Friedrich Pukelsheim: photo courtesy of his coauthor T. Klein.

Rounding Tables on My Bicycle

This morning, as I was riding my bicycle to the office, I was thinking about Howard Wainer's recent Visual Revelations column entitled "Rounding Tables" [*Chance*, Vol. 11, No. 1, Winter 1998 pp. 46–50] and it dawned on me that the handling of ties in Step I of the Webster method is a bit (too) brief. If the fractional part is equal to 1/2, then rounding to the nearest even integer works fine provided the resulting integers sum to 100.

There are tied cases, though, where more is needed. Imagine eight categories of equal size, so each contains 12.5 percent. Remembering Wainer's (1997b, p. 97) 2D-Theorem ("Two digits suffice"), we would like to round to full percentages. The nearest-even-integer advice (Eisenhart 1947) gives eight times 12 percent and sums up to 96 percent, thus falling short of 4 percent.

This is why Balinski and Young's (1982, p. 99) definition of the Webster "method" enumerates multiple solutions, if any. In the little contrived example above, we have 70 choices of four times rounding down to 12 percent and four times rounding up to 13 percent.

This sounds more intimidating than it really is. It has been my experience that looking at the data usually suggests a good way of resolving ties. I do not think that it is a good idea to have ties resolved mechanically by a computer program.

Here is an example from one of my unwritten papers. We are given two apportionments (counts) of 28 seats to four parties, together with their percentages:

15: 53.6	10: 35.7	2: 7.1	1: 3.6
14: 50.1−	10: 35.7+	2: 7.1+	2: 7.1+

In the first line, standardization to the tenth of a percent is unambiguous, as 15 out of 28 rounds to 53.6 percent etc. In the second line, rounding to the tenth of a percent produces a tie, with four equally legitimate solutions. They are the one quoted above, plus the three solutions where a tenth of a percent is transferred from 50.1 to either one of the following three percentages. What to do?

I propose the following. Since 2 is rounded to 7.1 percent in the first line, we are only inviting trouble if we round it to something else in the second line. Same with rounding 10 to 35.7 percent. Therefore, I shall round 14 to 50.1 percent.

Now, if my readers are not only literate but also numerate, as I hope they are, I may be getting E-mails pointing out a typo in my unwritten paper, to the effect that 14 out of 28 evidently equals 50.0, not 50.1 percent. What am I going to respond? I guess I would first try a friendly lie:

"When I rounded those numbers, I noticed a tenth of a percent was missing. So I added it on."

Should people get too inquisitive, I would have to hammer the truth into them:

"I did a minimum Chi-square fit among all distributions whose probabilities are integer multiples of 1/1000."

This stills everybody. The curious true fan has to read up on it in Balinski and Young (1982, pp. 103–104), anyways.

Last, if I may be forgiven an *obiter dictum*, I would like to advertise that more on the subject, like further Tech Reports and the RoundPro program itself, can be retrieved from the Internet at www1.math.uni-augsburg.de/sta/.

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