delta function $2\delta(2t+t_1+t_2)$. The integrations over ε_+ and ε_- can be treated as in the case of persistent currents, see Eq. (19). The current fluctuations are the sum over cooperon and diffuson contributions, hence

$$\langle I(t)I(0)\rangle_c = -\frac{1}{2\pi^2}\int_0^\infty dy y \coth\frac{y}{2T}$$

$$\times\left\{\sum_{qj}\mathrm{Re}\int dt_1 e^{iyt_1}\left[(\partial_\phi\partial_{\phi'}\Pi^C)2C_j(t+t_1,t-t_1)+(\partial_\phi\partial_{\phi'}\Pi^D)D_j(t_1,t)\right]\right.$$

$$+\sum_{qj}(\partial_\phi\Pi^C)(\partial_{\phi'}\Pi^C)\mathrm{Re}\int dt_1 dt_2 e^{iy(t_1+t_2)}2C_j(t+t_1,t-t_1)2C_j(t+t_2,t-t_2)$$

$$\left.+\sum_{qj}(\partial_\phi\Pi^D)(\partial_{\phi'}\Pi^D)\mathrm{Re}\int dt_1 dt_2 e^{iy(t_1+t_2)}D_j(t_1,t_2+t)D_j(t_2,t-t_1)\right\}. \quad (92)$$

The quantities $\Pi^{C,D}$ have been given in Eq. (15). The Fourier transformation

$$\mathrm{Re}\int_0^\infty dy y\coth\frac{y}{2T}e^{iyt_1} = \frac{1}{2}\int_{-\infty}^\infty dy y\coth\frac{y}{2T}e^{iyt_1} \quad (93)$$

can be performed by closing the path of integration in the upper half plane ($t_1>0$). We find a sum over Matsubara frequencies which can be evaluated analytically, with the result

$$\mathrm{Re}\int_0^\infty dy y\coth\frac{y}{2T}e^{iyt_1} = -\frac{\pi^2T^2}{[\sinh(\pi Tt_1)]^2}. \quad (94)$$

In the long time (or high temperature) limit, $t_1T\gg 1$, this expression equals $-4\pi^2T^2\exp(-2\pi t_1T)$, while in the zero temperature limit, $t_1T\ll 1$, it decays algebraically, $\sim -1/t^2$.

All time integrations in Eq. (92) will be cut off at a phase breaking time, τ_ϕ, which may assume different values in the different diffuson or cooperon channels. Phase breaking may be due to finite temperature, spin-flip scattering, or other processes not considered explicitly. Since we at least have thermal dephasing, i.e. $1/\tau_\phi \geq 2\pi T$, and also have to assume $T\gg 1/\tau_K$, the impurity spin dynamics is slow enough for the spin configuration to be considered static during the phase coherence time. Consequently, the diffuson and cooperon can be approximated such that an explicit calculation of (92) is straightforward: The solutions to the differential equations (86) and (88) are

$$2C_j(t+t_1,t-t_1) = \frac{1}{2\pi\mathcal{N}_0\tau^2}\exp\left\{-\frac{1}{2}\left[(Dq_+^2+\frac{1}{\tau_s})2t_1+\frac{c_j}{\tau_s}\int_{t-t_1}^{t+t_1}dt' e^{-|t'|/\tau_K}\right]\right\}$$

$$\approx \frac{1}{2\pi\mathcal{N}_0\tau^2}\exp\left\{-\left(Dq_+^2+\frac{1}{\tau_s}+c_j\frac{1}{\tau_s}e^{-|t|/\tau_K}\right)t_1\right\} \quad (95)$$

and

$$D_j(t_1, t-t_2) = \frac{1}{2\pi\mathcal{N}_0\tau^2}\exp\left\{-\left(Dq_-^2 + \frac{1}{\tau_s} + d_j\frac{1}{\tau_s}e^{-|t-t_2|/\tau_K}\right)t_1\right\}$$

$$\approx \frac{1}{2\pi\mathcal{N}_0\tau^2}\exp\left\{-\left(Dq_-^2 + \frac{1}{\tau_s} + d_j\frac{1}{\tau_s}e^{-|t|/\tau_K}\right)t_1\right\}. \tag{96}$$

On the formal level the approximations are valid since the time integrations in (92) have a cutoff at $t_1, t_2 \sim \tau_\phi < \tau_K$, and we can expand in lowest order in the ratios t_1/τ_K and t_2/τ_K.

Considering the current fluctuation for very long time differences t, the decay rate in all diffuson and cooperon channels is equal to $1/\tau_s$. In this limit Eq. (92) gives the stochastic fluctuations of the equilibrium current. If we neglect the impurity spin dynamics, $\tau_K \to \infty$, there are different rates in the singlet/triplet channels for diffuson and cooperon. In this case the spin subsystem is not ergodic. Although $\langle I(\phi)I(\phi')\rangle_c$ is time independent, it is different to the fluctuations in equilibrium (compare Subsect. 3.1.1). The static spin configuration breaks time reversal symmetry, and as a consequence we find that $\langle I(\phi)I(\phi')\rangle_c \neq \langle I(-\phi)I(\phi')\rangle_c$ similar to the spin glass case.

In the case of strong spin-flip scattering, $1/\tau_s \gg E_c$, the diffuson singlet component is the dominant contribution to the current fluctuation. Within the quasistatic limit discussed above the current fluctuations are given by ($\phi = \phi'$, $T = 0$)

$$\langle I(t)I(0)\rangle_c = \sum_{m=1}^{\infty}\frac{12e^2E_c^2}{\pi^2m^3}\left(1 + \sqrt{\Gamma_m} + \Gamma_m/3\right)\exp\left(-\sqrt{\Gamma_m}\right) \tag{97}$$

with $\Gamma_m = (1/\tau_s T_m)[1 - \exp(-|t|/\tau_K)]$. We conclude that in the presence of strong spin-flip scattering there are strong temporal fluctuations of the current. The current follows the actual spin-configuration. The time scale for changes in the spin configuration is the Korringa relaxation time τ_K. Since the current and the spin configuration are coupled nonlinearly, we find a much shorter time scale for the current fluctuations, $t \sim \tau_K(\tau_s E_c) \ll \tau_K$, from the condition $\Gamma_1(t) \sim 1$.

An extension to weak magnetic fields is straightforward. For $\omega_s \ll T$ the spin configuration can be considered static within the phase coherence time as before. In this case the current is found most conveniently using the methods of Chapter 3. The spin correlations are of the form

$$\langle S^z(t)S^z(0)\rangle = \frac{1}{3}S(S+1)e^{-|t|/\tau_K} \tag{98}$$

$$\langle S^-(t)S^+(0)\rangle = \frac{2}{3}S(S+1)e^{-i\omega_s t}e^{-|t|/\tau_K}, \tag{99}$$

compare Eq. (81). The bare diffuson vertex is

$$D^0_{\alpha\beta\gamma\delta} = \frac{1}{\tau}\delta_{\alpha\gamma}\delta_{\delta\beta} + \frac{1}{3\tau_s}e^{-|t|/\tau_K}\left(\sigma^z_{\alpha\gamma}\sigma^z_{\delta\beta} + 2e^{i\omega_s t}\sigma^-_{\alpha\gamma}\sigma^+_{\delta\beta} + 2e^{-i\omega_s t}\sigma^+_{\alpha\gamma}\sigma^-_{\delta\beta}\right). \tag{100}$$

The diffuson is nondiagonal in singlet/triplet representation, but it is sufficient to know the eigenvalues of the diffuson. The eigenvalues are independent of ω_s, which means that a weak magnetic field does not affect the current fluctuations.

By further increasing the magnetic field this approach breaks down at $\omega_s \sim T$. Here we have to take the full structure of the interaction into account, and a quasi-static approximation is not sufficient.

5 Conclusions

We studied the effect of paramagnetic impurities on the persistent current in normal metal rings. Magnetic impurities tend to destroy quantum coherence. In weak magnetic fields the persistent current is strongly reduced due to the impurity spin dynamics. Instead of the persistent current, there are temporal current fluctuations following the actual spin configuration. By freezing out the spin dynamics in a magnetic field, the amplitude of the typical current is of the same order as the amplitude without magnetic impurities. However the mechanism of restoring the persistent current works only poorly, and the maximum value for the current is only reached for magnetic fields, for which the Zeeman energy ω_s is larger than the spin-flip scattering rate $1/\tau_s$. The interaction contribution to the mean current is strongly reduced in the presence of magnetic impurities, whether or not the impurity spins are polarised.

If the Thouless energy E_c and the temperature T are below the Kondo temperature T_K, the impurity spins are effectively screened, and the magnetic impurities behave like nonmagnetic defects.

Finally we consider the observability of the temporal current fluctuations. A direct observation in a single ring experiment will be difficult. On the other hand, the temporal current fluctuations are related to the dynamic linear response $\chi(\omega)$ to a frequency-dependent flux $\phi(\omega)$. In the static limit, $\chi(\omega = 0)$ is the derivative of the persistent current with respect to the flux ϕ: $\chi(\omega = 0) = -\partial_\phi^2 K(\phi) = \partial_\phi I(\phi)$. In the presence of magnetic impurities, $\langle\chi(\omega = 0)\rangle$ is strongly suppressed. However, there are the temporal current fluctuations, which are of the order $\langle I(t)I(0)\rangle \sim (E_c/\phi_0)^2$, and which decay on the time scale $t \sim \tau_K(E_c\tau_s)$. As a consequence, we find for the dynamic response at frequencies $\omega \sim 1/(\tau_K E_c \tau_s)$ a contribution which is of the order $\langle\chi(\omega)\rangle \sim (E_c/\phi_0^2)(E_c/T)$. Note that this averaged susceptibility is measured in an experiment with many rings. The amplitude of the dynamic response, $\langle\chi(\omega)\rangle$, in the presence of magnetic impurities can be larger than $\langle\chi(\omega = 0)\rangle$ without magnetic impurities, by a factor $\sim E_c/T$.

Appendix A: The electron self energy

We calculate the self energy for electrons with spin up,

$$\Sigma_+^{(2)}(i\varepsilon_n) = N_s(-T)\sum_{\omega_l}\sum_{\mathbf{k}\alpha} V_{+\alpha\alpha+}(i\omega_l)G(i\varepsilon_n - i\omega_l, \mathbf{k}). \tag{101}$$

The relevant matrix elements of the interaction are

$$V_{++++}(i\omega_l) = -\frac{1}{T}\left(\frac{J}{\nu}\right)^2 \frac{1}{4}\delta_{0\omega_l} \tag{102}$$

and

$$V_{+--+}(i\omega_l) = -\left(\frac{J}{\mathcal{V}}\right)^2 \frac{\tanh(\omega_s/2T)}{i\omega_l + \omega_s}. \tag{103}$$

The frequency and momentum summation for the first type of interaction is easy, since the interaction does not transfer energy,

$$\frac{1}{4} n_s J^2 \frac{1}{\mathcal{V}} \sum_{\mathbf{k}} G(i\varepsilon_n, \mathbf{k}) \tag{104}$$

and

$$\frac{1}{\mathcal{V}} \sum_{\mathbf{k}} G(i\varepsilon_n, \mathbf{k}) = \int \frac{d^3k}{(2\pi)^3} G(i\varepsilon_n, \mathbf{k}) = -i\pi \mathcal{N}_0 \operatorname{sgn}(\varepsilon_n). \tag{105}$$

In the second term, the integration over $\mathbf{k}$ can be achieved as in Eq. (105). For $\varepsilon_n > 0$ we have

$$\Sigma_+^{(2)}(i\varepsilon_n) = -i n_s \pi \mathcal{N}_0 J^2 T \tanh \frac{\omega_s}{2T} \sum_{\omega_l} \frac{\operatorname{sgn}(\varepsilon_n - \omega_l)}{i\omega_l + \omega_s} - i n_s \pi \mathcal{N}_0 J^2 \frac{1}{4}. \tag{106}$$

The sum $\sum_{\omega_l}(\cdots)$ is cut off at the Fermi energy, and can be expressed using the digamma function Ψ:

$$T \sum_{\omega_l} \frac{\operatorname{sgn}(\varepsilon_n - \omega_l)}{i\omega_l + \omega_s} = \frac{i}{2\pi} \times \left\{ 2\Psi\left(\frac{\varepsilon_F}{2\pi T}\right) - \Psi\left(\frac{1}{2} - \frac{\varepsilon_n - i\omega_s}{2\pi T}\right) - \Psi\left(\frac{1}{2} + \frac{\varepsilon_n - i\omega_s}{2\pi T}\right) \right\}. \tag{107}$$

Using the property

$$\Psi(1 - z) = \Psi(z) + \pi \cot(\pi z), \tag{108}$$

we find

$$\Sigma_+^{(2)}(i\varepsilon_n) = \frac{i}{\pi} \left\{ \Psi\left(\frac{\varepsilon_F}{2\pi T}\right) - \Psi\left(\frac{1}{2} + \frac{\varepsilon_n - i\omega_s}{2\pi T}\right) - \frac{\pi}{2} \cot\left(\frac{\pi}{2} + \frac{\varepsilon_n - i\omega_s}{2T}\right) \right\} - i n_s \pi \mathcal{N}_0 J^2 \frac{1}{4}. \tag{109}$$

This can be simplified using

$$\cot\left(\frac{\pi}{2} + \frac{\varepsilon_n - i\omega_s}{2T}\right) = i \coth \frac{\omega_s}{2T}. \tag{110}$$

After analytic continuation, $i\varepsilon_n \to \varepsilon + i0$, we make use of another property of Ψ, namely $\mathrm{Im}\Psi(iy + 1/2) = (\pi/2)\tanh(\pi y)$, and we arrive at the expressions given in Eqs. (39) and (40). $\Sigma_-^R(\varepsilon)$ can be found in the same way.

Appendix B: Diffuson and cooperon in the presence of magnetic impurities

Rather than calculating $\langle\Omega\Omega\rangle_c$ and subsequently finding the persistent current in terms of the derivative with respect of the flux ϕ, the current can also be found directly, as described in Chapter 2. In this case we need the expressions for the diffuson and cooperon; the bare expressions C^0 and D^0 are given in Eqs. (44) and (45). Using the quantities N_j and M_j defined in Eqs. (48)–(53), we find for the non-zero components of the cooperon

$$2\pi\mathcal{N}_0\tau^2 C_{++++} = \left(N_2 + \frac{\gamma_{++} - \gamma_{--}}{2} - i\frac{\omega_s - \omega_s'}{2}\right)^{-1} \tag{111}$$

$$2\pi\mathcal{N}_0\tau^2 C_{----} = \left(N_2 - \frac{\gamma_{++} - \gamma_{--}}{2} + i\frac{\omega_s - \omega_s'}{2}\right)^{-1} \tag{112}$$

$$2\pi\mathcal{N}_0\tau^2 C_{+-+-} = \left(\frac{N_0 + N_1}{2} + \frac{\gamma_{-+} - \gamma_{+-}}{2} + i\frac{\omega_s + \omega_s'}{2}\right)/\mathcal{D}_+ \tag{113}$$

$$2\pi\mathcal{N}_0\tau^2 C_{-+-+} = \left(\frac{N_0 + N_1}{2} - \frac{\gamma_{-+} - \gamma_{+-}}{2} - i\frac{\omega_s + \omega_s'}{2}\right)/\mathcal{D}_+ \tag{114}$$

$$2\pi\mathcal{N}_0\tau^2 C_{+--+} = \left(\frac{N_0 - N_1}{2}\right)/\mathcal{D}_+ = 2\pi\mathcal{N}_0\tau^2 C_{-++-} \tag{115}$$

with

$$\mathcal{D}_+ = N_0 N_1 - \left(\frac{\gamma_{-+} - \gamma_{+-}}{2} + i\frac{\omega_s + \omega_s'}{2}\right)^2. \tag{116}$$

Using the Clebsch Gordan coefficients $\langle\sigma\sigma'|jm\rangle$ the components of the cooperon can be written in terms of singlets and triplets:

$$\langle jm|C|j'm'\rangle = \sum_{\sigma\sigma'\nu\nu'} \langle\sigma\sigma'|jm\rangle\langle\nu\nu'|j'm'\rangle C_{\sigma\sigma'\nu\nu'}. \tag{117}$$

For ω_s, $\omega_s' = 0$ the matrix $\langle jm|C|j'm'\rangle$ is diagonal, but in general there are off-diagonal elements mixing the $m = 0$ components.

The non-zero components of the diffuson are

$$2\pi\mathcal{N}_0\tau^2 D_{+-+-} = \left(M_2 - i\frac{\omega_s + \omega_s'}{2} + \frac{\gamma_{+-} - \gamma_{-+}}{2}\right)^{-1} \tag{118}$$

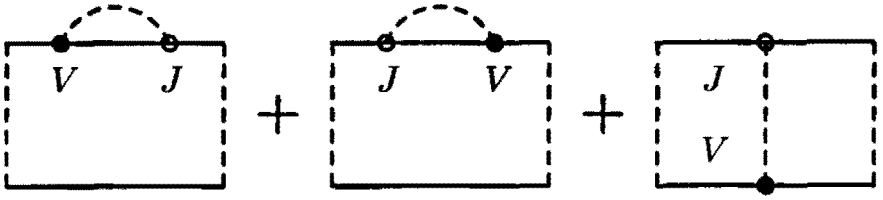

Fig. 8 Diagrams representing mixed potential and spin-flip scattering. The sum of the three diagrams is zero.

$$2\pi\mathcal{N}_0\tau^2 D_{-+-+} = \left(M_2 + i\frac{\omega_s + \omega_s'}{2} - \frac{\gamma_{+-} - \gamma_{-+}}{2}\right)^{-1} \tag{119}$$

$$2\pi\mathcal{N}_0\tau^2 D_{++++} = \left(\frac{M_0 + M_1}{2} + i\frac{\omega_s - \omega_s'}{2} - \frac{\gamma_{++} - \gamma_{--}}{2}\right)/\mathcal{D}_- \tag{120}$$

$$2\pi\mathcal{N}_0\tau^2 D_{----} = \left(\frac{M_0 + M_1}{2} - i\frac{\omega_s - \omega_s'}{2} + \frac{\gamma_{++} - \gamma_{--}}{2}\right)/\mathcal{D}_- \tag{121}$$

$$2\pi\mathcal{N}_0\tau^2 D_{++--} = \left(\frac{M_1 - M_0}{2}\right)/\mathcal{D}_- \tag{122}$$

with

$$\mathcal{D}_- = M_0 M_1 - (i\frac{\omega_s - \omega_s'}{2} - \frac{\gamma_{++} - \gamma_{--}}{2})^2. \tag{123}$$

Appendix C: Potential and spin-flip scattering

One may ask why we consider two distinct types of impurities, on the one hand a pure potential scatterer, on the other a pure spin scatterer. A real magnetic impurity will have both, potential and spin-flip scattering, described by a Hamiltonian

$$H_{sd} = -\frac{1}{\mathcal{V}}\sum_{\mathbf{R}}\sum_{\mathbf{k}\mathbf{k}'\sigma\sigma'} \exp[-i(\mathbf{k} - \mathbf{k}')\mathbf{R}]\left(V\delta_{\sigma\sigma'}c^+_{\mathbf{k}\sigma}c_{\mathbf{k}'\sigma'} + Jc^+_{\mathbf{k}\sigma}c_{\mathbf{k}'\sigma'}\vec{\sigma}_{\sigma\sigma'}\vec{S}_{\mathbf{R}}\right). \tag{124}$$

In particular, we cannot rule out that $V > J$. In second order in the interaction there are three types of contributions, proportional to V^2, VJ and $J^2 \sim 1/\tau_s$. The part $\sim V^2$ gives a correction to the nonmagnetic scattering rate $1/\tau$. In the absence of magnetic fields, the term $\sim VJ$ is equal to zero. In the presence of magnetic fields, however, this part can be much larger than the terms $\sim J^2$.

Corrections to the diffuson or cooperon from these mixed terms are shown in Fig. 8. The sum of these diagrams is exactly zero, which means contributions $\sim JV$ are unimportant for the questions considered.

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