Heterogeneity and Evolutionary Change

Empirical Conception, Findings and Unresolved Issues

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Abstract

In this position paper we deal with the conception of heterogeneity as both the force and the result of evolutionary change. We ask, how this heterogeneity can be measured empirically and how we can get a measure which allows to get a broad comparable empirical account especially on several levels of aggregation. Based on this discussion we suggest that for several questions the measures of total factor productivity (TFP) and local changes of TFP seem to be acceptable candidates for measuring heterogeneity and its dynamics. Examples out of a number of empirical investigations applying this measures show how interesting empirical facts about evolutionary change on several levels of aggregation can be detected. The paper concludes by raising a number of unresolved issues mainly related to the question about the relationship between evolutionary dynamics on several levels of aggregation.

1. Introduction

This paper summarizes part of the research performed during the last about 9 years at the University of Augsburg. This research has been concerned with the issue of heterogeneity of actors on various levels of aggregation, its sources and its consequences in the context of innovation and technological change. In this respect we have taken up the issue from a theoretical and from an empirical side. The theoretical work has dealt mainly with the question of how knowledge spillovers arising out of technologically different actors do influence the direction and intensity of technological change[1], the structural development in sectors[2], the cross-fertilization effects between sectors[3], as well as the comparative development of countries[4].

This position paper, however, will report on the empirical oriented research. There we have been concerned with the issue of measuring the heterogeneity within a population of actors and track its development over time.

This again has been performed on several levels of analysis, the intra-sectoral[5], the sectoral[6], the regional[7] as well as the international level[8]. Although quite different levels of aggregation have been approached, the respective research has in common a certain conception about heterogeneity with respect to the productive structure of the sample under consideration, i.e. the firms, the sectors or the countries. These structures are built up (a) by differences in *total factor productivity* and (b) by differences in *input-intensities, output-intensities* and/our *output coefficients*.

In order to identify such heterogeneous structures and to track their development and their change over time we suggest a specific empirical approach. For this purpose we introduce a two-step analysis consisting of the *non-parametric procedure* to constructing production or efficiency frontiers and the *Malmquist-productivity* index to tracking the change of this structure over time.

The plan of the presentation is as follows. In section 2 we will briefly refer to some theoretical issues describing, first, the role heterogeneity plays in the evolutionary framework; secondly, we ask for the criteria to be applied in order to determine the analytically relevant heterogeneity in empirical data and put forward four requirements for the empirical approach to be employed. In section 3 we refer to the measure total factor productivity, distinguish our procedure from the traditional way of computing this measure and briefly

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1 e.g. Cantner/Hanusch/Pyka (1999)
2 e.g. Cantner/Pyka (1998a, 1998b) Cantner (2000)
3 e.g. Cantner (1996)
4 Pyka/Krüger/Cantner (1999).
5 e.g. Cantner (1996), Cantner/Hanusch/Westermann (1998), Cantner/Westermann (1998)
6 Cantner/Hanusch (1999)
7 Bernard/Cantner (1999)

introduce the main methodological approach we have chosen in order to detect heterogeneity and to track its development over time. Section 4 presents a selected number of results pertaining to different levels of aggregation. The final section 5 summarizes our approach and discusses some as yet unresolved issues and problems.

2. The Concept of Heterogeneity

In this section, what we attempt to briefly discuss is the concept of heterogeneity both in a more general way and more specific with respect to the economic theory of technological change and innovation. For the further discussion this initial step allows us to propose a rather well defined analytical frame constructed on the basis of a conception of heterogeneity which aims at differences in the *technological performance of agents*. This, of course, is an extraction out of all possible sources and instances of heterogeneity, but it enables us to focus on that kind of heterogeneity we consider analytically relevant in the economic theory of technical change.

2.1 Theoretical Issues

In economics, there are a number of distinguishing elements of the neoclassical and the evolutionary approach. Probably one of the most important ones, however, is the *heterogeneity* of behavior, attitudes, characteristics etc. of agents.[9] Thus, what heterogeneity is all about is *asymmetry* among the agents in a set. However, it is not at all obvious whether this asymmetry matters for the description of the state of this set or for its development over time. In neoclassical approaches one would deny this in general with the consequence that the theoretical models suggested are characterized by symmetry of agents or even by a representative agent.[10] All the approaches designed in this way are justified by the attitude that for the final outcome of a certain process the differences in agents´ behavior during this process do not matter – it is just average behavior which is determining the result and which analytically is relevant and interesting. Hence, heterogeneity is of an only temporary nature and by this it is a phenomenon only showing up during transitory dynamics.

EVOLUTIONARY APPROACHES Contrariwise, heterogeneity or asymmetry is a fundamental principle in the theories of economic evolution. S*electionist approaches*, *synergetic approaches* and *developmental approaches* rely on it and discuss how the system's nature or structure, on the one hand, and how, on the other hand, – based on this – its

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[8] Cantner/Hanusch/Krüger (2000, 1999)

[9] Other distinguishing features are the different conception of uncertainty, of rationality, of equilibrium, etc.

[10] Analyses dealing with asymmetric information are an exception to this.

(structural) dynamics is affected or driven by it. However, in each approach the way heterogeneity affects evolutionary development is quite specific: In the selectionist approach it is heterogeneity which is reduced by competition and generated by innovation. In the synergetic approach it is heterogeneity which brings about specific structural, self-organizing features with respect to learning, co-operation etc. In the developmental approach, finally, heterogeneity is a matter of the stages of development (to be) passed.

HETEROGENEITY AS ASYMMETRY AND VARIETY Heterogeneity is a concept which refers to *the degree of difference within a population of observations*, let that be households, firms, sectors or even regions or countries which differ with respect to their efforts, behaviors and/or success due to – among others – the artefacts they consume or produce, the modes of production they employ, the direction and intensity of innovative activities they pursue, or the organizational setting they choose. This heterogeneity of agents is, on the one hand, considered the *result* of technological change, i.e. of different innovative/imitative/adaptive activities and differential innovative/imitative/adaptive success; on the other hand, it also serves as a *source* for further progress in the sense that this heterogeneity puts pressure on technologically backward actors to improve performance when the gaps become too large and on leaders when the gaps become too close; and that it provides for different kinds of learning processes (imitative and adaptive learning, cross-fertilization etc.).

In order to account for the heterogeneity of agents driven by and driving technological change one draws on the close relationship between the characteristics and the behavior of agents, on the one hand, and the kind of inputs which, on the other hand, they transform into outputs. In fact, in the theory of technological change the actors are characterized by the nature, the level and the degree of their innovative activities – either on the input or on the output side.

In this respect heterogeneity can be accounted for by a conception of *variety* (Saviotti 1996). This concept is based on the number of distinguishable elements of a set of artefacts.[11] In this sense Saviotti (1996, 94) distinguishes output and input variety, the former being the number of distinguishable outputs and the latter taking account of the number of distinguishable types of processes.

However, heterogeneity in general and within the context of innovativeness in particular is not only a matter of simply counting distinguishable elements. Any innovator attempts to perform better than his competitors, and this „better“ may show up in providing goods and services with superior price-quality ratios – compared to the ones of the competitors. Thus, more than often one would like to have a conception which allows for a quantifi-

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[11] An obvious candidate would be this number itsself. According to Saviotti (1991, 177) in information theory the variety of a set is just the logarithm in base 2 of the number of distinguishable elements.

cation of the differences on which heterogeneity rests. Hence, with respect to output variety one would be interested whether the variety observed is also built upon measurable quality differences (i.e. „higher“ and „lower“ quality) or whether this variety is found within a more narrow or more broad range of the specific characteristic under consideration (i.e. „more“ or „less“ built-in features). Equivalently, with respect to input or production variety we should have an account of whether the several techniques in use are rather similar or far different with respect to their efficiency (i.e. „more“ or „less“ efficient) or their relative input requirements (e.g. „more“ or „less“ capital-intensive).

An appropriate conception in this respect is found in Dosi (1988, 1155–7) who is concerned with the *asymmetry of activities* and distinguishes *variety* as a special case of asymmetry. Both are to be seen in a context where firms engaged in innovative activities are affected differently by technological change in terms of their process technologies and quality (or kind) of output. Whenever firms can be ranked as „better“ or „worse“ according to their distance to some technological frontier he refers to as *asymmetry*. The degree of asymmetry of an industry is then its dispersion of input efficiencies for a given (homogeneous) output and price-weighted performance characteristics of firms´ (differentiated) products. For all differences or asymmetries among firms which cannot be ranked as unequivocally better or worse he refers to *variety*. This may be the case when (a) firms producing the same good with identical costs employ different production techniques or (b) when firms´ search for their product innovations in different product spaces, embodying different product characteristics and aim at different corners of the markets.

THE CONCEPT OF HETEROGENEITY Based on this discussion, in the following we will be concerned with heterogeneity which is as closely as much related to technological performances and their differences – thus, heterogeneity is meant to be technological heterogeneity and it is based on the *local application of certain technologies*. By this it, first, includes performances which can be compared directly to each other and by this be ranked – i.e. producing a certain product with a higher/lower quality or running a specific production process more/less efficiently. Second, this conception comprises also technological performances which cannot be compared to each other directly – i.e. producing different products in the sense of old and new or running quite different production techniques. These latter performances cannot be compared directly (in terms of some physical measures) and one has to rely on some other measures such as the comparative economic success of those performances (as measured in terms of further technological change, of profitability, of market share, of growth rates, etc.).

TECHNOLOGICAL HETEROGENEITY ON HIGHER AGGREGATION LEVELS Moreover, the technological heterogeneity we are concerned with is not only confined to the technological performance of individual actors. It is also applicable to higher levels of aggregation such as the sectoral, the regional or the national level. By the way of aggregation, of course, any sub-level heterogeneity gets covered and only some – here no matter how defined – *average* characterizes the higher level unit. Despite this inevitable loss of information involved here, we nevertheless expect considerable and relevant heterogeneity in technological performance of sectors, regions or economies with respect to the product and quality range produced (e.g. agricultural products, Germany compared to India) and/or the kind and degree of certain production techniques employed (e.g. cotton production, US compared to Pakistan). Accordingly, what we mean with locally applied technologies does – with a loss of specific description – also refer in a more aggregate sense to sectors, regions and countries.

HETEROGENEITY AND DYNAMICS Heterogeneity as just introduced can be considered as a snapshot description of a sample of observations. Especially in the context of technological progress it is quite obvious and has to be expected that heterogeneity is subject to change. One could easily think of exogenous forces which quite equally affect all agents. However, for endogenous changes which are provoked by individual action it is just as much obvious that the respective changes are to a considerable degree specific to a certain agent or group of agents – that is the change we are concerned with is *local technological change*.[12] And even if we considered a number of agents to behave rather similar, e.g. in catching up to the technology leaders, such progress as well is local in the sense that only a subgroup of the agents under consideration achieved at that.

Equivalently to our discussion of heterogeneity the concept of *local technological change* is applicable to several levels of aggregation. In this respect, technological change is specific to a certain country (e.g. the US compared to Togo), to a certain region (e.g. East Asian Tigers compared to Western Europe), or to a certain sector (e.g. in machinery, Germany compared to Japan). Of course, and again equivalent to above, local change on higher levels of aggregation hides local changes on lower aggregation levels, so that only an „average“ change shows up.

2.2 Empirical Issues

HETEROGENEITY EVERYWHERE To state the importance of heterogeneity is one side of the coin, the other one is to clearly specify in which unit we should measure or observe heterogeneity and this in a way that it is also analytically relevant.

To clarify this, it is at all obvious that we are not all the same. But is this extreme degree of heterogeneity of analytical relevance for explaining e.g. any difference in language among us? Probably we have to be more crude or even more abstract. We thus could put

[12] See Atkinson/Stiglitz (1969); a good overview is found in Antonelli (1995).

forward that it is only nationality that matters – and for analyzing the differences in structure and content of the comments we could give here each one in his native language this might be a helpful distinction.

Consequently, to identify heterogeneity in empirical work is not an easy and straightforward task at all. In principle, one is facing a similar problem the typological approach is confronted with: what is essential for analyzing the issue under concern? Whereas the typological approach searches for some reliable average characteristic, the population perspective is confronted with the task to find characteristics which are diverse, i.e. heterogeneous, and by this essential for performance and the progress of the population under consideration. Looking for variables which can render this, one more than often has to be engaged in rather detailed analyses of an nearly case-study type. Although the results are often very illuminating and interesting, it is more than often not possible to transfer the methodology and the results of one study to another one; the aggregation of several results is often not possible, because the relevant variables are not of the same type; etc.

HETEROGENEITY AND INNOVATION Let us now look more carefully at the theory of technological change and innovation. How can we measure technological performance? To give an answer we start with another question: What does technological progress provoke, how do we distinguish an innovation from a well-known old artefact?

Here it is quite obvious that innovations provide for heterogeneity because something new – a „new combination“ in Schumpeter´s (1912) words – is introduced into the market. This may be a better technique of production, a better organizational structure, a better product quality, or an entirely new product. Hence, the innovator introducing this new combination can be distinguished from competitors just by his/her innovation.

Thus, more generally, technological heterogeneity is just the consequence of differential innovative success cumulated up to the present. In a dynamic context, with respect to several features of the process of technological change such as path dependency, cumulativeness etc., this heterogeneity can also serve as an indicator for the direction and success probability of further innovative activities – such as innovation, imitation, adoption etc..

The central question arising out of this is concerned with the measure we should apply in order to account for this innovation and technology related or determined heterogeneity.

SPECIFIC VERSUS GENERAL MEASURES Of course, we could have a long list of possible characteristics or features which perform the task of detecting the effects of technological change and innovations. All of the characteristics used in *technometrics* are based on rather technical issues. Look for example at the technological development of helicopters so well studied by Saviotti: technological progress here is represented by the development in technical characteristics such as engine power, rotor diameter, number of engines etc. Or, look at computer chips where we are informed about technological progress by steadily increasing storage capacities. Or, finally look at automobile production where technological progress or organizational progress shows up in the increased number of cars assembled within one hour (Fordism) or in the decreased number of bad quality cars assembled (Toyotism).

However, by all its merits this quite specific and quite exact technical measurement has also its drawbacks:

1. Despite the respective specific characteristics of a technology and despite its development can be represented relatively exactly, it also causes that the more exactly one measures specific features the less a comparison between different observations will be possible and meaningful.
2. Whenever different technologies and their respective progress are analyzed, a comparison of the results is less likely to be possible.
3. Any aggregation from the business unit to the firm, to the sector and industry, to the regional or even to the macroeconomic level is not possible anymore. The reason for that is quite obvious, because to aggregate the products of different firms in a sector, the products of different sectors in an economy cannot be accomplished when technical attributes are used such as pieces, kg, MB etc.

On the basis of the following four central requirements we suggest and introduce a measure and empirical procedure which attempts to circumvent the problematic issues just raised and which allows to analyze empirical observations within a theoretical framework aiming at locally applied technologies and local technological change. For each requirement we give a brief suggestion here – a completed discussion is found in section 3.

REQUIREMENT 1 The first task is to detect a measure which may help to overcome the problems mentioned. Thus, what one has to look for in this context is a measure which on the one hand is exact enough and on the other hand is not that specific so that the above-mentioned deficit will not show up. Hence, what we look for is a measure which serves this purpose and is *applicable to a broad range of innovative phenomena on different levels of aggregation*.[13]

SUGGESTION 1 In order to show the way for setting up a broadly applicable taxonomy we suggest the *measure of total factor productivity* (TFP) and its *change over time* to

[13] Saviotti (1996, 52) states:*“ ... a taxonomy at all levels of aggregation in such a way that the relationships of the various units of analysis within and between each level of aggregation can be analysed.”*

play a major or even pivotal role in this endeavor. This suggestion, on a first glance, might look somewhat old-fashioned as the concept of total factor productivity has been criticized intensely in the past – mainly in the context of growth accounting exercises where its construction is based on equilibrium assumption and conditions of traditional production theory combined with notion of the same production to be applied at all observations. This leads us to a second requirement.

REQUIREMENT 2 The way TFP is measured should differ considerably from standard procedures. By this, in a first step, it should allow to distinguish innovators from imitators and *account for better and for worse technological performance*. Moreover, it should deliver a quantitative account of these differences.

SUGGESTION 2 With respect to requirement 2 we suggest to apply a frontier analysis where the *frontier function* or *technology frontier* is set up by the best performing observations. All worse performing observations are in some distance to this technology frontier where this distance can be used as a measure for different technological performance.

REQUIREMENT 3 Related to the need to distinguish better from worse performance is the requirement that, following the evolutionary approach, the empirical analyses should not be restrictive in the sense that functional relationships, e.g. a specific production function, is a priori assumed to hold for all observations. One rather should allow for an open number of those relationships and take into account also *variety* in productions functions or output mixes.

SUGGESTION 3 For satisfying requirement 3 we suggest the computation of TFP measures by a no*n-parametric procedure to determine technology frontiers* which – at least compared to the traditional approaches of TFP index numbers, parametric production functions and parametric production frontiers[14] – are rather unrestrictive in the functional form employed for the aggregation of inputs and outputs, respectively. In principle, there are as much functional forms allowed for as a sample contains observations.

REQUIREMENT 4 The measure applied should be tracked over time. The respective measure of the change in TFP should be able to take account of *local technological change*.

SUGGESTION 4 In this respect we suggest to employ the procedure to compute the Malmquist productivity index which just measures the change in TFP. The important feature of this measure is that it allows to identify *local technological change* at both the technology frontier as well as the below best-practice observations.

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[14] For a discussion and comparison of the non-parametric approach with more traditional methods see Cantner/Krüger (1999).

3. TFP, technological process and evolutionary theory

3.1 TFP as a measure of technological performance

Referring to requirement and suggestion 1 we consider total factor productivity and its change over time as an appropriate measure for technological performance and technological change. This, of course, requires some qualifications.

GENERALITY As already claimed we are interested in a generally applicable measure which allows to track technological change on several levels of aggregation and in several fields of application. Thus, what we have to accept is a loss of specificity especially found if one applies the analysis on lower levels of aggregation often coming close to pure case studies. The loss of specificity, however, is counterbalanced (and in our view even over-compensated) by the opportunity to detect more general insights into structure and change whose driving elements are found on the individual level of actors and firms, whose collective outcome then shows up in a characteristic manner on the next level of aggregation and so forth. In this respect the measure of total factor productivity is applicable to all levels and areas of aggregation whenever we have at hand appropriate data on outputs and inputs.

CONSTRUCTION Index numbers for total factor productivity TFP have found a prominent application in growth accounting exercises. There it is aggregate output *Y*, prominently GDP, set into relation to an aggregate *X* of various input factors, prominently labor and capital:

$$TFP = \frac{Y}{X}$$

One can easily apply this measure to lower levels of aggregation such as to the sectoral level[15] and to the firm level. Any change in total factor productivity, in the sense that this indicator rises, is considered as the effect of technological progress, i.e. that change in output which cannot be accounted for with a change in aggregate inputs:

$$\Delta TFP = \Delta Y - \Delta X$$

It is this so-called *residual* which attracted so much research especially in the analysis of economic growth. And it is also this residual what Abramovitz called „*our measure of ignorance*“.

A first question arising in this context refers to whether TFP can be taken as a measure of technological performance and whether a change in total factor productivity can adequately account for technological change. Let us take up this issue accordingly.

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15 E.g. Wolff (1997).

TFP AS PERFORMANCE INDICATOR In order to account for the performance of an observation the indicator applied is to be interpreted always as a relative measure, either with respect to some known optimal performance or with respect to the best performance observed. In empirical work it is always the latter relativisation employed. For this comparison to work, however, one has to provide for that

1. the categories used for measuring inputs and outputs and
2. the respective way of aggregating of inputs and outputs in order to compute the TFP

are identical among the observations. Otherwise the comparison is inadequate.

To cope with the first problem, one has to look for measures which allow for homogeneous input and output categories. This is certainly not always achievable, but by some degree of abstraction or clever chosen units of measurement – in the sense of real units (e.g. hours worked, kg, etc.) – one might be able to cope with this problem – at least partly.

As to the aggregation functions for inputs and outputs, with respect to inputs it is just the production function what is searched for and which has a number of specific problems. We do not want to go into detail here but only remind that on the theoretically founded perceptions of techniques applied locally and local technological change a aggregation or production function identical for all observations cannot be expected a priori – contrariwise heterogeneity is to be expected.

With respect to outputs the problem is similar whenever we are not in the lucky situation to have to consider only one homogeneous output. Again, this is not the normal case and among the observations we normally have to expect both differences in the quality of the output as well as differences with respect to the number of outputs produced. A common way to deal with this is to accept *product prices* as weights which account for quality differences as well as differences in kind.[16] This leads to output measures such as GDP, sectoral sales or firm sales. Besides this, however, one might also be interested in dealing with output variety in an disaggregated way such as splitting up GDP into the output of various sectors or of firms sales into the sales of different products. A possible way of performing this is presented below.

CHANGE OF TFP AS MEASURE OF TECHNOLOGICAL CHANGE Interpreting the change of TFP as a measure for technological progress faces the same problems as just stated. Whenever we consider process innovations allowing the given resources to produce more of a homogeneous output, the change in TFP appropriately takes account of this.

[16] This comes close to what Dosi (1988, 1155-7) claims to be price-weighted performance characteristics of firms´ (differentiated) products.

However, dealing with quality improvements or new products, whenever quantity and price changes account for this in a proper way we can use the aggregate output. But if we were interested in the development beneath the level of aggregation it would be helpful to have the respective TFP change determined on the basis of a disaggregated TFP index.

OTHER INFLUENCES ON TFP A final remark here refers to differences in the TFP which are not due to the respective technological performance of the observation. Proper candidates are vintage structures as well as economies of scale. [17] For the change in TFP we should additionally be aware of substitution effects to work. With respect to substitution effects according to Rosenberg (1976) substitution along a traditional isoquant is to be considered as applying a technique not applied yet and this could also be considered as technical change.

Having given some justification and qualifications on the TFP measure as response to requirement 1 we now want to go one step further and introduce a method taking care of requirements 2 and 3.

3.2 Structure: a non-parametric frontier function approach

Requirements 2 and 3 ask for a method which allows to determine TFP in a way that technological heterogeneity in the sense of asymmetry and variety shows up. For this purpose we suggest a non-parametric frontier function approach.

UNRESTRICTED PERFORMANCE MEASURE The non-parametric frontier function approach (or DEA for **D**ata **E**nvelopment **A**nalysis) basically relies on index numbers to measure total factor productivity in a fashion similar to the one used in more standard productivity analysis. In a sample of n observations for each observation j ($j=1,...,n$) a productivity index h_j is given by:

$$h_j = \frac{u^T Y_j}{v^T X_j} \qquad (1)$$

Here Y_j is a s-vector of outputs ($r=1,...,s$) and X_j a m-vector of inputs ($i=1,...,m$) of observation j. The s-vector u and the m-vector v contain the aggregation weights u_r and v_i respectively.

The h_j in (1) is nothing else than an index of *total factor productivity*. The respective aggregation functions (for inputs and outputs respectively) are of a linear arithmetic type

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[17] Differences might occur also due to scale effects and/or vintage structures (Dosi 1988, 1156)

as also employed in the well-known Kendrick-Ott productivity index.[18] There, however, by special assumptions the aggregations weights, u_r and v_i, are given exogenously. The non-parametric approach does not rely on such assumptions – in particular, it is not assumed that all observations of the sample have a common identical production function. With this – at least to a certain degree – unconstrained way of aggregating both inputs and outputs we are able to account for requirement 3 above. The parameterization of the aggregation functions and thus the aggregation weights which may be specific to a certain observation are determined endogenously. They are the solution to a specific optimization problem (as discussed below), and therefore they are dependent on the empirical data of the sample. Critics often argue that a linear arithmetic aggregation nevertheless presupposes at least a special type of production function,[19] such as the Leontief-type production function.[20] Since the aggregation weights are determined endogenously and can be different between observations, there ultimately exists a number of parametrically different possible aggregation functions although they are all of the same type.[21] For the input side, moreover, the fact that the Leontief production function fits well into this framework suits well to the widely held assumption of short-run limited substitutability of production factors whenever techniques employed are of a local character.

This unrestricted form of the total factor productivity measure is *central to an application of this method to evolutionary analysis and to detecting heterogeneity in particular.* For computing this index we can include all different kinds of inputs and different types of outputs. This implies also that new products can be taken care of and equivalently new production factors. Since the non-parametric approach does not require all inputs to be employed or outputs to be produced by each observation we are readily able to take into account both product innovations and new techniques of production.

Having found a rather unrestricted mode for measuring the performance of an observation we would like to provide also a comparison of this performance with those of the other observations in the sense that we find statements about „unequivocally better“, or „unequivocally worse“ or even „not comparable“.

COMPARISON OF PERFORMANCE For doing so, the basic principle of the non-parametric approach is just to determine the indices h_j in such a way that they can be interpreted as efficiency ratings which implies a comparison of each observation with the best

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18 See Ott (1959).

19 See also Chang/Guh (1991, p.217).

20 Leontief (1947) has shown that a linear aggregation exists for a Leontief-type production function. Instead of a Leontief function one could also use a linear production function.

21 Employing parametric methods, e.g. the COLS or the EM-algorithm a specific production function is assumed. The coefficients of this function are estimated using the available data and the resulting production function is used to determine technical (in)-efficiencies of all the fims in the sample. This procedure, however, suggests that there is only one "*best-practice*"-technology. With the non-parametric approach a number of "*best-practice*"-technologies can be determined.

observation(s). The (relatively) most efficient observations of a sample are evaluated by h=1, less efficient observations by $h<1$. Hence, by comparing all observations with each other we achieve at an account of different technological performance where the differences are quantified in the measure h – this is just what requirement 2 asked for.

The following constrained maximization problem is used to determine such a h-value for a particular observation l, $l \in \{1,\ldots,n\}$:

$$
\begin{aligned}
\max \; h_l &= \frac{u^T Y_l}{v^T X_l} \\
s.t.: \; \frac{u^T Y_j}{v^T X_j} &\leq 1; \; j = 1,\ldots,n; \\
u,v &> 0.
\end{aligned}
\tag{2}
$$

Problem (2) determines h_l of observation l subject to the constraint that the h_j of all observations (including l itself) of the sample are equal or less to 1. The constraints provide that h is indexed on (0,1]. Moreover the elements of u and v have to be positive. This requirement is to be interpreted that for all inputs used and outputs obtained there must exist at least a positive efficiency value.[22]

BEST-PRACTICE OR FRONTIER FUNCTIONS Since we employ linear arithmetic aggregation functions for inputs and outputs, (2) is a problem of linear fractional programming.[23] To solve such optimization problems, there exist a number of methods the best known of which is Charnes and Cooper (1962). They suggest transforming (2) into a standard linear program which then can be solved with the well-known simplex algorithm. Performing this step and transforming the resulting primal to its corresponding dual problem, one arrives at the well-known Charnes/Cooper/Rhodes[24] envelopment form of the non-parametric approach:

$$
\begin{aligned}
& \min q_l \\
& s.t.: \\
& \qquad\quad Y\, l_l \geq Y_l \\
& q_l X_l - X\, l_l \geq 0 \\
& \qquad\qquad l_l \geq 0
\end{aligned}
\tag{3}
$$

22 This procedure is also known from activity analysis.

23 An overview over linear fractional programming is given in Böhm (1978).

24 There obviously exists a range of possible model specifications where the one chosen is known as CCR. Applying this one has to keep in mind that possible scale inefficiencies are included in the technical inefficiency measure.

Y_l and X_l are the *r*- and *s*-vectors of outputs and inputs respectively of observation *l*, *Y* and *X* are the $s \times n$-matrix of outputs and $m \times n$-matrix of inputs of all observations of the sample. The parameter θ_l to be minimized expresses the percentage level to which the inputs of observation *l* can be reduced proportionally, in order to have this observation producing on the production frontier representing the best practice technologies – it is identical to h_l and is a relative measure of technological performance. With $\theta_l = 1$ the respective observation belongs to the efficient observations on the frontier. Proceeding in this way and solving (3) for all observations in the sample, the non-parametric approach determines an *efficiency frontier* or *technology frontier* constructed by the best-practice observations. The efficiency rating of each observation is measured relative to this frontier.

Figure 1 states this result for a sample of observations which produce with two inputs, x_1 and x_2, one unit of output. The technology frontier determined is *DAB*. The technological performance is the relative distance of an observation from the technology frontier. In the case of observation C, the measure θ_C is given by the ratio *OC´* to *OC*.

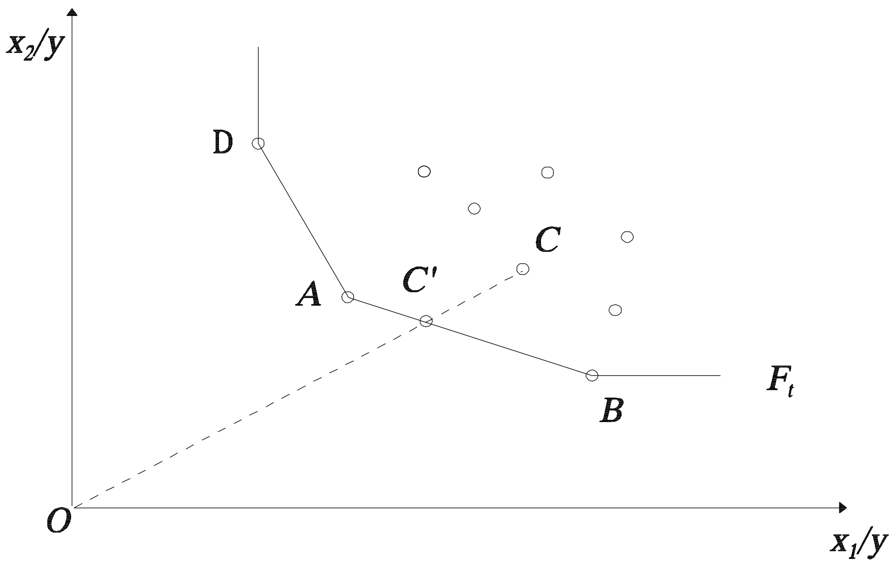


Figure 1: Technology frontier and the measure θ_C

The *n*-vector λ_l states the weights of all (efficient) observations which serve as reference for observation *l*. For the efficient observation *l* (with $\theta_l = 1$), we obtain 1 for the *l*th element of λ_l and 0 for all other elements. Grouping all observations according to their respective reference observations allows to detect fields of similarity. These fields are distinguished by different input intensities, output intensities or input coefficients. In terms of figure 1, for observation *C* the reference observations are *A* and *B*. Consequently only λ_A and λ_B are different from 0. The respective values state the degree to which *A* and *B* are used respectively to construct *C´*.

A FIRST CHARACTERISATION OF THE STRUCTURE OF A SAMPLE So far the discussion has delivered an account of a sample which allows to detect and quantify heterogeneity in productive performance. With program (3) we are now readily able to characterize the structure of a sample of observations:

1. θ as a measure of performance indicating and quantifying whether an observation is best-practice or below best-practice;
2. λ as a measure of structural (dis)similarity (Cantner 1996).

However, modifying the program (3) some measures can be computed which shed additional light on the structure of a sample.

COMPARISON OF BEST-PRACTICE Since the frontier function quite regularly is constructed by several best-practice observations which cannot be ranked as better or worse, one might additionally be interested in a comparison between them. The following modification of program (3) allows for this where now the observation under consideration l is not member of the reference set:

$$\begin{array}{rcrcl} & & \min \theta_l^* & & \\ & & s.t.: & & \\ & & Y_{-l}\,\lambda_l & \geq & Y_l \\ \theta_l^* X_l & - & X_{-l}\,\lambda_l & \geq & 0 \\ & & \lambda_l & \geq & 0 \end{array} \tag{4}$$

The matrices Y_{-l} and X_{-l} contain the outputs and inputs of all n observations except observation l. The modified efficiency measure is θ_l^*. For all below best-practice observations it is identical to θ_l determined by program (3). However, for all best-practice observations θ_l^* is different. It holds $\theta_l^* \geq 1$ and the difference $\theta_l^* - 1$ can be interpreted as the buffer or lead observation l holds compared to certain other observations. This θ_l^* is a measure to distinguish observations which with program (3) are determined as not comparable (Cantner/Westermann (1998)).

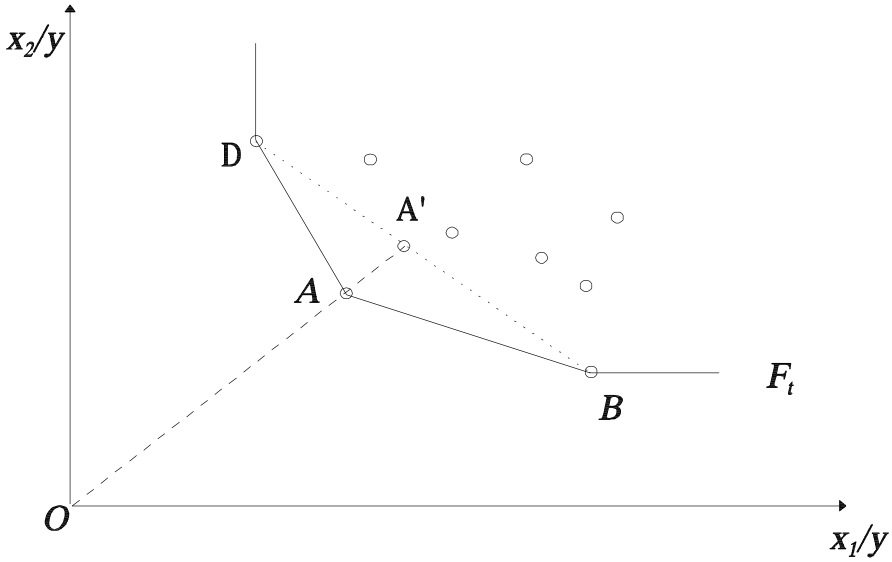


Figure 2: Comparison of best-practice observations

Figure 2 states the result of program (4) for observation *A*. The respective frontier for *A* in this case is *DB* and the θ_A^* is equal to the ratio *OA´* to *OA* which is larger than 1.

Besides this mode of comparing best-practice observations an alternative or additional way is to look at the dynamic performance, i.e. their comparative ability to shift the frontier function (by technological progress). This issue will be taken up below.

ACCOUNTING FOR SCALE EFFECTS Finally, since the programs used so far have been discussed under the assumption of constant returns to scale technologies one might be interested in taking into account size effects. This is done by first setting up a program allowing for non-constant returns to scale. This leads to a formulation where the elements of the λ_l vector have to sum up to 1 (e^T is a vector containing only elements 1):

$$
\begin{array}{rcrcl}
 & & \min q_l^v & & \\
 & & s.t.: & & \\
 & & Y\, l_l & \geq & Y_l \\
q_l^v X_l & - & X\, l_l & \geq & 0 \\
 & & e^T\, l_l & = & 1
\end{array}
\qquad (5)
$$

For the efficiency measure determined by program (5) we get $\theta_l^v \geq \theta_l$. Taking the ratio of these two measures, $\sigma_l = \theta_l / \theta_l^v$, states the level of efficiency which is due to scale with $1-\sigma_l$ accounting for that degree of below best practice which is caused by a size different to the minimum efficient scale size.

Besides these measures the non-parametric frontier approach does deliver a number of other measures allowing to deal with allocative efficiency, non-radial inefficiencies, specific forms of returns-to-scale etc. These are of minor importance in the context of this paper. More interesting, however, is the dynamic extension of the analysis.

3.3 Structural Dynamics: Local Technological Change, Catching-up and Falling-behind

The following discussion refers to requirement 4 asking for an appropriate way of dealing with localized technological change and thus the structural dynamics induced.

DYNAMIC ANALYSIS In order to track the structure – determined by the above introduced measures – it is by no means sufficient to compare the structural results of consecutive periods. Because for each period these measures are of an only relative type such a comparison makes no sense. Consequently, consecutive periods have to be set into relation implying that we have to compute relative measures which compare period t with $t+1$ and *vice versa*. Doing this pairwise for all consecutive periods allows to track structural change over time. The procedure chosen for this purpose is based on the *Malmquist index* which states a specific observation's change in productivity between two periods. A quite interesting feature of this index is that it can be decomposed into a measure for *technological change* and one for *catch-up* – or , of course, *technological regress* and *falling behind.*

MALMQUIST INDEX The theoretical basis of the Malmquist-productivity index is found in the work of Malmquist (1953), Solow (1957) and Moorsteen (1961). For productivity measurement this index has been applied by Caves/Christensen/Diewert (1982a, 1982b). Färe/Grosskopf/Lindgren/Roos (1994) have shown how the efficiency measure θ above can be used to compute the Malmquist index. We will follow this line of reasoning.

In order to explain what the Malmquist-productivity index measures we refer to figure 3 which contains a simple example of two non-parametric production frontiers F_t and F_{t+1} pertaining to period t and $t+1$. For measuring the productivity change of observation A from A_t to A_{t+1} consider the following: First evaluate A_t and A_{t+1} towards the frontier F_t and compute the ratio of the two results. For this we get Ob/OA_t divided by Od/OA_{t+1}; if this ratio is less than 1 the productivity of A increased. Second, and in addition to that we could also evaluate A_t and A_{t+1} towards the shifted frontier F_{t+1}; again we determine the ratio, here Oc/OA_t divided by Oe/OA_{t+1}; this ratio less than 1 implies a productivity improvement. In a final step the geometric mean of these two computations is taken.

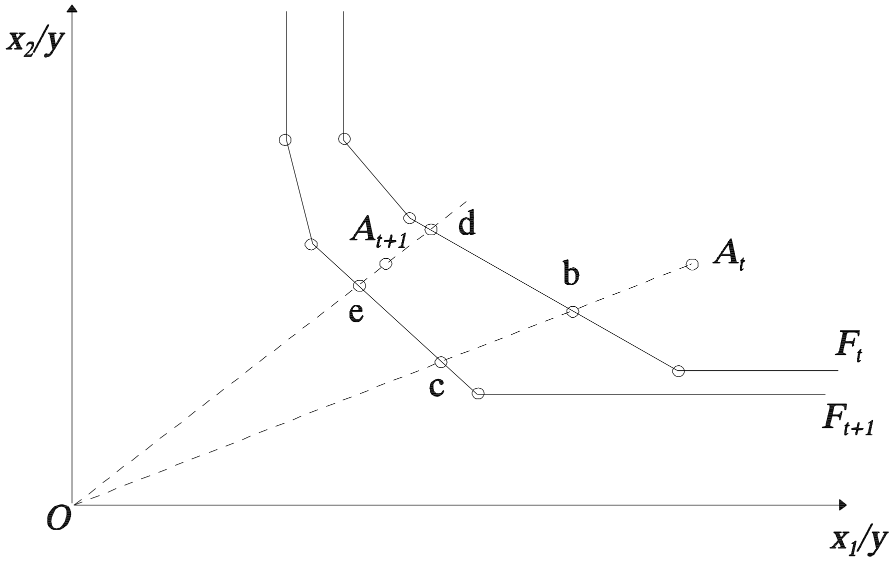


Figure 3: Malmquist productivity index

The resulting index

$$M_A^{t+1} = \left(\frac{Ob/OA_t}{Od/OA_{t+1}} \frac{Oc/OA_t}{Oe/OA_{t+1}} \right)^{0,5},$$

the *Malmquist-productivity index*, states the productivity change of *A* between *t* and *t+1*. In a general way, the Malmquist productivity index M_l^{t+1} measuring the productivity change of observation *l* from t to t+1 is defined as follows:

$$M_l^{t+1} = \left(\frac{\theta_l^{t,t}}{\theta_l^{t+1,t}} \frac{\theta_l^{t,t+1}}{\theta_l^{t+1,t+1}} \right)^{0,5} \tag{6}$$

$\theta^{t,s}$, $t,s \in T$, is the efficiency of observation *l* in period *t* whenever the frontier function of period *s* serves as reference measure. [25]

DECOMPOSITION OF THE MALMQUIST INDEX With some manipulation we can develop (6) to the following expression for the Malmquist index:

[25] For the respective programs required to compute the several θ-measures see appendix.

$$
\begin{aligned}
M_l^{t+1} &= \left(\frac{\theta_l^{t,t}}{\theta_l^{t+1,t}} \frac{\theta_l^{t,t+1}}{\theta_l^{t+1,t+1}} \right)^{0,5} \\
&= \left(\frac{\theta_l^{t,t}}{\theta_l^{t+1,t+1}} \right) \left(\frac{\theta_l^{t+1,t+1}}{\theta_l^{t+1,t}} \frac{\theta_l^{t,t+1}}{\theta_l^{t,t}} \right)^{0,5} \qquad (7) \\
&= \mathrm{MC} \cdot \mathrm{MT}\,.
\end{aligned}
$$

The second line in (7) states the decomposition of the productivity change into in *technological progress* MT and *change in the technology gap* MC.

Whenever MC<1 (MC>1) we find catch-up (falling-behind). The second term is MT and indicates the movement of the frontier. This is measured twice: first with the factor intensities of l in t, and a second time with those of l in $t+1$. With MT<1 (MT>1) we have technological progress (technological regress) at the frontier. Looking at our example in figure 3 this decomposition is given by the following ratio of distances:

$$
\begin{aligned}
M_A^{t+1} &= \left(\frac{Ob/OA_t}{Oe/OA_{t+1}} \right) \left(\frac{Oe/OA_{t+1}}{Od/OA_{t+1}} \frac{Oc/OA_t}{Ob/OA_t} \right)^{0,5} \\
&= \qquad \mathrm{MC} \qquad\qquad \mathrm{MT}.
\end{aligned}
$$

By this, we can state that the first bracket term measures the change in the distance of A towards the frontiers F_t and F_{t+1}. The second term in brackets takes account of the (geometric) mean change of the frontier part pertaining to A. In this example both terms will be smaller than 1 indicating that observation A performed technological progress and was able to catch-up to the frontier.

LOCAL CHANGE As is readily apparent from figure 3 the productivity change in (6) is local in the sense that it is specific to the observation under consideration. In this respect the degree of this local change depends (a) on the observation's ability to shift in direction to the origin and (b) on the behavior of the frontier. As to (b) the respective change is also local in the sense that for observation l it is only relevant how the respective part of the frontier assigned to l (by the way of the elements of the λ-vector) shifts. The decomposition of the index allows to distinguish these two movements.

Moreover, the decomposition allows also to evaluate best-practice observations in a dynamic context by comparing them by the way of the index MT and thus by the ability to locally shift the frontier function. An application of this is found in 4.3.

3.4 Summary of the issue

In face of the theoretical and empirical requirements stated in section 2 we have suggested to measure total factor productivity by a procedure which is as unconstrained as possible but nevertheless allows

1. to systematize heterogeneity and
2. to track its change accomplished by technological progress in general and local technological progress in particular.

For this purpose we apply a non-parametric procedure to determine frontier functions. These consist of the best-practice observations in a sample and do not rely on any common a priori parametrically given production function. We thus dispense with any notion of the neoclassical production function and rely entirely on production techniques which in the short-run show no substitutability among production factors, i.e. which could be described by a Leontief-type relationship between output and input.

For the dynamics we apply the Malmquist-index measuring productivity change by comparing the non-parametric production frontiers and observations of consecutive periods. By this we dispense with the notion that technological progress shifts the entire production frontier and instead we allow for (a) parts of the frontier to shift and for (b) this shift not to be proportional.

With respect to heterogeneity and its change this two-step procedure performs or detects the following. The first step of this two-step procedure allows to detect heterogeneity – here technological heterogeneity – and classify the observations into the following categories:

1. Heterogeneity in the performance of running a specific technique, class or range of techniques.
2. Heterogeneity in applying a specific technique out of a larger range of possible techniques.

The second step then tracks this heterogeneity over time and allows for the following:

1. Measuring local technological change.
2. Distinguishing between progress of the best-practice techniques or forging-ahead and dynamics of catching-up, falling-behind.

Taken literally, the procedure suggested does classify the observations in a specific way in both a static and a dynamic context. By this we do not have to a priori rely on restrictive assumptions or constraints which force the observations to behave in a certain way, e.g. to obey to the same parametrically given production function.

4. Selected Empirical Results

In the following we will briefly present some empirical results found by using the methods introduced above. In this respect, our focus will be on heterogeneity and its development over time. Of course, such kind of exercise *does not prove* any of the evolutionary concepts or theories. In order to perform this task, the respective results have to be analyzed in a further step by applying other statistical techniques such as regression analyses. Only then one can give answers on questions such as whether heterogeneity is the result of different innovative success, of different abilities to imitate etc.; or whether there are spillover effects arising out of heterogeneity and influence the structural development; or how macro growth is influenced by meso or micro dynamics; etc. Whenever available for our empirical analyses we will also briefly report on the results of those required third steps.

4.1 Intra-sectoral analysis of technological and structural change – Industrial Dynamics

The first of our selected empirical applications is concerned with the dynamics of productivity within industrial sectors (Cantner 1996, 1998). This dynamics is characterized by a certain structural stability with respect to best-practice performance as well as some regularities as to which firms are more likely to catch-up than others. We concentrate here on *heterogeneity* „defined“ on the basis of the performance differences between best- and below practice firms.

For the purpose of presentation we refer to a number of investigations into the German manufacturing sector. Here we report on the sectors plastics (22 firms), machinery (83 firms) and electronics (36 firms). The analysis is performed for the period 1981 to 1993. We use three inputs: labor (labor hours), capital (machinery and equipment, capacity adjusted) and materials. Output is sales corrected by change in stocks. Hence, the non-parametric frontier approach is run with 3 inputs and 1 output.

In figure 4 we present results for the three sectors. Using the efficiency values determined by the non-parametric approach we draw Salter curves of 1986 and 1993 for the three selected sectors. Here the order of firms on the abscissa is always in accordance to the efficiency ranking as found in 1986. Comparing the Salter curves for the two selected years we find

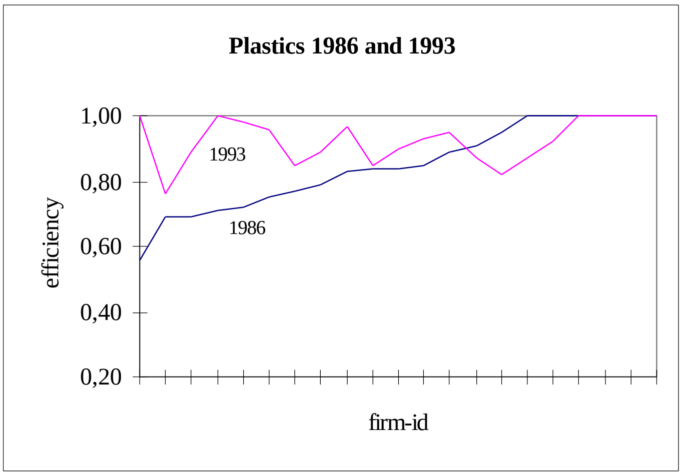
1. some degree of *persistency* because
    - a number of best-practice firms in 1986 are still ahead in 1993;
    - the efficiency ranking of firms is rather similar in 1993 compared to 1986 at least in plastics and machinery.

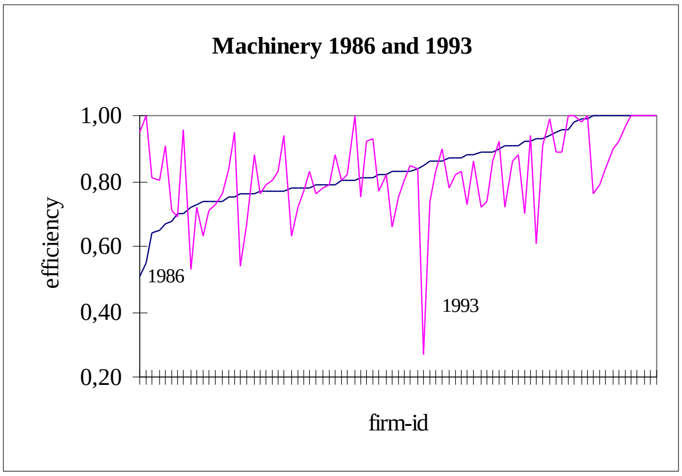
2. tendencies of overall *convergence or divergence*

   - where in plastics the 1993 curve is almost everywhere above the 1986 curve implying that the efficiency levels came closer together; the contrary applies to electronics; no clear answer is possible for machinery;

3. characteristic *structural dynamics*

   - where the „falling-back“ from 1986 to 1993 is more often the case in regions of higher efficiency in all three sectors;
   - where „catching-up“ from 1986 to 1993 is more likely in the lower regions of efficiency in plastics and machinery and only in regions of middle efficiency levels in electronics.





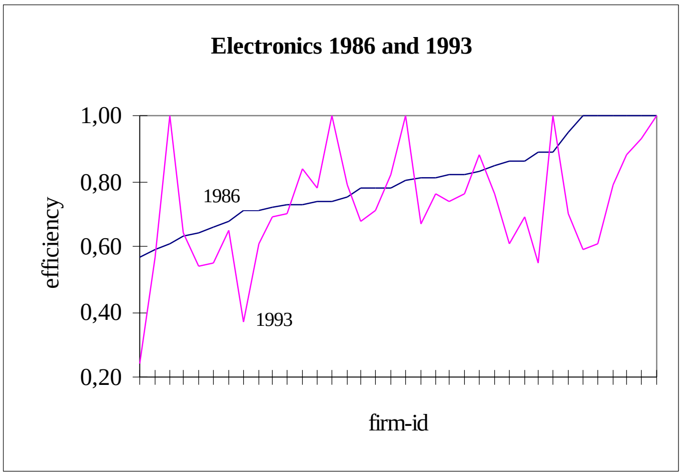


Figure 4: Salter curves 1986 compared to 1993

To explain those results additional analyses have to be performed. For the structural dynamics of catching-up and falling behind one could test for the hypothesis of „*advantage of backwardness*“ and the role of *absorptive capacities* for the followers in catching up.

Table 1a: Regression results for plastics (t-values in brackets)

dep.	const.	gap θ_i	absorptive capacity AC_i		R^2
MC_i			*SRDSL*	*SRDWORK*	F-value
OLS	0.117	-0.244			0.37
	(4.390)	(-6.415)			(11.47)
OLS	0.02	-0.267	0.151		0.47
	(0.497)	(-6.422)	(3.211)		(7.987)
OLS	0.027	-0.332		0.110	0.45
	(0.231)	(-6.172)		(1.288)	(7.17)
NLS	0.280	-0.893	-0.587		0.51
	(5.719)	(-4.687)	(-5.392)		(9.73)
NLS	0.297	-0.806		-0.371	0.54
	(6.556)	(-5.908)		(-5.871)	(10.56)

The results of this analysis are stated in tables 1a–c. We regress the catch-up variable MC_i (which measures the change in efficiency from 1986 to 1993) as the dependent variable on the technology gap in 1986, θ_i, and proxy variables for absorptive capacity, AC_i. SRDSL is the R&D capital stock per worker of firm *i* (determined by the capital inventory method) and SRDWORK is the number of R&D-personnel in total working force of firm *i*

Moreover we distinguish a linear relationship from a non-linear one; the latter presumes that, first, catching up is easier when the gap is larger because much more can be learned; this is the of „*advantage of backwardness*“-hypothesis. Alternatively and second, it is assumed that although much more can be learned with a larger gap, it becomes more difficult to absorb the respective knowledge when the gap increases. Thus, the ability to catch up is dependent on the firm´s absorptive capacity. For this case a bell-shaped relationship between catching-up and technology-gap can be deduced stating that up to some point a larger gap allows for higher spillovers but with the gap further increasing the absorptive capacity puts a constraint on the level of spillovers which then are decreasing.

For both specifications the signs of the coefficients except the constant are all expected to be negative. The linear version is estimated by using OLS whereas the bell-shaped relationship requires non-linear least squares (NLS).

The results show that the linear version of the catch-up hypothesis and thus the of „*advantage of backwardness*“ hypothesis holds in all sectors, whereas the non-linear one shows up in the expected way only in plastics and in electronics. This result fits quite well with the Pavitt classification (1984) of sectors where machinery is considered as specialized supplier implying that progress is mainly dependent on user-producer contacts rather than on knowledge flows among the machinery sector firms.

Table 1b: Regression results machinery (t-values in brackets)

dep.	const.	gap θ_i	absorptive capacity AC_i		R^2
MC_i			*SRDSL*	*SRDWORK*	(F-value)
OLS	0.148	-0.345			0.34
	(14.16)	(-12.88)			(32.02)
OLS	0.149	-0.357	-0.005		0.35
	(14.85)	(-12.98)	(-1.494)		(16.68)
OLS	0.150	-0.359		-0.002	0.36
	(14.33)	(-13.04)		(-1.841)	(17.44)
NLS	0.125	-0.249	0.296		0.37
	(9.358)	(-5.574)	(2.472)		(18.02)
NLS	0.129	-0.268		0.239	0.36
	(9.543)	(-5.633)		(1.935)	(17.90)

Similar results are found in other sectors of the German manufacturing industry (Cantner (1996)) in the French manufacturing sector for machinery, electronics and chemical products (Bernard/Cantner (1998)).

Table1c: Regression results for electronics (t-values in brackets)

dep.	const.	gap θ_i	absorptive capacity AC_i		R^2
MC_i			*SRDSL*	*SRDWORK*	F-value
OLS	0.048	-0.107			0.08
	(0.491)	(-1.45)			(2.26)
OLS	0.050	-0.107	-0.0001		0.08
	(0.497)	(-1.42)	(-0.246)		(1.17)
OLS	0.050	-0.107		-0.0001	0.08
	(0.495)	(-1.42)		(-0.229)	(1.17)
NLS	0.135	-0.318	-0.570		0.23
	(1.463)	(-2.874)	(-2.332)		(3.73)
NLS	0.107	-0.262		-0.657	0.19
	(1.103)	(-2.553)		(-1.834)	(2.93)

4.2 Comparative Macroeconomic Growth

The second group of empirical results refer to a study concerned with comparative macroeconomic growth of economies as analyzed by Cantner, Hanusch and Krüger (2000, 1999) and Cantner and Krüger (1999a, 1999b). Similar to the intra-sectoral analysis above we are here interested in a heterogeneity based on the performance differences among countries. Additionally we take into account the local character of progress and by this we explicitly consider internationally different „technological approaches" – meaning that countries differ in the technology mix they employ, where the input intensity is used as a proxy for those differences.

The data we use for these investigations are taken from Penn World Table 5.6. As input we use the labor force and the capital stock (computed by the perpetual inventory approach). Output is gross domestic product in international prices. We thus run a non-parametric frontier model with two inputs and one output.

Taking into account 87 countries, the frontier functions and their dynamics are computed for the 1960 to 1990. Figure 5 shows the world technology frontiers of the selected years 1960, 1970, 1975, and 1990.

The local character of change clearly shows up as best-practice performance increases only in the range of relatively high capital intensities – this range of capital intensities is where the G7 countries are located with the US as continuously being on the world technology frontier. In this range of capital intensity a continuous improvement of the respective frontier parts is observed. In the range of middle capital intensities the backward shift of the frontier in the late seventies and the eighties is considerable and quite obvious. Here we have mainly countries from Latin America, Northern Africa and Middle East, where Venezuela and Iran often have the leading position. At the lowest range of capital intensities we have countries from Africa; here some improvement of the frontier is to be observed which is mainly due to the development of Egypt.

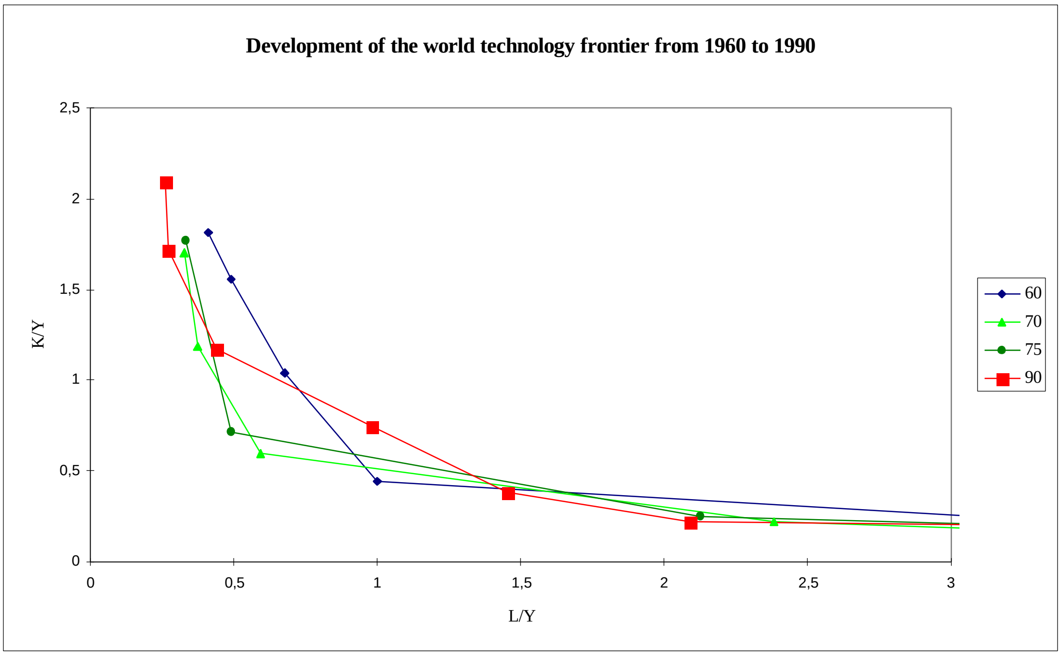


Figure 5: The world technology frontier for selected years

Figure 6 shows the development of several groups of countries. Most interesting is the development of the *Tiger states* compared to the countries from *Latin America*. During the whole period under consideration, the Tigers increase their capital intensity much more than the Latin Americas. By this they first where not able to achieve high productivities as they were falling behind the frontier in the first periods. Lateron, they have been able to catch-up to the frontier (e.g. Japan in the range of the G7 capital intensities), overtake the Latin Americas, and even come to dominate part of the world technology frontier as in the case of Hongkong.

For this phase of catching up one can distinguish two sub-groups within the Tiger states. A first one – consisting of Japan (which now is G7 but the 60s and 70s could be considered an early Tiger state) and Singapore and possibly South Korea – succeeded to achieve capital intensities as high as in the G7 countries – consequently their productivity development is rather similar to the G7 development. For this group there seems to hold the assimilation hypothesis at least for the second part of the period 1960-1990 – productivity growth and growth rates are considerably high although compared to 1960-1973 the intensity of investment slowed down.

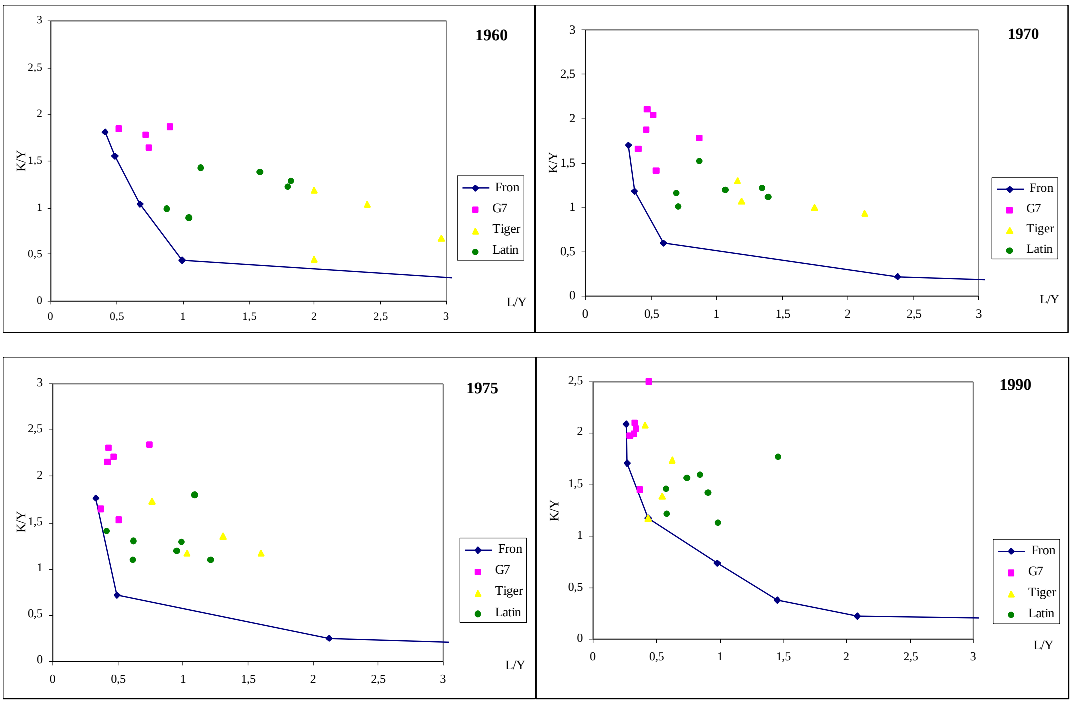


Figure 6: Tigers versus Latin Americas

A second group of Tiger states did not manage to raise capital intensity to G7 values. Those countries stick to „technologies on average" much more similar to those in Latin Americas. Here, however, they succeeded to catch up to the frontier and even to become best-practice. However the respective frontier parts do not show much technological progress and we even find considerable backward shifts. Thus, those Tigers succeeded in improving the application of relatively labor intensive technologies which did not show much technological improvements.

For explaining these developments in a third step an analysis is performed which attempts to explain the internationally different *technology levels* the countries achieved. These technology levels are computed by using the efficiency level in 1960 and then accumulating the productivity changes of each country from 1960 up to 1990. Doing this we distinguish between the productivity level, the efficiency level and the technology level, each one related to the measures M, MC and MT respectively.

As explanatory variables we used patents granted in the US, human capital, share of years open, investment ratio, etc.[26] Some selected results are stated in table 2.

26 *Patents grated* are the sum of the per capita number of patent grants for inhabitants from the country under consideration in the US over the periodd 1963-90 from the US Patent and Trademark

Table 2: Basic regressions on the technology levels

dependent variable→ regressors↓	Productivity level (M)	Efficiency level (MC)	Technology level (MT)
constant	0,50913***	0,67648***	0,47422***
	(9,595)	(11,374)	(7,640)
patents granted in the US	0,05478**	0,00898	0,08994***
	(2,606)	(0,799)	(2,989)
Human capital	0,04098***	0,03319***	0,04781***
	(4,434)	(3,340)	(4,310)
share of years open	0,17552***	0,20457***	0,02574
	(3,126)	(3,517)	(0,465)
investment ratio	-0,0082**	-0,0113***	-0,0099**
	(-2,071)	(-2,818)	(-2,377)
sample size	70	70	70
$\bar{R}^2$	0,558	0,346	0,492
RESET(3): F robust	0,1242	0,4335	0,2494
ANN test: F robust	1,3960	0,3631	2,1868*
White: F (no cross)	1,4584	4,1275***	0,1933
White: F (cross terms)	1,0721	2,6202***	0,2330
Jarque-Bera residuals	2,7820	0,5452	6,6695**

Note: t-statistics (in parentheses), the RESET and the ANN test are based on Jackknife corrected heteroscedasticity consistent covariance matrix; significance is indicated by * on 10%, ** on 5% and *** on 1% level.

Most interesting are differences in results between the efficiency and the technology levels. Patents granted are insignificant for the MC regression but significant on the 1% level in the MT regression with a much higher coefficient estimate. This implies that patents represent the amount of research activities leading to technological progress. In catching up through efficiency improvements there seems to be no strong case for activities that lead to inventions which are valuable enough to become granted in the US. For the years open to international trade we have exactly the reverse pattern. There is a substantially stronger relation between openness and the efficiency levels than between openness and the technology levels.

Office; *human capital* is the average schooling years in the total population over age 25 averaged over all six five-year values from 1960 to 1985 as reported in Barro/Lee (1993); *openess to foreign trade* is the fraction of years open to international trade between 1960 and 1990 according to a classification of Sachs/Warner (1995); *investment ratio* is the average percentage share of public and private investment in real GDP during 1960 to 1990 obtained from the Penn World Table 5.6.

Finally and contrary to the other variables, public and private investment in physical capital is significantly negative correlated with all total factor productivity levels. Positive externalities from capital accumulation seem to arise here. However, this result has to be taken with caution because the investment ratio data are the same as the ones used in the construction of the capital stocks for the non-parametric analysis and rapid accumulation of capital naturally depresses the efficiency parameter θ and also the Malmquist index.

4.3 Productivity growth – a macro-meso approach

The following results refer to a paper in preparation by Cantner/Hanusch (1999) which deals with the analysis of productivity growth for the OECD countries for the years 1970 to 1991. The analysis performed builds upon the advantage of the non-parametric frontier approach to allow the analyst to include output data in a disaggregated form. Look at equation (1) where we also have an aggregation function for outputs; the non-parametric frontier approach computes the respective aggregation weights – they are determined endogenously – and using them computes the productivity or efficiency index.

Referring to this feature of the non-parametric approach our analysis focuses on the difference between an analysis where the aggregated output is used and an analysis where output is included in a disaggregated way. In this respect we attempt to analyze whether and how „*heterogeneity below the aggregate*“ matters for the performance of the aggregate.

The data for the analysis are taken from the ISDB database of the OECD. For 13 countries[27] we run the following two computations:

1. A further on called „macro“-analysis with one output and two inputs. Output is the economy's real value added in international prices of 1990. Labor is the number of employed persons; capital is gross capital formation in prices of 1990.
2. A further on called „macro-meso“ analysis where the inputs are just the same as in the previous design. Output, however, is now disaggregated into 6 subsectors: natural resources, services, consumer goods, wood&paper, chemicals, remaining manufacturing.

These 6 subsectors have been selected in a first step in order to include all 13 countries in the analysis – which otherwise would not work. Obviously some more disaggregation as well as a focus on other subsectors would be preferable. This is future work.

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[27] These countries are Belgium (BEL), Canada (CAN), Germany (DEU), Denmark (DEN), Finland (FIN), France (FRA), the UK (GBR), Italy (ITA), Japan (JPN), the Netherlands (NDL), Norway (NOR), Sweden (SWE) and the US (USA).

For both analyses the efficiency indexes are computed for each year from 1970-1991. Then the Malmquist productivity index and its decomposition was computed for 21 years from 1971 to 1991. We here discuss mainly the results we obtain for the Malmquist computation. We obtain the following more general results:

1. *Best-practice performance*

Comparing the results of the two yearly efficiency analyses delivers that in the case of „macro-meso“ much more countries become best-practice than in „macro“. This, of course, is as expected for the non-parametric frontier analysis where the number of efficient observations (those with $\theta = 1$) increases with the number of outputs and inputs included. In the „macro“ we have for each year between 3 and 4 best-practice observations, whereas for the „macro-meso“ this number increases to between 5 and 9.

Table 3: Best-practice observations for the analyses „macro“ and „macro-meso“

	BEL	CAN	DEU	DNK	FIN	FRA	GBR	ITA	JPN	NDL	NOR	SWE	USA
Macro	89-91	70-91							70-77				70-91
Macro-Meso	75-91	70-91	70-91		80-91			73-91	70-91	80-91	70-91		70-91

For the purpose of interpretation we read this as follows: disaggregating the output real value added into subsectoral output implies to look at the specific importance each subsector has in a country's „portfolio“ – which also could be its international specialization. Take an example where several countries are compared to each other on an aggregate basis. As to figure 7 the ranking is B, C, A, ..., D,... .

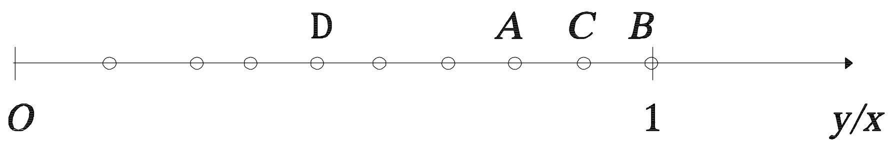

Figure 7: Ranking on the macro level

On a disaggregated basis we would get the following frontier function which in this case is a transformation function:

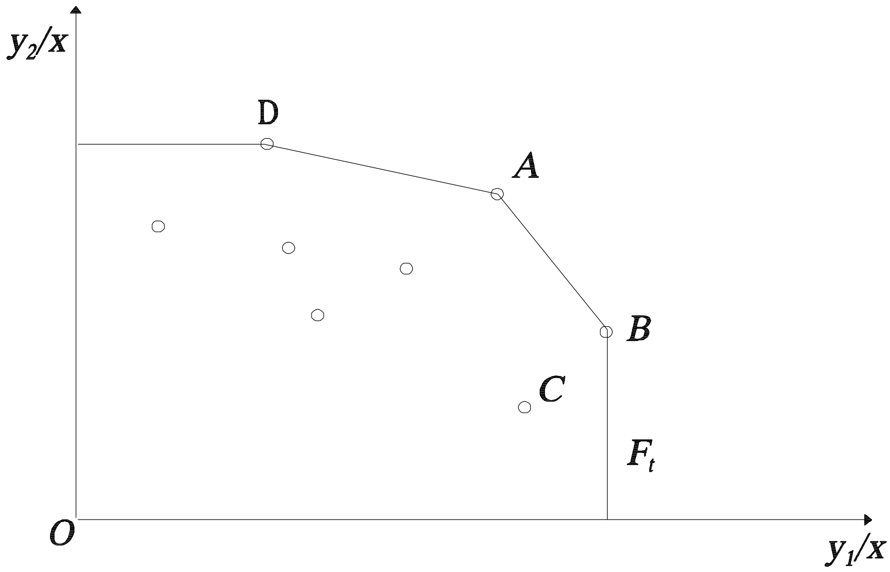

Figure 8: Performance on the disaggregated level

Obviously, there are more observations best-practice, because they differ in their output mix. Observations with an extreme output mix, just as D, become best practice and even overtake an observation which in the „macro“ performed better such as C compared to D.

Thus, if a country compared to the other countries specializes extremely in natural resource (as is the case for Norway) then it may well happen that it becomes efficient in the „macro-meso“ although it is not in the „macro“.

Consequently, the performance in the „.macro“ can be further analyzed in the „macro-meso“ where the following results may occur:

1. The result in “macro-meso” still states only below best-practice, indicating that some other country or a combination of countries have a better performance in producing just the output mix of the country under consideration. This applies to DNK, FRA, GBR and SWE.
2. The result delivers that in „macro-meso“ the country under consideration becomes best-practice. Consequently, its output mix shows some specificity to be considered further. This applies to GER and JPN which specialize in manufacturing with GER relatively capital intensive and JPN relatively labor intensive. Equivalently, NOR and FIN specialize in mining with FIN more labor intensive and NOR more capital intensive.

2. Performance dynamics

For both the „macro“ and the „macro-meso“ the Malmquist computations and the decomposition allow to look at the performance dynamics of the countries. Most interesting is to compare the results of the two settings. Take for example the average productivity changes as stated in table 4a. For countries where productivity change in „macro“ is less than in „macro-meso“ we conclude that the progress due to the specificity in the country’s output mix is larger than the overall rate of progress. Thus, the „specialization“ of the country is in sectors with relatively higher progress. This obviously holds for DNK, FIN, ITA and NDL. The contrary case, where progress in „macro-meso“ is less than in „macro“ is to be interpreted as a „specialization“ of country in less progressive sectors. This holds for BEL, DEU, GBR and JPN. For GBR with a positive macro progress, the „macro-meso“ is even negative stating that the „specialization“ contributes even negatively to overall progress. An equivalent argument holds for JPN.

Table 4a: Average productivity change

	BEL	CAN	DEU	DNK	FIN	FRA	GBR	ITA	JPN	NDL	NOR	SWE	USA
71-91													
macro	1,81	-0,26	0,86	0,85	1,05	0,97	0,55	0,76	-1,91	1,00	2,47	0,38	0,43
macro-meso	0,78	-1,10	0,13	1,38	1,79	0,97	-1,34	1,28	-2,51	2,35	2,92	0,55	0,41
71-80													
macro	2,91	0,31	0,90	0,59	1,10	1,39	0,38	1,39	-3,32	1,26	2,81	0,37	0,15
macro-meso	0,91	-0,54	0,04	1,88	2,81	1,27	-1,49	2,72	-4,82	3,71	0,62	0,32	-0,15
81-91													
macro	0,82	-0,77	0,82	1,09	1,01	0,59	0,71	0,18	-0,62	0,76	2,17	0,38	0,68
macro-meso	0,67	-1,60	0,22	0,92	0,87	0,69	-1,21	0,00	-0,37	1,12	5,05	0,77	0,92

The decomposition of the productivity change into technological progress and catching up sustains these findings. Tables 4b and 4c contain the respective changes. In table 4b the shaded cells indicate that the country under consideration is best-practice and is therefore responsible for the shift of the frontier.

For the subperiod 1981-91 the results show that there are countries which are able to manage a higher rate of technological progress in their „specialization“ than on the macro-level, such as BEL, FIN, NDL, NOR and USA. The contrary holds for CAN (regress), DEU and ITA.

Table 4b: Average technological progress

	BEL	CAN	DEU	DNK	FIN	FRA	GBR	ITA	JPN	NDL	NOR	SWE	USA
71-91													
macro	0,49	-0,26	0,37	0,28	0,37	0,30	0,24	0,36	-1,83	0,30	0,68	0,42	0,43
macro-meso	0,33	-1,18	-0,04	0,53	0,52	0,67	-1,14	0,90	-2,49	1,55	2,92	0,74	0,37
71-80													
macro	0,48	0,31	0,27	0,02	0,22	0,20	0,50	0,6	-3,22	0,13	0,39	0,29	0,16
macro-meso	0,09	-0,72	0,04	0,42	0,14	0,55	-0,91	1,8	-4,82	1,00	0,62	-0,04	-0,15
81-91													
macro	0,48	-0,77	0,46	0,52	0,51	0,39	0,02	0,46	-0,54	0,47	0,95	0,53	0,68
macro-meso	0,58	-1,60	-0,13	0,63	0,87	0,77	-,36	-0,09	-0,32	1,10	5,05	1,30	0,89

With respect to catching-up a comparison for the not best-practice countries shows again that in some cases the performance in the „macro-meso" is better than in the „macro" such as for DNK and NDL; the contrary holds for FRA, GBR and SWE. However, here one has to be careful with the interpretation because catching up is indicated also for a backward shift of the frontier.

Table 4c: Average Catching up

	BEL	CAN	DEU	DNK	FIN	FRA	GBR	ITA	JPN	NDL	NOR	SWE	USA
71-91													
macro	1,32	0	0,49	0,57	0,68	0,67	0,31	0,39	-0,09	0,69	1,78	-0,04	0
macro-meso	0,39	0	0	0,84	1,27	0,30	-0,20	0,40	0	0,79	0	-0,22	0
71-80													
macro	2,41	0	0,63	0,57	0,88	1,19	-0,11	1,13	-0,10	1,13	2,41	0,07	0
macro-meso	0,82	0	0	1,45	2,69	0,71	-0,59	0,85	0,00	1,67		0,12	0
81-91													
macro	0,33	0	0,36	0,58	0,50	0,20	0,69	-0,27	-0,08	0,29	1,21	-0,15	0
macro-meso	0	0	0	0,29	0	-0,08	0,15	0	0	0	0	-0,52	0

5. Conclusion

This position paper deals with empirical analysis in evolutionary economics in general and innovation economics as a prominent application of evolutionary ideas in particular. Within the latter, heterogeneity in the sense of different innovative activities, different production processes employed, different qualities or goods produced, is a major analytical element – even the more because innovative actors aim at creating heterogeneity and imitators attempt to reduce it again. This heterogeneity has an additional feature to be accounted for, the performance of the different techniques, activities, goods, etc. under consideration. Thus, it is not only a counting of different elements in a set but also the evaluation of these elements due to static or dynamic performance.

The task to be performed by empirical analyses contains three steps or problems: (1) Defining the heterogeneity which is analytically relevant; (2) evaluating the performance of the heterogeneous entities; (3) testing whether the structural development of the entities can be explained by evolutionary conceptions.

In this paper we focus mainly on the two first steps. The third one requires much more space and cannot be presented in an appropriate way here. With respect to steps (1) and (2) we suggest a measure and procedure which is applicable to all levels of aggregation – micro, meso and macro – and which rests on a comparison of total factor productivities of the entities under consideration. The procedure we suggests is as unrestricted as possible: in the static analysis of the non-parametric frontier function approach aiming at the identification of structures there is no restriction on the production technique employed or the output mix produced. In the dynamic analysis performed by the computation of Malmquist productivity indexes the local character of technological change is allowed to work and to be identified. By this „twin procedure“ the heterogeneity and the differences in performance, so central to innovation, can be accounted for.

By the help of three empirical analyses we show how the method suggested works and what results can de deduced. In an intra-sectoral study we focus on the stability and instability of certain technological structures. The study on macroeconomic growth throws some light on the dispute between accumulation and assimilation hypotheses concerning the East Asian Tigers. The macro-meso study finally shows how (meso-) heterogeneity below an (macro-)aggregate of countries may help to explain the differences in the macro-performance.

These examples already show what the future research agenda could look like:

On all three levels much more work has to be done especially referring to the step (3) analysis aiming at testing for evolutionary mechanisms. There the main problem is to find appropriate hypotheses to be tested. Some hypotheses are readily available, e.g. the relationship between market share dynamics and local technological progress, or spillover relationships in the international context, etc.

The third empirical example, however, shows an additional line of further research. Here we focus on the dependence of macro performance on the behavior or heterogeneity below the aggregate. In an evolutionary context, where the innovative activities of individuals or groups of individuals are the main driving force for progress on several levels of aggregation this focus seems to us of great importance. The following questions arise in this respect: What structures provide for which characteristic development? How does this development translate to the next level of aggregation? What performance is to be expected there? Which characteristic development will then be observed? How does this translate to the next level of aggregation?

Of course, the twin procedure presented provides opportunities for further development. For example stochastic elements could be included or the frontier conception chosen could be switched. An example of the latter is found in Cantner/Hanusch (1997) where we investigate a frontier function in the sense of best-practice up to period *t*.

A major problem is also the rather unrestricted form of the procedure which by definition allows as much production functions or output mixes as observations. Does this imply that a representative sample cannot be used to explain the behavior of the whole population?

Obviously, all the results presented and the future research agenda are dependent on the quality and the number of the data available. The coverage of the data with respect to the time period under consideration is one point. Another one refers to the degree to which the respective variables are an appropriate measure for the technology, the activity the outputmix under consideration.

Finally, the research we attempt to follow aims at shedding some light on the phenomenon of total factor productivity and its development. In many applications this still is a black box or residual. To achieve at a better understanding for this residual our procedure suggested might be an promising way to go.

References

Ali A.I. and C.S. Lerme (1990), Data Envelopment Analysis Models: A Framework, Working Paper, School of Management, University of Massachusetts at Amherst.

Antonelli C. (1994), The Economics of Localized Technological Change and Industrial Dynamics, Dordrecht et al.: Kluwer Academic Publishers, 1994.

Atkinson A. and J.E. Stiglitz (1969), A New View of Technological Change, Economic Journal, vol.79, 573-78.

Banker R.D., A. Charnes, W.W. Cooper (1984), Some Models for Estimating Technical and Scale Inefficienies in Data Envelopment Analysis, Management Science 30 (1984), 1078-92.

Barro R.J., J.W. Lee (1993), International Comparisons of Educational Attainment, Journal of Monetary Economics 32, 1993, 363-94.

Bernard J., U. Cantner (1999), French Regional Performance and Variety - A Non-Parametric Frontier Approach, forthcoming in: Badillo (eds.) (1999)

Bernard J., U. Cantner, G. Westermann (1996), Technological Leadership and Variety - A Data Envelopment Analysis for the French Machinery Industry, Annals of Operations Research 68, 1996, 361-377.

Bernard, J., U. Cantner (1998), Retombées technologiques, effects de rattrapage et dynamique intra-sectoriell, in: Bernard J., M. Catin (eds), Les Conditions Économiques du Changement Technologique, Paris, Montreal: L`Harmattan, 1998, 95-116.

Böhm K. (1978), Lineare Quotientenprogrammierung - Rentabilitätsoptimierung, Frankfurt a.M.: Haag+Herchen Verlag.

Cantner U. (1996), Heterogenität und technologische Spillovers - Grundelemente einer ökonomischen Theorie des technologischen Fortschritts, Habilitationsschrift, Universität Augsburg, 1996.

Cantner U. (1998), Industrial Dynamics and Structural Development: R&D, Spillovers and Absorptive Capacity, EARIE-Conference, Kopenhagen, 27.-30. August 1998

Cantner U. (2000) Heterogenität, Technologischer Fortschritt und Spillovers, forthcoming in: M. Lehmann-Waffenschmidt, H.W. Lorenz (2000).

Cantner U., A. Pyka (1998a) Absorbing technological spillovers - simulations in an evolutionary framework, Industrial and Corporate Change 7(2), 1998, 369-97

Cantner U., A. Pyka (1998b), Technological Evolution - An Analysis within the Knowledge-Based Approach, in: Structural Change and Economic Dynamics 9(1), 1998, 85-108.

Cantner U., G. Westermann (1998), Localized Technological Progress and Industrial Dynamics - An Empirical Approach, in: Antonelli C. (ed.), The Economics of Localized Technological Change, Special Issue of Economics of Innovation and New Technology, 1998.

Cantner U., G. Westermann (1998), Localized Technological Progress and Industrial Dynamics - An Empirical Approach, Economics of Innovation and New Technology 6, 1998, 121-45.

Cantner U., H. Hanusch (1996), Intrasektorale technologische Dynamik, in: Helmstädter E., G. Poser, H.J. Ramser (Hrsg.), Beiträge zur angewandten Wirtschaftsforschung, Festschrift K.H. Oppenländer, Berlin: Duncker&Humblot, 1996, 187-216.

Cantner U., H. Hanusch (1999), Productivity Development - A macro-meso approach, mimeo, University of Augsburg.

Cantner U., H. Hanusch, A Pyka (1998), Pushing Technological Progress Forward: a Comparison of Firm Strategies, in: J. Lesourne, A. Orlean (eds), Advances in Self-organization and Evolutionary Economics, London, Paris, Genéve: Economica, 1998, 114-45.

Cantner U., H. Hanusch, G. Westermann (1997), Technological Performance and Variety - The Case of the German Electronics Industry, in: Nielsen K., B. Johnson (eds), Institutions and Change - New Perspectives on Markets, Firms and Technology, Aldershot: Edward Elgar, 1997, 116-140.

Cantner U., H. Hanusch, J. Krüger (1999), Explaining International Productivity Differences, Congress of the International Joseph A. Schumpeter Society, Wien, 13.-16. Juni 1998

Cantner U., H. Hanusch, J.Krüger (2000), Total factor productivity, the East Asian Miracle and the World Production Frontier, forthcoming: Weltwirtschaftliches Archiv, 2000

Cantner U., J. Krüger (1999a), An empirical account of technological progress on the macro -level - DEA versus Growth Accounting, mimeo, University of Augsburg, 1999

Cantner U., J. Krüger (1999b), Assessing the Robustness of Malmquist Index based productivity changes and levels - an empirical investigation for 20 OECD countries, mimeo, University of Augsburg, 1999

Caves D.W., L.R. Christensen, W.E. Diewert (1982a), Multilateral Comparisions of Output, Input and Productivity Using Superlative Index Numbers, Economic Journal 92, 1982, 73-86.

Caves D.W., L.R. Christensen, W.E. Diewert (1982b), The Economic Theory of Index Numbers of the Measurement of Input, Output und Productivity, Econometrica 50, 1982, 1393-1414.

Chang K.-P. and Y.-Y. Guh (1991), Linear Production Functions and the Data Envelopment Analysis, European Journal of Operations Research, vol.52, 215-23.

Charnes A. and W.W. Cooper (1962), Programming with Linear Fractional Functionals, Naval Research Logistics Quarterly, vol.9, 181-86.

Charnes A., W.W. Cooper and E. Rhodes (1978), Measuring the Efficiency of Decision Making Units, European Journal of Operational Research, vol.2, 429-44.

David P. (1975), Technical Choice, Innovation and Economic Growth, Cambridge: Cambridge University Press.

Dosi G., (1988), Sources, Procedures, and Microeconomic Effects of Innovation, Journal of Economic Literature XXVI, 1988, 1120-71.

Färe R., S. Grosskopf, B. Lindgren, P. Roos (1994), Productivity Development in Swedish Hospitals: A Malmquist Output Index Approach, in Charnes et al. (1994), 253-72.

Färe R., S. Grosskopf, C.A.K. Lovell (1994), Production Frontiers, Cambridge, MA: Cambridge University Press, 1994.

Freeman C. and L. Soete (eds), (1990), New Explorations in the Economics of Technological Change, London, New York: Pinter Publishers.

Hanusch H. and U. Cantner (1991), Produktion Öffentlicher Leistungen, Effizienz und Technischer Fortschritt, Jahrbuch für Nationalökonomie und Statistik, vol.208(4), 369-84.

Leontief W.W. (1947), Introduction to a Theory of the Internal Structure of Functional Relationships, Econometrica, vol.15, 361-73.

Malmquist S. (1953), Index Numbers and Indifference Surfaces, Trabajos de Estatistica 4, 1953, 209-42.

Moorsteen R.H. (1961), On Measuring Productive Potential and Relative Efficiency, Quarterly Journal of Economics 75, 1961, 451-67.

Ott A.E. (1959), Technischer Fortschritt, in. Handwörterbuch der Sozialwissenschaften, Bd.10, Stuttgart, 302-16.

Pyka A., J. Krüger, U. Cantner (1999), .Twin-Peaks -What the knowledge based approach can say about the dynamics of the world income distribution, paper prepared for the European Meeting on Aplied Evolutionary Economics, June 7-9, 1999, Grenoble.

Rosenberg N. (1976), Perspectives on Technology, Cambridge: Cambridge University Press, 1976.

Sachs J.D., A. Warner (1995), Economic Reform and the Process of Global Integration, Brookings Papers on Economic Activity, no.1, 1995, 1-118.

Saviotti P.P. (1991), The role of Variety in Economic and Technological Development, in: Saviotti P.P., J.S. Metcalfe (eds) (1991), Evolutionary Theories of Economic and Technological Change, Chur: Harwood Academic Publishers, 1991, 172-208.

Saviotti P.P. (1996), Technological Evolution, Variety and the Economy, Edward Elgar, 1996.

Schumpeter J.A. (1935/1912), Theorie der Wirtschaftlichen Entwicklung, Duncker&Humblot, Berlin, 5.Auflage, 1935.

Solow R.M. (1957), Technical Change and the Aggregate Production Function, Review of Economics and Statistics 39, 1957, 312-20.

Verspagen B. (1990), Localized Technological Change, Factor Substitution and the productivity Slowdown, in: Freeman/Soete (1990).

Wolff E.N. (1992), Productivity Growth and Capital Intensity on the Sectoral and Industry Level: Specialization among OECD countries, 1970-1988, working paper, New York University, 1992.

Appendix A1: The enhanced linear program of non-parametric frontier analysis

The version of the envelopment form including possible excess inputs and output slacks reads as

$$\begin{aligned} \min \quad & \theta_l - \varepsilon e^T s_l^+ - \varepsilon e^T s_l^- \\ & s.t.: \\ & Y\lambda_l - s_l^- = Y_l \\ \theta_l X_l - {} & X\lambda_l - s_l^+ = 0 \\ & \lambda_l, s_l^+, s_l^- \geq 0 \end{aligned} \qquad \text{(A.1)}$$

A proportional reduction of inputs (as given by q_l) does not necessarily lead to efficiency in the Pareto-Koopmanns sense. In order to correct for this the remaining excess inputs (s^+) and output slacks (s^-) are taken into account in the objective function. Vector e^T contains only elements 1. (Of course, one should here distinguish two vectors e^T for inputs and output respectively which contain *s* and *i* elements respectively. To ease notation we do not take account of this. The further analysis is not affected.) ε is a positive non-Archimedean small number. Thus, additionally to *q* program (A.1) takes into account the remaining output slacks or excess inputs. Only then a clear-cut selection of efficient and inefficient observations is possible.

Appendix A2: Linear programs required for computing the Malmquist index

In computing the Malmquist-productivity index, for each observation *l* and for each periodical change four different linear programs have to be solved. In the case of $\theta^{t,t}$ and $\theta^{t+1,t+1}$ the programs being just the ones given by (3) and we will always get results obeying $\theta \leq 1$. In the case of $\theta^{t,t+1}$ the observation in period t will be compared to the frontier function of period t+1; and in the case of $\theta^{t+1,t}$ the observation in t+1 will be compared with the frontier in t. In both cases the efficiency values θ are not necessarily constrained to the interval]0,1] but they may be larger than 1. In this case technical progress would be detected.

For these four computations different linear programs are required. They are given as follows with t as the period under consideration and s as the period of the reference frontier:

$$
\begin{array}{rcrcl}
 & & \min\ \theta_l^{t,s} & & \\
 & & s.t.: & & \\
 & & Y^s \lambda_l & \geq & Y_l^t \\
\theta_l X_l^t & - & X^s \lambda_l & \geq & 0 \\
 & & \lambda_l & \geq & 0
\end{array}
\qquad (A.2)
$$

With these programs T-1 index number can be computed for all observations, with T being the length of the period under investigation.