# Quantum phase and magnetic flux

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THE Aharonov–Bohm effect, first described in 1959, is one example of the mysterious properties of quantum mechanics. Charged particles, propagating around a magnetic field in field-free vacuum, acquire a phase shift of their wavefunctions that has observable consequences. This has been confirmed — again, but now incontrovertibly — in a recent experiment[1]. Even more surprising, however, is the appearance of Aharonov–Bohm oscillations in some non-superconducting solid-state devices[2-5] which, as well as indicating possible technical applications[4], contributes significantly to our understanding of quantum interference on a macroscopic level. The most recent example, from G. Timp *et al.*[5], demonstrates that the effect occurs in high-mobility semiconductor samples.

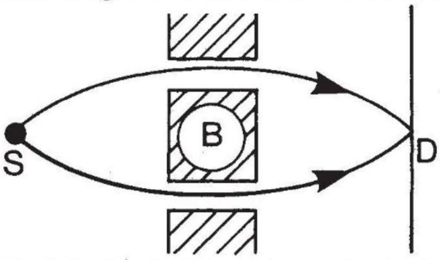


**Fig. 1** The idealized two-slit geometry for the Aharonov–Bohm effect. An electron beam from source S passes through two slits to arrive at the detector D. The magnetic field, which points into the page, is confined to the shaded regions. Despite this, it still affects the quantum-mechanical phase of the electrons.

In the typical (idealized) geometry (Fig. 1), an electron beam travels from a source to a detector by two different paths; the magnetic field (**B**) is confined to a long solenoid threading a region, inaccessible to the electrons, between the paths. The interference pattern thus detected is a periodic function of the magnetic flux, $\Phi$, with the period given by $\Phi_0 = hc/e = 4.135\times10^{-7}$ gauss cm$^2$, although the electrons never enter into a region of non-zero field. Alternatively, one can imagine an electron confined to a box that is transported very slowly (adiabatically) on a closed loop around the flux[6]. According to the laws of quantum mechanics, the phase, $\varphi$, of the electron wavefunction changes by $2\pi/\Phi_0$ times the line-integral of the vector-potential, **A**, along the loop. (**A** is to magnetic field roughly as electrostatic potential is to electrostatic field.) Thus $\Delta\varphi = 2\pi\Phi/\Phi_0$. These and similar considerations point to the fundamental role that electromagnetic potentials, rather than the fields, play in quantum physics of charged particles.

In one recent experiment[1] that was designed as a critical test of the effect, a tiny toroidal magnet was covered entirely by a superconductor (niobium), to prevent any leakage of the magnetic field into the electron path. The electron beam passing through the ring and the reference beam were brought together by an electron biprism to form an interference pattern, which was enlarged 1,000 times by electron lenses and recorded on film as a hologram. Because the magnetic flux enclosed by a superconductor is quantized in units of $\Phi_0/2$ (see the recent News and Views article by David Caplin[7]), the phase difference between the two beams is restricted to $\Delta\varphi = 0$ or $\pi$ (modulo $2\pi$). A typical result, with an odd number of flux quanta, is shown in Fig. 2.

In the past few years, the Aharonov–Bohm effect has also been detected in various non-superconducting solid-state devices, including metal cylinders[2] and rings[3], and in semiconductor microstructures[4,5], provided that the structures are small ('mesoscopic', scale $\sim 1$ μm) and the temperature is low ($< 1$ K). In a typical experiment, small oscillations of the resistance are recorded as an applied perpendicular field is varied, with a dominant period corresponding to $\Phi_0$ in thin rings, and $\Phi_0/2$ in the cylinder geometry.

Most results can be explained using Feynman's 'path summation' picture. Applied, for example, to a metal ring (see Fig. 3) with a diameter much larger than the metal width (so that the enclosed flux is well defined), this leads to the conclusion that the resistance of the device varies according to whether the electrons passing through the two semicircles recombine in phase, in antiphase or in between. Thus the resistance has a component that is periodic in $\Phi$, with period $\Phi_0$. Because of imperfections and impurities, one might expect the effect to be unobservable. But elastic scattering does not destroy quantum coherence, unlike inelastic processes such as scattering by phonons; the latter, fortunately, are negligible for mesoscopic samples at very low temperatures. On the other hand, because of the random impurity distribution, it is important that the phase difference to start with (at zero field) is also a random quantity.

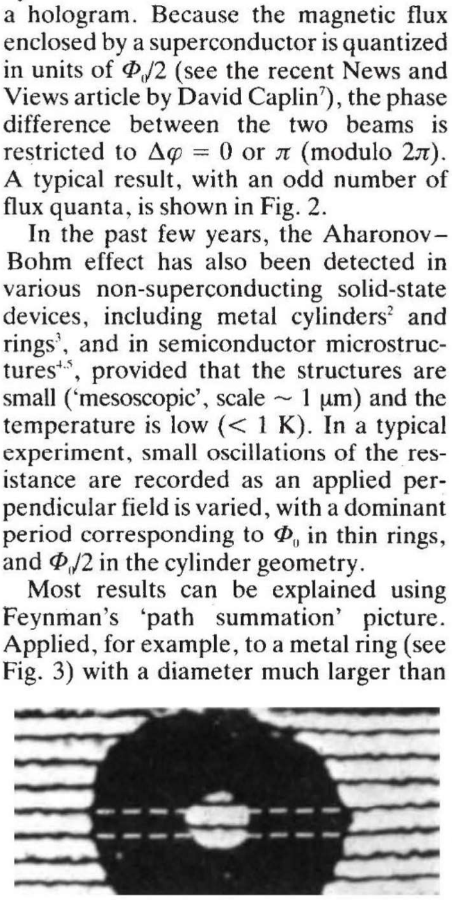

**Fig. 2** Interference micrograph of a toroidal magnet[1]. The interference fringes show that electrons passing through the middle of the magnet have had their phase shifted by $\pi$ relative to the electrons outside the rings — otherwise the fringes would be aligned.

Thus, the Aharonov–Bohm oscillations, as well as the slow random variations onto which they are superimposed[8], have to be considered as fingerprints of an individual sample. An average over many samples, as done effectively in the cylinder geometry[2], completely washes out the $\Phi_0$ oscillations, leaving only the $\Phi_0/2$ oscillations. The latter, in fact, arise from a higher-order interference of paths that encircle the magnetic flux, with their time-reversed counterparts[9].

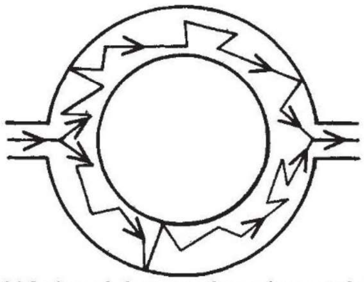

**Fig. 3** Motion of electrons through a metal ring. Elastic collisions off impurities do not destroy quantum coherence. Electrons take a more direct, or ballistic, path through high-purity, high-mobility semiconductors.

In their recent article[5], Timp *et al.* report the first unambiguous observation of the Aharonov–Bohm effect in semiconducting rings of interleaved GaAs and AlGaAs ('heterostructures'). In contrast to an earlier experiment[4] on similar device that saw only about three periods of the oscillation, the new experiment shows pronounced oscillations that persist up to $\Phi \sim 300\Phi_0$, with a period that can only be associated with $\Phi_0$.

That the oscillations are so strong, accounting for 10 per cent of the resistance is perhaps not so surprising, as the samples have very high charge mobility; the distance travelled between scattering events is comparable to the size of the device. Under favourable conditions, the variations are expected to be even stronger. Thus the experiment also indicates the possibility of potential applications in rapid-switching devices[4] that modulate resistance with a transverse electric field using the electrical Aharonov–Bohm effect. This effect, however, remains to be demonstrated experimentally. □

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