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Identification of Coulomb blockade and macroscopic quantum tunneling by noise

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Abstract. – The effects of Macroscopic Quantum Tunneling (MQT) and Coulomb Blockade (CB) in Josephson junctions are of considerable significance both for the manifestations of quantum mechanics on the macroscopic scale and potential technological applications. These two complementary effects are shown to be clearly distinguishable from the associated noise spectra. The current noise is determined exactly and a rather sharp crossover between flux noise in the MQT and charge noise in the CB regions is found as the applied voltage is changed. Related results hold for the voltage noise in current-biased junctions.

Generally, noise is considered undesirable and one searches for ways to suppress it. However, occasionally the observation of noise may provide valuable information. The presence of shot noise in electrical transport indicates the discreteness of the charge carriers and the ratio between noise and current directly measures their charge. This fact was exploited to demonstrate the fractional charge in the fractional quantum Hall effect [1, 2].

Noise may also be helpful in identifying a transport mechanism. Tunnel junctions often display linear current-voltage characteristics and are therefore indistinguishable from an ohmic resistor if only the current is measured. On the other hand, noise measurements exhibit clear differences. One finds shot noise in the first and Nyquist noise in the second case corresponding to discrete and continuous charge transport, respectively.

An even more interesting situation arises, when different physical mechanisms can occur as is the case for ultrasmall Josephson junctions. Such systems have been proposed as building blocks for quantum computers [3] and the operation of a superconducting box containing such a tunnel junction as a qubit has been demonstrated [4].

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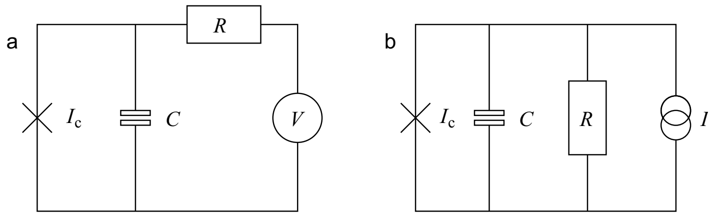


Fig. 1 – An ultrasmall Josephson junction is characterized by its critical current I_c and a capacitance C. a) Junction and ohmic resistor in series are voltage-biased. b) Junction and ohmic resistor in parallel are current-biased.

For a single ultrasmall Josephson junction at low temperatures, it has been theoretically predicted that one may change from transport dominated by macroscopic quantum tunneling (MQT) to the regime of Coulomb blockade (CB) just by changing the applied voltage [5]. These two regimes are qualitatively different as in MQT the phase difference across the Josephson junction is a good quantum variable while CB is governed by the conjugate charge variable. We propose to study the noise properties in order to experimentally identify the transport mechanism.

We will discuss the noise properties of a voltage-biased as well as a current-biased small Josephson junction with the effective circuits shown in figs. 1a and b, respectively. The Josephson junction may be characterized by its critical current I_c and its capacitance C which lead to two energy scales governing the behavior of the junction. The Josephson energy $E_\mathrm{J} = \hbar I_\mathrm{c}/2e$ determines the probability of Cooper pair tunneling while $E_\mathrm{c} = (2e)^2/2C$ is the charging energy of a capacitor carrying just one Cooper pair. The resistance $R = \rho R_\mathrm{Q}$ of the external resistor may be taken relative to the resistance quantum $R_\mathrm{Q} = h/4e^2$. Typically, the resistance will be small, *i.e.* $\rho \ll 1$. In the following, we will be interested in the behavior of the junction at voltages of the order of RI_c much smaller than the superconducting gap. Therefore, quasiparticle excitations may be neglected.

Exact results for the current-voltage characteristics and current noise are known [6–8] for some one-dimensional systems with ohmic dissipation corresponding to an ideal ohmic resistor. However, as can be seen from fig. 1, the external resistance is cut off by the junction's capacitance at high frequencies. For typical lead resistances, the cutoff frequency $1/RC$ is much larger than the frequency $(2e/\hbar)RI_\mathrm{c}$ corresponding to the typical voltages of interest and the assumption of an ohmic resistor is sufficient for these voltages. This implies an overdamped junction characterized by a McCumber parameter $\beta_\mathrm{c} = (2e/\hbar)R^2 I_\mathrm{c} C \ll 1$. In the following, we will focus on the overdamped regime.

We start with a discussion of the voltage-biased case (fig. 1a). In the overdamped limit and $\rho < 1$, the zero-temperature current-voltage characteristic displays an almost linear rise of the current for small voltages reflecting the fact that nearly the entire applied voltage drops across the external resistor. There are, however, deviations due to macroscopic quantum tunneling which causes phase slips by quantum tunneling of the phase accoss the barrier of the Josephson potential. This is responsible for the voltage drop across the junction captured by a perturbation theory in $E_\mathrm{c}/E_\mathrm{J}$ yielding the current-voltage characteristics [5]

$$\langle I_\mathrm{J}\rangle = \frac{V}{R}\left[1-\sum_{n=1}^{\infty} c_n(1/\rho)\left(\Gamma(1+\rho)^{1/\rho}\frac{\mathrm{e}^{\gamma}}{\pi^2\rho^2}\frac{E_\mathrm{c}}{E_\mathrm{J}}\right)^{2n}\left(\frac{V}{RI_\mathrm{c}}\right)^{2(1/\rho-1)n}\right], \tag{1}$$

where

$$c_n(\rho) = (-1)^{n-1} \frac{\Gamma(1+\rho n)\Gamma(3/2)}{\Gamma(1+n)\Gamma(3/2+(\rho-1)n)} \tag{2}$$

and $\gamma = 0.577\ldots$ is the Euler constant.

On the other hand, perturbation theory in $E_\mathrm{J}/E_\mathrm{c}$ yields the current-voltage characteristics

$$\langle I_\mathrm{J} \rangle = \frac{V}{R} \sum_{n=1}^{\infty} c_n(\rho) \left(\frac{\pi^2 \rho^2 \mathrm{e}^{-\gamma}}{\Gamma(1+\rho)^{1/\rho}} \frac{E_\mathrm{J}}{E_\mathrm{c}} \right)^{2n\rho} \left(\frac{V}{RI_\mathrm{c}} \right)^{-2(1-\rho)n}, \tag{3}$$

which describes incoherent tunneling of Cooper pairs across the oxide layer of the Josephson junction. The leading-order behavior $I \sim V^{2\rho-1}$ is typical for Coulomb blockade which for $\rho > 1$ manifests itself in a suppression of the current at low voltages. For $\rho < 1$, this term would correspond to a divergence at zero voltage and then (3) can only be valid for not too small voltages.

In fact, the two series have a finite radius of convergence. For $\rho < 1$, the expansions (1) and (3) converge for low and high voltages, respectively. They join smoothly and provide a full description of a peaked current-voltage characteristic. To the left of the peak, transport is therefore based on macrosopic quantum tunneling, while to the right of the peak, we find the regime of Coulomb blockade. The two regimes, even though the underlying physics is very different, are related to each other by a duality transformation [6–8]. A typical example for the current-voltage characteristics is depicted in fig. 2a for a Josephson junction with $E_\mathrm{c} = E_\mathrm{J}$ and a small environmental resistance $R = 0.1R_\mathrm{Q}$. The peak at a voltage of order RI_c is a remnant of the dc Josephson effect of a classical Josephson junction.

The question now arises how to identify the two transport regimes without making use of the theoretical results. We argue that a suitable way to achieve this goal is the observation of current noise

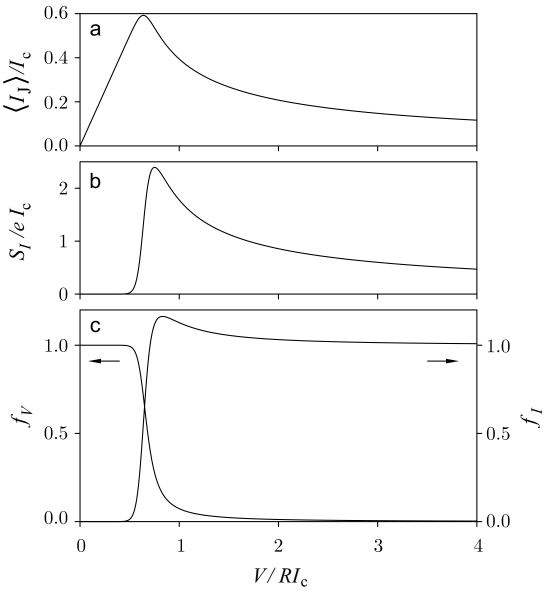
$$S_I = \int_{-\infty}^{+\infty} \mathrm{d}t \langle \delta I_\mathrm{J}(t) \delta I_\mathrm{J}(0) + \delta I_\mathrm{J}(0) \delta I_\mathrm{J}(t) \rangle \,. \tag{4}$$

Here, δI_J denotes the deviation of the current I_J from its mean value $\langle I_\mathrm{J} \rangle$. The noise may be determined by following the same line of reasoning employed previously to calculate the noise in fractional quantum Hall bars [6–8]. The time evolution of the density matrix may be written as a path integral on the Keldysh contour including an auxiliary field coupling to the current operator. Arbitrary current expectation values are then determined as functional derivatives of the path integral. Concrete results like the series (1) and (3) for the I-V curves may be obtained in the so-called Coulomb gas representation [9] of the real-time path integral. Second-order functional derivatives allow to determine the noise properties.

In the limit of zero frequency, the results can be expressed in closed form and one obtains for the current noise

$$S_I = \frac{2eV}{1-\rho}(G - G_\mathrm{d}). \tag{5}$$

Apart from the external voltage V and the dimensionless resistance ρ, the noise depends on the difference of absolute and differential conductance, $G = \langle I_\mathrm{J} \rangle / V$ and $G_\mathrm{d} = \partial \langle I_\mathrm{J} \rangle / \partial V$, where $\langle I_\mathrm{J} \rangle$ is the time-averaged current. Note that in the case of an ideal supercurrent the external voltage drops entirely across the resistor. Then, the current-voltage characteristic is linear in the external voltage and the noise vanishes due to the fully coherent transport of Cooper pairs. The result (5) allows us to obtain the current noise in the middle panel of fig. 2 from the current-voltage characteristic shown in the upper panel. The current noise in fig. 2c has been plotted as two different Fano factors. As will be explained in the following, these Fano factors are appropriate to identify the transport mechanisms.



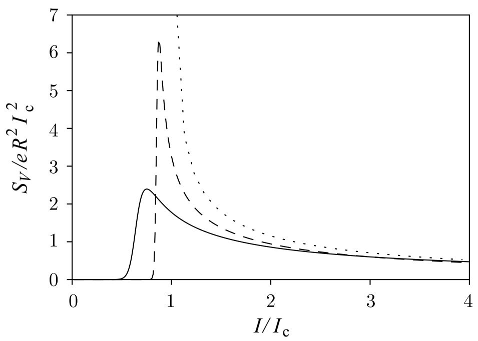


Fig. 2

Fig. 3

Fig. 2 – Current and current noise have been calculated for an ultrasmall Josephson junction with $E_c = E_J$ and an external resistance $R = 0.1R_Q$. a) Current-voltage characteristic. b) Current noise as a function of the voltage bias. c) Fano factors for flux noise (left scale) and charge noise (right scale) appropriate in the MQT and CB regime, respectively.

Fig. 3 – The voltage noise (10) as a function of the bias current is shown for a junction with $E_c = E_J$ and two external resistances $\rho = 0.1$ (full line) and $\rho = 0.02$ (dashed line). The result (12) in the limit $\rho \to 0$ is represented by the dotted line.

The current noise both in the CB and MQT regimes may be understood in terms of Poissonian shot noise where transport of the appropriate quantity occurs at uncorrelated random times. The shot noise is given by the product of the transported quantity and the corresponding current. In the CB regime, it is the charge flow of Cooper pairs which obeys Poissonian statistics. The current noise, $S_I = 4e\langle I_J \rangle$, is therefore proportional to the charge $2e$ of a Cooper pair and the average current $\langle I_J \rangle$ through the Josephson junction. The Fano factor

$$f_I = \frac{S_I}{4e\langle I_J \rangle} \tag{6}$$

plotted in fig. 2c clearly confirms the assumption of shot noise since it is very close to one in the CB region. On the other hand, in the regime of MQT, the charge flow becomes continuous and the corresponding shot noise is strongly suppressed. The change from the MQT to the CB regime is indicated by a remarkably sharp rise of the Fano factor f_I.

In the case of MQT, occasional phase slips lead to a voltage drop $V_J = (\hbar/2e)\dot{\varphi}$ across the junction and to voltage noise

$$S_V = \int_{-\infty}^{+\infty} \mathrm{d}t \langle \delta V_J(t) \delta V_J(0) + \delta V_J(0) \delta V_J(t) \rangle \,. \tag{7}$$

Since $V_{\mathrm{J}} = V - RI_{\mathrm{J}}$ and the external voltage does not fluctuate, the current noise is determined by (7) via $S_I = S_V/R^2$. The assumption of Poissonian statistics of the phase slips then allows us to evaluate the current noise. During a phase slip the phase changes by 2π leading to an integrated voltage pulse $h/2e$. The current noise thus becomes

$$S_I = \frac{1}{R^2}\frac{h}{e}\langle V_{\mathrm{J}}\rangle\,. \tag{8}$$

The corresponding Fano factor

$$f_V = \frac{eR^2}{h}\frac{S_I}{\langle V_{\mathrm{J}}\rangle} \tag{9}$$

therefore allows to identify MQT as is shown in the left part of fig. 2c. In contrast, in the CB regime the phase is strongly fluctuating and shot noise due to phase slips can no longer be detected. Again, the crossover between the two regimes is very distinct.

The results discussed so far for the voltage-biased case may be rewritten for a current-biased junction (fig. 1b). The voltage-biased case with applied voltage V and current I_{J} through the junction can be transformed to the current-biased case with applied current I and voltage drop V_{J} across the junction by means of the relations $I = V/R$ and $V_{\mathrm{J}} = V - RI_{\mathrm{J}}$. Then, the current noise (5) turns into voltage noise

$$S_V = \frac{2eRI}{1-\rho}\left(\frac{\partial\langle V_{\mathrm{J}}\rangle}{\partial I} - \frac{\langle V_{\mathrm{J}}\rangle}{I}\right), \tag{10}$$

which depends on the difference of the differential and the absolute resistance of the resistively shunted junction.

It is instructive to make connection to results known in the limit $\rho \to 0$, where the current-voltage characteristics for $I > I_{\mathrm{c}}$ is given by [10,11]

$$V_{\mathrm{J}} = R\left(I^2 - I_{\mathrm{c}}^2\right)^{1/2}. \tag{11}$$

From (10) one therefore finds for the voltage noise

$$S_V = 2eR^2 I_{\mathrm{c}}^2\left(I^2 - I_{\mathrm{c}}^2\right)^{-1/2} \tag{12}$$

in agreement with the results of ref. [12]. While this result diverges when I approaches the critical current I_{c}, the expression (10) for $\rho > 0$ yields a well-behaved voltage noise for the entire range of applied currents. Figure 3 compares the voltage noise according to (10) for a junction with $E_{\mathrm{c}} = E_{\mathrm{J}}$ and finite shunt resistance R corresponding to $\rho = 0.1$ and $\rho = 0.02$ with the result (12) for $\rho \to 0$. The divergence at $I = I_{\mathrm{c}}$ associated with the kink in the voltage-current characteristics (11) is smoothed as the external resistance increases.

In conclusion, we have studied noise properties of voltage-biased small junctions, which have been the subject of recent experimental investigations [13], as well as of the more standard current-biased junctions employed in SQUID technology [14]. Even though we started from analytical results valid for the overdamped limit at zero temperature, the reasoning leading to the Fano factors was completely independent of these results. They were only needed to confirm the validity of the assumption of Poissonian statistics for charge transport and phase slips. One may therefore conclude that the observation of noise allows to determine the transport mechanism independently of a theoretical result for the current-voltage characteristics and the current noise. As a consequence, noise measurements may well be useful to identify the transport mechanism beyond the overdamped limit and the limit of zero temperature.

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