A bi-proportional method applied to the Spanish Congress

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Draft version 22 February 2008

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Abstract

A bi-proportional divisor method is applied to allocate the seats of the 2004 Spanish Congress, thus achieving proportionality relative to the population counts in the fifty-two provinces, as well as proportionality relative to the vote counts for the political parties. Also, advantages and disadvantages of the method are discussed.

Keywords: proportional allotment, bi-proportional allotment, BAZI

1 Introduction.

For the election of their national parliaments, many countries subdivide the grand electoral region into smaller electoral districts, and assign seats to parties separately in each district. The idea is to bring representatives nearer to the electors. However, experience shows that separate district apportionments generally do not entail an equitable overall representation of political parties. It may happen that the overall proportion of seats of a party significantly deviates from their overall proportion of votes, be it to their advantage or to their disadvantage. For example, this has been continuously happening to the IU-party in Spain, since their total number of deputies consistently falls short of their proportional share of votes. The likelihood of such biases increases when there are many electoral districts that only have a small number of seats to fill.

In order to evade this problem, some countries have taken recourse to a mixed-member proportional system [10]. For instance, Germany elects 299 members of the Bundestag in single-seat districts. Yet the overall apportionment of the 598 regular Bundestag seats turns out to be very proportional. Here, proportionality applies to those parties that turn out to be eligible to participate in the apportionment process, that is, their vote share amounts to at least five-percent of the overall ballot count. In the German system proportionality is achieved by first allocating the 598 seats in proportion to the parties' vote counts. The seats that a party thus obtains are filled with this party's direct-seat winners, while the remaining seats are filled from this party's candidate list. A related proposal for the Spanish congress based on the vote total and seat total is discussed in [7], with the additional twist to reward big parties in order to improve upon governability.

In 1989 Balinski and Demange [2] proposed a bi-proportional apportionment method permitting a subdivision into several electoral districts, while at the same time securing overall proportionality with respect to vote counts. In [3] they propose an algorithm to obtain this bi-proportional apportionment solution. Other algorithms have been proposed by Pukelsheim and Ramírez. A total of thirteen algorithms is implemented in the software Bazi [9] that is made freely available by the Augsburg research group.

Section 2 explains the general idea underlying bi-proportional apportionment methods. In Section 3, we apply the bi-proportional divisor method with rounding down (Jefferson/D'Hondt/Hagenbach-Bischoff) to the 2004 election of the Spanish Congress. In our sample calculation, all valid votes are considered eligible to participate in the apportionment process. The final Section 4 discusses some advantages and disadvantages of the bi-proportional technique.

2 The bi-proportional method.

Suppose that the electoral region is subdivided into electoral districts $i = 1, \dots, k$, with district magnitudes M_i. The district magnitude M_i signifies the number of seats to fill in district i. Furthermore we assume that there are parties $j = 1, \dots, \ell$ campaigning in the electoral region. We assume that they are allocated P_j seats, in proportion to their overall vote totals. Clearly, the sum of the district magnitudes must be equal to the sum of the overall party seats, $\sum_{i=1}^{k} M_i = \sum_{j=1}^{\ell} P_j = H$, where H is the house size of the national parliament. Let v_{ij} be the number of votes obtained in district i by party j. A typical display of the data is shown in Table 1.

Overall party seats / District magnitudes		1 / P_1	$\dots$	j / P_j	$\dots$	ℓ / P_ℓ
1	M_1	v_{11}-s_{11}	$\dots$	v_{1j}-s_{1j}	$\dots$	$v_{1\ell}$-$s_{1\ell}$
$\vdots$		$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
i	M_i	v_{i1}-s_{i1}	$\dots$	v_{ij}-s_{ij}	$\dots$	$v_{i\ell}$-$s_{i\ell}$
$\vdots$		$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
k	M_k	v_{k1}-s_{k1}	$\dots$	v_{kj}-s_{kj}	$\dots$	$v_{k\ell}$-$s_{k\ell}$

Table 1: Input data and output data. In district $i = 1, \dots, k$ with district magnitude M_i, party $j = 1, \dots, \ell$ with overall party seats P_j wins v_{ij} votes and gets s_{ij} seats.

The task is how to obtain the number of seats s_{ij}, to be allocated in district i to party j. Row sums and column sums are required to achieve the pre-specified district magnitudes and overall party seats. That is, the seat numbers in district i must add up to M_i, and the seat numbers for party j must add up to the overall party seats P_j.

The main issue is to determine the seat numbers s_{ij} in such a way that they turn out to be proportional, in some sense or other, to the vote counts v_{ij}. It may be tempting to scale the vote count v_{ij} by some common constant, and then round the resulting quotient to a neighboring integer. It turns out that this approach does not assure that the pre-specified row and column sums are met correctly. A single common constant for re-scaling the vote matrix is insufficient to achieve the desired goal.

Instead, the double proportional methods proposed by Balinski and Demange use two sets of constants, namely, district divisors and party divisors. The divisors are found by an iterative procedure. Once they are obtained, the seat apportionment matrix is most easily verified, by dividing the vote count v_{ij}, in district i of party j, by the corresponding district divisor and the corresponding party divisor. We take the space to demonstrate the approach by example.

A three-district example. For the sake of simplicity we consider just three Spanish provinces with their actual district magnitudes for the Spanish Congress, Huesca with 3 deputies, Teruel with 3 deputies, and Zaragoza with 7 deputies. In line with the 2004 election we assume that the 13 seats are shared between three parties: PSOE 6, PP 5, and CHA 2.

Overall party seats		PSOE	PP	CHA
District magnitudes		*6*	*5*	*2*
Huesca	*3*	61 500	50 493	8 629
Teruel	*3*	36 152	35 920	4 463
Zaragoza	*7*	224 776	198 480	81 160

Table 2: Votes in Aragón 2004. There are 13 seats to be apportioned, for a total of 701 513 votes. Thus 61 500 votes yield an initial weight of $13 \times 61\,500/701\,513 = 1.140$, for the first cell of Table 3.

Table 3 illustrates that individual rounding does not result in a valid seat apportionment. To this end we subdivide all vote counts by the vote total (701 573) and multiply by the seat total (13). This calculation results in a fractional number that must be rounded to an integer value before it can be interpreted as a number of seats. Table 3 applies standard rounding, wherein a fractional number gets rounded to the nearest integer. The resulting apportionment is infeasible. It sums to a total of 14 seats instead of 13. Huesca and Teruel each get only 2 seats instead of 3, while Zaragoza is assigned 10 seats instead of 7. The PP party is allocated 6 seats instead of 5.

Weights(0)				Roundings		
1.140	0.936	0.160		1	1	0
0.670	0.666	0.083		1	1	0
4.165	3.678	1.504		4	4	2

Table 3: Initial weights(0) and roundings from Table 2. Row sums fail to match the prespecified district magnitudes, and the second column sum fails to match the overall party seats.

Since scaling the weights with a single common constant turns out not to be feasible, we instead proceed somewhat more sensitively row by row. Thus the first row is scaled by 1.60, whence the new weights 1.823 : 1.497 : 0.256 round to 2 : 1 : 0 and achieve the pre-specified district magnitude, 3. The multiplier 1.60 is not unique; in fact, any number in the range from 1.32 to 1.60 would do. We choose the maximum multiplier whenever the weights have to be scaled up, and the minimum multiplier whenever they need to be scaled down. See Table 4.

Multipliers	Weights(1)				Roundings		
$\times 1.60 =$	1.823	1.497	0.256		2	1	0
$\times 2.25 =$	1.507	1.498	0.186		2	1	0
$\times 0.68 =$	2.832	2.501	1.023		3	3	1

Table 4: A first scaling. The rows of the weights(0) are scaled by the given multipliers, to obtain weights(1) and their roundings. Row sums match the district magnitudes, but column sums still miss the overall party seats.

The row-wise adjustments lead to an intermediate seat allocation obeying the pre-specified district magnitudes. But the overall party seats are not met: PSOE is awarded 7 seats instead of 6, and CHA gets 1 seat instead of 2. In order to correct the column sums, the weights(1) are column-wise re-scaled to obtain weights(2), and then rounded. The intermediate seat apportionment meets the overall party seats, but simultaneously creates a new imbalance between Teruel and Zaragoza. See Table 5.

Multipliers	$\times 0.89 =$	$\times 1 =$	$\times 1.95 =$				
	Weights(2)				Roundings		
	1.623	1.497	0.499		2	1	0
	1.341	1.498	0.364		1	1	0
	2.521	2.501	1.994		3	3	2

Table 5: A second scaling. The columns of the weights(1) are scaled by the given multipliers, to obtain weights(2) and their roundings. Column sums match the overall party seats, but the last two rows do not fit.

It transpires that another re-scaling of rows is needed to obtain the final result. By multiplying the second row by 1.1 and the third row by 0.995, we obtain the weights(3) shown in Table 6. The rounding of these weights does indeed meet the pre-specified district magnitudes, as well as the overall party seats. Hence the rounded numbers in Table 6 constitute the seat apportionment of the bi-proportional divisor method with standard rounding.

Multipliers	Weights(3)				Roundings		
	1.623	1.497	0.499		2	1	0
$\times 1.1 =$	1.476	1.647	0.399		1	2	0
$\times 0.995 =$	2.508	2.488	1.984		3	2	2

Table 6: A final scaling. Multiplying the last two rows of the weights(2) produces the weights(3). Since their roundings obey the prespecified marginals, they represent the end result.

In conclusion, the bi-proportional apportionment that goes along with the input data from Table 2 are displayed in Table 7 in a compact form documenting the input data as well as exhibiting the output data. The prespecified district magnitudes and overall party seats are printed in italics, to the left and at the top. The district divisors and the party divisors are also shown in italics, to the right and at the bottom. The divisors are not unique, but may vary in small intervals, as long as the resulting quotient rounds to the seat numbers given in Table 7. In contrast, the seat numbers themselves are uniquely determined: There is just one seat apportionment which can be obtained from the input vote counts by first re-scaling rows and columns and then rounding. Of course, we could have displayed Table 7 with showing multipliers instead of divisors. But in a practical problem, like the present one, large vote counts must be scaled down into small seat numbers. It is then more convenient to communicate divisors rather than multipliers.

It is remarkably simple to double check the seat apportionment displayed in Table 7. All a voter needs to do, is to sub-divide the success of his or her party in his or her district by the corresponding divisors and than round the resulting quotient to obtain the number of seats. For instance, those who vote for PSOE in Huesca find that their party has quotient $61\,500/(1.1 \times 37\,000) = 1.511$, and hence is allocated 2 seats.

Thus a bi-proportional apportionment method operates such that rows and columns are re-scaled to obtain corrected weights which, when rounded, exhaust the pre-specified district magnitudes and overall party seats. That this approach yields a unique solution (except for ties) is proved in [2]. Not

Overall party seats District magnitudes		PSOE *6*	PP *5*	CHA *2*	District divisors
Huesca	*3*	61 500-2	50 493-1	8 629-0	*37 000*
Teruel	*3*	36 152-1	35 920-2	4 463-0	*23 000*
Zaragoza	*7*	224 776-3	198 480-2	81 160-2	*80 000*
Party divisors		*1.1*	*1*	*0.6*	

Table 7: Biproportional apportionment, for the data from Table 2. With the divisors displayed in the margins, the apportionment is easy to verify. For instance, PSOE in Huesca gets 2 seats since $61\,500/(1.1 \times 37\,000) = 1.511$.

surprisingly, the approach also works when standard rounding is re-placed by rounding down (Jefferson/D'Hondt/Hagenbach-Bischoff), or by rounding up (Adams). In the Swiss Canton of Zurich the bi-proportional method with standard rounding was made part of the electoral law, and has been successfully applied in the City of Zurich in 2006, and in the Canton of Zurich in 2007 [8]. The Swiss Cantons of Aargau and Schaffhausen have adopted initiatives to also incorporate the method into their electoral laws. An application of the bi-proportional method to Mexican elections is discussed in [4, 5, 6], to Italien elections in [11], to elections in the Färöer Islands in [12].

3 The 2004 election to the Spanish Congress.

We now apply the bi-proportional divisor method with rounding down (Jefferson/D'Hondt/Hagenbach-Bischoff) to the election of the Spanish Congress in March 2004. There are 52 electoral districts (circumscriptions), the 50 provinces plus the autonomic cities of Ceuta and Melilla. The Spanish Congress comprises 350 seats. The district magnitudes used are those from the 2004 election.

The first step is to obtain the overall party seats, across all of Spain, irrespective of the subdivision into the electoral districts. Table 8 shows the 20 parties that each received more than 30 000 nationwide votes, and their vote counts. The apportionment of the 350 congressional seats proportionally to these vote counts results in the columns labelled

- "Sainte-Laguë" when the divisor method with standard rounding (Webster/Sainte-Laguë) is used,

- "D'Hondt" when the divisor method with rounding down (Jefferson/ D'Hondt/Hagenbach-Bischoff) is used, and
- "Current" when the apportionment from the 2004 electoral law is used.

The total number of all valid votes, including those of over seventy minor parties not listed in Table 8, was 25 483 504. Note that party PAR with 36 540 votes represents less than 0.2 percent of all valid votes. On the other hand, a nationwide five percent threshold as in Germany would exclude all parties with fewer than 1 274 176 votes, and thus leave only the first three top runners. Rather than entering into a discussion which apportionment would result from a five, four, three, etc. percent threshold, we rely in the sequel on the divisor method with rounding down (Jefferson/D'Hondt/Hagenbach-Bischoff) which comes closest to the current results that are entailed by the pertinent electoral law. For the sake of demonstration, we then also use this method for the bi-proportional calculations.

In Table 8 we can compare the total number of seats the parties receive under D'Hondt with the number of seats they currently have. In any case, the PSOE is the party that obtains the plurality of seats. In order to build an absolute majority, it would need the support of other parties. In the current allotment, the support of any two parties amongst IU, CiU, ERC or PNV produce a majority. With D'Hondt, the support of IU would suffice.

The D'Hondt apportionment, though close to the current apportionment, is seen to be much more concordant with the actual vote counts. IU wins about twice as many votes as ERC, and is allocated twice as many seats. In contrast, the current allocation gives IU fewer seats than ERC. PA wins twice as many votes as CHA and D'Hondt allocates two seats as compared to one. The current law denies PA any representation in Congress, yet rewards CHA with one seat.

The bi-proportional divisor method with rounding down yields the seat numbers as shown in Table 9. For this data set, with 52 rows and 11 columns, five row scalings and four column scalings are needed to obtain the result.

Party	Votes	Sainte-Laguë	D'Hondt	Current
PSOE	11 026 163	152	158	164
PP	9 763 144	135	139	148
IU	1 284 081	18	18	5
CiU	835 471	12	11	10
ERC	652 196	9	9	8
PNV	420 980	6	6	7
CC	235 221	3	3	3
BNG	208 688	3	2	2
PA	181 868	3	2	0
CHA	94 252	1	1	1
EA	80 905	1	1	1
NA-BAI	61 045	1	0	1
EV	40 759	1	0	0
PSM...	40 289	1	0	0
CENB	40 208	1	0	0
ARALAR	38 560	1	0	0
LV-E	37 499	1	0	0
PAR	36 540	1	0	0
CDS	34 101	0	0	0
EV-AE	30 528	0	0	0
Total	25 142 498	350	350	350
Divisors		*72 400*	*69 760*	

Table 8: Seat apportionments for the 2004 Spanish Congress, based on the nationwide vote totals. For the Sainte-Laguë apportionment, the votes are divided by 72 400 and the resulting quotients are rounded in a standard fashion to obtain the seat numbers shown. For the D'Hondt apportionment, the divisor is 69 760 and all quotients get truncated to their integer parts. The D'Hondt method comes closest to the current electoral law, whence this method is also used in the biproportional apportionment in Table 9.

Province/City	**350**	**PSOE-158**	**PP-139**	**IU-18**	**Others-35**	Dist.div.
A Coruña	9	287 324-4	329 389-4	14 125-0	[5]86 459-1	*70 000*
Álava	4	561 374-1	48 992-1	14 181-1	[3]47 090-1; [8]7 838-0	*27 880*
Albacete	4	108 715-2	110 338-2	9 145-0		*40 000*
Alicante	11	374 631-5	434 812-6	34 774-0		*68 500*
Almería	5	145 868-3	135 434-2	9 522-0	[6]7 190-0	*45 000*
Asturias	8	305 240-4	307 977-3	59 253-1		*74 000*
Ávila	3	38 640-1	67 622-2	3 598-0		*30 000*
Badajoz	6	219 172-3	176 699-3	16 589-0		*55 000*
Barcelona	31	1 268 028-12	485 504-4	198 116-4	[1]586 854-6; [2]428 986-5	*97 000*
Burgos	4	91 727-2	122 415-2	7 703-0		*40 000*
Cáceres	4	137 654-2	118 627-2	7 569-0		*50 000*
Cádiz	9	326 152-4	216 416-3	38 611-1	[6]33 592-1	*65 000*
Cantabria	5	149 906-2	190 383-3	12 146-0		*50 000*
Castellón	5	139 236-3	142 462-2	10 322-0		*46 000*
Ceuta	1	12 769-0	21 142-1	218-0		*20 000*
Ciudad Real	5	147 271-3	142 508-2	8 581-0		*47 000*
Córdoba	7	246 324-4	166 665-2	47 908-1	[6]19 648-0	*60 000*
Cuenca	3	60 697-1	66 515-2	3 258-0		*31 000*
Girona	6	113 089-2	40 959-0	15 070-0	[1]96 928-2; [2]83 482-2	*40 000*
Granada	7	268 870-4	193 484-2	31 227-1	[6]14 030-0	*61 200*
Guadalajara	3	52 915-1	57 078-2	5 310-0		*26 600*
Guipúzcoa	6	98 100-1	56904-1	28 668-1	[3]115 402-2; [8]42 971-1	*50 000*
Huelva	5	154 579-3	84 173-2	15 097-0	[6]14 542-0	*39 000*
Huesca	3	61 500-2	50 493-1	3 650-0	[7]8 629-0	*30 000*
I. Baleares	8	185 623-4	215 737-4			*43 000*
Jaén	6	228 611-4	143 288-2	24 483-0	[6]15 493-0	*50 000*
La Rioja	4	81 390-2	92 441-2	5 115-0		*30 000*
Las Palmas	8	167 926-3	208 995-4	9 876-0	[4]89 420-1	*46 000*
León	5	156 786-3	150 688-2	7 160-0		*50 000*
Lleida	4	68 971-1	34 116-0	6 910-0	[1]68 735-2; [2]50 104-1	*35 000*
Lugo	4	92 708-2	123 986-2	2 570-0	[5]25 313-0	*40 000*
Madrid	35	1 544 676-16	1 576 636-15	225 109-4		*95 000*
Málaga	10	367 758-5	269 063-4	47 182-1	[6]32 368-0	*63 000*
Melilla	1	11 273-0	14 856-1	229-0		*13 000*
Murcia	9	252 246-3	413 902-6	30 787-0		*64 000*
Navarra	5	113 906-2	127 653-3	19 899-0		*40 000*
Ourense	4	74 636-1	132 631-3	2 055-0	[5]26 153-0	*40 000*
Palencia	3	51 824-1	60 449-2	3 415-0		*27 000*
Pontevedra	7	228 016-3	279 454-3	13 158-0	[5]70 763-1	*70 000*
Salamanca	4	94 655-2	128 932-2	4 713-0		*44 000*
Sta. C. Tenerife	7	165 158-3	133 677-2	8 736-0	[4]145 801-2	*50 000*
Segovia	3	39 976-1	52 500-2	3 470-0		*20 000*
Sevilla	12	639 293-7	306 464-3	73 344-1	[6]45 005-1	*80 000*
Soria	3	22 287-1	29 187-2	1 230-0		*12 000*
Tarragona	6	136 660-3	65 528-1	14 694-0	[1]82 954-1; [2]76 330-1	*45 000*
Teruel	3	36 152-2	35 920-1	2 514-0	[7]4 463-0	*17 000*
Toledo	5	167 807-3	171 325-2	12 707-0		*55 000*
Valencia	16	613 833-7	665 526-8	78 515-1		*78 000*
Valladolid	5	155 401-3	163 009-2	13 029-0		*51 420*
Vizcaya	9	185 514-3	129 889-2	59 493-1	[3]258 488-3; [8]30 096-0	*60 000*
Zamora	3	53 757-1	71 821-2	3 375-0		*30 000*
Zaragoza	7	224 776-3	198 480-3	15 672-0	[7]81 160-1	*60 000*
Party divisors		*1.007*	*1.05705*	*0.5084*		

Table 9: The biproportional divisor method with rounding down (Jefferson/D'Hondt/Hagenbach-Bischoff), applied to the March 2004 election of the Spanish Congress. To obtain the seats for a party in a Province, the votes are divided by the corresponding party and district divisors, and the resulting quotient is truncated to its integer part. The column "Others" refers to the smaller parties as enumerated in the main text.

The three largest parties, PSOE, PP, and IU, campaign in all districts. It so happens that no district features more than two other parties. These other parties, whose national totals range from about 800 000 (CiU) down to 80 000 (EA), are compactly displayed in just one column where a superscript number indicate their identity, as follows:

1. CiU in Catalunya (Barcelona, Girona, Lleida and Tarragona) has party divisor 0.97;

2. ERC in Catalunya has party divisor 0.87;

3. PNV in the Basque Country (Álava, Guipúzcoa and Vizcaya) has party divisor 1.1;

4. CC on the Canary Islands has party divisor 1.2;

5. BNG in Galicia (A Coruña, Lugo, Ourense and Pontevedra) has party divisor 1;

6. PA in Andalucía (Almería, Cádiz, Córdoba, Granada, Huelva, Jaén, Málaga and Sevilla) has party divisor 0.515;

7. CHA in Aragón (Huesca, Teruel and Zaragoza) has party divisor 1; and

8. EA in the Basque Country has party divisor 0.7.

A peculiar effect of any bi-proportional apportionment is the possible occurrence of *discordant seat assignments*. By definition, we speak of a discordant seat assignment in two cells of Table 9 whenever one cell features more votes but fewer seats than the other. This is particularly irritating within the same district. In the current system seats are assigned just within that district and with no regard to the rest of the nation, which makes discordant seat assignments within districts impossible. (But the current system has to pay its price. Securing more homogeneity within districts aggravates the heterogeneity between districts, and on a national level.)

Here are some examples. In Asturias, PSOE is weaker than PP (305 240 : 307 977), but wins more seats (4 : 3). Other discordant seat assignments between PSOE and PP occur in Castellón, Madrid, Toledo, and Valadolid. Within the PSOE party, a discordant seat assignment occurs between A Coruña and Álava (287 324 : 561 374 votes versus 4 : 1 seats).

Such frictions are unavoidable since a bi-proportional apportionment mediates between two goals that, at times, are conflicting. One goal is to achieve proportionality as pre-specified by the district magnitudes, the other, proportionality as pre-specified by the overall party seats. Since the turn-out in the 52 districts is not identical, and hence creates different proportional weightings than those based on the population and the district magnitudes, global balance cannot be achieved without local adjustments. In fact, it is this global view that represents the distinguished meritorious feature of a bi-proportional method. The method achieves proportionality among electoral districts, relative to population counts, as well as proportionality among parties, relative to vote counts. As such it brings about a "nationalization" of the electoral process for the major national institution, the Congress.

4 Conclusions.

The bi-proportional method is a new recent technique to solve a proportional representation problem that comes in a rectangular table of data, imposing restrictions in the direction of rows as well as in the direction of columns. For the application to political elections, the table is made up of the vote counts that various political parties receive in a number of electoral districts. The restrictions are the district magnitudes, and the overall party seats.

Usually the district magnitudes are determined in the middle of a legislative period, proportionally to the populations census data supplied by the statistical offices. In contrast, the overall party seats are calculated at the eve of election day, proportionally to the nationwide vote totals of the parties. This "super-apportionment" honors the popular vote irrespective of the subdivision into various districts. This guarantees that all voters contribute to the final result in an equal manner, without being advantaged or disadvantaged when casting their vote in a small, rural district or in a large, municipal district.

The bi-proportional method then proceeds to a sub-apportionment to obtain the number of seats of parties per districts. The principle of proportional representation persists, in that the results within a district are scaled by a common factor, the district divisor, as well as that the results within a party are scaled by a common divisor, the party divisor. However, since the method serves two goals, as dictated by district magnitudes and overall party seats, the interaction of the two sets of divisors is occasionally counter-intuitive.

The high degree of proportionality that is thus achieved on a national level would suggest to introduce a threshold of a minimum vote percentage before a party becomes eligible to participate in the apportionment process. Otherwise, since all votes contribute towards the final result, parties are well advised to adopt strategies of presenting themselves in all electoral districts. Therefore a bi-proportional method induces a "nationalization" of the election of a national political body such as the Congress.

A computer is needed to calculate a bi-proportional seat apportionment. However, once the final result is made public, verification is much easier than it used to be with the old system. All a voter has to do is to take the vote count of his or her party in his or her district, and divide it by the divisors that are published with the final apportionment. The increased transparency for the individual citizen goes along well with the increased equality on a national level [13].

Acknowledgements.

Authors wish to thank the Spanish Ministry of Education and Science and the FEDER for co-financing the Project SEC2001-3117. We also would like to thank the Andalusia Government for supporting the research group FQM191 and the Project FQM-01969, that permits to pay the expenses of our research on proportional representation and social election.

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