SLUGGISH DECAY OF PREPARATION EFFECTS IN LOW TEMPERATURE QUANTUM SYSTEMS

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1. Introduction

Dissipation is a widespread phenomenon in physics as well as in neighboring sciences. It arises from the coupling of the system under investigation to an environment with a large number of degrees of freedom. The familiar approach to discuss dissipative quantum systems, as e.g. in quantum optics or magnetic resonance, is based on master equations and quantum Langevin equations. However, in the last few years the interest has focussed on systems where the coupling to the heat bath may be strong or where the temperature may be very low as e.g. in superconducting devices. The familiar methods fail in these cases. It turned out that the functional integral representation of quantum mechanics provides an appropriate tool to describe such systems.

In the following we will discuss the effects of the preparation of the initial state on the dynamics of the system. Using a functional integral technique which allows for the description of a very large class of initial states we will discuss the time evolution of an exactly solvable model.

As an example we consider squeezed states which are of great interest in connection with quantum measurements near the quantum limit of resolution. It will turn out that there can be qualitative differences between the evolution of a factorizing initial state which neglects correlations between system and heat bath and an initial state where such correlations are present as it is usually the case. Then the relaxation may show algebraic long time tails which are not present for factorizing initial states and which cannot be obtained within a weak coupling theory.

2. The Model

A widely used model for dissipative systems represents the heat bath by harmonic oscillators which are linearly coupled to the system coordinate [1, 2]. The corresponding Hamiltonian

$$H = H_S + H_R + H_{SR} \tag{1}$$

consists of three parts describing the system

$$H_S = \frac{p^2}{2M} + V(q,t), \tag{2}$$

the heat bath or reservoir

$$H_R = \sum_{n=1}^{\infty} \left(\frac{p_n^2}{2m_n} + \frac{1}{2} m_n \omega_n^2 x_n^2 \right), \tag{3}$$

and the coupling

$$H_{SR} = -q \sum_{n=1}^{\infty} c_n x_n + q^2 \sum_{n=1}^{\infty} \frac{c_n^2}{2m_n \omega_n^2}. \tag{4}$$

Here we introduced the coordinate q and momentum p of the system, the coordinate x_n, momentum p_n and frequency ω_n of the n-th bath oscillator, and the coupling constant c_n between system and reservoir. The system is subject to an external potential $V(q,t)$ which may be time dependent for $t > 0$. This model is simple enough to be mathematically tractable. Furthermore it can often be used to describe even situations where the coupling between system and heat bath is nonlinear. In this case the model becomes exact if the perturbation of a single bath degree of freedom by the system is only weak and can be treated within a linear response theory. It should be emphasized that due to the large number of degrees of freedom, linear response of the bath does not imply a restriction to weak damping.

In general, one is not interested in the details of the behavior of the heat bath. If one integrates out these degrees of freedom one finds that the relevant quantity describing the heat bath is the spectral density of bath oscillators

$$I(\omega) = \pi \sum_{n=1}^{\infty} \frac{c_n^2}{2m_n \omega_n} \delta(\omega - \omega_n) \tag{5}$$

which is determined by the density of bath modes and the coupling strength between the system and the bath modes. For the reduced classical dynamics of the system one obtains the equation of motion

$$M\ddot{q} + M \int_0^t ds \gamma(t-s) \dot{q}(s) + \frac{\partial V(q,t)}{\partial q} = \xi(t) \tag{6}$$

where $\xi(t)$ is a noise term and the damping kernel

$$\gamma(t) = \frac{2}{M} \int_0^{\infty} \frac{d\omega}{\pi} \frac{I(\omega)}{\omega} \cos(\omega s) \tag{7}$$

is uniquely determined by the spectral density $I(\omega)$. This connection between the Hamiltonian (1) and the classical equation of motion (6) allows us to model a dissipative quantum system provided we know the phenomenological classical equation of motion of the system.

While the damping kernel $\gamma(t)$ in general describes damping with memory, it also contains the special case of memoryless or so-called Ohmic damping which is obtained for

$$I(\omega) = M\gamma\omega \tag{8}$$

which corresponds to

$$\gamma(t) = 2\gamma\delta(t). \tag{9}$$

In a real system the spectral density will not diverge as $\omega \to \infty$ but there will be a high frequency cutoff. We call a heat bath Ohmic if for low frequencies the spectral density is proportional to ω because it is the low frequency behavior which dominates the dynamics of the system.

3. Initial States

The dynamics of the system is governed by the Hamiltonian (1). Before calculating the time evolution explicitly, we have to discuss the preparation of an initial state. For $t < 0$ we assume that the total system consisting of the system degree of freedom and the heat bath be in thermal equilibrium at temperature $k_BT = 1/\beta$. It is then described by the density matrix

$$W_\beta = Z_\beta^{-1} \exp(-\beta H) \tag{10}$$

where Z_β is the partition function. At $t = 0$ we prepare the initial state by applying operators $\mathcal{O}_j$ and $\mathcal{O}'_j$ to W_β which act only in the Hilbert space of the system [2] according to

$$W_0 = \sum_j \mathcal{O}_j W_\beta \mathcal{O}'_j. \tag{11}$$

In coordinate representation the initial state is then given by

$$W_0(q, x_n, q', x'_n) = \int d\bar{q} \int d\bar{q}' \, \lambda(q, \bar{q}, q', \bar{q}') \, W_\beta(\bar{q}, x_n, \bar{q}', x'_n) \tag{12}$$

with the preparation function

$$\lambda(q, \bar{q}, q', \bar{q}') = \sum_j \langle q|\mathcal{O}_j|\bar{q}\rangle\langle\bar{q}'|\mathcal{O}'_j|q'\rangle. \tag{13}$$

For $t > 0$ the state propagates according to

$$W(t) = \exp(-\frac{i}{\hbar}Ht)W_0 \exp(\frac{i}{\hbar}Ht). \tag{14}$$

From the definition (11) of the initial state it is clear that in general it contains correlations between the system and the heat bath in contrast to the factorizing initial state [3]

$$W_F(q, x_n, q', x'_n) = \rho_0(q, q') \cdot W_R(x_n, x'_n) \tag{15}$$

where the system is in an arbitrary state ρ_0 and the heat bath is in thermal equilibrium. Such states can also be described within the framework of our more general approach and we can thus compare the dynamics of factorizing and nonfactorizing initial states. We note that although factorizing states are often used because the theory becomes simpler from a mathematical point of view [3, 4], this assumption is in most cases somewhat unrealistic since the heat bath usually cannot be decoupled from the system.

Let us now discuss a few examples which are contained in the class of initial states (11). First we can replace the operators $\mathcal{O}_j$ and $\mathcal{O}'_j$ by the identity. Obviously, we then get the equilibrium density matrix as initial state for which we may calculate the time evolution under a time dependent force for $t > 0$. An extension would be to interpret W_β as a constrained equilibrium state under a constant external force F and to discuss the relaxation into the unconstrained equilibrium with $F = 0$. Another possibility is to use projection operators for $\mathcal{O}_j$ and $\mathcal{O}'_j$. We then could make a position measurement and by choosing an appropriate weight function we could construct a wave packet. Below we will discuss the dynamics of a state where the density matrix was projected on a squeezed state. As a last example we mention the state BW_β which in general is not a proper density matrix. If B operates in the Hilbert space of the system only we still can calculate the dynamics and from the expectation value of another system operator A we are able to determine the equilibrium correlation function $\langle A(t)B\rangle_\beta$ which is connected to physically measurable quantities.

4. The Dynamics

We do not want to present the elimination of the heat bath within the functional integral formalism. Rather we only mention the results for the dynamics since the calculation is quite tedious and has been expounded elsewhere [2]. Since the heat bath is harmonic it can be eliminated exactly and after a lengthy calculation one is left with the so-called influence functional which describes the influence of the reservoir on the system. An exact solution for the time evolution of the system is only possible for a few special cases like the harmonic oscillator and the free damped particle. In the rest of the paper we will use for $V(q,t)$ in (2) the time independent harmonic potential

$$V(q) = \frac{1}{2}M\omega_0^2 q^2. \tag{16}$$

Then the dynamics for a Gaussian initial state is fully determined by the first and second moments and we will restrict the following discussion to these quantities. It is convenient to introduce sum and difference coordinates

$$r = \frac{q+q'}{2}, \quad x = q - q'. \tag{17}$$

The initial density matrix of the system then reads

$$\rho_i(x_i, r_i) = \int d\bar{r}d\bar{x}\,\lambda(x_i, r_i, \bar{x}, \bar{r})\rho_\beta(\bar{x}, \bar{r}) \tag{18}$$

where

$$\rho_\beta = Tr_R\, W_\beta \tag{19}$$

is the equilibrium density matrix traced over the reservoir. The first and second moments can be written in a compact form by introducing

$$q_1(t) = \dot{G}_+(t) q, \tag{20}$$

$$q_2(t) = G_+(t) \frac{p}{M}, \tag{21}$$

$$q_3(t) = [\frac{S(t)}{\langle q^2 \rangle} - \dot{G}_+(t)] \bar{r}, \tag{22}$$

$$q_4(t) = -\frac{i}{\hbar}[M \dot{S}(t) + \frac{\langle p^2 \rangle}{M} G_+(t)] \bar{x}. \tag{23}$$

We then have for the first moments at time t

$$\langle q \rangle_t = \sum_{\alpha=1}^{4} \langle q_\alpha(t) \rangle_0, \tag{24}$$

$$\langle p \rangle_t = M \sum_{\alpha=1}^{4} \langle \dot{q}_\alpha(t) \rangle_0 \tag{25}$$

where we need to calculate the following first moments at $t = 0$

$$\langle q \rangle_0 = \int dr_i d\bar{x} d\bar{r}\, \lambda(0, r_i, \bar{x}, \bar{r})\, r_i\, \rho_\beta(\bar{x}, \bar{r}), \tag{26}$$

$$\langle p \rangle_0 = \int dr_i d\bar{x} d\bar{r}\, [\frac{\hbar}{i} \frac{\partial}{\partial x_i} \lambda(x_i, r_i, \bar{x}, \bar{r}) \rho_\beta(\bar{x}, \bar{r})] \Big|_{x_i=0}, \tag{27}$$

$$\langle \bar{r} \rangle_0 = \int dr_i d\bar{x} d\bar{r}\, \lambda(0, r_i, \bar{x}, \bar{r})\, \bar{r}\, \rho_\beta(\bar{x}, \bar{r}), \tag{28}$$

$$\langle \bar{x} \rangle_0 = \int dr_i d\bar{x} d\bar{r}\, \lambda(0, r_i, \bar{x}, \bar{r})\, \bar{x}\, \rho_\beta(\bar{x}, \bar{r}). \tag{29}$$

For the second moments one obtains

$$\begin{aligned} \langle q^2 \rangle_t = \langle q^2 \rangle [1 - \frac{S^2(t)}{\langle q^2 \rangle^2}] + \frac{\langle p^2 \rangle}{M^2} G_+^2(t) + 2 G_+(t) \dot{S}(t) \\ + \sum_{\nu,\mu=1}^{4} \langle q_\nu(t) q_\mu(t) \rangle_0, \end{aligned} \tag{30}$$

$$\tfrac{1}{2} \langle pq + qp \rangle_t = \frac{1}{2} M \frac{d}{dt} \langle q^2 \rangle_t, \tag{31}$$

$$\begin{aligned} \langle p^2 \rangle_t = \langle p^2 \rangle [\dot{G}_+^2(t) + 1] - M^2 \frac{\dot{S}^2(t)}{\langle q^2 \rangle} + 2 M^2 \dot{G}_+(t) \ddot{S}(t) \\ + M^2 \sum_{\nu,\mu=1}^{4} \langle \dot{q}_\nu(t) \dot{q}_\mu(t) \rangle_0. \end{aligned} \tag{32}$$

The initial second moments are defined as obvious generalization of the expressions (26–29) for the first moments. The dynamics of the moments for factorizing initial conditions is obtained from these results by neglecting all moments containing $\bar{r}$ or $\bar{x}$ which is equivalent to vanishing initial correlations between system and reservoir.

In eqs. (20–23) and (30–32) we have made use of the functions $G_+(t)$ and $S(t)$ which represent the Green's function and the symmetrized position autocorrelation function, respectively. In the following section we will discuss their properties. Furthermore we introduced the equilibrium correlations $\langle q^2\rangle$ and $\langle p^2\rangle$ which can be obtained from the correlation function $S(t)$ by

$$\langle q^2\rangle = S(0) \tag{33}$$

$$\langle p^2\rangle = -M^2\ddot{S}(0). \tag{34}$$

5. The Position Autocorrelation Function of the Harmonic Oscillator

The explicit evaluation of the correlation function $S(t)$ and the Green's function $G_+(t)$ may proceed along various lines [2, 5, 6]. The most straightforward way to obtain these quantities is the original method [5] making use of Ehrenfest's theorem and the fluctuation dissipation theorem. The Laplace transform of the Green's function can easily be found from the equation of motion (6) to be

$$\hat{G}_+(z) = \frac{1}{z^2 + z\hat{\gamma}(z) + \omega_0^2}. \tag{35}$$

Since $\hat{G}_+(z)$ does not depend on temperature it always shows classical behavior even in the quantum regime. Therefore the linear response to an external force is classical. While in general it is difficult to explicitly calculate $G_+(t)$ from (35) it can be done for the Ohmic case where the Laplace transform of the damping kernel $\hat{\gamma}(z)$ takes the frequency independent value γ. One gets

$$G_+(t) = \Theta(t)\frac{1}{\zeta}\exp(-\frac{\gamma}{2}t)\sinh(\zeta t) \tag{36}$$

where $\Theta(t)$ is the unit step function which makes $G_+(t)$ a causal Green's function and where

$$\zeta = \sqrt{\frac{1}{4}\gamma^2 - \omega_0^2}. \tag{37}$$

For long times the behavior of $G_+(t)$ is therefore determined by an exponential decay with time constant $\gamma/2$ in the underdamped case ($\gamma < 2\omega_0$) and $\gamma/2 - \zeta$ in the overdamped case.

As a consequence of the fluctuation dissipation theorem, the Laplace transform $\hat{S}(z)$ of the symmetrized position autocorrelation function can be expressed through the Green's function $\hat{G}_+(z)$. Introducing the thermal frequencies

$$\nu_n = \frac{2\pi}{\hbar\beta}n \tag{38}$$

one gets

$$\hat{S}(z) = \frac{1}{\beta M}\sum_{n=-\infty}^{\infty}\frac{z}{\nu_n^2 - z^2}[\hat{G}_+(z) - \hat{G}_+(|\nu_n|)]. \tag{39}$$

This quantity is temperature dependent and its classical and quantum properties are quite different. In the Ohmic case one can again obtain an explicit expression for $S(t)$ which is given by [5]

$$S(t) = \frac{\hbar}{4M\zeta}[\cot(\frac{\beta\hbar\lambda_2}{2})e^{-\lambda_2 t} - \cot(\frac{\beta\hbar\lambda_1}{2})e^{-\lambda_1 t}] - \frac{2\gamma}{M\beta}\sum_{n=1}^{\infty}\frac{\nu_n e^{-\nu_n t}}{(\nu_n^2 - \lambda_1^2)(\nu_n^2 - \lambda_2^2)} \tag{40}$$

where

$$\lambda_{1,2} = \frac{\gamma}{2} \pm \zeta. \tag{41}$$

For high temperatures, i.e. in the classical regime, one again finds exponential decay of the correlation function with the time scale determined by the damping strength γ. As temperature is decreased the thermal frequencies ν_n become smaller and for low temperatures the long time behavior is dominated by the first thermal frequency ν_1. As temperature approaches absolute zero more and more thermal frequencies contribute in the sum in (40) and for $T = 0$ one finds the algebraic long time decay [5, 6]

$$S(t) \sim -\frac{\hbar\gamma}{\pi M\omega_0^4}\frac{1}{t^2}. \tag{42}$$

At finite temperatures this specific behavior can be found on an intermediate time scale before the exponential decay takes over [7]. The observation of these quantum effects may be difficult for very weak damping since the sum in (40) vanishes for $\gamma \to 0$.

6. Coherent and Squeezed States

In the rest of this paper we apply the theory of dissipative quantum systems to four different initial states where we project the equilibrium density matrix onto a squeezed state or where we let a displacement and squeezing operator act on it. In order to introduce the terminology we discuss a few properties of coherent and squeezed states which will be needed in the following [8].

We define the annihilation operator of the undamped quantum harmonic oscillator characterized by the potential (16) as

$$a = \sqrt{\frac{M\omega_0}{2\hbar}}q + i\sqrt{\frac{1}{2\hbar M\omega_0}}p \tag{43}$$

and the creation operator a^+ to be its hermitian conjugate. Then a coherent state $|\alpha\rangle$ is defined to be an eigenstate of a

$$a|\alpha\rangle = \alpha|\alpha\rangle \tag{44}$$

with complex eigenvalue α. This state can be generated from the ground state using the displacement operator

$$\mathcal{D}(\alpha) = \exp(\alpha a^+ - \alpha^* a), \tag{45}$$

i.e.

$$|\alpha\rangle = \mathcal{D}(\alpha)|0\rangle. \tag{46}$$

The coordinate representation of the displacement operator which will be needed later reads

$$\langle q|\mathcal{D}(p_0,q_0)|q'\rangle = \exp(\frac{i}{\hbar}p_0 q)\exp(\frac{i}{2\hbar}p_0 q_0)\delta(q-q_0-q'). \tag{47}$$

The effect of the displacement operator on the ground state is to displace the state in p–q space to the mean value of the coordinate

$$q_0 = \sqrt{\frac{2\hbar}{M\omega_0}}\,\mathrm{Re}(\alpha) \tag{48}$$

and the mean value of the momentum

$$p_0 = \sqrt{2\hbar M\omega_0}\,\mathrm{Im}(\alpha). \tag{49}$$

This can readily be verified by inspection of the coordinate representation of the coherent state

$$\langle q|\alpha\rangle = (\frac{M\omega_0}{\pi\hbar})^{1/4}\exp[-\frac{M\omega_0}{2\hbar}(q-q_0)^2 + i\frac{p_0}{\hbar}(q+\frac{q_0}{2})]. \tag{50}$$

This state represents a minimum uncertainty state with equal variances in p and q. In order to produce a minimum uncertainty state with different variances one needs the squeezing operator

$$\mathcal{S}(z) = \exp[\frac{z}{2}a^2 - \frac{z^*}{2}{a^+}^2] \tag{51}$$

which in coordinate representation is of the form

$$\langle q|\mathcal{S}(z)|q'\rangle = \zeta^{1/2}\delta(\zeta q - q') \tag{52}$$

where we introduced

$$\zeta = \exp(z). \tag{53}$$

In (52) we restricted ourselves to real z which means that the variance σ_{pq} vanishes initially. The squeezed state defined through

$$|\alpha,z\rangle = \mathcal{D}(\alpha)\mathcal{S}(z)|0\rangle \tag{54}$$

has the coordinate representation

$$\langle q|\alpha,z\rangle = (\frac{M\omega_0\zeta^2}{\pi\hbar})^{1/4}\exp[-\frac{M\omega_0}{2\hbar}\zeta^2(q-q_0)^2 + \frac{i}{\hbar}p_0(q+\frac{q_0}{2})]. \tag{55}$$

For $\zeta > 1$ the position uncertainty is reduced at the expense of a larger uncertainty in the momentum. We note that the squeezed state (55) for fixed q_0 and p_0 can be obtained from the coherent state (50) by formally scaling the frequency ω_0 of the harmonic oscillator by a factor of ζ^2. This is due to the fact that the ground state of a given harmonic oscillator represents a squeezed state with respect to a harmonic oscillator with different frequency. On the other hand, one can easily derive the results for coherent states by setting $\zeta = 1$ in the results for squeezed states.

7. Four Special Initial States

The preceding section dealt with coherent and squeezed states for an undamped harmonic oscillator. In real systems the oscillator may be coupled to a heat bath which introduces dissipation. Therefore one has to find an initial condition which replaces the squeezed state of the undamped case. One may think of two different preparations. The first one is a generalization of (54) to the form (11) with $\mathcal{O} = \mathcal{D}(\alpha)\mathcal{S}(z), \mathcal{O}' = \mathcal{O}^+$ where the vacuum is replaced by the equilibrium density matrix. This state reduces to a squeezed state in the limit of vanishing damping and zero temperature. A second possibility is to project the equilibrium density matrix onto a squeezed state. We may now define four different initial states

$$W_i^{(1)} \simeq \mathcal{D}(\alpha)\mathcal{S}(z)W_\beta\mathcal{S}^+(z)\mathcal{D}^+(\alpha), \tag{56}$$

$$W_i^{(2)} \simeq |\alpha, z\rangle\langle\alpha, z|W_\beta|\alpha, z\rangle\langle\alpha, z|, \tag{57}$$

$$W_i^{(3)} \simeq \mathcal{D}(\alpha)\mathcal{S}(z)\rho_\beta\mathcal{S}^+(z)\mathcal{D}^+(\alpha)\cdot W_R, \tag{58}$$

and

$$W_i^{(4)} \simeq |\alpha, z\rangle\langle\alpha, z|\rho_\beta|\alpha, z\rangle\langle\alpha, z|\cdot W_R, \tag{59}$$

where $\simeq$ means equal apart from an appropriate normalization factor. Here, ρ_β denotes the equilibrium density matrix of the damped harmonic oscillator. In coordinate representation this density matrix is given by

$$\rho_\beta(\bar{x}, \bar{r}) = (2\pi\langle q^2\rangle)^{-1/2}\exp\left[-\left(\frac{1}{2\langle q^2\rangle}\bar{r}^2 + \frac{\langle p^2\rangle}{2\hbar^2}\bar{x}^2\right)\right]. \tag{60}$$

$W_i^{(3)}$ ($W_i^{(4)}$) represents the factorizing initial state corresponding to the nonfactorizing initial state $W_i^{(1)}$ ($W_i^{(2)}$). While the initial reduced density matrices are equal, i.e.

$$\rho_i^{(1)} = \rho_i^{(3)} \tag{61}$$

and

$$\rho_i^{(2)} = \rho_i^{(4)}, \tag{62}$$

the dynamics of these states is different as will be shown below. The preparations 1 and 3 were discussed already earlier [2,9] while preparations 2 and 4 have not been considered as yet.

In the framework of the theory presented in section 3 we need the preparation function λ in order to describe the initial states and their dynamics. For the first initial state (56) we use the coordinate representations of the displacement operator (47) and the squeezing operator (52) and obtain with (13)

$$\lambda^{(1)}(x_i, r_i, \bar{x}, \bar{r}) = \zeta\exp\left(\frac{i}{\hbar}p_0x_i\right)\delta(\zeta x_i - \bar{x})\delta(\zeta(r_i - q_0) - \bar{r}) \tag{63}$$

where we used sum and difference coordinates as introduced in (17). From (55) we derive the coordinate representation of the projector $|\alpha, z\rangle\langle\alpha, z|$ and obtain

$$\lambda^{(2)}(x_i, r_i, \bar{x}, \bar{r}) = N \exp\left[-\frac{M\omega_0\zeta^2}{\hbar}[(r_i - q_0)^2 + \frac{x_i^2}{4} + (\bar{r} - q_0)^2 + \frac{\bar{x}^2}{4}]\right] \times \exp[i\frac{p_0}{\hbar}(x_i - \bar{x})] \tag{64}$$

where the normalization factor has to be chosen such that the initial density matrix is normalized to unity, i.e.

$$\int dr_i d\bar{x} d\bar{r}\, \lambda(0, r_i, \bar{x}, \bar{r}) \rho_\beta(\bar{x}, \bar{r}) = 1. \tag{65}$$

The corresponding factorizing initial states are obtained by setting $\bar{x}$ and $\bar{r}$ equals to zero. Thus we have

$$\lambda^{(3)}(x_i, r_i, \bar{x}, \bar{r}) \simeq \lambda^{(1)}(x_i, r_i, \bar{x}, \bar{r})\delta(\bar{x})\delta(\bar{r}) \tag{66}$$

and

$$\lambda^{(4)}(x_i, r_i, \bar{x}, \bar{r}) \simeq \lambda^{(2)}(x_i, r_i, \bar{x}, \bar{r})\delta(\bar{x})\delta(\bar{r}). \tag{67}$$

Together with the definitions of the moments (24), (25), (30–32) we have all what is needed in order to calculate the dynamics of the four initial states defined in this section.

8. Decay of Coherent and Squeezed States in the Presence of Dissipation

According to (24) and (25) the dynamics of the first moments $\langle q \rangle$ and $\langle p \rangle$ is determined by the initial first moments $\langle q \rangle_0$, $\langle p \rangle_0$, $\langle \bar{r} \rangle_0$, and $\langle \bar{x} \rangle_0$ which can easily be calculated using (26–29). For the first initial state (56) we obtain [2,9]

$$\langle q \rangle_0^{(1)} = q_0 \tag{68}$$

$$\langle p \rangle_0^{(1)} = p_0 \tag{69}$$

$$\langle \bar{r} \rangle_0^{(1)} = 0 \tag{70}$$

$$\langle \bar{x} \rangle_0^{(1)} = 0. \tag{71}$$

On the other hand, for the second initial state (57) we get

$$\langle q \rangle_0^{(2)} = q_0 \tag{72}$$

$$\langle p \rangle_0^{(2)} = p_0 \tag{73}$$

$$\langle \bar{r} \rangle_0^{(2)} = \frac{q_0}{1 + \dfrac{\hbar}{2M\omega_0\zeta^2\langle q^2 \rangle}} \tag{74}$$

$$\langle \bar{x} \rangle_0^{(2)} = -i\frac{2}{M\omega_0\zeta^2}\frac{p_0}{1 + \dfrac{2\langle p^2 \rangle}{M\hbar\omega_0\zeta^2}}. \tag{75}$$

For the factorizing inital states we again find the result (68–71) since the moments $\langle \bar{r} \rangle_0^{(3,4)}$ and $\langle \bar{x} \rangle_0^{(3,4)}$ vanish per definition.

The result for $\langle q \rangle_0$ and $\langle p \rangle_0$ for all four states is given by the displacements (48) and (49). As far as the first moments are concerned there is no difference between the nonfactorizing preparation 1 and its factorizing counterpart 3. In both cases one finds according to (20), (21), (24), and (25) that the first moments decay classically. This does however not mean that there are no initial correlations between system and heat bath in the initial state 1 as we will see below.

The situation is quite different in the case of the preparations 2 and 4. Here we have nonvanishing moments $\langle\bar{r}\rangle_0$ and $\langle\bar{x}\rangle_0$ for the nonfactorizing case. As a consequence the dynamics of $\langle q\rangle_t^{(2)}$ and $\langle p\rangle_t^{(2)}$ is not only determined by the Green's function $G_+(t)$ but also by the position autocorrelation function $S(t)$. This means that in contrast to the factorizing case we may observe in the evolution of the expectation values of q and p the specific quantum effects discussed in section 5. For instance we have

$$\begin{aligned}\langle q\rangle_t^{(2)} - \langle q\rangle_t^{(4)} =& [\frac{S(t)}{\langle q^2\rangle} - \dot{G}_+(t)]\frac{q_0}{1+\dfrac{\hbar}{2M\omega_0\zeta^2\langle q^2\rangle}} \\ & - \frac{1}{\hbar}[M\dot{S}(t) + \frac{\langle p^2\rangle}{M}G_+(t)]\frac{2}{M\omega_0\zeta^2}\frac{p_0}{1+\dfrac{2\langle p^2\rangle}{M\hbar\omega_0\zeta^2}}\end{aligned} \tag{76}$$

This difference between nonfactorizing and factorizing initial states is not only quantitative but both states show qualitatively different time behavior at low temperatures. While the factorizing state decays exponentially with a time constant determined by the damping constant γ, one has exponential decay with ν_1 or even algebraic decay $\propto t^{-2}$ for the nonfactorizing state.

The calculation of the second moments according to (30–32) is straightforward. We restrict the discussion to the second moments for preparations 1 and 3 in order to show that there are differences between the two preparations although they do not show up in the first moments.

The nonvanishing second moments for preparation 1 are obtained as [2,9]

$$\langle q^2\rangle_0 = q_0^2 + \zeta^{-2}\langle q^2\rangle \tag{77}$$

$$\tfrac{1}{2}\langle pq + qp\rangle_0 = p_0 q_0 \tag{78}$$

$$\langle p^2\rangle_0 = p_0^2 + \zeta^2\langle p^2\rangle \tag{79}$$

$$\langle\bar{r}^2\rangle_0 = \langle q^2\rangle \tag{80}$$

$$\langle q\bar{r}\rangle_0 = \zeta^{-1}\langle q^2\rangle \tag{81}$$

$$\langle p\bar{x}\rangle_0 = \frac{\hbar}{i}\zeta. \tag{82}$$

For preparation 3 the last three moments vanish. From (80–82) one concludes that there are indeed correlations between system and heat bath in the case of the nonfactorizing preparation 1. Therefore the dynamics of this initial state differs from the corresponding factorizing state.

We want to mention another effect concerning squeezed states which is due to damping and cannot be obtained within a weak coupling theory. While the squeezing due to the preparation decays on a certain time scale there exists a static contribution to squeezing which survives even in the limit $t \to \infty$. The reason is that for the harmonic potential (16) one always finds the equilibrium density matrix ρ_β (60) for long enough times. The variances in q and p differ from the corresponding values in the undamped case. Therefore there will be squeezing with respect to the undamped harmonic oscillator with a significant contribution for strong damping.

9. Conclusions

We have discussed the time evolution of squeezed states coupled to an environment. By way of example we showed that the influence of the initial preparation may decay on a time scale much longer than the time scale for the classical relaxation to equilibrium. We have proposed two preparations (2 and 4) with the same reduced density matrix at $t = 0$ where the initial correlations between system and bath influence the average relaxation for extremely long times at low temperatures. In another example (1 and 3) the correlation effects do not show up in the average relaxation of $\langle p \rangle$ and $\langle q \rangle$ but they are apparent in the time evolution of the second moments. These effects cannot be obtained within a weak coupling theory and become more pronounced as temperature is decreased. The commonly used approximation of factorizing initial states is then not necessarily well justified. It is expected that this behavior is of general relevance for dissipative quantum systems and not restricted to the special case considered here.

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For a more complete list of references related to this subject see [2].

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