A Gentle Majority Clause for the Apportionment of Committee Seats

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Abstract The divisor method with standard rounding (Sainte-Laguë/Schepers) is amended by a gentle majority clause, in order to map the government majority in parliament into a seat majority in committees.

Keywords: Divisor method with standard rounding (Sainte-Laguë/Schepers); D'Hondt; Hill; Hare/Niemeyer; Adams; Condorcet.

1. Itio in Partes

We address the problem of how to constitute legislative committees while attempting to reconcile two objectives that sometimes conflict in closely divided legistlatures: representing parties proportional to their seats in the legislature, and maintaining control of the committee by the party or coalition that enjoys a majority in the legislature. The problem arises in many legislatures at the national, state, and municipal levels. A notable recent instance occurred for the German Bundestag, and led to the December 2004 decision of the German Federal Constitutional Court, concerning the composition of the 16 seat Bundestag delegation in the Bundestag-Bundesrat Conference Committee.[1] On 17 February 2005, the Rules Committee of the German Bundestag conducted an expert hearing to elucidate the Court's decision. The present paper is the solution that the authors recommended to the Bundestag, and closely follows their testimony.[2]

[1] Decision of 8 December 2004 (Az. 2 BvE 3/02), here quoted using the marginal running numbers (Rn.) of the Internet publication `www.bverfg.de/entscheidungen/es20041208_2bve000302.html`. — As far as the German Bundestag-Bundesrat Conference Committee is concerned, a corrective action violating proportionality in order to preserve the government majority is considered inadmissible by J. Masing, who finds the contrary conclusion in the decision of the German Federal Constitutional Court inconsistent and nebulous, see Section C.I.3 of his commentary on Art. 77 GG in Mangoldt/Klein/Starck (2005). — See also Kämmerer (2003), Lovens (2003), Stein (2003), Lang (2005).
[2] Pukelsheim/Maier (2005). See also Meyer (2005).

Our proposal of a "gentle majority clause" builds on historic precedence. Inspired by the Pax Augustana of 1455, proclaimed in Augsburg some 450 years ago, the peace of Westphalia of 1648 codified constitutional clauses backing a peaceful coexistence of the two dominating Christian confessions. This included the procedural parity of an *itio in partes*.[3] The *splitting into parts* guaranteed an equal treatment of two unequal groups when the preservation of the mutual identities was considered essential for the whole body. In the then confessional age, the parts were the *Corpus Catholicorum* and the *Corpus Evangelicorum*. In today's democracies, the two groups are majority and minority.

For the expert hearing, the Bundestag Rules Committee compiled a catalogue of five questions. Question 1 concerns the constitutionality of obtaining a mirror image, or of preserving a parliamentary majority. Questions 3–5 aim at procedural and other legislative consequences. Mathematics cannot contribute to these questions. Question 2, adressed in the sequel, asks which operational options are available under the premise that a preservation of the parliamentary majority does conform with the Constitution:

2. If it is constitutionally legitimate to preserve the majority,

a) which measures (for example seat numbers of the factions; relation between majority and opposition),

b) which procedural possibilities (for example combination of one of the usual apportionment procedures with a correction factor; choice of a hitherto not practiced, but majority preserving apportionment procedure, other alternatives) and

c) which changes to the rules and standing orders of the Bundestag

would be called for in order to achieve a "gentle balance"?

The notion of a "gentle balance" [schonender Ausgleich] is taken from the Court decision.[4] However, we find the wording "balance" somewhat besides the point, and instead speak of *majority clauses*.

2. A Gentle Majority Clause

On 30 October 2002, right at the beginning of the legislative period, the Bundestag passed a motion on how to apportion committee seats.[5] The motion comprised two parts, of which Part 1) poses no particular problems:

1) The number of committee seats apportioned to a faction and the sequence of the allocation of chairpersons, of the Steering Committee and

[3] Heckel (1978), Burkhardt (1998). — The Court decision (Rn. 76) refers to the *itio in partes* in US Senate-House conference committees, see Riescher/Ruß/Haas (2000, page 39), or in the Internet `www.house.gov/rules/98-382.pdf`.

[4] BVerfGE 2 BvE 3/02, Rn. 64, 77, 84, 86. Dissenting: Rn. 112.

[5] BVerfGE 2 BvE 3/02, Rn. 8–10.

of the other committees of the German Bundestag, are determined by means of the procedure of mathematical proportions (Sainte-Laguë/Schepers), unless the Bundestag decides otherwise.
The same procedure is used for the apportionment of seats to other parliamentary bodys, unless a different procedure is stipulated by law.

Rather than using the term "procedure of mathematical proportions (Sainte-Laguë/Schepers)", we speak of the *divisor method with standard rounding (Sainte-Laguë/Schepers)*, thus providing some guidance about how the seat apportionments are calculated.[6] For example, for a delegation of size 16 the current faction sizes $249 : 247 : 55 : 47$ result in an apportionment of $7 : 7 : 1 : 1$ seats (divisor 37). Hence the government majority and the opposition minority are tied, with 8 seats each. Part 2) of the Bundestag motion serves as a tie breaking rule, to be called the *prevailing majority clause*:

2) If the parliamentary majority is not preserved, the method of D'Hondt is used. If this method also fails to preserve the parliamentary majority, the method of Sainte-Laguë/Schepers is used with the amendment that the number of seats to be apportioned is reduced by one and that the remaining seat is given to the largest faction.

For a delegation of size 16, the second sentence of Part 2) applies. Thus 15 seats are apportioned using the divisor method with standard rounding (Sainte-Laguë/Schepers), giving in an intermediate allocation of $7 : 6 : 1 : 1$ seats (divisor 38.2). The sixteenth seat is given to the largest faction, resulting in a final apportionment of $8 : 6 : 1 : 1$ seats.

The Court decision seems to indicate, or so we believe, that the prevailing majority clause secures a somewhat questionable advantage for the largest faction.[7] From the viewpoint of mathematics, the prevailing majority clause simply lacks general applicability.[8] The following proposal, to be called the *gentle majority clause*, applies quite generally:

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[6]In the Data Handbook of the German Bundestag, the method is called the "Proportional procedure (of Sainte-Laguë/Schepers)", see Schindler (1999, Volume II, page 2085). The method is attributed to *Daniel Webster* (1782–1813), see Balinski/Young (2001). — *André Sainte-Laguë* [sɛ̃t la′gy] (1882–1950) was professor of *Mathématiques générales en vue des applications* with the *Conservatoire national des arts et métiers* in Paris. *Hans Schepers* (* 1928) was Head of the Data Processing Group of the scientific staff of the German Bundestag (Pukelsheim 2002). Sainte-Laguë was not a saint, whence it is inappropriate to shorten his name to "St. Laguë" or "Ste. Laguë" — Sample calculation: The quotient $249/37 = 6.7$ is rounded in standard fashion to 7, as is $247/37 = 6.7 \nearrow 7$, and $55/37 = 1.49 \searrow 1$, as well as $47/37 = 1.3 \searrow 1$. The divisor 37 is indicative of 37 deputies being represented by one (up to rounding) delegate. — The government majority was composed by SPD (249 deputies) and Greens (55), the opposition minority by CDU/CSU (247) and Liberals (47).

[7]BVerfGE 2 BvE 3/02, Rn. 83, 85.

[8]For example, a transfer of ten opposition seats from FDP to CDU/CSU turns the faction sizes into $249 : 257 : 55 : 37$, making CDU/CSU the largest faction to be awarded the bonus seat. Hence the intermediate allocation $6 : 7 : 1 : 1$ (divisor 39) leads to the final apportionment of $6 : 8 : 1 : 1$ seats. Although government majority and opposition minority stay put at $304 : 294$ deputies, the prevailing majority clause produces a *majority reversal* of $7 : 9$ seats in the committee.

2) If the government majority is to be preserved, then first and foremost it is attempted to achieve the goal by selecting an appropriate committee size. Otherwise, the smallest possible committee majority is apportioned among the factions composing the government majority, while the remaining committee seats are apportioned among the remaining factions; both apportionments are calculated by means of the divisor method with standard rounding (Sainte-Laguë/Schepers).

Applying the gentle majority clause to a delegation of size 16, the government majority gets allocated 9 seats and the opposition minority 7 seats. The two factions forming the government majority have $249 : 55$ deputies whence they allocate their 9 seats into $7 : 2$ (divisor 35). The opposition minority, with $247 : 47$ parliamentary seats, share their 7 seats as $6 : 1$ (divisor 38.2). In summary, the 16 seats are apportioned into $7 : 6 : 2 : 1$.

In the first sentence, the gentle majority clause honors standard practice of the Bundestag. If feasible, the best way-out is to select a committee size evading a tie. The second sentence comes into play only when this road is blocked. In these exceptional cases, the committee is split into a majority part and a minority part, applying the divisor method with standard rounding (Sainte-Laguë/Schepers) to the two groups separately.[9]

3. Transparency, Calculability, and Abstract Generality

The Federal Constitutional Court demands of the Bundestag to formulate deviations from the majority principle in a transparent, calculable, and abstract-general manner.[10] As far as deviations from the majority principle are concerned, a transition from the prevailing majority clause to the gentle majority clause would not introduce any changes. The reservation at the end of the first paragraph in Part 1) of the Bundestag motion allows to enact other procedures for particular cases (Children's Commission, Conference Committee etc.), if so desired.

However, we find it appropriate to emphasize that a transition to the gentle majority clause generates deviations from the mirror image principle that conform with the Court's standards. Indeed, Part 2) of the gentle majority clause is transparent and explicit. It maintains a global mirror image as long as possible. The whole committee splits into a majority group and a minority group only when necessary. But even then the mirror image principle is followed as much as possible, by properly apportioning seats separately within each of the two groups. Moreover, the gentle majority clause is calculable and

[9]We emphasize that the same method is applied with and without a split, and, if split, within either group. A paradoxical seat transfer triggered by a change of apportionment methods is reported by M. Fehndrich, on the Internet site (`www.wahlrecht.de/systemfehler/zweiverfahren.html`).

[10]BVerfGE 2 BvE 3/02, Rn. 86.

abstract-general. Table 1 illustrates the usage of the gentle majority clause, with committee sizes from 1 up to 45. Every committee size preserves the government majority, requiring a split into majority and minority parts in the fifteen rows marked with a star $*$.[11]

The consistency of Table 1 is remarkable: There are no backward jumps![12] The seat apportionments for the majority group stay the same or increase, but never decrease; the same applies to the seat apportionments of the minority. Since divisor methods are coherent, a merger of the two within-group apportionments yields the same global apportionment that is obtained from a one-step calculation (without a split into majority and minority groups) whenever the latter is such that the majority is preserved.[13]

4. Success-Value Equality of the Deputies' Votes

Electoral systems should be judged not so much on the basis of such executive attributes as transparency, calculability, and abstract generality. Instead the judgment should focus on the question of whether the system satisfies the principle of electoral equality. The decision of the German Federal Constitutional Court touches this issue only in passing.[14]

The apportionment of committee seats involves three groups of actors that each can put forward a constitutional claim to equality: The deputies, the factions, and the committee members. From a mathematical viewpoint there is a structural similarity for the transitions, from Bundestag deputies to committee members via the apportionment method laid down in the Bundestag rules, and from voters to Bundestag deputies via the electoral system set forth in the Federal Electoral Law. For the Electoral Law, the Federal Constitutional Court interprets the abstract principle of electoral equality as "success-values equality" [Erfolgswertgleichheit] of the voters' ballots.

In the same vein, the problem of apportioning committee seats calls for an equal success-value of the deputies who are being represented in the commit-

[11]In a newly convening Bundestag it would then suffice to work with this one table of seat apportionments, only, rather than with the three tables used up to now: a first table with the Sainte-Laguë/Schepers apportionments, a second table with D'Hondt apportionments, and a third table with Hare/Niemeyer apportionments.
[12]Called "illogical jumps" in the Handbook of the German Bundestag, see Schindler (1999, Volume II, page 2084).
[13]Balinski (2004a, page 196; 2004b). Balinski/Young (2001, page 141) speak of *uniformity* in place of *coherence*. – Let $M = 1, 2, 3 \ldots, 45$ denote the committee size. The smallest possible majority then comprises $(M+1)/2$ seats when M is odd, and $(M+2)/2$ seats when M is even. Hence the minority is assigned $(M-1)/2$ or $(M-2)/2$ seats according as M is odd or even:

Committee size:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	...	45	M
Majority:	1	2	2	3	3	4	4	5	5	6	6	7	7	8	8	...	23	$\lceil (M+1)/2 \rceil$
Minority:	0	0	1	1	2	2	3	3	4	4	5	5	6	6	7	...	22	$\lfloor (M-1)/2 \rfloor$

Thus the "next" seat alternates between majority and minority, in the range of seats considered.
[14]BVerfGE 2 BvE 3/02, Rn. 82. Dissenting: Rn. 107–129.

Table 1: Apportionment of committee seats using the gentle majority clause[a]

Seats	SPD	CDU/ CSU	B90/Die Grünen	FDP	Divisor(s)
1	1	0	0	0	496
*2	2	0	0	0	165; 496
3	2	1	0	0	165
*4	2	1	1	0	100; 165
5	2	2	1	0	100
6	3	2	1	0	99
7	3	3	1	0	96
*8	4	3	1	0	71; 96
9	4	3	1	1	71
*10	5	3	1	1	55; 71
11	5	4	1	1	55
*12	6	4	1	1	45; 55
13	6	5	1	1	45
*14	7	5	1	1	38.2; 45
15	7	6	1	1	38.2
*16	7	6	2	1	35; 38.2
17	7	7	2	1	35
18	8	7	2	1	33
19	8	8	2	1	32
*20	9	8	2	1	29.2; 32
21	9	8	2	2	29.2
*22	10	8	2	2	26.1; 29.2
23	10	9	2	2	26.1
*24	11	9	2	2	23.6; 26.1
25	11	10	2	2	23.6
*26	11	10	3	2	21.8; 23.6
27	11	11	3	2	21.8
28	12	11	3	2	21.6
29	12	12	3	2	20
30	13	12	3	2	19.8
31	13	13	3	2	19
*32	14	13	3	2	18.4; 19
33	14	13	3	3	18.4
*34	15	13	3	3	17.1; 18.4
35	15	14	3	3	17.1
*36	16	14	3	3	16; 17.1
37	16	15	3	3	16
*38	16	15	4	3	15.4; 16
39	16	16	4	3	15.4
40	17	16	4	3	15
41	17	17	4	3	14.6
42	18	17	4	3	14.2
43	18	18	4	3	14
44	19	18	4	3	13.44
45	19	18	4	4	13.4

[a] on the basis of faction sizes on 1 February 2005: SPD 249, CDU/CSU 247, Bündnis 90/Die Grünen 55, FDP 47.

All apportionments are calculated using the divisor method with standard rounding (Sainte-Laguë/Schepers). In lines marked $*$ two separate calculations are carried out, one for the majority group and one for the minority group.

Sample calculation for committee size *16: The majority divisor 35 yields $249/35 = 7.1 \searrow 7$ and $55/35 = 1.6 \nearrow 2$. The minority divisor 38.2 leads to $247/38.2 = 6.47 \searrow 6$ and $47/38.2 = 1.2 \searrow 1$.

Sample calculation for committee size 18: The divisor 33 gives $249/33 = 7.55 \nearrow 8$ and $247/33 = 7.48 \searrow 7$ and $55/33 = 1.67 \nearrow 2$ and $47/33 = 1.4 \searrow 1$.

tee. Our preferred proposal of a gentle majority clause builds on the divisor method with standard rounding (Sainte-Laguë/Schepers). The reason is that this method produces seat apportionments that are in an exceptional harmony with the principle of success-value equality whence, in this very specific sense, the method is superior to other competing apportionment methods.[15]

The gentle majority clause is our preferred proposal because it does away with an ecclectic multitude of apportionment methods, and builds solely on the success-value oriented divisor method with standard rounding (Sainte-Laguë/ Schepers).

5. Preservation of the Majority by Means of D'Hondt

In the remaining sections we discuss other possibilities to respond to the Rules Committee's Question 2.[16]

For an appraisal of the following alternatives we recall that the prevailing majority clause, in its Part 2), resorts to the divisor method with rounding down (D'Hondt) for the reason that this method is known to be biased, in favor of larger participants and at the expense of smaller participants. These seat biases do not materialize every time the method is applied, but become clearly visible in repeated applications. As it happens, for the problem under discussion, with faction sizes $249 : 247 : 55 : 47$ and committee size 16, the D'Hondt method results in the already familiar tie $7 : 7 : 1 : 1$ (divisor 33).[17]

[15] See Pukelsheim (2000a, b, c).

[16] It would also be conceivable to apply the German Federal Electoral Law which (as of this writing) employs the quota method with residual fit by largest remainders (Hare/Niemeyer). — *Thomas Hare* (1806–1891) was a barrister and *Inspector of Charities* in London. *Horst F. Niemeyer* (*1931) is Professor emeritus for Mathematics with the Rheinisch-Westfälische Technische Hochschule Aachen. — The Federal Electoral Law (BWahlG) contains in its §6(3) a majority clause. Its constitutionality has been confirmed by NdsStGHE 1 (1978, pages 335–372). To apply this clause to a committee of size 16, the calculations are as follows. The faction sizes $249 : 247 : 55 : 47$ are divided by the quota $598/16$ and result in the ideal shares $6.66 : 6.61 : 1.47 : 1.26$. This gives rise to the main apportionment $6 : 6 : 1 : 1$, leaving two residual seats. According to §6(3) BWahlG, the majority is preserved by appropriately assigning the residual seats, leading to the final apportionment $7 : 6 : 2 : 1$. — Alternatively, one could carry out the calculations in two steps, with a split into two parts. Considering the majority and minority groups, of $304 : 294$ deputies, their ideal shares are $8.13 : 7.87$ and lead to the main apportionment $8 : 7$. According to §6(3) BWahlG, the remaining residual seat is allocated with the majority group, whence the two groups end up with $9 : 7$ seats. The sub-apportionments of the 9 seats within the majority, and of the 7 seats within the minority yield the same final apportionment $7 : 6 : 2 : 1$ as before. — For committees of size 8 and 12 either way leads to the apportionments $4 : 3 : 1 : 0$ and $6 : 4 : 1 : 1$, which coincide with those given in Table 1.

[17] In the Data Handbook of the Bundestag the method is called "Höchstzahlverfahren (nach D'Hondt)", see Schindler (1999, Volume II, page 2083). — *Victor D'Hondt* (1841–1901) was Professor for Civil Law and Financial Law with the University of Gent. He himself and his contemporaries spelled his name with a capital initial "D", librarians file the name under the letter "H". In Switzerland, the method is named after *Eduard Hagenbach-Bischoff* (1833-1910), Professor of Physics with the University of Basel. — Sample calculation: After subdivision by the divisor, all resulting quotients are rounded down: $249/33 = 7.5 \searrow 7$, and $247/33 = 7.5 \searrow 7$, and $55/33 = 1.7 \searrow 1$, and $47/33 = 1.4 \searrow 1$. — For the succession of apportionment methods the Bundestag has used so far, from D'Hondt via Hare/Niemeyer (from 1970 on) to Sainte-Laguë/Schepers (from 1980 on), see Fromme (1970), and Schindler (1999, Volume II, page 2081–

Of the fifteen tied rows in Table 1, ten persist under the divisor method with rounding down, while five ties are resolved. For instance, in a committee of size 32, the divisor method with standard rounding (Sainte-Laguë/Schepers) leads to the tie $13 : 13 : 3 : 3$ (divisor 18.7). In contrast, the divisor method with rounding down (D'Hondt) transfers a seat from the smallest to the largest participant and yields $14 : 13 : 3 : 2$ (divisor 17.8), which is the same apportionment resulting from the gentle majority clause in Table 1.

We may summarize the effects of the divisor method with rounding down (D'Hondt) as follows. At best it produces the same result as the gentle majority clause. Otherwise, it may preserve the majority without, however, securing within the majority and minority groups success-values as balanced as those coming with the gentle majority clause. And there is the third possibility that the method re-produces the tie it was suppose to resolve.

6. A Brutal Majority Clause

Technically, a split into majority and minority groups can also be implemented with the divisor method with rounding down (D'Hondt). The government majority, commanding $249 : 55$ deputies, would share their 9 committee seats in the proportion $8 : 1$ (divisor 30). The opposition minority, with $247 : 47$ Bundestag seats, would be allocated $6 : 1$ committee seats (divisor 40). The resulting apportionment is $8 : 6 : 1 : 1$, which is the seat allocation contested in Court. From our point of view as mathematicians, this majority clause is brutal and hard to defend. The split into majority and minority groups is aggravated by the seat biases inherent in the divisor method with rounding down (D'Hondt). The brutal majority clause comes with a greater deviation from proportionality than is needed for a gentle, minimal intervention.[18]

4). — When the D'Hondt method is applied to four participants, the largest participant can expect an advantage of $+0.5$ seat fractions, the second largest $+0.1$ fractions. To even out these advantages, the third participant misses its ideal share on the average by -0.2 fractions of a seat, the smallest participant by -0.4. See Schuster/Pukelsheim/Drton/Draper (2003, page 663).

[18] The German Federal Constitutional Court might well (presumably, at present) judge the brutal majority clause to be constitutional. In fact, the Court puts the divisor method with rounding down (D'Hondt) on a par with the divisor method with standard rounding (Sainte-Laguë/Schepers), even though the D'Hondt method exhibits noticable seat biases, while the Sainte-Laguë method is exceptionally concordant with the Court's imperative of success-value equality. Other German courts circumnavigate the shallowness in the decisions of the Federal Constitutional Court, by stating that the D'Hondt method is generally admissible, but then overruling its specific apportionment results as unlawful: due to multiple applications in separate electoral districts (BayVerfGHE 45, pages 12–23, 54–67, 85–89), due to a misuse of list combinations (BVerwG Az. 8 C 18.03 of 10 December 2003), due to a deviation from the ideal shares (BayVerwGH Az. 4 BV 03.117 and Az. 4 BV 03.1159 of 17 March 2004). We take this casuistry as a first evidence that the legal viewpoint is changing, as is implied by the State Court for the Land Baden-Württemberg (decision of 24 March 2003, Az. GR 3/01, Section B.III.2.b). A second evidence is the fact that appelants who lost their court case did not appeal to the top Federal courts although the contested facts were *not* unconstitutional (explicit: page 192 in BayVerfGH 47, 1994, 84-194; implicit: page 283 in BVerfGE 96, 1998, 264-288). A revision to the top Federal courts may induce these courts to turn to the Federal Constitutional Court for

7. Preservation of the Majority by Means of Hill et al.

If the divisor method with rounding down (D'Hondt) induces a tie-break, it does so for the reason that a seat of a smaller minority party is transferred to a larger majority party. Not surprisingly, there are counterparts resolving a tie by taking a seat away from a larger minority party and allocating it with a smaller majority party.[19]

A first such procedure is the divisor method with geometric rounding (Hill), used in the USA since 1941 for the apportionment of the 435 seats in the House of Representatives to the 50 States. Applying this method to a delegation of size 16, the faction sizes $249 : 247 : 55 : 47$ are mapped into $7 : 6 : 2 : 1$ seats (divisor 38.3). Size 16 is the only tie situation resolved by this method, for the range considered inTable 1.[20]

A second method is the divisor method with 0.4-rounding (Condorcet), which also produces the final result $7 : 6 : 2 : 1$ (divisor 38.8). This method resolves two of the fifteen ties listed in Table 1.[21]

A third procedure is the divisor method with rounding up (Adams), resolving five of the fifteen tie situations. The method is used in France to apportion the seats of the Assemblé Nationale to the Départments.[22]

There are committee sizes for which neither the divisor method with geometric rounding (Hill) nor the one with 0.4-rounding (Condorcet) resolves the tie. Moreover, it is possible that both methods do resolve a tie, but differently. An example is the German Bundestag 2002 at the beginning of the legislative period, with the then faction sizes $251 : 248 : 55 : 47$. For a committee of size 36, the divisor method with standard rounding (Sainte-Laguë/Schepers) leads to the tie $15 : 15 : 3 : 3$ (divisor 17). If we attempt to resolve the tie by using the two methods mentioned above, we get two conflicting answers: The divisor method with rounding down (D'Hondt) yields $16 : 15 : 3 : 2$ (divisor 15.68), while the divisor method with rounding up (Adams) leads to $15 : 14 : 4 : 3$ (divisor 17.8).[23]

clarification. But confronting the Court with the state-of-the-art raises the "danger", for the appellant, that the Court revokes not just a single D'Hondt apportionment, but the whole D'Hondt method.

[19] Marshall/Olkin/Pukelsheim (2002).

[20] Balinski/Young (2001, page 48). — *Joseph Adna Hill* (1860–1938) was Chief Statistician, Division of Revision and Results, US Bureau of the Census. — Sample calculation: The quotient $249/38.3 = 6.5$ lies above the decision point $\sqrt{6 \cdot 7} = 6.48$ and hence is rounded up to 7, while $247/38.3 = 6.45$ is rounded down to 6. The quotient $55/38.3 = 1.44$, when compared with the decision point $\sqrt{1 \cdot 2} = 1.41$, rounds up to 2, while $47/38.2 = 1.2$ goes down to 1. The decision points are *geometric means* of two neighboring integer numbers, whence the method receives its name.

[21] Balinski/Young (2001, page 63). — *Marie Jean Antoine Nicolas Caritat, Marquis de Condorcet* (1743–1794) was one of the leading politicians during the French Revolution. — Sample calculation: Fractions are rounded down when smaller than 0.4, and rounded up otherwise. Thus we get $249/38.8 = 6.42 \nearrow 7$ and $247/38.8 = 6.37 \searrow 6$ and $55/38.8 = 1.42 \nearrow 2$ and $47/38.8 = 1.2 \searrow 1$.

[22] See Balinski (2004a, page 190). — *John Quincy Adams* (1767–1848) was the sixth President of the USA.

[23] The gentle majority clause yields $16 : 14 : 3 : 3$ (majority divisor 16, minority divisor 17.1).

As a consequence we refrain from a proposal to remedy the prevailing majority clause by taking recourse to a multitude of different apportionment methods. When many methods are tendered like on a flea market, many answers are conceivable: at best a unique and clear-cut tie break, or otherwise no tie break at all, or else multiple but conflicting results. A methodological zoo degenerates into a game of numbers. Instead the focus ought to be on electoral principles such as success-value equality, set forth by the German Federal Constitutional Court in 1952 and since then having generated an impressively consistent body of constitutional decisions.

8. Minimum Seat Requirements

As a final point we would like to draw attention to the problem of guaranteeing each participant a minimum number of seats. With current faction sizes $249 : 247 : 55 : 47$ and for a committee of size 10, the divisor method with standard rounding (Sainte-Laguë/Schepers) results in the tie $4 : 4 : 1 : 1$ (divisor 60). The prevailing majority clause would resort to the divisor method with rounding down (D'Hondt), giving $5 : 4 : 1 : 0$ (divisor 49.6) and thus excluding the smallest party from representation.

However, the present problem concerns a committee of size 16, for which the Sainte-Laguë method yields the tie $7 : 7 : 1 : 1$. Considering how the divisor method with rounding down (D'Hondt) transfers seats from smaller to larger parties, there are just two possibilites: either the tie persists, or else it is broken into $8 : 7 : 1 : 0$. That is, the only way in which the prevailing majority clause could have resolved the tie would have deprived the smallest party of being represented at all. This may have set off some legal action of a different sort.[24]

It is easy to augment the gentle majority clause by the additional restriction that each participant be guaranteed representation. All that needs to be done is to modify the (unconditional) divisor method with standard rounding (Sainte-Laguë/Schepers), by demanding the minimum requirement that every participant receive at least one seat.[25]

We conclude with a *ceterum censeo*. The current topic, the apportionment of committee seats, is important. However, more important is the apportionment

[24]It is not clear to us how the Federal Constitutional Court would have settled the case. The Court sees the Conference Committee as a parliamentary body *sui generis*, for which the Constitution mandates neither a preservation of the majority (2 BvE 3/02, Rn. 67), nor a representation of all parliamentary groups, see BVerfGE 96 (1998) 264–288.

[25]The minimum committee size then is 5, of course, with the four parties filling one seat each and the fifth seat establishing a majority. The apportionments turn out to be $2 : 1 : 1 : 1$ for a committee of size 5, next $3 : 1 : 1 : 1$ for size 6, then $3 : 2 : 1 : 1$ for size 7, and finally $4 : 2 : 1 : 1$ for size 8. For committee sizes larger than 8 the apportionments of Table 1 apply.

of the Bundestag seats proper. The two-ballots electoral system of the German Federal Electoral Law is a top-quality product, enjoying high international esteem and serving as a prototype system.[26] But even top-quality products need be attended to. Negative weights of a ballot, doubly successful ballots, and overhang seats damage the image of the system.[27]

These deficiencies disappear when the idea of imposing minimum requirements is followed up. A simple adaptation of the divisor method with standard rounding would do, namely, imposing the minimum restrictions that each list receives at least as many seats as have been won in the constituencies. The *direct-seat restricted method* leaves no room for negative ballot weights, doubly successful ballots, nor overhang seats, and yet it stays in close harmony with the principle of success-value equality.[28] Whatever the requirements, the common denominator is the divisor method with standard rounding (Sainte-Laguë/Schepers). The method is so powerful that a few amendments suffice to adjust it to all practical purposes.

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[26] Shugart/Wattenberg (2001).

[27] Fehndrich (1999).

[28] Pukelsheim (2003, 2004a, b).

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