Alternating spin-orbital order in tetragonal Sr₂VO₄

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(Received 28 October 2011; published 27 December 2011)

Considering spin-orbit coupling, the tetragonal crystal field, and all relevant superexchange processes including quantum interference, we derive expressions for the energy levels of the vanadium ions in tetragonal Sr_2VO_4 . The used parameters of the model Hamiltonian allow us to describe well the excitation spectra observed in neutron scattering and optical experiments at low temperatures. The free energy exhibits a minimum which corresponds to a novel alternating spin-orbital order with strong thermal fluctuation of the orbital mixing parameter.

DOI: 10.1103/PhysRevB.84.212407

PACS number(s): 78.20.-e, 78.40.-q, 71.70.Ej

In many transition-metal compounds the orbital degrees of freedom play a decisive role in determining the groundstate properties of materials such as manganites, titanates, or vanadates.¹ When contributions of the orbital moment and spin-orbit coupling are not negligible, a separation between spin and orbital degrees of freedom is not adequate anymore and the system is better described by an effective total angular momentum.^{2,3} If spin-orbit coupling competes with electron-phonon or exchange interactions even strong fluctuation regimes can arise.^{4–6}

The system investigated here is the layered insulator Sr_2VO_4 with tetragonal symmetry, which early on has come into focus as an isostructural d^1 analog of La₂CuO₄.⁷ Consequently, it was suggested that Sr_2VO_4 could become superconducting upon applying chemical pressure by doping or external pressure.^{8,9} While the system could not be driven toward superconductivity, it turned out to be a model system for studying the interplay of orbital-lattice, spin-orbital, and superexchange interactions.^{10–13}

In tetragonal Sr₂VO₄ with space group I_4/mmm ,^{14,15} the octahedrally coordinated V⁴⁺ ions occupy a square lattice in the *ab* plane (see Fig. 1). The magnetic ground state has been claimed to be antiferromagnetic with transition temperatures in the range 10–100 K determined from susceptibility measurements, but long-range order has remained evasive on the basis of neutron-diffraction studies.^{14,16,17} Recent studies have established the occurrence of a magnetostructural phase transition extending over a temperature range from 94 K to 122 K. Both the high-temperature and the low-temperature structure are tetragonal and reportedly coexist within this range.¹¹ Specific heat data have revealed two distinct broad maxima occurring at 98 K and 127 K mirroring the borders of the two-phase regime.^{18,19}

The disappearance of the high-temperature phase is accompanied by a significant drop in the susceptibility at about 100 K, which has been attributed to the onset of long-range antiferromagnetic (AFM) and orbital order.¹¹ Theoretically, the ground state of Sr_2VO_4 has been interpreted in terms of stripelike orbital and collinear AFM spin order¹⁰ or an ordering of magnetic octupoles.¹² Inelastic neutron scattering has revealed two excitations at about 120 meV, which have been assigned to the highest lying doublet of the V⁴⁺ t_{2g} levels.¹³ Recent optical experiments have reported excitations at 31 meV (visible for T > 80 K) and 36 meV (visible for T < 120 K) and a two-peak structure at 100 meV and 108 meV, which remains visible from 13 K to room temperature.¹⁹

To elucidate the nature of the ground state and to understand the observed excitation spectrum we consider the effects of spin-orbit coupling, crystal field, and superexchange on the energy levels of the vanadium ions. The resulting free energy points toward a novel alternating spin-orbital order in the ground state.

To describe the system of V⁴⁺ ions we use the Hamiltonian $H = H_{si} + H_{ex}$, where H_{ex} describes the exchange coupling of neighboring ions and H_{si} contains the single-ion contributions in a tetragonal crystal field:

$$H_{si} = D[3l_z^2 - l(l+1)] + \lambda_c l_z s_z + \frac{\lambda_{a,b}}{2}(l_s + l_s - l_s).$$
(1)

Here *D* denotes the single-ion anisotropy and *l* the effective angular momentum l = 1 of the t_{2g} orbitals, which we describe using $|1\rangle = -\frac{1}{\sqrt{2}}[d_{yz} + id_{xz}], |-1\rangle = \frac{1}{\sqrt{2}}[d_{yz} - id_{xz}],$ and $|0\rangle = d_{xy}$ as a basis.² Moreover, we use anisotropic spinorbit coupling constants λ_c and $\lambda_{a,b}$ parallel and perpendicular to the *c* direction. Anisotropic spin-orbit coupling can arise due to covalency effects and has been observed in several d^1 systems in octahedral environment.^{2,20}

The superexchange coupling between V ions via oxygen ions in the *ab* plane is usually described via the corresponding hopping integrals,²¹ which in our case are given by

$$t_{1,1} = t_{-1,-1} = \frac{1}{2}(t_{xz,xz} + t_{yz,yz}),$$
(2)

$$t_{1,-1} = t_{-1,1} = \frac{1}{2}(t_{xz,xz} - t_{yz,yz}),$$
(3)

$$t_{0,0} = t_{xy,xy}.$$
 (4)

From the spatial distributions of the d_{xz} and d_{yz} orbitals it is clear that the signs of the transfer integrals $t_{xz,xz}$ and $t_{yz,yz}$ are different and, therefore, $|t_{1,-1}| > |t_{1,1}|$. This observation allows us to deduce the most likely ordering of the V states in the ground state.

Using the reported crystal structure of Sr_2VO_4 one finds that D < 0 and, therefore, the possible ground states of the V⁴⁺



FIG. 1. (Color online) Left: Unit cell of tetragonal Sr_2VO_4 with symmetry *I4/mmm* (Ref. 15) highlighting the VO₂ planes and the octahedral coordination of the V ions. Right: Splitting of the V⁴⁺ t_{2g} levels due the tetragonal crystal field and spin-orbit coupling.

ions are $|\pm 1,\pm 1/2\rangle$. The AFM superexchange coupling (see below) yields an additional gain in energy when $|-1,\pm 1/2\rangle$ states are surrounded by $|1,\pm 1/2\rangle$, or vice versa. Then, keeping in mind that the spin-orbit coupling parameter $\lambda_c < 0$, we arrive at a configuration in the *ab* plane of Sr₂VO₄ where each vanadium ion in the state $|1,1/2\rangle$ is surrounded by vanadium ions in the $|-1,-1/2\rangle$ state and vice versa. According to the third Hund rule, spin ($s_z = \pm 1/2$) and angular ($l_z = \mp 1$) momentum of the V⁴⁺ ground state configuration are in opposition. The corresponding combined spin-angular moment per site therefore possesses the peculiarity that the magnetic moment $m_z/\mu_B = 2s_z - \kappa l_z$ is almost completely muted, when the the covalency reduction factor κ is close to 1.² The resulting ordering scheme can be described as an alternating order of spin and orbital moments on each site.

First, we introduce the superexchange parameters

$$J_a = 4 \frac{t_{xz,xz}^2 + t_{yz,yz}^2}{U},$$
(5)

$$J_{\rm int} = -8 \frac{t_{x_{\mathcal{I}}, x_{\mathcal{I}}} t_{y_{\mathcal{I}}, y_{\mathcal{I}}}}{U},\tag{6}$$

where U denotes the on-site Coulomb repulsion. The signs of the transfer integrals $t_{xz,xz}$ and $t_{yz,yz}$ are different and, therefore, the cross term J_{int} is positive. This parameter describes the quantum interference effect in superexchange coupling. The part of the effective exchange Hamiltonian containing these parameters is written as

$$H_{ex}(1) = \frac{J_a}{8} \left(\mathbf{s}_i \mathbf{s}_j - \frac{1}{4} \right) \left(2l_{iz}^2 l_{jz}^2 + l_{i+1}^2 l_{j+1}^2 + l_{i-1}^2 l_{j-1}^2 \right) - \frac{J_{int}}{4} \left(\mathbf{s}_i \mathbf{s}_j - \frac{1}{4} \right) l_{iz} l_{jz}.$$
(7)

The ferromagnetic contributions to the superexchange interaction comprise two exchange integrals J_f and J'_f , which

for a pair of V ions along the x axis can be denoted as

$$J_f = J_p - 2\frac{I_f}{U^2} (t_{xy,xy}^2 + t_{xz,xz}^2) \quad \text{and}$$
 (8)

$$J'_{f} = J'_{p} - 2\frac{I_{f}}{U^{2}}(t^{2}_{xz,xz}), \quad \text{with}$$

$$e^{2}$$
(9)

$$T_f = \langle d_{xy}, d_{xz} | \frac{e^{-}}{r_{12}} | d_{xz}, d_{xy} \rangle.$$

Here I_f is an exchange integral, which can be estimated via Racah parameters as 3B + C = 0.9 eV and J_p and J'_p correspond to potential exchange contributions. In Eq. (9) the potential exchange parameter J'_p is expected to be small and is neglected in the following. Note that J'_f is called the Hundscoupling parameter in Ref. 12.

In momentum representation for l = 1 and s = 1/2, this part of the exchange Hamiltonian is written as

$$H_{ex}(2_x) = \left(\mathbf{s}_i \mathbf{s}_j + \frac{3}{4}\right) \left[J_f \left(-2 + l_{ix}^2 + l_{jx}^2 + l_{iz}^2 l_{jy}^2 + l_{iy}^2 l_{jz}^2\right) + J_f' \left(l_{ix}^2 + l_{jx}^2 - 2l_{ix}^2 l_{jx}^2\right)\right].$$
(10)

The effective Hamiltonian for a pair along the y axis can be obtained by a permutation of indices $x \rightarrow y$ and $y \rightarrow x$. For a detailed discussion of the differences in spin-dependent factors of ferromagnetic and AFM exchange terms we refer to Ref. 22. Let us consider now a two-sublattice configuration in which each V ion of sublattice *i* is described by the wave function $|\vartheta\rangle = \cos \vartheta/2|1,1/2\rangle + \sin \vartheta/2|-1, -1/2\rangle$ with $\vartheta = \vartheta_i$ and is surrounded by four V ions of sublattice *j* with $\vartheta = \vartheta_j$.²³ Using the effective exchange operator $H_{ex} = \sum [H_{ex}(1) + H_{ex}(2_x) + H_{ex}(2_y)]$ and assuming that only the ground states of the surrounding V ions are populated, we arrive at the following energy spectrum of the vanadium ions:

$$\varepsilon_{1,2} = D + \frac{\lambda_c}{2} - \frac{J_a + J_{\text{int}}}{4} + \frac{3J'_f}{2} \\ \pm \frac{1}{4} [(J_a + J_{\text{int}} + 2J'_f)u^2 + (J_a - J'_f)^2 v^2]^{\frac{1}{2}}, \quad (11)$$

$$\varepsilon_{3,4} = -\frac{D}{2} - \frac{\lambda_c}{4} + \frac{J_a - J_{int}}{8}(u-1) + \frac{J_f}{4}(3-u) + \frac{3J'_f}{2}$$
$$\pm \frac{1}{2} \left[\left(3D - \frac{\lambda_c}{2} + \frac{J_a - J_{int}}{4}(u-1) - \frac{J_f}{2}(3-u) + J'_f u \right)^2 + 2\lambda_{a,b}^2 \right]^{\frac{1}{2}}, \qquad (12)$$

$$\varepsilon_{5,6} = -\frac{D}{2} - \frac{\lambda_c}{4} - \frac{J_a - J_{\text{int}}}{8}(u+1) + \frac{J_f}{4}(3+u) + \frac{3J'_f}{2}$$
$$\pm \frac{1}{2} \left[\left(3D - \frac{\lambda_c}{2} - \frac{J_a - J_{\text{int}}}{4}(u+1) - \frac{J_f}{2}(3+u) - J'_f u \right)^2 + 2\lambda_{a,b}^2 \right]^{\frac{1}{2}}.$$
 (13)

Here we have introduced $u = \cos \vartheta_j$ and $v = \sin \vartheta_j$. From the expressions for $\varepsilon_{3,4}$ and $\varepsilon_{5,6}$ one finds that at $\vartheta = \pm \pi/2$ the excited states are degenerate doublets, i.e., $\varepsilon_3 = \varepsilon_5$ and $\varepsilon_4 = \varepsilon_6$. However, this choice of $\vartheta = \pm \pi/2$ cannot explain the observed splitting of the highest-lying doublet $\varepsilon_{4,6}$ which was observed by neutron scattering and optical spectroscopy.^{13,19}

Now let us turn to the ground state ε_1 . A minimum in energy of this level will occur at $\vartheta = \pm \pi/2$ only if $3|J'_f| > J_{\text{int}}$. If $J_{\text{int}} > 3|J'_f|$ the minimum will occur at $\vartheta = 0$ or $\dot{\vartheta} = \pi$. At these angles the excited states are split due to the exchange-molecular field in agreement with experiment.¹³ Using the experimentally observed splitting¹³ of the highest doublet of about 10 meV we estimate the value $|J_f + J'_f| \simeq$ 10 meV. Following Imai and co-workers the energy cost for moving a 3d electron between V ions in Sr₂VO₄ is about $U \simeq$ 11 eV and the effective transfer integrals $t_{xz,xz} \simeq -0.2$ eV and $t_{yz,yz} \simeq 0.05$ meV.¹⁰ Therefore, we estimate $J_a \simeq 15$ meV, $J_{\text{int}} \simeq 7.5 \text{ meV}, J'_f = -(3B + C)/2U * J_a \simeq -0.7 \text{ meV}, \text{ and}$ $J_f \simeq -9.3$ meV. The values D = -33 meV, $\lambda_c = -30$ meV, and $\lambda_{ab} = -28$ meV are in agreement with conventional estimates.^{2,12} Using these values we plot the energy levels as $\varepsilon_i(\vartheta) - \varepsilon_1(\vartheta = 0, \pi)$ in Fig. 2(a) as a function of the orbitalmixing angle ϑ . Note that $\varepsilon_1(\vartheta)$ is not constant but becomes minimal for $\vartheta = 0$ and π . The estimated excitation energies $\varepsilon_4 - \varepsilon_1 = 121$ meV, $\varepsilon_6 - \varepsilon_1 = 111$ meV, and $\varepsilon_3 - \varepsilon_1 =$ 36 meV for $\vartheta = 0$ (or corresponding values for $\vartheta = \pi$)



FIG. 2. (Color online) (a) Calculated energy levels as a function of the orbital mixing angle ϑ using Eqs. (13)–(15) and parameters $\lambda_c =$ -30 meV, $\lambda_{ab} = -28$ meV, D = -33 meV, $J_a = -15$ meV, $J_f =$ -9.3 meV, $J'_f = -0.7$ meV, and $J_{int} = 7.5$ meV. Excitation energies observed by neutron scattering (Ref. 13) and optical spectroscopy (Ref. 19) are shown as dashed and dash-dotted lines, respectively. (b) Free energy as a function of ϑ calculated with the same parameters 10 K and 100 K.



FIG. 3. (Color online) Sketch of the proposed alternating spinorbital order in the *ab* plane of tetragonal Sr_2VO_4 . Short and long arrows correspond to spin and orbital moments of the V ions, respectively.

are in agreement with the transitions observed by optical (dash-dotted lines) and neutron scattering experiments (dashed lines) at low temperatures shown in Fig. 2(a).^{13,19}

In Fig. 2(b) we compare the free energy per vanadium site at 10 K and 100 K (close to T_N) as a function of the parameter ϑ using the values for the exchange constants estimated above:

$$F(T,\vartheta) = -k_B T \ln \sum \exp\left(-\frac{\varepsilon_i(\vartheta)}{k_B T}\right).$$
 (14)

There is a minimum at $\vartheta = 0$ and $\vartheta = \pm \pi$ in both cases and the energy difference with respect to $\vartheta = \pm \pi/2$ decreases from 1.5 to 0.5 meV, respectively. Hence, one can anticipate that with increasing temperature considerable fluctuations of ϑ can be expected. We estimate these fluctuations by $\overline{\Delta \vartheta^2} =$ $k_B T (\partial^2 F / \partial^2)^{-1} \simeq (\pi/5)^2$ and $(\pi/3)^2$ for 10 K and 100 K, respectively. Certainly, the proposed spin-orbital ordered state as depicted in Fig. 3 will be destabilized at high temperatures. However, the splitting of the highest-lying doublet remains almost unchanged across the Neel temperature,¹³ and it is reasonable to expect the exchange splitting of $\varepsilon_{3,5}$ to survive as well, even though the exchange parameters might be somewhat reduced. We would like to mention that the value $\varepsilon_5 - \varepsilon_1 =$ 31 meV corresponds nicely to the optical excitation observed for T > 80 K, but given the strong fluctuations expected for this temperature range the interval $(\varepsilon_3 + \varepsilon_5)/2 - (\varepsilon_2 + \varepsilon_5)/2$ $\varepsilon_1)/2 \sim 29$ meV might provide a more suitable estimation of the high-temperature optical excitation.

In summary, we calculated the level scheme for the energy levels of the vanadium ions and proposed an alternating spin-orbital ordering with almost muted magnetic moment as the ground state for Sr_2VO_4 . The proposed scenario and parameter values allow one to obtain a consistent picture of the low-temperature excitation spectrum of Sr_2VO_4 , which was recently reported by neutron and optical experiments.

We thank G. Jackeli, D. I. Khomskii, and H.-A. Krug von Nidda for stimulating discussions. This work is partially supported by the SNSF and the National Center of Competence in Research (NCCR) MaNEP and by the DFG via the Collaborative Research Center TRR 80 (Augsburg-Munich). MVE acknowledges support by the Ministry Education of the Russian Federation via Grant No. 1.83.11.

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