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Angaben zur Veröffentlichung / Publication details:

Schultze, Wolfgang. 2004. "Valuation, tax shields and the cost-of-capital with personal taxes: a framework for incorporating taxes." *International Journal of Theoretical and Applied Finance* 7 (6): 769–804. https://doi.org/10.1142/S0219024904002621.

VALUATION, TAX SHIELDS AND THE COST-OF-CAPITAL WITH PERSONAL TAXES: A FRAMEWORK FOR INCORPORATING TAXES

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Abstract. This paper presents a general approach to deriving valuation models and the relevant cost of capital formulas independent of a particular tax environment. The value of a levered firm depends to a large extent upon the amount and value of the tax shields. The latter in turn differs from country to country and even from firm to firm, depending on its particular situation. It is subject to change with every change in the tax system of the country where the firm is located. Therefore, in an international environment a general approach is needed, which can be altered for any given situation. At the same time personal taxes play an increasing role in the valuation of companies. Therefore, their consideration is integrated into the models derived. Finally, the resulting generalized versions of the Modigliani/Miller- and Miles/Ezzell-formulas for adjusting the cost of capital to changes in leverage are applied to the situation of a corporation located in Germany after the Tax Reform Act of 2000.

Key Words: Capital Structure, Cost-of-Capital, Tax Shield, Valuation

1. Introduction

The cost of capital play a crucial role in the process of valuing companies. One major component in the determination of firm value is the tax shield resulting from debt financing, which is, in the majority of valuation models, integrated into valuation via the cost of capital. *Modigliani/Miller* (1963) were the first to show the effect of taxes and capital structure on firm value in the context of perpetuities and a given amount of debt. They introduced an approach to valuation which measures firm value by two separate components: the value of the unlevered firm and the financing side effects, which was later generalized and termed "Adjusted Present Value (APV)"-approach by *Myers* (1974).

Miles/Ezzell (1980) extended the analysis to the case of non-perpetuities and a constant market value leverage ratio, as the popular "textbook formula" of the weighted average cost of capital (WACC) is specified in these terms.[1] While their analysis is focused on the question of equivalence of the two valuation-approaches cited, it is ultimately linked with the derivation of the effects of tax shields and capital structure on the cost of capital.[2] *Inselbag/Kaufold* (1997) have reconciled the two approaches by extending the results of *Modigliani/Miller* (1963) and *Miles/Ezzell* (1980) to the case of non-perpetuities and fluctuating debt/equity ratios. They show that the methods are equivalent under different financing strategies. They conclude that the APV-approach is more practical to use in the case of a constant amount of debt, however, and the WACC-approach in the case of a fixed debt/equity-ratio.

The tax shield generally differs from country to country and is subject to change with every change in the tax system of the country where the firm is located and even from firm to firm, depending on its particular situation. The valuation approaches typically used do not account for such differences or changes. Therefore, a general approach to deriving the cost of capital is needed, which can be adjusted to any situation depending on the taxation characteristics of the given firm.

The results of the above-mentioned work were based on the US-tax system, but were applicable to the former German tax system with only minor adjustments.[3] A number of articles in the German literature have investigated the equivalence of different approaches to valuation with contradictory results. *Heitzer/Dutschmann* (1999) and *Wallmeier* (1999), have reconciled the two views by showing that clear relationships between the valuation approaches and the cost-of-capital-formulas arise, as long as the financing premises are modeled adequately.[4]

The German Tax Reform Act of the year 2000 has altered the basis of the above analysis. It introduced a system of double taxation in which only half of the equity income from corporations is subject to personal taxes.[5] Therefore, personal taxes can no longer be omitted when valuing German corporations. Personal income taxes on the level of the owners have increasingly been discussed for their inclusion in valuation models during the last decade. The German Institute of Certified Public Accountants ("Institut der Wirtschaftsprüfer in Deutschland e.V.", IDW) has lately decided to make personal taxes an integral part of their valuation standards (IDW S1).

This paper presents a general approach to deriving the cost of capital independently of a particular tax regime and including personal taxes. We follow the same line of reasoning as earlier work on this topic while using one single, consistent approach to deriving the cost-of-capital-formulas under different financing strategies. We show their interrelationship with different valuation approaches, yielding a general equivalence of the different models. In addition formulas for the determination of the unlevered cost of capital via the CAPM are derived. The application of these general results is demonstrated by analyzing the situation of German corporations after the Tax Reform Act 2000.

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1 See *Miles/Ezzell* (1980), p. 722.
2 See *Inselbag/Kaufold* (1997), p. 114; *Miles/Ezzell* (1980), p. 720.
3 See *Drukarczyk* (1993), pp. 177.
4 See *Heitzer/Dutschmann* (1999), p. 1463; *Wallmeier* (1999), p. 1476.
5 See *Miller* (1977) for a discussion of personal taxes and valuation within the US-tax system.

2. Leverage, Tax Shields and Valuation

The value of the firm is a function of its capital structure. *Modigliani/Miller* have shown that firm value is independent of capital structure under a set of specific assumptions.[6] When these assumptions do not hold, however, firm value will, by implication, not be independent of changes in capital structure.[7] The major source of changes in value with respect to a change in capital structure is the tax shield arising from debt financing.[8]

With increasing leverage, owners receive a higher rate of return on their invested capital, but they also bear higher risk. In a world without taxes, risk and therefore the costs of equity, increase by just as much as to offset the positive effect of higher rates of return. The cost of equity increases in proportion with increasing leverage. In almost any tax system, however, debt financing has an advantage over equity financing in a way that interest payments are tax deductible on the corporate level, and therefore, interest income is taxed more favourable than equity income, as long as both are taxed equally at the personal level of the recipients. As a consequence, total return after taxes to all investors of a levered firm increases compared to the unlevered firm and firm value increases. This increase is due to the fact that the investors in a levered firm, in total, pay fewer taxes than the investors in a unlevered firm. This difference is called "tax shield." The effect on firm-value of this increase in returns after taxes can most comprehensibly be described by the Adjusted Present Value (APV)-approach to valuation.

The APV-model, which was first introduced by *Modigliani/Miller* (1963) and later generalized by *Myers* (1974), builds the foundation for the analysis of changes in value, and therefore, the reaction of the cost of capital to changes in capital structure.[9] The value of a levered firm (V_0^ℓ) is composed of the value of an all-equity firm and the present value of the tax shield (PVTS). The value of the all-equity firm derives from discounting the expected unlevered free cash flows to the firm (X^u) at the cost of capital of an all-equity firm (r_E^u).[10] These free cash flows are determined by deducting all cash flows needed for future investments (ICF) from the operating cash flows to the firm (OCF) taxed under the assumption of all-equity financing (T^u). For the simple case of uniform, perpetual cash flows, this approach can be written as follows:[11]

$$V_0^\ell = \frac{X^u}{r_E^u} + PVTS \qquad \forall X^u = OCF + ICF - T^u \tag{1}$$

The present value of the future tax shields depends on the financing strategy followed by the firm. The different alternative financing strategies are presented in paragraph 2.3. In practice, however, other approaches to valuation are more popular. They are presented in paragraph 2.2. Their equivalence to the APV-approach results from the fact that the cost of capital used for these approaches derive directly form the APV-approach, as is demonstrated in Chapter 3. The relationships of the cost of capital to changes in capital structure

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6 See *Modigliani/Miller* (1958).

7 See *Miller* (1988), p. 100.

8 See *Modigliani/Miller* (1963); *Miller* (1988), p. 112.

9 See *Miles/Ezzell* (1980), p. 719.

10 See e.g. *Inselbag/Kaufold* (1997), p. 116. All symbols with a time subscript are expected values, the subscript 0 denotes present values at time 0. The riskiness of debt and interest payments is discussed and associated with explicit assumptions in Chapter 2.3.

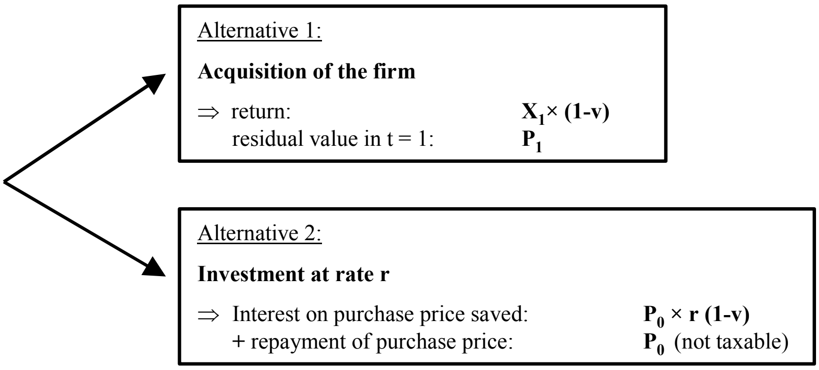
11 Free Cashflows are determined here by use of a cash flow statement. The cash flows from investing activities (ICF) are added to operating cash flows, but will in most cases carry a negative sign. See for a derivation *Coenenberg/Schultze* (1998).

are derived, which yields adjustment formulas as functions of leverage and the unlevered cost of capital. In Chapter 4 it is shown how these general formulas can be adapted to a particular situation. To do so, the German situation after the latest tax reform is analyzed. Before further investigating the different approaches to valuation, personal taxes need to be considered.

2.1. *Personal Taxes and Valuation*

Market value of equity derives from the comparison of the returns from the acquisition of the firm (X) with the rates of return from the best alternative investment (r_E).[12] Both alternative returns are determined after corporate taxes and are subject to personal taxes (v):[13]

Figure 1: Valuation by comparison of equivalent alternatives



A rational investor will be indifferent between the two alternatives, when the two are equivalent in all aspects:[14]

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12 See *Moxter* (1976), pp. 168.

13 See for a similar derivation *Ballwieser/Leuthier* (1986), p. 607. To be able to account for personal taxes, corporate income has to be taxable on the personal level of the owners. We therefore assume that there is an immediate flow of income to the owners (full distribution of profits), resulting in their immediate taxation. Or equivalently, that personal taxes are levied on the firm's income independently of their application for distribution or reinvestment, as it is often the case in partnerships. The assumption is equivalent to the irrelevance of dividend policy as long as retained earnings are reinvested at a rate equal to the cost of capital and distributed at a later point in time and then taxed at the same tax rate. We do therefore not account for changes in personal tax rates over time. Dividend irrelevance is a typical underlying assumption of Discounted Cash Flow-valuation. See e.g. *Damodaran* (1996), p. 233; *Penman* (1998), pp. 305. See for a discussion of the interactions of corporate and personal taxes *Miller* (1977); *Miller* (1988), pp. 116. We also assume that market values can be determined by discounting expected future cash flows, i.e. that capital markets determine the value of a levered firm as the sum of the unlevered component plus the value of the tax shield. See *Miles/Ezzell* (1980), p. 722. Differences in taxation of dividend income versus capital gains are not taken into consideration. We also do not account for credit risk in our analysis. We also assume that operating risk, reflecting the variance in future operating cash flows, remains the same over time, so that there is no need to reassess the cost of capital for that reason. This reduces the problem this paper is aimed at to financial risk.

14 To be able to assign a value to alternative 1 we have to find an alternative of which we know its value. To be able to transmit this value onto alternative 1 the two have to be equivalent. The price of the best alternative is transposed onto the asset to be valued by substituting its price into the investment program of the investor by holding constant his level of utility. Therefore both alternatives have to be perfect substitutes, i.e. one needs to identify the alternative which yields the equivalent level of utility for the investor as would alternative 1. *Ballwieser/Leuthier* (1986), p. 608 provide a list of necessary equivalences.

$$P_1 + X_1 (1 - v) = P_0 \times [1 + r (1 - v)] \tag{2}$$

Equity value as a marginal price can be determined by solving the equation for P_0:[15]

$$P_0 = \frac{P_1 + X_1(1-v)}{1 + r(1-v)} \tag{3}$$

P_1 can be explained by $P_1 = \frac{P_2 + X_2(1-v)}{1 + r(1-v)}$ etc., therefore, yielding the following general expression for the determination of equity value:

$$P_0 = \frac{X_1(1-v)}{1 + r(1-v)} + \frac{X_2(1-v)}{[1 + r(1-v)]^2} + ... = \sum_{t=1}^{\infty} \frac{X_t(1-v)}{[1 + r(1-v)]^t} \tag{4}$$

For a long time, the notion was generally held in literature that taxes are only relevant in valuation as long as they apply differently to the cash flows and required rates of return of the valuation model.[16] In the special case of perpetuities this is generally justified, as then the tax-term cancels out:

$$P_0 = \sum_{t=1}^{\infty} \frac{X(1-v)}{[1 + r(1-v)]^t} \rightarrow \frac{X(1-v)}{r(1-v)} = \frac{X}{r} \tag{5}$$

In the more general case of non-perpetuities, however, this will not generally be the case. Therefore, academics and practitioners in Germany have turned to the view that all taxes should be considered in valuation.[17] Besides the fact, that the omission of personal taxes can only be justified for perpetuities, some authors claim that cash flows and the required return of the investors are subject to different taxation, and therefore, the tax factors do not cancel out.[18] Discounting, however, as was shown above, is a comparison of two alternatives: buying the firm to be valued, represented by its cash flows in the numerator, or investing in a different asset whose returns are represented by the opportunity cost of capital.[19] The two alternatives have to be equivalent in all aspects as otherwise their price could not be the same in a rational market. This condition is termed the “principle of equivalence”[20] and is particularly important when it comes to the riskiness of the two alternatives. Therefore, the theoretically relevant alternative to acquiring a firm, and therefore the risk equivalent return needed for discounting, is the investment in a different firm with equivalent characteristics. Basis for the determination of the taxation of this return is an equity position in an alternative firm.[21]

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15 The resulting value is a marginal value that will turn the two alternatives equivalent, that is: any price paid in excess of the determined value will result in the alternative to be more desirable than acquiring the firm. See e.g. *Brealey/Myers* (2000), pp. 15.

16 See *Ballwieser* (1995), pp. 19; *Damodaran* (1996), p. 112; also *IDW* (1998), p. 68.

17 See *Günther* (1998), pp. 1834; *IDW* (1998), pp. 66; *Siepe* (1997), pp. 4.

18 See *Siepe* (1997), pp. 2.

19 See *Stehle* (1995), pp. 1111.

20 See *Ballwieser/Leuthier* (1986), p. 608.

21 For an investor to be indifferent between two alternatives, the comparison has to be done using a risk-equivalent alternative, i.e. investment in the equity of a different firm. Using securities for this purpose wouldn’t yield risk equivalence. Adjusting a risk free rate of return to account for risk (e.g. via the CAPM) doesn’t assume investment in a security, but in equity. Therefore the alternative needs to be taxed as equity, not like debt. We therefore differentiate between v_E and v_D.

The above valuation model so far does not yet account for debt financing and the resulting tax benefits. The different approaches to valuation used in practice differ from the general approach presented here by the inclusion of debt-financing.[22]

2.2. *Approaches to Valuation, Debt Financing and Personal Taxes*

All approaches currently used in DCF-valuation can be derived from the above comparison of alternatives altered to include debt-financing: For any given period the return from acquiring the company, represented by the free cash flows to the firm after taxes, have to meet the investors required rates of return after all taxes, including personal taxes (v).[23] Equity investors demand the levered return r_E^ℓ on the equity (E_0^ℓ) portion of firm value (V_0^ℓ), which is taxed at a personal tax rate v_E. Debt-holders require a return of r_D on debt (D_0), which is taxed at v_D. Therefore, the above comparison of alternatives can be written as follows:

$$V_1^\ell + OCF_1 + ICF_1 - T_1^u + TS_1 = V_0^\ell[1 + r_E^\ell(1 - v_E)\frac{E_0^\ell}{V_0^\ell} + r_D(1 - v_D)\frac{D_0}{V_0^\ell}] \qquad (6)$$

In practice, three other approaches to DCF-valuation are used besides the APV-approach: the WACC-, TCF-, and Equity-approach.[24] The WACC-approach to valuation is the most widely used approach of the different DCF-models.[25] It discounts the free cash flows to the firm X^u, calculated as if the firm were entirely equity financed, that is, after deduction of fictitious tax-payments without taking into account the tax advantage arising form debt-financing in the numerator: $X_t^u = OCF_t + ICF_t - T_t^u$. The tax advantage is completely integrated into the weighted average cost of capital, the WACC.[26] This can be derived from the above comparison as follows:

$$V_1^\ell + X_1^u = V_0^\ell[1 + r_E^\ell(1 - v_E)\frac{E_0^\ell}{V_0^\ell} + r_D(1 - v_D)\frac{D_0}{V_0^\ell}] - TS_1$$

$$\Leftrightarrow V_0^\ell = \frac{V_1^\ell + X_1^u}{1 + r_E^\ell(1 - v_E)\frac{E_0^\ell}{V_0^\ell} + r_D(1 - v_D)\left(1 - \frac{TS_1}{r_D(1 - v_D)D_0}\right)\frac{D_0}{V_0^\ell}} \qquad (7)$$

Successive substitution of the residual values by their discounted-cash-flow-equivalents yields the WACC-approach:

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22 See *Martin* (1987), pp. 51 for a discussion of the different DCF-approaches.

23 The required rates of return to the investors are defined here as rates of return after corporate, but before personal taxes v. The Tax Shield TS is the tax savings from debt and therefore (T^u – TS) is equal to the total tax payments of a levered firm (including personal taxes). Corporate and personal taxes are assessed on different bases. As the bases of assessment depend on the particular tax system it cannot be treated in a general framework. For the derivation of a general approach it is sufficient to define the tax shield as the difference between the total taxes paid by the investors in a levered (T^u–TS) and in an unlevered firm (T^u). The necessary assumptions on the dividend policy are treated in fn. 13. See *Brealey/Myers* (2000), pp. 500; *Miles/Ezzell* (1980), p. 721. The assessment of the tax shield and the taxes of a levered and an unlevered firm including personal taxes is exemplified for the new German tax system in chapter 4.

24 See *Martin* (1987), pp. 51; *Hachmeister* (1996).

25 See e.g. *Inselbag/Kaufold* (1997), p. 114.

26 See *Copeland/Weston* (1991), pp. 441.

$$V_0^\ell = \sum_{t=1}^{\infty} \frac{X_t^u}{[1 + r_{wacc}^\ell (1-v)]^t} \qquad \text{(WACC-approach)}$$

$$\text{with } r_{wacc}^\ell (1-v) = r_E^\ell (1-v_E) \frac{E_0^\ell}{V_0^\ell} + r_D (1-v_D) \left(1 - \frac{TS_1}{r_D (1-v) D_0}\right) \frac{D_0}{V_0^\ell} \qquad (8)$$

Note that this formula does not account for changes in capital structure. To be able to apply this approach, the ratio of debt to firm value has to remain constant over time.[27] This problem will be discussed in paragraph 2.3. The discount factor used here is a generalized version after personal taxes of what has become known as the "textbook-formula" for the WACC,[28] describing the relationship between the cost of debt and the levered cost of equity, observed at a given ratio of debt to equity. An increase in leverage results in an increase of the cost of equity capital. Return to shareholders increases by more than the increase in the cost of equity, the difference being the tax shield (TS). Therefore, firm value increases and the WACC decrease. This decrease of the weighted-average-costs of capital (WACC) results in an increase in firm value with increasing leverage.

Instead of using the WACC-approach for valuation one can also discount the total cash flows to the firm after actual tax payments, not fictitious taxes (T^u):

$$X^\ell = X^u + TS = (OCF + ICF - T^u + TS)$$

$$\Leftrightarrow V_1^\ell + X_1^u + TS_1 = V_0^\ell [1 + r_E^\ell (1-v_E) \frac{E_0^\ell}{V_0^\ell} + r_D (1-v_D) \frac{D_0}{V_0^\ell}] \qquad (9)$$

Solving the equation for the value of the levered firm yields the Total Cash Flow (TCF)-approach) to valuation:[29]

$$V_0^\ell = \sum_{t=1}^{\infty} \frac{X_t^u + TS_t}{\left[1 + r_E^\ell (1-v_E) \frac{E_0^\ell}{V_0^\ell} + r_D (1-v_D) \frac{D_0}{V_0^\ell}\right]^t} \qquad \text{(TCF-approach) (10)}$$

In this case, the tax shield will be considered in the numerator, not in the cost of capital. Instead of discounting the cash flows of an all-equity firm X^u, those of a levered firm X^ℓ after actual tax payments are discounted.

When applying the "Equity-approach", the levered cost of equity are used for valuation, deriving the value of equity by discounting the Net Free Cash Flows (NFCF). The cash flows to firm X^u are treated separately according to the holders of claims to those cash flows: Z represents the interest payments to debt-holders, NFCF represents the free cash flows net of payments from and to the debt-holders, equivalent to the claims of the shareholders to receive dividends net of increases in equity.[30] At the same time, actual tax payments are deducted instead of fictitious ones:

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27 The time-subscript is used to point out that capital structure is defined by the results of the valuation process, i.e. the resulting market values of debt and equity.

28 See e.g. *Inselbag/Kaufold* (1997), pp. 114.

29 Note that this formula just like the WACC-approach requires a constant capital structure.

30 See *Coenenberg/Schultze* (1998), pp. 290.

$$X^{\ell} = X^u + TS$$
$$X^{\ell} = NFCF - Z(1 - v_D) + \Delta D$$
$$\Rightarrow NFCF = X^u + TS - Z(1 - v_D) + \Delta D \tag{11}$$

This yields the following equivalence of alternatives:

$$V_1^{\ell} + X_1^u + TS_1 - Z(1 - v_D) + \Delta D = E_0^{\ell}[1 + r_E^{\ell}(1 - v_E)] \tag{12}$$

Solving for the levered value of equity and successive substitution of the residual values yields the Equity-approach[31] to valuation:[32]

$$E_0^{\ell} = \sum_{t=1}^{\infty} \frac{NFCF_t}{\left(1 + r_E^{\ell}(1 - v_E)\right)^t} = \sum_{t=1}^{\infty} \frac{X_t^u + TS_t - Z_t(1 - v_D) + \Delta D_t}{\left(1 + r_E^{\ell}(1 - v_E)\right)^t} \quad \text{(Equity-approach)} \tag{13}$$

All three approaches presented above are dependant upon the levered cost of equity. The influence of changes in capital structure on the cost of capital used in these models can be derived from the APV-approach. The resulting functions of the cost of capital for the WACC-, TCF- and Equity-approach describe the influence of changes in capital structure as a function of the unlevered cost of capital used in the APV-approach. By plugging in theses functions into the above approaches, it can be shown that they all yield the same results.[33]

2.3. *Financing strategies and the cost of capital*

The existing literature contains a number of contributions which discuss the equivalence and the advantages of the outlined approaches to valuation, in particular those of the WACC and APV-models.[34] Depending on the underlying financing premises, clear relationships between the different cost-of-capital-formulas arise.[35] While the cost of capital-formulas used in WACC-, TCF- and Equity-model describe the levered cost of capital at a given debt/equity-ratio, their change with respect to changes in capital structure have to be determined from basic relationship of firm value and tax shield, described by the APV-model. In other words: As long as the appropriate cost of levered equity and debt are determined, the firm's cost of capital at a particular level of debt can be calculated using the textbook-formula for the WACC- or TCF-approach and firm value can be assessed using the appropriate model. Therefore, a comparison of the WACC- and APV-models requires great care in assessing the cost of capital.

The value of a levered firm depends on the value of the interest tax shield. To determine the cost of capital for levered companies it is therefore necessary to integrate the

31 *Martin* (1987), p. 56 refers to the “flow through to equity”-approach in this context.

32 Note that the WACC-, TCF as well as the Equity approach include variables which depend on the value of equity, which is only known when the process of valuation is completed. This can be solved e.g. via iterations (see *Drukarczyk* (2001) p. 208). The problem of circularity of the different approaches, i.e. the fact that for the calculation of the cost of capital depends on the results of the process of valuation, is summarized by *Wallmeier* (1999). See paragraph 2.3. for further reference.

33 See below sections 3.1.2. and 3.3.1.

34 See *Inselbag/Kaufold* (1997), pp. 114; *Miles/Ezzell* (1980), p. 719. *Wallmeier* (1999) provides a list of references for the discussion within Germany. On the one hand some authors claim that all approaches lead to identical results as long as the financing premises are modelled adequately, whereas, on the other hand, some contributors are doubtful whether this holds true under realistic assumptions. These discussions are often contradictory, which is due to the fact that different models are mixed. See *Wallmeier* (1999), pp. 1481 for a discussion.

35 See *Wallmeier* (1999), p. 1476; *Wallmeier* (2001), pp. 283.

impact of debt-financing into the valuation model. This leads to different possible scenarios in which the complexities of possible real-world applications is reduced to a set of different assumptions:

Firstly, valuation depends on the financing strategies of the firm. Generally, two basic alternatives can be considered:[36]

- The firm targets the absolute value of debt outstanding.[37] The amount and development of debt is predetermined and independent of the development of the firm value. This can be the case when at the time of valuation the firm has agreed to a financing contract to adhere to a certain amortization schedule. In this case, the firm's ratio of debt to firm value $\Lambda_t = D_t / V_t$ will not remain constant. On the other hand, the fact that the level of debt in future periods is predetermined consequently results in a corresponding tax advantage already being known at the time of valuation, and therefore, being as risky as debt itself (so-called *F-model*).[38]
- The amount and development of debt is linked to the firm value. Future capital structure in market values is fixed such that the level of debt changes over time, but the ratio of debt to firm value remains constant. In this case, future tax advantages are just as uncertain and risky as firm value (so-called *L-model*).[39]

Secondly, we can distinguish two different cases, depending on whether future cash flows are determined as perpetuities or non-perpetuities.

Table 1: Different financing strategies and their influence on the discount rate

	F-Model	*L-Model*
1. perpetuities	discount rate remains constant	discount rate remains constant
2. non-perpetuities		
2.1. constant leverage	n.a.	discount rate remains constant
2.2. fluctuating leverage	discount rate fluctuates	discount rate fluctuates

Besides investigating the L- and F-models, two cases will be considered in the following discussion: Firstly, the special case of the perpetuity-model as starting point, secondly, the case of non-perpetuities. Within the non-perpetuities we can distinguish two subcases: a constant ratio of debt to firm value implying constant discount rates, and alternatively, the generalisation of the F- and L-models for fluctuating leverage and discount rates. In the case of perpetuities, future firm values are constant, and therefore, the ratio

36 See *Inselbag/Kaufold* (1997), p. 115; also *Kruschwitz/Löffler* (1999), p. 7; *Wallmeier* (1999), p. 1474. The two alternatives treated here can be viewed as extremes of the actual real world financing strategies which lie somewhere in between. There are no approaches developed yet that allow to treat these cases, in which the riskiness of the tax shield is not easily determined. E.g. *Löffler* (2000) derives the value of the tax shield in the special case of a levered buyout using a Martingale-process.

37 See *Inselbag/Kaufold* (1997), p. 115.

38 See *Inselbag/Kaufold* (1997), pp. 114, 116. This financing strategy requires the firm to stick to the predetermined path. Any additional financing needs, even if unexpected, occurring in the future, need to be covered by equity. Any surplus has to be paid out to equity instead of reducing debt.

39 See *Inselbag/Kaufold* (1997), pp. 115, 119. This financing strategy requires the firm to stick to the chosen debt/equity blend, independent of the actual financing needs resulting from the investment activities. This can mean that in periods when there are high surpluses from operations and low investing activities, the firm might have to raise debt and repay equity (debt-equity-swap).

of debt to firm value will be constant as well. As a result, the cost of capital will also be a constant. If, however, the expected future cash flows are allowed to fluctuate, future firm values will generally fluctuate as well. If debt levels follow a predetermined path independent of firm values (F-model), the ratio of debt and firm value will generally fluctuate, which will lead to changes in the cost of capital over time. Within the F-model, only perpetuities will yield constant discount rates. Only when debt-levels are a constant fraction of firm value (L-model), the ratio of debt to firm value will remain constant, allowing the use of a single discount rate for all future periods. If this fraction is allowed to vary over time, the cost of capital will again have to be adjusted for every change in leverage. Within the case of non-perpetuities we can therefore examine the special case of a constant ratio of debt to firm value implying constant discount rates.

3. Derivation of the cost of capital under different scenarios

The different approaches to valuation presented above depend on the specification of the tax shield. While the APV-approach can be specified by the determination of the present value of the tax shields, the results based on the other approaches, however, are only valid when the influence of changes in leverage on the levered cost of capital used in these models is adequately considered. The adjustments for changes in leverage can be found by modifying the APV-approach, adjusted for the particular financing strategy, to represent the valuation process typical for the valuation approach looked at.

3.1. *Valuation in the Perpetuity-model*

The APV-approach for perpetuities can be written as follows:

$$V_0^\ell = \frac{X^u}{r_E^u(1-v_E)} + PVTS \tag{14}$$

The value of the levered firm is determined as the sum of the value of the unlevered firm and the value of the tax shield (PVTS). The first is determined by discounting the unlevered free cash flows to the firm by the unlevered cost of capital (r_E^u). The value of the tax shield depends on whether the L- or F-Model is applied.

3.1.1. *F-Model*

Under the F-model financing strategy, the basic APV-valuation approach can be written as follows:[40]

$$V_0^\ell = \frac{X^u}{r_E^u(1-v_E)} + \frac{TS}{r_D(1-v_D)} \tag{15}$$

If, instead, valuation shall be performed using the WACC-approach, this basic identity has to be restated. In the WACC-approach levered firm value is determined by discounting the unlevered free cash flows at a single discount rate:[41]

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40 In the perpetuity model we have: $V_1 = V_t = V$.

41 This derivation is based on *Miles/Ezzell* (1980), p. 725.

$$V_0^\ell - \frac{TS}{r_D(1-v_D)} = \frac{X^u}{r_E^u(1-v_E)} \Leftrightarrow V_0^\ell = \frac{X^u}{\left(1-\frac{TS}{r_D(1-v_D)D}\frac{D}{V}\right)r_E^u(1-v_E)} \tag{16}$$

The denominator represents the levered cost of capital, needed for the WACC-approach as a function of the unlevered cost of capital and capital structure. It is a generalized version of what has become known in literature as the *Modigliani/Miller*-Formula for the firm's cost of capital for a levered firm in the perpetuity-F-model. It is written in a form which allows for a later specification of the tax shield:

$$\boxed{r_{WACC}^\ell(1-v) = r_E^u(1-v_E)\left(1-\frac{TS}{r_D(1-v_D)D}\frac{D}{V}\right)} \tag{17}$$

In the special case that $v_E = v_D = v$ we have:[42]

$$r_{WACC}^\ell(1-v) = r_E^u\left(1-v-\frac{TS}{r_D D}\frac{D}{V}\right) \tag{18}$$

The TCF-approach can be derived as follows:

$$V_0^\ell = \frac{X^u}{r_E^u(1-v_E)} + \frac{TS}{r_D(1-v_D)} = \frac{X^\ell - TS}{r_E^u(1-v_E)} + \frac{TS}{r_D(1-v_D)}$$

$$\Leftrightarrow V_0^\ell = \frac{X^\ell}{r_E^u(1-v_E) - \left(\frac{r_E^u(1-v_E)}{r_D(1-v_D)}-1\right)\frac{TS}{D}\frac{D}{V}} \tag{19}$$

with the cost of capital:

$$\boxed{r_{TCF}^\ell(1-v) = r_E^u(1-v_E) - \left(\frac{r_E^u(1-v_E)}{r_D(1-v_D)}-1\right)\frac{TS}{D}\frac{D}{V}} \tag{20}$$

In the special case that $v_E = v_D = v$ we have:[43]

$$r_{TCF}^\ell(1-v) = r_E^u(1-v) - (r_E^u - r_D)\frac{TS}{r_D D}\frac{D}{V} \tag{21}$$

The cost of equity for the Equity-approach can be derived as follows:[44]

$$E_0^\ell = V_0^\ell - D = \frac{X^u}{r_E^u(1-v_E)} + \frac{TS}{r_D(1-v_D)} - D = \frac{NFCF + Z(1-v_D) - TS}{r_E^u(1-v_E)} + \frac{TS}{r_D(1-v_D)} - D$$

$$\Leftrightarrow E_0^\ell = \frac{NFCF}{\left[r_E^u(1-v_E) - \left(r_D(1-v_D) - r_E^u(1-v_E) + \left(\frac{r_E^u(1-v_E)}{r_D(1-v_D)}-1\right)\frac{TS}{D}\right)\frac{D}{E}\right]} \tag{22}$$

42 See *Miller* (1977).

43 See *Modigliani/Miller* (1963), pp. 438.

44 X^u = NFCF + Z(1 – v) - TS + ΔD; The level of debt is assumed to be constant, therefore, ΔD = 0.

This leads to the generalized *Modigliani/Miller*-Formula for the cost of equity of a levered firm in the perpetuity-F-model:

$$\boxed{r_E^{\ell}(1-v_E) = r_E^u(1-v_E) + \left(r_E^u(1-v_E) - r_D(1-v_D)\right)\left(1 - \frac{TS/D}{r_D(1-v_D)}\right)\frac{D}{E}} \tag{23}$$

In the special case that $v_E = v_D = v$ we have:

$$r_E^{\ell}(1-v) = r_E^u(1-v) + (r_E^u - r_D)(1 - v - \frac{TS}{r_D D})\frac{D}{E} \tag{24}$$

Modigliani/Miller (1963) originally derived these different approaches to valuation based on the APV-approach, as functions of changes in leverage. The "textbook formula" of the WACC on the other hand was designed to determine the firm's cost of capital at a given level of debt-financing.[45] The "textbook formula" can be used to show that the formulas derived above, and therefore, the different approaches, are in accordance with each other: By plugging the cost-of-equity-function into the WACC-Formula, we obtain the MM-Formula for the firm's cost of capital:[46]

$$r_{WACC}(1-v) = \left(r_E^{\ell}(1-v_E)\frac{E}{V} + r_D(1-v_D - \frac{TS}{r_D D})\frac{D}{V}\right) \tag{25}$$

$$\forall r_E^{\ell}(1-v_E) = r_E^u(1-v_E) - \left(r_D(1-v_D) - r_E^u(1-v_E)\right)\left(1 - \frac{TS}{r_D(1-v_D)D}\right)\frac{D}{E}$$

$$\Leftrightarrow \; r_{WACC}(1-v) = r_E^u(1-v_E)\left(1 - \frac{TS}{r_D(1-v_D)D}\frac{D}{V}\right) = \text{Equation (17)}$$

The same can be shown for the TCF-model:

$$r_{TCF}(1-v) = \left(r_E(1-v_E)\frac{E}{V} + r_D(1-v_D)\frac{D}{V}\right) \tag{26}$$

$$\forall r_E^{\ell}(1-v_E) = r_E^u(1-v_E) - \left(r_D(1-v_D) - r_E^u(1-v_E)\right)\left(1 - \frac{TS}{r_D(1-v_D)D}\right)\frac{D}{E}$$

$$\Leftrightarrow \; r_{TCF}(1-v) = \left(r_E^u(1-v_E) - \left(\frac{r_E^u(1-v_E)}{r_D(1-v_D)} - 1\right)\frac{TS}{D}\frac{D}{V}\right) = \text{Equation (20)}$$

3.1.2. *L-model*

Under the assumption of a constant ratio of debt and firm value (L-model), the amount of debt will be known one period ahead of time. Therefore, the tax advantage is known one period in advance, and therefore, has to be valued at the cost of debt for that particular period. For all following periods, however, the tax shield is subject to the same risk as the value of the firm, requiring the unlevered cost of capital to be applied,[47] which leads to the following valuation model:[48]

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45 See *Kruschwitz/Löffler* (1999), pp. 12.

46 See *Copeland/Weston* (1992), p. 451; also *Drukarczyk* (2001), p. 407.

47 See *Myers* (1974), p. 22; *Wallmeier* (1999), p. 1481.

48 See *Brealey/Myers* (2000), pp. 561.

$$V_0^\ell = \frac{X^u}{r_E^u(1-v_E)} + \sum_{t=1}^{\infty} \frac{TS}{(1+r_D(1-v_D))(1+r_E^u(1-v_E))^{t-1}}$$

$$\Leftrightarrow V_0^\ell = \frac{X^u}{\left(r_E^u(1-v_E) - \frac{TS}{D}\frac{(1+r_E^u(1-v_E))}{(1+r_D(1-v_D))}\frac{D}{V}\right)} \tag{27}$$

This leads to the following adjustment formula for the cost of capital in the WACC-approach if the L-model is applied:

$$\boxed{r_{WACC}^\ell(1-v) = r_E^u(1-v_E) - \frac{TS}{D}\frac{(1+r_E^u(1-v_E))}{(1+r_D(1-v_D))}\frac{D}{V}} \tag{28}$$

Formula (28) is the generalized version of what has become known as the *Miles/Ezzell*-formula for the firm's cost of capital for a levered firm in the perpetuity-L-model. Applying the Equity-approach, valuation can be performed as follows:[49]

$$E_0^\ell = V_0^\ell - D = \frac{X^u}{r_E^u(1-v_E)} + \frac{TS}{r_E^u}\frac{(1+r_E^u(1-v_E))}{(1+r_D(1-v_D))} - D$$

$$\Leftrightarrow\ E_0^\ell = \frac{NFCF}{r_E^u(1-v_E) + \frac{\left(r_E^u(1-v_E) - r_D(1-v_D)\right)}{(1+r_D(1-v_D))}\left(1+r_D(1-v_D) - \frac{TS}{D}\right)\frac{D}{E}} \tag{29}$$

The cost of equity can be simplified to take the following form:[50]

$$\boxed{r_E^\ell(1-v_E) = r_E^u(1-v_E) + \left[r_E^u(1-v_E) - r_D(1-v_D)\right]\left(1 - \frac{TS/D}{1+r_D(1-v_D)}\right)\frac{D}{E}} \tag{30}$$

Formula (30) corresponds to the *Miles/Ezzell*-formula for the cost of equity capital. Plugging this result into the WACC-textbook-formula yields the firm's cost of capital derived above:[51]

$$r_{WACC}(1-v) = \left(r_E^\ell(1-v_E)\frac{E}{V} + r_D(1-v_D - \frac{TS}{r_D D})\frac{D}{V}\right) \tag{31}$$

$$\forall r_E^\ell(1-v_E) = r_E^u(1-v_E) + \left[r_E^u(1-v_E) - r_D(1-v_D)\right]\left(1 - \frac{TS/D}{1+r_D(1-v_D)}\right)\frac{D}{E}$$

$$\Leftrightarrow r_{WACC}(1-v) = r_E^u(1-v_E) - \frac{TS}{D}\frac{(1+r_E^u(1-v_E))}{(1+r_D(1-v_D))}\frac{D}{V} \ = \text{Equation (28)}$$

The perpetuity-case was chosen as a starting point as it highlights the procedure of derivation. In practice, fluctuating payments, at least during the first period of the planning process, will enter into the model. Therefore, we need to show the changes that come about when considering non-perpetuities. Two approaches are possible: one can derive discount rates for every single period and discount future cash flows step by step, or al-

[49] As cash flows are perpetuities, the value of the firm is expected to be a constant and so is debt, as a constant percentage of value, therefore, $\Delta D = 0$.

[50] See similar in *Wallmeier* (1999), p. 1478.

[51] See also *Miles/Ezzell* (1980), p. 726; *Wallmeier* (1999), p. 1476.

ternatively, we can attempt to find one uniform rate. The latter case shall be presented at first.

3.2. *Non perpetuities and constant discount rates*

The valuation approaches presented above require constant parameters for all future periods, i.e. expected cash flows, tax rates, cost of equity and debt. Therefore, the value of equity and debt as well as the underlying capital structure are constant over time (Perpetuity-model).[52] The requirement of constant expected cash flows, however, doesn't require cash flows to be fixed over time, and therefore, being known for sure, it merely requires their expected values to be the same for all future periods, leaving the future values of the firm uncertain.[53]

The underlying financing strategy has considerable impact on valuation and the cost of capital, as was clearly shown within the context of the perpetuity model. When the different components needed for valuation are no longer assumed to remain constant, firm value, and therefore capital structure, would fluctuate over time. Therefore, the cost of capital would need to be adjusted in each period. *Miles/Ezzell* (1980), however, have shown for the non-perpetuity case, that if a constant ratio (Λ = D/V) of debt to firm value is maintained (L-model), not only the basic approaches to valuation are applicable (WACC etc.),[54] but also the cost of capital will remain constant over time.[55]

The value of a levered firm is composed of the value of an all-equity firm plus the value of the tax advantages. The value of the all-equity firm can be obtained by discounting the unlevered free cash flows to the firm X^u by the cost of capital of the unlevered firm after taxes:

$$V_0^u = \sum_{t=1}^{T} \frac{X_t^u}{(1+r_E^u(1-v_E))^t} + \frac{V_T^u}{(1+r_E^u(1-v_E))^T} \tag{32}$$

For the levered firm, the value of the tax advantages (PVTS) needs to be added, which in the L-model is dependent for each period upon the value of the levered firm in the following period. This in turn makes simple addition of the values of the tax shields to the value of the all-equity firm impossible. The amount of debt is fixed at the beginning of each period, with the consequence that the value of the tax shield is fixed for the period, and is therefore, just as risky as debt, and therefore, has to be discounted at the cost of debt in that particular period. For the remaining periods the tax shield will be just as uncertain as the value of the firm, since the amount of the debt is not yet known, but is dependent on the firm value.[56] The difference between the levered value V_t^ℓ and the all-equity value V_t^u in the L-model in each period is merely the factor $1\Big/\left(1-\frac{\Lambda\, TS_t/D_{t-1}}{1+r_D(1-v_D)}\right)$. This means that the value of the levered firm and the value of the

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[52] See *Brealey/Myers* (2000), p. 546; *Myers* (1974), p. 12.
[53] See *Miles/Ezzell* (1980), p. 721.
[54] See *Miles/Ezzell* (1980), pp. 723, 726. See also *Brealey/Myers* (2000), p. 546.
[55] See *Miles/Ezzell* (1980), p. 726.
[56] See *Miles/Ezzell* (1980), p. 724. See also *Wallmeier* (1999), p. 1479.

all-equity firm are perfectly correlated, and therefore, identical in risk. Therefore, all V_t have to be discounted with r_E^u.[57] The levered value in t = 0 is determined as follows:

$$V_0^\ell = \frac{X_1^u}{1+r_E^u(1-v_E)} + \frac{TS_1}{1+r_D(1-v_D)} + \frac{V_1^\ell}{1+r_E^u(1-v_E)} \tag{33}$$

The WACC-approach can again be derived from this APV-valuation by solving for an equation that only discounts the unlevered expected free cash flows to the firm (X^u):

$$V_0^\ell = \frac{X_1^u + V_1^\ell}{1+r_E^u(1-v_E)} + \frac{\Lambda V_0^\ell \, TS_1/D_0}{1+r_D(1-v_D)} \Leftrightarrow V_0^\ell = \frac{X_1^u + V_1^\ell}{(1+r_E^u(1-v_E))\left(1-\frac{\Lambda\, TS_1/D_0}{1+r_D(1-v_D)}\right)} \tag{34}$$

The levered value in t = 1 can be determined in the same way:

$$V_1^\ell = \frac{X_2^u + V_2^\ell}{(1+r_E^u(1-v_E))\left(1-\frac{\Lambda\, TS_2/D_1}{1+r_D(1-v_D)}\right)} \tag{35}$$

Repeating this procedure and replacing V_t^ℓ with their present value equivalents yields the following valuation approach:

$$V_0^\ell = \sum_{t=1}^{T} \frac{X_t^u}{(1+r_V^\ell(1-v))^t} \frac{X_T^\ell}{(1+r_V^\ell(1-v))^T} \tag{36}$$

The relevant discount-factor for this approach is given by the following formula, a generalized version of the well-known *Miles/Ezzell*-formula for constant cost of capital of a levered firm, valid for all periods:[58]

$$\boxed{r_{WACC}^\ell(1-v) = r_E^u(1-v_E) - \frac{TS_t}{D_{t-1}}\left(\frac{1+r_E^u(1-v_E)}{1+r_D(1-v_D)}\right)\frac{D}{V}} \tag{37}$$

Since TS_t is a multiple of D_{t-1}, changes of D over time do not result in changes of TS/D, which, therefore, is a constant for the entire valuation period to be determined only once with the tax shield resulting from the amount of debt existing at the time of valuation. The above form for the levered cost of capital as a function of TS/D allows its adjustment to any kind of tax system by replacing TS/D by the specific tax shield resulting from the particular tax system.

Application of the Equity-approach yields the cost of equity: The cash flows are distributed among the investing parties - the shareholders have claims to receive the NFCF, the debt-holders to receive interest after taxes and debt repayments ($Z(1-v_D) - \Delta D_t$) net of increases in debt:

$$\begin{aligned} &NFCF_t = X_t^u + TS_t - Z_t(1-v_D) + \Delta D_t \\ &\Rightarrow X_t^u = NFCF_t - TS_t + Z_t(1-v_D) - \Delta D_t \end{aligned} \tag{38}$$

The following model results for period t = 0:

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57 See *Miles/Ezzell* (1980), p. 725.

58 See *Miles/Ezzell* (1980), p. 726.

$$E_0^{\ell} = V_0^{\ell} - D_0 = \frac{X_1^u}{1 + r_E^u(1 - v_E)} + \frac{TS_1}{1 + r_D(1 - v_D)} + \frac{V_1^{\ell}}{1 + r_E^u(1 - v_E)} - D_0^{\ell} \tag{39}$$

$$\Leftrightarrow \; E_0^{\ell} = \frac{NFCF_1 + E_1^{\ell}}{1 + r_E^u(1 - v_E)\left\{1 - \left(\frac{\left(1 - \frac{TS_1}{D_0} + r_D(1 - v_D)\right)}{1 + r_E^u(1 - v_E)} + \frac{\frac{TS_1}{D_0}}{1 + r_D(1 - v_D)} - 1\right)\frac{D_0}{E}\right\}} \tag{40}$$

Simplifying and gradually substituting the residual values by their present-value-equivalents yields the Equity-approach to valuation:

$$E_0^{\ell} = \sum_{t=1}^{T} \frac{NFCF_1}{(1 + r_E^{\ell}(1 - v_E))^t} + \frac{E_T^{\ell}}{(1 + r_E^{\ell}(1 - v_E))^T} \tag{41}$$

The levered cost of equity can be expressed as follows:[59]

$$\boxed{r_E^{\ell}(1 - v_E) = r_E^u(1 - v_E) + \left[r_E^u(1 - v_E) - r_D(1 - v_D)\right]\left(1 - \frac{TS/D}{1 + r_D(1 - v_D)}\right)\frac{D}{E}} \tag{42}$$

This is a general form of the well-known *Miles/Ezzell*-Formula and is used to adjust the cost of equity to changes in capital structure under the assumption of a constant ratio of debt and equity.[60] Substituting this into the „textbook formula“ yields again the function for the firm's cost of capital derived above (37), as was shown in equation (31).[61] If capital structure, however, cannot be assumed to remain constant over time in terms of market values, no single discount rate can be applied to all periods.

3.3. *Non-perpetuities and fluctuating capital structure*

In reality, the assumptions of a constant level of debt (MM-Perpetuity-model), and of a constant ratio of debt to firm value, respectively, are rarely fulfilled. Under realistic conditions, discount rates will often fluctuate over time. Generally, for the value in period t = 0 the following relationship holds:

$$V_0 = \frac{X_1 + V_1}{1 + r_0} \tag{43}$$

V_1 in turn can be expressed by $V_1 = \frac{X_2 + V_2}{1 + r_1}$ and so on, which leads to the following general relationship for the value of the firm:[62]

$$V_0 = \frac{X_1}{1 + r_0} + \frac{X_2}{(1 + r_0)(1 + r_1)} + ... = \sum_{t=1}^{T} \frac{X_t}{\prod_{j=0}^{t-1}(1 + r_j)} \frac{V_T}{\prod_{j=0}^{T-1}(1 + r_j)} \tag{44}$$

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59 See *Wallmeier* (1999), p. 1478.
60 See *Wallmeier* (1999), p. 1476.
61 See also *Miles/Ezzell* (1980), pp. 726, *Wallmeier* (1999), p. 1476.
62 See also *Miles/Ezzell* (1980), p. 723.

It is questionable whether a predetermined financing policy (F-model), or a value oriented strategy (L-model), is more adequate to represent real conditions. Both financing-strategies can result in contradictions: Strict application of the F-model does not allow for any further debt financing, even when necessary. In the L-model, however, situations can occur, in which an increase of debt-financing is required even when there is a surplus from internal financing. The financing strategy under which a firm maintains a constant capital structure in book values is neither adequately represented by the L-model nor the F-model. In such a case, the uncertainty of the tax shield, like the amount and timing of future financing needs, will depend on the projected (uncertain) future cash flows. Therefore, the increase in debt is polyvalent and as uncertain as the cash flows, unless the firm would, at the date of valuation, agree by contract, or otherwise, to a level of debt equal to the expected value. To date, the existing literature does, to our knowledge, not yet provide an approach for modeling capital structures planned in book values.[63] One first step in this direction is to take periodically adjusted cost of capital rates into consideration. Therefore, the F- and L-model under fluctuating cost of capital rates are considered below.

3.3.1. *F-model*

The F-model is, as before, characterized by a predetermined amount of debt for each planning period. The deterministic development of the level of debt implies that the value of the tax advantages will depend solely on the amount of debt, not on the expected future cash flows. Therefore, the cost of debt can be used to determine the value of the tax shield. In contrast to the perpetuity-F-model, a change in the amount of debt financing is possible (i. e. $\Delta D \neq 0$), in contrast to the L-model it is planned in advance and not only incurred afterwards through realisation of values of the firm differing from its expected value. The value of a levered firm is composed of the value of the all-equity firm and the present value of the tax shields:

$$V_0^{\ell} = V_0^u + PVTS_0 \tag{45}$$

The value of the all-equity firm is determined as before:[64]

$$V_0^u = \sum_{t=1}^{T} \frac{X_t^u}{(1 + r_E^u (1 - v_E))^t} + \frac{V_T^u}{(1 + r_E^u (1 - v_E))^T} \tag{46}$$

PVTS is based on the cost of debt $r_{D,t}$ which can now be subject to change over time:[65]

$$PVTS_0 = \sum_{t=1}^{T} \frac{TS_t}{\prod_{j=0}^{t-1} \left(1 + r_{D,j}(1 - v_D)\right)} + \frac{TS_T}{\prod_{j=0}^{T-1} \left(1 + r_{D,j}(1 - v_D)\right)} \tag{47}$$

The following relationship between the values of the firm holds true for any particular point of time:

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63 See for such an approach *Löffler* (2000); *Richter/Drukarczyk* (2001). See *Barclay/Smith/Watts* (1995), pp. 4, for a discussion of the determinants of debt-financing; also *Inselbag/Kaufold* (1997), p. 119.

64 The following discussion is based on *Inselbag/Kaufold* (1997) and *Heitzer/Dutschmann* (1999).

65 The fact that the unlevered cost of capital are invariant over time implies that operating risk has to remain the same.

$$V_{t-1}^{\ell} = V_{t-1}^{u} + PVTS_{t-1} = \frac{X_t^u + V_t^u}{1 + r_E^u(1 - v_E)} + \frac{PVTS_t}{1 + r_D(1 - v_D)} + \frac{TS_t}{1 + r_D(1 - v_D)} \tag{48}$$

and:

$$PVTS_{t-1} = \frac{PVTS_t}{1 + r_{D,t-1}(1 - v_D)} + \frac{TS_t}{1 + r_{D,t-1}(1 - v_D)} \tag{49}$$

The WACC-approach can now be derived as follows:

$$V_0^{\ell} = V_0^u + PVTS_0 = \frac{X_1^u + V_1^u}{1 + r_E^u(1 - v_E)} + \frac{PVTS_1}{1 + r_D(1 - v_D)} + \frac{TS_1}{1 + r_D(1 - v_D)}$$

$$\Leftrightarrow V_0^{\ell} = \frac{X_1^u + V_1^{\ell}}{1 + r_E^u(1 - v_E) + \dfrac{PVTS_1 - (1 + r_E^u(1 - v_E))PVTS_0}{V_0^{\ell}}} \tag{50}$$

Simplification and successive substitution of the residual values leads to the following valuation model (WACC-approach):

$$V_0^{\ell} = \sum_{t=1}^{T} \frac{X_t^u}{\prod_{j=0}^{t-1}\left(1 + r_{WACC,j}^{\ell}(1 - v)\right)} + \frac{V_T^{\ell}}{\prod_{j=0}^{T-1}\left(1 + r_{WACC,j}^{\ell}(1 - v)\right)} \tag{51}$$

with the formula for the firm's cost of capital:

$$\boxed{r_{WACC,t-1}^{\ell}(1 - v) = r_E^u(1 - v_E) - \frac{PVTS_t + (1 + r_E^u(1 - v_E))PVTS_{t-1}}{V_{t-1}^{\ell}}} \tag{52}$$

The Equity-approach yields the adjustment formula for the cost of equity:

$$E_0^{\ell} = V_0^u + PVTS_0 - D_0 = \frac{X_1^u}{1 + r_E^u(1 - v_E)} + \frac{V_1^u}{1 + r_E^u(1 - v_E)} + PVTS_0 - D_0$$

$$\Leftrightarrow E_0^{\ell} = \frac{NFCF_1 + E_1^{\ell}}{(1 + r_E^{\ell}(1 - v_E))} \tag{53}$$

Successive substitution of the residual values yields the generalized Equity-approach:

$$E_0^{\ell} = \sum_{t=1}^{T} \frac{NFCF_t}{\prod_{j=0}^{t-1}\left(1 + r_{E,j}^{\ell}(1 - v_E)\right)} + \frac{E_T^{\ell}}{\prod_{j=0}^{T-1}\left(1 + r_{E,j}^{\ell}(1 - v_E)\right)} \tag{54}$$

with the levered cost of equity per period:[66]

[66] See *Heitzer/Dutschmann* (1999), p. 1466; *Inselbag/Kaufold* (1997), p. 118.

$$\boxed{\begin{aligned} & r^{\ell}_{E,t-1}(1-v_E) = r^{u}_{E}(1-v_E) + \left(r^{u}_{E}(1-v_E) - r_{D,t-1}(1-v_D) + \frac{TS_1}{D_{t-1}} \right) \frac{D_{t-1}}{E^{\ell}_{t-1}} \\ & + \frac{PVTS_t - PVTS_{t-1}(1 + r^{u}_{E}(1-v_E))}{E^{\ell}_{t-1}} \end{aligned}} \tag{55}$$

Substituting this into the textbook formula yields again the formula for the firm's cost of capital derived above (formula (52)):

$$\begin{aligned} & r_{WACC,t-1}(1-v) = (r^{u}_{E}(1-v_E) + \left(r^{u}_{E}(1-v_E) - r_{D,t-1}(1-v_D) + \frac{TS_1}{D_{t-1}}) \right) \frac{D_{t-1}}{E^{\ell}_{t-1}} + \\ & \qquad \frac{PVTS_t - PVTS_{t-1}(1 + r^{u}_{E}(1-v_E))}{E^{\ell}_{t-1}}) \frac{E^{\ell}_{t-1}}{V^{\ell}_{t-1}} + r_D(1 - v_D - \frac{TS_1}{r_{D,t-1} D_{t-1}}) \frac{D_{t-1}}{V^{v}_{t-1}} \\ & \Leftrightarrow \; r_{WACC,t-1}(1-v) = \left(r^{u}_{E}(1-v_E) + \frac{PVTS_t - PVTS_{t-1}(1 + r^{u}_{E}(1-v_E))}{V^{\ell}_{t-1}} \right) \end{aligned} \tag{56}$$

The different valuation approaches therefore lead to identical results, even in the general case of fluctuating discount rates.

3.3.2. *L-model*

If capital structure in market values is changed, the above derivation of the ME-formula is applicable for the period in which a particular capital structure is maintained. If the target capital structure is changed from period to period, the following model can be derived. For every point in time, the following relationships hold:

$$\begin{aligned} V^{\ell}_{t-1} &= V^{u}_{t-1} + PVTS_{t-1} = \frac{X^{u}_{t}}{1 + r^{u}_{E}(1-v_E)} + \frac{V^{u}_{t}}{1 + r^{u}_{E}(1-v_E)} + PVTS_{t-1} \\ PVTS_{t-1} &= \frac{PVTS_t}{1 + r^{u}_{E}(1-v_E)} + \frac{TS_t}{1 + r_{D,t-1}(1-v_D)} \end{aligned} \tag{57}$$

Firm value in t = 0 therefore can be computed by:

$$\begin{aligned} V^{\ell}_{0} &= V^{u}_{0} + PVTS_0 = \frac{X^{u}_{1} + V^{u}_{1}}{1 + r^{u}_{E}(1-v_E)} + \frac{PVTS_1}{1 + r^{u}_{E}(1-v_E)} + \frac{TS_1}{1 + r_{D,0}(1-v_D)} \\ \Leftrightarrow V^{\ell}_{0} &= \frac{X^{u}_{1} + V^{\ell}_{1}}{1 + r^{u}_{E}(1-v_E) \left(1 - \frac{TS_1 / D_0}{1 + r_{D,0}(1-v_D)} \frac{D_0}{V^{\ell}_{0}} \right)} \end{aligned} \tag{58}$$

Simplification and successive substitution yields:

$$V^{\ell}_{0} = \sum_{t=1}^{T} \frac{X^{u}_{t}}{\prod_{j=0}^{t-1} \left(1 + r^{\ell}_{WACC,j}(1-v) \right)} + \frac{V^{\ell}_{T}}{\prod_{j=0}^{T-1} \left(1 + r^{\ell}_{WACC,j}(1-v) \right)} \tag{59}$$

$$\boxed{r^{\ell}_{WACC,t-1}(1-v) = r^{u}_{E}(1-v_E) - \frac{TS_t}{D_{t-1}} \frac{1+r^{u}_{E}(1-v_E)}{1+r_{D,t-1}(1-v_D)} \frac{D_{t-1}}{V^{\ell}_{t-1}}} \tag{60}$$

Formula (60) corresponds to the *Miles/Ezzell*-Formula applied to one single period. The same can be shown for the Equity-approach, which yields the levered cost of equity-capital:

$$E^{\ell}_0 = \frac{NFCF_1 - TS_1 + r_{D,0}(1-v_D)D_0 - \Delta D_1}{1+r^{u}_{E}(1-v_E)} + \frac{V^{u}_1}{1+r^{u}_{E}(1-v_E)} + PVTS_0 - D_0$$

$$E^{\ell}_0 - \frac{\left(1-\frac{TS_1}{D_0}+r_{D,0}(1-v_D)\right)D_0}{1+r^{u}_{E}(1-v_E)} - \frac{TS_1}{1+r_{D,0}(1-v_D)} + D_0 = \frac{NFCF_1 + E^{\ell}_1}{1+r^{u}_{E}(1-v_E)}\,. \tag{61}$$

Successive substitution yields the following general adjustment for the cost of equity-capital, which is again a generalized version of the *Miles/Ezzell*-formula:

$$\boxed{r^{\ell}_{E,t-1}(1-v_E) = r^{u}_{E}(1-v_E) + \left[r^{u}_{E}(1-v_E) - r_{D,t-1}(1-v_D)\right]\left(1-\frac{TS_t/D_{t-1}}{1+r_{D,t-1}(1-v_D)}\right)\frac{D_{t-1}}{E^{\ell}_{t-1}}} \tag{62}$$

3.4. *Overview*

The following list gives an overview over the different scenarios considered and the formulas derived:

<u>Cost of Equity (Equity-Approach):</u>
Perpetuity-model:

F-model $$r^{\ell}_{E}(1-v_E) = r^{u}_{E}(1-v_E) + \left(r^{u}_{E}(1-v_E) - r_D(1-v_D)\right)\left(1-\frac{TS/D}{r_D(1-v_D)}\right)\frac{D}{E}$$

L-model $$r^{\ell}_{E}(1-v_E) = r^{u}_{E}(1-v_E) + \left[r^{u}_{E}(1-v_E) - r_D(1-v_D)\right]\left(1-\frac{TS/D}{1+r_D(1-v_D)}\right)\frac{D}{E}$$

Constant discount rates:

L-model $$r^{\ell}_{E}(1-v_E) = r^{u}_{E}(1-v_E) + \left[r^{u}_{E}(1-v_E) - r_D(1-v_D)\right]\left(1-\frac{TS/D}{1+r_D(1-v_D)}\right)\frac{D}{E}$$

Fluctuating discount rates:

F-model $$r^{\ell}_{E,t-1}(1-v_E) = r^{u}_{E}(1-v_E) + \left(r^{u}_{E}(1-v_E) - r_{D,t-1}(1-v_D) + \frac{TS_1}{D_{t-1}}\right)\frac{D_{t-1}}{E^{\ell}_{t-1}} + \frac{PVTS_t - PVTS_{t-1}(1+r^{u}_{E}(1-v_E))}{E^{\ell}_{t-1}}$$

L-model

$$r^{\ell}_{E,t-1}(1-v_E) = r^u_E(1-v_E) + \left[r^u_E(1-v_E) - r_{D,t-1}(1-v_D)\right]\left(1 - \frac{TS_t/D_{t-1}}{1+r_{D,t-1}(1-v_D)}\right)\frac{D_{t-1}}{E^{\ell}_{t-1}}$$

<u>Firm's cost of capital (WACC-approach):</u>
Perpetuity-model:

F-model
$$r^{\ell}_{WACC}(1-v) = r^u_E(1-v_E)\left(1 - \frac{TS}{r_D(1-v_D)D}\frac{D}{V}\right)$$

L-model
$$r^{\ell}_{WACC}(1-v) = r^u_E(1-v_E) - \frac{TS}{D}\frac{(1+r^u_E(1-v_E))}{(1+r_D(1-v_D))}\frac{D}{V}$$

constant discount rates

L-model
$$r^{\ell}_{WACC}(1-v) = r^u_E(1-v_E) - \frac{TS}{D}\frac{(1+r^u_E(1-v_E))}{(1+r_D(1-v_D))}\frac{D}{V}$$

fluctuating discount rates

F-model
$$r^{\ell}_{WACC,t-1}(1-v) = \left(r^u_E(1-v_E) - \frac{PVTS_t + (1+r^u_E(1-v_E))PVTS_{t-1}}{V^{\ell}_{t-1}}\right)$$

L-model
$$r^{\ell}_{WACC,t-1}(1-v) = r^u_E(1-v_E) - \frac{TS_t}{D_{t-1}}\frac{1+r^u_E(1-v_E)}{1+r_{D,t-1}(1-v_D)}\frac{D_{t-1}}{V^{\ell}_{t-1}}$$

3.5. *Derivation of the unlevered cost of capital*

The application of the adjustment-formulas derived above is necessary in two cases: Either the capital structure of the firm being valued is assumed to change, e.g. to take better advantage of financial leverage, and therefore, its former cost of capital have to be adapted to the new situation. Or the cost of capital are determined by looking at comparable companies and applying their rates to the firm valued. In the latter case, capital structures of the comparable firms and of the firm will in most cases be different, making adjustments to the capital structure of the firm to be valued necessary.

In both cases, the application of the adjustment formulas requires the derivation of the unlevered cost of capital. This can be done by solving the relevant formula for the given case for the unlevered cost of capital:

Applying the *Modigliani/Miller* –formula yields the following formula for the unlevered cost of capital:

$$r_E^\ell(1-v_E) = r_E^u(1-v_E) + \left(r_E^u(1-v_E) - r_D(1-v_D)\right)\left(1 - \frac{TS/D}{r_D(1-v_D)}\right)\frac{D}{E}$$

$$\Leftrightarrow r_E^u(1-v_E) = \frac{r_{WACC}^\ell(1-v)}{\left(1 - \frac{TS}{r_D(1-v_D)D}\frac{D}{V}\right)} \tag{63}$$

For the *Miles/Ezzell*-formula we receive:

$$r_E^\ell(1-v_E) = r_E^u(1-v_E) + \left(r_E^u(1-v_E) - r_D(1-v_D)\right)\left(1 - \frac{TS/D}{1 + r_D(1-v_D)}\right)\frac{D}{E}$$

$$\Leftrightarrow r_{Ek}^u(1-v_E) = \frac{r_E^\ell(1-v_E) + \Psi r_D(1-v_D)}{(1+\Psi)} \text{ with } \Psi = \left(1 - \frac{TS/D}{1 + r_D(1-v_D)}\right)\frac{D}{E} \tag{64}$$

The levered cost of capital adjusted for the capital structure of the firm being valued can now be derived by plugging the resulting unlevered cost of capital into the relevant adjustment formula. If instead betas of comparable companies are to adjusted to the capital structure of the firm valued, unlevered betas will have to be derived first.

3.6. *Derivation of unlevered betas*

Unlevered Betas can be derived by plugging the formula for the security-market-line of the capital asset pricing model (CAPM):[67] $r_E^u(1-v) = R_f(1-v) + \beta_E^u(1-v)(R_m - R_f)$ into the relevant adjustment-formula. The resulting unlevered beta for the *Modigliani/Miller*–formula is (with $r = [R_f + \beta(R_m - R_f)]$):

$$r_E^u(1-v_E) = \frac{r_E^\ell(1-v_E)\frac{E}{V} + r_D(1-v_D)\frac{D}{V}\left(1 - \frac{TS}{r_D(1-v)D}\right)}{\left(1 - \frac{TS}{r_D(1-v)D}\frac{D}{V}\right)}$$

$$\Leftrightarrow$$

$$\beta_E^u(1-v_E) = \frac{\beta_E^\ell(1-v_E)\frac{E}{V} + \left(\beta_D(1-v_D) - (v_D - v_E)\frac{R_f}{(R_m - R_f)}\right)\left(1 - \frac{TS}{r_D(1-v_D)D}\right)\frac{D}{V}}{\left(1 - \frac{TS}{r_D(1-v_D)D}\frac{D}{V}\right)} \tag{65}$$

Only in the special case that $v_E = v_D = v$ we receive:

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67 See e.g. *Brealey/Myers* (2000), p. 483. R_m = return of the market-portfolio, R_f = risk-free rate of return. It should be noted that the CAPM-formula used here is the basic model which was derived without taking personal taxes into consideration. To our knowledge no model including personal taxes exists to date. However, *Brennan* (1970) has extended the standard CAPM to account for differences in taxation of capital gains and dividends. He concludes that beta is the appropriate measure of risk. His model, however, contains "an extra term that causes the expected return on an asset to depend on dividend yield as well as systematic risk." Copeland/Weston (1992), p. 211.

$$\beta_E^u = \frac{\beta_E^\ell \frac{E}{V} + \beta_D \frac{D}{V}\left(1 - \frac{TS}{r_D(1-v)D}\right)}{\left(1 - \frac{TS}{r_D(1-v)D}\frac{D}{V}\right)} \tag{66}$$

With $\Psi = \left(1 - \frac{TS/D}{1 + r_D(1-v_D)}\right)\frac{D}{E}$ the unlevered beta for the *Miles/Ezzell*-formula is:

$$\beta_E^u(1-v_E) = \frac{\beta_E^\ell(1-v_E) + \left(\beta_D(1-v_D) - (v_D - v_E)\frac{R_f}{R_m - R_f}\right)\Psi}{(1+\Psi)} \tag{67}$$

In the special case that $v_E = v_D = v$ we receive:

$$\beta_E^\ell = \frac{\beta_E^\ell + \Psi\beta_D^\ell}{(1+\Psi)} \tag{68}$$

The resulting unlevered betas after taxes can be used to determine the unlevered cost of capital after taxes by using the CAPM-formula after taxes:

$$r_E^u(1-v_E) = [R_f(1-v_E) + \beta_E^u(1-v_E)(R_m - R_f)] \tag{69}$$

The determined unlevered beta can also be relevered to the capital structure of the firm valued and then be plugged into the CAPM to determine the levered cost of capital. In both cases, the required beta of debt can be derived from the cost of debt by the following relationship:

$$r_D^\ell = R_f + \beta_D^\ell(R_m - R_f) \;\Leftrightarrow \beta_D^\ell = \frac{r_D^\ell - R_f}{R_m - R_f} \tag{70}$$

The following paragraph demonstrates the application of the general adjustments derived above by analyzing the situation of a corporation located in Germany after the Tax Reform Act 2000.

4. Tax Shields and the Cost of capital in Germany

4.1. *Tax Shield in Germany after the tax Reform Act 2000*

The German Tax Reform Act of the year 2000 has introduced a system of double taxation in which only the half of the equity income from corporations is subject to personal taxes. The first major source of tax advantages of debt in the German tax system, which already existed before the tax reform, is the communal income tax ("Gewerbeertragsteuer" GEST).[68] For calculating GEST, only 50% of the interest paid on longterm debt (Z_{lt}) is

[68] The following discussion analyzes only the basic characteristics of the new German tax system without being able to include every detail. We restrain our analysis to German corporations and the tax shields resulting from communal tax (GEST), corporate tax (KSt) and personal tax. For a description of the new tax system see e.g. *Eisgruber* (2000), pp. 1493.

deductible, while 100% of interests paid on shortterm debt (Z_{st}) can be deducted.[69] We start by analyzing the situation, in which debt consists entirely of longterm debt:[70]

GEST paid by a levered firm amounts to:[71] $GEST = (EBIT - 0{,}5Z_{lt})s$
GEST paid by the unlevered firm amounts to: $GEST = (EBIT)s$
The difference, and therefore the tax shield from GEST, is: $TS = 0{,}5sZ_{lt}$

If, instead, the firm were financed with shortterm debt, its tax payments would amount to: $GEST = (EBIT - Z_{st})s$
and the tax shield would amount to: $TS = sZ_{st}$
Therefore, the total tax shield from GEST of a firm with mixed debt will be a figure composed of the two tax shields $[0{,}5sZ_{lt}]$ and $[sZ_{st}]$. The total tax shield can also be calculated by the weighted average of interest-payments on longterm resp. shortterm debt as shares of total interest payments ($Z = Z_{st} + Z_{lt}$) as follows:

$$TS = (sZ_{st} + 0{,}5sZ_{lt}) = \left(\frac{Z_{st}}{Z} + 0{,}5\frac{Z_{lt}}{Z}\right) \times sZ = \phi sZ \qquad (71)$$

The tax shield from GEST therefore is represented by the term ϕsZ, the factor ϕ being the weighted average of interest payments $\phi = \left(\frac{Z_{st}}{Z} + 0{,}5\frac{Z_{lt}}{Z}\right)$ which is an amount in the bracket [0,5; 1].

Until the Tax Reform Act of 2000 consideration of the tax shield from GEST was sufficient for valuation as corporate taxes were deductable from personal taxes. Since this system was given up in favour of a new system termed „Halbeinkünfteverfahren", all taxes, i.e. GEST as well as corporate and personal taxes, need to be taken into account. Under the new system, corporate income after GEST is taxed at an income tax-rate (t_H) of 25%. After deduction of these corporate taxes, only 50 % of the income, received by the owners of the corporation, are taxable at their personal income tax rate (v). The income after taxes of the owners of a levered firm therefore amounts to:

$$\begin{aligned} &(EBIT - Z - GESt)(1 - t_H)(1 - 0{,}5v) \\ &= [EBIT - Z - (EBIT - \phi Z)s](1 - t_H)(1 - 0{,}5v) \\ &= [EBIT(1 - s) - (1 - \phi s)Z](1 - t_H)(1 - 0{,}5v) \end{aligned} \qquad (72)$$

[69] The tax shield is the tax savings occurred from debt financing. The investors of a levered firm pay less taxes than the investors in an unlevered firm. The absolute value of the difference is the tax shield. We follow the approach described in *Brealey/Meyers* (2000), pp. 504.

[70] The following derivation requires the assumption of full-distribution of profit. This assumption is equivalent to the irrelevance of dividend policy, when all retained earnings are invested and yield returns equal to the pretax cost of capital (see fn. 13). If dividend policy were considered relevant, one would have to define an explicit payout-ratio and make explicit assumptions on the rates of return on retained earnings (so-called Dividend-Discount Model, see *Damodaran* (1996), p. 233). But at any rate of return lower than the cost of capital, it would not be rational for the firm to invest and therefore retain earnings. At a rate above the cost of capital it would always pay off to retain earnings completely, even raise more money from the owners: a money machine. In a DCF-model we have already accounted for all necessary investments (ICF), so any additional investment would exceed the scope of foreseeable projects at t= 0. Only the assumption of a rate equal to the cost of capital is therefore sustainable in the long run, but it yields the same results as full-distribution of profit, so we might as well use this assumption. See *Schultze* (2001), pp. 291. The following derivation of the cost-of-capital adjustments therefore includes the possibility of retaining earnings for reinvestments, it requires a return equal to the cost-of-capital, however.

[71] The symbol s denotes the effective tax rate for GEST.

The debt-holders receive Z(1 − v). The income after taxes of all investors therefore amounts to:

$$[EBIT(1-s)-(1-\phi s)Z](1-t_H)(1-0{,}5v)+Z(1-v)$$
$$\Leftrightarrow EBIT(1-s)(1-t_H)(1-0{,}5v)+Z(1-v)-(1-\phi s)(1-t_H)(1-0{,}5v)Z \quad (73)$$

Total income after taxes of the investors in an unlevered firm is:

$$EBIT(1-s)(1-t_H)(1-0{,}5v) \quad (74)$$

The tax shield under the new tax regulations amounts to:

$$TS = Z(1-v)-(1-\phi s)(1-t_H)(1-0{,}5v)Z$$
$$\Leftrightarrow TS = \omega Z \text{ with } \omega=(1-v)+(1-0{,}5v)(1-t_H)(1-\phi s)=\frac{TS}{Z}=\frac{TS}{r_D D} \quad (75)$$

or equivalently:

$$TS = \tau Z(1-v)$$
$$\text{with } \tau=\frac{\omega}{(1-v)}=\frac{(1-v)+(1-0{,}5v)(1-t_H)(1-\phi s)}{(1-v)}=\frac{TS}{Z(1-v)}=\frac{TS}{r_D(1-v)D} \quad (76)$$

Both representations are useful for the integration in the valuation models. The latter is especially useful for representing the tax shield as a percentage factor of interest. Tables 3 and 4 exhibit the tax shields as percentage factors τ of interest expenses after taxes, depending on different tax rates of income tax and GEST, given 100% long-term debt (ϕ = 0,5) and alternatively of 100% short-term debt (ϕ = 1). From these tables it becomes clear that the tax rate of the GEST has a positive, the income tax rate, however, a negative effect on the amount of the tax shield.[72]Also it shows that the impact of the distinction between long-term and short-term debt is quite strong.

For every given GEST-rate s there is an income tax rate that will turn the tax shield to zero. That is, beyond a certain hurdle tax rate the tax shield from debt will actually turn into a tax disadvantage. This hurdle rate v* can be determined as follows:

$$(1-v)=(1-\phi s)(1-t_H)(1-0{,}5v)$$
$$\Leftrightarrow v\,(1-0{,}5(1-\phi s)(1-t_H))=1-(1-\phi s)(1-t_H)$$
$$\Leftrightarrow v^*=\frac{1-(1-\phi s)(1-t_H)}{1-0{,}5(1-\phi s)(1-t_H)} \quad (77)$$

Table 2 shows these hurdle income tax rates (v*) as a function of s. The third line states v* for long-term debt and the fourth line for short-term debt.

Table 2: Hurdle Income-tax Rates at which Tax Shields turn negative

h =	0%	40%	80%	120%	160%	200%	240%	280%	320%	360%	400%	440%	460%	500%
s =	0%	2,0%	3,8%	5,7%	7,4%	9,1%	10,7%	12,3%	13,8%	15,3%	16,7%	18,0%	18,7%	20,0%
0,5	40%	40,9%	41,8%	42,7%	43,5%	44,2%	45,0%	45,7%	46,4%	47,0%	47,6%	48,2%	48,5%	49,1%
1	40%	41,9%	43,6%	45,3%	46,8%	48,3%	49,7%	51,0%	52,2%	53,4%	54,5%	55,6%	56,1%	57,1%

[72] The communes can levy GEST at their own will. The symbol h (“Hebesatz”) stands for the variable tax factor that the communes have in their hands to vary. The effective tax rate s results from this.

Table 3: Tax factor τ for the Tax Shield from long-term debt (ϕ = 0,5)

	v =	15%	20%	25%	30%	35%	40%	45%	50%
h =	s =	τ =							
0%	0,00%	18,38%	15,63%	12,50%	8,93%	4,81%	0,00%	-5,68%	-12,50%
40%	1,96%	19,18%	16,45%	13,36%	9,82%	5,74%	0,98%	-4,65%	-11,40%
80%	3,85%	19,95%	17,25%	14,18%	10,68%	6,64%	1,92%	-3,65%	-10,34%
100%	4,76%	20,33%	17,63%	14,58%	11,10%	7,07%	2,38%	-3,17%	-9,82%
120%	5,66%	20,69%	18,01%	14,98%	11,51%	7,50%	2,83%	-2,69%	-9,32%
140%	6,54%	21,05%	18,38%	15,36%	11,91%	7,92%	3,27%	-2,22%	-8,82%
160%	7,41%	21,41%	18,75%	15,74%	12,30%	8,33%	3,70%	-1,77%	-8,33%
180%	8,26%	21,75%	19,11%	16,11%	12,69%	8,74%	4,13%	-1,32%	-7,86%
200%	9,09%	22,09%	19,46%	16,48%	13,07%	9,13%	4,55%	-0,88%	-7,39%
220%	9,91%	22,43%	19,81%	16,84%	13,44%	9,52%	4,95%	-0,45%	-6,93%
240%	10,71%	22,75%	20,15%	17,19%	13,81%	9,91%	5,36%	-0,02%	-6,47%
260%	11,50%	23,08%	20,48%	17,53%	14,17%	10,28%	5,75%	0,40%	-6,03%
280%	12,28%	23,39%	20,81%	17,87%	14,52%	10,65%	6,14%	0,81%	-5,59%
300%	13,04%	23,71%	21,13%	18,21%	14,87%	11,02%	6,52%	1,21%	-5,16%
320%	13,79%	24,01%	21,44%	18,53%	15,21%	11,37%	6,90%	1,61%	-4,74%
340%	14,53%	24,31%	21,75%	18,86%	15,54%	11,72%	7,26%	2,00%	-4,33%
360%	15,25%	24,61%	22,06%	19,17%	15,87%	12,07%	7,63%	2,38%	-3,92%
380%	15,97%	24,90%	22,36%	19,49%	16,20%	12,41%	7,98%	2,75%	-3,52%
400%	16,67%	25,18%	22,66%	19,79%	16,52%	12,74%	8,33%	3,13%	-3,13%
420%	17,36%	25,46%	22,95%	20,09%	16,83%	13,07%	8,68%	3,49%	-2,74%
440%	18,03%	25,74%	23,23%	20,39%	17,14%	13,39%	9,02%	3,85%	-2,36%
460%	18,70%	26,01%	23,51%	20,68%	17,44%	13,71%	9,35%	4,20%	-1,98%
480%	19,35%	26,28%	23,79%	20,97%	17,74%	14,02%	9,68%	4,55%	-1,61%
500%	20,00%	26,54%	24,06%	21,25%	18,04%	14,33%	10,00%	4,89%	-1,25%

Table 4: Tax factor τ for the Tax Shield from short-term debt (ϕ = 1)

	v =	15%	20%	25%	30%	35%	40%	45%	50%
h =	s =	τ =							
0%	0,00%	18,38%	15,63%	12,50%	8,93%	4,81%	0,00%	-5,68%	-12,50%
40%	1,96%	19,98%	17,28%	14,22%	10,71%	6,67%	1,96%	-3,61%	-10,29%
60%	2,91%	20,76%	18,08%	15,05%	11,58%	7,58%	2,91%	-2,60%	-9,22%
100%	4,76%	22,27%	19,64%	16,67%	13,27%	9,34%	4,76%	-0,65%	-7,14%
120%	5,66%	23,00%	20,40%	17,45%	14,08%	10,20%	5,66%	0,30%	-6,13%
140%	6,54%	23,72%	21,14%	18,22%	14,89%	11,04%	6,54%	1,23%	-5,14%
160%	7,41%	24,43%	21,88%	18,98%	15,67%	11,86%	7,41%	2,15%	-4,17%
180%	8,26%	25,12%	22,59%	19,72%	16,45%	12,67%	8,26%	3,04%	-3,21%
200%	9,09%	25,80%	23,30%	20,45%	17,21%	13,46%	9,09%	3,93%	-2,27%
220%	9,91%	26,47%	23,99%	21,17%	17,95%	14,24%	9,91%	4,79%	-1,35%
240%	10,71%	27,13%	24,67%	21,88%	18,69%	15,01%	10,71%	5,64%	-0,45%
260%	11,50%	27,77%	25,33%	22,57%	19,41%	15,76%	11,50%	6,48%	0,44%
280%	12,28%	28,41%	25,99%	23,25%	20,11%	16,50%	12,28%	7,30%	1,32%
300%	13,04%	29,03%	26,63%	23,91%	20,81%	17,22%	13,04%	8,10%	2,17%
320%	13,79%	29,64%	27,26%	24,57%	21,49%	17,94%	13,79%	8,89%	3,02%
340%	14,53%	30,24%	27,88%	25,21%	22,16%	18,64%	14,53%	9,67%	3,85%
360%	15,25%	30,83%	28,50%	25,85%	22,82%	19,33%	15,25%	10,44%	4,66%
380%	15,97%	31,41%	29,10%	26,47%	23,47%	20,01%	15,97%	11,19%	5,46%
400%	16,67%	31,99%	29,69%	27,08%	24,11%	20,67%	16,67%	11,93%	6,25%
420%	17,36%	32,55%	30,27%	27,69%	24,73%	21,33%	17,36%	12,66%	7,02%
440%	18,03%	33,10%	30,84%	28,28%	25,35%	21,97%	18,03%	13,38%	7,79%
460%	18,70%	33,64%	31,40%	28,86%	25,96%	22,61%	18,70%	14,08%	8,54%
480%	19,35%	34,18%	31,96%	29,44%	26,56%	23,23%	19,35%	14,77%	9,27%
500%	20,00%	34,71%	32,50%	30,00%	27,14%	23,85%	20,00%	15,45%	10,00%

In the following paragraphs the derived tax shield for German corporations is applied to the general formulas derived above.

4.2. *Derivation of the WACC for German corporations under the new tax regulations*

The generalized WACC-approach was derived above as:

$$V_0^\ell = \sum_{t=1}^{\infty} \frac{X_t^u}{[1+r_{WACC}^\ell(1-v)]^t} \qquad \text{(WACC-approach)}$$

$$\text{with } r_{WACC}^\ell(1-v) = r_E^\ell(1-v_E)\frac{E_0^\ell}{V_0^\ell} + r_D(1-v_D)\left(1-\frac{TS_1}{r_D(1-v)D_0}\right)\frac{D_0}{V_0^\ell} \qquad (78)$$

For a German corporation we have to note that equity income in the new tax system is only taxed to an extent of 50% on the personal level in order to reduce the effect of double taxation. This has to be taken into account for the determination of the after-tax return on equity. The cost of equity as derived e.g. from the CAPM is usually determined after corporate taxes but before personal taxes. Therefore, 50% of the personal income-tax rate needs to be applied to the return on equity. Therefore, we have: $v_E = 0{,}5v; v_D = v$.

$$r_{WACC}^\ell(1-v) = r_E^\ell(1-0{,}5v)\frac{E}{V} + r_D(1-v_D)\left(1-\frac{TS_1}{r_D(1-v)D_0}\right)\frac{D}{V} \qquad (79)$$

Substituting the value for the tax shield derived above into the general WACC-formula yields the WACC for a German corporation:[73]

$$r_{WACC}^\ell(1-v) = r_E^\ell(1-0{,}5v)\frac{E}{V} + r_D(1-v)(1-\tau)\frac{D}{V}$$

$$\Leftrightarrow\ r_{WACC}^\ell(1-v) = r_E^\ell(1-0{,}5v)\frac{E}{V} + r_D(1-0{,}5v)(1-t_H)(1-\phi s)\frac{D}{V} \qquad (80)$$

4.3. *Capital-structure-adjustments in Germany*

The general adjustment-formulas derived above can now be adapted to the German situation. Doing so we have to set

$$TS = \omega Z \text{ and } \phi = \left(\frac{Z_{st}}{Z} + 0{,}5\frac{Z_{lt}}{Z}\right) \text{ and } Z(1-v) = r_D(1-v)D$$

$$\frac{TS}{r_D D} = \omega = (1-v) - (1-\phi s)(1-t_H)(1-0{,}5v)$$

$$\frac{TS}{r_D(1-v)D} = \tau = \frac{\omega}{(1-v)} = 1 - \frac{(1-\phi s)(1-t_H)(1-0{,}5v)}{(1-v)}$$

$$(1-v)(1-\tau) = (1-\phi s)(1-t_H)(1-0{,}5v) \qquad (81)$$

Applying the above tax shield we receive the following adjustments:

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[73] See for a similar derivation *Baetge/Niemeyer/Kümmel* (2001), p. 326.

<u>Cost of Equity (Equity-Approach):</u>
Perpetuity-model:

F-model
$$r_E^\ell(1-v_E) = r_E^u(1-0{,}5v) + \left(r_E^u(1-0{,}5v) - r_D(1-v)\right)(1-\tau)\frac{D}{E}$$

$$\Leftrightarrow\ r_E^\ell(1-v_E) = r_E^u(1-0{,}5v) + \left(r_E^u(1-0{,}5v) - r_D(1-v)\right)\left(\frac{(1-\phi s)(1-t_H)(1-0{,}5v)}{(1-v)}\right)\frac{D}{E}$$

L-model

$$r_E^\ell(1-v_E) = r_E^u(1-0{,}5v) + \left[r_E^u(1-0{,}5v) - r_D(1-v)\right]\left(\frac{1+r_D(1-v)(1-\tau)}{1+r_D(1-v)}\right)\frac{D}{E}$$

$$\Leftrightarrow\ r_E^\ell(1-v_E) = r_E^u(1-0{,}5v) + \left[r_E^u(1-0{,}5v) - r_D(1-v)\right]\left(\frac{1+r_D(1-\phi s)(1-t_H)(1-0{,}5v)}{1+r_D(1-v)}\right)\frac{D}{E}$$

constant discount rates:

L-model

$$r_E^\ell(1-v_E) = r_E^u(1-0{,}5v) + \left[r_E^u(1-0{,}5v) - r_D(1-v)\right]\left(\frac{1+r_D(1-v)(1-\tau)}{1+r_D(1-v)}\right)\frac{D}{E}$$

fluctuating discount rates:

F-model
$$r_{E,t-1}^\ell(1-v_E) = r_E^u(1-0{,}5v) + \left(r_E^u(1-0{,}5v) - r_D(1-v)(1-\tau)\right)\frac{D_{t-1}}{E_{t-1}^v}$$
$$+\frac{PVTS_t - PVTS_{t-1}(1+r_E^u(1-v))}{E_{t-1}^v}$$

L-model
$$r_{E,t-1}^\ell(1-v_E) = r_E^u(1-0{,}5v) + \left[r_E^u(1-0{,}5v) - r_{D,t-1}(1-v)\right]\left(\frac{1+r_D(1-v)(1-\tau)}{1+r_D(1-v)}\right)\frac{D}{E}$$

<u>Firm's cost of capital (WACC-approach):</u>
Perpetuity-model:

F-model
$$r_{WACC}^\ell(1-v) = r_E^u(1-0{,}5v)\left(1-\tau\frac{D}{V}\right)$$

L-model
$$r_{WACC}^\ell(1-v) = r_E^u(1-0{,}5v) - \tau r_D(1-v)\frac{1+r_E^u(1-0{,}5v)}{1+r_D(1-v)}\frac{D}{V}$$

constant discount rates:

L-model $$r^{\ell}_{WACC}(1-v)=r^{u}_{E}(1-0{,}5v)-\tau r_{D}(1-v)\frac{1+r^{u}_{E}(1-0{,}5v)}{1+r_{D}(1-v)}\frac{D}{V}$$

fluctuating discount rates:

F-model $$r^{\ell}_{WACC,t-1}(1-v)=\left(r^{u}_{E}(1-0{,}5v)+\frac{PVTS_{t}-(1+r^{u}_{E}(1-0{,}5v))PVTS_{t-1}}{V^{\ell}_{t-1}}\right)$$

L-model $$r^{\ell}_{WACC,t-1}(1-v)=r^{u}_{E}(1-0{,}5v)-\tau r_{D}(1-v)\frac{1+r^{u}_{E}(1-0{,}5v)}{1+r_{D,t-1}(1-v)}\frac{D_{t-1}}{V^{\ell}_{t-1}}$$

4.4. *Unlevered cost of capital*

The unlevered cost of capital can be determined by applying the above tax shields to the general formulas derived above.

For the *Modigliani/Miller* –formula we receive the following expression for the German tax system:

$$r^{u}_{E}(1-v_{E})=\frac{r^{\ell}_{E}(1-0{,}5v)\frac{E}{V}+r_{D}(1-v)\frac{D}{V}(1-\tau)}{\left(1-\tau\frac{D}{V}\right)}\Leftrightarrow r^{u}_{E}(1-v_{E})=\frac{r^{\ell}_{wacc}(1-v)}{\left(1-\tau\frac{D}{V}\right)} \tag{82}$$

For the *Miles/Ezzell*-formula we receive:

$$r^{u}_{E}(1-v_{E})=\frac{r^{\ell}_{E}(1-0{,}5v)+\Psi r_{D}(1-v)}{(1+\Psi)}\quad\text{with }\Psi=\left(\frac{1+r_{D}(1-v)(1-\tau)}{1+r_{D}(1-v)}\right)\frac{D}{E} \tag{83}$$

For the unlevered beta after personal taxes, derived for the application of the *Modigliani/Miller*-formula, we receive:

$$\beta^{u}_{E}(1-v_{E})=\frac{\beta^{\ell}_{E}(1-0{,}5v)\frac{E}{V}+\left(\beta_{D}(1-v)-0{,}5v\frac{R_{f}}{(R_{m}-R_{f})}\right)(1-\tau)\frac{D}{V}}{\left(1-\tau\frac{D}{V}\right)} \tag{84}$$

For the *Miles/Ezzell*-formula we receive:

$$\beta^{u}_{E}(1-0{,}5v)=\frac{\beta^{\ell}_{E}(1-0{,}5v)+\left(\beta_{D}(1-v)-0{,}5v\frac{R_{f}}{R_{m}-R_{f}}\right)\Psi}{(1+\Psi)}$$

$$\text{with }\Psi=\left(\frac{1+r_{D}(1-v)(1-\tau)}{1+r_{D}(1-v)}\right)\frac{D}{E} \tag{85}$$

The resulting unlevered betas after taxes can be used to determine the unlevered cost of capital after taxes by using the following CAPM-formula:

$$r^{u}_{E}(1-0{,}5v)=[R_{f}(1-0{,}5v)+\beta^{u}_{E}(1-0{,}5v)(R_{m}-R_{f})] \tag{86}$$

5. Conclusion

This paper presents a comprehensive framework for the integration of taxes into valuation models. Based on the principle of equivalence, general versions of the WACC-, TCF- and the Equity-approach were derived. Applying the APV-model to different financing scenarios yielded different reaction functions of the unlevered cost of capital with respect to changes in capital structure. These cost-of-capital-formulas where used to show the equivalence of the different approaches to valuation.[74]

Generally, all approaches to valuation (WACC-, TCF-, Equity- and APV-approach) yield the same results as long as the consequences of changes in capital structure on the levered cost of equity are modeled adequately, using the relevant adjustment formula. The more complex APV-model is particularly useful in cases of unusual changes in capital structure. For the case of the value-oriented financing-strategy (L-model), it turned out that all considered cases could be treated with a generalized version of the *Miles/Ezzell*-formula. In the case of a constant level of debt financing even a uniform rate could be used. The WACC-approach is therefore especially suitable for the application of the L-model.[75]

The results correspond to previous work in the field.[76] The derived models are, however, independent of a particular tax system and provide for the inclusion of personal taxes.[77] They are designed to be adjusted to different tax systems. The use of the general results was demonstrated by their application to the situation of German corporations after the Tax Reform Act 2000. In addition formulas for the determination of the unlevered cost of capital via the CAPM were derived.

Acknowledgements

This paper was presented at the 25th Annual congress of the European Accounting Association 2002 in Copenhagen. I thank the participants for their comments and suggestions. Furthermore I thank the Arthur Andersen foundation within the Stifterverband der deutschen Wirtschaft, Essen as well as the Wissenschaftliche Gesellschaft für Prüfung und Controlling, Augsburg for financial support which enabled me to attend the Conference. I also wish to thank Martin Wallmeier for proofreading this paper and his helpful comments.

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[74] While the „textbook formula“ of the WACC describes the relationship between the levered cost of equity and debt at a given ratio of debt to firm value, the adjustment-formulas derived above are functions of the unlevered cost of capital with respect to changes in capital structure.

[75] See *Inselbag/Kaufold* (1997), p. 122.

[76] They are derived in a larger framework, which shows the interrelationship of the different approaches and the cost of capital. While the results are identical to the literature when applying the US-tax system or the former German tax system, they contradict recent work on the new German tax system. See *Baetge/Niemeyer/Kümmel* (2001), p. 326; *Peemöller* (2001), p. 1403; *Drukarczyk* (2000), pp. 199.

[77] The results, of course, are only valid within the framework given by the assumptions made in Fn. 13.

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Notation

β	Beta (measure for systematic risk)
Δ	change
ϕ	Weighted average of interest payments (short vs. long-term)
ω	Complex tax factor for calculation of tax shield for German Corporations
τ	Complex tax factor for calculation of tax shield for German Corporations
$\Lambda_t = D_t / V_t$	Ratio debt/firm-value at time t
D	Debt
D_t	Value of debt of a levered firm at time t
E	Equity
E_t^{ℓ}	Value of equity of a levered firm at time t
ICF	cash flows needed for investing
NFCF	Free cash flow to equity (net of payments to and from debt-holders)
OCF	operating cash flows to the firm
P	Price of the firm
PVTS	present value of the tax shield
r	Rate of return, cost of capital
r_D	Cost of debt
r_E^u	cost of capital of an all-equity firm
r_E^u	unlevered cost of equity
r_E^{ℓ}	levered cost of equity
r_{TCF}	Cost of capital for the TCF-approach
r_{WACC}	Weighted average cost of capital (WACC)
R_f	Risk-free rate of return
R_m	Rate of return of the market-portfolio
s	Communal tax rate for German Corporations
t	time
TCF	Total Cash Flow
t_H	Corporate tax rate for German Corporations
TS_t	Tax Shield at time t
T^u	taxes of the unlevered firm
V	Value of the firm
V_0^{ℓ}	Value of a levered firm at t = 0
v	Personal tax-rate
v_D	Personal tax rate of equity-investor
v_E	Personal tax rate of debt-investor
X	Free cash flow to the firm
X^u	unlevered free cash flow to the firm

Z	Interest payments
Z_{lt}	Interest payments on long-term debt
Z_{st}	Interest payments on short-term debt