Optimal strategy for controlling transport in inertial Brownian motors

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ADDENDUM AND ERRATUM

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Abstract
The expression for the effective diffusion of an inertial, periodically driven Brownian particle in an asymmetric, periodic potential is compared with the step number diffusion which is extracted from the corresponding coarse grained hopping process specifying the number of covered spatial periods within each temporal period. The two expressions are typically different and involve the correlations between the number of hops.

The expression used for the diffusion constant D_{eff} in equation (12) in [1], which will be denoted by D_N in what follows, coincides with the definition in equation (5) in [1] only under certain conditions; see figure 1. In order to understand the relation between these expressions we split the random distance $x(nT)$, which the particle has covered after n periods of duration T, into its integer multiple $N(nT)$ of spatial periods of length L and a remainder $\epsilon(nT)$:

$$x(nT) = N(nT)L + \epsilon(nT). \tag{A1}$$

Note that $\epsilon(nT)$ is non-negative and is bounded by L. From the definition (5) we then obtain

$$\begin{aligned} D_{\text{eff}} &= \lim_{n\to\infty} \left\{ \frac{L^2\langle(\delta N(nT))^2\rangle}{2nT} + \frac{L\langle\delta N(nT)\delta\epsilon(nT)\rangle}{nT} + \frac{\langle(\delta\epsilon(nT))^2\rangle}{2nT} \right\} \\ &= \lim_{n\to\infty} \frac{L^2\langle(\delta N(nT))^2\rangle}{2nT}. \end{aligned} \tag{A2}$$

Each fluctuation $\delta N(nT) = N(nT) - \langle N(nT)\rangle$ contributes to the averages with a factor growing as $n^{1/2}$ such that only the first term on the right-hand side of the first equation contributes in the limit $n \to \infty$. Next, we represent the number $N(nT)$ of spatial periods

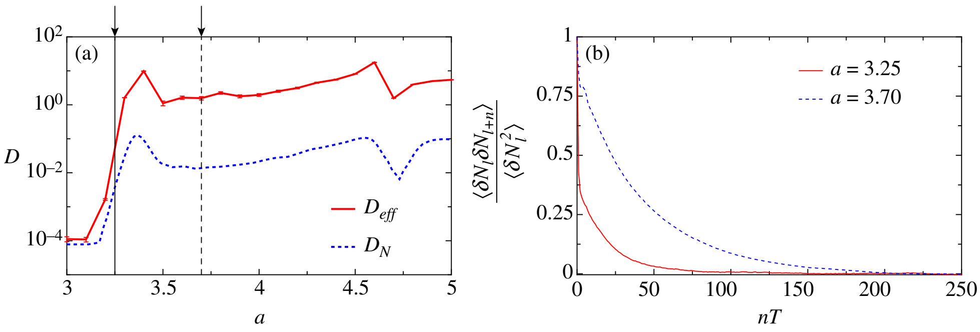

Figure 1. The step number diffusion D_N and the effective diffusion $D_{\rm eff}$ are depicted in panel (a) as functions of the driving amplitude a on a logarithmic scale for the driving frequency $\omega = 4.9$, noise strength $D = 0.001$ and potential parameters $c_1 = 0.425$ and $c_2 = 0.04$. In the locked regime for $a < 3.1$ both diffusion constants are comparably small. In the running regime for $a > 3.1$ both diffusion constants are large, with $D_{\rm eff}$ becoming larger than D_N roughly by a factor of ten. The normalized correlations $\langle \delta N_{l+n} \delta N_l \rangle / \langle (\delta N_l)^2 \rangle$ are depicted in panel (b) for two different values of a, which are marked by arrows in panel (a). They extend over many periods T, leading to the observed discrepancy of the two diffusion constants. For $a = 3.25$ the decay of the correlations is much faster than for $a = 3.7$. Accordingly the difference $D_{\rm eff} - D_N$ is more pronounced at the larger a value.

(This figure is in colour only in the electronic version)

covered within n temporal periods T as the sum of the number N_k of spatial periods which the particle passes through within the kth temporal period of the driving force, i.e.

$$N(nT) = \sum_{k=1}^{n} N_k. \tag{A3}$$

From equation (A2) we then obtain

$$D_{\rm eff} = \lim_{n\to\infty} \frac{L^2 \sum_{k,l}^{n} \langle \delta N_k \delta N_l \rangle}{2nT} = D_N + \lim_{n\to\infty} \frac{L^2 \sum_{k,l,k\neq l}^{n} \langle \delta N_k \delta N_l \rangle}{2nT} \tag{A4}$$

where

$$D_N = \lim_{n\to\infty} \frac{L^2 \sum_{k}^{n} \langle (\delta N_k)^2 \rangle}{2nT} = \frac{L^2 \langle (\delta N_k)^2 \rangle}{2T} \tag{A5}$$

is the quantity that was used in equation (12) in [1]. Here we took into account that in the limit $n \to \infty$ the increments δN_k become stationary and the variances $\langle (\delta N_k)^2 \rangle$ are independent of k. The difference between $D_{\rm eff}$ and D_N results from the sum over the correlations between the increments δN_k. In the limit $n \to \infty$ the double sum is dominated by terms with large values of k and l for which the correlations $\langle \delta N_k \delta N_l \rangle$ only depend on the difference $k - l$. If the correlations decay more quickly than $(k - l)^{-2}$ the limit can be simplified to read

$$D_{\rm eff} - D_N = \lim_{n\to\infty} \frac{L^2 \sum_{k,l,k\neq l}^{n} \langle \delta N_k \delta N_l \rangle}{2nT} = \frac{L^2}{T} \sum_{m=1}^{\infty} \langle \delta N_{k+m} \delta N_k \rangle. \tag{A6}$$

In principle, this sum may take positive as well as negative values. In figure 1 we display the dependence of $D_{\rm eff}$ and D_N on the amplitude of the driving force and illustrate the correlation functions $\langle \delta N_{k+m} \delta N_k \rangle$ in two selected cases.

References

[1] Machura L, Kostur M, Marchesoni F, Talkner P, Hänggi P and Łuczka J 2005 *J. Phys.: Condens. Matter* **17** S3741–52