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Coupled Brownian Rectifiers

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Abstract. Periodic structures that lack reflection symmetry act – when periodically rocked by external forces – as rectifiers for Brownian motion. Here we investigate the role of a global coupling (of the 'Kuramoto-type') among such rectifiers. We demonstrate that the coupling strength K acts as a control on the sign (it yields a realization for current reversal) and the magnitude of the directed average velocity of Brownian particles. Moreover, raising the coupling strength K, results in an effective reduction of ambient noise. This intriguing effect is revealed in a striking manner for the mean velocity vs. load characteristics.

INTRODUCTION

In recent years, the study of directional Brownian motion in periodic potentials that lack reflection symmetry (*ratchets*) has attracted considerable attention (for the major ideas and a recent review with more references, see [1]). Such ratchets provide models for molecular motors, but in addition also carry a potential for novel technological applications in the worlds of micro- and nano-physics [1,2]. Due to the fact that motion is directed, particles can move uphill against a load with finite velocity; thus they serve as archetype systems that are able to act as rectifiers (i.e., identical direction of the velocity upon varying the bias around zero) on a level of Brownian noise. In the previous literature a variety of ratchet mechanisms have been identified [1]. Here, our focus is on the interplay of ratchet devices that are mutually coupled with each other. Thus far, such coupling effects have rarely been addressed [3,4]. The focus in this work will be on globally coupled rocking ratchets.

MODEL

We start from the *overdamped* dynamics of N interacting particles in a ratchet potential $V(x)$, i. e.

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$$\dot{x}_i = -\frac{\mathrm{d}V(x_i)}{\mathrm{d}x_i} - \frac{K}{N}\sum_{j=1}^{N}\sin[2\pi(x_i - x_j)] + F + A\sin\Omega t + \sqrt{D}\xi_i(t), \tag{1}$$

where $i = 1, ..., N$. The dot denotes the derivative with respect to time t. The particles are forced by a static bias F and, additionally, by periodic driving with angular frequency Ω and amplitude A. Each particle experiences thermal fluctuations of strength D, modelled by independent Gaussian white noise sources $\xi_i(t)$ of zero average, and correlations $\langle\xi_i(t)\,\xi_j(s)\rangle = 2\delta_{i,j}\delta(t-s)$. We adopted the global interaction between particles of strength K/N in eq.(1) from the well known 'Kuramoto model' [5,6] describing excitable systems.

In this work we consider a ratchet potential being constructed from two Fourier modes [7], $V(x) \equiv -\frac{1}{2\pi}(\sin 2\pi x + \frac{1}{4}\sin 4\pi x)$. Equation (1) can be simulated numerically. This method has the disadvantage that for small noise strength D it is intrinsically rather time consuming. Thus, we look for an alternative route to the solution of our problem. An equivalent description of the noisy dynamics (1) is given by the N-dimensional Fokker-Planck equation, i.e.,

$$\frac{\partial}{\partial t}W(x_1,...,x_N;t) = \left\{\sum_{i=1}^{N}\frac{\partial}{\partial x_i}\left[\frac{\mathrm{d}V(x_i)}{\mathrm{d}x_i} + \frac{K}{N}\sum_{j=1}^{N}\sin 2\pi(x_i - x_j)\right.\right.$$
$$\left.\left. -F - A\sin\Omega t\right] + \sum_{i,j=1}^{N}\frac{\partial}{\partial x_i\partial x_j}D\right\}W(x_1,...,x_N;t). \tag{2}$$

For small N, this equation can effectively be solved by the method of *Matrix-Continued-Fractions* [2,7]. Further, to investigate the limit of many particles, $N \gg 1$, we turn to a *mean field description:* For the mean particle density $W(x;t) \equiv \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\delta(x - x_i(t))$ we find the one dimensional Fokker-Planck equation

$$\frac{\partial}{\partial t}W(x;t) = \left\{\frac{\partial}{\partial x}\left[\frac{\mathrm{d}V(x)}{\mathrm{d}x} + K\int_0^{2\pi}dx'W(x';t)\sin 2\pi(x-x') - F - A\sin\Omega t\right]\right.$$
$$\left. +\frac{\partial^2}{\partial x^2}D\right\}W(x;t), \tag{3}$$

which is *nonlinear* in $W(x;t)$. We solve this equation numerically for periodic boundary conditions by expanding into Fourier modes, i.e.,

$$W(x,t) = \sum_{l=-\infty}^{\infty} c_l(t)e^{il2\pi x}. \tag{4}$$

This yields an infinite set of nonlinear ordinary differential equations for the expansion coefficients $c_l(t)$, which are solved by standard methods.

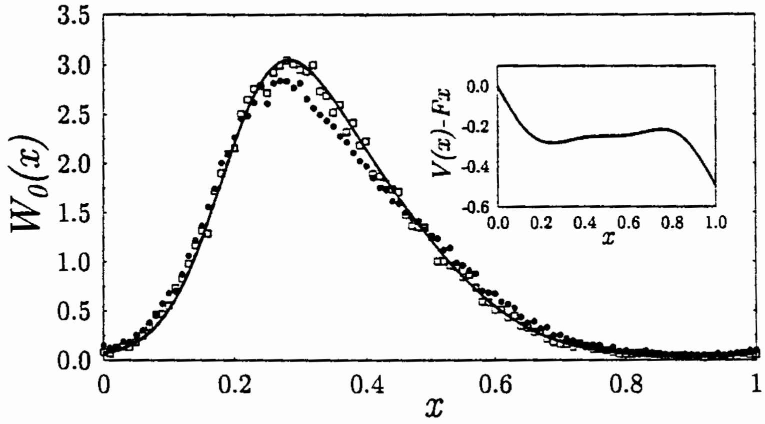

FIGURE 1. The asymptotic stationary mean field probability distribution $W_0(x)$ in a tilted potential (shown in the inset) is calculated within the mean field approximation (—) and compared to Langevin simulations for $N = 8(\bullet)$, $N = 256(\square)$ at model parameters $F = 0.5$, $D = 0.1$, and $K = 1$.

RESULTS

At this point we can ask, how large the particle number N must be, in order for the mean field to be a sensible valid approximation. For the undriven case $A = 0$ of a tilted potential $V(x) - Fx$ we compare in Fig. 1 the asymptotic stationary mean field solution $W_0(x) \equiv \lim_{t\to\infty} W(x;t)$ calculated by the solution of (3) with Langevin simulations, cf. Fig. 1.

Next, we turn to the investigation of the mean asymptotic particle velocity

$$\langle \dot{x} \rangle_{st} \equiv \lim_{t\to\infty, N\to\infty} \int_t^{t+2\pi/\Omega} dt' \sum_{i=1}^{N} \dot{x}_i(t')/N. \qquad (5)$$

Numerical results for the directed velocity versus coupling strength K are depicted with Fig.2. Clearly the interaction between particles can increase the current significantly, cf. Fig. 2(a). Moreover, the direction of transport can be *reversed* due to the coupling, cf. the large K-values in Fig. 2(a). We remark, that such a reversal emerges already for the periodically rocked thermal ratchet in absence of coupling [3,7]. In the limit of adiabatically slow driving, $\Omega \to 0$, we can apply an adiabatic approximation in order to calculate $\langle \dot{x} \rangle_{st}$ from (3), cf. [7], see Fig. 2(b).

In Fig. 3 we depict the dependence of $\langle \dot{x} \rangle_{st}$ in absence of a bias versus noise strength D for various coupling strengths K: The coupling can induce an increase for the absolute value of the current (e.g. $\langle \dot{x} \rangle_{st}$ at moderate-to-large

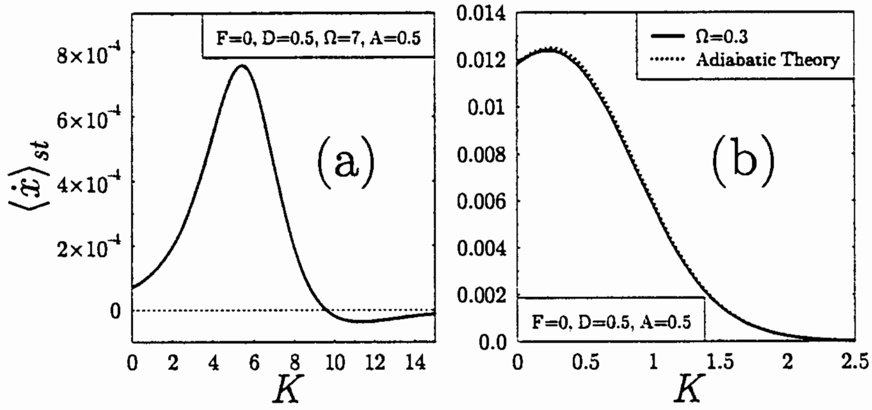

FIGURE 2. Numerically evaluated average particle velocity $\langle \dot{x} \rangle_{st}$ versus coupling strength K for parameter values given in the figures. (a) for non adiabatic driving, (b) for adiabatic rocking (—), which favourably compares with the adiabatic analytical theory (···).

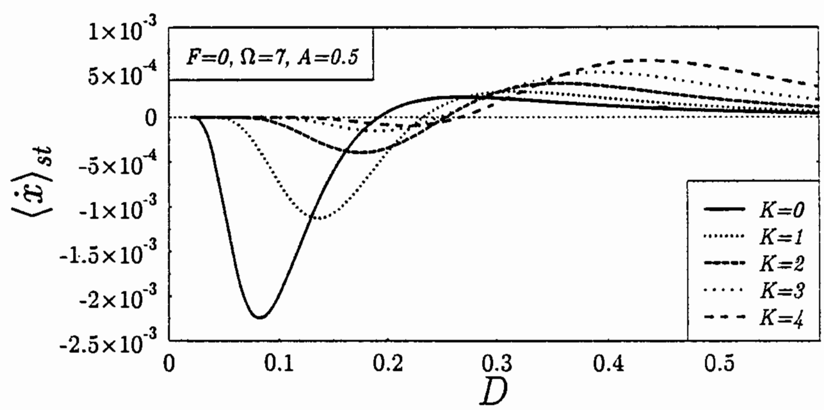

FIGURE 3. The numerically evaluated mean field particle velocity $\langle \dot{x} \rangle_{st}$ is shown versus the Gaussian white noise strength D, for various coupling strengths K.

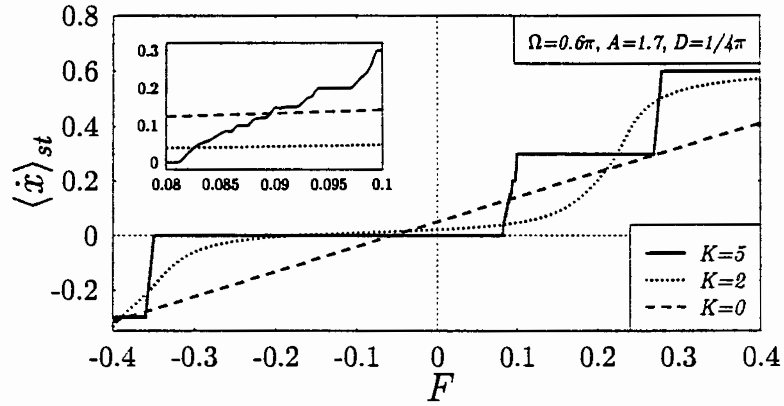

FIGURE 4. The numerically evaluated velocity $\langle \dot{x} \rangle_{st}$ in presence of an external bias force F versus different coupling strengths K. The inset presents an enlargement: The $\langle \dot{x} \rangle_{st}$-$F$ characteristics exhibits a devils staircase for large coupling K.

D in Fig. 3), as well as a decrease (at low noise level D). Increasing the coupling K shifts the point of current reversal to higher D-values.

In order to investigate the efficiency with which particles are performing work in a coupled rocking ratchet we study the mean velocity versus a nonvanishing external bias $F \neq 0$. In [2] we presented results for the velocity-bias characteristic for zero coupling K, which exhibits a devils staircase behavior, being increasingly smoothed out with increasing noise (not shown). In Fig. 4 we depict the load curves for the same parameters as in [2] for various values of coupling strength K. [1] We observe that an increase in K yields a more pronounced step-behavior. The coupling induced, extended plateaus can be utilized for the design of rectifiers operating with Brownian noise. Thus, the magnitude of the coupling strength can be used to control the smoothing level of load curves; a large value of K effectively reduces the bare noise level D.

CONCLUSIONS

In conclusion, we have investigated analytically (adiabatic limit) and numerically the physical role of mutual coupling among Brownian rectifiers that individually act as rocking ratchets. The effects of coupling are multifacetted: The presence of finite coupling K can induce a reversal of velocity; K acts

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1) The potential $\tilde{U}(\tilde{x})$ chosen in [2] (denoting all variables in [2] with a tilde) has a different periodicity and height than in this work: Rescaling the variables in [2] by $\tilde{x} = 2\pi x$, $\tilde{U}(\tilde{x}) = 2\pi V(x/2\pi)$, $\tilde{A} = A$, $\tilde{F} = F$, $\tilde{t} = 2\pi t$, $\tilde{\Omega} = \Omega/2\pi$, $\tilde{D} = 2\pi D$ yields $\langle \frac{d\tilde{x}}{d\tilde{t}} \rangle_{st} = \langle \frac{dx}{dt} \rangle_{st} = \langle \dot{x} \rangle_{st}$, when $K = 0$.

as a control for the sign and the magnitude for directed movement of Brownian particles. Its role for the load-characteristics is particularly intriguing: Increasing the global coupling strength effectively diminishes the influence of strong ambient noise forces, such as strong thermal noise D. Our system in (1) can likely be realized with arrays of Josephson junctions with internal asymmetry [2] due to the equivalence with a Kuramoto-dynamics as exemplified recently by Wiesenfeld, Colet, and Strogatz [6].

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