Nonadiabatic Electron Pumping: Maximal Current with Minimal Noise

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The noise properties of pump currents through an open double quantum dot setup with non-adiabatic ac driving are investigated. Driving frequencies close to the internal resonances of the double dot-system mark the optimal working points at which the pump current assumes a maximum while its noise power possesses a remarkably low minimum. A rotating-wave approximation provides analytical expressions for the current and its noise power and allows to optimize the noise characteristics. The analytical results are compared to numerical results from a Floquet transport theory.

In mesoscopic conductors, a cyclic adiabatic change of the parameters can induce a pump current, i.e., a non-vanishing dc current flowing even in the absence of any external bias voltage [1–3]. For adiabatic quantum pumps [4–8], the transfered charge per cycle is determined by the area enclosed in parameter space during the cyclic evolution [4, 5]. This implies that the resulting current is proportional to the driving frequency and, thus, suggests that *non-adiabatic* electron pumping is more effective. For practical applications, it is also desirable to operate the quantum pump in a low-noise regime. It has been found that adiabatic pumps can be practically noiseless [9]. This happens, however, on the expense of acquiring a small or even vanishing current [10]. Therefore, the question arises whether it is possible to boost the pump current by increasing the driving frequency while keeping the noise level very low.

Non-adiabatic electron pumping can be achieved experimentally with double quantum dots under the influence of microwave radiation [11–14]. In this letter we study the transport properties in this non-adiabatic regime. Our main aim is to find ideal parameter regimes in which a large pump current is associated with low current noise. For the optimization of the system parameters, it is beneficial to obtain, besides a numerical solution, also analytic expressions for the transport quantifiers. Therefore, within a rotating-wave approximation (RWA), we map the driven transport problem to a static one which we solve analytically. In doing so, a particular challenge represents the consistent RWA treatment of the connecting leads in the presence of ac fields.

The double-dot model.— We consider the setup sketched in Fig. 1 described by the time-dependent Hamiltonian $H(t) = H_{\text{dots}}(t) + H_{\text{leads}} + H_{\text{contacts}}$, where the different contributions correspond to the quantum dots, the leads, and the tunneling coupling to the respective lead. We disregard interaction and spin effects and assume that intra-dot excitations do not play a role such that each dot is well described by a single energy level.

Then, the double quantum dot Hamiltonian reads

$$H_{\text{dots}}(t) = -\frac{\Delta}{2}(c_1^\dagger c_2 + c_2^\dagger c_1) + \frac{\epsilon(t)}{2}(c_1^\dagger c_1 - c_2^\dagger c_2), \quad (1)$$

where the fermion operators $c_{1,2}$ and $c_{1,2}^\dagger$ annihilate and create an electron in the left and the right dot, respectively. The on-site energy difference $\epsilon(t) = \epsilon_0 + A\cos(\Omega t)$ is determined by the static internal bias ϵ_0, the driving amplitude A, and the frequency Ω. Typical driving frequencies range up to $100\,\text{GHz}$ [11] such that the wavelength exceeds the size of the setup and, thus, the implicitly assumed dipole approximation is well justified.

The leads are modeled as ideal electron gases, $H_{\text{leads}} = \sum_q \epsilon_q(c_{Lq}^\dagger c_{Lq} + c_{Rq}^\dagger c_{Rq})$, where $c_{\ell q}^\dagger$ creates an electron in lead $\ell = \text{L}, \text{R}$. The tunneling Hamiltonian

$$H_{\text{contacts}} = \sum_q \left(V_{Lq} c_{Lq}^\dagger c_1 + V_{Rq} c_{Rq}^\dagger c_2 \right) + \text{H.c.} \quad (2)$$

establishes the contact between the dot levels and the respective lead. Below, we shall assume within a so-termed wide-band limit that the coupling strengths $\Gamma_\ell = 2\pi \sum_q |V_{\ell q}|^2 \delta(\epsilon - \epsilon_q)$, $\ell = \text{L}, \text{R}$, are energy independent. To specify the dynamics, we choose as an initial condition for the lead electrons a grand canonical ensemble at temperature T and chemical potentials $\mu_{\text{L,R}}$. The influence of lead ℓ is fully determined by the lesser Green functions
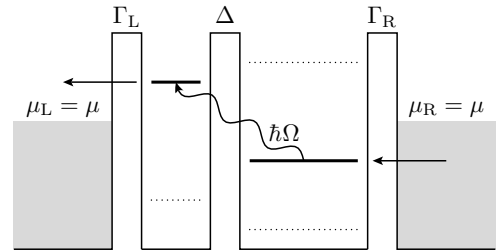

FIG. 1: Level structure of the asymmetric double quantum dot in a pump configuration. The solid lines mark the relevant levels $|1\rangle$ and $|2\rangle$ with the energies $\pm\epsilon_0/2$. The arrows indicate the dominating scattering process.

$g_{\ell q}^{<}(t, t') = (i/\hbar)\langle c_{\ell q}^{\dagger}(t')c_{\ell q}(t)\rangle$ and the tunnel matrix elements $V_{\ell q}$ [15]. More precisely, these quantities enter the expressions for the current and the noise in form of the correlation function

$$\langle \xi_{\ell}^{\dagger}(t - \tau)\xi_{\ell}(t)\rangle = \frac{\Gamma_{\ell}}{2\pi\hbar^2} \int d\epsilon \, e^{-i\epsilon\tau/\hbar} f_{\ell}(\epsilon) \qquad (3)$$

of the fermionic noise operator $\xi_{\ell} = -(i/\hbar)\sum_q V_{\ell q}^* c_{\ell q}$, where $f_{\ell}(\epsilon) = (1 + \exp[(\epsilon - \mu_{\ell})/k_B T])^{-1}$ denotes the Fermi function [16]. Henceforth, we consider the case of zero bias voltage with both chemical potentials located midway between the dot levels $\pm\epsilon_0/2$, i.e., $\mu_L = \mu_R = 0$.

Resonant electron pumping.— For harmonic driving, the Hamiltonian $H(t)$ obeys time-reversal symmetry and, hence, each individual scattering process has a time-reversed partner which occurs with the same probability. Thus, it is tempting to conclude that the net current of both partners and, consequently, the pump current vanishes. This, however, is not the case because the driving enables energy non-conserving scattering. In particular, there exist processes like the one sketched in Fig. 1: With the leads initially at equilibrium, an electron from the right lead with energy below the Fermi surface is scattered into a state in the left lead with energy above the Fermi surface. This process contributes to the current. By contrast, the time-reversed process does not transport an electron because the respective initial state is not occupied. The net effect is transport of electrons from the lower level to the higher level, i.e., from right to left.

None the less, the pump might vanish due to the presence of an additional symmetry, such as generalized parity $(x, t) \rightarrow (-x, t + \pi/\Omega)$ which relates two scattering processes with identical *initial* energies. Their contributions to the current cancel each other [16]. With equally strong coupling to the leads, $\Gamma_L = \Gamma_R = \Gamma$, generalized parity is satisfied for $H(t)$ at zero internal bias $\epsilon_0 = 0$. For finite bias $\epsilon_0 \neq 0$, however, this symmetry is broken and, consequently a finite pump current emerges. Moreover, this pump current exhibits resonance peaks including higher-order resonances [17].

Within our analytical approach, we focus on strongly biased situations, $\epsilon_0 \gg \Delta$, and driving frequencies close to the internal resonances of the double dot, $n\hbar\Omega = (\epsilon_0^2 + \Delta^2)^{1/2} \approx \epsilon_0$. In this regime, the dynamics of the dot electrons is dominated by the second term of the Hamiltonian (1) while the tunneling contribution, which is proportional to Δ, represents a perturbation. Consequently, a proper interaction picture is defined by the transformation $U(t) = \exp[-\frac{i}{2}(c_1^{\dagger}c_1 - c_2^{\dagger}c_2)\phi(t)]$ with the time-dependent phase

$$\phi(t) = n\Omega t + \frac{A}{\hbar\Omega}\sin(\Omega t). \qquad (4)$$

This yields the double-dot interaction-picture Hamiltonian $\widetilde{H}_{\rm dots}(t) = U^{\dagger}(t)H_{\rm dots}(t)U(t) - i\hbar U^{\dagger}(t)\dot{U}(t)$. The transformation $U(t)$ has been constructed such that $\widetilde{H}(t)$

obeys the time-periodicity of the original Hamiltonian (1) while all its other energy scales are significantly smaller than $\hbar\Omega$. Thus, we can separate time scales and replace $\widetilde{H}_{\rm dots}(t)$ within a rotating-wave approximation (RWA) by its time average

$$\bar{H}_{\rm dots} = -\frac{\Delta_{\rm eff}}{2}(c_1^{\dagger}c_2 + c_2^{\dagger}c_1) - \frac{\delta}{2}(c_1^{\dagger}c_1 - c_2^{\dagger}c_2) \quad (5)$$

with $\delta = n\hbar\Omega - \epsilon_0$ and the effective tunnel matrix element

$$\Delta_{\rm eff} = (-1)^n J_n(A/\hbar\Omega)\Delta, \qquad (6)$$

where J_n is the nth order Bessel function of the first kind.

While the lead Hamiltonian is unaffected by the transformation $U(t)$, the tunneling Hamiltonian acquires a time-dependence, $\widetilde{H}_{\rm contacts}(t) = \sum_q V_{Lq}c_{Lq}^{\dagger}c_1 e^{-i\phi(t)/2} + V_{Rq}c_{Rq}^{\dagger}c_2 e^{i\phi(t)/2}$. The lead elimination along the lines of Ref. [15], but here for a time-dependent contact Hamiltonian, reveals that the influence of the leads is no longer determined by the noise operators ξ_{ℓ} but rather by $\eta_{L/R}(t) = e^{\pm i\phi(t)/2}\xi_{L/R}(t)$. Its correlation function $\langle \eta_{L/R}^{\dagger}(t - \tau)\eta_{L/R}(t)\rangle$ depends not only on the time-difference τ, but also explicitly on t. The latter time-dependence is $2\pi/\Omega$-periodic and is therefore much faster than all other time scales. Hence, we can replace within the RWA the correlation function of $\eta_{L,R}$ by its t-average

$$\overline{\langle \eta_{\ell}^{\dagger}(t - \tau)\eta_{\ell}(t)\rangle} = \frac{\Gamma}{2\pi\hbar^2} \int d\epsilon \, e^{-i\epsilon\tau/\hbar} f_{\ell,\rm eff}(\epsilon), \quad (7)$$

where

$$f_{L/R,\rm eff}(\epsilon) = \sum_{k=-\infty}^{\infty} J_k^2\left(\frac{A}{2\hbar\Omega}\right) f_{L/R}\left(\epsilon + \left[k \mp \frac{n}{2}\right]\hbar\Omega\right) \quad (8)$$

can be interpreted as an effective electron occupation number of the levels in lead ℓ. At zero temperature, it exhibits steps at $\epsilon = \mu_{\ell} + (k \mp n/2)\hbar\Omega$ and is constant elsewhere.

The RWA provides a mapping of the originally time-dependent transport problem to a static one with renormalized parameters. This problem, in turn, can be solved by standard procedures: Both the current and the noise power can be expressed in terms of the transmission probability $T(\epsilon)$ of an electron with energy ϵ. For a two-level system in the wide-band limit, one obtains

$$T(\epsilon) = \Gamma^2|G_{12}(\epsilon)|^2 = \frac{\Gamma^2\Delta_{\rm eff}^2}{|(2\epsilon - i\Gamma)^2 - \Delta_{\rm eff}^2 - \delta^2|^2}. \quad (9)$$

Then, the current defined as the change of the charge in the, e.g., left lead, is given by the Landauer-like formula $I = (e/2\pi\hbar)\int d\epsilon \, T(\epsilon)[f_{L,\rm eff}(\epsilon) - f_{R,\rm eff}(\epsilon)]$; the corresponding expression for the noise power reads [18]

$$S = \frac{e^2}{\pi\hbar} \int d\epsilon \, T(\epsilon)\left\{ \sum_{\ell=L,R} f_{\ell,\rm eff}(\epsilon)[1 - f_{\ell,\rm eff}(\epsilon)] \right.$$
$$\left. + [1 - T(\epsilon)][f_{L,\rm eff}(\epsilon) - f_{R,\rm eff}(\epsilon)]^2 \right\}. \quad (10)$$

Note that in the presence of a driving field, even at zero temperature, the electron occupation $f_{\ell,\text{eff}}$ is not a simple step function and, thus, also the term in the first line of Eq. (10) contributes to the noise power. A convenient measure for the *relative* noise strength is the Fano factor $F = S/2eI$ which characterizes the noise with respect to the shot noise level given by $S = 2eI$ [18].

For the remaining evaluation of the energy integrals, it is important to note that the transmission (9) is practically zero for $\epsilon^2 \gtrsim \Delta_{\text{eff}}^2 + \Gamma^2 + \delta^2$. Thus, for $\hbar\Omega \gtrsim \Delta_{\text{eff}}, \Gamma, \delta$, the effective electron occupation (8) is constant in the relevant energy range and can be replaced by its value at $\epsilon = 0$. One obtains close to the nth resonance

$$I^{(n)} = \frac{e\Gamma}{2\hbar} \frac{\lambda_n \Delta_{\text{eff}}^2}{\Delta_{\text{eff}}^2 + \Gamma^2 + \delta^2}, \tag{11}$$

$$S^{(n)} = \frac{e^2\Gamma}{2\hbar} \frac{\lambda_n^2 \Delta_{\text{eff}}^2 [2(\Gamma^2 + \delta^2)^2 - \Delta_{\text{eff}}^2(\Gamma^2 - 3\delta^2) + \Delta_{\text{eff}}^4]}{(\Delta_{\text{eff}}^2 + \Gamma^2 + \delta^2)^3}$$
$$+ \frac{1 - \lambda_n^2}{\lambda_n} eI^{(n)}, \tag{12}$$

where $\lambda_n = f_{\text{L,eff}}(0) - f_{\text{R,eff}}(0) = \sum_{|k|\leq n/2} J_k^2(A/2\hbar\Omega)$ with $|\lambda_n| \leq 1$. Quite remarkably, for resonant driving ($\delta = 0$), the pump current assumes a *maximum* while the noise power S generally assumes a local *minimum*; cf. Fig. 2(a). This results in an even more pronounced minimum for the Fano factor.

Floquet transport theory.— Before developing an optimization strategy, we corroborate our analytical results by an exact numerical calculation within Floquet transport theory [16]: Starting from the Heisenberg equations of motion for the annihilation operators for both the lead and the dot electrons, one eliminates the lead operators and thereby obtains for the electrons on the dots a reduced set of equations. These are solved with the help of the retarded Green function obeying $[i\hbar d/dt - \mathcal{H}(t) + i\Gamma/2]G(t,t') = \delta(t - t')$, where $\mathcal{H}(t)$ is the single-particle Hamiltonian corresponding to the double-dot Hamiltonian (1). The coefficients of the equation of motion for $G(t,t')$ are $2\pi/\Omega$-periodic and, consequently, its solution can be constructed with the help of the Floquet ansatz $|\psi_\alpha(t)\rangle = \exp[(-i\epsilon_\alpha/\hbar - \gamma_\alpha)t]|\phi_\alpha(t)\rangle$. The Floquet states $|\phi_\alpha(t)\rangle$ obey the eigenvalue equation $[\mathcal{H}(t) - i\Gamma/2 - i\hbar d/dt]|\phi_\alpha(t)\rangle = (\epsilon_\alpha - i\hbar\gamma_\alpha)|\phi_\alpha(t)\rangle$. Its solution allows to construct the retarded Green function $G(t,t') = -(i/\hbar)\sum_\alpha |\psi_\alpha(t)\rangle\langle\psi_\alpha^+(t')|\Theta(t - t')$. Finally, one obtains for the pump current a convenient Landauer-like expression with an additional sum over the sidebands [16, 19]. Since the symmetrized noise correlation function $S(t,t') = \langle[I(t), I(t')]_+\rangle$ depends explicitly on both times, we characterize the noise by the time-average of its zero-frequency component, $S = (2\pi/\Omega)\int_0^{2\pi/\Omega} dt \int d\tau S(t, t - \tau)$.

Figure 2(a) depicts the numerically evaluated pump current and its noise power. For proper cooling, thermal excitations do not play a significant role. Therefore, we
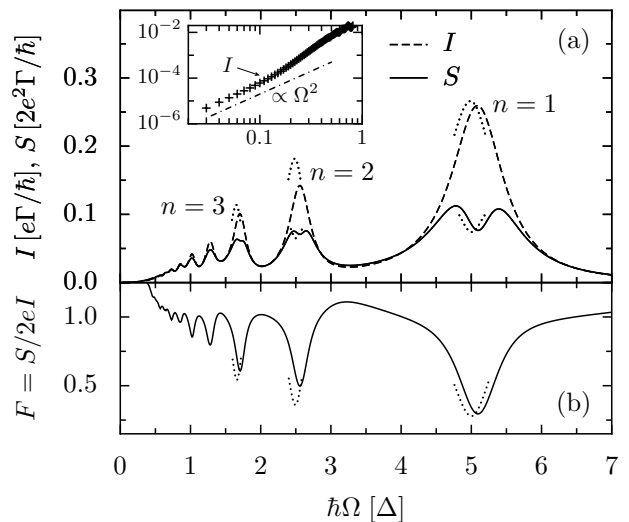

FIG. 2: (a) Pump current I (dashed line) and its noise power S (solid) at $k_BT = 0$ as a function of the driving frequency for coupling strength $\Gamma = 0.3\Delta$, driving amplitude $A = 3.7\Delta$, and internal bias $\epsilon_0 = 5\Delta$. The dotted lines mark the analytical results (11) and (12). Inset: Blow-up of the lower-left corner demonstrating that $I \propto \Omega^2$. (b) Corresponding Fano factor.

consider zero temperature only. We find that the current exhibits peaks located at the resonance frequencies $\Omega = (\epsilon_0^2 + \Delta^2)^{1/2}/n\hbar$. This agrees well with our analytical results (dotted lines), albeit the RWA predicts the location of the current maxima only to zeroth order in Δ, i.e., at the slightly shifted frequencies $\Omega = \epsilon_0/n\hbar$. In the adiabatic limit, the pump current vanishes proportional to Ω^2. For the chosen parameters, the noise power S possesses clear minima, each accompanied by two maxima. In the vicinity of the resonance, the noise is considerably below the shot noise level $2eI$; cf. Fig. 2(b). This feature is notably pronounced at the first resonance. Far from the resonances, the current becomes smaller and the Fano factor is close to $F = 1$. The comparison of the numerically exact results with the current (11) and the noise power (12) [dotted lines in Fig. 2(a)] leads to the conclusion, that the RWA predicts both the current maxima and the noise minima sufficiently well to employ these expressions for a parameter optimization towards low-noise pumping.

Tuning the electron pump.— We have already seen that the condition of large current and low noise is met at the internal resonances of the biased double-dot setup. Thus, we can restrict the search for optimal parameters to resonant driving. As a figure of merit for the noise strength we employ the Fano factor for $\delta = 0$

$$F^{(n)} = \frac{S^{(n)}}{2eI^{(n)}} = \frac{1}{2\lambda_n} - \frac{\lambda_n}{2} \frac{\Gamma^2(3\Delta_{\text{eff}}^2 - \Gamma^2)}{(\Delta_{\text{eff}}^2 + \Gamma^2)^2}, \tag{13}$$

which is a function of λ_n and $\Delta_{\text{eff}}/\Gamma$. The second term is minimal for $\Delta_{\text{eff}}/\Gamma = \sqrt{5/3}$, yielding $F^{(n)} = 1/(2\lambda_n) -$
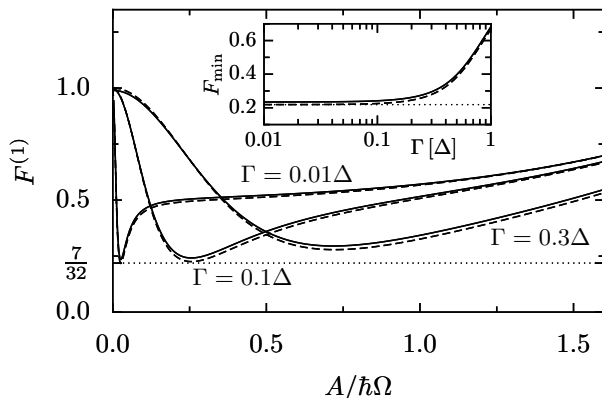
FIG. 3: Fano factor F at the first resonance for various coupling strengths Γ. The exact Floquet calculation (solid lines) is compared with the RWA for $\delta = 0$ (dashed). The inset depicts the minimal Fano factor in dependence of Γ for $\epsilon_0 = 5\Delta$ at the resonance $\Omega = \sqrt{\Delta^2 + \epsilon_0^2}/\hbar$. The dotted lines mark the optimal Fano factor $F_{\mathrm{opt}} = 7/32$.

$9\lambda_n/32$. Thus, the optimal Fano factor is assumed for $\lambda_n = 1$ and reads $F_{\mathrm{opt}} = 7/32 \approx 0.219$.

In the following, we restrict ourselves to the prime resonance ($n = 1$) for which $\Delta_{\mathrm{eff}} = J_1(A/\hbar\Omega)\Delta$ and $\lambda_1 = J_0^2(A/2\hbar\Omega)$ [20]. Then, the value $\lambda_1 = 1$ is assumed for $A = 0$ which means $\Delta_{\mathrm{eff}} = 0$; this unfortunately implies a vanishing current (11). Therefore, the central question is whether it is possible to find a driving amplitude providing on the one hand an appreciably large pump current, while on the other hand yielding a noise level close to F_{opt}. The numerical results depicted in Fig. 3 indeed suggest this possibility: The Fano factor is close to the optimal value already for a finite amplitude. A closer investigation reveals that the location of the minimum corresponds to $\Delta_{\mathrm{eff}} = J_1(A/\hbar\Omega)\Delta = \sqrt{5/3}\,\Gamma$, in compliance with our analytical considerations. In particular, the minimum is shifted towards smaller values of $A/\hbar\Omega$ for weaker coupling Γ. Moreover, the RWA solutions (11) and (12) agree very well with the numerically exact results, although they slightly underestimate the noise. This discrepancy diminishes as Ω^{-2} (not shown).

The data also reveal that in the interesting regime, the ratio $A/\hbar\Omega$ is considerably smaller than 1 and, hence, we can employ the approximations $J_0(x) \approx 1 - x^2/4$ and $J_1(x) \approx x/2$ valid for small arguments. It is now straightforward to obtain to lowest order in $A/\hbar\Omega$ the expressions $\Delta_{\mathrm{eff}} = A\Delta/2\hbar\Omega$ and $F^{(1)} = 7/32 + (5A/16\hbar\Omega)^2$. For instance, choosing $A = 0.3\hbar\Omega$, the noise level lies merely 5% above F_{opt} and the condition $\Delta_{\mathrm{eff}} = \sqrt{5/3}\,\Gamma$ corresponds to $\Gamma \approx 0.1\Delta$, i.e., to weak dot-lead coupling. This estimate is confirmed by the inset in Fig. 3 which, in addition, demonstrates that $F \approx F_{\mathrm{opt}}$ for $\Gamma \lesssim 0.1\Delta$. For such a small coupling Γ, interaction-induced electron-electron correlations typically play a minor role.

In the experiment of Ref. [11], a typical inter-dot coupling is $\Delta = 50\mu\mathrm{eV}$. Then, an internal bias $\epsilon_0 = 5\Delta$ corresponds to the resonance frequency $\Omega = 5\Delta/\hbar \approx 2\pi \times 60\mathrm{GHz}$. Tuning the lead coupling to $\Gamma = 0.1\Delta$ results in an optimized pump current of the order 200pA with a Fano factor $F \approx 0.23$.

Conclusions. — The analysis of ac-driven, asymmetric double quantum dots demonstrates that optimal pumping is achieved beyond the adiabatic regime. In particular, the ideal *modus operandi* requires a large internal bias at resonant driving in combination with a strong inter-dot coupling $\Delta \gtrsim 10\Gamma$. The resulting pump current then assumes a maximum while, interestingly enough, the (absolute) noise power assumes at the same time a minimum such that the Fano factor becomes remarkably small. In order to systematically tune the pump into a low-noise regime, we have derived analytical expressions for both the current and its noise power. The comparison with the numerically exact solution fully confirms the validity of this approach. Our findings convincingly suggest that coupled quantum dots are ideal for pumping electrons effectively and reliably at a low noise level.

[1] L. P. Kouwenhoven *et al.*, Phys. Rev. Lett. **67**, 1626 (1991).
[2] H. Pothier *et al.*, Europhys. Lett. **17**, 249 (1992).
[3] M. Switkes *et al.*, Science **283**, 1905 (1999).
[4] P. W. Brouwer, Phys. Rev. B **58**, R10135 (1998).
[5] B. L. Altshuler and L. I. Glazman, Science **283**, 1864 (1999).
[6] M. Wagner and F. Sols, Phys. Rev. Lett. **83**, 4377 (1999).
[7] M. Moskalets and M. Büttiker, Phys. Rev. B **70**, 245305 (2004).
[8] T. Brandes and T. Vorrath, Phys. Rev. B **66**, 075341 (2002).
[9] J. E. Avron *et al.*, Phys. Rev. Lett. **87**, 236601 (2001).
[10] M. L. Polianski, M. G. Vavilov, and P. W. Brouwer, Phys. Rev. B **65**, 245314 (2002).
[11] W. G. van der Wiel *et al.*, Physica B **272**, 31 (1999).
[12] W. G. van der Wiel *et al.*, Rev. Mod. Phys. **75**, 1 (2003).
[13] G. Platero and R. Aguado, Phys. Rep. **395**, 1 (2004).
[14] E. Cota, R. Aguado, and G. Platero, Phys. Rev. Lett. **94**, 107202 (2005).
[15] Y. Meir and N. S. Wingreen, Phys. Rev. Lett. **68**, 2512 (1992).
[16] S. Kohler, J. Lehmann, and P. Hänggi, Phys. Rep. **406**, 379 (2005).
[17] C. A. Stafford and N. S. Wingreen, Phys. Rev. Lett. **76**, 1916 (1996).
[18] Ya. M. Blanter and M. Büttiker, Phys. Rep. **336**, 1 (2000).
[19] S. Camalet *et al.*, Phys. Rev. Lett. **90**, 210602 (2003).
[20] At higher resonances, the same optimal Fano factor can be reached with a larger driving amplitude.