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# Adjustment Costs and Nominal Rigidities in a Small Open Economy

# Anpassungskosten und nominelle Rigiditäten in einer kleinen offenen Volkswirtschaft

# Von Alfred Maußner, Augsburg

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Adjustment costs, nominal rigidities, wage staggering, cyclical markups. Anpassungskosten, nominelle Rigidiäten, Lohnkontrakte, zyklische Aufschlagssätze.

# Summary

What does account for the persistence of monetary shocks in dynamic general equilibrium models of the business cycle? A number of papers have dealt with that question and point at labor market frictions besides those introduced by overlapping wage contracts.

In this paper I investigate an obvious source of persistence, namely small adjustment costs of labor at the firm level. These introduce indeed hump shaped impulse responses of hours worked in simulated time series. Compared with a benchmark model without nominal and real frictions my model outperforms the former in most respects.

However, its account of the time series properties of monetary variables is not satisfactory. This holds true for closely related models that change the current period utility function, that introduce money into the utility function, or that posit a cash in advance constraint. I take this as suggestive to think about more sophisticated models of money demand.

# Zusammenfassung

Was erklärt die Persistenz monetärer Schocks in dynamischen allgemeinen Gleichgewichtsmodellen des Konjunkturzyklus? Mit dieser Frage befaßt sich eine Anzahl von Arbeiten. Sie weisen auf Friktionen am Arbeitsmarkt hin, die über jene hinausgehen, die mit überlappenden Lohnkontrakten verbunden sind.

In dieser Arbeit betrachte ich eine naheliegende Ursache für die langsame Verarbeitung monetärer Schocks, nämlich geringfügige Kosten der Unternehmen bei der Variation der Beschäftigung. Diese Kosten führen in der Tat zu umgekehrt u-förmigen Imupulsantwortfunktionen. Verglichen mit einem Model ohne nominale und reale Friktionen, zeichnet mein Modell die stilisierten Fakten des Konjunkturzyklus in fast jeder Hinsicht weit besser nach.

Das Modell läßt allerdings im Hinblick auf die Eigenschaften monetärer Variablen zu wünschen übrig. Das trifft auch auf nahe verwandte Modelle zu, in denen anstelle der hier benutzten Nutzenfunktion die Standardversion gewählt wird, in denen die Geldhaltung über die Nutzenfunktion oder über eine Vorauskassenbedingung gerechtfertigt wird. Ich schließe daraus, daß die Geldnachfrage auf andere Weise motiviert werden muß.

# 1. Introduction

Nominal rigidities that arise from price or wage staggering or from costs of changing prices provide a major rational for the non neutrality of money (Blanchard 1990). A considerable number of papers has recently explored this non neutrality within dynamic general equilibrium models. Their success to replicate some well known stylized facts of the business cycle within these models has been mixed.

Cooley and Hansen (1998) consider a cash in advance model with nominal wage contracts that fails to account for the positive correlation of inflation and output and the negative correlation between the price level and output. Chari, Kehoe, and McGrattan (1998) demonstrate the weak internal propagation of monetary shocks in a price staggering model. Their finding is echoed by Christiano, Eichenbaum, and Evans (1997) who compare a model with preset prices to a limited participation model. In both models the real effects of monetary shocks fade out quickly. The authors attribute this failure to the absence of labor market frictions. Their conjecture is supported by Roeger (1999) who considers costs of price adjustment and wage staggering together with costly search in the labor market along the lines of Merz (1995). His model mimics the procyclical inflation rate and the countercyclical price level and displays stronger propagation of monetary shocks.

More recent papers by Bergin and Feenstra (2000) and by Huang, Liu, and Phaneuf (2000) highlight the role of the input-output production structure in generating persistence.

In this paper I explore a simpler but obvious propagation mechanism, namely costs of adjusting employment at the firm level. These costs have various sources: advertising job openings, screening applicants, training newly hired workers, overtime premia, and dismissal protection regulations. In a dynamic framework they cause firms to stretch decisions to hire or lay off workers over future periods. Sargent (1987) introduced convex costs of labor adjustment in his New Classical macroeconomic model, and I shall study their consequences in the context of a dynamic general equilibrium model of a small open economy. I borrow the basic structure of this model from Correia, Neves, and Rebelo (1995) and introduce money via transaction costs being proportional to consumption expenditures. Nominal rigidities arise from two sources: staggered wage setting and costs of price adjustment. I specify the process of wage setting according to the "expected-market-clearing-case": nominal wage rates are set in order to achieve the clearing of next period's labor market given today's expectations. Although there are no convincing microfoundations for that case<sup>1</sup> it has the advantage that it does not introduce other sources of persistence that emerge from imperfect substitutability between different types of labor as has been shown by Ascari and Garcia (1999). Costs of price adjustment arise if the rate of price change differs from average inflation (see Hairault and Portier 1995). Given these costs, the markup of prices over marginal costs decreases in the wake of an unexpected monetary shock and increases in response to a productivity shock. Hence, the cyclical properties of the markup depend on the relative size of both types of shocks. Recently, Linnemann (1999) showed that markups in Germany are weakly countercyclical. To account

<sup>&</sup>lt;sup>1</sup> Wage setters should enjoy market power. But in that case, wage rates can not be competitive. Furthermore, expectational errors force the wage setters off their supply curve, whereas in a monopoly framework it is ex post always optimal to satisfy labor demand.

for that fact, I introduce a second source of cyclical markups, previously studied by Galí (1994). If the price elasticity of investment demand exceeds the price elasticity of consumption demand, a supply shock that increases investment more than consumption raises the price elasticity of a firm's demand curve and lowers its markup. If adjustment costs of prices are moderate, this effect dominates and markups decrease.

This mechanism also intensifies the response of employment and output to changes of the world real interest rate, which are quite small in models without frictions, as those of Mendoza (1991) and Correia, Neves, and Rebelo (1995). Since higher world interest rates depress home investment they increase the markup, lower the real wage and depress employment. Nevertheless, the contribution of world interest rate movements to economic fluctuations remains negligible, quite in contrast to recent findings by Blankenau, Kose, and Yi (1999).

The combination of adjustment costs and nominal rigidities considered in this paper is able to explain observed patterns of cross correlations between real and monetary variables and improves the cyclical properties of real variables. However, nominal prices are much more volatile than in the data. And this finding seems to be robust against similar motivations of the demand for money, as, e. g., money in the utility function or a cash in advance constraint. I take this as suggestive for a more elaborate modeling of the economy's financial sector.

I develop my model of a small open economy in the next section and study its properties in Section 3. Section 4 concludes.

# 2. The Model

My starting point is the small open economy model of Correia, Neves, and Rebelo (1995). I introduce money demand into that model and extend it to allow for monopolistic competition on the product market and staggered wage setting on the labor market.

The economy is populated by a representative household and by a continuum of mass  $J_t$  of firms. There are three different types of assets held by the representative household: shares of domestic firms, bonds traded on the world capital market, and domestic money M. Bonds B are denoted in terms of a composite good, whose money price at time t is  $P_t$ . They pay a variable real rate of interest  $R_t^F$ , given exogenously to the home country. Since I will consider only symmetric equilibria, in which each individual firm earns the same amount of profits and has the same stock market value as any other firm, I will save on notation and let  $S_t$  denote the total number of shares in domestic firms priced at  $v_t$  and paying a variable dividend of  $d_t$  per unit.

# 2.1. Households

**Preferences** The representative household supplies labor N, receives wages and dividends from domestic firms and earns interest from her holdings of foreign bonds. She allocates her income net of taxes and adjustment costs – to which I will turn in a moment – to consumption C, additional stocks, bonds and money balances so as to maximize expected life time utility

$$E_0\left[\sum_{t=0}^{\infty}\beta^t u(C_t, N_t)\right], \ \beta \in (0, 1).$$
(2.1)

In this expression  $E_0$  denotes expectations as of period 0,  $u(C_t, N_t)$  is the period t flow of utility associated with consumption  $C_t$  and working hours  $N_t$ , and  $\beta^t$  is the subjective discount factor attached to period t utility. I specify the momentary utility function according to Greenwood, Hercowitz, and Huffman (1988):

$$u(C_t, N_t) := \frac{[C_t - \theta A_t N_t^{\nu}]^{1-\eta} - 1}{1 - \eta}, \ \theta, \ \eta > 0, \ \nu > 1.$$
(2.2)

In this function  $1/\eta$  measures the intertemporal elasticity of substitution of the term  $C_t - \theta A_t N_t^v$ , which can be thought of as the utility of consumption net of the disutility of work effort.<sup>2</sup> The parameter v controls the sensitivity of labor supply with respect to the real wage. Correia, Neves, and Rebelo (1995) find this preference structure more in line with the time series properties of a small open economy than the more standard current period utility function

$$U(C_t, N_t) := \frac{C_t^{1-\eta} (1-N_t)^{\theta(1-\eta)} - 1}{1-\eta}, \ \theta, \ \eta > 0.$$

**Consumption Demand** To motivate the monopolistically competitive output market I assume that  $C_t$  is not a single good but a basket of differentiated products  $C_{jt}$  indexed on the interval  $[0, J_t]$  and defined by

$$C_t = \left[\int_{0}^{J_t} C_{jt}^{(\varepsilon-1)/\varepsilon} dj\right]^{\varepsilon/(\varepsilon-1)}, \ \varepsilon > 1,$$
(2.3)

where  $\varepsilon$  is the elasticity of substitution between any two components  $j_1$  and  $j_2$  of the basket  $C_t$ . Let  $P_{jt}$  denote the money price of good j. For a given size of the basket  $C_t$ , the demand for brand j, denotes by  $C_{jt}$ , solves the expenditure minimization problem min  $\int_0^{J_t} P_{jt}C_{jt}dj$  subject to equation (2.3). The solution is

$$C_{jt} = \left(\frac{P_{jt}}{P_t^C}\right)^{-\varepsilon} \frac{C_t}{J_t},$$
(2.4)

with the consumer price index  $P_t^C$  defined by

$$P_{t}^{C} = \left[\frac{1}{J_{t}}\int_{0}^{J_{t}}P_{jt}^{1-\varepsilon}dj\right]^{1/(1-\varepsilon)}.$$
(2.5)

<sup>&</sup>lt;sup>2</sup> Another way to interpret (2.2) is as indirect utility function in an environment with home production, where the production function for home goods  $C_{ht}$  is given by  $C_{ht} = \psi_0 A N_{ht}$  and where the underlying direct utility function is  $\ln(C_t + C_{ht}) - \gamma \ln(N_t + N_{ht})$ .

Labor Supply To provide the basis for wage staggering it may be helpful to think of the household as composed of n + 1 members. Member 1 supplies labor on the spot market and members i = 1, 2, ..., n sign wage contracts with entrepreneurs. Let W denote the nominal wage per efficiency unit of labor AN, and let the superscripts s and c distinguish prices and quantities on the spot and the contract markets, respectively. Thus, labor sold in period t at the market clearing nominal wage  $W_t^s$  is  $N_t^s$ , and total employment at preset wages is  $N_t^c$ . There are n overlapping wage contracts. A contract signed i = 1, 2, ..., n quarters ago is in effect until period t + n - i. It specifies a nominal wage  $W_{it}^c$  for each quarter  $\tau = t - i + 1, ..., t + n - i$  of the contractual period and transfers the right to determine employment  $N_{it}^c$  at these wages to the entrepreneurs. Hence, the household's members work a total of  $N_t$  hours given by

$$N_t = N_t^s + \sum_{i=1}^n N_{it}^c.$$

At time t the contract labeled n expires and will be renegotiated. I assume that the contracting parties choose  $W_{1t+1}^c$  to meet the household's first order conditions with respect to labor supply given her expectations of next period's employment and prices. In the remaining n - 1 quarters this wage increases according to average inflation at the gross rate  $\pi$ .<sup>3</sup>

Budget Constraint Let  $P_t$  denote the money price of aggregate output<sup>4</sup> so that

$$\frac{W_t^s}{P_t} A_t N_t^s + \sum_{i=1}^n \frac{W_{it}^c}{P_t} A_t N_{it}^c$$

is the household's real wage income. Let  $B_t$  and  $S_t$  denote the households holdings of foreign bonds and shares of domestic firms, respectively. Bonds bear interest payments  $R_t^F$  per unit and shares deliver dividend payments  $d_t$  per unit. In addition to wages, interest and dividends the household receives transfers from abroad and must pay taxes  $T_t^G$  to the government.

Trade on consumption goods involves transaction costs  $TC_t$  that increase with the volume of trade  $C_t$  and decrease with the end of period stock of real balances  $M_{t+1}/P_t$ . I assume the following cost function in terms of the composite good:

$$TC_t = \gamma \left(\frac{C_t}{M_{t+1}/P_t}\right)^{\kappa} C_t, \ \gamma, \kappa > 0.$$

Using end of period real money holdings instead of beginning of period real balances  $M_t/P_t$  implies that money is neutral in the loglinearized model, if the money supply

<sup>&</sup>lt;sup>3</sup> Alternatively, as in Fischer (1977), an expected market clearing wage for each of the *n* quarters could be specified, or, as in Taylor (1979), the average market clearing wage. These hypotheses imply a slightly higher degree of wage flexibility but burden the model with additional state variables. Cooley and Hansen (1995) and Cho and Cooley (1995) solve the market clearing model numerically for the function relating the wage rate to the model's states and employ that function in the wage setting process.

<sup>&</sup>lt;sup>4</sup> It is not necessary to be precise on the aggregator function, since we will consider only symmetric equilibria, when  $P_t$  is equal to the individual money price of a (typical) firm *j*.

shock is not autocorrelated and if nominal rigidities are absent. The intuition behind this result is simple: Consider a lump sum, one time cash transfer to the household. The household will increase her consumption demand and lower her labor supply. Therefore, the money price of output will increase and beginning of period money balances depreciate raising transaction costs. End of period real money balances, however, do not change, and thus, the real variables of the model remain unaltered.

Income net of consumption expenditures  $(P_t^C/P_t)C_t$  and transaction costs is used to acquire newly issued shares  $S_{t+1} - S_t$  at relative price  $v_t$  and to increase holdings of foreign bonds and money balances. Summing up, the household's budget constraint reads:

$$v_{t}(S_{t+1} - S_{t}) + B_{t+1} - B_{t} + \frac{M_{t+1} - M_{t}}{P_{t}} \leq \frac{W_{t}^{s}}{P_{t}} A_{t}N_{t}^{s} + \sum_{i=1}^{n} \frac{W_{it}^{c}}{P_{t}} A_{t}N_{it}^{c} + d_{t}S_{t} + R_{t}^{F}B_{t} + T_{t}^{F} - T_{t}^{G} - \frac{P_{t}^{C}}{P_{t}} C_{t} - \gamma \left(\frac{C_{t}}{M_{t+1}/P_{t}}\right)^{\kappa} C_{t}.$$

$$(2.6)$$

First Order Conditions The household maximizes (2.1) subject to (2.6). The solution to this program must satisfy the following first order conditions:

$$[C_t - \theta A_t N_t^{\nu}]^{-\eta} = \Lambda_t \left[ \frac{P_t^C}{P_t} + \gamma (1 + \kappa) \left( \frac{C_t}{M_{t+1}/P_t} \right)^{\kappa} \right],$$
(2.7*a*)

$$\frac{W_t^s}{P_t} = \nu \theta N_t^{\nu-1} \left[ C_t - \theta A_t N_t^{\nu} \right]^{-\eta}, \qquad (2.7b)$$

$$0 = E_t \left( \Lambda_{t+1} \; \frac{W_{1t+1}^c A_{t+1}}{P_{t+1}} - \nu \theta \, A_{t+1} N_{t+1}^{\nu-1} \; [C_{t+1} - \theta \, A_{t+1} N_{t+1}^{\nu}]^{-\eta} \right), \tag{2.7c}$$

$$\Lambda_t = \beta E_t \Lambda_{t+1} (1 + R_{t+1}^F), \tag{2.7d}$$

$$\Lambda_t = \beta E_t \Lambda_{t+1} (1 + R_{t+1}^H), \quad R_{t+1}^H := \frac{d_{t+1} + v_{t+1}}{v_t} - 1, \qquad (2.7e)$$

$$\beta E_t \ \frac{\Lambda_{t+1}}{P_{t+1}/P_t} = \Lambda_t \left[ 1 - \kappa \gamma \left( \frac{C_t}{M_{t+1}/P_t} \right)^{1+\kappa} \right], \tag{2.7f}$$

where  $\Lambda_t$  is the lagrange multiplier associated with the budget constraint and  $E_t$  denotes expectations as of period t. Equation (2.7a) combined with (2.7d) equates the expected marginal rate of intertemporal substitution in consumption with the expected gross real interest rate  $1 + R_{t+1}^F$ . Equation (2.7b) determines labor supply on the spot market, and equation (2.7c) defines the new contract wage replacing the contract expired in period t. Equations (2.7d) and (2.7e) taken together require that the expected

return on foreign bonds equals the expected return on domestic stocks. Condition (2.7f) equates the discounted expected marginal return on money holdings with current period costs of increasing the stock of money.

#### 2.2. Firms

Firm j's Investment Demand The output market is monopolistically competitive. At each quarter t there is a continuum of mass  $J_t$  of firms. Each firm buys products from all other firms to increase its stock of capital. Let investment of firm  $j \in [0, J_t]$  be given by

$$I_{jt} = \left[\int_{0}^{J_{t}} I_{kjt}^{(\zeta-1)/\zeta} dk\right]^{\zeta/(\zeta-1)}, \ \zeta > 1,$$
(2.8)

where  $\zeta$  denotes the elasticity of substitution between any two components  $k_1$  and  $k_2$  of the basket  $I_{jt}$ . Firm j's demand of good k, denotes by  $I_{kjt}$ , solves the expenditure minimization problem min  $\int_0^{J_t} P_{kt}I_{kjt}dk$  subject to (2.8). The solution is:

$$I_{kjt} = \left(\frac{P_{kt}}{P_t^l}\right)^{-\zeta} \frac{I_{jt}}{J_t},\tag{2.9}$$

with price index

$$P_t^l = \left[\int_0^{J_t} P_{kt}^{1-\zeta} dk\right]^{1/(1-\zeta)}.$$
(2.10)

Since, in a symmetric equilibrium, all firms will choose the same amount of total investment,  $I_{jt} =: (I_t/J_t)$ , this function can be aggregated over all  $j \in [0, J_t]$  to deliver the demand for investment goods for each firm k:

$$I_{kt} = \left(\frac{P_{kt}}{P_t^l}\right)^{-\zeta} \frac{I_t}{J_t}.$$
(2.11)

The Demand for Firm j's Output Assume that the remaining components of aggregate demand  $Y_t$  (i. e. government's purchases of goods and the trade balance) have the same price elasticity as consumption goods. Equation (2.4) and (2.11) – with *j* replacing k –, then, imply the following demand schedule for firm *j*:

$$Y_{jt} = \left(\frac{P_{jt}}{P_t^C}\right)^{-\varepsilon} \frac{Y_t - I_t}{J_t} + \left(\frac{P_{jt}}{P_t^I}\right)^{-\zeta} \frac{I_t}{J_t}.$$
(2.12)

**Labor Demand of Firm** *j* Each firm  $j \in [0, J_t]$  uses labor and capital services,  $L_{jt}$  and  $K_{jt}$ , respectively, to produce output  $Y_{jt}$  according to

$$Y_{jt} = Z_t (A_t L_{jt})^{\alpha} K_{jt}^{1-\alpha} - F, \ \alpha \in (0, \ 1), \ F > 0,$$

$$L_{jt} := (\varphi/n)^{-\varphi} (1-\varphi)^{(\varphi-1)} \left[ \prod_{i=1}^{n} (N_{ijt}^{\epsilon})^{\varphi/n} \right] (N_{jt}^{s})^{1-\varphi}, \quad \varphi \in (0, 1).$$
 (2.13)

The constant F specifies set up costs, and the role of the parameter  $\varphi$  in the Cobb-Douglas index for labor input  $L_{jt}$  will become obvious in a moment. Labor augmenting technical progress  $A_t$  grows at the constant rate a - 1:

$$A_{t+1} = aA_t, \ a \ge 1. \tag{2.14}$$

 $Z_t$  denotes a random shock to productivity with mean  $E(Z_t) = 1$ . The deviations from that mean,  $\hat{Z}_t \approx \ln(Z_t)$ , follow an AR(1)-process

$$\hat{Z}_t = \rho_Z \hat{Z}_{t-1} + \varepsilon_t^Z, \ \varepsilon_t^Z \sim n(0, \ \sigma_{\varepsilon^Z}).$$
(2.15)

Minimization of wage costs,

$$W_t^s N_{jt}^s + \sum_{i=1}^n W_{it}^c N_{ijt}^c,$$

for a given level of employment  $L_{it}$  implies the demand for the various types of labor:

$$N_{ijt}^{c} = \frac{\varphi}{n} \frac{W_{t} L_{jt}}{W_{it}^{c}},$$
(2.16a)

$$N_{jt}^{s} = (1 - \varphi) \; \frac{W_{t}L_{jt}}{W_{t}^{s}}, \qquad (2.16b)$$

$$W_t := \left[\prod_{i=1}^{n} (W_{it}^c)^{\varphi/n}\right] (W_t^s)^{1-\varphi}.$$
 (2.16c)

Note, that if the wage rates of all different kinds of labor are equal, as they will be under perfect foresight since labor is homogeneous from the household's perspective, these imply

$$N_{jt} := N_{jt}^s + \sum_{i=1}^n N_{ijt}^c = \varphi L_{jt} + (1 - \varphi) L_{jt} = L_{jt}.$$

Therefore,  $\varphi$  is a measure of the size of the contract sector and determines the degree of nominal rigidity.

Adjustment Costs Firms face three kinds of adjustment costs. Those associated with capital accumulation are introduced as in Correia, Neves, and Rebelo (1995).

$$K_{jt+1} - (1-\delta)K_{jt} = \Phi(I_{jt}/K_{jt})K_{jt},$$
(2.17)

where  $\delta$  denotes the depreciation rate of the firm's capital stock. The function  $\Phi(\cdot)$  is a concave function of its argument and has the following properties:<sup>5</sup>

<sup>&</sup>lt;sup>5</sup> If  $\Phi$  were identically equal to  $I_{it}/K_{it}$ , there were no adjustment costs of capital.

$$\Phi(a+\delta-1) = a+\delta-1, \tag{2.18a}$$

$$\Phi'(I_{it}/K_{it}) > 0 \quad \text{and} \quad \Phi'(a+\delta-1) = 1.$$
 (2.18b)

Price adjustment is costly if it departs from the average inflation factor  $\pi$ :

$$ACP_{jt} = \frac{\psi_1}{2} \left(\frac{P_{jt}}{P_{jt-1}} - \pi\right)^2, \quad \psi_1 \ge 0.$$
 (2.19)

Costs of adjusting the firm's working hours are linked to the rate of change of effective employment according to

$$ACH_{jt} = \frac{\psi_2}{2} \left( \frac{A_t L_{jt}}{A_{t-1} L_{jt-1}} - 1 \right)^2, \quad \psi_2 \ge 0.$$
(2.20)

These costs origin from various sources: screening, hiring and training expenses, overtime premia, and dismissal protection laws. It seems reasonable to assume a convex function. The German Industrial Constitution Law (Betriebsverfassungsgesetz), e. g., requires entrepreneurs to negotiate the terms of large lay offs with its employees. Workers that were dismissed under these regulations received on average a settlement of more than four months' salary (Franz 1996, p. 135). These costs drive a wedge between the marginal product of labor and the real wage. Faced with an unexpectedly increasing real wage firms keep a fraction of the workers they would have laid off otherwise. This introduces a second channel of impulse propagation in addition to the costs of adjusting the capital stock, which are a general feature of small open economy models.<sup>6</sup> **Firm j's Objective Function** The firm pays dividends

$$D_{jt} := d_{jt}S_{jt} = \Pi_{jt} - RE_{jt}$$
(2.21)

out of its profits

$$\Pi_{jt} := \frac{P_{jt}}{P_t} Y_{jt} - \frac{W_t}{P_t} A_t L_{jt} - ACH_{jt} - ACP_{jt}$$
(2.22)

and finances its investment expenditures from retained earnings  $RE_{jt}$  and from issuing new equities  $v_{jt}(S_{jt+1} - S_{jt})$ :

$$\frac{P_t^l}{P_t} I_{jt} = RE_{jt} + v_{jt}(S_{jt+1} - S_{jt}).$$
(2.23)

Let  $V_{jt} := v_{jt-1}S_{jt}$  denote the firm's stock market value and

$$C F_{jt} = \Pi_{jt} - \frac{P_t^l}{P_t} I_{jt}$$

<sup>&</sup>lt;sup>6</sup> See, e.g., Turnovsky (1997).

its current cash flow. Then,

$$V_{jt+1} + \Pi_{jt} = D_{jt} + \frac{P_t^l}{P_t} I_{jt} + v_{jt} S_{jt}$$

or

$$V_{jt+1} + CF_{jt} = (1 + R_{jt}^H)V_{jt}, \quad R_{jt}^H := \frac{d_{jt} + v_{jt}}{v_{jt-1}} - 1.$$

Iterating this equation forward and assuming

$$\lim_{T\to\infty}E_t\rho_{jT}V_{jT}=0,$$

where

$$\rho_{jT} = \left[\prod_{\tau=0}^{T} (1 + R^{H}_{jt+\tau})\right]^{-1}$$

is the appropriate discount factor for time T revenues, implies that the firm's value is given by the expected discounted present value of its future cash flows:

$$V_{jt} = E_t \sum_{\tau=0}^{\infty} \rho_{j\tau} C F_{jt+\tau}.$$
 (2.24)

First Order Conditions The firm chooses sequences of employment  $L_{jt}$ , investment expenditures  $I_{jt}$  and output prices  $P_{jt}$  that maximize (2.24) subject to the demand function (2.12), the production function (2.13), and the capital accumulation equation (2.17). The solution must satisfy the following first order conditions, where  $\vartheta_{jt}$ ,  $q_{jt}$ , and  $e_{jt}$  denote the shadow price of output, the shadow price of investment goods, and the share of investment demand in total demand, respectively:

$$0 = \frac{P_{jt}}{P_t} Y_{jt} - \vartheta_{jt} Y_{jt} [\varepsilon(1 - e_{jt}) + \zeta e_{jt}] - \psi_1 \frac{P_{jt}}{P_{jt-1}} \left(\frac{P_{jt}}{P_{jt-1}} - \pi\right) + E_t \frac{\psi_1}{1 + R_{jt+1}^H} \frac{P_{jt+1}}{P_{jt}} \left(\frac{P_{jt+1}}{P_{jt}} - \pi\right),$$
(2.25a)

$$e_{jt} = \left(\frac{P_{jt}}{P_t^I}\right)^{-\zeta} \frac{I_t/J_t}{Y_{jt}},$$
(2.25b)

$$0 = \alpha Z_t A_t (A_t L_{jt})^{\alpha - 1} K_{jt}^{1 - \alpha} \left( \frac{P_{jt}}{P_t} - \vartheta_{jt} \right)$$

$$-\frac{W_{t}}{P_{t}}A_{t} - \psi_{2} \frac{A_{t}}{A_{t-1}L_{jt-1}} \left(\frac{A_{t}L_{jt}}{A_{t-1}L_{jt-1}} - 1\right)$$
$$+E_{t} \frac{\psi_{2}}{1 + R_{jt+1}^{H}} \frac{A_{t+1}L_{jt+1}}{A_{t}L_{jt-1}^{2}} \left(\frac{A_{t+1}L_{jt+1}}{A_{t}L_{jt}} - 1\right), \qquad (2.25c)$$

$$q_{jt} = \frac{P_t^l / P_t}{\Phi'(I_{jt} / K_{jt})},$$
(2.25d)

$$q_{jt} = E_t \frac{1}{1 + R_{jt+1}^H} \left[ (1 - \alpha) Z_{t+1} (A_{t+1} L_{jt+1})^{\alpha} K_{jt+1}^{-\alpha} \left( \frac{P_{jt+1}}{P_{t+1}} - \vartheta_{jt+1} \right) - \left( \frac{P_{t+1}^I I_{jt+1}}{P_{t+1} K_{jt+1}} \right) + q_{jt+1} (1 - \delta + \Phi(I_{jt+1}/K_{jt+1})) \right].$$
(2.25e)

#### 2.3. Government

The government creates money,  $M_{t+1} - M_t$ , buys goods,  $G_t$ , and levies taxes,  $T_t^G$ . Its budget is balanced, if the following equation holds:

$$T_t^G = \frac{P_t^C}{P_t} \ G_t - \frac{M_{t+1} - M_t}{P_t}.$$
 (2.26)

Government expenditures grow deterministically with the same rate as labor augmenting technical progress  $A_t$ . Money supply develops according to

$$M_{t+1} = \mu_t M_t \tag{2.27}$$

with mean rate  $\mu$ . Deviations from that rate are stochastic and follow an autoregressive process with normally distributed innovations  $\varepsilon_t^{\mu}$ :

$$\hat{\mu}_t = \rho_\mu \hat{\mu}_{t-1} + \varepsilon_t^\mu, \ \varepsilon_t^\mu \sim n(0, \ \sigma_{\partial^\mu}).$$
(2.28)

# 2.4. Symmetric Temporary Equilibria

Symmetric Equilibria in Stationary Variables Since all firms face the same demand schedule and share the same costs for their various inputs, they will choose the same prices:

$$P_{jt} = P_{kt} \forall j, \ k \in [0, \ J_t].$$

Therefore, all relative prices,  $P_{jt}/P_t$ ,  $P_t^C/P_t$ , and  $P_t^l/P_t$  equal unity and the economy's price level  $P_t$  equals the selling price of a typical firm  $j \in [0, J_t]$ . Furthermore, all firms employ the same amount of resources and produce the same quantity of output:

$$L_{jt} = L_t/J_t, \ K_{jt} = K_t/J_t, \ Y_{jt} = Y_t/J_t,$$

where variables not indexed by j denote aggregate quantities.

To study fluctuations around the economy's deterministic growth path, it is convenient to work with stationary variables. Towards that end I use the following definitions: If not otherwise explicitly stated, smaller case letters denote variables scaled by the level of technical progress  $A_i$ , i.e.,

$$x_t := \frac{X_t}{A_t}.$$

The inflation factor of period t is denoted by

$$\pi_t := \frac{P_t}{P_{t-1}},$$

beginning of period real money balances are given by

$$m_t := \frac{M_t}{A_{t-1}P_{t-1}}$$

contract wages in terms of their beginning of period purchasing power are

$$w_{it}^c := \frac{W_{it}^c}{P_{t-1}}.$$

Finally, the following transformation of the marginal utility of wealth,  $\Lambda_t$ , proves to be stationary:

$$\lambda_t = \Lambda_t A_t^{\eta}.$$

The System of Stochastic Difference Equations Restated Condition (2.7a) may then be written as

$$\lambda_t \left[ 1 + (1+\kappa)\gamma \left(\frac{c_t}{m_{t+1}}\right)^{\kappa} \right] = (c_t - \theta N_t^{\nu})^{-\eta}.$$
(2.29)

Combining the definition of the wage index in (2.16c) with the first order condition on spot market labor supply (2.7b) yields:

$$w_t := \frac{W_t}{P_t} = \left[\prod_{i=1}^n \left(\frac{w_{it}^c}{\pi_t}\right)^{\varphi/n}\right] \left(\frac{\theta v N_t^{v-1} \left[c_t - \theta N_t^v\right]^{-\eta}}{\lambda_t}\right)^{1-\varphi}.$$
(2.30)

Equation (2.25d) reduces to

$$q_t = \frac{1}{\Phi' (I_t / K_t)},$$
 (2.31)

the definition of beginning of period money balances together with the assumption on money growth implies:

$$m_{t+1} = \frac{\mu_t m_t}{a \pi_t},\tag{2.32}$$

and the pricing equation (2.25a) can be written as<sup>7</sup>

$$0 = \frac{y_t}{j_t} \left[ 1 - \vartheta_t (\varepsilon (1 - (i_t/y_t)) + \zeta(i_t/y_t)) \right]$$
$$-\psi_1 \pi_t (\pi_t - \pi) + E_t \frac{\psi_1}{1 + R_{t+1}^H} \pi_{t+1} (\pi_{t+1} - \pi).$$
(2.33)

In the scaled variables, equation (2.17) reads:

$$ak_{t+1} - (1 - \delta)k_t = \Phi(i_t/k_t)k_t.$$
(2.34)

Aggregation of the dividends payed to the household using (2.21), (2.22), and (2.23) implies

$$d_t S_t = C F_t + v_t (S_{t+1} - S_t).$$

After substitution of  $d_tS_t$  by the right hand side of this expression and of  $T_t^G$  by the right hand side of (2.26) the household's budget constraint implies that net foreign assets change according to

$$ab_{t+1} - b_t = y_t + R_t b_t - c_t - g_t - i_t + t_t^F$$
  
- $\gamma \left(\frac{y_t}{m_t/(a\pi_t)}\right)^{\kappa} y_t - \frac{\psi_1}{2} j_t (\pi_t - \pi)^2 - \frac{\psi_2}{2} j_t \left(\frac{j_{t-1}}{j_t} \frac{L_t}{L_{t-1}} - 1\right)^2.$  (2.35)

The exogenous sequence of world interest rates determines the time path of the marginal utility of wealth: In the stationary variables (2.7d) can be written as:

$$\lambda_t = \beta a^{-\eta} E_t \lambda_{t+1} (1 + R_{t+1}^F).$$
(2.36)

Analogously, (2.7e) now reads

$$\lambda_t = \beta a^{-\eta} E_t \lambda_{t+1} (1 + R_{t+1}^H)$$
(2.37)

This condition induces the household to keep domestic shares together with foreign bonds in her portfolio. Given the firm's decisions on prices, employment, and investment, it determines the time path of the home stock price. The equations that govern the time sequences of employment and capital, (2.25c) and (2.25e), read in stationary variables:

$$q_{t} = E_{t} \frac{1}{1 + R_{t+1}} \left[ (1 - \alpha) Z_{t+1} L_{t+1}^{\alpha} k_{t+1}^{-\alpha} (1 - \vartheta_{t+1}) - (i_{t+1}/k_{t+1}) + q_{t+1} (1 - \delta + \Phi(i_{t+1}/k_{t+1})] \right],$$

$$(2.38)$$

<sup>7</sup> Note that  $j_t := J_t / A_t$ .

and

$$0 = \alpha Z_t L_t^{\alpha} k_t^{1-\alpha} (1-\vartheta_t) - w_t L_t - \psi_2 \frac{L_t}{L_{t-1}} j_{t-1} \left( \frac{j_{t-1}}{j_t} \frac{L_t}{L_{t-1}} - 1 \right)$$

$$+E_t \frac{\psi_2}{1+R_{t+1}} \frac{L_{t+1}}{L_t} \frac{j_t^2}{j_{t+1}} \left(\frac{j_t}{j_{t+1}} \frac{L_{t+1}}{L_t} - 1\right).$$
(2.39)

Using (2.7d), the money demand equation (2.7f) can be written as

$$0 = E_t \frac{\lambda_{t+1}}{\pi_{t+1}} \left[ 1 - \pi_{t+1} \left( 1 + R_{t+1}^F \right) \left( 1 - \kappa \gamma \left( \frac{c_t}{m_{t+1}} \right)^{1+\kappa} \right) \right].$$
(2.40)

Observe from (2.7c) that at any period *t*, irrespective of what group of workers is about to negotiate a new contract, that group will choose an expected real wage

$$\frac{W_{1t+1}^c}{P_{t+1}} = \frac{w_{1t+1}^c}{\pi_{t+1}}$$

satfisfying

$$0 = E_t \left( \lambda_{t+1} \; \frac{w_{1t+1}^c}{\pi_{t+1}} - v\theta \, N_{t+1}^{\nu-1} \; [c_{t+1} - \theta \, N_{t+1}^{\nu}]^{-\eta} \right).$$
(2.41)

The wage  $W_{1t+1}^c$  replaces the wage from the expired contract in the wage index of period t + 1. All other wage rates are increased by average inflation. Figure 2.1 illustrates the change of the wage index from quarter t to quarter t + 1. Thus, next period's structure of contract wages is determined by

$$w_{1t+1}^c$$
 and  $w_{i+1t+1}^c = \frac{\pi}{\pi_t} w_{it}^c$ ,  $i = 1, 2, ..., n-1$ . (2.42)

#### 2.5. The Balanced Growth Path

The following derivation of a balanced growth path serves two purposes. In the first place, it provides the point around which the system of stochastic difference equations is linearized to study its approximate dynamics. In the second place, the equations characterizing the deterministic balanced growth path allow us to determine the values of some of the model's key parameters.



Figure 2.1: Change of Wage Index

Thus, assume that there are no productivity shocks,  $Z_t = 1 \forall t$ , that money supply grows steadily at the rate  $\mu_t = \mu \forall t$ , that the world interest rate  $R^F$  is time invariant, and that government expenditures as well as foreign transfers increase at the rate of technical progress a - 1. Equations (2.29) to (2.42) may then be solved for the balanced growth path of the economy along which all scaled variables are constant, firms make no extra profits, and adjustment costs do not matter. Hence, ignore time indices and the expectation operator E for the moment being.

It is immediate from (2.36) and (2.37) that along a deterministic perfect foresight path bonds and stocks must yield the same rate of return

$$\mathbf{R}=\mathbf{R}^{F}=\mathbf{R}^{H},$$

and since the transformed marginal utility of wealth  $\lambda$  is constant along that path, (2.36) implies

$$\mathbf{R} = a^{\eta}/\beta - 1. \tag{2.43}$$

The inflation factor  $\pi$  follows from (2.32) and is equal to

$$\pi = \frac{\mu}{a}.\tag{2.44}$$

Hence, the Euler equation for money demand (2.40) can be solved for the velocity of money in terms of consumption goods, yielding

$$\frac{C}{M/P} = \mu \left[ \frac{1}{\kappa \gamma} \left( 1 - \frac{\beta a^{1-\eta}}{\mu} \right) \right]^{1/(1+\kappa)}.$$
(2.45)

It is obvious from equations (2.41) and (2.42) that the real wage per efficiency unit is equal for all groups of workers along a perfect foresight path. In that case, as noted above, equations (2.16a) and (2.16b) imply that employment is given by

$$N_t^s = (1 - \varphi)L_t = (1 - \varphi)N_t,$$

$$N_{it}^c = \frac{\varphi}{n} \ L_t = \frac{\varphi}{n} \ N_t,$$

equation (2.30) reduces to

$$\lambda w = v\theta N^{\nu-1} \left[ c - \theta N^{\nu} \right] \tag{2.46}$$

and the Euler condition on employment (2.39) simplifies to

$$w = \alpha N^{\alpha - 1} k^{1 - \alpha} (1 - \vartheta),$$

so that the stationary real wage w is a function of the capital-labor ratio k/N. The markup 1/(1 - 9) is determined by equation (2.33):

$$\vartheta = \frac{1}{\varepsilon(1 - i/y) + \zeta(i/y)} \tag{2.47}$$

with

$$\frac{i}{y} = \frac{a+\delta-1}{y/k}$$

from the properties of  $\Phi(i/k)$  stated in (2.18). In addition, these assumptions imply via (2.38), (2.31), and (2.43) that the user costs of capital exceed the marginal product of capital:

$$R + \delta = (1 - \alpha) N^{\alpha} k^{-\alpha} (1 - \vartheta).$$

We are now in the position to determine the long run number of firms relative to productivity growth, j = J/A. In the long run the value of a firm should equal the value of its capital stock. Given the assumptions on adjustment costs, this holds true if in the stationary cash flow (aggregated over firms)

$$CF = \underbrace{N^{\alpha}k^{1-\alpha} - jF - wN}_{(R+\delta)k} - (\alpha + \delta - 1)k$$

revenues minus the payroll equal the user costs of capital  $(R + \delta)k$ . This condition delivers

$$j = \frac{N^{\alpha}k^{1-\alpha}}{F} \quad \vartheta. \tag{2.48}$$

The production function (2.13), thus, implies

$$\frac{1}{1-9} y = N^x k^{1-x}.$$
 (2.49)

This in turn implies that the long run wage share is equal to the elasticity of production waith respect to labor:

$$\frac{wN}{y} = \alpha,$$

thus,  $1 - \alpha$  equals capital's share, and (2.36) implies that the output-capital ratio must equal

$$\frac{y}{k} = \frac{a^n - \beta(1-\delta)}{\beta(1-\alpha)}.$$
(2.50)

This solution determines the markup  $1/(1 - \vartheta)$ , the capital-labor ratio

$$\frac{k}{N} = \left(\frac{y/k}{1-\vartheta}\right)^{-1/\alpha},$$

and, thus, labor productivity  $y/N = (1 - \vartheta)(k/N)^{1-\alpha}$ . Using the stationary version of (2.29) to replace  $\lambda$  in (2.46) gives a condition on employment N:

$$\alpha \ \frac{y}{N} = v\theta N^{v-1} \left[ 1 + (1+\kappa)\gamma \left( \frac{C}{\mu M/P} \right)^{\kappa} \right].$$

The term in square brackets in this equation is a decreasing function of the money growth factor  $\mu$ , as can be seen from (2.45). Since the left hand side is independent of  $\mu$ , the stationary value of employment and, hence, the level of output relative to the level of technological progress, y, is a decreasing function of money growth. Finally, equation (2.35) gives:

$$\frac{y}{b} = [a - a^{\eta}/\beta] \left[ (1 - \gamma(Y/(M/P))^{\kappa}) - \frac{c}{y} - \frac{a + \delta - 1}{y/k} - \frac{g}{y} + \frac{t^F}{y} \right]^{-1}.$$

Given the exogenous shares of government expenditures, g/y, and net foreign transfers,  $t^F/y$ , this system can be solved for each given ratio of output to net foreign wealth, y/b.<sup>8</sup> I will now turn to the short run dynamics of the model.

# 3. Real and Monetary Business Cycles

# 3.1. Solution and Calibration

There is no convenient analytic solution of the system of non linear stochastic difference equations (2.29) to (2.42). Yet, there are different numerical procedures to derive approximate solutions. In the following, I employ the approach set out in King, Plosser, and Rebelo (1988). They rely on a log linear approximation of the above equations at the stationary state described in the previous subsection and replace the perfect foresight time paths of the forcing variables by their respective expected time paths. This linear system can be solved numerically. My simulations of the model rely on the parameter values presented in Table 3.1.

There are two groups of parameters. I picked the values of the parameters in the first group from the literature and calibrated the parameters in the second group from seasonally adjusted quarterly West German data covering the period 70.i to 89.iv. Although available, I did not include data before 1970. The investment-output ratio and the average propensity to consume show a marked trend during the sixties, which is clearly at odds with the assumption of a steady state. Also, I want to exclude the

Preferences			Production	_	
$\beta = 0.998$ $\epsilon = 5.0$ c/y = 0.54	$\begin{array}{l} \eta = 2.0 \\ \kappa = 5.0 \end{array}$	$v = 3.0$ $\frac{c}{M/P} = 0.86$	a = 1.0068 $\xi = 0.02$	$\begin{array}{l} \alpha = 0.65 \\ \rho_z = 0.93 \end{array}$	$\delta = 0.011$ $\sigma_{\rm z} = 0.0046$
Economic Policy		1	Market Structu	re	
g/y = 0.2 $\rho_{\mu} = 0.0$	$t^{\rm F}/y = - 0.017$ $\sigma_{\mu} = 0.018$	μ = 1.019	$\frac{1}{1-\vartheta} = 1.2$	φ = 0.38	n = 4

Table 3.1: Simulation Parameters

<sup>&</sup>lt;sup>8</sup> The model is underdetermined, since, in a small open economy, the real rate of interest is exogenously given. This restricts the model's parameters according to  $a^{\eta}/\beta = 1 + R$ .

impact of the reunification shock at the beginning of the nineties on the West German economy.<sup>9</sup>

Parameters from the first group are  $\beta$ ,  $\eta$ , v, the elasticity of q with respect to *i/k*, which I denote by  $\xi$ , and the markup  $1/(1 - \vartheta)$ . I chose the value of  $\beta$  to deliver a real rate of interest of 6.5 percent per year using equation (2.43).<sup>10</sup> My choice of  $\eta = 2$  follows, among others, Greenwood, Hercowitz, and Huffman (1988), but is also consistent with recent evidence provided by Ogaki and Reinhart (1998). The estimates of compensated wage elasticities in Franz (1996), p. 73, imply on average v = 3. The value of  $\xi = 0.02$  is in the range of values considered by Greenwood, Hercowitz, and Huffman (1988) and smaller than 1/15 which is used by Correia, Neves, and Rebelo (1995). Recently, Linnemann (1999) estimated an aggregate markup of  $1/(1 - \vartheta) = 1.2$  from German data. In the simulations with a cyclical price elasticity of demand I set  $\varepsilon$  arbitrarily to 5.0 and calculated  $\zeta$  from (2.47), using the value of  $i/k = a + \delta - 1$ , to give  $1/(1 - \vartheta) = 1.2$ .

West German real GDP per hour grew on average between 70.i and 89.iv at the rate of 0.68 % per quarter. This provides a = 1.0068. The average share of labor income in GDP at factor prices is  $\alpha = 0.65$ . This number rests on the assumption that self employed persons earn the same wage income as the average worker.

The production function in equation (2.13) implies that aggregate GDP evolves according to

$$Y_t = Z_t (a^t H_t)^{\alpha} K_t^{1-\alpha} - J_t F.$$

If the zero profit condition were always met,

$$J_t F = \vartheta_t Z_t (a_t H_t)^{\alpha} K_t^{1-\alpha},$$

this would imply

$$Y_t = (1 - \vartheta_t) Z_t (a_t H_t)^{\alpha} K_t^{1 - \alpha}.$$

I used the average value of  $\vartheta$  to compute a first approximation to the productivity shock  $Z_t$  from

$$Z_t = \frac{Y_t}{(1-\vartheta)(a_t H_t)^{\alpha} K_t^{1-\alpha}}$$

using the time series on hours  $H_t$  and capital  $K_t$ .<sup>11</sup> This procedure ignores the cyclical nature of the markup. As a partial remedy, I purged the series for  $\hat{Z}_t$  from the influence

<sup>&</sup>lt;sup>9</sup> The data are from the database of the German Institute of Economic Research (DIW) and the CD "50 Jahre Deutsche Bundesbank" compiled by the German Bundesbank and distributed by the Vahlen Verlag, Munich.

<sup>&</sup>lt;sup>10</sup> See King, Plosser, and Rebelo (1988).

<sup>&</sup>lt;sup>11</sup> Since quarterly data of the capital stock are not available, I computed this series from quarterly investment expenditures net of depreciation and from the annual capital stock series. The sum of net investment over the four quarters of a year,  $I_t := \sum_{i=1}^4 IN_{ti}$ , differs usually from the annual increase of the capital stock  $\Delta K_t = K_{t+1} - K_t$ . Hence, I scaled quarterly investment proportionately with  $I_t / \Delta K_t$ .

of monetary shocks following the procedure outlined in Hairault and Poitier (1995). The original  $\hat{Z}_t$  series has an AR(1)-coefficient of  $\rho_Z = 0.96$  and the estimated standard deviation of innovations is  $\sigma_z = 0.006$ . The AR(1) properties of the purged series are those presented in Table 3.1.<sup>12</sup> The quarterly rate of capital depreciation is the mean ratio of depreciation to the capital stock. The shares of consumption expenditures, government expenditures and net foreign transfers in GDP, c/y, g/y, and  $t^F/y$ , respectively, are time series averages of the respective ratios.

My measure of money is German M1 per capita, which has grown between 70.i and 89.iv at the average rate of  $\mu = 1.0186$ . An AR(1)-process fitted to the deviations from that rate provides  $\sigma_M = 0.018$  and an autocorrelation coefficient  $\rho_{\mu}$  that is not significantly different from zero. The value of  $\gamma$  is taken to replicate the average C/(M/P) being equal to 0.86. The parameter  $\kappa$  is related to the interest elasticity of money demand. Along a balanced growth path the Euler equation (2.7f) implies that this elasticity is given by

$$-\frac{1}{\pi(1+R)}\,\frac{1}{1+\kappa}.$$

I choose  $\kappa = 5$ , which is well in accordance with the estimates of the interest rate elasticity of German money demand provided by Hoffman, Rasche, and Tieslau (1995). There is another motivation for choosing this value. If  $\gamma$  is chosen to imply the empirical value of C/(M/P), the share of transactions costs in GDP is inversely related to  $\kappa$ . This share should be small, since interest forgone by holding cash instead of interest bearing deposits amounts to around 0.5 and 2 percent of GDP. For  $\kappa = 5$  transactions costs are less than 0.3 percent of GDP. Simulations with  $\kappa$  between 4 and 10 show no noteworthy sensitivity of the main results.

Finally, my measure of nominal wage stickiness is the average degree of unionization in (West) Germany between 1977 and 1988, which provides  $\varphi = 0.38$ . Since German wage contracts usually cover four quarters and are in fact overlapping, I set n = 4.

# 3.2. Adjustment Costs and Persistence

I will now reveal the contribution of wage staggering and adjustment costs of labor to the persistence of monetary shocks.

Figure 3.2 and Figure 3.3 display the effect of a time t = 3 monetary shock on employment and output, respectively. The size of the shock is one standard deviation of the innovations to the money growth rate. The solid lines correspond to a baseline case where costs of price adjustment are the single source of nominal rigidities. ACH = 0 depicts the impulse responses in the case of wage staggering without adjustment costs of labor. The last two lines show impulse responses if a one percent increase of the labor force incurs costs of 0.01 and 0.02 percent of value added, respectively. Obviously, wage staggering introduces important real effects which are considerably dampened by very small adjustment costs. Without those costs, output and employment return

<sup>&</sup>lt;sup>12</sup> The procedure of *Hairault* and *Poitier* (1995) is based on a VAR model in the percentage deviations of Z and  $\mu$  from their respective means. I selected a VAR order of 8, at which the residuals showed no further signs of autocorrelation.



Figure 3.2: Impulse Response of Hours



Figure 3.3: Impulse Response of Output

quickly to the balanced growth path shortly after the last cohort of contract workers was able to adjust their wage.

The time series moments in Table 3.2 give a quantitative account of this pattern. They reflect the average values of 500 simulations based on the parameter values in Table 3.1, if monetary shocks were the single driving force. These numerical experiments

Variable	I	Empirical <sup>a</sup>	Simulated <sup>b</sup>						
			$\varphi$ :	arphi=0		ACH = 0.0 %		= 0.01 %	
	s <sub>x</sub> c	r <sub>x</sub> <sup>d</sup>	s <sub>x</sub>	r <sub>x</sub>	s <sub>x</sub>	r <sub>x</sub>	S <sub>x</sub>	r <sub>x</sub>	
Production Hours	1.34 0.98	0.86 0.73	0.20 0.26	- 0.06 - 0.06	0.56 0.72	0.27 0.27	0.25 0.32	0.61 0.60	

Tab	le	3.	2:	Propagation	of	Monetary	/ SI	hocl	ks
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\* HP-filtered quarterly time series, 70.i to 89.iv.

<sup>b</sup> Average over 500 stimulated and HP-filtered time series with 80 quarters.

 $s_x :=$  standard deviation of variable x.

<sup>d</sup> First order autocorrelation.

assume adjustment costs of prices of 0.01 percent of revenue for a one percent deviation from average inflation  $\pi$ . Given only these costs, the real effects of money supply shocks are small. With wage staggering they are three times as large and weakly autocorrelated. Small adjustment costs of 0.01 percent of value added restore the original variability but introduce strong positive autocorrelation of the size empirically observed.

Figures 3.4 and 3.5 depict the effect of a productivity shock on hours and output under various circumstances. The solid lines present the benchmark case where only price adjustment is costly. The hump shaped graph of this function reflects the costs of price adjustment: the increased productivity raises the shadow price of output and for one period the markup jumps upwards and dampens the increase of the real wage. In the next period the markup has almost returned to its long run value and, therefore, the real wage increases further, raising labor supply and employment.

The case  $\varepsilon < \zeta$  reveals the influence of a cyclical price elasticity, if  $\varepsilon$  equals 5 and  $\zeta$  is chosen to imply a gross markup of 1.2. Since investment expenditures increase relatively more than consumption demand firms face an increased price elasticity if  $\varepsilon < \zeta$  and lower their markup. Thus, employment increases and reinforces output expansion slightly.

The graph labeled  $\varphi = 0.38$  displays the impulse responses under wage staggering, and ACH = 0.01 marks adjustment costs of labor. Hours' response is increasing with wage rigidity and, again, markedly reduced and spread out by adjustment costs. Table 3.3 confirms this visual impression numerically. It presents the outcome of simulations that assume productivity shocks as the single driving force of the business cycle. Compared to the results displayed in 3.2 it becomes obvious that the standard deviation and auto-correlation of output and hours are relatively insensitive to the various nominal and real frictions in the case of productivity shocks.

Hump shaped impulse response functions of output require a highly persistent productivity shock. This is illustrated by the impulse responses to productivity shocks for different values of  $\rho_Z$  in Figure 3.6.

Uncorrelated shocks to the foreign interest rate induce an income effect: if the home country is a net debtor, the increased interest payments reduce consumption and thereby increase the trade balance. The associated reduction in transactions costs is offset by a one time upward jump of the price level so that the velocity of money is unchanged. Output and production remain unaltered, too. If the interest rate shock is positively



Figure 3.4: Impulse Response of Hours



Figure 3.5: Impulse Response of Output

autocorrelated it also triggers substitution effects. The domestic household saves more in foreign bonds, thus, reducing funds available for home investment. Since firms invest less, the marginal product of labor drops, and, thus, employment and production decrease. If investment demand is more price elastic than consumption demand, the price elasticity of demand decreases, raising the markup and, thus, reinforcing the downturn of hours and production. Nevertheless, the real effects of interest rate shocks remain

Variable <sup>a</sup>	$\mathbf{\epsilon} = \mathbf{\zeta} = 6$		$arepsilon=5<\zeta$		$\phi = 0.38$		ACH = 0.01 %	
	s <sub>x</sub> <sup>b</sup>	r <sub>x</sub> <sup>c</sup>	s <sub>x</sub>	rx	5 <sub>x</sub>	rx	s <sub>x</sub>	r <sub>x</sub>
Output	0.88	0.72	0.92	0.71	0.93	0.71	0.89	0.74
Hours	0.23	0.79	0.28	0.76	0.29	0.75	0.24	0.85

Table 3.3: Propagation of Productivity Shocks

<sup>a</sup> Averages over 500 stimulated and HP-filtered time series with 80 quarters.

<sup>b</sup>  $s_x :=$  Standard deviation of variable x.

<sup>c</sup> First order autocorrelation of variable x.



Figure 3.6: Output Response as a Function of  $\rho_z$ 

negligible: A one percent increase of the world real interest rate reduces employment by less than 0.01 percent. Nominal rigidities and adjustment costs reduce this effect further as can be seen from Figure 3.7, which depicts the response of several variables to an one percent increase of the world interest rate that is autocorrelated with coefficient 0.5.

#### 3.3. Properties of Simulated Time Series

In this section I will assess the model's overall ability to replicate the key characteristics of business cycles. As a benchmark I use a model without nominal rigidities and adjustment costs of labor. In the loglinear approximation of the model described in Section 2 this amounts to setting  $\psi_1 = \psi_2 = \varphi = 0$  and  $\varepsilon = \zeta$ . Table 3.4 displays the second moments of simulated time series. They are averages of 500 runs.

With respect to the real variables, the numbers confirm what is known from the standard real business cycle model: consumption is less variable than output, whereas the



Figure 3.7: Impulse Response to a World Interest Rate Shock

standard deviation of investment expenditures is more than six times larger than that of GDP.<sup>13</sup> Consumption, hours, and the real wage are almost perfectly correlated with output. The trade balance is countercyclical. This finding is sensitive to both the size and persistence of productivity shocks, since it depends on two opposing forces. The temporary increase in total factor productivity raises the return on capital relative to the costs of external funds and firms build up more capital. This negative effect on the trade balance is counteracted by the household's desire to smooth her consumption path by increasing her savings. With highly autocorrelated supply shocks the first effect dominates. Since investment demand can be arbitrarily smoothed by increasing the value of  $\xi$ , the sign of the cross correlation between the trade balance and GDP eventually turns positive, if  $\xi$  is increased. With respect to the monetary variables the model predicts a weakly countercyclical price level and inflation rate. The price level as well as the inflation rate are far more volatile than in the data. Consequently, the real money stock is much smoother than empirically observed.

Table 3.5 presents the time series properties of the model with nominal rigidities and small adjustment costs of labor. The model outperforms the benchmark model in almost all respects. From the 39 second moments in Table 3.4 28 (about 72 %) get closer

<sup>&</sup>lt;sup>13</sup> The discrepancy between the simulated and the empirically observed standard deviation of private consumption is, at least in part, due to the fact that measured private consumption is the sum of expenditures on non durables and durables, the latter being more in the nature of investment goods. The standard deviation of expenditures on food, clothing, and housing, being 0.88, is much closer to the simulated value of 0.4.

Variable <sup>a</sup>	s <sub>x</sub> <sup>b</sup>	s <sub>x</sub> /s <sub>y</sub>	r <sub>x</sub>	r <sub>x</sub>	r <sub>×tb</sub>	r <sub>xm</sub>
Output	0.75	1.00	0.69	1.00	0.73	1.00
•	(1.34)	(1.00)	(0.86)	(1.00)	(-0.23)	(0.36)
Consumption	0.40	0.54	0.69	1.00	- 0.72	1.00
	(1.28)	(0.96)	(0.86)	(0.70)	(- 0.67)	(0.69)
Investment	4.53	6.06	0.63	0.92	- 0.94	0.92
	(3.72)	(2.78)	(0.88)	(0.82)	(-0.54)	(0.47)
Hours	0.25	0.33	0.69	1.00	- 0.73	1.00
	(0.98)	(0.73)	(0.51)	(0.69)	(- 0.18)	(0.08)
Real Wage	0.50	0.67	0.69	1.00	- 0.73	1.00
-	(0.92)	(0.69)	(0.49)	(0.13)	(0.15)	(-0.22)
Trade Balance	24.09	32.20	0.65	0.73	1.00	- 0.73
	(26.06)	(19.46)	(0.81)	(-0.23)	(1.00)	(0.63)
Inflation	1.75	2.34	- 0.08	- 0.06	0.10	- 0.06
	(0.33)	(0.25)	(0.14)	(0.15)	(0.33)	(-0.32)
Price Level	2.25	3.00	0.68	-0.10	0.03	- 0.10
	(0.65)	(0.49)	(0.86)	(-0.74)	(0.25)	( 0.56)
Real Money	0.35	0.47	0.68	1.00	- 0.73	1.00
	(3.01)	(2.24)	(0.83)	(0.36)	(- 0.63)	(1.00)

Table 3.4: Time Series Properties: Model Without Nominal Regidities

<sup>a</sup> Empirical values from HP-filtered German data in parenthesis.

<sup>b</sup>  $s_x :=$  standard deviation of HP-filtered simulated series of variable x,  $s_x/s_y :=$  standard deviation of variable x relative to standard deviation of output,  $r_x :=$  first order autocorrelation of variable x,  $r_{xy} :=$  cross correlation of variable x with output,  $r_{xtb} :=$  cross correlation of variable x with trade balance,  $r_{xm} :=$  cross correlation of variable x with real end of period money balances.

to their empirical counterparts, 3 remain unchanged and 8 (20.5 %) depart more from the estimated moments. In particular, the variability of output, consumption, hours, the real wage, inflation, the price level, and real money balances are closer to their empirical counterparts than in the benchmark model. The cross correlations with GDP are much more credible than in the baseline model, where consumption, hours, and the real wage are almost perfectly correlated with output, and where the trade balance is strongly countercyclical. The correlation of the markup with GDP is near the value of -0.4 reported by Linnemann (1999), and the rate of inflation is now procyclical.

Nevertheless, the model still implies too much price variability, and, hence, too less variability of real money balances as compared to West German data. I examined some related assumptions to motivate money holdings to check how robust this finding is. A model with standard preferences including real money holdings according to

$$u(C_t, N_t, (M_t/P_t)) = \frac{(\gamma C_t^{\kappa} + (1-\gamma)(M_t/P_t)^{\kappa})^{(1-\eta)/\kappa} (1-N_t)^{\theta(1-\eta)} - 1}{1-\eta}$$

provides results similar to those reported above. When I considered a cash in advance constraint, I found sunspot equilibria even for flexible wages, moderate adjustment costs of hours and prices. In the simulations where the equilibrium is determinate, inflation and the price level are still to volatile.

Variable <sup>a</sup>	s <sub>x</sub> <sup>b</sup>	s <sub>x</sub> /s <sub>y</sub>	r <sub>x</sub>	r <sub>xy</sub>	r <sub>xtb</sub>	r <sub>xm</sub>
Output	0.92	1.00	0.73	1.00	- 0.52	1.00
	(1.34)	(1.00)	(0.86)	(1.00)	(0.23)	(0.36)
Consumption	0.53	0.58	0.74	0.92	- 0.38	0.91
•	(1.28)	(0.96)	(0.86)	(0.70)	(- 0.67)	(0.69)
Investment	4.78	5.21	0.62	0.87	- 0.87	0.88
	(3.72)	(2.78)	(0.88)	(0.82)	(- 0.54)	(0.47)
Hours	0.36	0.40	0.73	0.80	- 0.28	0.79
	(0.98)	(0.73)	(0.51)	(0.69)	( 0.18)	(0.08)
Real Wage	0.65	0.70	0.64	0.56	- 0.23	0.53
-	(0.92)	(0.69)	(0.49)	(0.13)	(0.15)	( 0.22)
Trade Balance	24.02	26.14	0.67	- 0.52	1.00	- 0.56
	(26.06)	(19.46)	(0.81)	( 0.23)	(1.00)	(-0.63)
Inflation	1.67	1.82	- 0.08	0.08	0.05	0.12
	(0.33)	(0.25)	(0.14)	(0.15)	(0.33)	( 0.32)
Price Level	2.19	2.39	0.69	- 0.07	0.06	-0.10
	(0.65)	(0.49)	(0.86)	( 0.74)	(0.25)	(- 0.56)
Real Money	0.40	0.44	0.69	1.00	- 0.56	1.00
-	(3.01)	(2.24)	(0.83)	(0.36)	( 0.63)	(1.00)
Mark Up	0.35	0.38	- 0.00	0.38	0.30	- 0.42

Table 3.5: Time Series Properties:  $\phi = 0.38$ , ACP = 0.01 %, and ACH = 0.005 %

<sup>a</sup> Empirical values from HP-filtered German data in parenthesis.

<sup>b</sup>  $s_x :=$  standard deviation of HP-filtered simulated series of variable x,  $s_x/s_y :=$  standard deviation of variable x relative to standard deviation of output,  $r_x :=$  first order autocorrelation of variable x,  $r_{xy} :=$  cross correlation of variable x with output,  $r_{xtb} :=$  cross correlation of variable x with trade balance,  $r_{xm} :=$  cross correlation of variable x with real end of period money balances.

# 4. Conclusion

What does account for the persistence of monetary shocks in dynamic general equilibrium models of the business cycle? A number of papers have dealt with that question and point at labor market frictions besides those introduced by overlapping wage contracts.

In this paper I investigate an obvious source of persistence, namely small adjustment costs of labor at the firm level. These introduce indeed hump shaped impulse responses of hours worked in simulated time series. Compared with a benchmark model without nominal and real frictions my model outperforms the former in most respects.

However, its account of the time series properties of monetary variables is not satisfactory. As a number of experiments revealed, closely related models that introduce money into the utility function or that posit a cash in advance constraint fare not much better. I take this as suggestive to think about more sophisticated models of money demand.

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