Optimal Design of High Power Electronic Devices by Topology Optimization

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Abstract. High power electronic devices such as converter modules are frequently used as electric drives for high power electromotors. The efficient and reliable operating behaviour of such devices requires an optimal design with regard to a minimization of power losses due to parasitic inductivities caused by eddy currents. The mathematical modelling gives rise to a topology optimization problem where the state variables are required to satisfy the quasistationary limit of Maxwell's equations and the design variables are subject to both equality and inequality constraints. Based on appropriate finite element approximations involving domain decomposition techniques, the discretized optimization problem is solved by a primal-dual Newton interior-point method.

1 Introduction

The design of innovative high power electronic devices and systems based on the pulse width modulation technique has become an industrially relevant issue in recent years due to the wide range of applications. In particular, pulsed DC-AC converter modules are used both for energy generation and/or transmission and as electric drives for high power electromotors in public transportation systems such as trams and, as the most spectacular example, in high speed trains (cf. Fig. 1). Such converter modules consist of modern fast switching semiconductor devices, e.g. IGBTs (Insulated Gate Bipolar Transistors) or GTOs (Gate Turn-Off Thyristors) interconnected and linked with the power source and load by copper made bus bars (see Fig. 2). Due to the use of the IGBTs or GTOs, switching times of less than 100 ns and switched currents of one up to five kA can be realized. However, as a side-effect of the fast switching times and steep current ramps, eddy currents build up inside the bus bars that lead to parasitic inductivities causing possible overvoltages and significant power losses (cf. [9,11]). Therefore, one of the prime objectives of the electrical engineers is to design the bus bars in such a way that the total inductivity is minimized. It is known that the geometrical shape and topology of the bus bars play a prominent role in so far as they have a significant impact on the distribution and size of the generated eddy

Fig. 1. Applications of high power converter modules: Energy generation (left), public transportation (top right), high speed trains (bottom right)

currents. Consequently, the task is to distribute the material in an optimal way. From a mathematical point of view, the problem can be stated as a topology optimization problem with constraints on the state and design variables. Here, the state variables are the generated electromagnetic fields, or associated potentials, and the design variable is chosen as the conductivity of the material. In this paper, we will present a primal-dual Newton-type interior-point method for the solution of the topology optimization problem (Sect. 4). Since this algorithm requires the frequent solution of the underlying field equations, we need advanced numerical solution methods for their efficient computation. In particular, we will discuss in some detail multilevel and domain decomposition techniques using adaptive curl-conforming edge element discretizations (Sect. 3). These are applied to potential equations for the scalar electric potential and the magnetic vector potential which can be derived from the quasi-stationary limit of Maxwell's equations as shown in Sect. 2. Finally, in Sect. 5 we will document the results of numerical computations that can lead to a considerable reduction of the parasitic inductivities.

2 Parasitic Inductivities in Converter Modules

We consider a converter module consisting of N bus bars Ω_ν, $1 \leq \nu \leq N$, and M IGBTs (Insulated Gate Bipolar Transistors) connecting a power source with the load (cf. Fig. 2 (left)). Each bus bar contains N_ν ports $T_{\nu\alpha}$, $1 \leq \alpha \leq N_\nu$, where currents are either supplied or taken off the bar (cf. Fig. 2 (right); the ports are marked by different colours). During operation of the module, eddy currents are generated that can be described by the quasistationary limit of Maxwell's equations

$$\frac{\partial \mathbf{B}}{\partial t} + \operatorname{curl} \mathbf{E} = \mathbf{0} \ , \quad \operatorname{div} \mathbf{B} = 0 \ , \quad \operatorname{curl} \mathbf{H} = \mathbf{J} \ , \tag{1}$$

$$\mathbf{B} = \mu \mathbf{H} \ , \quad \mathbf{J} = \sigma \mathbf{E} \ . \tag{2}$$

Here, $\mathbf{E}$ and $\mathbf{H}$ denote the electric and the magnetic field, $\mathbf{B}$ and $\mathbf{J}$ stand for the magnetic induction and the current density, μ is the magnetic permeability and σ refers to the electric conductivity (cf. [1] for a justification of the quasistationary limit in the computation of eddy currents).

We use a potential formulation by introducing a scalar electric potential φ and a magnetic vector potential $\mathbf{A}$ according to

$$\mathbf{E} = -\operatorname{grad} \varphi - \frac{\partial \mathbf{A}}{\partial t} \quad , \quad \mathbf{B} = \operatorname{curl} \mathbf{A}$$

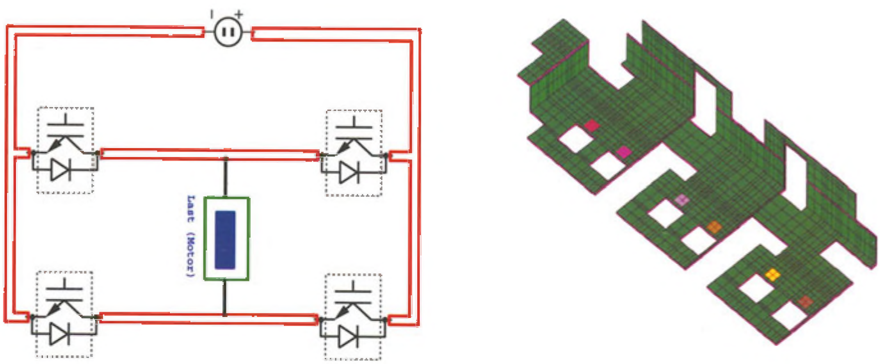

Fig. 2. Converter Module (left); Geometry of a bus bar (right)

(cf., e.g., [6]). For the electromagnetic potentials φ and $\mathbf{A}$, from (1),(2) we obtain a coupled system of PDEs consisting of an elliptic boundary value problem

$$\mathrm{div}\,(\sigma\ \mathrm{grad}\ \varphi) \;=\; 0 \quad \text{in} \quad \Omega\ , \tag{3}$$

$$\sigma\,\mathbf{n}\cdot\mathrm{grad}\ \varphi \;=\; \begin{cases} -I_{\nu\alpha}(t) & \text{on } T_{\nu\alpha} \\ 0 & \text{else} \end{cases} \tag{4}$$

with $\sum_{T_{\nu\alpha}} I_{\nu\alpha} = 0$, and a parabolic PDE

$$\sigma\frac{\partial\,\mathbf{A}}{\partial\,t} \;+\; \mathrm{curl}\,\mu^{-1}\,\mathrm{curl}\,\mathbf{A} \;=\; \begin{cases} -\,\sigma\,\mathrm{grad}\,\varphi & \text{in } \Omega \\ 0 & \text{in } \mathbf{R}^3\setminus\Omega \end{cases} \tag{5}$$

with appropriate initial and boundary conditions. Note that (5) has to be considered in the interior and exterior domain.

The total inductivity is given by

$$L_{\mathrm{tot}} := \left(\sum_{\nu,\alpha}\sum_{\mu,\beta}\int_0^T |\,L_{\nu\alpha,\mu\beta}(t)\,|^2\,dt\right)^{1/2} \tag{6}$$

with the generalized transient inductivity coefficients

$$L_{\nu\alpha,\mu\beta}(t) := \sigma^{-1}\int_{\Omega_\nu}\mathbf{J}_{\nu\alpha}(x)\cdot\mathbf{S}(t)\mathbf{J}_{\mu\beta}(x)\,dx$$

where $J_{\nu\alpha}$ denotes the current associated with the bar Ω_ν at the port $T_{\nu\alpha}$ and $\mathbf{S}(\cdot)$ is the solution operator associated with (5).

3 Numerical Solution of the Field Equations

Structural optimization algorithms with PDE constraints like the primal-dual Newton interior-point method to be described in the subsequent section require the frequent solution of the PDEs, i.e., in this context the solution of the equations (3),(4) and (5) for the electric potential φ and the magnetic vector potential $\mathbf{A}$. Therefore, we have to provide efficient numerical tools for their iterative solution. For that purpose we will use adaptive multilevel and domain decomposition methods based on curl-conforming edge element discretizations for the computation of the magnetic vector potential $\mathbf{A}$ and on nonconforming P1 approximations for the electric potential φ.

Discretizing the interior domain problem associated with (5) implicitly in time by the backward Euler scheme, at each time step $t_m := t_{m-1} + \Delta t$ we are faced with an elliptic boundary value problem for the double curl-operator whose variational formulation takes the form:

Find $\mathbf{j}^m \in \mathbf{V} \subset H(\mathbf{curl};\Omega) := \{\mathbf{q} \in L^2(\Omega)^3 \mid \mathrm{curl}\,\mathbf{q} \in L^2(\Omega)^3\}$ such that

$$a(\mathbf{j}^m, \mathbf{q}) = \ell(\mathbf{q}) \quad , \quad \mathbf{q} \in \mathbf{V} \ , \tag{7}$$

where

$$a(\mathbf{j}, \mathbf{q}) := \int_\Omega [\Delta t\, \mu^{-1} \mathbf{curl}\,\mathbf{j} \cdot \mathbf{curl}\,\mathbf{q} \ + \ \sigma \mathbf{j} \cdot \mathbf{q}]\, dx \quad , \quad \mathbf{j}, \mathbf{q} \in \mathbf{V} \ ,$$

$$\ell(\mathbf{q}) := \int_\Omega \sigma\, [\mathbf{j}^{m-1} \cdot \mathbf{q} \ - \ \Delta t\, \mathbf{grad}\, \varphi_m \cdot \mathbf{q}]\, dx \quad , \quad \mathbf{q} \in \mathbf{V} \ .$$

Edge elements, originally due to Whitney, have been systematically introduced into the finite element methodology by Nédélec (cf. [18,19]) and have become an appropriate tool in the computation of electromagnetic fields (see e.g. [7,8,10]).

We consider the lowest order edge elements with respect to a simplicial triangulation $\mathcal{T}_h$ of the computational domain Ω

$$Nd_1(K) \ := \ \{\mathbf{q} = \mathbf{a} + \mathbf{b} \wedge \mathbf{x} \mid \mathbf{a}, \mathbf{b} \in \mathbf{R^3}\} \quad , \quad K \in \mathcal{T}_h$$

with the degrees of freedom given by the tangential components with respect to the edges E_ν of K

$$\ell_{E_\nu}(\mathbf{q}) \ := \ \int_{E_\nu} \mathbf{t}_{E_\nu} \cdot \mathbf{q}\, d\sigma \quad , \quad 1 \le \nu \le 6 \ .$$

The global edge element space

$$Nd_1(\Omega; \mathcal{T}_h) \ := \ \{\mathbf{q} : \Omega \to \mathbf{R^3} \mid \mathbf{q}\,|_K \in Nd_1(K)\, ,\, K \in \mathcal{T}_h\}$$

is then a proper subspace of $H(\mathbf{curl};\Omega)$ and we choose $\mathbf{V}_h := Nd_1(\Omega;\mathcal{T}_h) \cap \mathbf{V}$. The problem with the solution of the finite dimensional analogue of (7)

$$a(\mathbf{j}_h^m, \mathbf{q}_h) = \ell(\mathbf{q}_h) \quad , \quad \mathbf{q}_h \in \mathbf{V}_h \ , \tag{8}$$

is that the curl-operator has a nontrivial kernel which is the subspace of irrotational vector fields. We take advantage of the fact that in the discrete regime this very subspace can be identified with $\mathbf{grad}\, V_h$ for some $V_h \subset S_1(\Omega;\mathcal{T}_h)$, where $S_1(\Omega;\mathcal{T}_h)$ is the finite element space associated with the standard conforming P1 approximation. To be more precise, we can set up the following hybrid iterative scheme:

Given some iterate $\mathbf{j}_h^{m,\nu} \in \mathbf{V}_h\, ,\, \nu \in \mathbf{N}_0$, we first perform some SOR sweeps on (8) resulting in $\tilde{\mathbf{j}}_h^{m,\nu}$ and consider the defect correction problem on the subspace of irrotational vector fields

$$\int_{\Omega} \sigma \, \mathbf{grad}\, u_h^m \cdot \mathbf{grad}\, v_h \, d\mathbf{x}$$
$$= \ell(\mathbf{grad}\, v_h) - a(\tilde{\mathbf{j}}_h^{m,\nu}, \mathbf{grad}\, v_h) \quad , \quad v_h \in V_h \; . \tag{9}$$

Performing some SOR sweeps on (9) gives $\tilde{u}_h^m$, and we obtain $\mathbf{j}_h^{m,\nu+1} := \tilde{j}_h^{m,\nu} + \mathbf{grad}\, \tilde{u}_h^m$.

In a multilevel framework, having a hierarchy $(\mathcal{T}_k)_{k=0}^{\ell}$ of simplicial triangulations at our disposal, the above hybrid iteration is used both as a smoother on all levels $1 \le k \le \ell$ and as an iterative solver on the coarsest grid $k = 0$ whereas the intergrid transfers are handled in a canonical way (cf. [12] for a detailed analysis and [2] for various applications).

Mortar edge element methods (cf. [4,13]) are appropriate for an FEM-FEM coupling of the interior and exterior domain problems as well as for a doamin decomposition approach to the interior domain problem with respect to a nonoverlapping geometrically conforming decomposition

$$\bar{\Omega} = \bigcup_{i=1}^{n} \bar{\Omega}_i \quad , \quad \Omega_i \cap \Omega_j = \emptyset \quad , \quad i \neq j$$

according to the geometrical structure of the bus bars. We refer to the union of the interfaces between adjacent subdomains

$$S := \bigcup_{i \neq j} \Gamma_{ij} \quad , \quad \Gamma_{ij} = \bar{\Omega}_i \cap \bar{\Omega}_j$$

as the skeleton of the decomposition.

We further consider individual simplicial triangulations of the subdomains Ω_i, $1 \le i \le n$, allowing nonconforming nodal points on the interfaces $\Gamma_{ij} \subset S$ and discretize the subdomain problems by the lowest order curl-conforming edge elements. In order to guarantee consistency of the overall approximation, we have to impose weak continuity constraints on the skeleton S which can be realized by appropriately chosen Lagrangian multipliers. We denote by $\mathbf{V}_h(\Omega_i)$, $1 \le i \le n$, the subdomain based edge element spaces and by $\mathbf{M}_h(\Gamma_{ij})$, $\Gamma_{ij} \subset S$, the local multiplier spaces. We define the product spaces

$$\mathbf{V}_h(\Omega) := \prod_{i=1}^{n} \mathbf{V}_h(\Omega_i) \quad , \quad \mathbf{M}_h(S) := \prod_{\Gamma_{ij} \subset S} \mathbf{M}_h(\Gamma_{ij}) \tag{10}$$

as well as bilinear forms $a_h : \mathbf{V}_h(\Omega) \times \mathbf{V}_h(\Omega) \to \mathbf{R}$

$$a_h(\mathbf{u}_h, \mathbf{v}_h) := \sum_{i=1}^{n} a_{\Omega_i}(\mathbf{u}_h, \mathbf{v}_h) \quad , \quad \mathbf{u}_h, \mathbf{v}_h \in \mathbf{V}_h(\Omega) \; , \tag{11}$$

where $a_{\Omega_i} := a \mid_{\Omega_i}$, and $b_h : \mathbf{V}_h(\Omega) \times \mathbf{M}_h(S) \to \mathbf{R}$

$$b_h(\mathbf{q}_h, \boldsymbol{\mu}_h) \ := \ \sum_{\Gamma_{ij} \subset S} \int_{\Gamma_{ij}} \boldsymbol{\mu}_h \cdot [\mathbf{n} \wedge \mathbf{q}_h] \mid_{\Gamma_{ij}} \, ds \tag{12}$$

for edge element discretizations where $[\mathbf{n} \wedge \mathbf{q}_h] \mid_{\Gamma_{ij}}$ denote the jumps of the tangential components of $\mathbf{q}_h$ across the interfaces $\Gamma_{ij} \subset S$.

The domain decomposition approaches give rise to the discrete saddle point problems:

Find $(\mathbf{u}_h, \lambda_h) \in \mathbf{V}_h(\Omega) \times \mathbf{M}_h(S)$ such that

$$a_h(\mathbf{u}_h, \mathbf{v}_h) \ + \ b_h(\mathbf{v}_h, \lambda_h) = \ell(\mathbf{v}_h) \quad , \quad \mathbf{v}_h \in \mathbf{V}_h(\Omega) \ , \tag{13}$$

$$b_h(\mathbf{u}_h, \mu_h) \qquad = \quad 0 \quad , \quad \mu_h \in \mathbf{M}_h(S) \ . \tag{14}$$

In order to guarantee the ellipticity of the bilinear form $a_h(\cdot,\cdot)$ on $\operatorname{Ker} B_h$

$$a_h(\mathbf{v}_h, \mathbf{v}_h) \ \geq \ \alpha \, \|\mathbf{v}_h\|^2_{\mathbf{V}_h(\Omega)} \quad , \quad \mathbf{v}_h \in \operatorname{Ker} B_h \ , \ \alpha > 0 \ , \tag{15}$$

where $B_h : \mathbf{V}_h(\Omega) \to (\mathbf{M}_h(S))^*$ is the operator associated with $b_h(\cdot,\cdot)$ as well as the discrete inf-sup condition (LBB-condition)

$$\inf_{\mu_h \in \mathbf{M}_h(S)} \ \sup_{\mathbf{v}_h \in \mathbf{V}_h(\Omega)} \ \frac{b_h(\mathbf{v}_h, \mu_h)}{\|\mathbf{v}_h\|_{\mathbf{V}_h(\Omega)} \ \|\mu_h\|_{\mathbf{M}_h(S)}} \ \geq \ \beta \ > \ 0 \ , \tag{16}$$

the multiplier spaces $\mathbf{M}_h(\Gamma_{ij})$, $\Gamma_{ij} \subset S$, have to be chosen in an appropriate way. This can be done by a suitable modification of the basis functions associated with edges in the interior of Γ_{ij} that have neighboring edges on $\partial\Gamma_{ij}$. The modification consists in adding a convex cobination of the basis functions corresponding to the neighbors on $\partial\Gamma_{ij}$. For details of the construction as well as a verification of (15) and (16) we refer to [4,13].

The numerical solution of (13),(14) has again to take into account a defect correction in subspaces of irrotational vector fields which now has to be adapted to the domain decomposition setting. We refer to [14] for details as well as to [3,14] for the realization of grid adaptation strategies based on efficient and reliable residual type a posteriori error estimators.

4 Minimization of the Total Inductivity by Topology Optimization

Significant power losses during operation of the converter module can be avoided by a reduction of the parasitic inductivities that can be achieved by an optimal distribution of the material in the bus bars. Such topology optimization problems are well-known in structural mechanics (cf., e.g., [5] and the references therein), but have not yet been investigated in the framework of an optimal design of high power electronic devices.

We use the conductivity σ as a material parameter and choose an SIMP-methodology (Simple Isotropic Material with Penalization) where we consider a scaled version of the potential equations and their discrete analoga, respectively, with $\eta(\sigma) := ((\sigma - \sigma_{min})/(\sigma_{max} - \sigma_{min}))^q$ and an appropriately chosen $q \in \mathbf{N}$ (e.g., $q = 2$) for a penalization of intermediate conductivities. Here, σ_{max} stands for the conductivity of the basic material (copper) and $0 < \varepsilon =: q_{min} \ll 1$ to avoid difficulties due to a loss in ellipticity.

The potential equations are discretized implicitly in time by the backward Euler scheme and by Nédélec's lowest order curl-conforming edge elements resp. the nonconforming P1 approximation in space with respect to a simplicial triangulation $\mathcal{T}_h$ as described in the previous section. Denoting the discretized total inductivity by L_h and comprising the discrete state variables φ_h and $\mathbf{A}_h$ in a single vector $\mathbf{u}_h := (\varphi_h, \mathbf{A}_h)^T$ for the sake of notational convenience, we are thus led to the following nonlinear minimization problem:

Find σ_h^*, $\mathbf{u}_h^*$ such that

$$L_h(\sigma_h^*, \mathbf{u}_h^*) = \min_{\sigma_h, \mathbf{u}_h} L_h(\sigma_h, \mathbf{u}_h) \tag{17}$$

subject to the equality constraints

$$\mathbf{C}_h(\sigma_h)\,\mathbf{u}_h = \mathbf{b}_h \ , \tag{18}$$

$$g(\sigma_h) := \sum_{K \in \mathcal{T}_h} \text{meas}\,(K)\,\sigma_h\,|_K = C \tag{19}$$

and the inequality constraints

$$\sigma_{min}\,\mathbf{e}_h \le \sigma_h \le \sigma_{max}\,\mathbf{e}_h \ , \tag{20}$$

where $\mathbf{e}_h := (e_{h,i})_{i=1}^{d_h}$, $e_{h,i} = 1$, $1 \le i \le d_h := \text{card}\,\mathcal{T}_h$. Note that the system (18) represents the discretized potential equations in compact form.

For the numerical solution of (17)–(20) we use a primal-dual Newton interior-point method where the inequality constraints (20) are taken care of by logarithmic barrier functions. To be more precise, we consider a sequence of minimization subproblems of the form

$$\min_{\sigma_h, \mathbf{u}_h} B_h(\sigma_h, \mathbf{u}_h, \rho_h) \ , \tag{21}$$

$$B_h(\sigma_h, \mathbf{u}_h, \rho_h) := L_h(\sigma_h, \mathbf{u}_h) - \rho_h[\,\log\,(\sigma_h - \sigma_{min}\mathbf{e}_h) + \,\log\,(\sigma_{max}\mathbf{e}_h - \sigma_h)]$$

subject to the equality constraints (18), (19) where $B_h(\sigma_h, \mathbf{u}_h, \rho_h)$ is the barrier function and $\rho_h > 0$ the barrier parameter.

The equality constraints are coupled by Lagrangian multipliers giving rise to the Lagrangian

$$\mathcal{L}_h(\sigma_h, \mathbf{u}_h, \boldsymbol{\lambda}_h, \mu_h) := B_h(\sigma_h, \mathbf{u}_h, \rho_h) + \boldsymbol{\lambda}_h \cdot (\mathbf{C}_h(\sigma_h)\mathbf{u}_h - \mathbf{b}_h) +$$

$$+ \mu_h \left(g_h(\sigma_h) - C\right) .$$

The first-order Karush–Kuhn–Tucker (KKT) conditions for the associated saddle point problem are solved for decreasing values of the barrier parameter ρ_h by a Newton-type method with line-search. We choose a primary merit function based on the logarithmic barrier function and an augmented Lagrangian as well as a secondary merit function in terms of the ℓ_2-norm of the residual with respect to the KKT conditions combined with a watchdog strategy for convergence monitoring (for details we refer to [16,17]).

5 Numerical Results

The application of the design algorithm typically results in a material distribution which can be displayed by a grey-scale ranging from black ($\sigma = \sigma_{max}$) to white ($\sigma = \sigma_{min}$). Fig. 3 shows the results in a 2D situation where the design objective is to minimize the total amount of dissipated electric energy. The initial design was chosen as a uniform distribution. The performance of the primal-dual Newton interior-point method as described in Sect. 4 depends on the number of ports and individual contact currents (for details we refer to [16,17]). For an individual bus bar with prescribed currents at the ports, Fig. 4 displays the distribution of both the computed electric potential (left) and the computed electric currents (right). We refer to [9,11] for a detailed documentation.

Finally, Fig. 5 contains the visualization of the computed magnetic induction between two ports of the bus bar where the computation has been done by the adaptive multilevel method described in Sect. 3. Again, more details can be found, e.g., in [15].

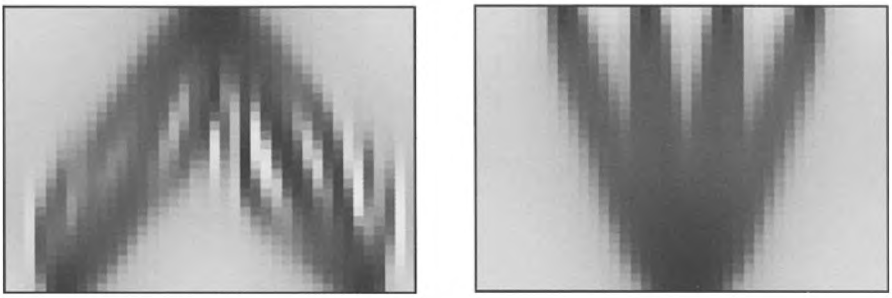

Fig. 3. Material distribution: 3 contacts (left); 5 contacts (right)

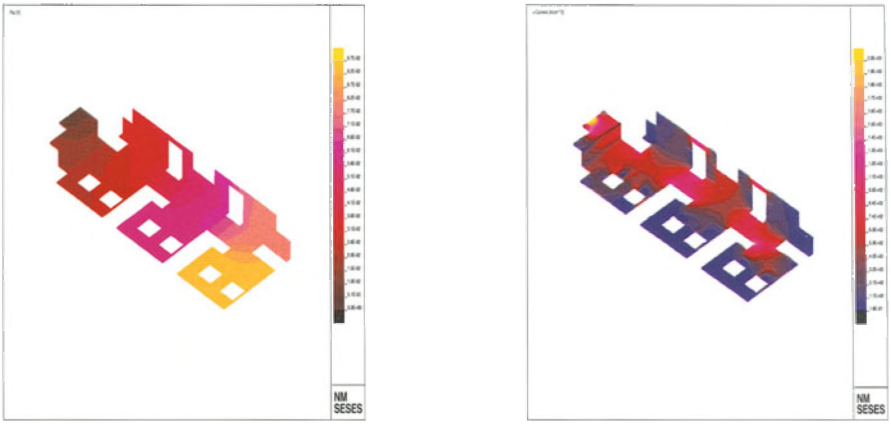

Fig. 4. Electric potential (left); Electric currents (right)

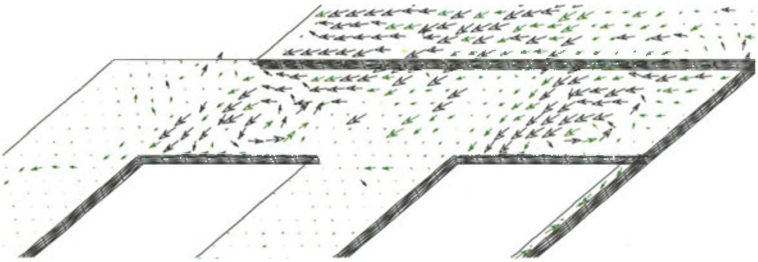

Fig. 5. Magnetic induction in converter module (zoom)

6 Conclusions

In a combined way, we have used modern mathematical methods from structural optimization and the numerical solution of PDEs in general and the eddy current equations in particular to develop an efficient algorithmic tool for the optimal design of high power electronic devices. The numerical computations reveal that the design can be improved by a margin ranging between 10% and 20%.

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