Observability of the Coulomb Blockade in Single Tunnel Junctions

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The finite temperature I–V–characteristic of a normal tunnel junction coupled to its electromagnetic environment is calculated. It is shown that due to quantum fluctuations of the charge the Coulomb gap is only observable provided the impedance $Z(\omega)$ of the environment exceeds the resistance quantum $R_Q = h/2e^2$. The observability of the Coulomb blockade in single junctions is also severely limited by finite temperature effects. Apart from general results for an environment with arbitrary impedance, detailed predictions are presented for a case of practical interest where the leads attached to the junction may be represented through a series inductance and a shunt capacitance.

Introduction

The state of the art of microfabrication techniques and cryogenics makes it possible to investigate ultrasmall tunnel junctions with capacitance C such that the charging energy of a single electron $e^2/2C$ exceeds the characteristic energy k_BT of thermal fluctuations. Simple energy considerations suggest that the tunneling of a single electron is completely blocked when the junction capacitor holds a charge less than $e/2$ [1]. This Coulomb blockade of tunneling is clearly seen in multijunction configurations while the existence of elementary charging effects in single tunnel junctions is still questionable.

Recently [2] we have shown that the quantum mechanical nature of the electromagnetic environment can severely reduce Coulomb charging effects in single junctions. For a tunneling electron to change effectively the charge on the junction capacitor and thus lead to the Coulomb blockade effect it has to excite electromagnetic modes of the coupled system formed by the junction and its electromagnetic environment. Since the energy $\hbar\omega$ of these modes is quantized, they will not be excited unless the voltage V across the junction reaches $\hbar\omega/e$. The junction couples more strongly to low frequency modes when it is placed in a high impedance environment and charging effects will only then be observable.

The I–V–Characteristic and the Coulomb Gap

Our treatment of electron tunneling in a normal junction imbedded in an electrical circuit starts from the tunneling Hamiltonian

$$H_T = \sum_{\sigma k q} T_{kq} c^{\dagger}_{k\sigma} c_{q\sigma} \Lambda_e + h.c.$$

where $T_{kq} c^{\dagger}_{k\sigma} c_{q\sigma}$ is the usual tunneling term transfering an electron from one side of the junction to the other and where Λ_e is an operator changing the charge Q on the capacitor plates of the junction: $\Lambda_e Q \Lambda_e^{\dagger} = Q - e$. Using this model, we find [2] that the tunneling current may be written

$$I = \frac{1}{eR_T} \int_{-\infty}^{+\infty} dE\, E \frac{1 - e^{-\beta eV}}{1 - e^{-\beta E}} P(eV - E).$$

where $1/R_T$ is the tunneling conductance. The novel feature here is the appearance of the function

$$P(E) = (2\pi\hbar)^{-1} \int_{-\infty}^{+\infty} dt \exp\left[J(t) + iEt/\hbar\right]$$

which gives the probability that a tunneling electron creates an excitation with energy E of the electromagnetic environment. In the conventional treatment where the coupling to the environment is disregarded one has $P(E) = \delta(E)$ and $I(V)$ reduces to the well-known Ohmic characteristic $I = V/R_T$. The function $J(t)$ is intimately connected with the spontaneous charge fluctuations on the junction capacitor arising from the coupling to the electromagnetic environment. For an environment with impedance $Z(\omega)$ one has

$$J(t) = \int_0^{\infty} \frac{d\omega}{\omega} \frac{\mathrm{Re} Z_t(\omega)}{R_Q} \left\{\coth\left(\frac{1}{2}\beta\hbar\omega\right) \left[\cos(\omega t) - 1\right] - i \sin(\omega t)\right\}$$

where

$$Z_t(\omega) = \frac{1}{i\omega C + Z^{-1}(\omega)}$$

is the total impedance of the junction in parallel with the environmental impedance. These formulae allow for the determination of the I–V–characteristic for arbitrary frequency dependence of the electromagnetic environment at finite temperatures.

Let us extract the I–V–characteristic at large voltages. Using $J(t) = -(i\pi/2C)t - (e/\hbar C)^2(\delta Q^2)t^2$ for $t \to 0$, we find $I = R_T^{-1}(V - e/2C)$ for $eV \gg k_BT$, $e^2/2C$. This describes an offset of magnitude $e/2C$, the so-called Coulomb gap. For low voltages the behavior of the I-V-characteristic depends on the spectrum of charge fluctuations at frequencies below $eV/\hbar$. Only when the impedance $\mathrm{Re}[Z_t(\omega)]$ in this frequency range exceeds the quantum resistance will the offset still be noticeable.

A Simple Model

As a concrete example let us consider a tunnel junction connected to leads that may be represented by a series inductance L and a shunt capacitance C_s. The practical observation of Coulomb charging effects requires junction capacitances less than a few fF and the leads attached to such a junction will easily produce shunt capacitances that are several orders of magnitude larger, i.e. $C_s \gg C$. The change of the charge Q_s on the shunt capacitor caused by a tunneling event is therefore entirely negligible and the current-biased junction can thus effectively be replaced by a voltage-biased junction. For this circuit, we have

$$J(t) = \rho\left\{\coth\left(\frac{1}{2}\beta\hbar\omega_L\right)[\cos(\omega_L t) - 1] - i\sin(\omega_L t)\right\}$$

where $\omega_L = (LC)^{-1/2}$ is the oscillation frequency of the environmental mode and $\rho = \pi/2CR_Q\omega_L$ is the ratio of the single electron charging energy $e^2/2C$ and the mode excitation energy $\hbar\omega_L$. This leads to an I–V–characteristic of the form [2]

$$I = (1/eR_T)\exp\left[-\rho\coth\left(\frac{1}{2}\beta\hbar\omega_L\right)\right]$$

$$\sum_{n=-\infty}^{+\infty} \epsilon_n \frac{\sinh\left(\frac{1}{2}\beta eV\right)}{\sinh\left(\frac{1}{2}\beta\epsilon_n\right)} I_n\left(\rho/\sinh\left(\frac{1}{2}\beta\hbar\omega_L\right)\right)$$

where $\epsilon_n = eV - n\hbar\omega_L$ and $I_n(x)$ is the modified Bessel function. Each term corresponds to a tunneling channel where the electron creates or absorbs n quanta of the environmental mode. At $T = 0$, the differential conductance dI/dV displays a series of steps at voltages $V_n = n\hbar\omega_L/e$ (Fig. 1a). Usually, for low voltages, the I–V–characteristic will be dominated by the elastic channel ($n = 0$). Inelastic processes are negligible when the mode energy $\hbar\omega_L$ exceeds the single electron charging energy, that is for $\rho \ll 1$. This will mostly be the case since typical series inductances will be well below $(\hbar^2/e^4)C$ even for ultrasmall junctions. At least

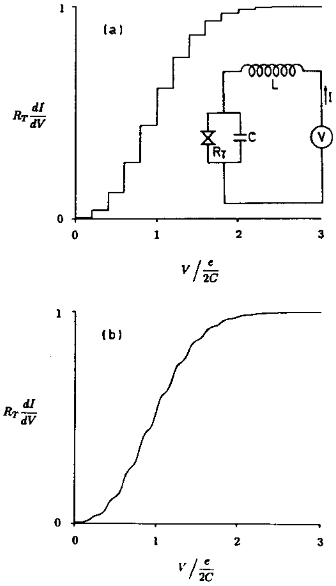

Figure 1. The differential conductance of the circuit shown in the inset with $\rho = 5$ for (a) $T = 0$ and (b) $k_BT = e^2/100C$

for low voltages the Coulomb charging effects are then suppressed. Finite temperatures make the deviation from Ohmic behavior even less pronounced (Fig. 1b).

Conclusions

By a quantum mechanical treatment of the electromagnetic environment of a tunnel junction we have calculated the I-V-characteristic as a function of the environment impedance $Z(\omega)$. We have shown that the junction capacitance C will be revealed in the I-V-characteristic as a voltage offset $e/2C$ only if the impedance of the environment $Z(\omega)$ exceeds the resistance quantum R_Q. We thus predict that in experimental setups designed to provide a high impedance environment for a single junction, the Coulomb gap should be observable.

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References

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