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Angaben zur Veröffentlichung / Publication details:

Hida, Kazuo, and Ulrich Eckern. 1985. "Influence of dissipation on quantum soliton pair creation in the Sine-Gordon model." *Molecular Crystals and Liquid Crystals* 119 (1): 467–70. https://doi.org/10.1080/00268948508075199.

INFLUENCE OF DISSIPATION ON QUANTUM SOLITON PAIR CREATION IN THE SINE-GORDON MODEL

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Abstract The influence of dissipation on the quantum soliton-antisoliton pair creation rate P is calculated for the sine-Gordon model using the path integral formulation. The external field dependence of P is shown to change drastically in the weak field limit. The relation to the electric conductivity of commensurability pinned CDW is discussed.

INTRODUCTION

It was suggested by Maki[1] that the quantum creation of soliton-antisoliton pairs in the sine-Gordon model is related to the electric conduction in commensurate CDW system. In solid state physics, solitons are usually coupled to a heat bath and subject to damping. The purpose of the present work is to investigate the influence of dissipation on the quantum pair creation rate of solitons.

THE MODEL

We consider the one-dimensional quantum sine-Gordon model in an external field whose Lagrangian is given by

$$\mathcal{L} = A \int_{-\infty}^{\infty} dx\{(\frac{\partial\varphi}{\partial t})^2 - (\frac{\partial\varphi}{\partial x})^2 + 2\omega^2(:\cos\varphi-1:) + \varepsilon\varphi\} \qquad (1)$$

where φ is the bose field, ε the external field, A a positive constant, and ω the frequency of small amplitude oscillations. The notation :: denotes the normal product.

When ε is smaller than $\varepsilon_c = 2\omega^2$, this system has an infinite series of metastable states with values of $\varphi = \varphi_0 + 2n\pi$ (n:integer), where φ_0 vanishes if $\varepsilon = 0$. The decay rate P of one of those states, in the semiclassical approximation, is given by[2] $P=K\cdot\exp(-S_B)$, where K is a determinantal factor, and S_B the Euclidean action S, evaluated for a single "bounce"[1,2,3]. Specifically, S is given by

$$S = -\int_{-\infty}^{\infty} \mathcal{L}_E \, d\tau + S_d \quad , \tag{2}$$

where $\mathcal{L}_E$ is obtained from $\mathcal{L}$ by $t\to-i\tau$, and the coupling to the heat bath is described by[3]

$$S_d = \frac{\eta_0}{2\pi} \iiint_{-\infty}^{\infty} d\tau d\tau' dx \left\{\frac{\varphi(x,\tau)-\varphi(x,\tau')}{\tau-\tau'}\right\}^2 \tag{3}$$

In the limit of small ε , the bounce is a bubble with $\varphi=2\pi$ on the x-τ plane in a background with $\varphi=0$, and the size of the bubble is much larger than its wall (thin wall approximation).

RESULTS

For weak damping ($\eta\equiv\eta_0/A \ll \omega$) we find the following results:

$$P \sim \exp(-128\omega^2 A/\varepsilon) \tag{4a}$$

$$\text{for } \varepsilon_c \gg \varepsilon \gtrsim \varepsilon_c \exp(-4\omega/\pi\eta)$$

$$P \sim (4\omega\eta/\pi\varepsilon)^{\frac{-128\omega\eta A}{\varepsilon}} \tag{4b}$$

$$\text{for } \varepsilon_c(\eta/\omega)\exp(-4\omega/\pi\eta) \gtrsim \varepsilon$$

The expression (4a) coincides with that of Maki[1]. In the presence of dissipation, however, this does not hold for small ε , and the crossover to the expression (4b) is found.

For strong damping ($\eta \gg\omega$), we find

$$P \sim (8e\omega^2/\pi^2\varepsilon)^{-128\eta\omega A/\varepsilon} \tag{5}$$

DISCUSSION

The correspondence between our Lagrangian and that of the commensurability pinned CDW is given in Table I [1,4-6]. In the latter system, P is proportional to the electric conductivity except a prefactor which is weakly dependent on ε . In the absence of dissipation, the typical scale of the electric field is given by $\varepsilon_1 = 64A\varepsilon_c = (16v_F/\pi c_o N^2)\varepsilon_c$. Although $v_F \simeq 10...10^2 c_o$, quantum tunneling phenomena could be observed before the CDW is depinned classically if commensurability is high enough. On the other hand, our results shows that the typical scale of electric field in the dissipative case is given by $\varepsilon_d = 128\eta\omega A$, which is smaller than ε_1 for small η . The detailed calculation will be published elsewhere[7].

ACKNOWLEDGEMENTS

We thank A. Schmid and W. Wonneberger for useful discussions. One of us (K.H.) is grateful to the Alexander von Humboldt-Stiftung for financial support.

TABLE I Correspondence between the commensurability pinned CDW and the present system. N: commensurability; c_o: phason velocity; e: electronic charge; E: electric field; v_F: Fermi velocity; τ_2: backward scattering time due to impurities.

Commensurability pinned CDW	$N\cdot\phi$	$\frac{v_F}{4\pi N^2 c_o}$	$\frac{4eNc_o^2 E}{v_F}$	$\frac{c_o^2}{\tau_2 v_F^2}$
present system	φ	A	ε	η

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