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A Posteriori Estimates for Optimal Control Problems with Control Constraints

Alexandra Gaevskaya
(joint work with Ronald H.W. Hoppe, and Sergey I. Repin)

In this talk, we present two approaches to a posteriori analysis of optimal control problems associated with elliptic type partial differential equations with control constraints. The analysis is performed for distributed and boundary control problems of the following form:

Problem 1. Given $\psi \in L_\infty(\Omega)$, $y^d \in L_2(\Omega)$, $u^d \in L_2(\Omega)$, $f \in L_2(\Omega)$, and $a \in \mathbb{R}_+$, consider the distributed control problem

$$\begin{aligned}
&\text{minimize } J(y(u),u) \ := \ \frac{1}{2}\ \|y-y^d\|^2 \ + \ \frac{a}{2}\ \|u-u^d\|^2 \\
&\text{over } (y,u) \in Y := H_0^1(\Omega) \times L^2(\Omega)\ , \\
&\text{subject to } -\Delta\ y \ = \ u+f \quad \text{a.e. in } \Omega\ , \\
&\qquad u \in K \ := \ \{v \in L^2(\Omega) \mid v \leq \boldsymbol{\psi} \text{ a.e. in } \Omega\}\ .
\end{aligned}$$

Problem 2. Given $\psi_1, \psi_2 \in L_\infty(\Gamma_N)$, $y^d \in L_2(\Omega)$, $u^d \in L_2(\Gamma_N)$, $f \in L_2(\Omega)$, and $a \in \mathbb{R}_+$, consider the boundary control problem

$$\begin{aligned}
&\text{minimize } J(y(u),u) := \frac{1}{2}\|y-y^d\|^2 + \frac{a}{2}\|u-u^d\|^2_{L_2(\Gamma_N)} \\
&\text{over } (y,u) \in Y := H^1_{0,\Gamma_D}(\Omega) \times L^2(\Gamma_N) , \\
&\text{subject to } -\Delta\, y \;=\; f \quad \text{a.e. in } \Omega , \\
&\qquad\qquad \frac{\partial y}{\partial n} \;=\; u \quad \text{on} \quad \Gamma_N , \\
&\quad u \in K \;:=\; \{v \in L^2(\Gamma_N)\ |\psi_1 \le\ v \le \psi_2 \text{ a.e. on } \Gamma_N\} \ .
\end{aligned}$$

The first approach is based on residual-type a posteriori error estimators and incorporates data oscillations. In the framework of this approach, we construct an adaptive finite element method that consists of successive loops of the sequence

$$\text{SOLVE} \rightarrow \text{ESTIMATE} \rightarrow \text{MARK} \rightarrow \text{REFINE}$$

Here, the step SOLVE stands for the numerical solution of the finite element discretized problem, whereas the following three steps include the implementation of the residual-type estimator (ESTIMATE), appropriate marking strategy (MARK), and refinement process (REFINE). Up to data oscillations, the residual-type error estimator does provide an upper and a lower bound for the errors in the state, the co-state, the control, and the co-control (see, e.g., [1–3]). In [1], we provide convergence of the adaptive scheme based on the residual-type estimator in terms of guaranteed error reduction in the state, the co-state, the control, and the co-control for the case of distributed control problem.

The second approach involves the so-called functional type a posteriori error estimates that provide sharp upper bounds for the error with respect to any feasible approximation of the state (see, e.g., [4], [5]). Using functional type estimates, we obtain directly computable upper bounds for the cost functionals of the respective optimal control problems (the majorants). It is proved that a numerical strategy based upon using the majorants produces sequences of control and state functions which provide a value of the cost functional as close to the optimal value as it is required. Moreover, the respective sequences of control and state functions tend to the desired solution of the original problem (see [6]). We further note that the majorants can be used to find guaranteed and easily computable upper bounds for the cost functional when the optimization problem is solved by known methods.

At the end, we present results of numerical experiments that illustrate the performance of both approaches.

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