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Directed Sector and Skill-Specific Technological Change: The Development of Wages for the High and Low Skilled*

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Abstract

This paper presents a dynamic two sector, two skill groups model of endogenous skill and sector specific technological change. The sectors refer to a "high-tech" and a "low-tech" sector of an economy. The direction of technological change is driven by market forces determined by the skill composition of the work force. It is shown that a change in this skill composition - a higher growth rate of the high skilled workforce in the "high-tech" sector than in the "low-tech" sector - leads to an increasing relative wage of the high skilled despite the fact that the aggregate supply of the high skilled might rise. This directed technological adjustment can easily overcome the usual substitution effect which would lead the relative wage to fall. The important result of the model is that the result does not depend on high values of the elasticity of substitution as necessary in other models of directed technological change, e.g. Acemoglu (1998, 2001). Further some of these models can be interpreted as special cases of the present model. Some open economy extensions show how effects of the mentioned change in the skill composition of the work force can spill over from one country to another if both countries engage in free trade and if the state of technology is determined globally.

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1. Introduction

The growing wage inequality between high educated and less educated workers in the U.S. and other major countries has been a field of high interest for economists. For a recent review of the corresponding literature see Acemoglu (2002). The relative number of college graduates in the American working population has increased from 6 percent in 1939 to over 28 percent in 1996. By the same time the proportion of workers not having a high school degree dropped from 68 to 10 percent (Autor, Katz and Krueger, 1998). Despite this rise in the relative supply of educated workers it is well known that the wage mark up for education measured by the College wage premium has also increased during this period (Acemoglu (1998)), with the exception of the 1970s where the college premium actually fell. This premium, compared with workers having only a high school degree, enabled college graduates to earn a 55 percent higher wage in the 1970s. During the 1970s this difference fell to 41 percent but increased thereafter to 62 percent in 1995. One popular view in the light of these facts is that the technological change which took place during the last decades was skill-biased, favoring the high skilled relatively more than the low skilled workers[1]. Other theories explaining the rising relative wage for the high skilled focus on the institutional change that took place in the labor market. Notably these are the declining minimum wage and the declining unionization (see Freeman (1991), DiNardo, Fortin and Lemieux (1995) or Lee (1999)). Yet another possible reason for the rise in the relative wage is the impact of increasing trade with low developed countries, see among others e.g. Fenestra and Hanson (2001). Although these arguments might be important, the focus of this paper lies on the change of technology which, as will be shown, can have major consequences on the distribution of wages.

The aforementioned skill bias hypothesis obviously raises the question why the technological development was shaped in favor of the high skilled. Are there good economic reasons guiding the research sectors of an economy to invent relatively more technological advances for the high skilled? This question is addressed by the literature concerning the so-called *directed technological change* (see Acemoglu (1998, 1999a,b, 2001), Acemoglu and Zilibotti (2001) and Kiley (1999)). The models in these articles argue that the direction of technological development is influenced by the demand of producing firms using the technology. It is assumed that the economy consists of two sectors, of which one uses only high skilled workers and the other one only low skilled. Both sectors produce intermediate goods which are combined in the final stage of production to yield the final output. Further the articles focus on the

[1] Also objections against this hypotheses can be found in the literature, see e.g. Card and DiNardo (2002).

situation where the technology used in the different sectors is skill specific, i.e. the high skilled work with a different set of production technology than the low skilled. In other words skill and sector have the same meaning in these models. If the number of potential users of one specific technology to be developed increases, the profit for the research facility who is to invent that technology increases as well. This is the so-called *market size effect*. This effect is accompanied by the *price effect*: If the sector employing only high skilled personal is growing through an increase of the high skilled proportion of the work force and technological advances, the relative price of its output used in the final good production decreases because of the usual substitution process. This effect counteracts the market size and the overall effect depends on the absolute value of the elasticity of substitution in the final production stage. The main result of these articles is that, if the mentioned elasticity of substitution is larger than at least two, then the *directed technological change* leads to an increase of the wage mark up for high skilled if the proportion of the high skilled in the working population increases.

Although the suggestion of the cited articles is very appealing, it has two drawbacks. First the result relies on the elasticity of substitution being larger than two plus a term whose value is positive and unknown. Second, the above models make an important strong assumption: The skill bias in technological progress is by the same time the sector bias. This is because only high skilled workers are present in the high skilled sector and only low skilled workers are employed in the low skilled sector. This rules out the possibility of the occurrence of different skill and sector biased technological changes.

The literature concerning the implication of sector and skill biased technological change comes mostly from the field of international economics. The analysis in this body of literature focuses on the effects of a different skill bias of technological change across sectors. Xu (2001) analyses exogenous skill and sector biased technological change in a two country, two goods, two factors Heckscher-Ohlin model. He shows how changes in the exogenous technology parameters affect the relative factor prices under different sets of assumptions about the trade environment under which the economy acts. Krugmann (2000) also addressed the question how relative factor prices of high and low skilled workers are affected by sector specific skill biased exogenous technological change in a Heckscher-Ohlin model. Recently Haskel and Slaughter (2002) used a model with exogenous technological changes which can take the form of sector specific skill biased technological change as well as sector specific skill neutral technological progress. These authors show how these different sources of technological changes affect the relative wage of high and low skilled workers. What is missing seems to be a unifying approach which takes account of the different mentioned technological changes as well as of their possible endogenity.

The model developed in this paper aims to add to the literature by filling this gap. This is done by presenting a framework which allows for different endogenous skill biased technological changes in different sectors as well as different endogenous technological changes which are skill neutral but sector biased. Therefore it might be termed a model of directed sector and skill specific technological change. It is shown that for the relative wage of the high skilled workers to rise in response to a change in the skill structure not the absolute value but the differences of the elasticities of substitution matter. This result stems from an easy formulation of the innovation possibility frontier used below. But also a more generalized formulation of the research process shows that the mentioned high elasticities of substitution may not be required for a rising relative wage for the high skilled. Further it will be shown that some of the above cited models of directed technological change can be seen as special cases of the model in the present paper, and therefore might be interpreted carefully.
Section 2 sets up the basic model and section 3 examines the direction of endogenous skill and sector specific technological change. Section 4 shows the relationship of the presented model and some existing models. Some quite interesting open economy extensions are presented in section 5. Finally section 6 concludes.

2. The basic model

2.1 The production technology

To analyze how the technological changes mentioned in the introduction can affect the relative wage of high and low skilled workers, a two sector, two factor model is used. Firms in both sectors of the economy are producing, using both high and low skilled workers denoted by $H(j)$ and $L(j)$ respectively. The index *i*=*H*, *L* corresponds to a "high-tech" and a "low-tech" sector, the index *j* denotes firm *j*. Firm *j* in sector *i* produces output $Y_i(j)$ with following production technology:

$$(1)\qquad Y_i(j)=\left\{\left[\left(\int_{A(i,j)} x_i(l,j)^{\beta}\,dl\right)L(j)^{1-\beta}\right]^{\rho}+\mu_i\left[\left(\int_{A'(i,j)} x_i(h,j)^{\beta}\,dh\right)H(j)^{1-\beta}\right]^{\rho}\right\}^{1/\rho}.$$

Besides the use of labor there are also other inputs involved in production. For each type of labor there is a continuous set of technological equipment denoted by $A(i,j)$ and $A'(i,j)$ that can be used. The quantity of each particular machine to be used by firm *j* together with low and high skilled labor is denoted by $x_i(l,j)$ and $x_i(h,j)$. To keep the analysis tractable it is assumed that these machines fully depreciate after use in one particular period of time. The two sets of technological equipment play an important role in the model. Similar as in Stiglitz (1969) it is assumed that they both together form the support $[0;a_i]$ and that $A(i,j)=[0;\gamma_i(j)a_i]$, $A'(i,j)=[\gamma_i(j)a_i;a_i]$ and $\gamma_i(j)\in[0;1]$. These two disjoint sets might be interpreted as the technological resources which are devoted to each type of labor. Since this is a fundamental assumption of the model it seems necessary to elaborate on this issue a little bit more. To justify the assumption it is first necessary to think about these two disjoint sets maybe not literally as sets of machines, as in usual growth models, but more of technological resources. As will be shown later the demanded quantity of each variant, $x_i(l,j)$ and $x_i(h,j)$, will be the same regardless with which kind of labor it is combined. Therefore the assumption can be interpreted as a budgetary problem. The firm j is willing to spend, as will be shown later, a certain fraction of its revenues on technological resources and has to decide how much of this budget it will devote to the high and low skilled department.

The technological equipment and labor are combined according to a Cobb-Douglas production function with output elasticities β and $1-\beta$. This intermediate output which comes from the high and low skilled departments of firm j is then combined according to a CES production function to yield the final output $Y_i(j)$ of firm j in sector i. The production technology in this final stage is characterized by two parameters μ_i and ρ. μ_i is a sector specific parameter which determines the productivity of the high skilled intermediate output relative to the low skilled intermediate output. It is plausible to assume that $\mu_H > \mu_L \geq 1$, i.e. high skilled intermediate products are more productive in the "high-tech" sector and are in general not less productive than low skilled intermediate products. The parameter ρ determines the elasticity of substitution $\sigma = (1-\rho)^{-1}$ between high and low skilled intermediate products. If $\sigma = 0$ then there is no substitution possible and the sector stage of production is Leontief. The case $\sigma = 1$ is the Cobb-Douglas case and with $\sigma = \infty$ the final production stage is linear and the two intermediate products are perfect substitutes. If $\sigma < 1$ then the two intermediate products might be termed as in Acemoglu (2001) as gross compliments, if $\sigma > 1$ they are gross substitutes. For now it is only assumed that $\sigma > 0$ is fulfilled, which seems to be a quite reasonable assumption. Finally, it is clear that the production function (1) has constant returns to scale with respect to all four inputs $x_i(l,j)$, $x_i(h,j)$, $H(j)$ and $L(j)$.

From the preceding discussion of the production technology it is obvious that the parameter $\gamma_i(j)$ characterizes the nature of technological change. A rise in $\gamma_i(j)$ is by construction low skilled labor augmenting whereas a rise in 1-$\gamma_i(j)$ is necessarily high skilled labor augmenting.

To proceed in the analysis it is necessary to make some assumptions about the environment in which firm j acts. To simplify the computation, let firm j be one of many in its sector so that competition between these firms is perfect. Furthermore, if there is a large number of firms, the wages for high and low skilled labor can be seen as exogenous to the individual firm. All firms are faced with the same wages which are identical for high and low skilled workers regardless in which sector they are employed. The wages are entirely determined on the labor market which is also assumed to be perfectly competitive so that wages adjust to clear this market. No unemployment can occur. Regarding the market for technological resources I abstract from perfect competition. This is necessary to motivate a research sector which invents and sells the different variants $x_i(l,j)$, $x_i(h,j)$. The firms engaging in research will benefit from their inventions by a monopoly over the particular variant which is guaranteed through an everlasting patent.

2.2 The demand for technological resources

First, to simplify the notation, some additional terms should be introduced. The sector stage of production combines the high and low skilled intermediate products which are termed in the following by $Y_{H,i}(j) = (\int_{A'(i,j)} x_i(h,j)^{\beta} dh) H(j)^{1-\beta}$ and $Y_{L,i}(j) = (\int_{A(i,j)} x_i(l,j)^{\beta} dl) L(j)^{1-\beta}$, the costs of firm *j*, associated with the production of these intermediate products, are denoted by $c_{L,i}(j)$ and $c_{H,i}(j)$. The production function (1) can then be formulated as

(2) $$Y_i(j) = \left[Y_{L,i}(j)^{\rho} + \mu_i Y_{H,i}(j)^{\rho} \right]^{1/\rho},$$

and has a usual corresponding unit cost function if the intermediate products are used in a cost minimizing way. This unit cost function is equal to the price of final output in each sector due to the assumption of perfect competition:

(3) $$P_i(j) = \left[c_{L,i}(j)^{-\frac{\rho}{1-\rho}} + \mu_i^{\frac{1}{1-\rho}} c_{H,i}(j)^{-\frac{\rho}{1-\rho}} \right]^{-\frac{1-\rho}{\rho}},$$

where $P_i(j)$ is the price of final output of firm *j* in sector *i*. Later it will be clear that all firms in one sector will charge the same price. Let $\chi_{H,i}(h)$ and $\chi_{L,i}(h)$ be the prices for the variants of $x_i(h,j)$ and $x_i(l,j)$ and let these prices be identical for all firms in one sector. The demand for each variant is determined by the first order condition which equates the marginal product of each variant with its price. Since the variants combined with one kind of labor enter the production function symmetrically it is clear that the demand for all these variants by firm *j* will be identical. Standard calculation gives the following demand functions:

(4) $$x_{H,i}(h,j) = H(j) c_{H,i}(j)^{\frac{1}{1-\beta}} \beta^{\frac{1}{1-\beta}} \chi_{H,i}(h)^{-\frac{1}{1-\beta}},$$

(5) $$x_{L,i}(l,j) = L(j) c_{L,i}(j)^{\frac{1}{1-\beta}} \beta^{\frac{1}{1-\beta}} \chi_{L,i}(l)^{-\frac{1}{1-\beta}},$$

which have a constant price elasticity[2]. Since the inventor and producer of the particular variant is a monopolist, he sets the price as a mark up on marginal costs of production. Let these marginal costs be constant and normalize them to one, then the price for each unit of a variant is $\chi_{H,i}(h) = \chi_{L,i}(l) = 1/\beta$.

2.3 Determinants of the relative wage for high skilled

Given perfect competition it is clear that the relative wage, the ratio of the high to the low skilled wage, is given by the ratio of the corresponding marginal products

$$(6) \qquad \frac{w_H}{w_L} = \mu_i \left(\frac{1-\gamma_i(j)}{\gamma_i(j)} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{c_{H,i}(j)}{c_{L,i}(j)} \right)^{\frac{\beta}{1-\beta}\frac{\sigma-1}{\sigma}} \left(\frac{H(j)}{L(j)} \right)^{-\frac{1}{\sigma}}.$$

What still needs to be determined is the ratio between the costs of high and low skilled intermediate products. Since in the first production stage the variants $x_i(h, j)$ and $x_i(l, j)$ are combined with the corresponding kinds of labor using a Cobb-Douglas production technology, this ratio is given by

$$(7) \qquad \frac{c_{H,i}(j)}{c_{L,i}(j)} = \left(\frac{\gamma_i(j)}{1-\gamma_i(j)} \frac{w_H}{w_L} \right)^{1-\beta}.$$

Equation (7) assumes that the input factors in the first production stage are used in a cost minimizing way. Using (6) and (7) the relative wage can be written as

$$(8) \qquad \frac{w_H}{w_L} = \mu_i^{\frac{\sigma}{\sigma(1-\beta)+\beta}} \left(\frac{1-\gamma_i(j)}{\gamma_i(j)} \right)^{\frac{(\sigma-1)(1-\beta)}{\sigma(1-\beta)+\beta}} \left(\frac{H(j)}{L(j)} \right)^{-\frac{1}{\sigma(1-\beta)+\beta}}.$$

From this equation it can first be seen that the relative wage decreases if the relative skill structure measured by the ratio of high to low skilled workers in firm j increases. This is the usual substitution effect. The effect of high and low skilled augmenting technological change depends on the elasticity of substitution between high and low skilled intermediate products. If $\sigma > 1$ then the two just mentioned input factors are gross substitutes and high (low) skilled augmenting technological change will also be high (low) skilled biased. If $\sigma < 1$ the input factors are gross compliments and high (low) skilled augmenting technological change will be low (high) skilled biased. If production in the final stage is Cobb-Douglas, $\sigma = 1$, then high and low skilled augmenting technological change has no bias.

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[2] Here it is assumed that there is some sort of cost controlling active in the individual firm so that unit costs of producing intermediate goods can be taken as given for the high and low skilled department which demand the technological equipment. It is also assumed that the number of workers is determined outside the departments. This corresponds to a hierarchical organization structure.

The analysis so far is quite standard and the results are not very surprising. What happens if the skill bias represented in the model by a change in the parameter $\gamma_i(j)$ becomes endogenous will be analyzed in the following section.

3. Endogenous technological change

In this section first the implications of endogenous skill specific technological change will be examined. With the obtained results in hand, the model will then be extended by taking also into account that sector specific technological change is driven by market forces.

3.1 Skill specific technological change

Using the preceding results the unit cost function corresponding to the production function (1), given that firm j is a cost minimizer, can be written as

$$(9) \qquad P_i(j) = \beta^{-2\beta}\left[\left(\frac{w_L}{\gamma_i(j)a_i(1-\beta)}\right)^{-(\sigma-1)(1-\beta)} + \mu_i^{\frac{1}{1-\rho}}\left(\frac{w_H}{(1-\gamma_i(j))a_i(1-\beta)}\right)^{-(\sigma-1)(1-\beta)}\right]^{-\frac{1}{\sigma-1}}.$$

Note that in (9) the price for the variants of technological equipment is already substituted and factored out. The use of the variants of technological equipment has the effect of lowering the effective wage costs by a factor which is given by the number of these variants used with each kind of the two types of labor. Facing this relationship, firm j is now assumed to endogenously determine the amount of technological resources which is to be devoted to each kind of labor by choosing the appropriate value for $\gamma_i(j)$. Differentiating (9) with respect to $\gamma_i(j)$, setting this derivative equal to zero and using Shepard's Lemma, the following condition must be satisfied by firm j

$$(10) \qquad \frac{1-\gamma_i(j)}{\gamma_i(j)} = \frac{w_H H(j)}{w_L L(j)}.$$

That is, the ratio of the technological resources devoted to high and low skilled workers is equal to the ratio of their wage costs. Checking the second derivative it turns out that this can be cost minimum or maximum depending on whether $\sigma < 2 + \beta/(1-\beta)$ or $\sigma > 2 + \beta/(1-\beta)$. The economic reasoning behind the first case is, given the relative wage, if firm j decides to hire more high skilled workers relative to low skilled workers, the marginal product of each variant $x_i(h, j)$ increases. This makes it profitable for the firm to use more of these variants in combination with high skilled labor. At some point this incentive stops because high and low skilled workers are relatively essential in the production process. If the second case is true, it pays for firm j to concentrate only on one skill group because the elasticity of substitution is so high that the other skill group can be easily replaced. Since the first case is the interesting one the focus of the following analysis will be on that case. Section 4 will examine a special case where the second case occurs.

Combining equation (9) and (10) it follows that the ratio of technological resources for high and low skilled workers can be written as a function of the relative wage and μ_i alone. This means that the distribution of technological resources is identical for all firms in one sector:

$$(11) \qquad \frac{1-\gamma_i}{\gamma_i} = \mu_i^{-\frac{\sigma}{(\sigma-1)(1-\beta)-1}} \left(\frac{w_H}{w_L}\right)^{\frac{(\sigma-1)(1-\beta)}{(\sigma-1)(1-\beta)-1}} .$$

There are some interesting special cases arising from different values of the elasticity of substitution. If high and low skilled intermediate products are gross compliments then a higher relative high skilled wage means a higher ratio of technological resources for the high skilled employees. The same applies for the productivity parameter μ_i. The amount of variants of technological equipment used in combination with high skilled labor will always be larger than 0.5 if wages for high skilled are higher than for low skilled and will be higher in the "high-tech" sector. If high and low skilled intermediate products are gross substitutes, it is still true that a higher productivity parameter μ_i leads to a higher ratio of technological resources for the high skilled. But the opposite is true with respect to the relative wage. If this relative wage is increasing, more variants of equipment will be devoted to the low skilled workers.

If γ_i is identical for all firms in sector *i*, it can be seen from equation (8) that the skill composition must also be identical for all firms in sector *i*. The relative wage is then entirely determined by the ratio of high to low skilled workers in each sector.

$$(12) \qquad \frac{w_H}{w_L} = \mu_i^{\sigma} \left(\frac{H_i}{L_i}\right)^{(\sigma-1)(1-\beta)-1} ,$$

where H_i and L_i denote the high and low skilled workforce in each sector. Since the technology distribution parameter γ_i is identical for all firms in one sector as is the relative wage, all firms in one sector employ the same skill composition. This skill composition must then also be equal to the skill composition of the whole sector. Equation (12) is exactly the result of Acemoglu (1998), but here it applies only to one sector not economy wide. Note that the possibility of rising relative wage in response to an increase in the ratio of high to low skilled workers can never occur on the sector level because of the implied condition $\sigma < 2 + \beta/(1-\beta)$. In the following sections it will be shown that is not hindering the relative wage for the high skilled from rising if if sector specific technological change is taken into account.

To guarantee that the relative wage is identical in each sector it must further be assumed that high and low skilled workers can freely choose in which sector they work. The ratio of the relative skill compositions of the two sectors is given by

$$(13) \quad \frac{H_H / L_H}{H_L / L_L} = \left(\frac{\mu_H}{\mu_L} \right)^{-\frac{\sigma}{(\sigma-1)(1-\beta)-1}} .$$

From this equation it can be seen that the "high-tech" sector always has the higher ratio of high to low skilled workers.

Substituting the relative wage as determined by equation (10), in equation (11) it turns out that the relative distribution of the variants of technological equipment can be written as

$$(14) \quad \frac{1-\gamma_i}{\gamma_i} = \mu_i^{\sigma} \left(\frac{H_i}{L_i} \right)^{(\sigma-1)(1-\beta)} .$$

Regarding the distribution of the variants of technological equipment the following conclusions can be drawn. If $\sigma < 1$, the "high-tech" sector has the higher ratio of high skilled to low skilled employees. In addition to this, this sector has the higher productivity parameter μ_i. From equation (11) it is clear that the "high-tech" sector has the higher relative distribution of the variants of technological equipment. So the effect of the productivity parameter μ_i outweighs the counteracting effect of the higher relative skill composition in (14). From the point on where the elasticity of substitution is larger than one both effects work in the same direction

3.2 Endogenous sector biased technological change

At the heart of the following analysis lies the unit cost function of the firms in the two sectors. Since all firms in each sector are identical, it is sufficient to concentrate on the sector production function. After the endogenous adjustment of the distribution parameter γ_i and taking the demand for variants of technological equipment into account, the production function can be written in reduced form as[3]

$$(15) \quad Y_i = a_i \beta^{\frac{2\beta}{1-\beta}} P_i^{\frac{\beta}{1-\beta}} \left[L_i^{(\sigma-1)(1-\beta)} + \mu_i^{\sigma} H_i^{(\sigma-1)(1-\beta)} \right]^{1/(\sigma-1)(1-\beta)}$$

To determine the corresponding cost function it is also necessary to compute the expenditures for the different variants of technological equipment. Surprisingly, it turns out using equations (4), (5), (9), (10) and (15) that the demand for each variant is the same, regardless of which kind of labor it is combined with[4]

[3] The reduced form can be obtained by using (1), the marginal product for each type of labor and the demand for the variants of technological equipment (4) and (5).

[4] The demand for each variant for all firms in one sector depends linearly on the produced sector output. Therefore the demand functions can easily be aggregated on the sector level.

(16) $x_i(l) = x_i(h) = \frac{1}{a_i}\beta^2 P_i Y_i$.

Now using equations (12), (15) and (16), the unit cost function can be computed as

(17) $$P_i = a_i^{-(1-\beta)}(1-\beta)^{-(1-\beta)}\beta^{-2\beta}\left(w_L^{\frac{(\sigma-1)(1-\beta)}{(\sigma-1)(1-\beta)-1}} + \mu_i^{-\frac{\sigma}{(\sigma-1)(1-\beta)-1}} w_H^{\frac{(\sigma-1)(1-\beta)}{(\sigma-1)(1-\beta)-1}} \right)^{\frac{(\sigma-1)(1-\beta)-1}{\sigma-1}}.$$

Up to now the sector wide technology parameter a_i was treated as exogenous and before turning to the case where a_i becomes endogenous let's see what equation (16) can tell about the development of wages. Building the total differential of (16) for each sector and subtracting the results yields the following relationship between the development of wages, the development of prices and the states of sector technology

(18) $$\hat{w}_H - \hat{w}_L = \frac{1}{\varpi_H - \varpi_L}\left[\frac{1}{1-\beta}(\hat{P}_H - \hat{P}_L) + \hat{a}_H - \hat{a}_L\right].$$

From now on the notation $(\hat{\ })$ denotes the percentage change of a variable. In equation (18) ϖ_i denotes the wage bill share of the high skilled in sector i, $\varpi_i = w_H H_i /(w_L L_i + w_H H_i)$ which can equivalently be written by using equation (12) as

(19) $$\varpi_i = 1 - \left[1 + \mu_i^{-\frac{\sigma}{(\sigma-1)(1-\beta)-1}}\left(w_H / w_L\right)^{\frac{(\sigma-1)(1-\beta)}{(\sigma-1)(1-\beta)-1}}\right]^{-1}.$$

The only difference between the two wage shares of the high skilled in the two sectors comes from the productivity parameter μ_i. Here again a discussion about the effects of the elasticity of substitution is in order. From equation (11) and (19) it can be seen that the wage share is influenced by the same forces as the relative distribution of the variants of technological equipment and therefore the same arguments apply. If high and low skilled intermediate products are gross compliments, a higher productivity parameter μ_i implies a higher skilled wage share in the "high-tech" sector. A rise in the relative wage of the high skilled increases this wage share by less in the "high-tech" sector than in the "low-tech" sector in percentages. The change in absolute value is larger in the "high-tech" sector if the high skilled wage share is smaller than 50 percent in the "low-tech" sector. In the second case where high and low skilled intermediate products are gross substitutes but $\sigma < 2 + \beta/(1-\beta)$ a higher μ_i implies still a higher high skilled wage share but a rise in the relative wage of the high skilled now lowers this share. Furthermore it lowers ϖ_i in the "high-tech" sector by less than in the "low-tech" sector in percentages. In absolute value this change is larger for the "high-tech" sector than for the "low-tech" sector if again the wage share for the high skilled is smaller than 50 percent in the "low-tech" sector.

To summarize, if the elasticity of substitution is smaller than $2 + \beta/(1-\beta)$, the term $\varpi_H - \varpi_L$ must be positive. A rise in the relative price of the "high-tech" good and a rise in the relative state of technology given by a_H / a_L have a positive impact on the relative wage of the high skilled.

We have seen in the preceding section that the skill composition has effects which influence the skill bias in technological change. The next step in the analysis is now to find out what effects the skill composition of the work force has on the development of the sectoral technological change.

Using equation (16) the one period profits of the inventors of new variants of technological equipment for the "high-tech" and the "low-tech" sector are given by $\pi_i = (1/a_i)\beta(1-\beta)P_iY_i$ and the value of the discovery of a new variant is determined by the dynamic programming equation $rV_i - \dot{V}_i = \pi_i$, where r is the interest rate which is possibly time varying. This equation relates the discounted present value of future profits V_i to the flow of profits π_i. The term $\dot{V}_i$, the derivative with respect to time, reflects the possibility that the present value might be time varying. Focusing on a balanced growth path where the present values are constant, they are given by

$$(20) \quad V_i = \frac{1}{r}(1-\beta)\beta^{\frac{2}{1-\beta}+1} P_i^{\frac{1}{1-\beta}} \left(L_i^{(\sigma-1)(1-\beta)} + \mu_i^{\sigma} H_i^{(\sigma-1)(1-\beta)} \right)^{1/(\sigma-1)(1-\beta)} .$$

There a two in the literature of directed technical change quite well known effects present. First the price effect: A higher price of the final output using the particular variant increases the profits of the inventing monopolist. Second the market size effect: The larger the market for a variant, i.e. the larger the number of workers who are to use the technology, the higher the profits for the inventor are. If the number of employees in one sector increases, naturally output of this sector will increase as well. As a consequence of this higher supply of final products its relative price will fall, so the price effect works in the opposite direction as the market size effect. Note that regardless of the value of the elasticity of substitution the market size effect is always positive. A more interesting formulation of equation (20) can be obtained by using (17) and (12) to yield

$$(21) \quad V_i = \frac{1}{a_i r}\beta(w_H H_i + w_L L_i)$$

which states that the profits of inventing a new variant increase with the wage bill of the sector. In other words it will be more profitable to invent for the sector which has the higher wage costs.

To determine the possible sector bias in the development of new variants of technological equipment one first has to make some assumptions about the environment under which these new variants are to be discovered. The innovation possibilities frontier can take two forms following the literature on endogenous growth if the case of sustainable growth is desired. A first possibility is the so called lab equipment specification of Rivera-Batiz and Romer (1991). In this specification only final output is used in the production of new blue prints for new variants. The second possibility is the knowledge based R&D specification of Rivera-Batiz and Romer (1991). Here long run balanced growth is produced via positive spill over effects from past research which increase the productivity of current R&D activities.

3.2.1 Sectoral technological change with the lab equipment specification

Here only final output is used in the production of new designs for the variants of technological equipment. Since in the model there are two kinds of final output one has to decide how these two types of goods are used in the production of new ideas. One possible assumption would be that only "high-tech" products are used in research, yet another is that a certain combination of the two goods enter the production of new variants of technological equipment. To keep the analysis tractable this last possibility will be used, I will return to this issues later on.

To close the model there needs to be one more assumption about the financing of R&D activities. It is reasonable to assume that the households or consumers of the economy save a part of their income which is then used by the research sector for R&D. To keep the analysis simple only consumers with a constant marginal propensity to save are considered[5].

The innovation possibility frontier in the lab equipment specification takes the form $\dot{a}_i = \eta_i R_i$, where $\dot{a}_i$ is the time derivative of the number of variants of technological equipment in sector *i*. R_i is the quantity of the combination of final "high-tech" and "low-tech" products used for research activities concerning new variants for sector *i*. η_i is a parameter determining the productivity of R&D. From investing one unit in R&D η_i new variants will be discovered and the profit stream induced by them has a present value on the balanced growth path of $\eta_i \pi_i / r$. If the R&D sector is to be profit maximizing it coordinates its research activities so that the present value of the profit streams for innovations to each sector equalize. This implies that on the balanced growth path $\eta_H \pi_H = \eta_L \pi_L$ is satisfied, which might be called the technological market clearing condition. Using equation (21) it turns out that on the balanced growth path it must be true that

$$(22) \qquad \frac{a_H}{a_L} = \frac{\eta_H}{\eta_L} \frac{w_H H_H + w_L L_H}{w_H H_L + w_L L_L} \quad ,$$

which is analogous to the optimality condition for the distribution of variants of technological equipment.

To see what effect a change of the skill composition and a change of the wage structure has on the development of new variants of technological equipment on the balanced growth path, it is useful to totally differentiate equation (22). This leads to

5 The choice of the preferences of the consumers has no influence on the results since they are determined alone by the production side of the economy.

(23) $\hat{a}_H - \hat{a}_L = (\varpi_H - \varpi_L)\dfrac{(\sigma-1)(1-\beta)}{(\sigma-1)(1-\beta)-1}(\hat{w}_H - \hat{w}_L) + \hat{H}_H - \hat{H}_L$,

where the result (12) is used and the fact that, if high and low skilled wages are each the same in both sectors, then $\hat{H}_H - \hat{L}_H = \hat{H}_L - \hat{L}_L$ has to be fulfilled. Further it is useful to note again that the terms $\varpi_H - \varpi_L$ and $(\sigma-1)(1-\beta)-1$ always carry the opposite sign (as long as $\mu_H > \mu_L$ is satisfied). Equation (23) says first that, if high and low skilled intermediate products are gross compliments, a rise in the relative wage of the high skilled leads to a rise in the relative state of the "high-tech" sector technology. If "high-tech" and "low-tech" products a gross compliments the opposite occurs. But more important if the term $\hat{H}_H - \hat{H}_L$ is positive, i.e. the high skilled work force in the "high-tech" sector grows faster than in the "low-tech" sector, this leads unambiguously to an increase of the relative state of technology of the "high-tech" sector.

From equation (18) it is obvious that it is still to be determined how relative prices react in response of changes in the skill composition in order to draw conclusions about the net effects on wages. For this it is necessary to make some assumptions about how the output of the "high-tech" and "low-tech" sector is used in the economy. Assume that in a very final stage the output of the "high-tech" and the "low-tech" sector is combined to yield a final good Y which is then used for consumption and R&D activities. The price for this final good is assumed to equal its marginal costs[6] and is normalized to one.

(24) $Y = \left[\delta Y_H^{\alpha} + (1-\delta)Y_L^{\alpha}\right]^{1/\alpha}$,

where the parameter δ determines how important the "high-tech" and "low-tech" products are in the production of the final good Y. The parameter α is assumed to lie in the interval $]-\infty;1]$. Equation (24) then implies relative prices given that the two types of goods are used in a cost minimizing way

(25) $\dfrac{P_H}{P_L} = \dfrac{\delta}{1-\delta}\left(\dfrac{Y_H}{Y_L}\right)^{-(1-\alpha)}$.

Now using the production function in reduced form, equation (15), the present value of the discovery of new variants (20) and the technological market clearing condition, the relative price on the balanced growth path can be written as a function of technological terms:

(26) $\dfrac{P_H}{P_L} = \left(\dfrac{\delta}{1-\delta}\right)^{\frac{\varepsilon}{\varepsilon-1}}\left(\dfrac{a_H}{a_L}\dfrac{\eta_L}{\eta_H}\right)^{-\frac{1}{\varepsilon-1}}$.

[6] This assumption can be justified within a framework of many firms engaging in the production of Y which stand in perfect competition with each other.

In (26) $\varepsilon = 1/(1-\alpha)$ denotes the elasticity of substitution between "high-tech" and "low-tech" products in the final stage of production. The important implication is that if "high-tech" and "low-tech" products are gross substitutes in production of the final good, then the relative price of "high-tech" products depends negatively on the ratio of the states of technology in the "high-tech" and "low-tech" sector. To use equation (26) to complete the analysis of this section it is necessary to compute the relationship between the growth rates of the variables

$$(27) \quad \hat{P}_H - \hat{P}_L = -\frac{1}{\varepsilon - 1}(\hat{a}_H - \hat{a}_L)$$

With this result in hands the effect of a change in the skill composition of the working population via technological adjustment of the economy on relative wages can be computed. This is done using equation (18), the reaction of wages in response technological and price changes, equation (23), the reaction of the sector technologies in response to changes in the skill composition of the working population and equation (27), the reaction of prices.

Proposition: There exists a balanced growth path on which consumption, final output, sector output and the number of variants of technological equipment in the two sectors grow with the same constant rate.

Provided that the parameters of the model fulfil the condition stated below, this balanced growth path is stable.

$$\varepsilon < 2 + \frac{\beta}{1-\beta} + \frac{1}{\phi}\left(2 + \frac{\beta}{1-\beta} - \sigma\right),$$

where ϕ is always positive, depends on the levels of endogenous variables of the model but stays constant on the balanced growth path. See appendix A at the end of the paper for details.

In response to a shock in the skill composition of the working population the relative wage of the high skilled adjusts by

$$(28) \quad \hat{w}_H - \hat{w}_L = \frac{(\sigma-1)(1-\beta)-1}{\varpi_H - \varpi_L}\frac{(\varepsilon-1)(1-\beta)-1}{(1-\beta)(\sigma-\varepsilon)}(\hat{H}_H - \hat{H}_L),$$

to reach the new steady state value. Equation (28) follows from the results (18), (23) and (27).

Since the first term on the right hand side of (28) is always negative the sign of the second term determines whether the relative wage of the high skilled rises when $\hat{H}_H - \hat{H}_L$ is positive. There are several cases where this can occur: First if the elasticity of substitution in the sector production is smaller than in the final stage, $\sigma < \varepsilon$, this requires ε to be larger than $2 + \beta/(1-\beta)$. Second if the elasticity of substitution in the final stage does not possess such a high value, $\varepsilon < 2 + \beta/(1-\beta)$, the elasticity in the sector production has to be larger than in the final stage, $\sigma > \varepsilon$. Figure 1 shows the parameter regions where the mentioned rise in the relative wage of the high skilled can occur.

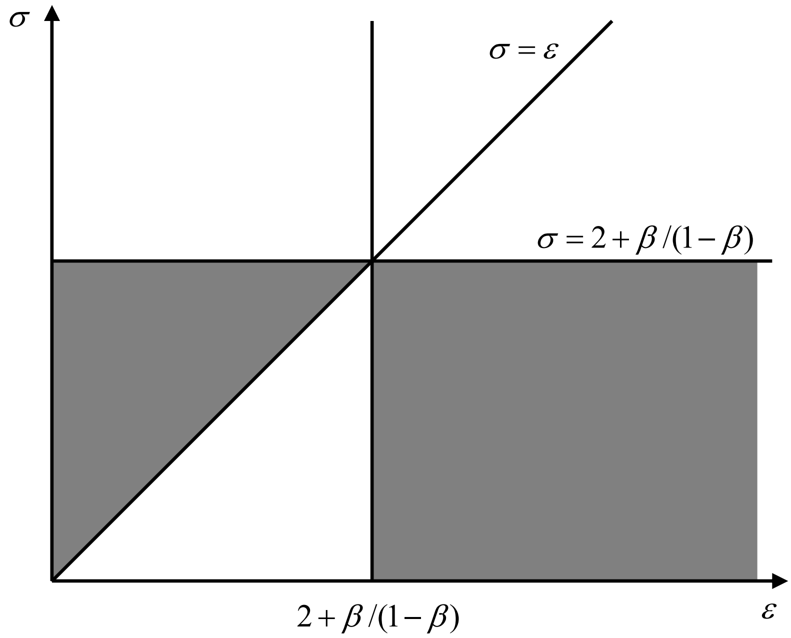

Fig. 1: Regions (shaded) for the elasticities of substitution where a rising relative wage for the high skilled occurs when the high skilled working population in the "high-tech" sector grows faster than in the "low-tech" sector.

The empirical evidence for this result is also supportive. The Industrial Statistics Yearbook (UN 1993, 1989) gives data on the total number of employees and the number of operative workers for 37 three-digit ISIC sectors of the manufacturing industry of the US. Identifying the high skilled by the non-operative and the low skilled by the operative workers gives a clue about the changes in the skill composition although this measure might by imprecise[7]. Furthermore let the "high-tech" sectors be the sectors with the higher ratio of high to low skilled workers. Then looking at the data for the time period from 1983 to 1991 it turns out that the correlation between the growth rate of the high-skilled work force in the different sectors and their ratio of high to low skilled workers is 0.13. Although this is not overwhelmingly large it gives an argument in favor of the presented model.

3.2.2 Sectoral technological change with the knowledge-based R&D specification

In this section a more general formulation for the innovation possibility frontier will be used. The so called knowledge-based R&D specification of Rivera-Batiz and

[7] Due to the lack of other data this drawback is unavoidable. This measure is also used in Berman, Bound and Machin (1997).

Romer (1991) makes assumptions about the state dependence of the productivity of research activities. The difference with respect to the lab equipment specification of the preceding section is that R&D is now conducted by scientists rather than by use of final output. A natural assumption is that these resources are a scarce factor and can not be accumulated over time. Then in order to achieve sustainable growth in the steady state, there must be state dependence in the R&D process, i.e. past discoveries must create a positive spill-over effect onto current research. This process, often illustrated by the metaphor that scientists can "stand on shoulders of giants", leads to ever increasing productivity of scientists giving rise to a constant growth rate in the number of variants of technological equipment. A flexible formulation, also used by Acemoglu (2001), for the innovation possibility frontier is

(29) $\dot{a}_H = \eta_H a_H^{(1+\kappa)/2} a_L^{(1-\kappa)/2} S_L$ and $\dot{a}_L = \eta_L a_L^{(1+\kappa)/2} a_H^{(1-\kappa)/2} S_H$.

Here S_H and S_L are the numbers of scientists engaging in the discovery of new variants of technological equipment for the "high-tech" and "low-tech" sector. It is assumed that $S_H + S_L = \overline{S}$ and that $\overline{S}$ is constant over time. The parameter $\kappa \in [0;1]$ determines the degree of state dependence. If $\kappa = 0$ then there is no state dependence – neither an increase in a_H or a_L makes R&D activities of scientists relatively more productive. The opposite case is $\kappa = 1$ where there is extreme state dependence and current research in one sector makes future R&D in that sector relatively more productive. This alternative specification of the innovation possibility frontier has an important impact on the technological market clearing condition. This condition states that the impact of one researcher should lead in both sectors to the same profits, $\eta_H a_H^\kappa \pi_H = \eta_L a_L^\kappa \pi_L$. Note that the case of no state dependence leads to the same market clearing condition as in the case of the lab equipment specification.

On the balanced growth path where each type of R&D is equal profitable equation (22) now becomes

(30) $$\frac{a_H}{a_L} = \left(\frac{\eta_H}{\eta_L} \frac{w_H H_H + w_L L_H}{w_H H_L + w_L L_L} \right)^{1/(1-\kappa)} .$$

Changes in the relative wage bill of the sectors now lead to larger effects on the relative state of the sector technology. Totally differentiating equation (30) yields

(31) $$\hat{a}_H - \hat{a}_L = \frac{(\varpi_H - \varpi_L)}{(\sigma-1)(1-\beta)-1} \frac{(\sigma-1)(1-\beta)}{1-\kappa} (\hat{w}_H - \hat{w}_L) + \frac{1}{1-\kappa} (\hat{H}_H - \hat{H}_L) ,$$

which now replaces equation (23).

Relative prices in terms of the technology parameters and variables are now determined by

(32) $$\frac{P_H}{P_L} = \left(\frac{\delta}{1-\delta} \right)^{\frac{\varepsilon}{\varepsilon-1}} \left(\frac{a_H}{a_L} \right)^{-\frac{1-\kappa}{\varepsilon-1}} \left(\frac{\eta_H}{\eta_L} \right)^{\frac{1}{\varepsilon-1}} ,$$

and therefore equation (27) now becomes

$$(33) \quad \hat{P}_H - \hat{P}_L = -\frac{1-\kappa}{\varepsilon - 1}(\hat{a}_H - \hat{a}_L).$$

Finally the relationship between relative wages, the relative state of technology and the skill composition of the work force, determined by the unit costs of producing in the two sectors, equation (18) remains valid. To put things together, the reaction of the relative wage for the high skilled in the steady state in response to a change in the skill composition of the work force is now given by (18), (31) and (33). In reduced form this yields

$$(34) \quad \hat{w}_H - \hat{w}_L = \frac{(\sigma-1)(1-\beta)-1}{\varpi_H - \varpi_L} \frac{(\varepsilon-1)(1-\beta)-(1-\kappa)}{(1-\beta)[(\sigma-\varepsilon)(1-\kappa)-(\varepsilon-1)(\sigma-1)\kappa]}(\hat{H}_H - \hat{H}_L)$$

For the relative wage of the high skilled to rise, if $\hat{H}_H - \hat{H}_L$ is positive, there are two possibilities to consider. First, if $\varepsilon > 1 + (1-\kappa)/(1-\beta)$ then the denominator of the second term on the right hand side of equation (34) has to be negative. This leads to the condition $(\sigma-\varepsilon)(1-\kappa)-(\varepsilon-1)(\sigma-1)\kappa < 0$. If instead the elasticity of substitution in the final stage of production is smaller, i.e. $\varepsilon < 1 + (1-\kappa)/(1-\beta)$, then the opposite has to be true. These are unfortunately highly non-linear restrictions on the parameters of the model and the stability conditions for all values of κ are not always easily interpretable. I therefore abstract in the following from the general formulation and focus on the special case of extreme state dependence, $\kappa = 1$[8]. Appendix B at the end of the paper shows that for stability the condition $\varepsilon < 1$ has to be fulfilled.

Now turning to equation (34). If $\kappa = 1$, the second term of the right hand side of equation (34) becomes $-(\sigma-1)^{-1}$. Therefore the relative wage of the high skilled increases if σ is larger than one if the growth rate of the high skilled work force in the "high-tech" sector is larger than in the "low-tech" sector, provided the stability conditions is satisfied. Furthermore in equilibrium consumption, output and the number of variants of technological equipment grow with the same constant rate.

4. A special case

In this section a special case of the derived model is examined and related to the existing literature. Acemoglu (1998) and (2001) uses models of directed technological change which can nicely be nested into the above framework[9]. First in these articles it is assumed that there are two sectors in the economy. One uses only high skilled

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[8] Note that the other extreme of no state dependence gives the same results as in the lab equipment specification.

[9] Acemoglu (1998) uses the quality ladder approach instead of increasing numbers of variants. However his model can be formulated with the variants specification and still leads to the same results.

workers, the other only low skilled. This can be achieved in the present model using the lab equipment R&D specification by setting the elasticity of substitution of sector production to a value larger than $2+\beta/(1-\beta)$ and imposing that the productivity parameter μ_L equals zero. This obviously leads to the "low-tech" sector using only low skilled workers. Cost minimization with respect to the parameter γ_H then leads to a corner solution for the "high-tech" sector. If unemployment is ruled out the only possibility is then that the "high-tech" sector only employs high skilled workers. Consequently all variants of technological equipment will be used only with the one type of labor which is employed. These assumption simplify equation (23) to yield

(35) $\hat{a}_H - \hat{a}_L = \hat{w}_H - \hat{w}_L + \hat{H}_H - \hat{L}_L$.

Together with the results (18) and (27) now the reaction of wages in response to a shock in the skill composition of the working population becomes

(36) $\hat{w}_H - \hat{w}_L = [(\varepsilon-1)(1-\beta)-1](\hat{H}_H - \hat{L}_L)$.

This is exactly the final result of the above cited articles: The relative wage for the high skilled increases with a growing relative number of high skilled workers if the elasticity of substitution in the final stage of production is larger than $2+\beta/(1-\beta)$.

5. Directed technological change in the open economy

So far the analysis concentrated on the case of the closed economy. This section will focus on an open economy specification of the model. A relevant scenario for this is that the relative state of technology measured by a_H / a_L is a global variable, i.e. it is identical for all countries. The incentives for innovations are set world wide by all the potential users of the variants of technological equipment regardless of which country they come from. Another reasonable assumption is that wages for high and low skilled workers are determined on local labor markets; the factor labor is immobile between countries.

Unfortunately this realistic setup is in general too complicated to be analysed within the above model. It is however possible to examine some special cases which are also quite interesting. The following analysis focuses on the two country case where the two countries can freely trade "high-tech" and "low-tech" products. The first scenario to be examined is the case of two identical countries having the same relative and absolute supply of high and low skilled workers in both sectors at the beginning. The second scenario is that of one small and one large country, where small means that the relative incentives for innovations can be approximated by the situation in the larger country.

5.1 Two identical countries

At the outset, both countries face the same situation. The relative state of sector technology is globally determined and is therefore identical in both countries. Furthermore the two countries have the same relative and absolute supply of high and low skilled workers in both sectors. This means that the relative distribution of technological equipment is identical in the two countries and the costs of production in the two sectors are identical as are the wages for high and low skilled employees. In addition to this, it is assumed that all parameters determining the production processes in the two economies are identical.

To begin the analysis, let‘s turn to the innovation possibility frontier. For reasons of tractability, this section will only use the lab equipment specification of section 3.2.1. Taking account of the global market for innovations, the technology market clearing condition is now given by

$$(37)\qquad \frac{a_H}{a_L}=\frac{\eta_H}{\eta_L}\frac{w_H^1H_H^1+w_L^1L_H^1+w_H^2H_H^2+w_L^2L_H^2}{w_H^1H_L^1+w_L^1L_L^1+w_H^2H_L^2+w_L^2L_L^2},$$

where the super-script 1 and 2 denotes country one and country two. Note that at the beginning of the analysis, the corresponding variables for the two countries have the same value. Totally differentiating equation (37) yields

$$(38)\qquad \begin{aligned}\hat{a}_H-\hat{a}_L=&\frac{1}{2}(\varpi_H-\varpi_L)\frac{(\sigma-1)(1-\beta)}{(\sigma-1)(1-\beta)-1}\left(\hat{w}_H^1-\hat{w}_L^1+\hat{w}_H^2-\hat{w}_L^2\right)+\\&+\frac{1}{2}\left(\hat{H}_H^1-\hat{H}_L^1+\hat{H}_H^2-\hat{H}_L^2\right)\end{aligned},$$

which is quite analogous to equation (23) in the closed economy case. Here the relative state of technology is influenced by the development of relative wages and the supply of high skilled workers in the two sectors of the two countries.

Now turning to the development of relative prices. Since the products of sector production can be traded freely, their prices have to be identical in the two countries

$$(39)\qquad \frac{P_H}{P_L}=\frac{\delta}{1-\delta}\left(\frac{Y_H^1-E_H^1}{Y_L^1+E_L^2}\right)^{-\frac{1}{\varepsilon}}=\frac{\delta}{1-\delta}\left(\frac{Y_H^2+E_H^1}{Y_L^2-E_L^2}\right)^{-\frac{1}{\varepsilon}}.$$

In equation (39) the two terms E_H^1 and E_L^2 denote the exports of "high-tech" and "low-tech" products of country one and country two respectively[10]. Note that at the beginning the exports are both equal to zero since the two economies are identical. This however changes when the skill structure of the economies changes. From theses two equations three conditions for the growth rate of the variables of interest can be

[10] Of course the choice of the origins of the exports is arbitrary, so no assumptions about the signs of the two terms are being made.

deducted. First there is a condition guaranteeing the equality of prices in the two economies, which determines the necessary exports of the intermediate products

$$\text{(40)}\quad \begin{aligned} \frac{dE_H^1}{Y_H^1}+\frac{dE_L^2}{Y_L^2} &= \frac{1}{2}(\varpi_H-\varpi_L)\frac{(\sigma-1)(1-\beta)}{(\sigma-1)(1-\beta)-1}\left[(\hat{w}_H^1-\hat{w}_L^1)-(\hat{w}_H^2-\hat{w}_L^2)\right]+ \\ &+\frac{1}{2}\left[(\hat{H}_H^1-\hat{H}_L^1)-(\hat{H}_H^2-\hat{H}_L^2)\right] \end{aligned}.$$

Since there are no exports and imports before changes in the skill composition of the working population take place, the terms dE_H^1 and dE_L^2 give the absolute value of exports of the two intermediate goods. Equation (40) says that the sum of the export shares of production of "high-tech" and "low-tech" products is a function of the development of relative wages and the relative composition of the high skilled work force of the two countries. If the relative wage for the high skilled in country one increases faster than in country two, the export shares increase if high and low skilled intermediate products are gross compliments in the sector production. They decrease in the case of these intermediate products being gross substitutes. If the ratio of high skilled workers in the "high-tech" sector to the high skilled workers in the "low-tech" sector grows faster in country one than in country two, this unambiguously leads to an increase in the trade activities between these two economies.

In addition to the equal prices conditions, there are two conditions showing the development of prices in terms of the variables of the two countries

$$\text{(41)}\quad \hat{P}_H-\hat{P}_L=-\frac{1}{\varepsilon-1}\left((\varpi_H-\varpi_L)\frac{(\sigma-1)(1-\beta)}{(\sigma-1)(1-\beta)-1}(\hat{w}_H^1-\hat{w}_L^1)+\hat{H}_H^1-\hat{H}_L^1-\frac{dE_H^1}{Y_H^1}-\frac{dE_L^2}{Y_L^2}\right),$$

$$\text{(42)}\quad \hat{P}_H-\hat{P}_L=-\frac{1}{\varepsilon-1}\left((\varpi_H-\varpi_L)\frac{(\sigma-1)(1-\beta)}{(\sigma-1)(1-\beta)-1}(\hat{w}_H^2-\hat{w}_L^2)+\hat{H}_H^2-\hat{H}_L^2+\frac{dE_H^1}{Y_H^1}+\frac{dE_L^2}{Y_L^2}\right).$$

Whether a rise in the relative wage of the high skilled in one country leads to an increasing relative price depends on whether "high-tech" and "low-tech" products are gross substitutes or compliments and whether the elasticity of substitution in the final stage of productions is larger or smaller than one. If this elasticity is smaller than one in absolute value, a rise in the ratio between the high skilled workers in both sectors leads to a rising relative price of "high-tech" products. Exports of "high-tech" and "low-tech" products are working to equalize the development of prices in both countries as described by equation (40).

Finally, to close this two economy model, still two additional equations are needed. As in the model of the closed economy, these are the two total differentials of the unit cost function (18) for the two countries. Now using equations (38), (40), (41), (42) and equation (18) for both economies leads to the following result. The development of the relative wage for the high skilled is identical in both countries and this development is given by

(43) $$\hat{w}_H^1 - \hat{w}_L^1 = \hat{w}_H^2 - \hat{w}_L^2 = \frac{1}{2}\frac{(\sigma-1)(1-\beta)-1}{\varpi_H - \varpi_L}\frac{(\varepsilon-1)(1-\beta)-1}{(\sigma-\varepsilon)(1-\beta)}\left[(\hat{H}_H^1 - \hat{H}_L^1) + (\hat{H}_H^2 + \hat{H}_L^2)\right].$$

This result is analogous to the development of the relative wage in the closed economy using the lab equipment specification for the innovation possibility frontier and the same arguments apply. The same conditions apply especially to the relative wage to rise in response to a higher growth rate of the high skilled working population in the "high-tech" sector than in the "low-tech" sector.

5.2. One large and one small economy

This section deals with the situation of one large and one small economy engaging in trade with each other. As in the preceding section "high-tech" and "low-tech" products can be freely exchanged. The parameters of the model are identical for the two economies but their supply of high and low skilled workers now differ. Assuming that the larger economy is relatively more important, the incentives for innovations can be approximated solely by the profits obtained in the larger economy. New variants of technological equipment are produced again by the lab equipment specification. Therefore the relative state of sector technology is determined by an equation of the type of (22) for the large economy. Consequently the relative price of "high-tech" and "low-tech" products is also determined in the larger country. From this it directly follows that for the larger economy all results of the closed economy apply. However with respect to the smaller country things are quite different. A changing skill composition in the small economy now does not have any effect on the relative state of technology, but a changing skill composition in the larger country has spill-over effects onto the smaller country via a change in the state of the relative sector technology. For the small economy this leads to a change of the relative wage in response to a change in the skill composition of the larger country. Using equations (18), (23) and (27) this effect is

(44) $$\hat{w}_H^S - \hat{w}_L^S = \frac{(\sigma-1)(1-\beta)-1}{\varpi_H^S - \varpi_L^S}\frac{(\varepsilon-1)(1-\beta)-1}{(\sigma-\varepsilon)(1-\beta)}(\hat{H}_H^L - \hat{H}_L^L),$$

where the super script "*S*" and "*L*" denote the variables of the smaller and the larger country. Equation (44) is analogous to the closed economy case and the same arguments apply. Therefore if one country is the technology leader the effects of a change in the skill composition in this economy carry over to the small country.

6. Conclusion

The model presented in this paper has examined the impact of changes in the skill composition of the workforce in different sectors via induced technological change on the relative wage of the high skilled workers. The direction of this technological change is endogenously determined and can have a different skill and sector specific component. The bias in the development of relative technology for skills and sectors comes from the different profitability of new discoveries with respect to sectors and the distribution of them with respect to skill groups.

Endogenizing the innovation process with the lab equipment specification for R&D yields nice results which can easily be interpreted. For the relative wage of the high skilled to rise in response to a higher growth rate of the high skilled workers in the "high-tech" sector than in the "low-tech" sector, all what matters is the sign of the difference of the elasticity of substitution in the sector production and the final production stage. Furthermore the model is stable for a reasonable range of parameter constellations. Things however get more complicated using a more flexible formulation of the innovation possibility frontier. With the so called knowledge-based R&D specification one introduces an additional parameter. The model now has four exogenous elasticities and the conditions for stability and the aforementioned rise in the relative wage of the high skilled all involve a non-linear combination of three parameters. Examining the special case of extreme state dependence has shown that nevertheless there exists a parameter region over reasonable values which leads to a rising relative wage and guarantees a stable balanced growth path of the model.

Furthermore it has been shown that quite popular models of the literature on directed technological change can be seen as special cases of the presented model. Their results should therefore be interpreted having in mind the implied assumptions in the light of the presented framework.

Although the case of the open economy is of major interest, only some special cases can be examined within the model of the present paper. It has been shown that in the case of two identical economies, the effects of a change in the skill composition of the work force are smaller but carry over from one country to another symmetrically. The effect on wages is the same in both countries because the relative state of sector technology is determined globally and both economies can trade in sector products. If the relative state of sector technology is determined solely in one large country, effects of a change in the skill composition of the large country spill over onto the small country.

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Appendix A: Stability conditions for the lab equipment specification

To simplify the analysis it is assumed that the labor market is always in equilibrium, i.e. the relative wage is always the same in both sectors. Furthermore it is assumed that relative prices in the economy adjust instantaneously.

Free entry into the R&D sector implies that the reward to research is not larger than its costs, $1 \geq \max[\eta_H V_H, \eta_L V_L]$. The case $1 > \eta_i V_i$ only occurs if and only if $R_i = 0$ and consequently $\dot{a}_i = 0$. In order for the variants of technological equipment to expand in both sectors one would need $1 = V_i(t)$ for an interval of time. Now from dynamic programming it follows that $r\eta_i V_i(t) = \eta_i \pi_i(t) + \eta_i \dot{V}_i(t)$ and that $1 = \eta_H V_H(t) = \eta_L V_L(t)$ is only possible for $0 = \dot{V}_H(t) = \dot{V}_L(t)$. For this to be true it would be necessary that $\eta_H \pi_H(t) = \eta_L \pi_L(t)$ for that interval of time. However from equation (15), (20) and (25) it follows that always

(45)
$$\frac{P_H}{P_L} = \left(\frac{\delta}{1-\delta}\right)^{\frac{1-\beta}{1-\alpha\beta}} \left[\frac{\left(L_H^{(\sigma-1)(1-\beta)} + \mu_H^\sigma H_H^{(\sigma-1)(1-\beta)}\right)^{1/(\sigma-1)(1-\beta)}}{\left(L_L^{(\sigma-1)(1-\beta)} + \mu_L^\sigma H_L^{(\sigma-1)(1-\beta)}\right)^{1/(\sigma-1)(1-\beta)}} \frac{a_H}{a_L} \right]^{-\frac{(1-\alpha)(1-\beta)}{1-\alpha\beta}},$$
$$\equiv \left(\frac{\delta}{1-\delta}\right)^{\frac{1-\beta}{1-\alpha\beta}} \left[\frac{\tilde{H}}{\tilde{L}} \frac{a_H}{a_L} \right]^{-\frac{(1-\alpha)(1-\beta)}{1-\alpha\beta}}$$

(46)
$$\frac{\pi_H}{\pi_L} = \left(\frac{\delta}{1-\delta}\right)^{\frac{1}{1-\alpha\beta}} \left(\frac{a_H}{a_L}\right)^{-\frac{1-\alpha}{1-\alpha\beta}} \left(\frac{\tilde{H}}{\tilde{L}}\right)^{\frac{\alpha(1-\beta)}{1-\alpha\beta}}.$$

Totally differentiation of $\tilde{H}/\tilde{L}$ gives

(47)
$$\hat{\tilde{H}} - \hat{\tilde{L}} = \frac{(\varpi_H - \varpi_L)}{(\sigma-1)(1-\beta)-1}(\hat{w}_H - \hat{w}_L) + \hat{H}_H - \hat{H}_L.$$

The adjustment of wages is given by (equation (18))

(48)
$$\hat{w}_H - \hat{w}_L = \frac{1}{\varpi_H - \varpi_L}\left[\frac{1}{1-\beta}(\hat{P}_H - \hat{P}_L) + \hat{a}_H - \hat{a}_L\right],$$

and prices react according to equation (45) by

(49)
$$\hat{P}_H - \hat{P}_L = -\frac{(1-\alpha)(1-\beta)}{1-\alpha\beta}(\hat{\tilde{H}} - \hat{\tilde{L}} + \hat{a}_H - \hat{a}_L).$$

Now using equations (47), (48) and (49) the change in $\tilde{H}/\tilde{L}$, given the relative state of technology ($\hat{a}_H - \hat{a}_L = 0$), can be computed as

(50)
$$\hat{\tilde{H}} - \hat{\tilde{L}} = \frac{[(\sigma-1)(1-\beta)-1](1-\alpha\beta)}{[(\sigma-1)(1-\beta)-1](1-\alpha\beta)+1-\alpha}(\hat{H}_H - \hat{H}_L),$$

and the corresponding change in the relative profitability of R&D is given by

(51)
$$\hat{\pi}_H - \hat{\pi}_L = \frac{[(\sigma-1)(1-\beta)-1]\alpha(1-\beta)}{[(\sigma-1)(1-\beta)-1](1-\alpha\beta)+1-\alpha}(\hat{H}_H - \hat{H}_L).$$

If $\hat{H}_H - \hat{H}_L$ is positive, then the relative change in the profits can either be positive or negative. It will turn out that this does not matter for the stability of the balanced growth path. If $\hat{\pi}_H - \hat{\pi}_L$ is positive (negative) this will give rise to technological adjustment $\hat{a}_H > 0$ and $\hat{a}_L = 0$ ($\hat{a}_L > 0$ and $\hat{a}_H = 0$). Now assume that after the shock ($\hat{H}_H - \hat{H}_L > 0$) occurred, the total high skilled and low skilled working population is constant, $dH_H + dH_L = dL_H + dL_L = 0$. The assumption that the labor market is always in equilibrium implies the conditions $\hat{w}_H - \hat{w}_L = [(\sigma-1)(1-\beta)-1](\hat{H}_H - \hat{L}_L)$ and $\hat{H}_H - \hat{L}_H = \hat{H}_L - \hat{L}_L$. Using these conditions the equations (47) and (48) now become

$$(52) \quad \hat{\tilde{H}} - \hat{\tilde{L}} = \hat{H}_H[(\varpi_H - \varpi_L)(1-(H/L)(L_L/H_L)) + H/H_L],$$

$$(53) \quad \begin{aligned} &\hat{H}_H[(\sigma-1)(1-\beta)-1](\varpi_H - \varpi_L)[1-(H/L)(L_L/H_L)] = \\ &= -\frac{1-\alpha}{1-\alpha\beta}(\hat{\tilde{H}} - \hat{\tilde{L}}) + \frac{\alpha(1-\beta)}{1-\alpha\beta}(\hat{a}_H - \hat{a}_L) \end{aligned},$$

where $H = H_H + H_L$ and $L = L_H + L_L$.

Equations (49), (52) and (53) now give the effects of the technological adjustment on $\tilde{H}/\tilde{L}$

$$(54) \quad \hat{\tilde{H}} - \hat{\tilde{L}} = \frac{\alpha(1-\beta)}{1-\alpha\beta} \frac{\overline{\dfrac{\phi}{(\sigma-1)(1-\beta)-1}}}{\dfrac{1-\alpha}{1-\alpha\beta}\overline{\dfrac{\phi}{(\sigma-1)(1-\beta)-1}} - 1}(\hat{a}_H - \hat{a}_L),$$

where $\phi = -\dfrac{(\varpi_H - \varpi_L)(H_L/H - L_L/L) + 1}{(\varpi_H - \varpi_L)(H_L/H - L_L/L)}$.

It can be shown that ϕ is always positive.

With (54) it is now clear from equation (46) that the relative profitability responds to technological adjustment by

$$(55) \quad \hat{\pi}_H - \hat{\pi}_L = \left[\left(\frac{\alpha(1-\beta)}{1-\alpha\beta}\right)^2 \frac{\overline{\dfrac{\phi}{(\sigma-1)(1-\beta)-1}}}{\dfrac{1-\alpha}{1-\alpha\beta}\overline{\dfrac{\phi}{(\sigma-1)(1-\beta)-1}} - 1} - \frac{1-\alpha}{1-\alpha\beta}\right](\hat{a}_H - \hat{a}_L).$$

To stabilize the economy now $\hat{\pi}_H - \hat{\pi}_L$ has to be negative if $\hat{a}_H - \hat{a}_L$ is positive during adjustment and vice versa. But this requires in both cases the first term on the right hand side to be negative. For this to be true the following condition has to be satisfied

$$\varepsilon < 2 + \frac{\beta}{1-\beta} + \frac{1}{\phi}\left(2 + \frac{\beta}{1-\beta} - \sigma\right)$$

If this condition is satisfied the ratio π_H / π_L always returns to its equilibrium level η_L / η_H after a shock occurred.

In equilibrium it is then true that

$$(56)\qquad \frac{\eta_L}{\eta_H}=\left(\frac{\delta}{1-\delta}\right)^{\frac{1}{1-\alpha\beta}}\left(\frac{a_H}{a_L}\right)^{-\frac{1-\alpha}{1-\alpha\beta}}\left(\frac{\tilde{H}}{\tilde{L}}\right)^{\frac{\alpha(1-\beta)}{1-\alpha\beta}}$$

has to be fulfilled. But then it must also be true that $(\hat{a}_H-\hat{a}_L)=(\varepsilon-1)(1-\beta)(\hat{\tilde{H}}-\hat{\tilde{L}})$. At the same time equation (54) has to be satisfied. In general this system of two equations has only the solution $\hat{a}_H-\hat{a}_L=\hat{\tilde{H}}-\hat{\tilde{L}}=0$. It has an infinite number of solutions if it happens that $\varepsilon=2+\beta/(1-\beta)+(1/\phi)(2+\beta/(1-\beta)-\sigma)$ which is the borderline of the stability conditions. Also $\hat{\tilde{H}}-\hat{\tilde{L}}=0$ implies that $\hat{H}_H=\hat{H}_L$ (equation (50)). If H is to be constant then this only possible for $\hat{H}_H=\hat{H}_L=\hat{L}_H=\hat{L}_L=0$ which leads directly to $\hat{\tilde{H}}=\hat{\tilde{L}}=0$.

In equilibrium therefore a_H and a_L grow with the same rate θ and $\tilde{H}$ and $\tilde{L}$ do not change. From the production functions (15) and (24) it can be seen that then also Y_H, Y_L and Y have to grow with this rate if prices are constant. They however must be constant since the relative price on in equilibrium can be expressed as a function of a_H/a_L (equation (26)) and the price of final output is normalized to one. The produced final output is used to finance wages and the investments for technological equipment. Since the demand for technological equipment is linear in sector output (equation (16)) these investments grow at the rate θ.

Consumers receive interest payments as a reward for their savings. The zero profit condition for the R&D sector implies that all profits must be used for these interest payments. Since these profits are a fixed proportion of the turnover of the monopolists, which in turn is linear in the sector output, they must grow at rate θ.

Then the total income of the consumers which equals total output minus investments in technological equipment plus interest payments must grow also with rate θ. It is assumed that consumers have a constant marginal propensity to save which implies that consumption and savings grow with same rate as income.

On the balanced growth path savings are divided into spendings for R&D leading to equal growth rates of the number of variants which implies $\eta_H R_H/a_H=\eta_L R_L/a_L$ and all savings go into R&D, $S=R_H+R_L$. This yields $R_H=[(\eta_L/a_L)/(\eta_H/a_H+\eta_L/a_L)]S$ and $R_L=[(\eta_H/a_H)/(\eta_H/a_H+\eta_L/a_L)]S$. But a_H and a_L grow with the same rate so the first terms on the right hand sides are constant on the balanced growth path and R_H and R_L grow with rate θ. Finally from the innovations possibility frontier it can be seen that this growth rate has to be constant since $\dot{a}_i/a_i=\eta_i R_i/a_i$ and the right hand side is a constant because R_i and a_i grow with rate θ.

Appendix B: Stability conditions for the knowledge-based R&D specification

Essentially the same steps have to be taken as in the proof of Appendix A and the same arguments apply. The only difference is that equilibrium is now defined by

(57) $$\frac{\eta_L}{\eta_H} = \left(\frac{a_H}{a_L}\right)^{\kappa} \frac{\pi_H}{\pi_H}.$$

To reach this equilibrium after a shock which lead to a positive $\hat{\pi}_H - \hat{\pi}_L$ and therefore to $\hat{a}_H > 0$ and $\hat{a}_L = 0$, $\hat{\pi}_H - \hat{\pi}_L$ has not only to be negative but at least as large as to compensate the effect $\kappa(\hat{a}_H - \hat{a}_L)$ from the adjustment process. The necessary condition therefore is

(58) $$\left[\left(\frac{\alpha(1-\beta)}{1-\alpha\beta}\right)^2 \frac{\frac{\phi}{(\sigma-1)(1-\beta)-1}}{\frac{1-\alpha}{1-\alpha\beta}\frac{\phi}{(\sigma-1)(1-\beta)-1}-1} - \frac{1-\alpha}{1-\alpha\beta}\right] < -\kappa .$$

In the special case $\kappa = 1$ this leads to the condition $\varepsilon < 1$ be satisfied if the model should be stable. If this condition is satisfied the economy arrives at the equilibrium condition given in equation (57) and stays there because then R&D for each scientist is equally profitable in both sectors. This demands that in equilibrium the following relation is satisfied

(59) $$\hat{\pi}_H - \hat{\pi}_L = -\kappa(\hat{a}_H - \hat{a}_L).$$

But equation (55) of appendix A has also to be fulfilled at all points in time. These two equations imply that if equation (58) is fulfilled with equality, which is the borderline case of stability, the system has a unlimited number of solutions. If this possibility is ruled out, the only solution is $\hat{\pi}_H - \hat{\pi}_L = (\hat{a}_H - \hat{a}_L) = 0$ which states that the number of variants grow with the same rate. Furthermore from the innovations possibility frontier it directly follows that this rate is constant. It is then easy to verify that also output and consumption grow at this rate.