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Optimal pensions in aging economies

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Abstract:
We derive the optimal replacement ratio of the pay-as-you-go public pension system for the US economy in a life-cycle model that 1) replicates the empirical wage heterogeneity and 2) endogenizes the individual's labor supply decision. The optimal net pension replacement ratio is found to be in the range of 0%–43% depending on demographic parameters and, in particular, the Frisch labor supply elasticity. Reducing the pensions from the present to the optimal pension policies implies considerable welfare gains amounting to approximately 0.1%–4.1% of total consumption. The welfare increase is particularly pronounced for the greyer US population that is projected for the time after the demographic transition.

Keywords: demographic transition, income and wealth distribution, optimal social security

JEL classification: C68, D31, D91, H55, J11, J26

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1   Introduction

İmrohoroğlu, İmrohoroğlu, and Joines (1999) shows that the optimal unfunded pay-as-you-go (PAYG) public pensions in the US should be zero in the stationary state. In order to derive this result, they consider an Overlapping Generations model where land is used as a (constant) production factor in addition to capital and labor. As a consequence, the economy is characterized by dynamic efficiency and a decrease in savings and, hence, capital also implies lower total consumption. In their model, PAYG pensions help to insure against the risk of low income that result from times of unemployment and against uncertain lifetime. However, pensions also distort the savings decisions of the households and, therefore, reduce welfare. The overall effect of pensions is found to be negative for the US economy so that pensions should be abolished.[1]

In our model, we extend the analysis of İmrohoroğlu, İmrohoroğlu, and Joines (1999) along multiple dimensions: First, we assume a more realistic distribution of wages. In İmrohoroğlu, İmrohoroğlu, and Joines (1999), all employed households of a cohort receive the same wage and, therefore, heterogeneity in income is only caused by periods of unemployment. Second, we assume that the worker is able to adjust its labor supply along the intensive margin. In the presence of contributions to a pay-as-you-go pension system, the worker will decrease his labor supply in our model. Third, we assume that a government also imposes an income tax that distorts labor supply additionally. Since the welfare costs of a distortionary tax increase non-linearly with the tax rate, a pension contribution rate that acts like a tax on wage income becomes more welfare-reducing. Accounting for all these effects, we confirm the results of İmrohoroğlu, İmrohoroğlu, and Joines (1999). We find an optimal value of the net pension replacement ratio close to zero amounting to 8%. Moreover, the result of zero or close to zero optimal pensions is rather robust with respect to the specification of the preferences, but sensitive with respect to the value of the Frisch labor supply elasticity.[2]

Our research makes two contributions to the existing literature. 1) We study a wide range for the optimal pension replacement rate that covers the interval [0%, 100%]. Previous related studies only consider a narrow range of possible policy changes in the pension level. For example, Nishiyama and Smetters (2007) only analyzes a privatization of social security by 50%, while İmrohoroğlu and Kitao (2009) only examine two changes of the pension levels, by 50% and by 100%. Similarly, Kitao (2014) compares four different financing policies to keep social security sustainable, but does not derive the optimal pension level either.[3] 2) We also analyze the robustness of our result with respect to a greying economy where the population ages due to the demographic change. Therefore, we compare the optimal allocations for the US population in the years 2015 and 2050 using projections from UN (2015). The effect of aging on welfare is not unanimous. On the hand, the pension contribution rate (for given net replacement ratio of pensions relative to wages) will increase between 2015 and 2050 due to the demographic change and, as a consequence, the tax wedge will be higher so that labor supply and welfare decrease. On the other hand, retirees are getting older on average so that the (discounted) loss in old-age utility as a consequence of possible negative income shocks and zero pensions is magnified. We find

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that the distortionary effect of higher pension contributions on the labor supply dominates and that pensions should optimally be zero in 2050. Moreover, quantitative welfare effects from abolishing pensions are even more pronounced than in 2015.

Our research is closely related to other studies on the welfare effects of public pensions. Fehr, Kallweit, and Kindermann (2013) compute the optimal mix between flat and earnings-related pensions for the German pension system. In addition to our model, Fehr, Kallweit, and Kindermann (2013) endogenize the decision on the retirement age and also allow for disability risk reflecting the fact that 20% of new entries into the German pension system are due to disability. They find that the flat-rate pension share should equal 30% in total pensions in order to optimize the trade-off between the increased labor supply distortion and the benefit from increased insurance provision against labor market risk. Different from our analysis, however, the pension contribution rate is set constant so that these authors do not study the optimal amount of pensions.[4]

Storesletten, Telmer, and Yaron (1999) also conduct a welfare analysis of the social security system in a large-scale Overlapping Generations (OLG) model, but focus on the distortion of social security contributions on the accumulation of capital. The main channel emphasized in their model is the financing of pensions with a distortionary income tax that is levied on labor and capital income. Since labor supply is exogenous, the only distortion is on capital accumulation. The authors compare the current US pension system (as of 1996) to alternative scenarios including the abolition of the social security system and a system that is partially pay-as-you-go and partially fully-funded. They find the considered alternatives to imply significant welfare gains if general equilibrium effects are taken into account.

While we study the optimal pension policy and search for the optimal (non-negative) replacement ratios, De Nardi, Imrohoroğlu, and Sargent (1999) concentrate their analysis on 8 different specific public policy measures in a similar OLG model. In order to finance additional expenditures on pensions due to the demographic transition, they consider policies that raise different taxes (on consumption and labor income), reduce pension benefits, or increase the mandatory retirement age. The authors also account for the welfare of the cohorts during the transition. In order to keep the model tractable and computable, they assume a special functional form of utility from consumption and disutility of labor (both quadratic and additive). In addition, the insurance properties of the social security are not motivated by a temporary shock on individual labor productivity, but rather a shock to the individual's wealth endowment. As a consequence of these assumptions, individual policy functions (e.g., individual consumption) are a linear function of individual state variables (in particular, wealth) so that aggregation is straightforward and does not depend on the distribution of wealth (different from our model). De Nardi, Imrohoroğlu, and Sargent (1999) find that the only policy of those considered in the paper that raises the welfare of all generations is one that switches to a purely defined contribution system.

The paper is organized as follows. The next section describes the model that we use to derive quantitative policy implications. Section 3 discusses the calibration of the model, while Section 4 presents our simulation results. Section 5 concludes. In the Appendix, we present the description of the stationary equilibrium and its properties.

2 The model

The model is described by a life-cycle model comprising a large number of overlapping generations with a finite life, a representative firm and government. Households are heterogeneous with respect to their individual productivity and their assets. The firm uses aggregate capital and labor to maximize profits, while operating a neoclassical production function. Income from labor and capital are subject to proportional taxes. The government uses income tax revenues in order to finance the provision of public consumption goods and lump-sum transfers to households. In addition, a proportional social security contribution is levied upon wage income in order to finance pay-as-you-go pension and unemployment insurance payments.

2.1 Demographics and timing

A period, t, corresponds to 1 year. At each t, a new generation of households is born. Newborns have a real life age of 20 denoted by $s = 1$. All generations retire at age $s = R = 46$ (corresponding to real life age 65) and live up to a maximum age of $s = J = 75$ (real life age 94).

Let $N_t(s)$ denote the number of agents of age s at t. We denote total population at t by N_t. At t, all agents of age s survive until age $s + 1$ with probability $\phi_{t,s}$ where $\phi_{t,0} = 1$ and $\phi_{t,J} = 0$.

2.2 Households

Each household comprises one (possibly retired) worker. At working age, the household is either employed or unemployed. We introduce unemployment following Kaplan (2012) .[5] There are two types of employment statuses $\iota \in \{e, u\}$. The employed households (with $\iota = e$) supply labor l. The other households with status $\iota = u$ become unemployed for a duration equal to d, $0 < d < 1$ so that their labor supply is only equal to $l(1 - d)$. Furthermore, the unemployment duration $d(s, \epsilon)$ depends on the age and his permanent productivity type ϵ. Leisure of the unemployed is equal to $1 - l(1 - d)^\zeta$. When $\zeta = 1$, the agent derives leisure from the whole period of unemployment so that leisure amounts to $1 - l(1 - d)$. When $\zeta = 0$, the period of unemployment does not provide leisure. The employment status is exogenous and a household belongs to the employed (unemployed) households with probability $(1 - p)$ (p), where $p = p(s, \epsilon)$ also depends on age and the permanent productivity type. The household chooses his labor supply l in each period after he observes his employment type.

Besides their employment status $\iota \in \{e, u\}$, households are also heterogeneous with regard to their age s, their individual labor efficiency, $\eta \epsilon_j \bar{y}_s$, and their wealth, a_t^s. We stipulate that an agent's efficiency depends on its age, $s \in \mathcal{S} \equiv \{1, 2, ..., 75\}$, and its efficiency type, $\epsilon_j \in \mathcal{E} \equiv \{\epsilon_1, \epsilon_2\}$. We choose the age-efficiency profile, $\{\bar{y}_s\}$, in accordance with the US wage profile. The permanent efficiency types ϵ_1 and ϵ_2 are meant to capture differences in education and ability. In addition, we follow Krueger and Ludwig (2007) and assume that a household's labor productivity is affected by an idiosyncratic shock, $\eta \in \Gamma \equiv \{\eta_1, \eta_2\}$, that follows a time-invariant Markov chain with transition probabilities

$$\pi^\eta(\eta'|\eta) = \begin{pmatrix} \pi_{11}^\eta & \pi_{12}^\eta \\ \pi_{21}^\eta & \pi_{22}^\eta \end{pmatrix}. \tag{1}$$

The net wage income in period t of an s-year old employed household with efficiency type $\eta \epsilon$ is given by $(1 - \tau_w - \tau_{pen} - \tau_{ui})w_t A_t \eta \epsilon \bar{y}_s l_t(s)$, where w_t and A_t denote the wage rate per efficiency unit in period t and the state of technology. The wage income is taxed at the constant rate τ_w. Furthermore, the worker has to pay contributions to the pension and the unemployment insurance system at the rates τ_{pen} and τ_{ui}, respectively. The unemployed household receives unemployment benefits b_t during times of unemployment. During retirement, the worker receives pensions *pen*. Both payments are provided lump-sum and irrespective of the individual's past contributions.

Households are born without assets at the beginning of age $s = 1$, hence $a_t^1 = 0$. In addition, household are not allowed to borrow so that assets $a_t^s \geq 0$ for all ages s. Parents do not leave bequests to their children, and all accidental bequests are confiscated by the government. The household earns interest r_t on his wealth $a_t \in ^+$. Interest income is taxed at the constant rate τ_r. In addition, households receive lump-sum transfers tr_t from the government. As a result, the budget constraint of an s-year old household with productivity type $\eta \epsilon$ and employment status ι during working age, and wealth a_t^s in period t is presented by:

$$c_t^s + a_{t+1}^{s+1} = \begin{cases} (1 - \tau_w - \tau_{pen} - \tau_{ui})w_t A_t \eta \epsilon \bar{y}_s l_t^s + \left[1 + (1 - \tau_r)r_t\right] a_t^s + tr_t, \\ \text{for } s \leq 45, \iota = e \\ (1 - \tau_w - \tau_{pen} - \tau_{ui})w_t A_t \eta \epsilon \bar{y}_s l_t^s(1 - d_t^s) + l_t^s d_t^s b_t(s, \epsilon, \eta) + \left[1 + (1 - \tau_r)r_t\right] a_t^s + tr_t, \\ \text{for } s \leq 45, \iota = u \\ pen_t + \left[1 + (1 - \tau_r)r_t\right] a_t^s + tr_t, \text{ for } s > 45. \end{cases} \tag{2}$$

We consider a utility function that allows to separately study attitudes towards risk and intertemporal subsitution. We use Epstein-Zin preferences which are given by the following expression:[6]

$$V_t(a_t^s, s, \iota, \epsilon, \eta) = \max_{a_{t+1}^{s+1}, c_t^s, l_t^s} \Big\{ u\left(c_t^s, l_t^s, d_t^s\right)^{1-\sigma} +$$
$$\beta \phi_{t,s} E_t \left[V_{t+1}(a_{t+1}^{s+1}, s+1, \iota', \epsilon, \eta')^{1-\mu} \right]^{\frac{1-\sigma}{1-\mu}} \Big\}^{\frac{1}{1-\sigma}}, \tag{3}$$

where $V_t(.)$ is the value function of the s-year old with individual productivity parameters ϵ and η, employment status ι, and wealth a_t^s. The parameters $1/\sigma$ and μ denote the intertemporal elasticity of substitution and the coefficient of relative risk aversion. In the case $\sigma = \mu$, we are back to the case characterized by the time-separable expected utility specification.

Per-period utility $u(c, l, d)$ is a function of consumption c, labor l, and unemployment duration d:

$$u(c, l, d) = c^\gamma (1 - l(1 - d)^\zeta)^{1-\gamma}, \quad \gamma \in (0, 1). \tag{4}$$

2.3 Production

Production is characterized by constant returns to scale and assumed to be described by a Cobb-Douglas function in the production factors capital K_t and efficient labor, $A_t L_t$:

$$Y_t = K_t^\alpha (A_t L_t)^{1-\alpha},$$

where the rate of technological progress is exogenous and equal to g_A:

$$A_t = (1 + g_A) A_{t-1}. \tag{5}$$

Capital K_t depreciates at the rate δ, and L_t denotes aggregate labor. The profit maximization conditions of the firm are given by:

$$r_t = \alpha K_t^{\alpha-1} (A_t L_t)^{1-\alpha} - \delta \tag{6}$$

$$w_t = (1 - \alpha) K_t^\alpha (A_t L_t)^{-\alpha}. \tag{7}$$

In the stationary equilibrium, the aggregate growth rate of output is given by $g = (1 + g_A)(1 + n) - 1$, while the per capita growth rate amounts to g_A.

2.4 Government

The government collects income taxes T_t in order to finance its expenditure on government consumption G_t and transfers Tr_t. In addition, it confiscates all accidental bequests Beq_t. The government budget is balanced in every period t, i. e.,

$$G_t + Tr_t = T_t + Beq_t. \tag{8}$$

In view of the tax rates τ_w and τ_r, the government's tax revenues are given by

$$T_t = \tau_w w_t A_t L_t + \tau_r r_t K_t. \tag{9}$$

Government spending is exogenous and grows at the equilibrium economic growth rate g:

$$G_t = G_{t-1}(1 + g). \tag{10}$$

2.5 Social security

The social security system comprises a pay-as-you-go pension system and an unemployment insurance system.

Pension system. The social security authority collects contributions from the workers to finance its pension payments to the retired agents. The pension is provided lump-sum[7] with the net replacement ratio being denoted by θ_{pen}:

$$pen_t = \theta_{pen}(1 - \tau_w - \tau_{pen} - \tau_{ui}) w_t A_t \bar{l}. \tag{11}$$

In equilibrium, the social security budget is balanced so that total expenditures on pensions, Pen_t, are equal to total contributions:

$$Pen_t = \tau_{pen} w_t A_t L_t. \tag{12}$$

Unemployment insurance. Individual unemployment insurance depends on the efficiency type and the wage rate as follows:

$$b_t = \theta_{ui} w_t A_t \epsilon \eta y^s \bar{l}, \tag{13}$$

where θ_{ui} denotes the replacement ratio of unemployment insurance payments.

In equilibrium, the budget for the unemployment insurance is balanced so that aggregate payments on unemployment insurance, B_t, are equal to revenue from unemployment insurance contributions:

$$B_t = \tau_{ui} w_t A_t L_t. \tag{14}$$

2.6 Stationary equilibrium

In the stationary equilibrium, the age composition of the population is constant, individual behavior is consistent with the aggregate behavior of the economy, firms maximize profits, households maximize intertemporal utility, and factor and goods' markets clear. Since we study a closed economy, total output has to equal private consumption C_t, public consumption G_t, and investment in the goods market equilibrium:

$$Y_t = C_t + G_t + K_{t+1} - (1 - \delta)K_t. \tag{15}$$

A detailed description of the stationary equilibrium is provided in an Appendix that is available from the author upon request. Stationary aggregate variables are denoted by $\tilde{X}_t = X_t/(A_t N_t)$, $X_t \in \{K_t, Y_t, C_t\}$, and $\tilde{L}_t = L_t/N_t$.

3 Calibration

3.1 Demographics

We calibrate the parameters of the model in accordance with the US economy. The forecast for the US population development until 2050 is taken from UN (2015). We use the two sets of the survival probabilities, $\{\phi_{t,s}\}_{s=1}^{75}$, $t \in \{2015, 2050\}$, and the corresponding population growth rates, $n = 1.1\%$ and $n = 0.2\%$, to study the optimal public pension policy. For simplification, we assume that the economy is in stationary steady states in 2015 and 2050, respectively.

3.2 Preference and production parameters

The parameter γ, which reflects the relative weight of consumption and leisure in utility, is set equal to 0.31 so that the average working hours $\bar{l}$ amount to approximately 0.30 in the benchmark equilibrium for the year 2015. We choose $\sigma = 2.0$ in accordance with İmrohoroğlu, İmrohoroğlu, and Joines (1995) and Huggett and Ventura (1999). A sensitivity analysis for $\sigma = 4$ is also reported in Section 4. For the relative risk aversion μ, DSGE studies commonly consider a wide range of values. For example, Caldara, Fernández-Villaverde, and Rubio-Ramiírez (2012) consider values between 2 and 40. We will use a conservative estimate $\mu = 2.0$ in the benchmark case and study the effects of higher risk aversion $\mu = 4.0$ in our sensitivity analysis below. We set the discount factor $\beta = 1.011$ in accordance with the empirical estimates of Hurd (1989) who explicitly accounts for mortality risk.[8] The preference parameter ζ that determines the value of unemployment time in utility is set to zero in accordance with Kaplan (2012) so that the household does not derive leisure from the part of the period when he is unemployed. Our calibration parameters are summarized in Table 1.

Table 1: Calibration of benchmark parameters.

Parameter	Value	Description
β	1.011	subjective discount factor

γ	0.31	weight of leisure in utility
$1/\sigma$	$\{1/2, 1/4\}$	parameter determining the IES
μ	$\{2, 4\}$	risk aversion
ζ	0	intertemporal separability of hours worked and unemployment periods
$\bar{l}$	0.3	steady state labor supply
α	0.36	share of capital income
δ	8.0%	rate of capital depreciation
g_A	2.0%	growth rate of technological progress
$\{\epsilon_1, \epsilon_2\}$	$\{0.57, 1.43\}$	permanent productivity types
$\{\eta_1, \eta_2\}$	$\{0.727, 1.273\}$	stochastic idiosyncratic productivity types
G/Y	19.5%	share of government spending in steady state production
τ_w	24.8%	wage income tax
τ_r	42.9%	capital income tax
θ_{pen}	50%	pension net replacement ratio
θ_{ui}	30%	unemployment insurance replacement ratio
$\pi_{11}^{\eta} = \pi_{22}^{\eta}$	0.98	persistence of idiosyncratic productivity shock

Our value of $\alpha = 0.36$ reflects the share of capital income and has been employed in numerous studies on the U.S. economy. In accordance with Trabandt and Uhlig (2011), we set the annual depreciation rate of capital equal to $\delta = 8\%$ and technology A_t grows at the rate $g_A = 2\%$.

3.3 Individual productivity

The s-year old household has the idiosyncratic productivity $\eta\epsilon\bar{y}_s$. The age-efficiency profile $\{\bar{y}_s\}_{s=1}^{45}$ is taken from Hansen (1993), interpolated to in-between years and normalized to one. The set of the equally distributed productivity types $\{\epsilon_1, \epsilon_2\} = \{0.57, 1.43\}$ is taken from Storesletten, Telmer, and Yaron (2004). Our choice of the stochastic individual productivity component, $\eta \in \{\eta_1, \eta_2\}$, is also motivated by Storesletten, Telmer, and Yaron (2004). In particular, the two-state Markov chain is calibrated so that the annual persistence amounts to 0.98 with an implied conditional variance of 8%. Accordingly, $\{\eta_1, \eta_2\} = \{0.727, 1.273\}$ and

$$\pi^{\eta}(\eta'|\eta) = \begin{pmatrix} \pi_{11}^{\eta} & \pi_{12}^{\eta} \\ \pi_{21}^{\eta} & \pi_{22}^{\eta} \end{pmatrix} = \begin{pmatrix} 0.98 & 0.02 \\ 0.02 & 0.98 \end{pmatrix}.$$

Our modeling of individual productivity is in accordance with the large-scale OLG models of Storesletten, Telmer, and Yaron (2004), Conesa and Krueger (1999), and Krueger and Ludwig (2007). We acknowledge that our specification of the individual productivity process and, hence, the labor earnings is rather parsimonious and a simplification with respect to recent empirical evidence. Guvenen et al. (2015), for example, study the dynamics of individual labor earnings over the life cycle. In contrast to previous studies on income inequality such as Heathcote, Perri, and Violante (2010) and Guvenen (2009) that apply data from the Current Population Survey, the Panel Study of Income Dynamics or the Survey of Consumer Finance, Guvenen et al. (2015) employ the more comprehensive data set from the Master Earnings File of the U.S. Social Security Administration records. They find, among others, that 1) earnings shocks display substantial deviations from lognormality in the form of an extremely high kurtosis and that 2) the statistical properties of the labor earnings process vary over the life cycle. In our analysis, we neglect the modeling of the top earnings percentiles. We conjecture that consideration of the top income households would not significantly affect our welfare results on social security. First, the maximum taxable earnings for social security only amount to a small fraction of high incomes so that the substitution and income effect of the social security contribution on the labor supply is rather negligible.[9] Second, the welfare effect from social security is only of subordinate order for these households since the pension income from social security is a relatively small share of total savings for the top income earners.[10] With respect to the lower tail of the labor earnings distribution, our model is able to replicate the empirical income and wealth heterogeneity closely as presented in Appendix A.1. In particular, we are also able to model the fact that a substantial fraction of households is credit-constrained. These households will be subject to considerable changes in lifetime utility if the public pension system is altered.[11]

The arrival probability and duration of unemployment, $p(s, \epsilon)$ and $d(s, \epsilon)$, depend on the age s and the permanent productivity type ϵ of the worker and are chosen in accordance with the the estimates provided in Table 1 of Peterman and Sommer (2016) which are replicated in Table 2. In particular, unemployment probability

is higher for the younger (aged between 20 and 45 years) and low-skilled worker (with $\epsilon_1 = 0.57$), while the duration increases with age s.

Table 2: Calibration of unemployment parameters.

Age	Productivity ϵ	$p(s, \epsilon)$
$20 \le s \le 45$	ϵ_1	7.5%
	ϵ_2	3.1%
$s > 45$	ϵ_1	4.4%
	ϵ_2	2.7%
Age	**Productivity ϵ**	$d(s, \epsilon)$
$20 \le s \le 45$	ϵ_1	18.2%
	ϵ_2	15.5%
$s > 45$	ϵ_1	20.6%
	ϵ_2	22.6%

Calibration of unemployment parameters as in Peterman and Sommer (2016) .

3.4 Government policy and social security

Government expenditures $\tilde{G}$ are set so that the government share G/Y is equal to the average ratio of government consumption in GDP, $G/Y = 19.5\%$, in the US economy during 1959–1993 according to the Economic Report of the President (1994). The tax rates $\tau_w = 24.8\%$ and $\tau_r = 42.9\%$ are computed as the average values of the effective US tax rates over the time period 1965–1988 that are reported by Mendoza, Razin, and Tesar (1994). Government transfers, tr, are computed using the equilibrium condition of the government budget (8).

The social security contribution rate on wage income τ_{pen} and the unemployment insurance rate τ_{ui} are set so that the (net) replacement ratio of pensions and unemployment insurance payments relative to wage income, θ_{pen} and θ_{ui}, are equal to 50% and 30% following İmrohoroğlu, İmrohoroğlu, and Joines (1999). In the benchmark case, pensions are provided lump-sum, while unemployment benefits are paid relative to the average wage income of the worker with the same productivity $\eta \, \epsilon \, y_s$. In our policy analysis below, we study how a change in the replacement ratio θ_{pen} affects the equilibrium allocation and welfare.

The properties of the benchmark equilibrium and the computation of the model are described in the Appendices A.1 and A.2. As our measure of welfare change, we use the consumption equivalent change Δ that is computed as the percentage by which we need to increase (or reduce) the consumption in the benchmark case (with $\theta_{pen} = 50\%$) to get the same welfare as under the policy θ_{pen}. Noticing the functional form of our utility function, Δ can be computed with the help of:

$$(1 + \Delta)^{\gamma(1-\sigma)} W(0.50) = W(\theta_{pen}), \tag{16}$$

where $W(\theta_{pen})$ denotes average stationary lifetime-utility under social security policy θ_{pen}. Notice that we keep government consumption G constant for different policies θ_{pen} to have a meaningful comparison as government consumption neither affect utility nor productivity in our model and constitute a pure waste.

4 Results

In this section, we present our results on the optimal level of pensions. In Section 4.1, we point out the distortion of the present US public pension system by comparing it to the case without public pensions. Abolishing public pensions results in a large welfare gain of 2.3% of total consumption in stationary state. Next, we show our main result that the optimal net replacement ratio of pensions should be equal to 8%. In Section 4.3, we examine the effects of aging on optimal pensions and compute the optimal pension for the (projected) population in the year 2050. Optimal pensions decrease with higher old-age dependency ratios that are associated with the demographic change. The transitional effects of lower pensions on generational welfare are considered in Section 4.4. Welfare results are shown to depend on the timing and implementation period of the pension policy. In Section 4.5, we study the sensitivity of our results with respect to the assumptions on preferences, lump-sum pensions, and exogenous technological growth. Our result that long-run pension should be close

to zero is shown to be robust with one exception. A low Frisch labor supply elasticity increases the optimal pension level significantly.

4.1 Abolition of social security

In Table 3, the stationary-state allocation of the benchmark (with the net pension replacement ratio $\theta_{pen} = 50.0\%$) is compared to the case without social security ($\theta_{pen} = 0\%$). The abolition of social security increases savings for old age considerably so that the aggregate capital stock $\tilde{K}$ rises by 38.5%, from $\tilde{K} = 1.108$ to $\tilde{K} = 1.534$. In addition, the abolition of distortionary pension contributions from the initial level $\tau_{pen} = 8.51\%$ increases the labor supply (which is also augmented because of the rise in the marginal product of labor) so that the average working hours rises by 7.1%, from $\bar{l} = 0.297$ to $\bar{l} = 0.318$. As a consequence, both equilibrium output $\tilde{Y}$ and consumption increase by 17.4% and 12.7% in response to the abolition of social security.

Table 3: Allocation effects of social security.

θ_{pen}	50%	8%	0%
$\tilde{Y}$	0.423	0.480	0.497
$\tilde{K}$	1.108	1.428	1.534
$\tilde{L}$	0.247	0.260	0.264
$\bar{l}$	0.297	0.312	0.318
$\tilde{C}$	0.221	0.244	0.249
τ_{pen}	8.51%	1.50%	0%
Gini coefficients			
Net income	0.362	0.411	0.425
Gross income	0.390	0.414	0.420
Wealth	0.664	0.611	0.604
Consumption	0.276	0.285	0.289
Liquidity-constrained	34.5%	27.5%	26.3%
Δ	0%	2.32%	2.17%

Welfare is measured by the average lifetime utility of the newborn generation in stationary state. The welfare change Δ is computed as the consumption equivalent change relative to the benchmark case ($\theta_{pen} = 50\%$).

Without social security, wage income is less concentrated because the substitution effect of a higher net wage rate affects the labor supply of the low-productivity workers to a larger extent than that of the high-productivity workers. However, gross income is nevertheless more concentrated than in the case with social security because retired households with only interest income do not receive any income from pensions payments. Therefore, the Gini coefficient of gross income increases from 0.390 to 0.420 if pensions are abolished. The inequality of the wealth distribution decreases without pensions because, in this case, many low-income workers have to save in order to provide for old age and the number of households without any savings decreases from 34.5% to 26.3%.[12] Accordingly, the Gini coefficient of wealth decreases from 0.664 to 0.604 if social security is abolished. Even though the social security system redistributes from the income-rich to the income-poor, the distortionary effect of public pensions dominates, and welfare increases significantly by a consumption equivalent of 2.17% in the case without social security.

4.2 Optimal public pensions

There are multiple effects of a pay-as-you-go pension system on welfare. 1) Social security provides partial insurance by redistributing income between generations and among cohorts. On the one hand, annuity markets are missing and the public pension system helps to insure against a long life.[13] On the other hand, social security insures against individual income uncertainty to some extent as it redistributes from those workers with high-income to those workers with low income if pensions are not proportionally linked to contributions. Furthermore, the welfare effect is aggravated because a large fraction of the households in the economy is credit-constrained and cannot use debt in order to finance consumption during times of low income. 2) The social security contribution introduces a welfare-reducing distortion on the labor supply. 3) In addition, savings decline and if the economy is dynamically efficient (as in the present model) aggregate consumption declines, too. Whether the positive insurance effect compensates for the negative distortions that result from both lower labor supply and savings can only be determined quantitatively.

The welfare effect of the net replacement ratio θ_{pen} is illustrated in Figure 1. Welfare is measured as the ex-ante expected life-time utility of the newborn household or, equally, the weighted average of the life-time utility of the different agent types in the model with the weights being equal to the fraction of each type at birth (as described in Section 3). The optimal net replacement ratio amounts to $\theta_{pen}^* = 8\%$ and results in a welfare gain of $\Delta = 2.32\%$ of total consumption compared to the benchmark case with a replacement ratio of 50%.[14]
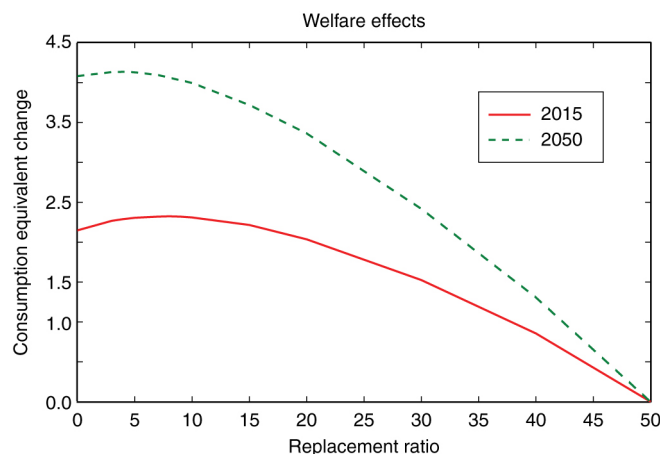
Figure 1: Welfare effects of pension policies θ_{pen}.

The life-time utilities of the 8 newborn household types with permanent productivity $\epsilon \in \{0.57, 1.43\}$, transitory productivity $\eta \in \{0.727, 1.273\}$ and employment status $\iota \in \{e, u\}$ are affected differently by a change of the net pension replacement ratio from the benchmark with $\theta_{pen} = 50\%$ to the optimal pension policy with $\theta_{pen}^* = 8\%$. Table 4 provides a welfare decomposition for the different types. Evidently, all high-productivity workers benefit from lower pensions. The welfare gains of the high-productivity workers with $\epsilon_2 = 1.43$ are considerable amounting to 4.4%–5.1% of consumption, while the losses of the low-productivity workers are relatively small and amount to 0.4%–0.7% depending on the idiosyncratic productivity shock η. Notice that the employment status ι has little effect on the welfare effect from pension policy. One obvious reason for this latter observation is presented by the transitory nature of unemployment in comparison with the much more pronounced persistence of the individual income shock η. In addition, periods of unemployment spell are rather short-lived and do not exceed 3 months so that the drop in income is rather subdued.

Table 4: Decomposition of welfare effects.

Household type	Δ
Low permanent productivity $\epsilon = 0.57$	
$\{\iota, \eta\} = \{e, 0.727\}$	−0.40%
$\{\iota, \eta\} = \{e, 1.273\}$	−0.72%
$\{\iota, \eta\} = \{u, 0.727\}$	−0.40%
$\{\iota, \eta\} = \{u, 1.273\}$	−0.71%
High permanent productivity $\epsilon = 1.43$	
$\{\iota, \eta\} = \{e, 0.727\}$	5.08%
$\{\iota, \eta\} = \{e, 1.273\}$	4.39%
$\{\iota, \eta\} = \{u, 0.727\}$	5.11%
$\{\iota, \eta\} = \{u, 1.273\}$	4.35%

The entries in the second column represent the consumption equivalent change that accrue to the 20-year-old household with the efficiency type $\{\eta, \epsilon\}$ and employment status ι resulting from a reduction of θ_{pen} from 50% to the optimal rate $\theta_{pen}^* = 8.0\%$.

4.3 The effect of aging

How does a greyer population affect the optimal amount of pension payments? On the one hand, an increase of the old-age dependency ratio increases the welfare distortion from the pension system. For given net replacement ratio $\theta_{pen} = 50\%$, the contribution rate τ_{pen} will increase from 8.5% to 10.5% between 2015 and 2050; as a consequence, the tax wedge will be higher and labor supply decreases.[15] For this reason, the quantitative effects of abolishing pensions are also much more pronounced in the greyer economy where capital and working hours

would increase by 46.7% and 9.1% in 2050, while the effect of eliminating social security would only amount to 38.4% and 7.1% in 2015 (compare Section 4.1). On the other hand, retirees are getting older on average so that the (discounted) loss in old-age utility as a consequence of possible negative income shocks and zero pensions is magnified. The overall effect can only be computed numerically.

For the population in 2050 that is characterized by a higher old-age dependency ratio,[16] the welfare effects of lower pensions are illustrated by the broken line in Figure 1. The optimal social security policy consists of low pensions with a replacement ratio equal to $\theta_{pen}^{2050,*} = 4\%$. If pensions are cut from 50% of average net wage income to 4%, the stationary welfare gain amounts to 4.1% of total consumption as reported in the column entitled "Δ" in Table 5.[17] Our welfare result supports the hypothesis that the demographic transition makes the provision of a public pension system even less desirable as the distortions on labor supply and savings increase.

Table 5: Optimal pension policies.

Case	θ_{pen}	τ_{pen}	Δ
Benchmark	8%	1.50%	2.32%
Year 2050	4%	0.97%	4.12%
Sensitivity analysis			
1. $\sigma = 4.0$	0%	0%	13.7%
2. Frisch labor supply	43%	8.06%	0.11%
elasticity $\eta_{l,w} = 0.3$			
3. Recursive preferences	9%	1.69%	2.23%
4.	0%	0%	4.76%
Productivity-dependent pensions			

Cases 1 and 2 correspond to the sensitivity analysis with respect to a lower intertemporal elasticicty of substitution, $1/\sigma = 0.25$, and a lower Frisch labor supply elasticity, $\eta_{l,w} = 0.3$. The optimal pension policies for the sensitivity cases (1)–(4) are computed for the demographics prevailing in the year 2015. In the column entitled by "Δ", the stationary state welfare change from moving to the optimal pension policy is reported.

4.4 Transition dynamics of pension reforms

Hitherto, our analysis has focused on the steady state where we assume a stationary population as implied by the survival probabilities and population growth rates prevailing in 2015 and 2050, respectively. In the following, we also consider the transitional dynamics. For this reason, we assume that the economy is in steady state in 2015 and that the population evolves according to that in the US economy. In particular, we assume that the survival probabilities and the population growth rates of the model population are equal to those forecasted by UN (2015). Starting in 2100, the demographic variables are constant and equal to those prevailing in the year 2100 with a population growth rate equal to 0.2%. We further assume that the transition to the new steady state is complete by the year 2200. In addition, the levels of government expenditures and transfers are kept constant at the levels prevailing in the steady state in 2015.[18]

In 2015, the government announces an unexpected policy change that becomes effective in the year $\tilde{t}$. In the benchmark, we choose $\tilde{t} = 2015$, but we will also consider $\tilde{t} \in \{2030, 2045\}$ below. The new pension policy consists in a change of the replacement ratio from 50% to the level $\tilde{\theta}_{pen} = 8\%$ that was found optimal for the stationary state population in 2015. We also present the results for the case of no policy change, $\tilde{\theta}_{pen} = 50\%$ and two intermediate cases, $\tilde{\theta}_{pen} \in \{20\%, 35\%\}$. We assume that the policy is implemented gradually and stretched over a period of n_θ years. In the benchmark, we choose $n_\theta = 45$ so that the number of years accord with the length of the working life. Furthermore, the replacement ratio is increased over the implementation period linearly.

Figure 2 plots the evolution of aggregate capital, aggregate labor, aggregate income, the wage tax rate τ_w, the pension contribution rate τ_{pen}, and the old-age dependency ratio of the 65+/(20–65)-years old during the transition. The results are in line with those of similar transition experiments in the literature. During the demographic transition, the population is aging due to increasing life expectancy and the old-age dependency ratio of the 65+/(20–65) years-old is increasing from 27% to 43% between 2015 and 2200. As a consequence, the labor force share and, hence, aggregate $\tilde{L}$ is shrinking.[19] There are multiple effects of the demographic transition on savings. On the one hand, a higher share of the population is retired and decumulates savings. On the other hand, households live longer and, therefore, workers build up higher retirement savings. If pensions are also decreased, the latter effect even compensates the former effect and aggregate savings and the capital stock even increase over time. Since capital increases relative to labor over time, the real interest rate decreases, while the wage rate increases (not presented).
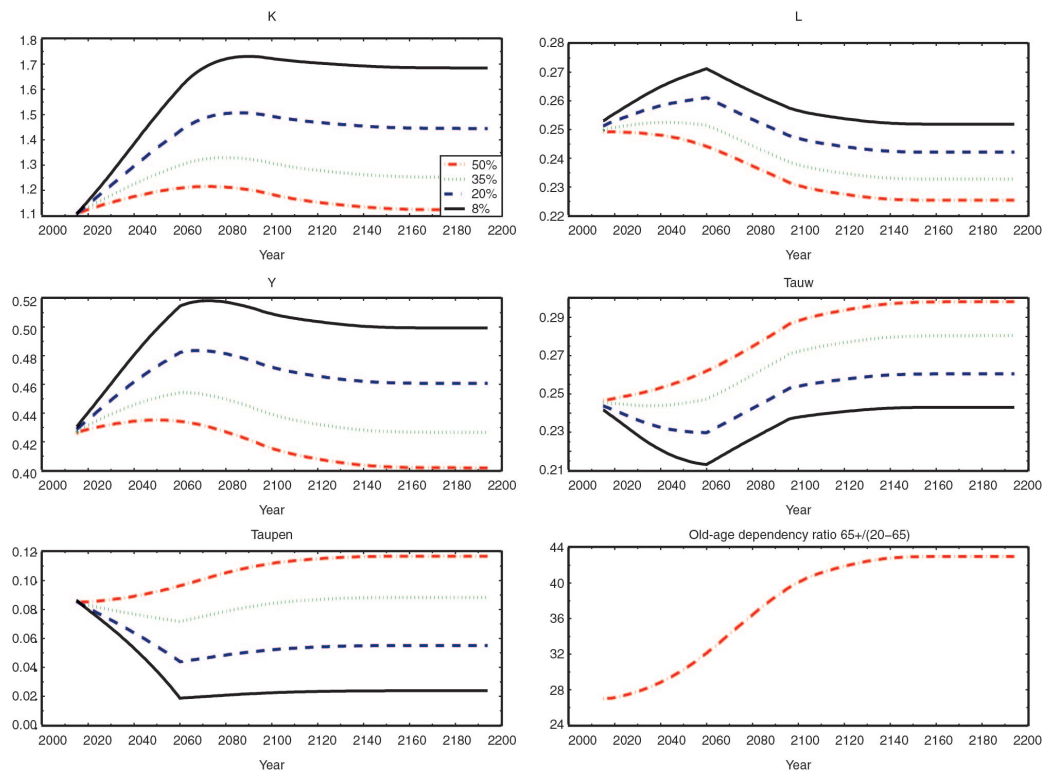
Figure 2: Transitional dynamics: aggregate variables.

When the pension replacement ratio θ_{pen} is reduced over time, both pensions *pen* and the contribution rate τ_{pen} fall until the year 2060. As a consequence, the net wage rate, $(1 - \tau_w - \tau_{pen} - \tau_{ui})w_t$, increases and the individual augments his labor supply. Therefore, aggregate labor even increases during the initial phase of the transition during the years 2015–2060 for the pension policies $\theta_{pen} \in \{8\%, 20\%, 35\%\}$. Afterwards, the effect of a shrinking labor force dominates and $\tilde{L}$ declines to its new long-run equilibrium value. The capital stock is also hump-shaped and peaks at a latter year in these cases due to the sluggishness of the capital stock.[20] In the case of medium to high pensions, $\theta_{pen} \in \{20\%, 35\%, 50\%\}$, wage taxes τ_w even have to increase in the medium and long run relative to the year 2015 because the share of government expenditures (government consumption and transfers) increases relative to GDP.[21] The adjustment in the savings is sluggish so that the aggregate capital stock $\tilde{K}$ and output $\tilde{Y}$ display a hump-shaped profile over time.

Figure 3 presents the welfare effects of the different policies $\theta_{pen} \in \{35\%, 20\%, 8\%\}$ for the individual generations that enter the labor force in the years 1946–2200. Welfare is computed as the average value of the newborn generation and expressed as the consumption equivalent change that makes the generation indifferent to the case of the old policy characterized by a replacement ratio $\theta_{pen} = 50\%$. The first generation that is affected by this change of policy is the one that is still alive in the year 2015 and entered the labor force at age 20 in the year 1946. Since this generation is only affected in the last period of her life, the effect on life-time utility is negligible and close to zero. Later generations, however, suffer substantial welfare losses which obtain a maximum for those agents who enter the labor force around the year 1995 and are in the mid-period of their working life when the policy change is implemented in the year 2015. For those households, average life-time utility drops by a consumption equivalent of 2%–10% depending on the policy $\theta_{pen} \in \{35\%, 20\%, 8\%\}$. These households receive a lower pension in old-age, but still have to provide for the retirees with the higher pension during the transition. Households born after the year 2025–2040 benefit from the new pension policy. Notice that the generations who will benefit from such a policy depends on the boldness of the reform. The stronger the reform, the longer it takes for its benefits to manifest itself in an increasing generational welfare. The implementation of a policy that maximizes steady-state life-time utility, $\theta_{pen} = 8\%$, therefore, implies significant costs for current and even the initial future generations.
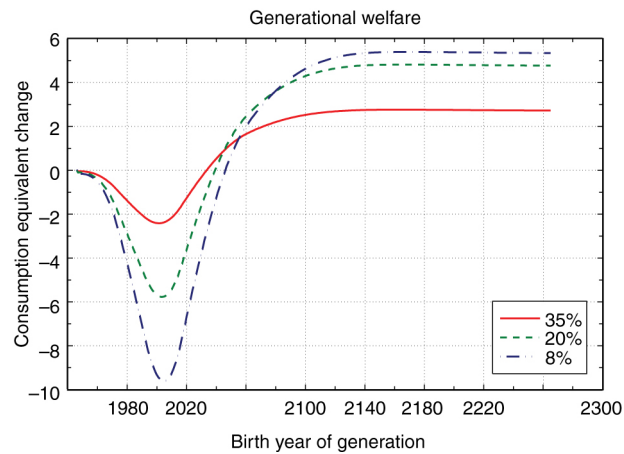
Figure 3: Transitional dynamics: generational welfare.

In order to lighten the burden of the lower pension policy on the transitional generations, the government can either postpone the introduction of the policy change, $\tilde{t}$, or increase the number of implementation periods, n_θ. Both policies imply that the transitional cohorts can adjust their behavior over a longer stretch of time. Figure 4 displays the generational welfare consequences of a policy with a replacement rate $\theta^*_{pen} = 8\%$ that only introduces the policy in $\tilde{t} = 2030$ with implementation periods n_θ of 45 and 90 years and a policy with $\tilde{t} = 2045$ and $n_\theta = 45$. Later implementation of the policy in the years 2030 and 2045 basically results in a shift of the welfare burden to later generations. The maximum burden only shrinks by half a percentage point if the pension is introduced in 2045 rather than 2015 and remains at a level of approximately 9–10% of total consumption. The number of generations that suffer from such a pension policy of a postponed reduction to a replacement ratio $\theta^*_{pen} = 8\%$ in 2045 rather than 2015 declines from 99 to 88. A longer spread of the pension reduction over 90 years helps to reduce the maximum burden by approximately half so that the loss in total consumption is below 5% during the transition for all generations; in this case, however, the number of generations who experience a loss in life-time utility increases to 99 again and affects those generations that are born between 1986 and 2084.
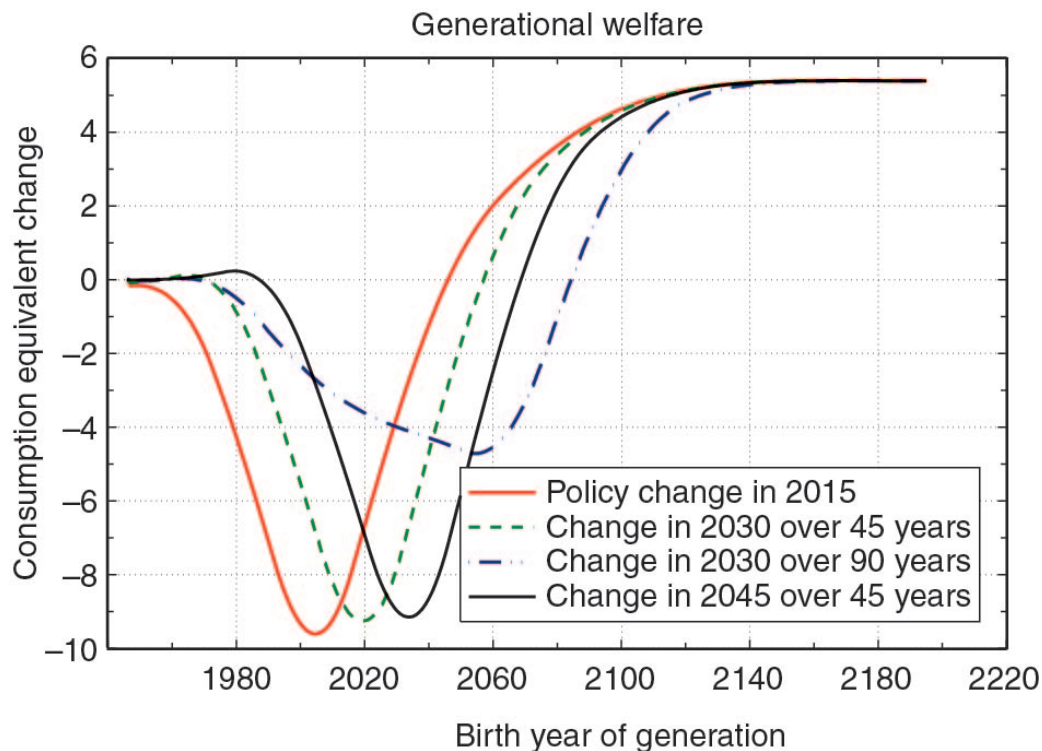

Figure 4: Generational welfare and different implementation policies.

4.5   Sensitivity analysis

In the following, we study the sensitivity of our stationary-state results with regard to the choice of preferences and the pension progressivity. In particular, two parameters of the utility function that are often found to be crucial for quantitative results are the 1) intertemporal elasticity of substitution of consumption and leisure in different years, $1/\sigma$, that determines the utility-costs of the variation of income over the life-cycle (and, therefore, consumption and instantaneous utility in each period) and 2) the utility parameter γ that determines the Frisch elasticity of labor supply and, therefore, the utility costs of substituting consumption by leisure in times of negative income shocks. In addition, we study the sensitivity of our results with respect to 3) the assumption on (relative) risk aversion μ. Finally, we consider 4) the case that pensions are not provided lump-sum to the retirees but depending on their permanent productivity. The sensitivity of our results on the optimal pension policy is summarized in the bottom four rows of Table 5. In essence, in all cases considered, the optimal replacement ratio is equal or close to zero except for the case of a low Frisch labor supply elasticity η_{lw}.[22]

1. Intertemporal elasticity of substitution. In our sensitivity analysis, we present results for $\sigma = 4.0$ (keeping $\gamma = 0.31$ constant) which is usually considered an upper value for σ in related studies, e.g., in İmrohoroğlu, İmrohoroğlu, and Joines (1995). For higher values of σ, precautionary savings increase to a larger extent if pensions are abolished. As a consequence, aggregate savings even increase by 45.1% relative to the benchmark case if the pension replacement ratio θ_{pen} drops from 50% to 0.0%. Therefore, the change in output is also quantitatively more significant compared to the case with $\sigma = 2$ and output increases by 19.1%. The most marked change, however, is the one on welfare. Abolishing social security results in stationary-state welfare gains equal to $\Delta = 13.7\%$ of total consumption. In addition, it is even optimal to abolish pension contributions completely and set $\theta_{pen} = 0\%$.

2. Frisch elasticity of labor supply. For our choice of the functional form of instantaneous utility (4), we cannot calibrate separately for the steady-state value labor supply, $\bar{l}$, and the Frisch elasticity of labor supply, $\eta_{l,w}$, since the parameter γ of the utility function determines both values. In the stationary equilibrium without pensions and with the preference parameter $\zeta = 0$, the Frisch labor supply elasticity for the instantaneous utility function (4) is given by

$$\eta_{l,w} = \frac{\sigma}{\sigma - (1 - \gamma)(1 - \sigma)} \frac{1 - l}{l}. \tag{17}$$

Therefore, in the benchmark case with $\gamma = 0.31$ and $\sigma = 2.0$, the Frisch elasticity is equal to $\eta_{l,w} = 1.53$ for a household with labor supply $l = 0.30$.

In the following, we choose a different functional form for instantaneous utility that allows for the separate calibration of $\bar{l}$ and $\eta_{l,w}$. In particular, we assume that preferences are time-separable, $\sigma = \mu = 2.0$, so that we can write intertemporal utility as the discounted sum of per-period utilities:

$$\max E_t \sum_{s=1}^{J} \beta^{s-1} \left(\Pi_{j=1}^{s} \phi_{t+j-1,j-1} \right) u(c_{t+s-1}^{s}, l_{t+s-1}^{s}), \tag{18}$$

and choose instantaneous utility as follows:[23]

$$u(\tilde{c}, l) = \frac{\tilde{c}^{1-\sigma}}{1 - \sigma} - \psi_0 \frac{l^{1+1/\psi_1}}{1 + 1/\psi_1}. \tag{19}$$

For this utility function, the Frisch intertemporal labor supply elasticity $\eta_{l,w}$ is equal to ψ_1. Estimates of $\eta_{l,w}$ implied by microeconometric studies vary considerably. MaCurdy (1981) and Altonij (1986) both use PSID data in order to estimate values of 0.23 and 0.28, respectively, while Killingsworth (1983) finds an US labor supply elasticity equal to $\eta_{l,w} = 0.4$.[24] We will use the conservative estimate $\eta_{l,w} = 0.3$ and choose $\psi_1 = 0.30$ accordingly. We calibrate $\psi_0 = 965$ so that equilibrium labor supply is equal to 30% of available time, $\bar{l} = 0.30$, in the benchmark equilibrium with $\theta_{pen} = 50\%$.

For the lower Frisch labor supply elasticity, $\eta_{lw} = 0.3$, the distortionary effects of pension contributions on the labor supply are reduced and it is optimal to provide pensions in the stationary state at a net replacement ratio equal to 43%. The associated welfare gain from reducing θ_{pen} from 50% to 43% amounts to $\Delta = 0.11\%$ of total consumption.[25] As an intermediate result, we, therefore, observe that the optimal pension policy is sensitive with respect to the labor supply elasticity.

Our results are related to those obtained by İmrohoroğlu and Kitao (2009) who study the effect of the Frisch labor supply elasticity on aggregate labor and the labor-age profile. İmrohoroğlu and Kitao distinguish between

two different scenarios for the pension reform consisting in the downsizing of the system by 50% and a total elimination of social security. They show that the effect of pension reforms on aggregate labor is rather insensitive with regard to the Frisch elasticity, while the profile of hours over the life-cycle is highly sensitive. In contrast to us, they find substantial welfare gains from the reduction of pensions even in the case of a low labor supply elasticity. According to their Table 2, the long-run welfare gain of half-privatization amounts to 4.3% of total consumption for the utility function (17) and a low Frisch elasticity equal to $\eta_{ltw} = 0.5$. Different from us, however, they do not model permanent productivity differences between the workers so that income heterogeneity is smaller in their model than in ours. In addition, they assume that pensions are earnings-dependent and, thus, provide less income redistribution among households in their model. Therefore, our results in this section can be interpreted as a sensitivity analysis to their welfare findings. Under the assumption of permanent skill differences among the workers and a lump-sum redistributive pension, the abolition of social security implies smaller welfare gains or even losses in the case of rather inelastic labor supply.

3. Recursive preferences. As a third sensitivity analysis of our preferences, we consider the attitudes towards risk as presented by the risk aversion parameter μ.[26] As pointed out in Section 3, DSGE studies commonly consider a wide range of values for μ. For example, Caldara, Fernández-Villaverde, and Rubio-Ramiírez (2012) consider values between 2 and 40. We will use a conservative intermediate estimate $\mu = 4.0$ as in Fehr, Kallweit, and Kindermann (2013) for the comparison with our benchmark case $\mu = 2.0$.

We find that our optimality result of a low pension replacement ratio to be insensitive with respect to the assumption of higher risk aversion. If risk aversion μ increases relative to the inverse of the intertemporal elasticity of substitution, σ, the utility costs of income uncertainty increases for the individual. Accordingly, a higher pension helps to increase (average) lifetime utility (3). However, the general equilibrium effects on aggregate savings and, therefore, income almost compensates for this positive welfare effect, and we find that it is optimal to drastically reduce pensions in this case as well with the optimal replacement ratio amounting to $\theta_{pen}^* = 9.0\%$. The welfare gain in stationary state from reducing pensions from the present level of 50% to the optimal level of 9% amounts to 2.2% of total consumption and, hence, is found to be comparable to the case of time-separable preferences with $\sigma = \mu = 2.0$.

4. Heterogenous pensions. So far, we have assumed that pensions are provided lump-sum. In the following, we analyze the case that pensions of the household with productivity type ϵ_j, pen_t^j, depend proportionally on the permanent productivity ϵ_j, $j = 1, 2$, according to:

$$pen_t^j = \theta_{pen}(1 - \tau_w - \tau_{pen} - \tau_{ui})\epsilon_j w_t A_t \bar{l}. \tag{20}$$

As a consequence, capital accumulation is smaller in the economy with a pension replacement rate of 50% than under the lump-sum pension (by 2.5%) because the high-income households with $\epsilon = \epsilon_2$ have smaller incentives to accumulate savings for old age.[27] In addition, wealth is less concentrated and the Gini coefficient falls from 0.664 (lump-sum pensions) to 0.621 (heterogeneous pensions).

The considered pension system (20) redistributes less income from the high-earnings to the low-earnings households and provides only (partial) income insurance against the idiosyncratic income shock η. We find that it is optimal to eliminate social security complete and set $\theta_{pen} = 0\%$ (compare Table 5). Welfare gains are considerable in this case and amount to 4.76% of consumption.[28]

5 Conclusion

We find that the optimal US pension replacement ratio relative to net earnings should be much lower than found empirically and should be equal to 0–9% for the present US population for our benchmark calibration and 0% for the projected US population in 2050. Our result is robust to a variety of sensitivity analyses including higher risk aversion, lower intertemporal substitution elasticity or heterogeneous pensions. There is only a welfare-enhancing role of a substantial public pay-as-you-go system if labor supply is rather inelastic and, hence, higher pension contributions do not imply considerable tax distortions. For a Frisch labor supply elasticity equal to 0.3, the optimal replacement ratio increases to 43%.

In conclusion, we would like to point out the direction for our future research. In the present paper, we analyze the insurance effect of redistributive pensions if individuals are subject to idiosyncratic income, unemployment, and longevity risk. However, the financing of the pensions in a pay-as-you-go system with the help of a tax on wage income also introduces distortions on the labor supply. In essence, the social security tax on wage income redistributes income from the young to the old and, depending on the progressivity of the pension system, from the income-rich to the income-poor households. In related research, Grant et al. (2010) have analyzed the empirical magnitude of the insurance versus the distortionary effect of the US income tax system.

They find strong evidence for the former, but milder evidence for the latter.[29] In future work, we would like to incorporate these findings into out model so that we are able to consider both lump-sum redistribution to all agents and (possibly means-tested) redistribution that is specifically targeted at certain age or income groups in order to determine the optimal redistributive policy.

A Appendix

A.1   Properties of the Benchmark Equilibrium

In stationary equilibrium of the benchmark case, the average wealth $\tilde{a}^s$, working hours l^s, and consumption $\tilde{c}^s$ of the s-year-old cohort over the life cycle (or working life respectively) are graphed in Figure 5, Figure 6, and Figure 7. The solid (broken) lines of the graphs present the low (high) productivity type ϵ_1 (ϵ_2) and correspond to the lower (upper) curves in the figures.
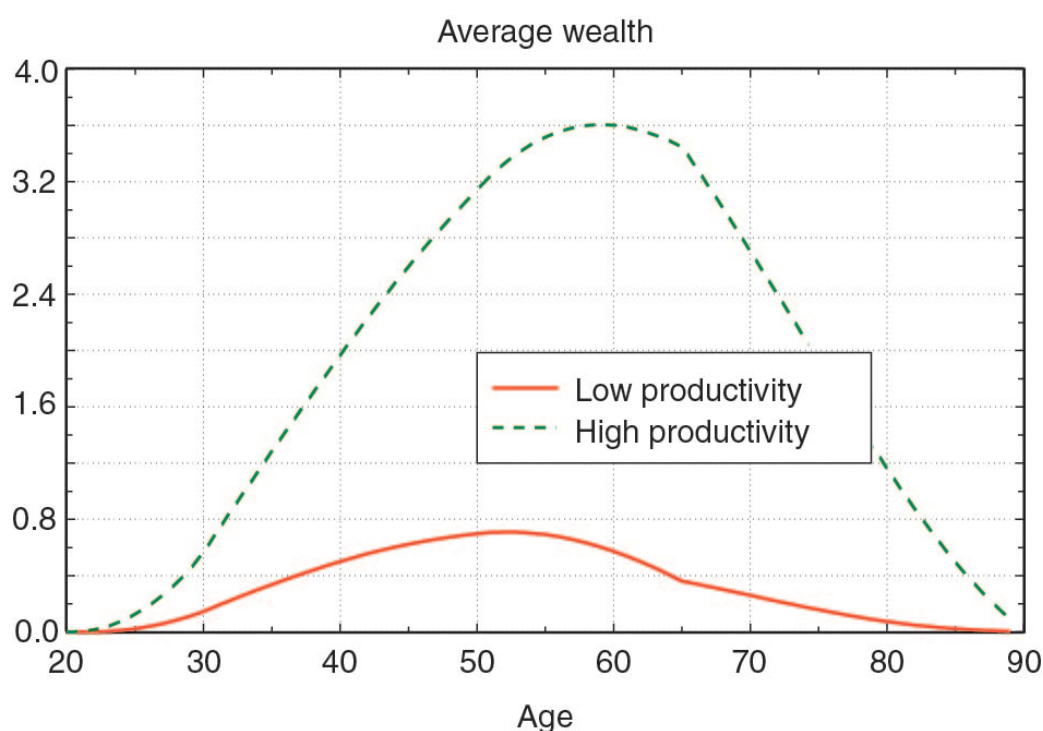
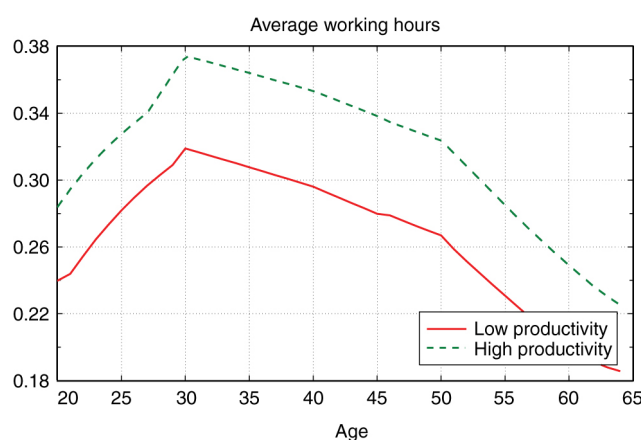

Figure 5: Wealth-age profile.
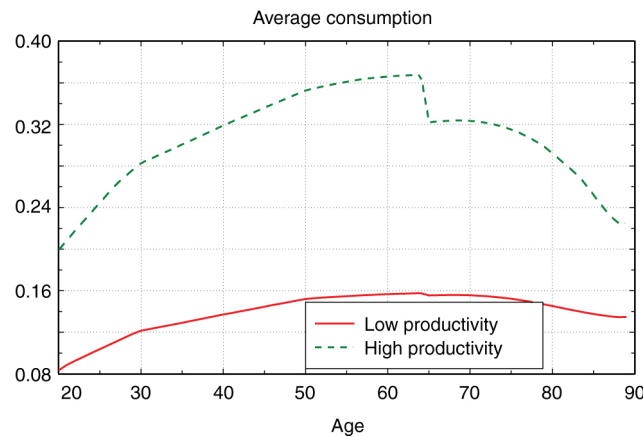

Figure 6: Labor-supply-age profile.

Figure 7: Consumption over the life cycle.

Households with high (low) productivity accumulate savings until the age of 59 (52) before they start to dissave. In their effort to smooth consumption over their lifetime, households start to consume part of their savings as their income drops. The drop in income from wages is caused by the decrease of age-dependent efficiency $\bar{y}_s$ which peaks at age 50 (not illustrated). The decline in wealth is accelerated for the high-productivity households as soon as the households retire because pensions are below the former wage income.

The profile of working hours in Figure 6 also mirrors the age-productivity profile because the substitution effect of higher wages dominates the income effect. However, the peak of working hours (at age 30) takes place prior to the peak in age-dependent efficiency $\bar{y}_s$ because of increasing wealth (prior to age 52) which reduces the labor supply.

Labor supply l and wealth $\tilde{a}$ also depend on the permanent and temporary productivity types $\{\epsilon, \eta\}$. Both variables increase with higher productivity $\epsilon = \epsilon_2$ and $\eta = \eta_2$. The household with $\epsilon = \epsilon_1$ who experiences a negative productivity shock, $\eta = \eta_1$, is also liquidity-constrained, $\tilde{a} = 0$, if he has not accumulated sufficient savings in former periods. In fact, the percentage of households without savings amounts to 34.5% in our benchmark calibration.

The heterogeneity with regard to individual productivity, $\epsilon\eta\bar{y}_s$, results in inequality of the household's income and wealth distribution. The dsitributions of income and wealth are characterized by Gini coefficients of 0.362 (net income after taxes), 0.390 (gross income before taxes), and 0.664 (wealth). Notice that the OLG model is able to generate much more inequality in wealth than in income as observed empirically.[30] However, all our inequality measures fall short of values observed empirically. For example, Budría Rodriguez et al. (2002) report Gini coefficients of (gross) income and wealth equal to 0.553 and 0.803. Our model values fall short of the empirical ones for mainly three reasons: 1) We do not consider self-employed workers and entrepreneurs. Quadrini (2000) presents empirical evidence that the concentration of income and wealth is higher among entrepreneurs and that the introduction of an endogenous entrepreneurial choice in a dynamic general equilibrium model helps to reconcile the inequality in the model with that of the US economy. 2) We neglect the top income percentile of the wage earners in our model. 3) We omit bequests.[31]

Figure 7 displays the average consumption of the two productivity types over the life cycle. The profile is hump-shaped in both cases and declines after retirement. The profile accords with empirical observations in its qualitative features. For the US economy, Fernández-Villaverde and Krueger (2007) find that the empirical consumption-age profile display a significant hump over the life cycle even after correcting for the change of the family size. For the high-education households (that are roughly corresponding to the high-productivity households in our model), the peak occurs at age 55, while the low-education households attain their consumption maximum at an earlier age close to 50 and the hump is much smaller. Therefore, in our model, the hump occurs too late in the life cycle and the increase in consumption from age 20 to age 50 is too high for the low-productivity households. Since consumption and leisure are substitutes and leisure increases to 100% during retirement, consumption is only reduced at the beginning of retirement.[32] In addition, the consumption of the very old low-education households in the US economy drops to 70–80% of the consumption of the corresponding 20-year old, while this is not the case in the model.[33]

With regard to the cross-sectional distribution of consumption, our model is able to replicate the fact that consumption inequality is muss less than income inequality. Using US data from the Consumer Expenditure Survey, Krueger and Perri (2006) present evidence that the Gini coefficient of consumption amounts to 0.26 in 2003, while it is equal to 0.28 in the model.

A.2   Computation

The main computational problem is the numerical solution of the intertemporal household decision problem. We use value function iteration as described in Chapter 9.3 of Heer and Maußner (2009).[34] We apply Golden Section Search in each step to find the optimal next-period assets a' for each type $\{\iota, \eta, \epsilon, s\}$ of the household. Our reason for this approach is that the Golden Section Search is a very robust method that can easily handle non-negativity constraints such as $l \geq 0$ or $a \geq 0$. Between gridpoints, we interpolate linearly.

Notes

1 In İmrohoroğlu, İmrohoroğlu, and Joines (1995), the optimal pension replacement ratio is found to be around 30%. However, as pointed out by İmrohoroğlu, İmrohoroğlu, and Joines (1999), this high value for optimal pensions results from the fact that their model is characterized by dynamic inefficiency in the absence of social security. Higher pensions and, hence, lower savings even increase total consumption for low replacement ratios. In addition, İmrohoroğlu, İmrohoroğlu, and Joines (1999) argue that the US economy is dynamically efficient as shown by Abel et al. (1989). We, therefore, follow these authors and only consider dynamically efficient economies.

2 One modification that we do not consider is the introduction of altruism. Fuster, İmrohoroğlu, and İmrohoroğlu (2003) find that in the case of two-sided altruism towards the predecessors and descendants the welfare effects of social security are enhanced. Altonij, Hayashi, and Kotlikoff (1997), however, present empirical evidence that the implications of altruism on the intergenerational risk-sharing behavior are rejected.

3 Kitao (2014) compares polices that i) increases the payroll tax while keeping the benefit level constant, ii) keeps the payroll tax constant, iii) increases the retirement age, and iv) introduces means-tested benefits. In accordance with our results, he finds that the reduction of the benefit is the most efficient policy in the long run. Heer and Irmen (2014) also analyze the three policies (i)–(iii), but economic growth is endogenous in their model. During the demographic transition, firms' incentives to invest in labor-saving technological progress increase and depend on the pension policy.

4 Moreover, they abstract from population growth.

5 This modeling device helps to ensure that the average unemployment duration can also be chosen to less than 1 year as observed empirically.

6 These preferences were introduced by Epstein and Zin (1989). Epstein and Zin (1991) uses time series data on consumption and asset returns to test the representative-agent model. The preferences allow, among others, for a better explanation of some asset-price puzzles [see also Bansal and Yaron (2004)].

7 In our sensitivity analysis, we study the case that pension contributions depend on the permanent productivity type of the worker.

8 Related research that uses such a value for β includes İmrohoroğlu, İmrohoroğlu, and Joines (1995) and Huggett (1996). With this value of β, the effective time discount factors display an increasing weight to instantaneous utility until real lifetime age 67, before they decline again and even fall below one after the real lifetime age 87 (for the survival probabilities of the year 2015).

9 In 2016, the maximum taxable income for social security in the U.S. amounted to $118,500. According to the data of the Current Population Survey, individual annual earnings above $288,000 fall in the top percentile in the same year.

10 In 2016, the maximum social security benefit amounted to $2,663 per month.

11 In Section 4.2, we will point out that the low-earnings workers will be affected most severely from an abolition of the pay-as-you-go pension system.

12 Budría Rodriguez et al. (2002) report that 2.5% of the households have zero wealth, and even 7.4% have negative wealth in the 1998 Survey of Consumer Finances.

13 Hubbard and Judd (1987) show that a fully-funded social security system can increase welfare in the absence of liquidity constraints and annuity markets because it provides insurance against longevity. In a recent study, however, Caliendo, Guo, and Hosseini (2014) demonstrate that this result is sensitive with respect to the assumption whether 1) bequest income is fixed or endogenous and 2) bequest income is redistributed anonymously or through a direct linkage between deceased parents and surviving children.

14 In the sensitivity analysis below, we point out that pensions should optimally be zero if pensions are provided proportional to the permanent efficiency type ϵ rather than lump-sum.

15 For our choice of the utility function and parameters, the substitution effect dominates the income effect for all policy experiments under consideration.

16 The old-age dependency ratio measures the number of people in the population aged 65 and above as a percentage of those aged between 20 and 64. In our computation, the dependency ratio increases from 27% to 36% between 2015 and 2050. Even though we assume the population to be stationary in 2015 and 2050, respectively, our values of the dependency ratio compare favorably with those reported by UN (2015) which amount to 25% and 41% in 2015 and 2050.

17 For the computaton of the stationary equilibrium, we have assumed that absolute government expenditures $\tilde{G}$ (relative to the technology level A_t and population N_t) remain at the constant level of 2015. We refrain from comparing the welfare of the generations born in 2015 with that of the generation born in 2050 because, due to the different lifetime expectations, lifetime utilities of the two generations born in 2015 and 2050 would be different even if the individual cohorts would consume exactly the same and work equal hours.

18 More exactly, we assume that the per-capita government expenditures and transfers grow at the exogenous technological growth rate.

19 Aggregate labor L_t in period t is expressed relative to total population N_t.

20 For example, young workers in the years 2050–2060 supply the highest number of working hours during the whole transition period, but their wealth peaks only at the end of their working life in later years.

21 Remember that we assumed G and tr to remain at their 2015 level. In addition, government revenue from accidental bequests decline due to higher survival probabilities.

22 Our result of optimal pensions close to zero is also insensitive with respect to the introduction of the production factor land as in the model of İmrohoroğlu, İmrohoroğlu, and Joines (1999) and the absence of economic growth. These results are not reported for reason of space, but are available from the author upon request.

23 Notice that we use stationary consumption $\tilde{c} \equiv c/A$ as an argument of the utility function in (19). If we had used c instead of $\tilde{c}$, utility would not be stationary (labor l would converge to zero in the long-run for a per capita growth rate $g > 0$ and $\sigma > 1$). In addition, $\zeta = 0$ applies so that periods of unemployment provide the same disutility as periods of employment.

24 Domeij and Floden (2006) argue that these estimates are biased downward due to the omission of borrowing constraints.

25 Notice that we cannot use (16) in order to compute the consumption equivalent welfare change because the function is no longer multiplicatively, but only additively separable in the utility from consumption and leisure, and the lifetime profiles of leisure depend on the pension policies. Instead, we compute the average value of the discounted lifetime (dis)utility from consumption (labor)

$$W_c(\theta_{pen}) = E_t \sum_{s=1}^{J} \beta^{s-1} \left(\Pi_{j=1}^{s} \phi_{t+j-1,j-1} \right) \frac{(\bar{c}^s)^{1-\sigma}}{1-\sigma},$$
$$W_l(\theta_{pen}) = -E_t \sum_{s=1}^{J} \beta^{s-1} \left(\Pi_{j=1}^{s} \phi_{t+j-1,j-1} \right) \psi_0 \frac{(l^s)^{1+1/\psi_1}}{1+1/\psi_1}.$$

The consumption equivalent change Δ from a change of the pension policy θ_{pen} to θ_{pen}^* can then be computed with the help of:

$$(1+\Delta)^{1-\sigma} = \frac{W(\theta_{pen}^*) - W_l(\theta_{pen})}{W_c(\theta_{pen})}.$$

26 Instantaneous utility is specified as (4) in the benchmark model.

27 This effect dominates the effect from higher savings of the low-productivity workers.

28 Our results confirm the findings of Fehr, Kallweit, and Kindermann (2013) that pensions should be provided rather flat than earnings-dependent. In order to see this, notice that in the two economies with and without productivity-dependent pensions, the allocations coincide for $\theta_{pen} = 0\%$. Since the absence of social security is optimal for the pension-dependent case, but welfare can be increased for the lump-sum case if $\theta_{pen} = 8\%$, welfare is higher in the lump-sum pension case.

29 In Grant et al. (2006), these authors also find that the optimal income tax rate amounts to 16% in the Aiyagari (1994) model with idiosyncratic income shocks.

30 One of the first studies that pointed out the role of the OLG model to account for observed wealth heterogeneity was Huggett (1996) .

31 Among others, Heer (2001) analyzes the effect of endogenous bequests in a life-cycle model. De Nardi and Yang (2016) set up a model that considers both bequests of wealth and inheritance of abilities from the parents and is able to match the skewness of the distribution of income, wealth, and bequests.

32 In order to get a smoother consumption profile at the beginning of the retirement period at age 65, we could allow for early retirement and endogenize retirement along the lines of Fehr, Kallweit, and Kindermann (2013) or Kitao (2014) .

33 One obvious way to overcome the latter shortcoming of the model would be the introduction of education-dependent survival probabilities. For example, Peracchi and Perroti (2010) present empirical evidence from Europe that the subjective survival probabilities of high-income households are higher than those of the low-income households.

34 The computer programs are available from the author upon request.

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