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LETTER TO THE EDITOR

Bethe Ansatz solution of a new class of Hubbard-type models

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Abstract. We define one-dimensional particles with generalized exchange statistics. The exact solution of a Hubbard-type Hamiltonian constructed with such particles is achieved using the coordinate Bethe Ansatz. The chosen deformation of the statistics is equivalent to the presence of a magnetic field produced by the particles themselves, which is also present in a 'free gas' of these particles.

Studies on strongly correlated one-dimensional (1D) systems have deeply influenced modern concepts in many-particle physics. These systems may be of strictly linear extension like quantum wires, quantum Hall bars, (quasi-)1D organic metals and spin chains, or higher-dimensional systems where rotational degrees of freedom are not relevant, as in the Kondo problem [1]. Exactly solvable models [2] play a crucial role in the theoretical description of 1D systems since there most of the results obtained by approximate methods are not reliable (especially at low temperature). They also serve as a reliability test for various approximations, which are applied to models in higher dimensions.

An important example is the Bethe Ansatz (BA) solvable 1D-Hubbard model (HM) [3]. The HM, originally proposed in [4], describes electrons hopping on a D-dimensional lattice, while experiencing an on-site interaction. The two-dimensional HM is believed to capture important features of high-T_c superconductivity [5]. Away from half-filling, the 1D HM exhibits Luttinger liquid (LL) behaviour (for a review, see [6]).

Various generalizations of the HM have been proposed; most of them emerge from changing explicitly the form of the Hamiltonian (for a review, see [7]). The coupling of the fermionic degrees of freedom to a phononic bath has been studied in [8]. Recently, Schulz and Shastry [9] proposed a class of solvable Hubbard-type models in which the hopping amplitude of the particles with a given spin orientation is modulated by a gauge field (entering the Hamiltonian via a Peierls-like substitution), which depends on the density of particles with opposite spin orientation. A similar gauge-like potential has been employed to couple several Hubbard chains [10], where 'M-coloured' electrons in M chains perform intra-chain hopping, which is (gauge-like) affected by the density of electrons in the other chains [10]. The effective inter-chain interaction resembles an electromagnetic field due to the motion of electrons moving along each chain. The models studied in [9] show LL behaviour even in the absence of the interaction: the asymptotic behaviour of correlation functions depends on the coupling strength between the gauge field and the particles, which is responsible for the change

in periodicity of the wavefunction. The gauge potential breaks the time reversal symmetry of the Hamiltonian. Charge and spin excitations have anyon-like character, and the scaling properties deviate from the 'fermionic' LL theory [9].

Another way of extending a model is to modify the 'nature' of the particles entering the Hamiltonian, without changing its formal structure†. In this paper we choose this approach to extend the HM: we keep the formal structure of the Hubbard Hamiltonian unaltered, but we change the particles from fermions to particles whose wavefunctions obey a generalized exchange symmetry. We call the statistics of such particles deformed exchange statistics (DES) [12]. Here, a variant of the DES introduced in [13] is used. The corresponding particles constitute a non-Abelian realization of the symmetric group S_N (see [12]). The 'deformed' 1D HM is solved exactly by means of the coordinate BA (CBA) and the Bethe equations (BE) are compared with the results in [9–14].

We define the HM Hamiltonian for particles obeying DES as

$$H = -t \sum_{i,\sigma} (f^{\dagger}_{i,\sigma} f_{i+1,\sigma} + \text{h.c.}) + U \sum_{i} \nu_{i,\uparrow} \nu_{i,\downarrow} \tag{1}$$

where $f_{i,\sigma}$, $f^{\dagger}_{i,\sigma}$ ($\sigma \in \{\uparrow, \downarrow\}$, respectively $\sigma \in \{\frac{1}{2}, -\frac{1}{2}\}$) obey the deformed relations

$$\begin{aligned} f^{\dagger}_{j,\sigma} f_{k,\sigma'} + \mathcal{Q}^{\sigma,\sigma'}_{j,k} f_{k,\sigma'} f^{\dagger}_{j,\sigma} &= \delta_{j,k}\delta_{\sigma\sigma'} \\ f_{j,\sigma} f_{k,\sigma'} + \mathcal{Q}^{\sigma',\sigma}_{k,j} f_{k,\sigma'} f_{j,\sigma} &= 0 \end{aligned} \tag{2}$$

$$\mathcal{Q}^{\sigma,\sigma'}_{j,k} = (\mathcal{Q}^{\sigma',\sigma}_{k,j})^{-1} = \mathcal{Q}^{\dagger\,\sigma',\sigma}_{k,j} \tag{3}$$

$$[f^{\dagger}_{i,\sigma}, \mathcal{Q}^{\sigma,\sigma'}_{j,k}] = [f_{i,\sigma}, \mathcal{Q}^{\sigma,\sigma'}_{j,k}] = 0. \tag{4}$$

Relations (2) are formally analogue to quon commutation rules [15], but here the deformation parameter depends on indices $(j, \sigma | k, \sigma')$. The operators $\nu_{j,\sigma} \doteq f^{\dagger}_{j,\sigma} f_{j,\sigma}$ are the particle-number operators. Equation (3) guarantees that the particles are representations of S_N, whereas (4) ensures the standard commutation relations $[\nu_{i,\sigma}, \nu_{j,\sigma'}] = 0$, $[\nu_{i,\sigma}, f^{\dagger}_{j,\sigma'}] = \delta_{i,j}\delta_{\sigma,\sigma'} f^{\dagger}_{j,\sigma'}$, and $[\nu_{i,\sigma}, f_{j,\sigma'}] = -\delta_{i,j}\delta_{\sigma,\sigma'} f_{j,\sigma'}$, providing a well defined Fock representation of the algebra defined in (2). We point out that DES is defined for any $\mathcal{Q}^{\sigma,\sigma'}_{j,k}$ fulfilling (3), (4); we choose the deformation parameter depending on spins and coordinates, but also such that the solvability of the model (1) is ensured:

$$\mathcal{Q}^{\sigma,\sigma'}_{j,k} := \begin{cases} \mathrm{e}^{\mathrm{i}\psi(\sigma+\sigma')} \mathrm{e}^{\mathrm{i}\tilde{\Phi}\cdot(\sigma-\sigma')} & j > k \\ \mathrm{e}^{\mathrm{i}\Phi\cdot(\sigma-\sigma')} & j = k \\ \mathrm{e}^{-\mathrm{i}\psi(\sigma+\sigma')} \mathrm{e}^{\mathrm{i}\tilde{\Phi}\cdot(\sigma-\sigma')} & j < k. \end{cases} \tag{5}$$

For fixed j, k, σ, σ', $\mathcal{Q}^{\sigma,\sigma'}_{j,k}$ is a $\mathbb{C}$-number.

The deformation defined above becomes integrable if $\Phi - \tilde{\Phi} \pm \psi(0) = 0 \bmod 2\pi$ holds. This implies $(\Phi - \tilde{\Phi}; \psi(0)) = (0; 0) \vee (\pi; \pi) \bmod 2\pi$. Note that inserting definition (5) in (2) implies the Pauli principle since $\mathcal{Q}^{\sigma,\sigma}_{j,j} = 1$, and that two particles with different spin on the same site obey deformed anticommutation relations since $\mathcal{Q}^{\uparrow,\downarrow}_{j,j} \neq 1$. Equation (4) is trivially fulfilled as well as the consistency relation (3) together with (5). The fermionic case is obtained setting $\Phi = \tilde{\Phi} = 0$; $\psi \equiv 0$, whereas the hard-core bosonic case is covered via $\Phi = \tilde{\Phi} = 0$; $\psi(\pm 1) \equiv \pi$. Furthermore, for spin-$\frac{1}{2}$ particles and factorizing spin- and coordinate dependence of the deformation, every spin dependence in the deformation can be written in the linear form used here.

† Such an approach has been also employed by Maassarani, where he proposed a HM in which the electronic degrees of freedom were generalized to being generators of $su(n)$ [11].

It is worth mentioning that (5) implies fixing an order on the ring. Such an order distinguishes one (given) lattice site from the others in that it defines the 'beginning' of the ring, where one starts counting. Periodicity means not only that the wavefunction has to be periodic, but simultaneously demands the result being independent of where the starting point was set. What is equivalent, is fixing a period $P_0 \doteq \{j_1, \ldots, j_1 + L\}$ on the infinite periodic chain. Consistency of the periodic boundary condition (see below) with this induced order is given if the results are independent of P_0. In what follows it will be seen that this condition is fulfilled.

Now we show that the Hamiltonian (1) is exactly solvable by means of CBA. We apply periodic boundary conditions (PBC), which means $f_{j+L,\sigma} \equiv f_{j,\sigma}$. The general N-particle state in a chain with L sites can be written as

$$|\Psi\rangle = \sum_{\pi \in S_N} \sum_{j_1 \ldots j_N} \Psi(\vec{j}_{\mathbb{I}}|\pi)\mathcal{F}^{\dagger}(\vec{j}_{\mathbb{I}}|\pi)|0\rangle \tag{6}$$

where we have used the abbreviations $\vec{j}_\gamma \doteq (j_{\gamma(1)}, \ldots, j_{\gamma(N)})$, $\mathcal{F}^{\dagger}(\vec{j}_\gamma|\pi) \doteq f^{\dagger}_{j_{\gamma(1)},\sigma_{\pi(1)}} \cdots f^{\dagger}_{j_{\gamma(N)},\sigma_{\pi(N)}}$, for arbitrary $\gamma \in S_N$; $\mathbb{I}$ is the identity in S_N. For indistinguishable particles in $D \neq 2$, a Fock base state is uniquely determined by the coordinates and the spin-configuration; it must not depend on the order in which the particles are created. This requires

$$\Psi(\vec{j}_\gamma|\gamma \circ \pi)\mathcal{F}^{\dagger}(\vec{j}_\gamma|\gamma \circ \pi) \equiv \Psi(\vec{j}_{\gamma'}|\gamma' \circ \pi)\mathcal{F}^{\dagger}(\vec{j}_{\gamma'}|\gamma' \circ \pi) \tag{7}$$

for any $\gamma, \gamma' \in S_N$, since changing the order in creating particles means permuting both the coordinate and spin indices of the operators $f^{\dagger}_{j_{\gamma(l)},\sigma_{\pi(l)}}$ in $\mathcal{F}^{\dagger}$. Equation (7) fixes the symmetry of the wavefunction Ψ. In the case of fermionic statistics, $\Psi(\vec{j}_\gamma|\gamma \circ \pi) = \mathrm{sign}(\gamma)\Psi(\vec{j}_{\mathbb{I}}|\pi)$, where $\mathrm{sign}(\gamma) = +/-$ for even/odd permutations. Every permutation can be decomposed into a product of neighbour exchanges $\chi_k \doteq k \leftrightarrow (k+1)$. Taking $\gamma = \chi_n \circ \cdots \circ \chi_2 \circ \chi_1$ as a relevant example to fix the idea and further defining the partial products as $\gamma_l \doteq \chi_l \circ \cdots \circ \chi_2 \circ \chi_1$ for $l \leqslant n$, we obtain

$$\Psi(\vec{j}_\gamma|\gamma \circ \pi) = \prod_{l=1}^{n}[-\mathcal{Q}^{s'(l),s(l)}_{x'(l),x(l)}]\Psi(\vec{j}_{\mathbb{I}}|\pi) \tag{8}$$

where $x(l) = j_{\gamma_l \circ \pi(l)}$, $s(l) = \sigma_{\gamma_l \circ \pi(l)}$, $x'(l) = j_{\gamma_l \circ \pi(l+1)}$, and $s'(l) = \sigma_{\gamma_l \circ \pi(l+1)}$.

Due to the DES (2), special care must be taken when employing the symmetry of Ψ to restrict the coordinate space to a sector of ordered coordinates (called 'order region' in the following). An order region can be characterized by a permutation $\tau \in S_N$ which connects the actual set of coordinates $(j_1, \ldots, j_N)$ with the ordered one so that $(j_1, \ldots, j_N) \doteq (i_{\tau(1)}, \ldots, i_{\tau(N)})$, where $i_1 \leqslant \cdots \leqslant i_n$. The corresponding wavefunction, which is defined in this order region, is labelled by τ: $\Psi_\tau(\vec{j}_{\pi'}|\pi) \doteq \Psi_{\mathbb{I}}(\vec{j}_{\tau \circ \pi'}|\tau \circ \pi)$. At first, it is necessary keeping this label for writing the Schrödinger equation, since the hopping term can connect different order regions. Due to the Pauli principle, only two order regions are needed to solve the Schrödinger equation (let these two order regions be connected by χ_k). For $\gamma = \chi_k$, equation (8) simplifies to

$$\Psi_{\chi_k \circ \tau}(\vec{i}|\pi) = -\mathcal{Q}^{\sigma,\sigma'}_{x,x'}\Psi_\tau(\vec{i}|\pi) \tag{9}$$

where $x = i_{\pi(k)}$, $\sigma \doteq \sigma_{\pi(k)}$ and $x' = i_{\pi(k+1)}$, $\sigma' \doteq \sigma_{\pi(k+1)}$. In the special case of a doubly occupied site, $i_k = i_{k+1}$, the symmetry relation on the border of an order region leads to

$$\Psi_{\chi_k}(\vec{i}|\pi) = \Psi_{\mathbb{I}}(\vec{i}|\chi_k \circ \pi) = -\mathcal{Q}^{\sigma,\sigma'}_{x,x}\Psi_{\mathbb{I}}(\vec{i}|\pi). \tag{10}$$

Equations (9) and (10) allow writing the Schrödinger equation in terms of a single Ψ_τ†. Since each Ψ_τ can be obtained from $\Psi_{\mathbb{I}}$ using (8), we consider $\Psi_{\mathbb{I}}$ in the following. The label $\mathbb{I}$ will be omitted, and the coordinate vector $\vec{j}$ is chosen being ordered. We insert the Bethe wavefunction

$$\Psi(\vec{j}|\pi) = \sum_{\pi' \in S_N} A(\pi'|\pi) \exp \ \mathrm{i} \sum_{m=1}^{N} j_m p_{\pi'(m)}$$

into $H|\Psi\rangle = E|\Psi\rangle$. For pairwise distinct coordinates we obtain the energy E in terms of the momenta p_l: $E = -2t \sum_{l=1}^{N} \cos p_l$. Its form is unaltered by the deformation (5). Using (9) and (10), the Schrödinger equation corresponding to doubly occupied sites explicitly contains the deformation parameter, and the scattering matrix S reads

$$S(\lambda, \lambda'; m) = -\frac{\mathrm{i}(\lambda - \lambda')\mathrm{e}^{\mathrm{i}\Phi\cdot(\mu-\mu')}\mathbb{P}_{m,m+1} - \frac{U}{2t}}{\mathrm{i}(\lambda - \lambda') - \frac{U}{2t}} \tag{11}$$

where we have defined $\lambda \doteq \sin(p_{\pi'(m)})$, $\lambda' \doteq \sin(p_{\pi'(m+1)})$, $\mu \doteq \sigma_{\pi(m)}$, and $\mu' \doteq \sigma_{\pi(m+1)}$. The permutation operator $\mathbb{P}_{m,m+1}$ is defined by its action on the amplitudes $A(.|.)$: $\mathbb{P}_{m,m+1} A(\pi'|\pi) \doteq A(\pi'|\chi_m \circ \pi)$. The scattering matrix S fulfils the Yang–Baxter equation (YBE)

$$S(\lambda, \lambda'; m+1)S(\lambda, \lambda''; m)S(\lambda', \lambda''; m+1) = S(\lambda', \lambda''; m)S(\lambda, \lambda''; m+1)S(\lambda, \lambda'; m) \tag{12}$$

where $\lambda'' \doteq \sin(p_{\pi'(m+2)})$. For the validity of the YBE, it is important to notice that $\mathbb{P}$ and $(\mu - \mu')$ do not commute, since μ and μ' contain the spin permutation, which is affected by $\mathbb{P}$: $\mathbb{P}_{m,m+1}(\mu - \mu') = (\mu' - \mu)\mathbb{P}_{m,m+1}$. Applying PBC, which also demands independence from the pre-chosen order, we obtain

$$\Psi(j_1, \ldots, j_N|\pi) = Q_{PBC}\Psi(j_{k+1}, \ldots, j_N, j_1 + L, \ldots, j_k + L|\tilde{\pi}) \tag{13}$$

where $\tilde{\pi}$ is the cyclic permutation $\tilde{\pi} \doteq \binom{1\ldots N-1\,N}{k+1\ldots k-1\,k}$, and the function Q_{PBC} reads

$$\begin{aligned} Q_{PBC} &= (-1)^{k(N-1)} \prod_{m=1}^{k} \prod_{l=1}^{N-1} \mathcal{Q}^{\sigma_{\pi(m+l)},\sigma_{\pi(m)}}_{i_{m+l},i_m} \\ &\equiv (-1)^{k(N-1)} Q_{B,0}^{k} \prod_{m=1}^{k} Q_{B,\sigma_{\pi(m)}} \end{aligned} \tag{14}$$

where $Q_{B,\sigma} \doteq \mathrm{e}^{\mathrm{i}[(N_\sigma - 1)\psi(2\sigma) - 2\sigma\Phi N_{-\sigma}]}$ is the spin-dependent part, and $Q_{B,0} = 1$ the spin-independent part of Q_{PBC}‡. Thus, the deformation affects the BE in two distinct ways: (a) via the prefactor Q_{PBC} by imposing PBC: its form arises from the deformed symmetry of the wavefunction induced by the algebra (2); (b) due to a modified S-matrix in case of on-site deformation. The BE are

$$\begin{aligned} \mathrm{e}^{\mathrm{i}p_j L} &= \frac{Q_S^{N_\downarrow}}{Q_{B,\uparrow}Q_{B,0}} \prod_{a=1}^{N_\downarrow} \frac{\mathrm{i}(\sin p_j - \zeta_a) - \frac{U}{4t}}{\mathrm{i}(\sin p_j - \zeta_a) + \frac{U}{4t}} \\ \prod_{\substack{b=1 \\ b\neq a}}^{N_\downarrow} \frac{\mathrm{i}(\zeta_a - \zeta_b) + \frac{U}{2t}}{\mathrm{i}(\zeta_a - \zeta_b) - \frac{U}{2t}} &= \frac{Q_{B,\downarrow}Q_S^N}{Q_{B,\uparrow}} \prod_{l=1}^{N} \frac{\mathrm{i}(\sin p_l - \zeta_a) - \frac{U}{4t}}{\mathrm{i}(\sin p_l - \zeta_a) + \frac{U}{4t}} \end{aligned} \tag{15}$$

† Equations (9) and (10) guarantee that two wavefunctions defined in neighbouring regions are single valued on the intersection of their domains.

‡ The choice of the functional form of q, which was integrable in [13], would destroy the CBA solvability of the HM, since Q_{PBC} would become configuration dependent.

where the factor $Q_S \doteq \mathrm{e}^{-\mathrm{i}\Phi}$ arises from the S matrix in diagonalizing the transfer matrix of the 'spin problem', equation (13). Using the expression for $Q_{B,\sigma}$, $Q_{B,0}$, Q_S, and the integrability condition $\Phi - \tilde{\Phi} \pm \psi(0) = 0 \bmod 2\pi$, the factors due to the DES are

$$\frac{Q_S^{N_\downarrow}}{Q_{B,\uparrow} Q_{B,0}} = \mathrm{e}^{-\mathrm{i}(N_\uparrow - 1)\psi(1)} \tag{16}$$

$$\frac{Q_{B,\downarrow} Q_S^N}{Q_{B,\uparrow}} = \mathrm{e}^{\mathrm{i}[(N_\downarrow - 1)\psi(-1) - (N_\uparrow - 1)\psi(1)]}. \tag{17}$$

We see how the above contributions involve only the spatial part of the deformation (5); a purely spin-dependent deformation of the statistics does not affect the spectrum of the model. Even in this case, however, the eigenstates are different due to the modification in the S matrix. This can also be made transparent by representing the f, $f^\dagger$ by fermionic operators c, $c^\dagger$: $f_{j,\sigma} \doteq c_{j,\sigma} \prod_{l=1}^{L} \mathrm{e}^{-\mathrm{i}\sigma\Phi n_{l,\sigma}}$. In a forthcoming paper [16] we will describe in more detail the connection between fermionization and DES, as well as the connections to [9, 14]. Here we mention only that, defining $\Phi_\sigma \doteq -(N_\sigma - 1)\psi(2\sigma)$, the phases appearing in the BEs decompose into $\Phi_\uparrow$ and $\Phi_\uparrow - \Phi_\downarrow$, found in [14]. In contrast to the result in [9], the factors $N_\sigma - 1$ appear. The reason for this is that the phases in [9] were produced by particles having opposite spin, whereas here the relevant phase comes from particles having equal spin projections. Thus the particle feeling that phase is excluded.

From the BEs (15), the total momentum can be extracted:

$$P \doteq \sum_{i=1}^{N} p_j = -\sum_\sigma N_\sigma (N_\sigma - 1)\frac{\psi(2\sigma)}{L} = \sum_\sigma N_\sigma \frac{\Phi_\sigma}{L}. \tag{18}$$

From this it is seen that the total momentum, though periodicity is implied, is not necessarily a multiple of $2\pi/L$, which is the case for the undeformed model. This is not surprising, since the spectrum is that of a twisted fermionic model. The total momentum can be obtained in shifting $N_\uparrow$ momenta of the undeformed model, which become $2\pi n/L$ for $U \to 0$, by $-(N_\uparrow - 1)\psi(1)$ and the remaining $N_\downarrow$ momenta by $-(N_\downarrow - 1)\psi(-1)$. That means

$$E = -2t \sum_\sigma \sum_{i_\sigma=1}^{N_\sigma} \cos \quad \frac{2\pi}{L} l_{i_\sigma} + \frac{\Phi_\sigma}{L} \tag{19}$$

with Φ_σ as defined above in comparison with [14] and l_{i_σ}, for fixed σ, being distinct integers modulo L. Expression (19) shows the physical meaning of DES: the deformation of the particles' statistics is equivalent to a magnetic field generated from the particles themselves. Such a magnetic field depends on how many particles are in the system. In particular, (19) shows how the energy of a gas of free ($U \to 0$) particles is not simply the sum of the single particles' energies, but it describes a system of interacting particles. Such interaction purely comes from the deformed statistics. However, the noninteracting limit is to be taken carefully here. This can be understood by facing the second BE. In order to keep the phase, two or more spin rapidities are forced to either coincide with each other or with one or several $\sin(p_j)$. Such contributions have to be linear in $U/4t$.

In conclusion, we have extended the BA for particles with deformed exchange statistics or, which is equivalent, for solutions with generalized exchange symmetry. A method already introduced in [13] is here generalized for problems including inner degrees of freedom like spin. Using this technique, a new class of generalized Hubbard models could be shown being exactly solvable. The spectrum equals that of a fermionic model with spin dependent boundary phases, which in fact turns out being a general feature of integrable deformed exchange statistics. The eigenfunctions, however, are different due to the non-fermionic exchange symmetry. A

preliminary study showed that these phases in general already contribute to the thermodynamic limit. We will discuss the details elsewhere [16].

A systematic study of deformed models is most interesting for at least two reasons. First, 'integrable deformed statistics' can be an alternative way of handling complicated interactions of integer statistics' particles. In a forthcoming paper [16] we will review these results from another viewpoint: namely, fermionic models with correlated hopping. There, a general statement on integrability of deformed exchange statistics will be drawn as well as for models with correlated hopping.

Second, it might be relevant for modelling the edge states in fractional quantum Hall effect physics, the chiral LL behaviour of which has recently been questioned [17].

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