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Microscopic Theory of Charge-Density Wave Systems

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An effective action (in imaginary time) for the phase of the order parameter is derived using the path integral formulation of the microscopic theory. After analytic continuation, the classical equation of motion is derived. Dissipation is found to arise from impurity scattering and from the screened Coulomb interaction with normal electrons. Similarities to a recent theory of Josephson junctions are discussed.

1. Introduction

In recent years, it has become clear that the unusual low temperature transport properties of linear-chain compounds such as $NbSe_3$, TaS_3 etc. [1] are related to the formation of a charge-density wave (CDW), i.e. a periodic distortion of the lattice accompanied by a periodic modulation of the electron density. The transition into a CDW state which was suggested by Peierls [2] more than thirty years ago, is related to the instability of onedimensional metals against perturbations with wavevector $Q=2p_F/\hbar$, where p_F is the Fermi momentum. In addition, Fröhlich [3] argued that in the incommensurate case a CDW could slide through the lattice without friction. Subsequent work [4] has shown that impurities or lattice imperfections are important since they lead to pinning of the charge-density wave. Consequently, for small electric fields, the charge-density wave does not contribute to the dc conduction. In general, a strong frequency and field (above a threshold field) dependent response is observed, as well as other phenomena characteristic for non-linear systems: interference phenomena, hysteresis, narrow-band noise, etc. Recent reviews of experimental and theoretical developments can be found in [5] (see also [1]).

Different models have been proposed in order to explain the experimental results. While it is accepted that the phase of the charge-density wave is the relevant variable, there is a controversy between classical models [6–11] and models employing concepts of quantum tunneling [12–14]. Among the classical models, it was noted that the model of a particle in a periodic potential [6, 1] describes qualitatively many of the experimental features, and an improvement can be expected by including, for example, a kind of shot noise in the equation of motion [7]. In detail, however, it is clear that local deformations of the charge-density wave have to be taken into account [8–11]. Near threshold, for example, it seems to be useful to view the CDW depinning as a dynamical critical phenomenon [11]. In contrast to the classical models, which are based on a classical equation of motion, the quantum approach [12–14] starts with the idea of Zener tunneling, and combines it with the concept of photon-assisted tunneling which is well known from the theory of tunneling between two weakly coupled superconductors.

The controversy between classical and quantum models was one of our original motivations to undertake the research presented in this article. Therefore, our aim is, starting from microscopic theory, to derive an effective action for the phase of the order parameter (which describes the distortion of the lattice) thus including the full quantum mechanics of this collective variable. This approach is similar to the phase Hamiltonian [15–17]. As a new feature, dissipative effects are included in the action, and we propose to calculate the damping constant usually introduced phenomenologically in the classical equation of motion. This is of particular importance since dissipation is known to decrease quan-

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tum effects [18]. In fact, the present approach is analogous to recent investigations of the tunneling between superconductors [19, 20], in which path integral methods have been applied successfully. Similar methods, in the context of charge-density wave systems, have been used before [21, 22]. We remark that dissipative effects can also be investigated within what we call the kinetic equation approach [23–25], which is particularly suited for studies of quasiparticle nonequilibrium effects [15, 26].

This article is organized as follows. In Sect. 2, we briefly present the microscopic model and derive the effective action for the phase of the order parameter. The analytic continuation to real times and the classical equation of motion, are discussed in Sect. 3. Section 4 contains a discussion of the interaction with normal electrons, which are known to be present in some materials due to an incomplete destruction of the Fermi surface in the Peierls transition. A conclusion is given in Sect. 5. Some of the results presented here have been discussed briefly [27, 28] (see also [29]).

2. Model Hamiltonian and Effective Action

2.1. The Model

We consider the standard model where electrons are strongly coupled to phonons of wave-vectors close to $\pm Q$, and where they are scattered by impurities and driven by an external field. In particular, it is assumed that the Fermi surface consists of two parallel planes (perpendicular to the x-axis), while the phonon spectrum is taken to be three-dimensional. In addition, for electrons close to the Fermi surface, the electron spectrum can be replaced by a linear one.

Ignoring for a moment impurity scattering and the external field, we take the Hamiltonian to be

$$H = H_{\mathrm{el}} + H_{\mathrm{el-ph}} + H_{\mathrm{ph}} \tag{1}$$

where

$$H_{\mathrm{el}} = \sum_{\mathbf{p},s} v_F p_x a^+_{\mathbf{p},s} a_{\mathbf{p},s} \tag{2a}$$

and*

$$H_{\mathrm{ph}} = \sum_{\mathbf{q}} \omega(\mathbf{q}) b^+_{\mathbf{q}} b_{\mathbf{q}}. \tag{2b}$$

The notation is standard: v_F denotes the Fermi velocity, s the electron spin, $\omega(\mathbf{q})$ the phonon dispersion, and a^+, a and b^+, b are the electron and phonon creation and annihilation operators, respectively. In the following, we ignore the spin index except the usual factor two to account for multiplicity. It is convenient to work in the coordinate representation, in which the electron-phonon interaction is given by

$$H_{\mathrm{el-ph}} = g \int \mathrm{d}^3 r\, \varphi(\mathbf{r})\, \hat{\psi}^+(\mathbf{r})\, \hat{\psi}(\mathbf{r}) \tag{2c}$$

where g is the coupling constant, and $\varphi(\mathbf{r})$ the phonon field operator. Considering only phonons with wave-vectors $\mathbf{q} \simeq \mathbf{Q} \equiv (Q, 0, 0)$, it is useful to introduce the order parameter Δ according to

$$g\varphi(\mathbf{r}) \to \Delta(\mathbf{r}) \mathrm{e}^{\mathrm{i}Qx} + \Delta^*(\mathbf{r}) \mathrm{e}^{-\mathrm{i}Qx} \tag{3}$$

such that $\Delta(\mathbf{r})$ is slowly varying in space. Taking only slowly varying contributions (compared to Q^{-1}) in the Hamiltonian into account, we arrive at*

$$H_{\mathrm{el}} + H_{\mathrm{el-ph}} = \int \mathrm{d}^3 r\, \boldsymbol{\psi}^+(\mathbf{r})\, \hat{h}\, \boldsymbol{\psi}(\mathbf{r}). \tag{4}$$

In this expression, the components of the vector $\boldsymbol{\psi} = (\hat{\psi}_+, \hat{\psi}_-)$ are the field operators for the two electron branches, where $\alpha = +(-)$ refers to positive (negative) momenta. The matrix $\hat{h}$ is given by

$$\hat{h} = v_F(-\mathrm{i}\partial_x - \mathrm{e}A)\hat{\tau}_3 + e\phi\hat{1} + \begin{bmatrix} \eta & \Delta + \xi \\ \Delta^* + \xi^* & \eta \end{bmatrix}. \tag{5}$$

Here, A and ϕ are the electromagnetic potentials, and $\hat{\tau}_3$ is the third Pauli matrix. In addition, impurity scattering is taken into account through a real field $\eta(\mathbf{r})$, describing processes with small momentum transfer, and a complex field $\xi(\mathbf{r})$ for momentum transfer close to $\pm Q$. We assume that η and ξ have a Gaussian distribution with average mean, and correlations given by

$$\pi N(0) \langle \eta(\mathbf{r})\, \eta(\mathbf{r}') \rangle = \frac{1}{2\tau_1} \delta(\mathbf{r} - \mathbf{r}'), \tag{6a}$$

$$\pi N(0) \langle \xi(\mathbf{r})\, \xi^*(\mathbf{r}') \rangle = \frac{1}{2\tau_2} \delta(\mathbf{r} - \mathbf{r}') \tag{6b}$$

while $\langle \xi\, \xi \rangle = \langle \xi^*\, \xi^* \rangle = 0$. Here, $\langle \cdot \rangle$ denotes the impurity average, and $N(0)$ is the normal state density of states at the Fermi surface (for one spin). Clearly, τ_1 and τ_2 are the impurity scattering times for small and large momentum transfer, respectively. Equations (1)–(6) define the model we investigate in this and the following section.

2.2. Effective Action

In the next step, we consider the path integral representation [19–22] of the partition function Z for the

* Units are such that $\hbar = k_B = 1$

* A hat indicates a 2×2 matrix

present model. After integrating over the electron fields, we arrive at the following expression:

$$Z=\int \mathscr{D}\Delta\, \mathscr{D}\Delta^* \exp(-S) \tag{7}$$

where the path integral is over functions periodic in imaginary time with period β, with β the inverse temperature, such that $\Delta(\mathbf{r},\tau=0)=\Delta(\mathbf{r},\tau=\beta)$. The action $S=S[\Delta,\Delta^*]=S_{\rm ph}+S_{\rm el}$ has two contributions, given by

$$S_{\rm ph}=\frac{N(0)}{\lambda\omega_Q^2}\int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3 r\, \Delta^*(\mathbf{r},\tau)\, D^{-1}\Delta(\mathbf{r},\tau) \tag{8a}$$

and

$$S_{\rm el}=\mathrm{Tr}\log \hat{G}. \tag{8b}$$

Here λ is the dimensionless electron-phonon coupling constant, and $\omega_Q=\omega(\mathbf{Q})$. The inverse of the phonon Green's function, D^{-1}, Fourier transformed with respect to the spatial coordinate, is given by

$$D^{-1}=-\partial_\tau^2+\omega_q^2 \tag{9a}$$

which upon expansion with respect to $\delta\mathbf{q}=\mathbf{q}-\mathbf{Q}$ leads to

$$D^{-1}\simeq -\partial_\tau^2+\omega_Q^2-c_\perp^2 V_\perp^2. \tag{9b}$$

In this expression, gradients in x-direction have been neglected, since the sound velocity ($\simeq c_\perp$) turns out to be much smaller than the phason velocity (see below). Finally, $\hat{G}$ is the electron Green's function, which depends on the order parameter as well as on the external and impurity fields. We obtain the following result:

$$\hat{G}^{-1}=(-\partial_\tau-e\phi-\eta)\,\hat{\tau}_3+v_F(\mathrm{i}\partial_x+eA)\,\hat{1} +\begin{bmatrix} 0 & \Delta+\xi \\ -(\Delta^*+\xi^*) & 0 \end{bmatrix}. \tag{10}$$

In (8b), the Tr operation is with respect to all variables as there are, space, time, branch and spin indices. As an illustration, consider the Green's function in equilibrium and without impurity fields, $\hat{G}_0$. This quantity depends only on the relative coordinates, and its Fourier transform is given by*

$$\hat{G}_0(p,\omega_n)=[\mathrm{i}\omega_n\hat{\tau}_3-v_F p\hat{1}+\mathrm{i}|\Delta|\hat{\tau}_1]^{-1} \tag{11}$$

where the phase of Δ was fixed such that $\Delta=\mathrm{i}|\Delta|$. Then we obtain

$$\mathrm{Tr}\log\hat{G}_0=2\cdot N(0)\int_{-p_0}^{p_0}\frac{v_F\,\mathrm{d}p}{2}\,T\sum_{\omega_n}\mathrm{tr}\log\hat{G}_0(p,\omega_n). \tag{12}$$

* No confusion should arise between $\hat{\tau}_1$, the Pauli matrix, and τ_1, the scattering time

Here, the first factor two is due to the spin, p_0 is a cut-off momentum, and tr denotes the trace with respect to the branch index.

In order to deal with situations in which the order parameter varies in space and time, we introduce an expansion with respect to gradients of the phase of the order parameter. We put $\Delta=\mathrm{i}|\Delta|\exp(-\mathrm{i}\chi)$, and transform the Green's function according to

$$\tilde{\hat{G}}(\mathbf{r},\mathbf{r}',\tau,\tau')=\hat{S}^+(\mathbf{r},\tau)\,\hat{G}(\mathbf{r},\mathbf{r}',\tau,\tau')\,\hat{S}(\mathbf{r}',\tau) \tag{13}$$

where

$$\hat{S}(\mathbf{r},\tau)=\exp[-\mathrm{i}\hat{\tau}_3\chi(\mathbf{r},\tau)/2]. \tag{14}$$

As a result, the inverse of $\tilde{\hat{G}}$ is given by an expression similar to (10), with the replacements $\Delta\to\mathrm{i}|\Delta|$, $\xi\to\xi\exp(\mathrm{i}\chi)$, $\eta\to\eta-v_F\partial_x\chi/2$, $\mathrm{i}\partial_x\to\mathrm{i}\partial_x+\mathrm{i}\partial_\tau\chi/2v_F$. Thus, we may write

$$\tilde{\hat{G}}^{-1}=\hat{G}_0^{-1}-\hat{V} \tag{15}$$

where, with $\hat{V}=\sum_{i=1}^{4}V_i\hat{\tau}_i$ and $\hat{\tau}_4\equiv\hat{1}$:

$$V_1=-\mathrm{i}\,\mathrm{Im}(\xi\,\mathrm{e}^{\mathrm{i}\chi}), \tag{16a}$$

$$V_2=-\mathrm{i}\,\mathrm{Re}(\xi\,\mathrm{e}^{\mathrm{i}\chi}), \tag{16b}$$

$$V_3=e\phi+\eta-\tfrac{1}{2}v_F\partial_x\chi, \tag{16c}$$

$$V_4=-\left(ev_F A+\frac{\mathrm{i}}{2}\partial_\tau\chi\right). \tag{16d}$$

Small variations of the magnitude of the order parameter could be easily included by letting $V_1\to V_1-\mathrm{i}\delta$, where δ is the deviation of $|\Delta|$ from its equilibrium value. Note that

$$S_{\rm el}=\mathrm{Tr}\log\tilde{\hat{G}} =\mathrm{Tr}\log\hat{G}_0-\mathrm{Tr}\log(\hat{1}-\hat{G}_0\hat{V}). \tag{17}$$

In addition, the phonon part of the action follows from (8a) to be given by (in the following $|\Delta|$ will be denoted by Δ):

$$S_{\rm ph}=\frac{N(0)}{\lambda}\int \mathrm{d}\tau\,\mathrm{d}^3 r\,\Delta^2 +\tfrac{1}{2}N(0)\int \mathrm{d}\tau\,\mathrm{d}^3 r\, m_F[\tfrac{1}{2}(\partial_\tau\chi)^2+\tfrac{1}{2}c_\perp^2(V_\perp\chi)^2] \tag{18}$$

where $m_F=4\Delta^2/\lambda\omega_Q^2$, and variations of the magnitude of the order parameter have been ignored for simplicity. The quantity (m_F+1) is the Fröhlich mass divided by the electron band mass (typically $m_F\gg 1$). Considering the first term in (17) and (18), respectively, namely

$$S_0=\frac{N(0)}{\lambda}\int_0^\beta \mathrm{d}\tau\,\mathrm{d}^3 r\,\Delta^2+\mathrm{Tr}\log\hat{G}_0 \tag{19}$$

we remark that in equilibrium $S_0=\beta F$ where F is the free energy. Surprisingly, the condition $\partial F/\partial \Delta=0$ turns out to lead to an equation similar to the BCS gap equation in the theory of superconductivity.

2.3. Expansion of the Action

In the following, we concentrate on the low frequency long wavelength behavior of the charge-density wave. The magnitude mode can be omitted, since it has a finite gap ($\sim\omega_Q$) in the excitation spectrum. In addition, $S_{\rm el}$ is expanded in second order in the "perturbation" $\hat{V}$. The result is of the form:

$$S_{\rm el}^{(1)}=\mathrm{Tr}\,\hat{G}_0\,\hat{V}, \tag{20}$$

$$S_{\rm el}^{(2)}=\tfrac{1}{2}\mathrm{Tr}\,\hat{G}_0\,\hat{V}\hat{G}_0\,\hat{V}. \tag{21}$$

Inspecting the first order term, $S_{\rm el}^{(1)}$, we realize that only the contribution related to the backward scattering impurity field is relevant, the others being either independent of the phase or total derivatives. Using the gap equation, we obtain

$$S_{\rm el}^{(1)}=\rho_1\int_0^\beta \mathrm{d}\tau\int \mathrm{d}^3r\,\mathrm{Re}[\mathrm{i}\,\xi(\mathbf{r})\,\mathrm{e}^{\mathrm{i}\chi(\mathbf{r},\tau)}] \tag{22}$$

where ρ_1 is the CDW amplitude, given by $\rho_1=2N(0)\Delta/\lambda$. [$\rho_1/\rho_0\simeq\Delta/\lambda E_F$, where ρ_0 is the equilibrium electron density, and E_F the Fermi energy.] Equation (22) is the so-called "classical" impurity pinning contribution [15, 16].

Considering the second order terms, we realize that (21) can be written in the following form:

$$S_{\rm el}^{(2)}=\tfrac{1}{2}\sum_{i,j}\int \mathrm{d}y\,\mathrm{d}y'\,V_i(y)\,V_j(y')\,s_{ij}(y-y') \tag{23}$$

where $y\equiv(\mathbf{r},\tau)$, $y'=(\mathbf{r}',\tau')$, and

$$s_{ij}(y)=2\,\mathrm{tr}\,\hat{G}_0(y)\,\hat{\tau}_i\,\hat{G}_0(-y)\,\hat{\tau}_j. \tag{24}$$

The factor two in (24) is due to the trace over the spin index. Equation (23) can be simplified further by assuming that $\hat{V}$ is slowly varying compared to the variation of the response functions s_{ij}, which leads to

$$S_{\rm el}^{(2)}\simeq\tfrac{1}{2}\sum_{i,j}\int \mathrm{d}y\,V_i(y)\,V_j(y)\,\bar{s}_{ij} \tag{25}$$

where $\bar{s}_{ij}$ is the low frequency long wave-length limit of s_{ij} (for the off-diagonal parts of $\hat{V}$, however, see below). Due to symmetry with respect to $\omega_n\to-\omega_n$ or $p\to-p$, we find that $\bar{s}_{ij}=0$ for $i\neq j$. One obtains, for example, $\bar{s}_{33}=2N(0)\cdot(1-Y)$ while $\bar{s}_{44}=0$, where the Yoshida function Y is given by

$$Y=1-\pi T\sum_{\omega_n}\frac{\Delta^2}{(\omega_n^2+\Delta^2)^{3/2}}. \tag{26}$$

Combining the results obtained so far, we write $S=S_0+S_1+S_2$, and ignore for a moment S_2 which contains the contributions of second order in the backward scattering impurity field. For S_1, the following result is found:

$$S_1[\chi]=\tfrac{1}{2}m_F N(0)\int_0^\beta \mathrm{d}\tau\int \mathrm{d}^3r[\tfrac{1}{2}(\partial_\tau\chi)^2+U(\chi)] \tag{27a}$$

with

$$U(\chi)=\tfrac{1}{2}c_\perp^2(\nabla_\perp\chi)^2+\tfrac{1}{2}c^2(\partial_x\chi)^2 -2e^*v_F(\varepsilon-\partial_x\eta/e)\chi+\tilde{\rho}_1\,\mathrm{Re}(\mathrm{i}\,\xi\,\mathrm{e}^{\mathrm{i}\chi})]. \tag{27b}$$

Here, the phason velocity, c, is given by $c^2=v_F^2(1-Y)/m_F$, $e^*=e(1-Y)/m_F$ is an effective charge, $\varepsilon=-\partial_x\phi$ the electric field, and $\tilde{\rho}_1=2\rho_1/m_F N(0)$. The coupling between ε and χ as apparent in (27), also implies the following expression for the charge density:

$$\delta\rho=-eN(0)\,v_F(1-Y)\,\partial_x\chi. \tag{28}$$

Note that in one dimension $N(0)=(\pi v_F)^{-1}$, while for a quasi-onedimensional model of electrons moving along individual chains, $N(0)=(\pi v_F d^2)^{-1}$, where d^2 is the area for one chain perpendicular to the chain direction. The effective action (27) or the corresponding phase Hamiltonian is the standard result [17, 30].

We wish to add a few remarks. As a first point, note that the above result, (27), has been derived in a gauge where $A=0$. In this gauge, the evaluation of the coefficients is straightforward. Of course, the general expression, (8b), is gauge invariant, and we could have worked with a gauge in which $\phi=0$, with the same result. We feel, however, that gauge invariance is most easily discussed within the framework of Boltzmann-like transport equations [25].

Secondly, though (27) is expected to be the correct result for low temperatures, the temperature dependence of c^2 and e^* also requires a more detailed discussion. Essentially, at finite temperatures and especially close to the critical temperatures, T_c, a careful treatment of the quasiparticles becomes necessary [25]. An inspection of the transport equations shows that (27), or more precisely the corresponding classical equation of motion, is recovered provided the deviation of the quasiparticle distribution function from equilibrium is small. Considering the kinetic equations in the absence of inelastic collisions, this can be realized in the low frequency

limit, $\omega \ll v_F q,\ v_F^2 q^2 \tau_2$, where ω and q are the external frequency and wave-vector, respectively. On the other hand, for $q \to 0$ and $\omega \tau_2 \ll 1$, one obtains [23–25] $m_F c^2 \sim m_F e^* \sim \Delta$ for temperatures close to T_c, a result which is also known to hold in the high frequency limit [26], in contrast to the behavior $\sim \Delta^2$ as follows from (27).

2.4. *Non-Local Elastic Forces* [31]

As a brief extension of our results, we consider the q-dependence of the phason velocity, represented in the effective action (27) by a non-local contribution of the form

$$\int \mathrm{d}x\, c^2 (\partial_x \chi)^2 \to \int \mathrm{d}x\, \mathrm{d}x' (\partial_x \chi)\, c^2 (x - x') (\partial_{x'} \chi). \tag{29}$$

Note that x, x' refers to the spatial coordinate along the chain direction. This term is obtained directly from the zero-frequency component of s_{33}, and turns out to be given by

$$c^2(x) = \frac{v_F^2}{m_F} \pi v_F T \sum_{\omega_n} \mathrm{tr}\, \hat{G}_0(\omega_n, x)\, \hat{\tau}_3\, \hat{G}_0(\omega_n, -x)\, \hat{\tau}_3. \tag{30}$$

This quantity is even in x, and inserting (11), we obtain for $x > 0$:

$$c^2(x) = \frac{v_F^2}{m_F} \cdot \frac{\pi T}{v_F} \sum_{\omega_n} \frac{\Delta^2}{\omega_n^2 + \Delta^2} \cdot \exp[-2(\omega_n^2 + \Delta^2)^{1/2} \chi / v_F] \tag{31}$$

Clearly, $c^2 = \int \mathrm{d}x\, c^2(x)$. For zero temperature, one finds [31]

$$c^2(x) = c^2 \cdot \xi_0^{-1} \int_{2x/\xi_0}^{\infty} \mathrm{d}z\, K_0(z) \tag{32}$$

where $\xi_0 = v_F/\Delta$ is the coherence length, and $K_0(z)$ the modified Bessel function. In addition, the Fourier transform of $c^2(x)$ is given by ($q > 0$):

$$c^2(q) = c^2 \cdot \frac{2}{q \xi_0} \frac{\log\{q\xi_0/2 + [1 + (q\xi_0/2)^2]^{1/2}\}}{[1 + (q\xi_0/2)^2]^{1/2}}. \tag{33}$$

Note that $\xi_0/2$ plays the role of a screening length*.

2.5. *Impurity Fields (in 2nd Order): S_2*

We discuss now in more detail the contributions of second order in the backward scattering impurity fields [28]. These have been called [32] "Josephson" terms, since there is an analogy between the group of electrons with momenta close to $+p_F$ and $-p_F$, with the electrons in the two superconductors, respectively, which are weakly coupled in a Josephson junction. However, it will become clear that this analogy has only a limited validity.

From the general expression (23), we obtain the following result:

$$S_2[\chi] = \tfrac{1}{8} \int \mathrm{d}y\, \mathrm{d}y' \{\tilde{\xi}(y)\, \tilde{\xi}(y')[s_{11} - s_{22} + \mathrm{i}(s_{12} + s_{21})]_{y-y'} - \tilde{\xi}(y)\, \tilde{\xi}^*(y')[s_{11} + s_{22} - \mathrm{i}(s_{12} - s_{21})]_{y-y'} + \text{c.c.}\} \tag{34}$$

where $\tilde{\xi}(y) = \xi(\mathbf{r}) \cdot \exp \mathrm{i}\chi(\mathbf{r}, \tau)$, and $y' \equiv (\mathbf{r}', \tau')$ as before. The first (second) term in this expression depends on $\chi(y)_{(-)}^{+} \chi(y')$, and thus is related to pinning (dissipation) of the charge-density wave. In the low frequency long wave-length limit, we can approximate the sum and difference of the phase as follows:

$$\begin{aligned} \chi(y) + \chi(y') &\simeq 2\chi\left(\frac{y + y'}{2}\right), \\ \chi(y) - \chi(y') &\simeq \chi\left(\frac{\mathbf{r} + \mathbf{r}'}{2}, \tau\right) - \chi\left(\frac{\mathbf{r} + \mathbf{r}'}{2}, \tau'\right). \end{aligned} \tag{35}$$

Then we obtain

$$S_2[\chi] = -(2d^2)^{-1} \int \mathrm{d}\tau\, \mathrm{d}^3 r\, \mathrm{Re}[\mu(\mathbf{r})\, \mathrm{e}^{2\mathrm{i}\chi(\mathbf{r}, \tau)}] + \int \mathrm{d}\tau\, \mathrm{d}\tau'\, \mathrm{d}^3 r\, \mathrm{Re}[\nu(\mathbf{r}, \tau - \tau')\, \mathrm{e}^{\mathrm{i}\chi(\mathbf{r}, \tau) - \mathrm{i}\chi(\mathbf{r}, \tau')}] \tag{36}$$

where $\mu(\mathbf{r})$ and $\nu(\mathbf{r}, \tau - \tau')$, which depend on the impurity field, are easily found from (34), (35). The prefactor in the first term has been chosen for convenience (see [28]), and we have written d^{-2} instead of $\pi v_F N(0)$ for simplicity. Considering the pinning term, and taking into account that the Green's functions extend in the perpendicular direction only over a length of the order of d, we find the following result:

$$\mu(\mathbf{r}) = \int \mathrm{d}x'\, \xi\left(\mathbf{r}_\perp, x + \frac{x'}{2}\right) \xi\left(\mathbf{r}_\perp, x - \frac{x'}{2}\right) \beta_0(x') \tag{37}$$

where $\mathbf{r} = (\mathbf{r}_\perp, x)$, and $\beta_0(x)$ is the function introduced earlier [28]. [Note that (31) can be written in the form $c^2(x) = (v_F^2/m_F) \cdot \pi v_F \beta_0(x)$.] In addition

$$\beta_0(x = 0) = (\Delta / 2 v_F^2) \tanh(\Delta / 2T) \tag{38}$$

which is an expression familiar from the critical current of Josephson junctions [33]. The impurity average of μ is zero, and correlations are given by

$$\langle \mu(\mathbf{r})\, \mu^*(\mathbf{r}') \rangle = \Gamma_\mu \delta(\mathbf{r} - \mathbf{r}'). \tag{39}$$

The first term in (36) was called Josephson contribution [32], and it is periodic in χ with period π, compared to the period 2π for the classical term. To

* See [31] for a detailed discussion

estimate its relative importance, note that

$$\Gamma_\mu \simeq (\xi_0/d^2)[\Gamma_\xi \beta_0(0)]^2 \tag{40}$$

where $\Gamma_\xi = v_F d^2/2\tau_2$. Comparing the Josephson term with (22), we note that

$$\frac{d^{-4}\Gamma_\mu}{\rho_1^2 \Gamma_\xi} \simeq 0.1 \cdot \left(\frac{\rho_0}{\rho_1}\right)^2 \frac{\Delta/\tau_2}{E_F^2}. \tag{41}$$

Typical values are $\rho_1 \sim 0.1\rho_0$ and we conclude, even if Δ is relatively large ($\Delta \sim 0.1 E_F$), that the ratio of the Josephson to the classical contribution is very small, $\sim (E_F \tau_2)^{-1} \ll 1$. The Josephson term becomes even smaller close to T_c, where we have an additional factor $\sim(\Delta/T_c)^3$ in (41) (note that for $T \simeq T_c$, $\xi_0 \to v_F/T_c$ in (40)).

On the other hand, consider now the dissipative contribution to S_2, which adds a new feature to the effective action. This term is rather complicated, and we suggest the following procedure. Inspecting the expression for $\nu(\mathbf{r}, \tau - \tau')$, we realize that it depends on $\xi(\mathbf{r}+\mathbf{r}'/2)\,\xi^*(\mathbf{r}-\mathbf{r}'/2)$; thus the impurity average of ν gives a non-zero result. Consequently, we propose to replace ν by its impurity average, which gives an average description of dissipation. Thus (36) is replaced by

$$S_2[\chi] = m_F N(0) \int_0^\beta d\tau \int_0^\beta d\tau' \int d^3 r \cdot \alpha(\tau - \tau') \sin^2\left[\frac{\chi(\mathbf{r},\tau) - \chi(\mathbf{r},\tau')}{2}\right] \tag{42}$$

where a constant has been omitted. The quantity $\alpha(\tau - \tau')$ is given by

$$\alpha(\tau - \tau') = \{4\pi m_F [N(0)]^2 \tau_2\}^{-1} \cdot [s_{11} + s_{22}](\mathbf{r} = 0, \tau - \tau'). \tag{43a}$$

This result is most conveniently presented in terms of the momentum integrated Green's function (because of the condition $\mathbf{r} = 0$ in (43a)). We define

$$g_3(\tau) = \frac{i v_F}{\pi} \int dp \frac{1}{2} \operatorname{tr} \hat{\tau}_3 \hat{G}_0(p, \tau) = T \sum_{\omega_n} \frac{\omega_n}{(\omega_n^2 + \Delta^2)^{1/2}} e^{-i\omega_n \tau} \tag{44}$$

and obtain, with $\gamma_0 = (m_F \tau_2)^{-1}$:

$$\alpha(\tau) = \gamma_0 \frac{\pi}{2} g_3(\tau)\, g_3(-\tau). \tag{43b}$$

Especially, in the normal state ($T > T_c$), and for zero temperatures, this leads to [20, 27]

$$\alpha(\tau) = \frac{\gamma_0}{2\pi} \begin{cases} (\pi T)^2/\sin^2(\pi T \tau) & T > T_c \\ [\Delta K_1(\Delta|\tau|)]^2 & T = 0 \end{cases} \tag{45}$$

with K_1 the modified Bessel function. Considering $\Delta = 0$ but nevertheless $T \to 0$, we find $\alpha(\tau) = \gamma_0/2\pi\tau^2$, which is the relation proposed by Caldeira and Leggett [18].

As a conclusion of this section, we remark that, for the model discussed so far, the quantity

$$S[\chi] = S_1[\chi] + S_2[\chi] \tag{46}$$

where S_1 and S_2 are given in (27) and (42), respectively, is the appropriate (imaginary time) action for the phase of the order parameter. Some consequences of the dissipative contribution, S_2, namely dissipation and fluctuations in the classical equation of motion, are explored in the next section.

2.6. Comparison with the Theory of Josephson Junctions

As mentioned before, similar techniques as discussed here have been applied to the theory of tunneling between two weakly coupled superconductors [19, 20]. The following results has been obtained:

$$S_1[\varphi] = \int_0^\beta d\tau [(C/8e^2)(\partial_\tau \varphi)^2 + U(\varphi)], \tag{47}$$

$$S_2[\varphi] = 2 \int_0^\beta d\tau \int_0^\beta d\tau' \alpha(\tau - \tau') \sin^2\left[\frac{\varphi(\tau) - \varphi(\tau')}{4}\right]. \tag{48}$$

The total effective action is given by $S = S_1 + S_2$; φ denotes the phase difference across the junction, C the capacitance, and $U(\varphi) = -(I_c/2e)\cos\varphi - I\varphi/2e$, where I is the external current, and I_c the critical current. In addition, $\alpha(\tau - \tau')$ is given by an expression similar to (43b), with γ_0 replaced by a factor proportional to the inverse of the normal state resistance of the junction. Note also that

$$I_c/I_c(T=0) = [\Delta/\Delta(T=0)] \cdot \tanh(\Delta/2T) \tag{49}$$

for tunneling between identical superconductors [33]. Comparing (47) and (48) with (27) and (42), some formal similarities are apparent.

However, we wish to discuss in more detail the differences between the two systems. First of all, in the CDW case, the phase is a spatially varying quantity, even in equilibrium situations, because of the impurity potential. Secondly, the electrons with momenta close to $+p_F$ and $-p_F$ are strongly coupled by phonons, and impurity scattering is assumed to be only a small correction to this. As a consequence, the dominant contribution to the pinning

potential arises from the contribution of *first order* in the impurity potential, while the second order pinning term is very small. However, the second order term leads to a dissipative contribution, as discussed above.

On the other hand, in Josephson junctions, the two superconductors are *uncoupled* in the absence of tunneling. Therefore, the washbord potential as well as the dissipative contribution are of *second order* in the tunneling matrix element i.e. proportional to the inverse of the resistance. Note also the factor 1/4 in the argument of the sine in (48), compared to the corresponding 1/2 in (42).

Finally, we wish to emphasize another aspect, which charge-density wave systems and Josephson junctions have in common. This is the appearance of the trigonometric function in the dissipative contribution of the action, S_2, as given in (42) and (48). In this respect, the present results *differ* from the model discussed in [18], in which dissipation is introduced by coupling a particle linearly to an environment of oscillators. In fact, only if the sine is replaced by its argument, do we recover the result of [18]. This has some consequences in the quasiclassical limit (see [20], and below), and may change the results qualitatively in the quantum limit [34].

3. Real-Time Description and Equation of Motion

3.1. Analytic Continuation

In the real time description, we are concerned with the temporal evolution of the density matrix $\rho(t)$. As an illustration, we consider an object having only one coordinate, say q; then we may write

$$\rho(q_1, q_2, t_f) = \int \mathrm{d}q_1' \,\mathrm{d}q_2' J(q_1, q_2, t_f; q_1', q_2', t_i)\, \rho(q_1', q_2', t_i) \tag{50}$$

where t_i and t_f denote the initial and final time, respectively. The quantity J can be expressed as a path integral [35]:

$$J = \int \mathscr{D}q_1 \,\mathscr{D}q_2 \exp \mathrm{i}\mathscr{A}[q_1, q_2]. \tag{51}$$

Here, the integration is over paths $q_1(t)$, $q_2(t)$ such that $q_1(t_i)=q_1'$, $q_1(t_f)=q_1$, and $q_2(t_i)=q_2'$, $q_2(t_f)=q_2$. In addition, for the present example, we have $\mathscr{A}[q_1, q_2]=S[q_1]-S[q_2]$, where $S[q]$ is the classical action of the object. As is well known, $\mathscr{A}$ is modified if the particle interacts with an environment [35].

Returning to our investigation of charge-density wave systems, we consider the appropriate generalization of (51), i.e. a quantity J defined as

$$J = \int \mathscr{D}\chi_1 \,\mathscr{D}\chi_2 \exp \mathrm{i}\mathscr{A}[\chi_1, \chi_2] \tag{52}$$

and $\mathscr{A}[\chi_1, \chi_2]$ can be obtained by well-known analytic continuation procedures, which have been discussed in detail [20], from the effective action derived above. The results can be obtained without difficulty, by comparison with [20].

An essential concept is the deformation of the time integration path, such that it runs along a contour C, from t_i to t_f to t_i, along the real axis, and from t_i to $t_i - \mathrm{i}\beta$. This leads to a doubling of the variables, according to $\chi(\tau) \to (\chi_1(t), \chi_2(t))$. Analogous to (46), we define

$$\mathscr{A} = \mathscr{A}_1 + \mathscr{A}_2. \tag{53}$$

Taking into account that (see (27))

$$S_1[\chi] = M \int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3 r \left[\tfrac{1}{2}(\partial_\tau \chi)^2 + U(\chi)\right]$$

where $M \equiv m_F N(0)/2$, we immediately arrive at the following result:

$$\mathscr{A}_1[\chi_1, \chi_2] = M \int \mathrm{d}t \,\mathrm{d}^3 r \left[\tfrac{1}{2}(\partial_t \chi_1)^2 - U(\chi_1) - (\chi_1 \to \chi_2)\right]. \tag{54a}$$

The dissipative contribution, on the other hand, leads to a coupling between χ_1 and χ_2. Therefore, it is advantageous to introduce center-of-mass and relative variables $\phi = (\chi_1 + \chi_2)/2$, $\chi = \chi_1 - \chi_2$, in which (54a) assumes the form

$$\mathscr{A}_1[\phi, \chi] = M \int \mathrm{d}t \,\mathrm{d}^3 r \left[\dot{\phi}\,\dot{\chi} - U\left(\phi + \frac{\chi}{2}\right) + U\left(\phi - \frac{\chi}{2}\right)\right] \tag{54b}$$

where $\dot{\phi} = \partial_t \phi$, $\dot{\chi} = \partial_t \chi$. The analytic continuation in the dissipative contribution is more difficult. Especially, the quantities $\alpha^>(t-t')$ and $\alpha^<(t-t')$ appear, which are analytic continuations of $\alpha(\tau - \tau')$ for $\tau > \tau'$ and $\tau < \tau'$, respectively. We define the real and imaginary part, α_R and α_I by

$$\alpha^>(t) = \alpha_R(t) + \mathrm{i}\,\alpha_I(t) \tag{55}$$

and $\alpha^<(t) = [\alpha^>(t)]^*$. Consequently, $\mathscr{A}_2$ has a real and imaginary part, and we obtain the following result:

$$\mathrm{Re}\,\mathscr{A}_2[\phi, \chi] = 8M \int \mathrm{d}t \int \mathrm{d}t' \,\mathrm{d}^3 r\, \theta(t-t') \cdot \alpha_I(t-t') \sin(\phi - \phi') \sin\frac{\chi}{2} \cos\frac{\chi'}{2}, \tag{56}$$

$$\mathrm{Im}\,\mathscr{A}_2[\phi, \chi] = 4M \int \mathrm{d}t \,\mathrm{d}t' \,\mathrm{d}^3 r \cdot \alpha_R(t-t') \cos(\phi - \phi') \sin\frac{\chi}{2} \sin\frac{\chi'}{2} \tag{57}$$

where $\phi \equiv \phi(\mathbf{r}, t)$, $\phi' \equiv \phi(\mathbf{r}, t')$, etc., to simplify the notation. $\theta(t-t')$ is the step function. Since $\mathscr{A}$ has an imaginary part, it is not possible to derive the classical equation of motion directly by a "least action" principle.

3.2. Langevin Equation

We proceed by introducing two independent Gaussian random fields $f_1(\mathbf{r}, t)$ and $f_2(\mathbf{r}, t)$ such that [20]

$$J = \int \mathscr{D}\phi\, \mathscr{D}\chi \langle \exp \mathrm{i}\tilde{\mathscr{A}} \rangle_f \tag{58}$$

where $\langle \cdot \rangle_f$ represents the average with respect to f_1 and f_2, and $\tilde{\mathscr{A}} = \mathscr{A}_1 + \mathrm{Re}\,\mathscr{A}_2 + \mathscr{A}_f$, with

$$\mathscr{A}_f = 2M \int \mathrm{d}t\, \mathrm{d}^3 r \sin\frac{\chi}{2}(f_1 \cos\phi + f_2 \sin\phi). \tag{59}$$

From the condition $\langle \exp \mathrm{i}\mathscr{A}_f \rangle_f = \exp(-\mathrm{Im}\,\mathscr{A}_2)$, we find that the correlations of f_1 and f_2 have to be chosen as follows:

$$\langle f_1(\mathbf{r}, t) f_1(\mathbf{r}', t') \rangle_f = \langle f_2(\mathbf{r}, t) f_2(\mathbf{r}', t') \rangle_f$$
$$= \frac{2}{M} \delta(\mathbf{r}-\mathbf{r}')\, \alpha_R(t-t') \tag{60}$$

and $\langle f_1 \rangle_f = \langle f_2 \rangle_f = 0$. Note the non-locality with respect to the time argument, expressed by $\alpha_R(t-t')$. Then the "classical" equation of motion can be obtained by minimizing $\tilde{\mathscr{A}}$ with respect to the relative variable, χ, and putting $\chi = 0$ (which is a solution of the equation $\delta\tilde{\mathscr{A}}/\delta\phi = 0$). We find the following result:

$$\ddot{\phi} - 4 \int_{-\infty}^{t} \mathrm{d}t'\, \alpha_I(t-t') \sin[\phi(\mathbf{r}, t) - \phi(\mathbf{r}, t')]$$
$$= -\frac{\partial U}{\partial \phi} + f_1 \cos\phi + f_2 \sin\phi \tag{61}$$

where the force, $-\partial U/\partial\phi$, is understood to include the gradient terms. Note the rather complicated dissipative term on the lhs of (61), and the corresponding fluctuating force on the rhs, which depends on ϕ as indicated. The procedure which led to (61) is well-known in the limit of high temperatures $T \gg \omega$, and has also been examined in the quantum limit [36] $\omega \gg T$, for a simpler model. Equation (61) is called a quantum Langevin equation. Though its range of applicability in the quantum case is not completely clarified, we remark that dissipation should not be too small in the limit of low temperatures [36].

3.3. Dissipation and Fluctuations: Specific Results

As a general result, we note that $\alpha_R(t)$ and $\alpha_I(t)$ are even and odd functions of t, thus their Fourier transforms $\alpha_R(\omega)$ and $\alpha_I(\omega)$ are real and imaginary, respectively; also, they are related by the fluctuation dissipation theorem:

$$\alpha_R(\omega) = \mathrm{i}\,\alpha_I(\omega) \coth(\omega/2T). \tag{62}$$

Explicit expressions are found as follows. From (43b), we conclude that

$$\alpha^>(t) = \frac{\pi\gamma_0}{2} g_3^>(t)\, g_3^<(-t) \tag{63}$$

where the Fourier transforms of the Green's functions, $g_3^>(E)$ and $g_3^<(E)$, are given by

$$g_3^>(E) = 2N_1(E)\, f_0(-E),$$
$$g_3^<(E) = -2N_1(E)\, f_0(E) \tag{64}$$

such that $g_3^>(E) - g_3^<(E) = 2N_1(E) = g_3^R(E) - g_3^A(E)$, where $R(A)$ denotes the retarded (advanced) function, $N_1(E)$ is the normalized BCS density of states ($N_1(E) = 1$ for $\Delta = 0$), and $f_0(E)$ the Fermi function. Some straightforward algebra leads to the following result:

$$\alpha^>(t) = -\int \frac{\mathrm{d}\omega}{2\pi} \mathrm{e}^{-\mathrm{i}\omega t} b(-\omega)\, I(\omega) \tag{65a}$$

where $b(\omega)$ is the Bose function, and $I(\omega)$ is given by (recall that $\gamma_0 = (m_F \tau_2)^{-1}$):

$$I(\omega) = \gamma_0 \int_{-\infty}^{\infty} \mathrm{d}E\, N_1\left(E + \frac{\omega}{2}\right) N_1\left(E - \frac{\omega}{2}\right)$$
$$\cdot \left[f_0\left(E - \frac{\omega}{2}\right) - f_0\left(E + \frac{\omega}{2}\right) \right]. \tag{65b}$$

Apart from a constant factor, $I(\omega)$ is just the quasiparticle current in Josephson junctions. Since $I(-\omega) = -I(\omega)$, we recover (62) from (65), and in addition $2\mathrm{i}\,\alpha_I(\omega) = I(\omega)$. Given the quantity $I(\omega)$, which can be calculated from (65b), the dissipative contributions as well as the fluctuating forces in the equation of motion (61) can be determined.

Of particular interest is the low frequency limit, $\omega \ll \Delta$, in which case

$$I(\omega) = \gamma \cdot \omega \tag{66}$$

or equivalently, $2\alpha_I(t) = \gamma\,\delta'(t)$, where the damping constant γ is given by

$$\gamma = \gamma_0 \int_{-\infty}^{\infty} \mathrm{d}E [N_1(E)]^2 \left(-\frac{\partial f_0}{\partial E}\right). \tag{67}$$

In the BCS limit of a sharp density of states, this integral is, strictly speaking, logarithmically divergent, and a finite smearing of $N_1(E)$ has to be assumed (for example, due to inelastic electron-phonon scattering). Clearly, for $\Delta \to 0$, we have $\gamma \to \gamma_0$, while at low temperatures $T \ll T_c$, γ depends strongly on T because of the gap in the single particle excitation spectrum. To summarize, we obtain the following equation of motion provided $I(\omega) = \gamma\omega$:

$$\ddot{\phi} + \gamma\dot{\phi} = -\frac{\partial U}{\partial \phi} + f(\mathbf{r}, t, \phi) \tag{68}$$

where $f = f_1 \cos\phi + f_2 \sin\phi$ denotes the total fluctuating force. Note that the correlations of f are given by the following expression:

$$\Gamma_f(\mathbf{r}-\mathbf{r}', t-t') \equiv \langle f(\mathbf{r}, t, \phi) f(\mathbf{r}', t', \phi') \rangle_f = \frac{2}{M} \delta(\mathbf{r}-\mathbf{r}') \alpha_R(t-t') \cos(\phi - \phi') \tag{69}$$

with $\alpha_R(\omega) = (\gamma\omega/2) \coth(\omega/2T)$. As an example, consider the charge-density wave in the sliding mode, in which case we can assume that the time derivative of ϕ is fixed and independent of the spatial coordinate. Then we may put $\phi - \phi' \simeq \bar{\dot{\phi}} \cdot (t-t')$ in (69) and obtain the following result:

$$\Gamma_f(\mathbf{q}, \omega) = \frac{\gamma}{m_F N(0)} \sum_{+,-} (\omega \pm \bar{\dot{\phi}}) \coth \frac{\omega \pm \bar{\dot{\phi}}}{2T}. \tag{70}$$

For comparison we note that in Josephson junctions, the time derivative of the phase can be fixed by the voltage, V. This leads to $(\omega \pm eV)$ in the expression corresponding to (70), which is the characteristic feature of shot noise. Note that for high temperatures such that $\omega, \bar{\dot{\phi}} \ll T$, the standard white noise power spectrum is obtained:

$$\Gamma_f(\mathbf{q}, \omega) = \frac{4T\gamma}{m_F N(0)} \qquad \omega, \bar{\dot{\phi}} \ll T. \tag{71}$$

In this limit, the phase dependence of the fluctuating force becomes unimportant. On the other hand, for low frequencies and low temperatures such that $\omega, T \ll \bar{\dot{\phi}}$, we obtain the following expression:

$$\Gamma_f(\mathbf{q}, \omega) = \frac{2\gamma}{m_F N(0)} \bar{\dot{\phi}} \qquad \omega, T \ll \bar{\dot{\phi}}. \tag{72}$$

Thus the noise spectrum is proportional to the average current in this case. For most experimental situations, we expect (71) to be applicable.

3.4. *Comparison with Kinetic Theory*

We briefly summarize our results, and compare them with the results of the kinetic equation approach. In an expansion of the action with respect to gradients and impurity fields, we have recovered the standard model, and also found additional contributions of the "Josephson" type. While the second order contribution to pinning turned out to be very small, we found that the analog of a "normal current" led, on the average, to a dissipative contribution in the action, which is given in (42). Generally, such a term shows a rather non-ohmic behavior, because of the non-linear dependence on the variable, and because of the characteristic dependence on time of the kernel $\alpha(\tau - \tau')$.

However, considering low frequencies such that $\omega \ll \Delta$, an ohmic damping term $\gamma\dot{\phi}$ emerged in the equation of motion, after analytic continuation to real times. We have found that the damping constant γ is strongly temperature dependent for low T, roughly given by $\gamma \sim \gamma_0 \exp(-\Delta/T)$, see (67). In general, the corresponding fluctuations in the equation of motion include quantum fluctuations, and depend on ϕ. However, for low frequencies such that $\omega, \dot{\phi} \ll T$, the fluctuations are of the well-known form, namely white noise whose strength is proportional to the temperature.

As mentioned before, dissipation has been investigated by using Boltzmann like transport equations [25]. In particular, consider the results for the damping constant derived from the kinetic equations. For a situation with elastic scattering only, the following results is obtained (see [24] and [25], Eq. (95)):

$$\gamma = 2\gamma_0 \int_{\Delta}^{\infty} dE [N_1(E)]^2 \left(-\frac{\partial f_0}{\partial E}\right) \cdot Z(E) \tag{73}$$

which differs from (67) by $Z(E)$, given by

$$Z(E) = \frac{\Delta^2 \tau_2/\tau}{E^2 - \Delta^2 + \Delta^2 \tau_2/\tau}, \qquad \frac{1}{\tau} = \frac{1}{\tau_1} + \frac{1}{2\tau_2}. \tag{74}$$

Therefore, both results agree for $\tau_2 \to \infty$ or more precisely if $\tau_2/\tau_1 \gtrsim T/\Delta$, which is realized for temperatures $T \lesssim 0.9\, T_c$, provided $\tau_1 \sim \tau_2$. On the other hand, for $\Delta \ll T$, (73) leads to $\gamma \simeq \gamma_0 \Delta/4T_c$, where a logarithmic factor was omitted. In the kinetic equations, however, it has been possible also to include *inelastic* collisions through the corresponding collision operators in the Boltzmann equation [25]. Especially in the limit of strong inelastic collisions, it was found in agreement with (67) that $\gamma \simeq \gamma_0$ for $\Delta \to 0$. Note that the kinetic equation approach predicts γ/e^* to be independent of Δ for $\Delta \to 0$, for weak *and* strong energy relaxation. We expect that a formulation of the action in real times which explicitly introduces as variables the phase *and* the quasiparticle distribution function, should shed further light on the differences between the two approaches.

4. Interaction with Normal Electrons

Up to now, we considered the model of a Fermi surface consisting of two parallel planes, in which case all electrons participate in the Peierls transition. However, in some materials [1], for example, $NbSe_3$, only part of the Fermi surface is involved in the transition, and we expect some new features by the interaction of the "Peierls" electrons with the "normal" electrons, the latter having no gap in the single particle excitation spectrum. Especially, we wish to investigate impurity scattering, and the screening of the Coulomb interaction by the normal electrons.

4.1. Impurity Scattering

Similar to the discussion in Sect. 2.1, we consider the following contribution to the Hamiltonian operator due to the impurity potential:

$$\hat{H}_{i,n}=\int \mathrm{d}^3 r[\hat{\psi}_+^+ v \hat{\psi}_n+\hat{\psi}_-^+ v^* \hat{\psi}_n+\text{h.c.}] \tag{75}$$

where $v(\mathbf{r})$ is another Gaussian random field, independent of the η and ξ fields (see (6)), and $\hat{\psi}_n^+(\mathbf{r})$, $\hat{\psi}_n(\mathbf{r})$ are the field operators of the normal electrons. The correlations of v are taken to be

$$\pi N_n(0)\langle v(\mathbf{r})\, v^*(\mathbf{r}')\rangle=\frac{1}{2\tau_n}\,\delta(\mathbf{r}-\mathbf{r}') \tag{76}$$

where $N_n(0)$ is the density of states of n-electrons. This model implies an extension of the Green's function in (8b) to a 3×3 matrix. We expand the electron action with respect to v up to second order, and in the spirit of the discussion in Sect. 2.5, consider the impurity average (for fixed χ) of the resulting contribution. We arrive at a term similar to (42), given by

$$S_3[\chi]=4m_F N(0)\int \mathrm{d}\tau\, \mathrm{d}\tau'\, \mathrm{d}^3 r \cdot \alpha_n(\tau-\tau')\sin^2\left[\frac{\chi(\mathbf{r},\tau)-\chi(\mathbf{r},\tau')}{4}\right]. \tag{77}$$

Note the additional factor 1/2 in the argument of the sine, which arises because of the coupling of Peierls to normal electrons. The kernel in (77) is given by:

$$\alpha_n(\tau)=\gamma_n\frac{\pi}{2}\,g_3(\tau)\,g_n(-\tau) \tag{78}$$

where $\gamma_n=(m_F\tau_n)^{-1}$, and g_n is the quasiclassical Green's function of the normal electrons. We have used the relation $\mathrm{i}\,G_n(\mathbf{r}=0,\tau)=\pi N_n(0)\,g_n(\tau)$ to arrive at this result. Besides an obvious factor γ_n instead of γ_0, $\alpha_n(\tau)$ differs from $\alpha(\tau)$ by the fact that one of the Green's functions in (43b) assumes its normal state value. Keeping this in mind, or repeating the procedure of Sect. 3 for $S_3[\chi]$ we conclude that (77) leads to an additional dissipative term in the equation of motion, and the total damping constant is given by

$$\gamma=\int_{-\infty}^{\infty}\mathrm{d}E\,N_1(E)\left(-\frac{\partial f_0}{\partial E}\right)\cdot[\gamma_0 N_1(E)+\gamma_n]. \tag{79}$$

The additional contribution has a temperature dependence given by Y, the Yoshida function. Note that in the presence of a finite line-width in the density of states, we have $N_1(E=0)\neq 0$, and find γ to be given by $\gamma=\gamma_0[N_1(0)]^2+\gamma_n N_1(0)$ in the zero temperature limit.

4.2. Screened Coulomb Interaction

Finally, we wish to examine some aspects of the Coulomb interaction (see also [37]), in particular with respect to dissipation of the charge-density wave. We imagine that the interaction between the electrons participating in the CDW is screened by the presence of normal electrons: This picture is in close correspondence with [38], where the friction in the motion of a particle interacting with a Fermi fluid is investigated (see also [39]). Thus we are led to consider the following contribution to the effective action:

$$S_c=\tfrac{1}{2}\int \mathrm{d}\tau\, \mathrm{d}\tau'\, \mathrm{d}^3 r\, \mathrm{d}^3 r'\, n(\mathbf{r},\tau)\, V(\mathbf{r}-\mathbf{r}',\tau-\tau')\, n(\mathbf{r}',\tau') \tag{80}$$

where $V(\mathbf{r}-\mathbf{r}',\tau-\tau')$ denotes the screened Coulomb interaction, i.e. $V(\mathbf{q},\omega_n)=V_0(\mathbf{q})/\varepsilon_n(\mathbf{q},\omega_n)$, where $\varepsilon_n(\mathbf{q},\omega_n)$ is the dielectric function of the normal electrons, related in the usual way to the normal electron density-density correlation function. In addition, the CDW density is given by $n(\mathbf{r},\tau)=-N(0)\,v_F\,\partial_x\chi+\rho_1\cos(\mathbf{Q}\mathbf{r}-\chi)$, where the second term is the rapidly oscillating contribution, and the factor $(1-Y)$ was omitted (in the first term, see (28)) for simplicity. Neglecting terms varying as $\exp(\pm\mathrm{i}\,\mathbf{Q}\mathbf{r})$, we obtain two contributions which we denote by S_4 and S_5:

$$S_4[\chi]=\tfrac{1}{2}[N(0)\,v_F]^2\int(\partial_x\chi)_{\mathbf{r},\tau}\,V(\mathbf{r}-\mathbf{r}',\tau-\tau')(\partial_{x'}\chi)_{\mathbf{r}',\tau'} \tag{81}$$

and

$$S_5[\chi]=\tfrac{1}{4}\rho_1^2\int \mathrm{Re}\{\mathrm{e}^{\mathrm{i}[\chi(\mathbf{r},\tau)-\chi(\mathbf{r}',\tau')]}\,\tilde{V}(\mathbf{r}-\mathbf{r}',\tau-\tau')\} \tag{82}$$

where the integration is with respect to the same variables as in (80), and $\tilde{V}=V\cdot\exp\mathrm{i}\,\mathbf{Q}(\mathbf{r}'-\mathbf{r})$. Thus,

for the Fourier transforms of V and $\tilde{V}$, we have the relation $\tilde{V}(\mathbf{q}, \omega_n) = V(\mathbf{q}+\mathbf{Q}, \omega_n)$. Following [38], we consider the low frequency expansion of the normal electron density-density correlation function, for simplicity at zero temperatures, which leads to the following result:

$$2N(0)[V(\mathbf{q},\omega) - V(\mathbf{q},0)] = |\omega|\,\tau_c(\mathbf{q}) + \ldots \tag{83}$$

where the factors have been chosen for convenience. Omitting the static part, we thus obtain S_4 to be given by:

$$S_4[\chi] = \frac{m_F N(0)}{4} \int \frac{\mathrm{d}\omega\, \mathrm{d}^3 q}{(2\pi)^4} |\chi_{\mathbf{q},\omega}|^2 c^2 q_x^2 \cdot |\omega|\,\tau_c(\mathbf{q}) \tag{84}$$

(recall that $c^2 = v_F^2/m_F$). Comparison with (27) demonstrates that S_4 can be considered a ω- and $\mathbf{q}$-dependent correction to the phason velocity, given by $\delta c^2 = c^2 \cdot |\omega|\,\tau_c(\mathbf{q})$. For a discussion of S_5, on the other hand, we can use $\chi(\mathbf{r}', \tau') \simeq \chi(\mathbf{r}, \tau')$ in (82), and in addition assume $\chi(\mathbf{r}, \tau) - \chi(\mathbf{r}, \tau') \ll 1$. Then we find the relevant contribution to be given by the following expression:

$$S_5[\chi] = \frac{1}{4}\rho_1^2 \int \frac{\mathrm{d}\omega\, \mathrm{d}^3 q}{(2\pi)^4} |\chi_{\mathbf{q},\omega}|^2 V(\mathbf{Q},\omega). \tag{85}$$

Assuming again a linear behavior for small ω, we can write (85) as follows:

$$S_5[\chi] = \frac{m_F N(0)}{4} \int \frac{\mathrm{d}\omega\, \mathrm{d}^3 q}{(2\pi)^4} |\chi_{\mathbf{q},\omega}|^2 \cdot \gamma_c |\omega| \tag{86}$$

where $\gamma_c = [\rho_1^2/m_F N(0)] \cdot (\partial V(\mathbf{Q},\omega)/\partial\omega)_{\omega \to +0}$. To find the corresponding term in the equation of motion, we minimize $S_4 + S_5$ with respect to χ, and perform the analytic continuation according to the rule $|\omega| \to -\mathrm{i}\omega$. The following modification of the damping term (see (68)) is found:

$$[\gamma \dot{\phi}]_{\mathbf{q},\omega} \to -\mathrm{i}\omega[\gamma + c^2 q_x^2 \tau_c(\mathbf{q}) + \gamma_c]\, \phi_{\mathbf{q},\omega}. \tag{87}$$

A reliable estimate of $\tau_c(\mathbf{q})$ and γ_c seems to be very difficult, especially for the anisotropic materials under consideration. As an illustration, we quote the standard RPA result for an isotropic system, which leads for small $\mathbf{q}$ to the following expression:

$$\tau_c(q) = \pi N(0)/2N_n(0)\, v_F^n\, q \tag{88}$$

where v_F^n is the Fermi velocity of the normal electrons, and we used the relation $q_{TF}^2 = 4e^2 p_F^n m/\pi = 8\pi e^2 N_n(0)$. Consequently, with $\omega \sim c\,q$ and assuming $N(0) \simeq N_n(0)$, $v_F \simeq v_F^n$, we conclude $\omega\tau_c \sim c/v_F \ll 1$; thus this contribution to damping is small. It must be kept in mind that the behavior of $V(\mathbf{q},\omega)$ linear in ω for $\omega \to 0$ is related to the possibility to create particle-hole pairs in the normal electron system, of arbitrarily small energy. Though this mechanism is likely to exist for the small wave-vectors which have to be considered in $\tau_c(\mathbf{q})$, we expect the result in general to be very anisotropic, and possibly smaller than our estimate.

The situation is even more complicated for the large wave-vectors which are considered in $V(\mathbf{Q},\omega)$, see (85). Of course, assuming the normal electrons to have a *spherical* Fermi surface with diameter $2p_F^n > Q$, we conclude from the RPA results that

$$\gamma_c = \frac{\rho_1^2}{m_F N(0)} \frac{8\pi m^2 e^4}{Q(Q^2 + q_{TF}^2)^2}. \tag{89}$$

Note that the range in frequency of the linear behavior shrinks to zero for $Q \to 2p_F^n$ and $\gamma_c = 0$ for $2p_F^n < Q$. As an estimate, assume $q_{TF} \sim p_F \sim p_F^n$, $N(0) \sim N_n(0)$, $m_F \sim 10^2$, and $\rho_1 \sim 0.1\,\rho_0^n$, which leads to $\gamma_c \sim 10^{-5} E_F \sim 10^{10}\,s^{-1}$. However, from the above discussion, it is clear that an estimate of γ_c depends very much on the details of the Fermi surface.

5. Conclusion

Several contributions to the damping of charge-density waves have been investigated, especially the effects of impurity scattering, and the particle-hole pair creation through the screened Coulomb interaction. With respect to elastic scattering, we remark that for high temperatures compared to the Debye temperature, Θ, electron-phonon scattering also contributes to momentum relaxation, with a rate given by $\hbar/\tau_{ep} \simeq 4\pi\lambda k_B T$ (in the normal state). Accordingly, (79) has to be modified. Considering recent experimental results [40], typical values for the damping constant γ are $\gamma_{\mathrm{exp}} \simeq 5 \times 10^{11}\,\mathrm{s}^{-1}$, which implies with $m_F \sim 10^2$ a single particle scattering rate ($\sim m_F \gamma$) which is comparable to or even larger than the value quoted for Δ [40]. If this interpretation can be confirmed, some aspects of the weak coupling theory have to be revised.

We briefly discuss the extension of our results to this case, i.e. to situations in which a large line-width (compared to Δ) is present in the density of states (strong pair-breaking limit). It is well known that $N_1(E)$, the normalized density of states, assumes its normal state value $N_1(E) = 1$; then, from (79), we conclude that γ is given by the *total* elastic scattering rate, divided by m_F. Assuming that electron-phonon scattering is the dominant process ($T \gg \Theta$), we thus obtain $\hbar\gamma \simeq 4\pi\lambda k_B T/m_F$. For example, using

the values for λ and m_F as quoted in [40] for $NbSe_3$ at $T=120\,K$, we find $\gamma \simeq 2\times 10^{11}\,s^{-1}$; thus our estimate is in reasonable agreement with the experimental result.

Finally, we remark that the damping of the charge-density wave motion due to interaction with thermal phasons was studied recently [41]. We note that the phason-phason interaction, which was discussed in [41], can also be derived easily with the methods described in this article, by expanding the action, (17), in higher orders with respect to $\hat{V}$. This procedure was used in [21], and applied to the low temperature limit. As stated in [41], the resulting damping constant is slightly smaller than the result found above (at $T\simeq 150\,K$), however, it is proportional to the *square* of the temperature. Clearly, additional experimental and theoretical investigations are desirable, to illuminate further the relaxation processes relevant for the dissipation in the motion of charge-density waves.

Returning to the question alluded to in the introduction, namely the controversy between classical and quantum models, it seems to us that under most circumstances, the transport properties of charge-density wave systems can be adequately described by the classical equation of motion as discussed above. In this context, it is also important that dissipation decreases quantum effects: For example, it was shown that the decay rate of a single particle out of a metastable minimum of the potential decreases upon increasing the coupling to an environment [18]. Similar results have been found for the soliton-antisoliton pair creation rate in the sine-Gordon model [42, 43], which has been discussed in relation to the nonlinear conductivity [44, 30]. Furthermore, for a single particle moving in a *periodic* potential, quantum coherence is destroyed by increasing the dissipation beyond a critical value [45, 46]; i.e. the particle behaves classically if dissipation is large. In the context of charge-density wave systems, these ideas remain to be investigated.

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