Relaxation Processes in Charge-Density-Wave Systems

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Starting from the quasiclassical formulation of the microscopic theory, the kinetic equations describing low-frequency, long-wavelength phenomena in quasi-one-dimensional conductors at low temperatures are derived. The solution of the equations in which electron–phonon and electron–impurity collisions are taken into account is used to derive the classical equation of motion for the phase of the order parameter.

1. INTRODUCTION

Some time ago, it was demonstrated by Peierls and Fröhlich[1] how the interaction between electrons and phonons in quasi-one-dimensional metals at low temperatures leads to the formation of a charge-density wave (CDW), which entails a periodic modulation of the electron density as well as a distortion of the lattice. In the presence of impurities, or if the CDW is commensurable with the underlying lattice, the CDW will be pinned. On the other hand, if an electric field above a well-defined threshold is applied, the CDW slides through the system. Many experimental investigations (see ref. 2 and references therein) have demonstrated the nonlinear current–voltage curve related to this effect, as well as the narrowband noise in the sliding mode, and other phenomena characteristic for nonlinear systems. Theoretically, the results have been interpreted in terms of a single classical particle model,[3] or as Zener tunneling,[4] or in terms of an effective Hamiltonian for the phase of the complex order parameter.[5,6] Recent approaches to the problem of impurity pinning discuss the limit of large electric fields by perturbation theory[7] and the behavior close to threshold as a critical phenomenon.[8]

Starting from microscopic theory,[9] I discuss in this paper two relaxation mechanisms expected to play an important role in the dynamics of a CDW: scattering of electrons by impurities and scattering by phonons. While

scattering by defects is typically an elastic and momentum-nonconserving process, it is known that electron–phonon scattering in general contributes to dissipation of energy and momentum. Consider, for example, a simple normal metal, and let θ be its Debye temperature. Then, for high temperatures, $T \gg \theta$, electron–phonon processes are to a good approximation elastic, and the relaxation rate is of the order of $\sim \lambda k_B T/\hbar$, where λ is the dimensionless electron–phonon coupling constant; thus, in this limit, electron–phonon processes act similar to impurity scattering, and the energy relaxation rate is expected to be smaller than the momentum relaxation rate by a factor θ/T.

On the other hand, for very low temperatures such that $T \ll \theta$, electron–phonon scattering is more effective in dissipating energy than momentum. Typically, the energy relaxation rate is of the order of $\sim \lambda k_B T^3/\hbar\theta^2$, while the momentum relaxation rate is smaller by a factor $(T/\theta)^2$. This situation will be investigated in detail in this paper. To be definite, I consider a model where momentum dissipation is due to impurity scattering, and electron–phonon processes are assumed to contribute to energy relaxation only.

In the case where impurity scattering can be neglected completely, it was noted earlier[6,10] that a particular type of nonequilibrium between condensate and excitations can be established. This nonequilibrium state is of similar form as found in connection with the branch imbalance in superconductors[11] or the spin dynamics[12] of superfluid ^{3}He. This case also corresponds to the "mixed" dissipation discussed recently by Leggett[13] in the context of dissipative quantum tunneling. The goal of the present paper is to clarify the relevance of these concepts for CDW systems.

In this paper, I use the quasiclassical approach to derive kinetic equations for CDW systems, i.e., equations describing the low-frequency and long-wavelength behavior (where the scale is set by the Fermi energy and Fermi wavelength). This method, which has been applied successfully[14–18] to a variety of nonequilibrium phenomena in superconductors and superfluid ^{3}He, was used in the present context by Artemenko and Volkov.[19,20] Essentially, I follow their line of argument, except that the equations are brought into a more transparent form of a Boltzmann-like transport equation by introducing quasiparticle distribution functions. As an additional feature, energy relaxation by phonons is included.

The model is presented in Section 2. Section 3 discusses the approximation that leads to the equation of motion for the quasiclassical Green's function, which is a Green's function integrated with respect to the magnitude of the momentum. Since this Green's function obeys a normalization condition, quasiparticle distribution functions can be introduced, and a Boltzmann equation is derived (Section 4). In Section 5, I solve the transport equation and discuss the dynamics of the phase of the order parameter in various limits. Some conclusions and a discussion are given in Section 6.

2. MICROSCOPIC THEORY

The starting point is the equation of motion for the Green's function, which, since my main interest is a description of nonthermal states, is conveniently modified according to the theory of Keldysh.[21] In addition, I distinguish as usual between positive and negative momentum states. Thus one is led to define the following (4×4) matrix Green's function:

$$\bar{\hat{G}}=\begin{pmatrix}\hat{G}^{\mathrm{R}} & \hat{G}^{\mathrm{K}}\\ 0 & \hat{G}^{\mathrm{A}}\end{pmatrix} \tag{1}$$

where $\hat{G}^{\mathrm{R(A)}}$ is the retarded (advanced) Green's function and $\hat{G}^{\mathrm{K}}$ is called Keldysh's function. These quantities are related to $\hat{G}^{>}$ and $\hat{G}^{<}$ in the following way:

$$\begin{aligned}\hat{G}^{\mathrm{R}}&=[\hat{G}^{>}-\hat{G}^{<}]\Theta(t_1-t_2)\\ \hat{G}^{\mathrm{A}}&=-[\hat{G}^{>}-\hat{G}^{<}]\Theta(t_2-t_1)\\ \hat{G}^{\mathrm{K}}&=\hat{G}^{>}+\hat{G}^{<}\end{aligned} \tag{2}$$

Note that all Green's functions depend on two spatial and two time coordinates x_1, x_2 and t_1, t_2, respectively. The quantities $\hat{G}^{>}$ and $\hat{G}^{<}$, which are 2×2 matrices, are defined by

$$\hat{G}^{<}_{\alpha\beta}=iz_{\alpha\beta}\langle\psi^{+}_{\beta}(x_2,t_2)\psi_{\alpha}(x_1,t_1)\rangle \tag{3}$$

$$\hat{G}^{>}_{\alpha\beta}=-iz_{\alpha\beta}\langle\psi_{\alpha}(x_1,t_1)\psi^{+}_{\beta}(x_2,t_2)\rangle \tag{4}$$

where the branch index $\alpha,\beta=+\,(-)$ refers to the states with positive (negative) momentum, and the ψ's are the corresponding fermion operators. Note that I ignore here and in the following the spin index, except the usual factors 2 to account for multiplicity. In addition, note that one can concentrate on states with momenta close to the Fermi surface, i.e., in the range $(-p_{\mathrm{F}}-p_0)\cdots(-p_{\mathrm{F}}+p_0)$ and $(p_{\mathrm{F}}-p_0)\cdots(p_{\mathrm{F}}+p_0)$, where p_{F} is the Fermi momentum, and $p_0\ll p_{\mathrm{F}}$ is a cutoff, which is assumed to be much larger than all momenta of interest. Therefore, I included in Eqs. (3) and (4) the factor $z_{\alpha\beta}$, which removes the rapid spatial variation of the Green's function:

$$z_{\alpha\beta}=\alpha\exp\left[-ip_{\mathrm{F}}(\alpha x_1-\beta x_2)\right] \tag{5}$$

In other words, the momentum will be measured relative to the Fermi momentum. In the following, it is convenient to work with the variables $(p, E; x, t)$, where p and E are the Fourier conjugate variables to x_1-x_2 and t_1-t_2, respectively, and $x=(x_1+x_2)/2$ and $t=(t_1+t_2)/2$. For two functions A and B, I define the functional product "$*$" by

$$A*B=\exp\left[\frac{i}{2}(\partial^A_E\partial^B_t-\partial^A_t\partial^B_E-\partial^A_p\partial^B_x+\partial^A_x\partial^B_p)\right](AB) \tag{6}$$

where the notation ∂_E^A, e.g., indicates that A is differentiated with respect to E. The equation of motion has the following form:

$$\check{Q} * \check{G} = \check{1} + \check{\Sigma} * \check{G} \tag{7}$$

where $\check{Q} = \hat{Q} \cdot \bar{1}$, and

$$\hat{Q} = (E - e\phi)\hat{\tau}_3 - \hat{\xi} + \hat{\Delta} \tag{8}$$

Also, I introduced

$$\hat{\xi} = \frac{1}{2m}[(p_F\hat{\tau}_3 + p - eA)^2 - p_F^2]\hat{\tau}_3 \tag{9}$$

and

$$\hat{\Delta} = \begin{pmatrix} 0 & \Delta \\ -\Delta^* & 0 \end{pmatrix} \tag{10}$$

Here, ϕ and A are the electromagnetic potentials, $\hat{\tau}_k$ is the Pauli matrix, and one introduces the complex order parameter[9] Δ to describe the coupling between the branches with positive and negative momenta through phonons with momentum close to $\pm Q = \pm 2p_F$ (and corresponding energy ω_Q). The self-energy on the rhs of Eq. (7) contains the scattering with thermal phonons, which can be separated from the effect of high-energy phonons ($\sim\omega_Q$), provided the temperature is low enough, $T \ll \omega_Q$, as well as impurity scattering. Accordingly, one writes

$$\check{\Sigma} = \check{\Sigma}_{ph} + \check{\Sigma}_{imp} \tag{11}$$

For the following, it will be of central importance that the self-consistency relation for the order parameter, as well as the expression for the self-energy, can be expressed in terms of the so-called quasiclassical or momentum integrated Green's function $\check{g}$:

$$\check{g}(E; x, t) = \frac{i}{\pi} v_F \fint dp\, \check{G}(p, E; x, t) \tag{12}$$

From the standard expression, it is clear that the self-consistency relation can be written in terms of $\check{g}$, and is given by

$$[1 + \omega_Q^{-2}\partial_t^2]\Delta = \frac{1}{4}\lambda \int dE\, \hat{g}_{+-}^K \tag{13}$$

where λ is the dimensionless electron–phonon coupling constant. As discussed above, I neglect the (small) momentum transfer in electron–phonon collisions, in which case it is possible to introduce the momentum averaged phonon Green's function B, which is assumed to depend only on energy,

since the phonons are considered to be in thermal equilibrium. Analogous to the theory of superconductors,[15] one obtains the following expression:

$$\hat{\Sigma}^X_{\rm ph} = \frac{1}{4}\int dE' \begin{cases} B^{\rm K}\hat{g}^{\rm K} - (B^{\rm R} - B^{\rm A})(\hat{g}^{\rm R} - \hat{g}^{\rm A}) & X = {\rm K} \\ B^{\rm A}\hat{g}^{\rm K} + B^{\rm K}\hat{g}^{\rm R} & X = {\rm R} \\ B^{\rm R}\hat{g}^{\rm K} + B^{\rm K}\hat{g}^{\rm A} & X = {\rm A} \end{cases} \tag{14}$$

where the arguments are such that $B^X\hat{g}^Y \equiv B^X(E')\hat{g}^Y(E+E'; x, t)$ in this equation. Note that

$$B^{\rm K}(E) = [B^{\rm R}(E) - B^{\rm A}(E)]\coth(E/2T) \tag{15}$$

In addition, for a Debye model, one has

$$2\pi\alpha^2 F(E) \equiv i(B^{\rm R} - B^{\rm A}) \sim E^2 \times \operatorname{sgn} E.$$

Finally, for the impurity self-energy, use is made of the expression obtained by neglecting intersecting impurity lines. Also, it is useful to distinguish between impurity collisions with small (~ 0) and large ($\sim \pm Q$) momentum transfers, which are approximately independent, and their corresponding scattering times τ_1 and τ_2, respectively. The result is given by[20]

$$i\hat{\Sigma}_{\rm imp} = \frac{1}{4\tau_1}\hat{\tau}_3\hat{g}\hat{\tau}_3 - \frac{1}{8\tau_2}[\hat{\tau}_1\hat{g}\hat{\tau}_1 + \hat{\tau}_2\hat{g}\hat{\tau}_2] \tag{16}$$

For illustration, consider the electrons in thermal equilibrium, in which case

$$\hat{g}^{\rm K}(E) = [\hat{g}^{\rm R} - \hat{g}^{\rm A}]\tanh(E/2T) \tag{17}$$

and in the normal state, where $\hat{g}^{\rm R} - \hat{g}^{\rm A} = 2\hat{\tau}_3$. In this limit, one obtains, e.g., the following results:

$$\frac{1}{\tau^{\rm N}_{\rm ep}} \equiv i(\hat{\Sigma}^{\rm R}_{\rm ph} - \hat{\Sigma}^{\rm A}_{\rm ph})_{++,E=0} = 2\pi\int dE \frac{\alpha^2 F(E)}{\sinh(E/T)} \tag{18}$$

where $\tau^{\rm N}_{ep}$ is the normal-state electron-phonon collision time at the Fermi surface ($E = 0$), and

$$i(\hat{\Sigma}^{\rm R}_{\rm imp} - \hat{\Sigma}^{\rm A}_{\rm imp})_{++} = \frac{1}{2}\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right) \tag{19}$$

While this approximation for the impurities is standard in three dimensions, and leads (for normal metals) to the usual Boltzmann equation, its application for one-dimensional systems is hard to justify.[22] In particular, the localization of the electrons in the random potential and the pinning of the CDW by the impurities are neglected. One must note, however, that actual charge-density-wave systems do have three-dimensioal character, i.e., there is a coupling between different chains through phonons, which introduces

perpendicular gradients into the equation of motion for the phase,[23] and the interaction between electrons leads to a small distortion of the otherwise flat Fermi surface. In the case of commensurability pinning, further arguments can be given.[20,23] Note that the present approximation is used here to derive the dissipative terms in the equation of motion for the phase, and it is expected that a reasonable approximation is obtained by *adding* in this equation the random pinning force due to impurities.*

It must be noted that the equations given above depend explicitly on the phase of the order parameter, which is introduced according to $i\Delta = |\Delta| \exp(-i\chi)$. (In the following, the magnitude sign will be omitted.) To remove this dependence, the transformation

$$\hat{Q}' = \hat{S}^+ * \hat{Q} * \hat{S}, \qquad \bar{\check{G}}' = \bar{\check{S}}^+ * \bar{\check{G}} * \bar{\check{S}} \tag{20}$$

is used, where $\bar{\check{S}} = \hat{S} \cdot \bar{1}$, and $\hat{S} = \exp(-i\hat{\tau}_3\chi/2)$. For example,

$$E' = E + \tfrac{1}{2}\hat{\tau}_3 \partial_t \chi, \qquad p' = p - \tfrac{1}{2}\hat{\tau}_3 \partial_x \chi \tag{21}$$

from which $\hat{Q}'$ is obtained without difficulty. While this transformation leaves the phonon self-energy unchanged, i.e., in Eq. (14), $\bar{\check{g}}$ is replaced by $\bar{\check{g}}'$, one obtains for the impurities

$$i\bar{\check{\Sigma}}'_{\text{imp}} = \frac{1}{4\tau_1}\hat{\tau}_3 \bar{\check{g}}' \hat{\tau}_3 - \frac{1}{8\tau_2}[\hat{\tau}'_1 * \bar{\check{g}}' * \hat{\tau}'_1 + (1 \to 2)] \tag{22}$$

where $\hat{\tau}'_1(\chi) = (\cos\chi)\hat{\tau}_1 - (\sin\chi)\hat{\tau}_2$ and $\hat{\tau}'_2 = \hat{\tau}'_1(\chi - \pi/2)$. For example, if the functional product is expanded in first order, one finds

$$\begin{aligned}&[\hat{\tau}'_1 * \bar{\check{g}}' * \hat{\tau}'_1 + (1 \to 2)] \\ &\quad \approx [\hat{\tau}_1 \bar{\check{g}}' \hat{\tau}_1 + (1 \to 2)] - i(\partial_t \chi)\, \partial_E [\hat{\tau}_1 \bar{\check{g}} \hat{\tau}_2 - (1 \leftrightarrow 2)]\end{aligned} \tag{23}$$

Physically, the correction $\sim(\partial_t \chi)$ is related to the fact that a moving charge-density wave is damped by impurity collisions with large momentum transfer, which are represented by τ_2.

In the following, I investigate the quasiclassical approximation for the model defined above, i.e., by Eqs. (7), (8), (13), (14), and (16); it is assumed that the transformation (20) has been performed.

3. QUASICLASSICAL GREEN'S FUNCTIONS

From the equations given in the preceding section the quasiclassical equations are obtained according to the following steps.

1. As noted above, for the present model, the self-energies can be expressed in terms of the quasiclassical Green's function $\bar{\check{g}}$.

*This point of view was also taken in ref. 25.

2. For electrons close to the Fermi surface, the quadratic spectrum can be approximated by a linear one, and one obtains

$$\hat{Q} \approx (E - e\phi + \tfrac{1}{2} v_F \partial_x \chi)\hat{\tau}_3 - [v_F(\mathrm{p} - eA) - \tfrac{1}{2}\partial_t \chi]\hat{1} + i\Delta\hat{\tau}_1 \tag{24}$$

3. From Eq. (7), it is clear that the equation of motion can be written in commutator form, namely

$$[\bar{\bar{Q}} \underset{;}{*} \bar{\bar{G}}] = [\bar{\bar{\Sigma}} \underset{;}{*} \bar{\bar{G}}] \tag{25}$$

In particular,

$$-v_F[p \underset{;}{*} \bar{\bar{G}}] = iv_F \partial_x \bar{\bar{G}} \tag{26}$$

i.e., the strong momentum dependence has dropped out of (25).

4. With quasiclassical accuracy, all spatial gradients, except in (26), can be omitted. Clearly, the resulting equation can be integrated with respect to the momentum, and one obtains the equation of motion for the quasiclassical Green's function, which is of the following form:

$$[\bar{\bar{Q}} \underset{;}{\circ} \bar{\bar{g}}] + iv_F \partial_x \bar{\bar{g}} = [\bar{\bar{\Sigma}} \underset{;}{\circ} \bar{\bar{g}}] \tag{27}$$

which has to be supplemented by the normalization condition

$$\bar{\bar{g}} \circ \bar{\bar{g}} = \bar{\bar{1}} \tag{28}$$

where the operation $\circ$ is defined by

$$A \circ B = \exp\left[\frac{i}{2}(\partial_E^A \partial_t^B - \partial_t^A \partial_E^B)\right](AB) \tag{29}$$

In addition, the $\circ$ operation will be approximated by the ordinary product on the rhs of Eq. (27), in the spirit of Boltzmann's ansatz. Equations (27) and (28), together with the self-energy equations, form a closed set. In particular, the self-consistency relation in a form in which the phase dependence has been made explicit is given by:

$$\Delta + \omega_Q^{-2}(\ddot{\Delta} - \Delta\dot{\chi}^2) - i\omega_Q^{-2}(\Delta\ddot{\chi} + 2\dot{\Delta}\dot{\chi}) = \frac{\lambda}{4i}\int dE\, \hat{g}_{+-}^{K} \tag{30}$$

where $\dot{\chi} \equiv \partial_t \chi$, etc. While gauge invariance and conservation laws are, of course, a direct consequence of the full equations, this has to be checked for the quasiclassical approximation. Considering first a gauge transformation, define

$$\bar{\bar{S}} = \exp[i\theta(x, t)] \cdot \bar{\bar{1}} \tag{31}$$

and

$$\bar{\bar{g}}' = \bar{\bar{S}}^{+} \circ \bar{\bar{g}} \circ \bar{\bar{S}} \tag{32}$$

Applying (29), one immediately obtains

$$\bar{\bar{Q}}' = \bar{\bar{S}}^{+} \circ \bar{\bar{Q}}(\phi, A) \circ \bar{\bar{S}} = \bar{\bar{Q}}\left(\phi + \frac{1}{e}\dot{\theta}, A\right) \tag{33}$$

In addition, from the gradient term

$$(iv_F\partial_x\bar{\hat{g}})' = iv_F\partial_x\bar{\hat{g}}' - v_F[\partial_x\theta \circ \bar{\hat{g}}'] \tag{34}$$

By noting that

$$\bar{\hat{Q}}' - v_F\partial_x\theta = \bar{\hat{Q}}\left(\phi + \frac{1}{e}\dot{\theta}, A - \frac{1}{e}\partial_x\theta\right) \tag{35}$$

it can be concluded that the quasiclassical equations *are* gauge covariant. A similar observation could be made for superconductors.[15] Note, however, the formal differences in the matrix structure of the present theory as compared to superconductors. In particular, ϕ and A enter here in a reversed way; also, a gauge transformation is described by a unit matrix, and is not equivalent to the transformation that changes the phase of the order parameter. Physically, this is related to the fact that the quantity conjugate to the phase is the momentum, which will become evident shortly, and not the charge, as in the theory of superconductors.

To illustrate this more clearly, consider the conservation of particle number and total momentum (electrons plus phonons), ignoring, for the moment, collision processes. Therefore, I consider the Keldysh component of the equation of motion, and expand the $\circ$ product in first order. The result is

$$[\hat{Q}, \hat{g}^K]_- + \tfrac{1}{2}[\hat{\tau}_3, \partial_t\hat{g}^K]_+ - \tfrac{1}{2}i[\partial_t\hat{Q}, \partial_E\hat{g}^K]_+ + iv_F\partial_x\hat{g}^K = 0 \tag{36}$$

where $[\cdot,\cdot]_{-(+)}$ denotes the commutator (anticommutator). Take the trace of this equation, integrate with respect to energy, and multiply the result by $-N(0)/4$, where $N(0) = (\pi v_F)^{-1}$ is the normal state density of states at the Fermi surface (for one spin). By noting that

$$\int dE\ \partial_E\hat{g}^K = 4\hat{\tau}_3 \tag{37}$$

one obtains the particle number conservation in the usual form:

$$\partial_t\rho + \partial_x j_{el} = 0 \tag{38}$$

where the electron density and current ρ and j_{el} are given by

$$\rho = -\frac{1}{4}N(0)\,\mathrm{tr}\,\hat{\tau}_3 \int dE\,\hat{g}^K - 2N(0)e\phi \tag{39}$$

$$j_{el} = -\frac{1}{4}N(0)v_F\,\mathrm{tr}\int dE\,\hat{g}^K + N(0)v_F\dot{\chi} \tag{40}$$

In the next step, multiply (36) by $\hat{\tau}_3$ before applying the same procedure. Then, however, it is crucial that the self-consistency relation is taken into account, and one finds

$$\partial_t P + v_F^2 \partial_x \rho = (e/m)\rho_0 \varepsilon \tag{41}$$

where the total momentum P is given by

$$P = j_{el} + \frac{1}{\pi}\frac{4\Delta^2}{\lambda\omega_Q^2}\dot{\chi} = -\frac{1}{4\pi}\mathrm{tr}\int dE\,\hat{g}^K + \frac{1}{\pi}m_F\dot{\chi} \tag{42}$$

Here, I introduced $m_F = 1 + 4\Delta^2/\lambda\omega_Q^2$, which is the Fröhlich mass divided by the electron mass; the equilibrium particle density is $\rho_0 = 2p_F/\pi$, and the electric field is $\varepsilon = -\partial_t A - \partial_x \phi$. Note that a constant ($=\rho_0$) has to be added in (39). From these expressions, it becomes apparent that the quasiclassical Green's function only allows the calculation of difference quantities with respect to a suitable reference state, which is usually taken as the normal state in the same external field. This leads to the second term in (39) and (40), respectively; by considering the conservation laws, these expressions have emerged in a natural way.[17]

4. KINETIC EQUATIONS

4.1. Quasiparticle Distribution Function

As a consequence of the normalization condition, it is possible to define a quasiparticle distribution function. Writing out the normalization (28) in explicit form

$$\hat{g}^R \circ \hat{g}^R = \hat{g}^A \circ \hat{g}^A = \hat{1} \tag{43}$$

$$\hat{g}^R \circ \hat{g}^K + \hat{g}^K \circ \hat{g}^A = 0 \tag{44}$$

one realizes that the second equation is solved by the ansatz

$$\hat{g}^K = \hat{g}^R \circ \hat{h} - \hat{h} \circ \hat{g}^A \tag{45}$$

Also, one can argue[15] that $\hat{h}$ can be chosen diagonal, namely

$$\hat{h} = h^L \hat{1} + h^T \hat{\tau}_3 \tag{46}$$

and the longitudinal (L) and transverse (T) parts of the distribution function are introduced by the relations

$$f^L = \tfrac{1}{2}(1 - h^L); \qquad f^T = -\tfrac{1}{2}h^T \tag{47}$$

Note that L (T) refers to changes of the order parameter considered as a vector in the complex plane, to which $f^L(f^T)$ are mainly coupled (see below). In equilibrium [see (17)], one has $h^T = 0$ and $h^L_{th} = \mathrm{th}(E/2T) \equiv h_0$, such that $f^L_{th} \equiv f_0$ is the Fermi function.

In the following, I consider slow variations in space and time compared to Δ^{-1} and $\xi_0 \sim v_F/\Delta$, respectively, and expand the $\circ$ product in first order. Explicitly,

$$A \circ B = A \cdot B + \frac{i}{2}\hbar\left(\frac{\partial A}{\partial E}\frac{\partial B}{\partial t} - \frac{\partial A}{\partial t}\frac{\partial B}{\partial E}\right) + O(\hbar^2) \tag{48}$$

where the dependence on $\hbar$ in ordinary units is indicated. It will be convenient to refer to this expansion as an expansion in $\hbar$; as a rule, scattering rates are of first order, and so are collision operators. On the other hand, derivatives of the phase, as appearing in (24), must be considered of order $\hbar^0$.

4.2. Regular Green's Function

Consider first the equations for the regular functions (R, A) in *thermal equilibrium.* Then one can use the ansatz

$$\hat{g}_0^{R(A)} = \alpha^{R(A)}\hat{\tau}_3 + \beta^{R(A)}\hat{\tau}_1 \tag{49}$$

and obtain from (27) and (43)

$$(E \,{}_{(}{}^{\pm}{}_{)}\, i\Gamma_{ep})\beta^{R(A)} - i\Delta\alpha^{R(A)} = -i\Gamma_i\alpha^{R(A)}\beta^{R(A)} \tag{50}$$

$$(\alpha^{R(A)})^2 + (\beta^{R(A)})^2 = 1 \tag{51}$$

where Γ_{ep} describes the linewidth due to collisions with phonons.* While this quantity depends on energy, it is known that a good estimate is obtained by evaluating it for $E \approx \Delta$. On the other hand, impurity scattering enters into (50) in the same ay as *paramagnetic* impurities in the theory of superconductors; one finds

$$\Gamma_i = \frac{1}{2}\left(\frac{1}{\tau_1} + \frac{1}{2\tau_2}\right) \equiv \frac{1}{2\tau} \tag{52}$$

i.e., both types of scattering contribute to Γ_i. I define for further use so-called spectral densities N_i and R_i $(i = 1, 2)$ by

$$\begin{aligned} N_1 + iR_1 &= \alpha^R = -(\alpha^A)^* \\ N_2 + iR_2 &= \beta^R = (\beta^A)^* \end{aligned} \tag{53}$$

The results for weak scattering (BCS limit), namely $\Gamma_{ep}, \Gamma_i \ll \Delta$, are

$$N_1(E) \approx (|E|/\xi)\Theta(E^2 - \Delta^2) \tag{54a}$$

$$R_2(E) \approx (\Delta/E)N_1(E) \tag{54b}$$

and, for $E^2 > \Delta^2$,

$$N_2(E) \approx (\Delta N_1/\xi^2)(\Gamma_{ep} + N_1\Gamma_i) \tag{54c}$$

*Here and in the following, normalization effects are neglected, which amounts to the replacement $\hat{\Sigma}^{R(A)} \to {}_{(}{}^{\pm}{}_{)}\frac{1}{2}(\hat{\Sigma}^R - \hat{\Sigma}^A)$.

where $\xi^2 = E^2 - \Delta^2$ and $\Theta(\cdot)$ is the step function. In this range of energy, N_2 is of order $\hbar^1$.

From the expression given in (24), it follows that the *local equilibrium* expression ($\sim\hbar^0$) of the regular Green's function is of the form (49), with the replacement $E \to \tilde{E} = E - e\phi + v_F\chi'/2$. Writing $\hat{g}^{R(A)} = \hat{g}_0^{R(A)} + \hbar^1 \hat{g}_1^{R(A)}$, one obtains the following equation for the first-order correction $\hat{g}_1$ (the superscripts R, A are omitted for simplicity, as are collision effects):

$$\begin{aligned} &\tilde{E}[\hat{\tau}_3, \hat{g}_1]_- + i\Delta[\hat{\tau}_1, \hat{g}_1]_- + \tfrac{1}{2}i[\hat{\tau}_3, \dot{\hat{g}}_0]_+ \\ &\quad -\tfrac{1}{2}i\dot{\tilde{E}}[\hat{\tau}_3, \partial_E \hat{g}_0]_+ + \tfrac{1}{2}\dot{\Delta}[\hat{\tau}_1, \partial_E \hat{g}_0]_+ \\ &\quad + iv_F\partial_x \hat{g}_0 - i(ev_F\dot{A} + \ddot{\chi}/2)\partial_E \hat{g}_0 = 0 \end{aligned} \tag{55a}$$

Equation (55) simplifies considerably if variations in the magnitude Δ are ignored. In this case, $\hat{g}_1$ has to be calculated from

$$\tilde{E}[\hat{\tau}_3, \hat{g}_1]_- + i\Delta[\hat{\tau}_1, \hat{g}_1]_- = i(\partial_E \hat{g}_0)\psi \tag{55b}$$

where I defined

$$\psi = \tfrac{1}{2}(\ddot{\chi} - v_F^2\chi'') - ev_F\varepsilon \tag{56}$$

One immediately concludes that $\hat{g}_1 = \gamma\hat{\tau}_2$, and

$$\gamma = \psi(\partial_E \alpha)/(2i\Delta) \tag{57}$$

Note that γ does not depend on the electromagnetic potentials separately, but rather on the combination which gives the electric field.

4.3. Transport Equations

For the derivation of the transport equation, consider the diagonal part of the Keldysh component of (27), into which (45) is inserted. Multiplying by $i/4$, one obtains in a first tep for the first term of the lhs of (27)

$$\tfrac{1}{4}i\{\hat{g}^R \circ [\hat{Q} \overset{\circ}{,} \hat{h}] + [\hat{Q} \overset{\circ}{,} \hat{g}^R] \circ \hat{h} - \hat{h} \circ [\hat{Q} \overset{\circ}{,} \hat{g}^A] - [\hat{Q} \overset{\circ}{,} \hat{h}] \circ \hat{g}^A\} \tag{58}$$

Clearly, one can use the equations for the regular functions to simplify this expression. Then one inserts for $\hat{g}^{R(A)}$ the following ansatz:

$$\hat{g}^{R(A)} = \hat{\tau}_3\alpha^{R(A)} + \hat{\tau}_1\beta^{R(A)} + \hat{\tau}_2\gamma^{R(A)} \tag{59}$$

and defines, similar to (53),

$$N_3 + iR_3 = \gamma^R = (\gamma^A)^* \tag{60}$$

Then one expands the $\circ$ product through first order in $\hbar$, and obtains two equations, corresponding to the $\hat{1}$ and $\hat{\tau}_3$ components of the equation of motion. To simplify the notation, I introduce

$$\tilde{\phi} = e\phi - \tfrac{1}{2}v_F\chi', \qquad \tilde{A} = ev_F A + \tfrac{1}{2}\dot{\chi} \tag{61}$$

and

$$\{\{A, B\}\} = \frac{\partial A}{\partial E}\frac{\partial B}{\partial t} - \frac{\partial A}{\partial t}\frac{\partial B}{\partial E} \tag{62}$$

One obtains the following transport equations:

$$\begin{aligned} &N_1(\partial_t f^{\mathrm{L}} + \dot{\tilde{\phi}}\partial_{\mathrm{E}} f^{\mathrm{L}} - \dot{\tilde{A}}\partial_{\mathrm{E}} f^{\mathrm{T}} + v_{\mathrm{F}}\partial_x f^{\mathrm{T}}) \\ &\quad + 2\Delta R_3 f^{\mathrm{T}} + R_2\dot{\Delta}\partial_E f^{\mathrm{L}} + \{\{N_3, \Delta f^{\mathrm{T}}\}\} = K^{\mathrm{L}} \end{aligned} \tag{63}$$

$$\begin{aligned} &N_1(\partial_t f^{\mathrm{T}} + \dot{\tilde{\phi}}\partial_{\mathrm{E}} f^{\mathrm{T}} - \dot{\tilde{A}}\partial_{\mathrm{E}} f^{\mathrm{L}} + v_{\mathrm{F}}\partial_x f^{\mathrm{L}}) \\ &\quad + 2\Delta N_2 f^{\mathrm{T}} - N_3\dot{\Delta}\partial_E f^{\mathrm{L}} - \{\{R_2, \Delta f^{\mathrm{T}}\}\} = K^{\mathrm{T}} \end{aligned} \tag{64}$$

Here, the remaining effects of collision processes are included through K^{L} and K^{T}. In these equations, the expansion with respect to $\hbar$ of the spectral quantities has to be inserted. In particular, since $N_3 \sim \hbar^1$, the terms proportional N_3 are of higher order.

Before discussing the collision operators, I present the expressions for density and momentum in terms of the distribution functions. Working with accuracy $\sim\hbar^0$, one obtains from (39) and (42)

$$\rho = \frac{2}{\pi v_{\mathrm{F}}}\left[\fint dE\, N_1\left(f^{\mathrm{L}} - \frac{1}{2}\right) - e\phi\right] \tag{65}$$

$$P = \frac{2}{\pi}\int dE\, N_1 f^{\mathrm{T}} + \frac{m_{\mathrm{F}}}{\pi}\dot{\chi} \tag{66}$$

Finally, for the self-consistency equation (30), the off-diagonal part of the Keldysh component is expressed in terms of $f^{\mathrm{L(T)}}$ as follows:

$$\begin{aligned} \frac{1}{4i}[\hat{g}^{\mathrm{K}}]_{+-} = &\frac{1}{2}R_2(1-2f^{\mathrm{L}}) + N_3 f^{\mathrm{T}} - i\left[N_2 f^{\mathrm{T}} + \frac{1}{2}R_3(1-2f^{\mathrm{L}})\right] \\ &- \frac{1}{2}\{\{N_2, f^{\mathrm{L}}\}\} + \frac{i}{2}\{\{R_2, f^{\mathrm{T}}\}\} + \cdots \end{aligned} \tag{67}$$

Note, especially, the equation obtained by taking the imaginary part of (30), namely

$$\begin{aligned} &\omega_Q^{-2}(\Delta\ddot{\chi} + 2\dot{\Delta}\dot{\chi}) \\ &\quad = \Delta\omega_Q^{-2}\cdot F(\chi) + \lambda\int dE\left[N_2 f^{\mathrm{T}} - \frac{1}{2}\{\{R_2, f^{\mathrm{T}}\}\} + \frac{1}{2}R_3(1-2f^{\mathrm{L}})\right] \end{aligned} \tag{68}$$

Here, I also included the force arising in the case of commensurability pinning,[9] which is derived by an extension of the methods discussed.[20] On the other hand, the standard BCS gap equation is recovered from the real part of (30) by ignoring time derivatives and commensurability effects and inserting for f_{L} the Fermi function as well as $R_2 = \Delta N_1/E$.

4.4. Collision Operators

From the derivation of the kinetic equations, it has become clear that one has to consider the following expression:

$$\begin{pmatrix} K^{\mathrm{L}} \\ K^{\mathrm{T}} \end{pmatrix} = \frac{i}{8} \mathrm{tr} \left\{ \begin{pmatrix} \hat{1} \\ \hat{\tau}_3 \end{pmatrix} [\hat{\Sigma}, \hat{g}]^{\mathrm{K}} \right\} \tag{69}$$

In the self-energy, one has contributions from electron–phonon and impurity scattering, and therefore one writes

$$K^{\mathrm{L(T)}} = K_{ep}^{\mathrm{L(T)}} + K_{\mathrm{imp}}^{\mathrm{L(T)}} + P^{\mathrm{L(T)}} \tag{70}$$

where K_{imp} and P arise from impurity contributions. In particular, consider the limit $\dot{\chi} \ll \Delta$, in which case P is related to the *second* term of the rhs of (23).

4.4.1. K_{ep}

From the analogy with the theory of superconductors, discussed in detail in Section 2, it is clear that one arrives at collision operators of similar form. Here the linearized version of the collision integral is considered, i.e., one puts

$$f^{\mathrm{L}} = f_0 + \delta f^{\mathrm{L}} \tag{71}$$

and assumes that δf^{L} and f^{T} are small. The result can be found, e.g., in refs. 14 and 15. An important property of the collision integrals is

$$\int dE\, K_{ep}^{\mathrm{L,T}} = 0 \tag{72}$$

which in the present case is related to the conservation of particle number and momentum.* Also, it must be noted that K_{ep} consists of so-called "scattering-out" and "scattering-in" contributions, the latter being a complicated integral operator. On the other hand, the "scattering-out" is given by

$$\begin{pmatrix} K^{\mathrm{L}} \\ K^{\mathrm{T}} \end{pmatrix}_{\mathrm{out}} = -N_1 \begin{pmatrix} \delta f^{\mathrm{L}}/\tau_{ep}^{\mathrm{L}} \\ f^{\mathrm{T}}/\tau_{ep}^{\mathrm{T}} \end{pmatrix} \tag{73}$$

For T close to T_c, $\tau_{ep}^{\mathrm{L}} \approx \tau_{ep}^{\mathrm{T}} \approx \tau_{ep}^{\mathrm{N}}$, while for low temperatures,[26] $T \ll T_c$,

$$\tau_{ep}^{\mathrm{L}} \approx (T_c/T)\tau_{ep}^{\mathrm{T}} \approx (T_c/T)^{1/2}\tau_{ep}^{\mathrm{N}} \tag{74}$$

where the scattering times† have been evaluated for $E \approx \Delta$. In the following, an approximation is considered for $K_{ep}^{\mathrm{L,T}}$ that preserves the conservation laws and agrees in the scattering-out contribution with the exact expression.

*Note that momentum conservation in electron–phonon collisions in only approximate (see Section 1, and also Section 6).

†Note that the linewidth [see (50)] is given by $\Gamma_{ep} = (2\tau_{ep}^{\mathrm{L}})^{-1}$.

Also, it is a reasonable approximation to take the rates independent of energy. This approximation, which elsewhere[26] was called the "reduced" collision operator, has the following form:

$$K^{\mathrm{L}}_{ep} \to K^{\mathrm{L}}_{\mathrm{red}} = -\frac{N_1}{\tau^{\mathrm{L}}_{ep}}\left[\delta f^{\mathrm{L}} - \frac{(-f_0')}{Y}\int dE'\, N_1(E')\delta f^{\mathrm{L}}_{E'}\right] \tag{75}$$

and similarly for the T mode, where $f_0' = \partial f_0/\partial E$, and Y is the Yoshida function:

$$Y = \int dE\, N_1(-f_0') \tag{76}$$

This approximation, which provides a considerable simplification, but nevertheless gives very good results, will be used later.

4.4.2. P

The quantity P is given by

$$\begin{pmatrix} P^{\mathrm{L}} \\ P^{\mathrm{T}} \end{pmatrix} = \frac{i}{8}\frac{\dot{\chi}}{8\tau_2}\,\mathrm{tr}\left\{\begin{pmatrix} \hat{1} \\ \hat{\tau}_3 \end{pmatrix}[\hat{\tau}_1\partial_E\bar{\hat{g}}\hat{\tau}_2 - (1\leftrightarrow 2), \bar{\hat{g}}]^{\mathrm{K}}\right\} \tag{77}$$

Inserting (49) and $\hat{h} = h_0\hat{1}$, one finds $P^{\mathrm{L}} = 0$, and

$$P^{\mathrm{T}} = (\dot{\chi}/2\tau_2)N_1^2\,\partial f_0/\partial E \tag{78}$$

Note that P^{T} is proportional to the backward scattering rate. The general result, where $\dot{\chi}$ is not assumed to be small, is

$$P^{\mathrm{T(L)}} = \frac{N_1(E)}{4\tau_2}\{N_1(E+\dot{\chi})[f_0(E+\dot{\chi}) - f_0(E)]_{(\mp)}[\dot{\chi}\to -\dot{\chi}]\} \tag{79}$$

Here, however, higher derivatives of the phase have been neglected. Note the analogy to the theory of tunneling between superconductors.[27]

4.4.3. K_{imp}

Finally, I evaluate what is called here the impurity collision operator, namely (69) into which the expression [see (16)]

$$-\frac{i}{4\tau_1}\hat{\tau}_3\bar{\hat{g}}\hat{\tau}_3 + \frac{i}{8\tau_2}[\hat{\tau}_1\bar{\hat{g}}\hat{\tau}_1 + (1\to 2)] \tag{80}$$

is inserted for the self-energy. It is straightforward to see, however, that forward scattering does not contribute to K_{imp}, and that the L-mode collision operator is identically zero. Thus, one obtains the result

$$K^{\mathrm{L}}_{\mathrm{imp}} = 0, \qquad K^{\mathrm{T}}_{\mathrm{imp}} = -(1/\tau_2)N_1^2 f^{\mathrm{T}} \tag{81}$$

Physically, this is connected to the fact that forward scattering cannot equilibrate a nonequilibrium distribution related to a change of density or momentum, and this is also true for backward scattering with respect to density fluctuations. Thus, τ_2^{-1} appears only in (81), and only in the T mode related to changes of the momentum.

5. PHASE MODE

Here the kinetic equations are considered in more detail in order to derive the equation of motion for the phase of the order parameter in the presence of electron-phonon and impurity scattering. From an inspection of the Boltzmann equations, it becomes apparent that a solution should be sought in which $\tilde{E} = E - \tilde{\phi}$ instead of E appears as an independent variable. Defining new distribution functions by

$$\tilde{f}(\tilde{E}; x, t) = f(E; x, t) \tag{82}$$

one can use the following relation:

$$\partial_t f(E; x, t) = \partial_t \tilde{f}(\tilde{E}; x, t) - \dot{\tilde{\phi}}\, \partial \tilde{f}/\partial \tilde{E} \tag{83}$$

where it is understood that the time derivative is for fixed E ($\tilde{E}$) on the left (right) hand side of this equation. A similar relation holds for spatial derivatives. Clearly, the combination

$$\psi = v_F\, \partial_x \tilde{\phi} + \partial_t \tilde{A} \tag{84}$$

arises in the equation [see (56)]. Note that the Poisson bracket $\{\{\cdot, \cdot\}\}$ is not affected by this transformation. *The "~" is neglected in the following.* Ignoring also the terms $\sim N_3$, as discussed above, and derivatives of the magnitude of the order parameter, one obtains

$$N_1(\partial_t f^{\mathrm{L}} - \psi \partial_E f^{\mathrm{T}} + v_F \partial_x f^{\mathrm{T}}) + 2\Delta R_3 f^{\mathrm{T}} = K_{ep}^{\mathrm{L}} \tag{85}$$

$$N_1(\partial_t f^{\mathrm{T}} - \psi\, \partial_E f^{\mathrm{L}} + v_F \partial_x f^{\mathrm{L}}) + 2\Delta[N_2 - \tfrac{1}{2}(\partial_E R_2)\partial_t] f^{\mathrm{T}} = K_{ep}^{\mathrm{T}} + K_{\mathrm{imp}}^{\mathrm{T}} + P^{\mathrm{T}} \tag{86}$$

where the collision terms are given in Section 4.4. Inserting $f^{\mathrm{L}} = f_0$ and the expression for R_3 obtained from (57) in the Ginzburg-Landau equation (68), one finds

$$\begin{aligned}(m_F - 1)[\ddot{\chi} - F(\chi)] + (1 - Y)(\ddot{\chi} - v_F^2 \chi'' - 2ev_F \varepsilon) \\ = 4\Delta \int dE \left[N_2 - \frac{1}{2}(\partial_E R_2)\partial_t \right] f^{\mathrm{T}}\end{aligned} \tag{87}$$

where

$$F(\chi) = -\frac{2\pi v_F}{m_F - 1} \frac{\partial V(\chi)}{\partial \chi} \tag{88}$$

is the force due to the commensurability pinning potential $V(\chi)$. Note that the transformation defined in (82) changes the expression for the density into the following:

$$\rho = \frac{2}{\pi v_F} \int dE\, N_1 \left(f^L - \frac{1}{2} \right) - \frac{1}{\pi} \chi' \tag{89}$$

while the momentum, Eq. (66), is not modified. Note that the (charge) current is given by

$$j = e j_{\ni 1} = \frac{e}{\pi} \left(2 \int dE\, N_1 f^T + \dot{\chi} \right) \tag{90}$$

Clearly, by putting $f^T = 0$ and thereby ignoring dissipative effects, one recovers the well-known equation of motion for the phase from (87), namely $\ddot{\chi} - c_0^2 \chi'' = F(\chi) + 2 e_0^* v_F \varepsilon$, where $c_0^2 = v_F^2 (1 - Y)/(m_F - 1)$ and $e_0^* = e(1 - Y)/(m_F - 1)$.

For the present case, the solution of the transport equations can be further simplified, since deviations from equilbrium are small. Therefore, one may use

$$\partial_E f^L \approx \partial_E f_0$$

in (86), and this term as well as P^T can be considered a "drive" term for the T-mode distribution function. In addition, one can ignore the terms $\sim R_3$ and $\sim \partial_E f^T$ in (85), since these will produce corrections quadratic in ψ. Fourier transforming the resulting equations yields

$$N_1(-i\omega \delta f^L + i v_F q f^T) = K_{ep}^L \tag{91}$$

$$\begin{aligned} N_1(-i\omega f^T + i v_F q \delta f^L) + 2\Delta N_2(\omega) f^T \\ = K_{ep}^T - \frac{1}{\tau_2} N_1^2 f^T + N_1 \left[\psi + \frac{N_1}{2\tau_2} (\dot{\chi})_{\omega,q} \right] (\partial_E f_0) \end{aligned} \tag{92}$$

where $N_2(\omega) = N_2 + i\omega\, \partial_E R_2/2$. Note that τ_1 enters only through N_2 into these equations. Equations (91) and (92) can be solved, provided the electron–phonon collision integral is replaced by the "reduced" operator (75) in terms of a few integrals that have to be evaluated numerically (except in limiting cases). Note the N_2 term, which is characteristic for the transverse mode, and especially its sharp increase for energies close to the gap [see (54c)].

5.1. Momentum Relaxation (only)[20]

As a first application, consider a situation in which energy relaxation is negligible. Putting $K_{ep} = 0$, it is straightforward to solve the kinetic

equations, and one finds

$$\{-i\omega N_1[1-(v_F q/\omega)^2]+N_1^2/\tau_2+2\Delta N_2(\omega)\}f^T$$
$$= N_1[\psi+N_1(\dot{\chi})_{\omega,q}/2\tau_2]f_0' \tag{93}$$

Clearly, there are two contributions to f^T, corresponding to the two "drive" terms on the rhs of this equation. Therefore, inserting the solution into (87) gives two terms, which are proportional to ψ and $\dot{\chi}$, respectively, with, however, rather complicated (ω, q, T)-dependent prefactors.

As an illustration, consider the limit ω, $(v_F q)^2/\omega \ll \tau_2^{-1}$. In this case, (93) simplifies since the ω and q dependences can be neglected on the lhs of this equation. The resulting equation of motion is of the following form:

$$\ddot{\chi}+\gamma_i\dot{\chi}=F(\chi)+\kappa_i(c_0^2\chi''+2e_0^* v_F\varepsilon) \tag{94}$$

Note that the appearance of γ_i and the fact that $\kappa_i \neq 1$ are related to the nonthermal distribution of quasiparticles, as expressed by $f^T \neq 0$. Since $m_F \gg 1$, one could neglect $\ddot{\chi}$ in ψ. The γ_i and κ_i are given by the following expressions:

$$\gamma_i=\frac{1}{m_F\tau_2}\int dE\,(-f_0')\frac{E^2}{\xi^2}\frac{\Delta^2/\tau}{\xi^2/\tau_2+\Delta^2/\tau} \tag{95}$$

$$\kappa_i=1+(1-Y)^{-1}\int dE\,(-f_0')\frac{|E|}{\xi}\frac{\Delta^2/\tau}{\xi^2/\tau_2+\Delta^2/\tau} \tag{96}$$

where the integration is over the region $E^2 \geq \Delta^2$; recall that $\xi^2=E^2-\Delta^2$ $(\xi>0)$, and $\tau^{-1}=\tau_1^{-1}+(2\tau_2)^{-1}$. Also, in (95) one has to take into account a finite linewidth Γ_{ep}, since the integral is divergent for $\Gamma_{ep}\to 0$. (The authors of Ref. 20 consider, instead, the bending of the Fermi surface.) For high and low temperatures, these expressions can be evaluated, and one obtains:

$$\gamma_i \approx \frac{1}{m_F\tau_2}\begin{cases}(\Delta/4T)\ln(\Delta/\Gamma_{ep}) & \Delta \ll T\\ (\Delta/T)\,e^{-\Delta/T}\ln(T/\Gamma_{ep}) & \Delta \gg T\end{cases} \tag{97}$$

$$\kappa_i \approx \begin{cases}(1-Y)^{-1}(\pi\Delta/4T)(\tau_2/\tau)^{1/2} & \Delta \ll T\\ 1 & \Delta \gg T\end{cases} \tag{98}$$

where, for $\Delta \ll T$, use has also been made of the condition $\Delta^2/\tau \ll T^2/\tau_2$. The Yoshida function is given by

$$Y \approx \begin{cases}1-[7\zeta(3)/4\pi^2](\Delta/T)^2 & \Delta \ll T\\ (2\pi\Delta/T)^{1/2}\,e^{-\Delta/T} & \Delta \gg T\end{cases} \tag{99}$$

Taking into account that $m_F \sim \Delta^2$, it is found that close to the transition temperature, $\gamma_i \sim \Delta^{-1}\ln\Delta$, and $\kappa_i c_0^2 \sim \Delta^{-1}$, $\kappa_i e_0^* \sim \Delta^{-1}$. These results also

indicate the limitations of the techniques used elsewhere.[25,*] The results obtained here are in agreement with ref. 20.

5.2. Energy Relaxation (only)

For illustration, I also consider the limit of dominant energy relaxation, such that momentum relaxation can be neglected completely. For this case, it has been demonstrated[14–16,26] (see also ref. 28) that the following local equilibrium ansatz gives an excellent description of the energy dependence of the distribution functions:

$$f^{\mathrm{T}} = (\xi^2/E^2)(-f_0')\eta^{\mathrm{T}}\Theta(E^2-\Delta^2) \tag{100}$$

$$\delta f^{\mathrm{L}} = (-f_0')\eta^{\mathrm{L}} \tag{101}$$

where η^{L} and η^{T} are constants to be determined. Here the linear decrease of f^{T} for energies close to the gap reflects the increase of the function N_2, (54c). By integrating the kinetic equations with respect to energy, taking the conservation laws into account, one obtains the following equations, which can be easily solved for $\eta^{\mathrm{L,T}}$:

$$-i\omega Y\eta^{\mathrm{L}} + iv_{\mathrm{F}}qZ\eta^{\mathrm{T}} = 0 \tag{102}$$

$$[-i\omega Y + (1/\tau_{ep})(Y-Z)]\eta^{\mathrm{T}} + iv_{\mathrm{F}}qY\eta^{\mathrm{L}} = -Y\psi \tag{103}$$

where I have defined the function Z:

$$Z = \int dE\, N_1 \frac{\xi^2}{E^2}(-f_0') \tag{104}$$

whose limiting values are given by

$$Z \approx \begin{cases} 1-(\pi\Delta/4T)+[7\zeta(3)/4\pi^2](\Delta/T)^2 & \Delta \ll T \\ (T/\Delta)Y & \Delta \gg T \end{cases} \tag{105}$$

In addition, $\tau_{ep} \equiv \tau_{ep}^{\mathrm{L}}$. The solution of (102) and (103) leads to the following result:

$$2\Delta \int dE\, N_2(\omega) f^{\mathrm{T}} = -\frac{Y\psi}{1-i\omega\tau_{\mathrm{R}}(\omega)[1-(v_{\mathrm{F}}q/\omega)^2]} \tag{106}$$

where I have introduced (also for historical reasons) the quantity $\tau_R(\omega)$:

$$\tau_R(\omega) = \tau_{ep}\frac{Z}{Y-Z}(1-i\omega\tau_{ep})^{-1} \tag{107}$$

Note that $\tau_R(0)$ is the relaxation time characteristic for the branch imbalance;[11,14] in particular, close to T_c, it is given by $\tau_R(0) \approx \tau_{ep}(4T/\pi\Delta)$. On

*For a review of the phase Hamiltonian approach, see ref. 29; see also ref. 30.

the other hand, in the limit $\omega\tau_{ep} \gg 1$, $q=0$, one immediately makes contact with the results discussed in refs. 6 and 10. Namely, from (103), one can conclude that in this case the leading term is given by $\eta^{\mathrm{T}} = \psi/i\omega$, which leads to the following relation between current [see (90)] and time derivative of the phase:

$$j = (e/\pi)\nu\dot{\chi}, \qquad \nu = 1 - Z \tag{108}$$

where the electric field was ignored in this relation. Note the characteristic temperature dependence of the ratio $j/\dot{\chi} \sim \Delta$ (near T_c), which has been confirmed experimentally.[2] The appropriate equation of motion in this limit turns out to be given by ($\omega\tau_{ep} \gg 1$)

$$\ddot{\chi} = F(\chi) + 2e_\nu^* v_{\mathrm{F}}\varepsilon \tag{109}$$

where $e_\nu^* = e\nu/(m_{\mathrm{F}} - 1)$.

5.2.1. Equation of Motion ($q \to 0$)

In the limit $q \to 0$, but arbitrary $\omega\tau_{ep}$, one finds an equation of motion which is the analogue of the equation governing the longitudinal A-phase spin dynamics[12,16] of superfluid ^{3}He. Here, of course, it is modified by the coupling between electrons and photons ($m_{\mathrm{F}} > 1$) and by the appearance of the electric field. In an intermediate step, one obtains

$$\begin{aligned}(m_{\mathrm{F}} - 1)&\left\{\ddot{\chi}\left[1 + \frac{1-Y}{m_{\mathrm{F}}-1}\right] - F(\chi) - 2e_0^* v_{\mathrm{F}}\varepsilon\right\}_\omega \\ &+ [1 - i\omega\tau_R(\omega)]^{-1} Y(\ddot{\chi} - 2ev_{\mathrm{F}}\varepsilon)_\omega = 0\end{aligned} \tag{110}$$

which can be transformed back to the time variable after multiplying with appropriate frequency-dependent factors. Defining $\gamma_{ep} = (Y - Z)/Y\tau_{ep}$, one finds

$$\begin{aligned}&(\partial_t + \gamma_{ep})[\ddot{\chi} - F(\chi)] + (\nu\partial_t + \gamma_{ep})\ddot{\chi}/(m_{\mathrm{F}} - 1) \\ &\quad = 2e_\nu^* v_{\mathrm{F}}(\partial_t + \gamma_{ep}/\nu)\varepsilon\end{aligned} \tag{111}$$

which can be written in the following form:

$$\dddot{\chi} + \gamma_{ep}\ddot{\chi} - \frac{m_{\mathrm{F}}}{m_{\mathrm{F}} + \nu - 1}\dot{\chi}\frac{\partial F}{\partial \chi} - \gamma_{ep}F = 2e_\nu^* v_{\mathrm{F}}\left(\partial_t + \frac{\gamma_{ep}}{\nu}\right)\varepsilon \tag{112}$$

More precisely, in this equation one should also multiply γ_{ep} by $m_{\mathrm{F}}/(m_{\mathrm{F}} + \nu - 1)$, and F by $(m_{\mathrm{F}} - 1)/m_{\mathrm{F}}$, on the lhs, and e_ν^* by $(m_{\mathrm{F}} - 1)/(m_{\mathrm{F}} + \nu - 1)$. These factors are close to unity. Note that, close to T_c, $\gamma_{ep} \approx (\pi\Delta/4T)\tau_{ep}^{-1}$, while for $T \ll T_c$, $\gamma_{ep} \approx \tau_{ep}^{-1}$. On the other hand, $\nu/\gamma_{ep} \approx \tau_{ep}$ for all temperatures.

As an illustration, consider the linear response to a field of the form $\varepsilon = \varepsilon_\omega \exp(-i\omega t)$. In this case, one may put $F(\chi) \approx -\omega_0^2 \chi$, where ω_0 is the characteristic frequency of the commensurability potential. From Eq. (112), one obtains

$$\chi_\omega = -\frac{2e_\nu^* v_F}{\gamma_{ep}} \frac{-i\omega + \gamma_{ep}/\nu}{(1 - i\omega/\gamma_{ep})(\omega^2 - \omega_0^2) + i\omega\Delta\omega_{ep}} \varepsilon_\omega \tag{113}$$

where, with $\nu = 1 - Z$,

$$\Delta\omega_{ep} = \frac{\omega_0^2 Z}{\gamma_{ep}(m_F - Z)} = \frac{\omega_0^2 \tau_{ep}}{m_F - Z} \frac{ZY}{Y - Z} \tag{114}$$

For high and low temperatures, one finds ($m_F \gg 1$)

$$\Delta\omega_{ep} \approx \frac{1}{m_F} \omega_0^2 \tau_{ep} \begin{cases} 4T/\pi\Delta & \Delta \ll T \\ (2\pi T/\Delta)^{1/2} e^{-\Delta/T} & \Delta \gg T \end{cases} \tag{115}$$

Consequently, one obtains $\Delta\omega_{ep}/\omega_0^2 \sim (T_c - T)^{-3/2}$, close to the transition temperature; on the other hand, $\Delta\omega_{ep}$ is exponentially small for $T \ll T_c$, which reflects the gap in the quasiparticle excitation spectrum.

5.2.2. Conductivity ($q \to 0$; $\omega\tau_{ep} \ll 1$; $\Delta \ll T$)

It has become apparent from the preceding discussion, and is also intuitively clear, that quasiparticle effects are suppressed at low temperatures, while important changes can be found close to T_c. The latter case is considered in more detail, and especially the conductivity in the limit $q \to 0$, $\omega\tau_{ep} \ll 1$. From Eq. (103),

$$\eta^T = -\frac{\tau_R}{1 - i\omega\tau_R} \frac{1}{2}(\ddot{\chi} - 2ev_F\varepsilon)_\omega \tag{116}$$

where $\tau_R \equiv \tau_R(0) = \tau_{ep} \cdot 4T/\pi\Delta \gg \tau_{ep}$. This can be combined with the expression for the current [see (90)], namely

$$j \approx (e/\pi)(2\eta^T + \dot{\chi}) \tag{117}$$

and the with the result obtained above, (113), to give the conductivity. One finds $\sigma_\omega = \sigma_\omega^{(1)} + \sigma_\omega^{(2)}$, with (note that $2v_F/\pi = \rho_0/m$)

$$\sigma_\omega^{(1)} = \frac{2e^2 v_F}{\pi} \frac{\tau_R}{1 - i\omega\tau_R} \tag{118}$$

and

$$\sigma_\omega^{(2)} = \frac{2e^2 v_F}{\pi(m_F - 1)} \frac{i\omega/(1 - i\omega\tau_R)}{(1 - i\omega\tau_R)(\omega^2 - \omega_0^2) + i\omega\,\Delta\omega_{ep}} \tag{119}$$

Note that $\sigma_\omega^{(1)}$ diverges for $\omega = 0$, $T \to T_c$, which reflects the approximation of neglecting the momentum-nonconserving contribution in electron-phonon collisions.

5.3. Energy and Momentum Relaxation

This section investigates the dynamics in the presence of electron–phonon and impurity collisions. In general, there are three parameters, namely τ_{ep}^{N}, τ_1, and τ_2, characterizing the strength of the electron–phonon and impurity scattering (recall that τ_{ep}^{N} is the scattering time at T_c). In most practical cases, however, $\tau_1 \sim \tau_2$, and this will be the main concern here. To simplify the discussion, the kinetic equations are investigated close to the transition temperature, i.e., for $\Delta \ll T$, and in the long-wavelength limit $q \to 0$.

As has become apparent from the preceding subsection, a more detailed investigation is necessary in the presence of momentum-conserving and nonconserving collisions. I start with a general discussion of the solution of the Boltzmann equation, within the approximation of the reduced operator. By noting that, close to T_c, $\tau_{ep}^{T} \approx \tau_{ep}^{L} \equiv \tau_{ep}$, one immediately obtains the solution of (92) in the following form:

$$\left[\left(-i\omega+\frac{1}{\tau_{ep}}\right)\frac{E^2}{\xi^2}+\frac{|E|}{\xi}\left(\frac{1}{\tau_2}+\frac{\Delta^2}{\xi^2}\frac{1}{\tau}\right)\right]f^{T}=f_0'\left(\tilde{\psi}+\frac{|E|}{\xi}\frac{(\dot{\chi})_\omega}{2\tau_2}\right) \tag{120}$$

Here, only the range $E^2 > \Delta^2$ is considered, and $N_1 = |E|/\xi$. The quantity $\tilde{\psi}$ has to be determined self-consistently, namely from

$$\tilde{\psi} = \psi - \frac{1}{\tau_{ep}}\int dE\, N_1 f^{T} \tag{121}$$

where the second term arises from the "scattering-in" term of the reduced operator, and Y has been put equal to ~ 1, since $\Delta \ll T$. Clearly, without impurity collisions, the local equilibrium solution (100) emerges. Close to the transition temperature, the integrals in (121) can be evaluated, and one finds

$$\tilde{\psi}\left(-i\omega+\frac{1}{\tau_R^{\alpha}}+\frac{1}{\tau_2}\right)=\psi\left(-i\omega+\frac{1}{\tau_{ep}}+\frac{1}{\tau_2}\right)+\frac{1}{\tau_{ep}}\frac{(\dot{\chi})_\omega}{2\tau_2} \tag{122}$$

where τ_R^{α} is given by

$$\tau_R^{\alpha} = \tau_R \frac{1-\alpha}{1-\alpha^{3/2}} \tag{123}$$

where, as before, $\tau_R = \tau_{ep} \cdot 4T/\pi\Delta$, and

$$\alpha = \frac{1/\tau}{-i\omega + 1/\tau_{ep} + 1/\tau_2} \tag{124}$$

Clearly, in the clean limit, $\alpha \to 0$, and the results of the preceding subsection are recovered. On the other hand, for relatively dirty samples and dominating

forward scattering, such that $\tau_1^{-1} \gg \tau_{ep}^{-1} \gg \omega, \tau_2^{-1}$, one obtains $\alpha \approx \tau_{ep}/\tau \gg 1$, and (123) is identical to the relaxation rate calculated by Schmid and Schön.[14]

To derive the equation of motion for the phase, these results have to be inserted into (87), which, for $\Delta \ll T$(but $m_F \gg 1$), can be considered in the following simplified form:

$$m_F(\ddot{\chi} - F)_\omega = 4\Delta \int dE\, N_2(\omega) f^T \tag{125}$$

In addition, in evaluating the rhs of this equation, an approximation has been introduced which formally replaces a density of states multiplying the impurity rates at some places by unity.* Then one obtains

$$(\ddot{\chi} + \gamma_i \dot{\chi} - F)_\omega = -\frac{\tilde{\psi}}{m_F} \frac{\pi\Delta}{2T} \left(\frac{-i\omega + 1/\tau_{ep} + 1/\tau}{-i\omega + 1/\tau_{ep} + 1/\tau_2} \right)^{1/2} \tag{126}$$

where γ_i and $\tilde{\psi}$ are given by Eqs. (97) and (122), respectively. As an application, consider now the expansion with respect to the frequency ω of the rhs of this equation. As a result, one obtains the standard equation of motion, namely (after transforming to the time variable)

$$\ddot{\chi} + \gamma\dot{\chi} = F + 2e^* v_F \varepsilon \tag{127}$$

where γ and e^* are given by the following expressions (note that α is to be evaluated for $\omega = 0$):

$$\gamma = \gamma_i + \tilde{\gamma} \tag{128}$$

$$\tilde{\gamma} = \frac{A}{m_F} \frac{1/\tau_R}{1 + \tau_2/\tau_R^\alpha} \tag{129}$$

and

$$e^* = \frac{e}{m_F} A \frac{\pi\Delta}{4T} \frac{1 + \tau_2/\tau_{ep}}{1 + \tau_2/\tau_R^\alpha} \tag{130}$$

where A is a dimensionless constant close to unity, namely $A^2 = (\tau_{ep}^{-1} + \tau^{-1})/(\tau_{ep}^{-1} + \tau_2^{-1})$. Clearly, the values of the damping constant as well as the effective charge are modified by the presence of electron-phonon collisions. In particular, for strong energy relaxation, i.e., $\tau_R \ll \tau_2$,

$$\gamma \approx \tilde{\gamma} = (m_F \tau_2)^{-1} \tag{131}$$

*Nevertheless, this approximation ensures the correct results for $\tau_{ep} \gg \tau$ and $\tau_{ep} \ll \tau$, and is expected to be sufficiently accurate in between.

$$e^* = e/m_F \tag{132}$$

which should be contrasted with the results for the opposite limit, namely $\gamma = \gamma_i$ and $e^* = (e/m_F)(\pi\Delta/4T)(\tau_2/\tau)^{1/2}$ [see (98)].

As mentioned above, (127) is a valid description in the hydrodynamic regime, more precisely if $\omega \ll \tau_R^{-1} + \tau_2^{-1}$ and $\omega^2 \ll (\tau_2\tau_R)^{-1}$. Here, the second condition, which applies only if $\tau_R \ll \tau_2$, ensures that a dissipative correction of the type discussed in Section 5.2, which is typical for energy-relaxing processes, is negligible compared to $\gamma\dot{\chi}$.

Physically, the results as represented by Eq. (131) and (132) can be understood as follows. In a situation of dominant energy relaxation, the quasiparticle distribution function will be close to the local equilibrium form (100). With the help of this ansatz, the above results can be obtained directly by integrating the kinetic equations, similar to Section 5.2; note that (131) and (132) apply only if $\tau_{ep}/\tau_2 \ll \Delta/T$.

Finally, consider the current under the same condition of small frequencies (and $q \to 0$, $\Delta \ll T$). From Eqs. (90) and (121),

$$j = (e/\pi)[2\tau_{ep}(\psi - \tilde{\psi}) + \dot{\chi}] \tag{133}$$

which can be written as follows:

$$j = \sigma^{(1)}\varepsilon + (e/\pi)\mu\dot{\chi} \tag{134}$$

where $\sigma^{(1)}$ is given by

$$\sigma^{(1)} = \sigma_N/(1 + \tau_2/\tau_R^\alpha) \tag{135}$$

and $\sigma_N = 2e^2 v_F \tau_2/\pi$ is the normal state conductivity. In addition,

$$\mu = \frac{(\tau_R^\alpha)^{-1} + \tau_2^{-1}(\pi\Delta/4T)[(\tau_2/\tau)/(1+\tau_2/\tau_{ep}]^{1/2}}{(\tau_R^\alpha)^{-1} + \tau_2^{-1}} \tag{136}$$

Note that a correction $\sim\Delta\dot{\chi}$ in the relation between $\tilde{\psi}$ and ψ had to be taken into account here, which could be ignored before [and was not included in (122)]. In particular, the results for dominant energy and momentum relaxation, respectively, are

$$\mu \approx \begin{cases} 1 & \tau_R \ll \tau_2 \\ (\pi\Delta/4T)(\tau_2/\tau)^{1/2} & \tau_R \gg \tau_2 \end{cases} \tag{137}$$

Of course, the equation of motion (127) combined with the expression for the current (134) are just the more or less standard set of equations often used to interpret experiments on charge-density-wave systems. The derivation of these equations has been discussed in detail here to demonstrate

the role momentum-conserving and nonconserving collisions play in determining the parameters entering in such a description. The results demonstrate that careful investigations close to the transition temperature could shed further light on the mechanism of dissipation in CDW systems.

6. SUMMARY AND DISCUSSION

In this paper, I have derived the quasiclassical equations of motion for quasi-one-dimensional systems at low temperatures in order to discuss some aspects of the dynamics of charge-density-wave systems. It turns out that, as in superconductors and superfluid ^{3}He, where this method has been applied very successfully,[14-18] the quasiclassical approach is an elegant and relatively simple (compared to full microscopic theories) tool for investigating nonequilibrium phenomena. Physically, its simplicity relies on the fact that "unnecessary" information about the momentum can be integrated out. More precisely, the equations for the Green's function, which is integrated with respect to the magnitude of the momentum, can be derived, and this function satisfies a normalization condition: this condition allows the definition of quasiparticle distribution functions, and Boltzmann-like transport equations emerge in the limit of low frequencies and long wavelengths (discussed here), and in cases of a linear perturbation above a time-independent reference state. The collision operators appearing in these equations can be treated by standard methods. Note that I have discussed the kinetic equations with emphasis on the BCS limit, i.e., situations in which the scattering rates as well as the frequency are small compared to the magnitude of the order parameter, and the wave vector is small compared to the inverse of the magnitude coherence length. In this limit, the kinetic equations can also be derived from "golden rule" arguments, and the correspondence between the present approach and the "excitation picture" has been discussed in detail elsewhere.[16]

In particular, I have investigated here the dynamics of the phase of the order parameter, and considered the influence of (elastic) impurity scattering and (inelastic) electron–phonon scattering. To be definite, I assumed the temperatures low enough that the *momentum* relaxation due to electron–phonon processes can be neglected ($T \ll \theta$). However, no use was made of any special properties of electron–phonon collisions, *except* that they *do* contribute to energy relaxation, and that the collision operators in the L- and T-mode equations can be characterized by an energy-independent rate (which can be different, in general, in the L and the T mode). In fact, the "reduced" collision operator has just the eigenvalue zero eigenfunctions that correspond to particle and momentum conservation and, in addition, has the property of equilibrating all other eigenfunctions within

a certain relaxation time. Therefore, the model applies to situations in which a reasonable separation of energy and momentum relaxation can be made, and should be a good interpolation formula in between. For example, the equations also apply for $T \gg \theta$, in which case electron-phonon scattering is almost elastic (a case considered in ref. 31), and one has to identify $\tau_1^{-1} \sim \tau_2^{-1} \sim \lambda k_B T/\hbar$.

For illustration, I consider in more detail the parameters characteristic for systems in which CDW transitions are observed (see Grüner and Zettl,[2] Section 3). For example, for $NbSe_3$, some typical values are $E_F \sim 1$ eV, $\Delta(T=0)/k_B \sim T_c \sim 100$ K, $\hbar\omega_Q/k_B \sim \theta \sim 100$ K, and $\lambda \sim 1/4$. Also, one obtains a Fröhlich mass of the order of $m_F \sim 10^2$. [In addition, a BCS-like behavior of the temperature dependence of the gap is found, with $\Delta^2 \sim k_B^2 T_c(T_c - T)$ close to the transition temperature.] Considering the temperature dependence of the resistivity found in the early experiments,[32] which is almost linear for high temperatures, one can conclude that momentum relaxation is due to electron-phonon scattering, with $\theta \lesssim T$, at least close to and above the upper CDW transition[31] ($T_c \approx 144$ K). Impurity scattering seems to be negligible, except at very low temperatures.[32] Furthermore, since $\theta \approx 100$ K, one can conclude that energy relaxation is of the same order (or even larger) than momentum relaxation close to the lower transition ($T_c \approx 59$ K). Generally, the results obtained in this paper suggest that careful measurements close to the transition temperature should be able to clarify the role of energy versus momentum relaxation. Of course, the momentum relaxation rate can also be varied experimentally, by changing the concentration of impurities.

For completeness, I discuss the limit in which momentum relaxation is neglected completely. Then, one has a situation similar to the branch imbalance in superconductors[11,14,28] and to the spin dynamics[12,16] of superfluid ^{3}He. Considering the branch imbalance, it has to be noted that the variable conjugate to the phase is the electron density, and the branch imbalance relaxation is due to electron-phonon collisions, with the characteristic behavior of the relaxation time $\tau_R \sim \tau_{ep} \cdot 4k_B T/\pi\Delta$ (close to T_c). This result is not affected by (nonmagnetic) impurities. On the other hand, for CDW systems, the phase is conjugate to the *momentum*, and even weak momentum relaxation invalidates the analogy to the branch imbalance (see Section 5.3). Note, however, that the parameters in the equation of motion (127) are different, and especially their temperature dependence close to T_c, in cases when energy relaxation dominates over momentum relaxation compared to the opposite limit.

Finally, note again that the *pinning* of charge-density waves by the random impurity potential has not been considered here. I believe that by adding the random impurity pinning force to the equation of motion

obtained in this paper a reasonable description of transport in CDW systems is obtained. Also, I have neglected the interaction with "normal" electrons, i.e., electrons not affected by the Peierls transition. Some aspects of this problem have been discussed recently.[33]

In conclusion, I have presented a detailed discussion of the dynamics of quasiparticles in CDW systems and their impact on the dynamics of the phase of the order parameter. In particular, the quasiclassical technique, as in superconductors and superfluid ^{3}He, is a convenient and elegant tool for the description of nonequilibrium states in quasi-one-dimensional conductors, and may have further applications.

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