Efficient parallel domain decomposition methods for fluidmechanical problems on nonmatching grids

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Abstract

Domain decomposition methods that are based on overlapping or nonoverlapping partitions of the computational domain are appropriate tools for the design and implementation of parallel algorithms in the numerical solution of PDEs and systems thereof. We consider such techniques on nonmatching grids also known as mortar element methods which are nonconforming in nature and thus require additional continuity constraints on the interfaces between adjacent subdomains. The methods have been implemented on the IBM SP2 and applied to fully potential flow problems around airfoils.

Keywords: Domain decomposition methods; nonmatching grids; parallel algorithms

1 Introduction

Among the most powerful iterative solvers for PDEs of optimal resp. suboptimal computational complexity, domain decomposition methods have attracted considerable interest during the past decade. Originating from early work done by H.A. Schwarz more than a century ago related to the study of harmonic functions in complex-shaped domains, these techniques have experienced an extensive growth since the late seventies of this century in the need for appropriate parallel computing platforms. Actually, the scope of the methodology ranges from the theory of partial differential and integral equations, numerical mathematics and parallel computation to the mathematical modeling and numerical simulation of complex technological processes.

Recently, a new powerful approach within this class of methods has been provided by what has become known as domain decomposition on nonmatching grids. This approach stems from macro-hybrid formulations of differential problems with Lagrange multipliers on the interfaces of the subdomains. The finite element discretization of the macro-hybrid formulation results in large scale algebraic systems in saddle point form with special block-structured matrices.

Here, we consider the development, analysis, and parallel implementation of adaptive mortar finite element approximations of linear elliptic boundary value problems featuring block-structured multilevel preconditioners and adaptive grid refinement based on efficient and reliable residual and hierarchical type a posteriori error estimators (cf. [1]– [5]).

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2 Numerical and Computational Aspects

For the numerical solution of linear second order elliptic boundary value problems of the form

$$\begin{aligned} -\nabla\cdot(a\nabla u) + bu &= f \quad \text{in } \Omega \;, \\ u &= 0 \quad \text{on } \Gamma_1 \;, \\ \mathbf{n}\cdot a\nabla u &= g \quad \text{on } \Gamma_2 \end{aligned}$$

we start from a nonoverlapping geometrically conforming decomposition of the computational domain into mutually disjoint subdomains. We consider individual simplicial triangulations of the subdomains and corresponding discretizations of the subdomain problems by continuous, piecewise linear finite elements regardless the situation on the interfaces between the subdomains so that typically nonconforming nodal points arise on the interfaces. To ensure consistency of the overall nonconforming approach, we impose weak continuity constraints by means of suitably chosen Lagrangian multipliers. The resulting saddle point problem is solved by a preconditioned generalized Lanczos algorithm featuring a block-structured multilevel preconditioner which involves appropriate preconditioners for the subdomain problems and the Schur complement arising from static condensation of the unknowns associated with the subdomains. Adaptive features are taken into account by means of easily computable a posteriori error estimators of both residual and hierarchical type.

3 Parallelization and Performance

The parallel implementation of the algorithm is based on the following concepts. First, to minimize the communications, we duplicate the multiplier data to both subdomains sharing the respective interface. It slightly increases the data storage but results in only one communication operation during the residual computation. The solution procedure for the preconditioner only exploits communications within the evaluation of the interface preconditioner. Here, two types of communications are used, the interface exchange data, and the coarse mesh problem solution at a root processor for which we use gathering data and broadcasting the solution.

To overcome the problem of load balancing we merge the subdomains into clusters, each of them corresponding to a process. We use a greedy type algorithm for allocating the subdomain tasks to the processors.

Concerning the technical issues, for the sake of wide portability we have applied the message passing interface (MPI) library. The code is single program multiple data (SPMD) written in Fortran 77 and has been ported on IBM SP2 (LRZ), CRAY T3E (Stuttgart), and Dec TruCluster (Augsburg).

We have applied the code in the computation of flow problems around airfoils. For, the benchmark problem NACA0012, the asymptotic optimality in terms of the iteration time per unknown as a function of the total number of unknowns is shown in Figure 1a (IBM SP2) and Figure 1b (CRAY T3E) for the partition of the subdomains on different numbers of processors, respectively. Moreover, Figures 1c (IBM SP2) and 1d (CRAY T3E) display the parallel efficiencies in dependence on the number of processors for different levels of the refinement process. We oberserve that on the highest level the efficiencies slightly decrease to 0.93 in case of 39 processors on the IBM SP2 (Fig. 1c) and to 0.90 for 116 processors on the CRAY T3E (Fig. 1d) which indicates a reasonably well parallel performance of the algorithm.

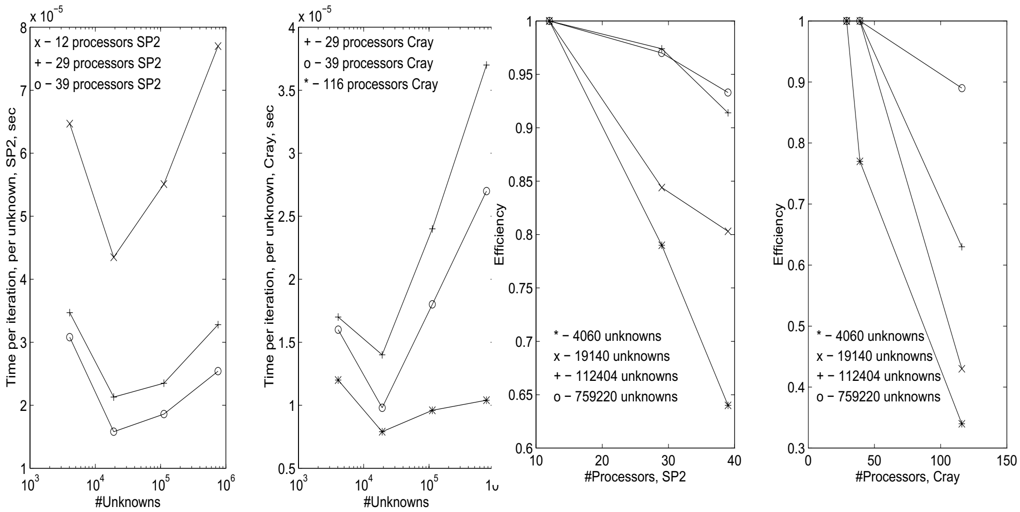

Fig. 1a: Asympt. Opt. Fig. 1b: Asympt. Opt. Fig. 1c: Par. Eff. Fig. 1d: Par. Eff.

4 Remarks and References

For further information see URL: `http://wwwhoppe.math.uni-augsburg.de/index.html`

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