Kinetic theory of charge-density wave systems in the presence of long-range Coulomb forces

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The coupling between the charge-density wave condensate, and the quasiparticles and normal electrons, due to long-range Coulomb forces is investigated within the framework of the kinetic theory, for finite temperatures and in the presence of impurity scattering. The temperature dependence of the parameters, which enter the phenomenological equations for the transport in CDW systems, is determined. In the absense of normal electrons, a strong enhancement of the damping and the velocity of the phase mode is found for low temperatures, which is due to the freezing out of quasiparticles.

1. Introduction

Though many years of experimental and theoretical investigations[1] have established that the unusual phenomena found in many quasi-onedimensional conductors at low temperatures are related to the formation of a charge-density wave (CDW) state, the question of the transport in these systems is still an open one. In particular, the controversy between explanations based on a quantum approach, or on a classical model, seems to be unsettled [3–5]. Furthermore, recent experiments [6–10] have revealed that there is a rather strong coupling between the CDW condensate and the remaining free carriers, namely the quasiparticles which are excited thermally across the Peierls gap, and other normal electrons (from those parts of the Fermi surface which do not participate in the formation of the CDW state). A possible explanation for these results invokes the long-range Coulomb interaction: The charge fluctuations associated with the motion of the CDW are screened by quasiparticles and, since the number of quasiparticles is strongly temperature dependent, a strong increase of the CDW viscosity results upon lowering the temperature [11].

In fact, it is well-known [12–13] that long-range Coulomb forces have a drastic effect on the phase mode at zero temperatures (in the absence of normal electrons): The (acoustic) phason becomes an optic mode if the Coulomb interaction is taken into account. Considering finite temperatures, on the other hand, the phason velocity is found to diverge with decreasing temperature [14–17], a feature which has been used [18, 19] to explain the strong temperature dependence of several quantities in KCP.

In this article, I wish to investigate the effect of the long-range Coulomb forces within the framework of the kinetic theory [14, 20, 21], i.e. by explicitly considering the coupling between the CDW condensate, and the thermally excited quasiparticles and/or normal electrons. The derivation of the kinetic equations, which has been discussed in detail in [21], starts from microscopic theory and utilizes the concept of so-called quasiclassical Green's functions, i.e. Green's functions which have been integrated with respect to the magnitude of the momentum. The equations represent a rather general approach to the dynamics of CDW systems, and they are equivalent to microscopic theory, as long as the variation in space and time is slow compared to the atomic scale. As a special feature of the momentum integrated Green's functions, these quantities turn out to be normalized which allows to define quasiparticle distribution functions. Furthermore, various scattering processes can be included easily [21]. Note also that in the limit of well-defined quasiparticles, the kinetic theory as

[1] Some reviews are given in [1]; see also [2]

discussed here can be related to the more familiar transport equation studied in [22].

In this paper, I will concentrate on the derivation of the (temperature dependence of the) parameters entering the phenomenological equation of motion for the phase of the order parameter [11]. Long-range Coulomb forces are taken into account by coupling the equations to Poisson's equation, which corresponds to the RPA approximation for the Coulomb interaction. In particular, impurity scattering is included, thereby generalizing earlier results from microscopic theory [15, 16], especially to the most important hydrodynamic regime in which the frequency is small compared to the scattering rate (see also [17]). It turns out to be straightforward to recover the above mentioned results, and to discuss their range of validity.

This article is organized as follows. In the next section (Sect. 2), I briefly review the kinetic theory, concentrating on the low-frequency long-wavelength limit (to be specified below). The results, in the presence of Coulomb forces, are presented in Sect. 3. In the final section (Sect. 4), the results are summarized, and the relation to experiment is discussed.

2. Review of the kinetic theory

2.1. The kinetic equations

In this section, I give a brief review of the kinetic theory [14, 21] in the limit of low frequencies and long wavelengths such that $\omega \ll \Delta$, ω_Q; $v_F q \ll \Delta$, where ω and q are the external frequency and wave-vector, respectivley, Δ is the magnitude of the order parameter, ω_Q the characteristic phonon frequency, and v_F the Fermi velocity.[2] Under these conditions, it is possible to consider variations of the phase of the order parameter, χ, only, and to linearize the kinetic equations with respect to small deviations of the distribution functions f^L and f^T from equilibrium: $f^L = f_0 + \delta f^L$, where f_0 is the Fermi function, while $f^T = 0$ in equilibrium. In addition, the impurity scattering rates (τ_1^{-1}: forward scattering, τ_2^{-1}: backward scattering) are also assumed to be small compared to Δ. Concentrating on the one-dimensional case, the following Boltzmann-like transport equations have been derived ((E91) and (E92)):

$$N_1(-\mathrm{i}\omega\delta f^L + \mathrm{i} v_F q f^T) = 0, \tag{1}$$

$$N_1(-\mathrm{i}\omega f^T + \mathrm{i} v_F q \delta f^L) + 2\Delta N_2^\omega f^T + \frac{1}{\tau_2} N_1^2 f^T = N_1\left[\psi + \frac{N_1 \dot{\chi}_{\omega,q}}{2\tau_2}\right]\left(\frac{\partial f_0}{\partial E}\right), \tag{2}$$

[2] Units are such that $\hbar = k_B = 1$. Also, I will refer to equations given in [21] as, for example, (E91): This denotes Eq. (91) of [21]

Note that δf^L and f^T are functions of the energy E, ω, and q, and I sometimes use an obvious mixed notation; for example $\dot{\chi}_{\omega,q} \equiv -\mathrm{i}\omega\chi_{\omega,q}$. Also, N_1 denotes the (normalized) BCS density of states, and

$$N_2^\omega = N_2 + \mathrm{i}\omega\partial_E R_2/2 \tag{3}$$

where N_2 and R_2 are related to the regular (retarded and advanced) Green's functions (see (E53)). For the present discussion, the approximation

$$2\Delta N_2^\omega \simeq (\Delta^2 N_1/\xi^2)\cdot(-\mathrm{i}\omega + N_1/\tau) \tag{4}$$

will be sufficient, for $\xi^2 = E^2 - \Delta^2 > 0$; $1/\tau = 1/\tau_1 + 1/2\tau_2$. The physical meaning of δf^L and f^T will be transparent when considering the expressions for the charge-density and the current ((E89) and (E90)):

$$\rho = 2N(0)\, e[\int \mathrm{d}E N_1\, \delta f^L - \tfrac{1}{2} v_F \chi'], \tag{5}$$

$$j = 2N(0)\, v_F\, e[\int \mathrm{d}E N_1 f^T + \tfrac{1}{2}\dot{\chi}] \tag{6}$$

where $N(0)$ is the normal-state density of states at the Fermi surfaces (for one spin; $N(0) = (\pi v_F)^{-1}$ in one dimension). Thus δf^L and f^T describe variations of the density (and the energy), and the current (and the energy current), respectively. Also, I defined

$$\psi = (\ddot{\chi} - v_F^2 \chi'' - 2e v_F \mathscr{E})/2 \tag{7}$$

where $\mathscr{E}$ is the electric field.

The quasiparticle distribution function, more precisely, f^T, is coupled to the equation of motion for the phase via a kind of time-dependent Ginzburg-Landau equation, namely (see (E87)):

$$m_F \ddot{\chi} + (1 - Y)(\ddot{\chi} - v_F^2 \chi'' - 2e v_F \mathscr{E}) = 4\Delta \int_{-\infty}^{\infty} \mathrm{d}E N_2^\omega f^T. \tag{8}$$

Here m_F, the ratio of the Fröhlich mass to the electron mass, is given by $m_F = 4\Delta^2/\lambda\omega_Q^2$, where λ is the (dimensionless) electron-phonon coupling constant. Note that (8) does not contain the impurity pinning potential, which can be obtained by other techniques (see, for example, [23]).

2.2. Some results

An inspection of the transport equations reveals a special feature which is due to the neglect of *inelastic* scattering processes. Clearly, Eq. (1) leads to the relation $\omega \cdot \delta f^L = v_F q \cdot f^T$, which means that the quasiparticle charge is conserved for each energy: In the absence of inelastic processes, there is no communication between different energies. Furthermore, the rhs of (2) can be considered as a drive term for the distribution functions, and it consists of two contributions, $\sim\psi$, and $\sim\dot{\chi}$. Obviously, (1) and (2) can be solved for f^T, and inserted into (8) to yield the equation of motion for the phase, and into (5) and (6) to express the charge-density and the current in terms of the phase and the electric field. The result is written as follows:

$$4\Delta\int \mathrm{d}E N_2^{\omega} f^T = -a\cdot 2\psi - b\cdot\dot{\chi}/\tau_2 \tag{9}$$

and

$$j=\sigma_{qp}\cdot\left[\mathscr{E}-\frac{\ddot{\chi}-v_F^2\chi''}{2ev_F}\right]+\frac{e}{\pi}(1-\mu_{qp})\cdot\dot{\chi} \tag{10}$$

where a, b, σ_{qp}, and μ_{qp} are relatively complicated ω, q, T dependent expressions which, however, can be calculated by a single integration with respect to energy. Defining $\mathscr{K}(E)$ by

$$\mathscr{K}^{-1}=-\mathrm{i}\omega N_1[1-(v_F q/\omega)^2]+N_1^2/\tau_2+2\Delta N_2^{\omega}. \tag{11}$$

I obtain the following results (note that because of the density of states, N_1, the integration is over the range $E^2>\Delta^2$ only):

$$\begin{pmatrix}a\\b\end{pmatrix}=2\Delta\int_{-\infty}^{\infty}\mathrm{d}E\left(-\frac{\partial f_0}{\partial E}\right)\cdot N_1 N_2^{\omega}\mathscr{K}\cdot\begin{pmatrix}1\\N_1\end{pmatrix} \tag{12}$$

and

$$\begin{pmatrix}\sigma_{qp}/\sigma_N\\ \mu_{qp}\end{pmatrix}=\int_{-\infty}^{\infty}\mathrm{d}E\left(-\frac{\partial f_0}{\partial E}\right)\cdot\frac{N_1^2\mathscr{K}}{\tau_2}\cdot\begin{pmatrix}1\\N_1\end{pmatrix} \tag{13}$$

where $\sigma_N=2e^2v_F\tau_2/\pi$ is the normal state dc conductivity. Clearly, the quantities a, b, σ_{qp}, and μ_{qp} characterize the quasiparticle effects in the dynamics of the charge-density wave.

As an illustration, I consider σ_{qp} and μ_{qp} in more detail, and especially in the hydrodynamic regime where the frequency is small compared to the elastic scattering rate. Actually, because of an extra density of states factor multiplying $1/\tau_2$ and $1/\tau$, it is sufficient to require (τ_1 is assumed to be of the same order as τ_2) that $\omega\tau\ll(T_c/T)^{1/2}$. Then Eq. (13) simplifies to the following one:

$$\frac{\sigma_{qp}}{\sigma_N}=\int_{-\infty}^{\infty}\mathrm{d}E\left(-\frac{\partial f_0}{\partial E}\right)\frac{\xi^2}{\xi^2+\Delta^2\tau_2/\tau}\cdot\frac{-\mathrm{i}\omega}{-\mathrm{i}\omega+D(E)q^2} \tag{14}$$

where I defined the (energy-dependent) diffusion constant, $D(E)$, by the relation ($D_N=v_F^2\tau_2$):

$$D(E)=D_N\cdot\left(\frac{\partial E}{\partial\xi}\right)^2\cdot\frac{|E|\cdot\xi}{\xi^2+\Delta^2\tau_2/\tau}. \tag{15}$$

A similar expression holds for μ_{qp}, which differs from (14) only by an additional factor N_1 under the integral. Considered as a function of the wave-vector, or more precisely of $x\equiv D_N q^2/(-\mathrm{i}\omega)$, σ_{qp} is a decreasing function for increasing x, with $\sigma_{qp}(x=0)\equiv\sigma_0$, and $\sigma_{qp}\simeq\sigma_N Y/x$ for $x\to\infty$, where $Y=Y(T)$ is the Yosida function[3]. As a *reasonable interpolating formula*, I propose to replace $D(E)$ by a constant, $\bar{D}$, such that

$$\sigma_{qp}\simeq\sigma_0\cdot(-\mathrm{i}\omega)/(-\mathrm{i}\omega+\bar{D}q^2) \tag{16}$$

where $\bar{D}$ is chosen such that the correct limit is obtained for $x\to\infty$, leading to the relation $\bar{D}=D_N\cdot(\sigma_0/\sigma_N)/Y$. The temperature dependence of these quantities, as well as of $\mu_0\equiv\mu_{qp}(x=0)$, is shown in Figs. 1 and 2. For completeness, I summarize the re-

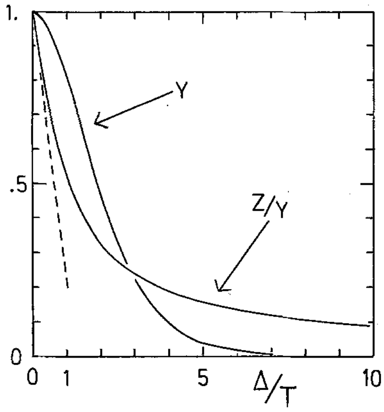

Fig. 1. Temperature dependence of Y and Z/Y, where $Z=\mu_0(\tau=\tau_2)$. The dashed line indicates the behavior of Z close to T_c (see Eq. (18))

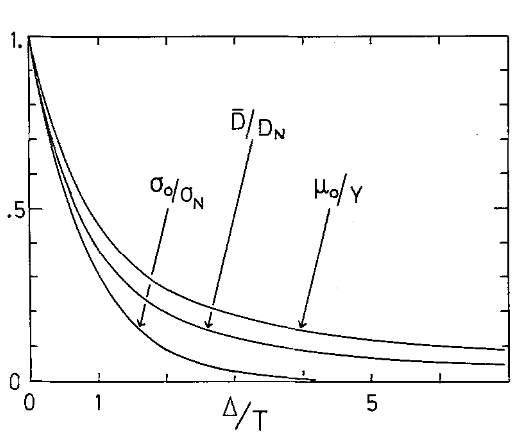

Fig. 2. Temperature dependence of σ_0/σ_N, $\bar{D}/D_N$, and μ_0/Y, for $\tau_1=\tau_2$

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[3] The paramagnetic susceptibility of an ideal superconductor, relative to its normal state value, was found by Yosida [24] to be given by this function, now commonly denoted Y

sults which are obtained analytically for $T \simeq T_c$, and for $T \ll T_c$:

(i) Close to T_c:

$$\sigma_0/\sigma_N \simeq \bar{D}/D_N \simeq 1 - k_1 \cdot \Delta/2T_c \tag{17a}$$

where, with $k_2 = (\tau_2/\tau - 1)^{-1/2}$, k_1 is given by

$$k_1 = 1 + k_2 \cdot \frac{\tau_2}{\tau}\left(\frac{\pi}{2} - \operatorname{arctg} k_2\right). \tag{17b}$$

Furthermore,

$$1 - \mu_0 \simeq (\pi\Delta/4T_c)(\tau_2/\tau)^{1/2}. \tag{18}$$

For example, for $\tau_1 = \tau_2$ (i.e. $\tau_2/\tau = 3/2$), one finds $k_1 \simeq 2.31$, and $1 - \mu_0 \simeq 0.96 \cdot \Delta/T_c$.

(ii) Low temperatures:

$$\sigma_0/\sigma_N \simeq 4(\tau/\tau_2)(T/\Delta)\,\mathrm{e}^{-\Delta/T}, \tag{19}$$

$$\mu_0 \simeq (\tau/\tau_2)(2\pi T/\Delta)^{1/2}\,\mathrm{e}^{-\Delta/T}, \tag{20}$$

$$\bar{D}/D_N \simeq 2(2/\pi)^{1/2}(\tau/\tau_2)(T/\Delta)^{3/2}. \tag{21}$$

Note that for the special case $\tau = \tau_2$, μ_0 agrees with the function $Z(T)$ defined earlier (see (E104)), and that $\mu_0 = Z(T)$ is also recovered in the collisionless regime [22]. Remarkably, in these cases, the quasiparticle distribution function, f^T, is similar to the one discussed in connection with the branch imbalance in superconductors [25] (see also [21]).

Finally, the second term in the square bracket and the last term in Eq. (10), can be combined to the relation

$$j = \sigma_{qp} \cdot \mathscr{E} + (e/\pi) \cdot v \cdot \dot{\chi} \tag{22}$$

where v is related to σ_{qp} and μ_{qp} in an obvious way:

$$v = 1 - \mu_{qp} + \frac{\sigma_{qp}}{\sigma_N} \cdot \frac{\tau_2}{-\mathrm{i}\omega}(\omega^2 - v_F^2 q^2). \tag{23}$$

Thus, from the above results, one finds $v \simeq 1 - Y$ in the static limit ($\omega \to 0$), while $v \simeq 1 - \mu_0$ in the opposite case ($q \to 0$). Note also that, from the continuity equation, the charge-density is given by $\rho = q \cdot j/\omega$.

2.3. The phase mode

Returning to the equation of motion for the phase, it is evident that (8) and (9) can be combined to the following equation:

$$\ddot{\chi} - c^2\chi'' + \gamma\dot{\chi} = 2e^\star v_F \mathscr{E} \tag{24}$$

where c^2, $e^\star$, and γ (which are functions of ω and q) are given by

$$\frac{c^2}{v_F^2} = \frac{e^\star}{\mathrm{e}} = \frac{1 - Y + a}{m_F + 1 - Y + a} \simeq \frac{1 - Y + a}{m_F} \tag{25}$$

and

$$\gamma = \frac{b/\tau_2}{m_F + 1 - Y + a} \simeq \frac{b/\tau_2}{m_F}. \tag{26}$$

The last equality in (25) and (26) holds for all temperatures, provided $m_F^0 \equiv m_F(T=0) \gg 1$, except in a small region close to T_c, given by $T_c - T_1 \simeq T_c/(m_F^0)^2$. [However, all results have also been derived under the assumption $\Delta \cdot \tau \gg 1$.] This is justified by noting that in the hydrodynamic limit, Eq. (12) simplifies to the following one:

$$a = \int_{-\infty}^{\infty} \mathrm{d}E\left(-\frac{\partial f_0}{\partial E}\right)\frac{N_1\,\Delta^2\tau_2/\tau}{\xi^2 + \Delta^2\tau_2/\tau} \cdot \frac{\mathrm{i}\omega}{-\mathrm{i}\omega + D(E)\,q^2}. \tag{27}$$

In particular, this leads to the relation $a_0 \equiv a(q=0) = Y - \mu_0$; thus $a_0 \simeq 1 - \mu_0 \sim \Delta/T_c$ close to the critical temperature.

The quantity b, entering the "viscosity" γ, is given by an expression which differs from (27) only by an additional factor N_1 under the integral. In order to determine the velocity and the damping of the phase mode, in the limit $q \to 0$, I expand b with respect to q as follows:

$$b(q \to 0) \simeq b_0 - b_1 \cdot D_N q^2/(-\mathrm{i}\omega)$$

which leads to

$$\gamma\dot{\chi} \simeq \gamma_0\dot{\chi} + b_1 \cdot v_F^2 \chi''/m_F \tag{28}$$

where $\gamma_0 = \gamma(q=0)$ is discussed briefly below. Thus, in the limit $q \to 0$, the phason velocity is found to be given by

$$q \to 0: \quad \frac{c_\chi^2}{v_F^2} = \frac{1 - \mu_0 - b_1}{m_F + 1 - \mu_0} \tag{29}$$

where the full denominator was included for completeness. In particular, the phason velocity increases with increasing temperature,[4] and is proportional to Δ^{-1} in the region $T_c - T_1 < T_c - T \ll T_c$. This is in strong contrast to the static limit, where

$$\omega \to 0: \quad \frac{c_\chi^2}{v_F^2} = \frac{1 - Y}{m_F + 1 - Y}. \tag{30}$$

[4] Close to T_c, one finds $b_1 \simeq (1 - \mu_0)/2$

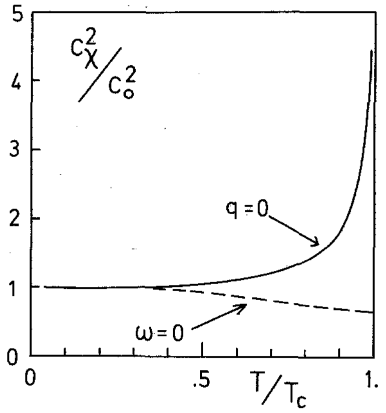

Fig. 3. Temperature dependence of the square of the phason velocity, normalized to the zero temperature value, in the static ($\omega=0$) and dynamic ($q=0$) limit, for $m_F^0=10^2$ and $\tau_1=\tau_2$

In this limit, c_χ^2 decreases (weakly) with increasing temperature, to the value [15] $c_\chi^2 \simeq 0.66 \cdot c_\chi^2(T=0)$ for $T \to T_c$. At low temperature, of course, the standard result $c_0^2 \equiv c_\chi^2 T(=0) = v_F^2/(1+m_F)$ is obtained. For illustration, the temperature dependence of the phason velocity is shown in Fig. 3, where I have chosen $m_F^0 = 10^2$.

Finally, the damping constant γ_0 (which is given explicitly in (E95)) describes dissipation due to the scattering of thermally excited quasiparticles at impurities. Therefore, γ_0 is strongly temperature dependent, and decreases $\sim \exp(-\Delta/T)$ at low T. In order of magnitude, one finds

$$\gamma_0 \sim \frac{1}{m_F \tau_2} \begin{cases} \Delta/4T_c & \Delta \ll T \\ (\Delta/T)\, e^{-\Delta/T} & \Delta \gg T. \end{cases} \tag{31}$$

In this expression, I have omitted logarithmic corrections which arise from a divergence in the integral, for $\xi^2 \to 0$, which can be handled by introducing a finite linewidth in the density of states. Note, in particular, that γ_0 increases $\sim \Delta^{-1}$ for $T_c - T_1 < T_c - T \ll T_c$, with $\gamma_0 \sim \tau_2^{-1}$ for $T_c - T_1 \sim T_c - T$ (as an order of magnitude estimate). I wish to remark at this point that an additional damping mechanism, namely phason-phason scattering, which results in a somewhat weaker temperature dependence of the damping constant, was studied in [26].

3. Long-range Coulomb forces

Before taking into account the effects of the long-range Coulomb forces, I briefly discuss the extension of the results of the previous section to *quasi*-onedimensional situations. First of all, the transverse dispersion of the phase mode is taken into account by the replacement $c^2 \partial_x^2 \to c^2 \partial_x^2 + c_\perp^2 \nabla_\perp^2$ in (24), where $c_\perp (\ll c)$ is of the order of the phonon velocity, and I have chosen the CDW wave-vector to be along the $\hat{x}$-direction. Consequently, $\mathscr{E} \to \mathscr{E}_x$ in (24), too. Secondly, the current is generalized as follows:

$$\mathbf{j} = j \cdot \hat{\mathbf{x}} + \boldsymbol{\sigma}_n \cdot \mathscr{E} \tag{32}$$

where [5]

$$j = \sigma_{qp} \cdot \mathscr{E}_x + j_{\mathrm{CDW}}; \qquad j_{\mathrm{CDW}} = e v_F N(0) \cdot v \cdot \dot{\chi} \tag{33}$$

is the current due to the CDW and the quasiparticles which are excited thermally across the Peierls gap, and the last term in (32) describes the effect of normal electrons, i.e. electrons from those parts of the Fermi surface which are not affected by the CDW transition. The quantities $v(q_x, \omega)$ and $\sigma_{qp}(q_x, \omega)/\sigma_N$ have been discussed above; furthermore, $N(0)$ is the (3-D) density of states, and $\sigma_N = n_0 e^2 \tau_2/m$, where n_0 is the density of Peierls electrons.

Following the standard procedure, long-range Coulomb forces are taken into account by adding Poisson's equation:

$$\nabla \cdot (\mathscr{E} - \bar{\mathscr{E}}) = 4\pi(\rho + \rho^\star) \tag{34}$$

where $\bar{\mathscr{E}}$ is the (externally controlled) spatial average of the electric field, and ρ is the charge density related by the continuity equation to the current in (32). For completeness, I also included $4\pi\rho^\star \equiv -\varepsilon^\star \partial_x \mathscr{E}_x$, where $\varepsilon^\star$ describes virtual excitations across the Peierls gap [12, 13]. It is found that the standard zero temperature result [12, 15, 16], namely $\varepsilon^\star = \omega_p^2/6\Delta^2$, where ω_p is the Plasma frequency, follows from the quasiclassical approach [21] by considering higher order terms (in the sense of the gradient expansion) in the equations given above.

Defining $\delta\mathscr{E} \equiv \mathscr{E} - \bar{\mathscr{E}}$, and putting $\delta\mathscr{E} = -\nabla\phi$ in an intermediate step, the following relation between the phase and the electric field is found:

$$\{q^2[1 + 4\pi i \hat{\sigma}/\omega] + \varepsilon^\star q_x^2\}\, \delta\mathscr{E}_x = -4\pi e v_F N(0) \cdot v \cdot q_x^2\, \delta\chi \tag{35}$$

where $\delta\chi = \chi - \bar{\chi}$; $\bar{\chi}$ is the spatial average of the phase, and $\hat{\sigma}$ is defined by

$$\hat{\sigma} \equiv \mathbf{q} \cdot \boldsymbol{\sigma} \cdot \mathbf{q}/q^2 = [q_x^2(\sigma_{qp} + \sigma_{n\parallel}) + q_\perp^2 \sigma_{n\perp}]/q^2 \tag{36}$$

where $\sigma_{n\parallel}$ and $\sigma_{n\perp}$ are the parallel and perpendicular components of $\boldsymbol{\sigma}_n$. Combining (35) and (24) I thus obtain the following equation of motion:

$$\ddot{\chi} + \gamma \dot{\chi} - c_\perp^2 \nabla_\perp^2 \chi - \tilde{c}^2 \partial_x^2 \chi = 2 e^\star v_F \bar{\mathscr{E}}_x \tag{37}$$

[5] Note that $\dot{\chi}/2p_F$ can be identified with the CDW velocity

where $\tilde{c}^2_{\omega,q}$ is given by ($q^2_{TF}=8\pi e^2 N(0)$):

$$\frac{\tilde{c}^2}{c^2}=1+\frac{q^2_{TF}\cdot v}{q^2(1+4\pi i\hat{\sigma}/\omega)+\varepsilon^\star q_x^2}. \tag{38}$$

From this equation, and considering the static limit ($\omega\to 0$) in which case one has the relation

$$\lim_{\omega\to 0} q^2\hat{\sigma}/(-i\omega)=2e^2[N(0)\cdot Y+N_n(0)] \tag{39}$$

where $N_n(0)$ is the normal electron density of states, the renormalization of the phason velocity is easily determined. Taking into account that $v(\omega=0)=1-Y$, I find the following result:

$$\tilde{c}^2\simeq c^2\frac{1+\alpha}{Y+\alpha}, \qquad \alpha\equiv\frac{N_n(0)}{N(0)}. \tag{40}$$

Here, in addition, the long wave-length limit ($q^2+\varepsilon^\star q_x^2\ll q^2_{TF}(Y+\alpha)$) is considered. Of course, Eq. (40) is in agreement with the results of [15]. Remarkably, the phason velocity *increases* upon lowering the temperatures, in a rather dramatic way if α is small, to the value $\tilde{c}^2=c^2\cdot(1+\alpha)/\alpha$ for $T\to 0$. (See [18] and [19] for a comparison with experiment.) Of course, (40) becomes incorrect in the absence of any screening ($T=0$ and $\alpha=0$), where the acoustic phase mode is shifted to a finite frequency [15, 16], as follows immediately from (38).

On the other hand, in the dynamic limit $\omega\gg\bar{D}q_x^2$ (but nevertheless $\omega\ll 4\pi(\sigma_0+\sigma_{n\parallel})/\varepsilon^\star$), the effect of the screening current is best described in terms of an effective damping constant. Using $v(q_x=0)\simeq 1-\mu_0$, and $c^2(q_x=0)/v_F^2\simeq(1-\mu_0)/m_F$, I obtain the following expression:

$$\tilde{\gamma}=\gamma_0+\frac{(1-\mu_0)^2}{m_F\tau_2}\frac{\sigma_N}{\sigma_0+\sigma_{n\parallel}+q_\perp^2\sigma_{n\perp}/q_x^2}. \tag{41}$$

Combining (41) with (31), I conclude that the damping due to the screening currents is small close to the critical temperature, since $m_F^0\tau_2\gamma_0\sim T_c/\Delta$, while $m_F^0\tau_2(\tilde{\gamma}-\gamma_0)\sim 1$. On the other hand, assuming for simplicity that there are no normal electrons, one finds for $T\ll T_c$:

$$\tilde{\gamma}\simeq\frac{1}{m_F\tau_2}\frac{\sigma_N}{\sigma_0}=\frac{\omega_p^2}{4\pi m_F\sigma_0} \tag{42}$$

where $\sigma_N/\sigma_0\simeq(\Delta/T)(\tau_2/4\tau)\exp(\Delta/T)$. As an important point[6], $\tilde{\gamma}$ *increases* upon decreasing the temperature, and $\tilde{\gamma}\sim\sigma_0^{-1}$.

[6] This result was first obtained by Sneddon [11], while the corresponding increase of the phason velocity was first discussed by Artemenko and Volkov [14]

In summary, neglecting for simplicity any normal carriers, but taking into account the expression (16) for σ_{qp}, the equation of motion is thus given by

$$\ddot{\chi}+\gamma\dot{\chi}-c_\perp^2\nabla_\perp^2\chi+\tilde{\gamma}\delta\dot{\chi}-\tilde{c}^2\partial_x^2\chi=2e^\star v_F\bar{\mathscr{E}}_x \tag{43}$$

where

$$\tilde{\gamma}=\frac{\omega_p^2}{4\pi}\frac{c^2}{v_F^2}\frac{v}{\sigma_0}; \qquad \tilde{c}^2=c^2(1+v/Y). \tag{44}$$

Note that $\tilde{c}^2/\tilde{\gamma}\simeq\bar{D}$ in the low temperature regime. Under the conditions specified above ($q^2+\varepsilon^\star q_x^2\ll 4\pi q^2\hat{\sigma}/\omega$), the electric field can be determined directly from the charge neutrality condition, which in the absence of normal electrons leads to the relation

$$\sigma_0\,\delta\mathscr{E}_x=-ev_F N(0)\cdot v\cdot(\delta\dot{\chi}-\bar{D}\partial_x^2\delta\chi). \tag{45}$$

As an important result, it is thus confirmed that the damping and the velocity of the phase mode (for finite wave-vectors) is strongly enhanced at low temperatures, due to long-range Coulomb forces, provided the concentration of normal electrons is small.

4. Discussion

In this article, I have presented a detailed discussion of the coupling between the CDW condensate, and the thermally excited quasiparticles and normal electrons, in the long-wavelength low-frequency limit. Within the framework of the kinetic theory, which is based on the equation of motion of the quasiclassical Green's function,[7] it is straightforward to determine the parameters entering the equation of motion for the phase of the order parameter, in the presence of impurity scattering and for all temperatures. The Coulomb forces are taken into account by adding Poisson's equation which can be reduced, in the limit considered, to the condition of charge neutrality. The results confirm the hydrodynamic approach [11], and also illuminate and extend the investigations of the microscopic theory [15, 16]. Recently, the microscopic theory of the combined effect of disorder and Coulomb interaction has also been studied in great detail [17]. I emphasize again that in the absence of normal electrons, the inclusion of (diffusive) screening currents leads to a strong enhancement of the phason velocity and the damping constant, in the low temperature limit. In particular, the damping is found to scale with the inverse of the low-field quasiparticle conductivity, σ_0^{-1}, and thus increases $\sim\exp(\Delta/T)$.

[7] Further applications of the quasiclassical equations are given in [27]

For a discussion of the consequences of these results for the CDW dynamics, it is of course essential to include (in the incommensurate case) the pinning due to impurities in the equation of motion, (43). For easy reference, I have summarized the weak pinning limit results in the appendix. I remark that such an equation gives a hybrid description of the effect of impurities, since the coefficients in the equation of motion have been determined by the standard impurity averaging technique, while on the other hand, the pinning force depends on the actual impurity configuration. Nevertheless, I expect that such an equation of motion should give an adequate description of the dynamics, *provided one accepts the assumption that a classical description is possible at all.* Of course, the classical equation has been analysed in great detail.[8]

Returning to the strong temperature dependence of the damping constant and the phason velocity, I remark that in the *weak pinning* limit, the characteristic length, field and frequency $(L_0, \mathscr{E}_0, \omega_0)$ scale with the phason velocity as follows: $L_0 \sim c^2$, $\mathscr{E}_0 \sim c^{-2}$, and $\omega_0 \sim c^{-1}$. Thus, as a consequence of the renormalization of the phason velocity, one should expect a strong temperature dependence of these quantities. On the other hand, note that in the *strong impurity pinning* case, L_0 is determined by the average distance between impurities, and the results of [17] also indicate that ω_0 is not renormalized. However, it has to be emphasized that the leading contribution to the large field CDW current is *not* affected by the screening currents, provided the impurity pinning can be treated perturbatively [11]. (In contrast, the next to leading order contribution is strongly modified [11].)

As mentioned in the introduction, recent experiments [6–10] have shown quite unexpected features which may be related to the screening mechanism discussed above. Most notably, it has been found that the (strongly temperature dependent) low field conductivity is also the relevant parameter for the large field CDW conductivity, the latter being measured, e.g. in $K_{0.30}MoO_3$, at fields as high as about 30 times the threshold field [9]. Both quantities show a temperature activated behavior, with a characteristic temperature of $\sim 10^3$ K, and increase by five orders of magnitude between 25 and 125 K (for this compound). This result suggests that the distortions of the CDW remain undiminished, even for very large fields, such that the renormalized damping constant $(\tilde{\gamma})$ effectively determines the viscosity, in contrast to the prediction of the classical model. The threshold field, on the other hand, is only weakly temperature dependent [9], which indicates that pinning is due to strong impurities. Furthermore, computing the characteristic time scale, τ_0, from the relation $\tau_0 = \tilde{\gamma}/(2e^\star v_F \mathscr{E}_0)$ (note that the over-damped limit is adequate), into which the measured high-field value of the damping constant (as well as $\mathscr{E}_0$) is inserted, gives good agreement with τ_0 as measured in the low-frequency regime [8, 9] (see also [10]). For the latter measurements, it seems to be clear that the renormalized damping constant is the relevant quantity.

While these experimental results, as well as results concerning interference phenomena and mode locking [3] have been interpreted as strong support for the quantum tunneling model [5], another explanation invoking very strong impurity pinning has been offered recently [30]. Though the apparent importance of long-range Coulomb forces in the semiconducting materials has brought new elements into the discussion about the transport mechanism in CDW systems, it is presently only a hope that this question can be settled in the near future.

It is a pleasure to thank L.K. Hansen, K. Maki, and H. Smith for interesting discussions.

Appendix

For completeness and for easy reference, I summarize in this appendix some well-known results about the characteristic scales (length, frequency, electric field) in the equation of motion for the phase of the order parameter, in the presence of weak impurity pinning. Coulomb effects are ignored for simplicity. I start from (24), generalized to include the perpendicular dispersion and the impurity pinning [23] (compare also [4, 11, 28, 29]):

$$\ddot{\chi} + \gamma \dot{\chi} - c^2 \partial_x^2 \chi - c_\perp^2 \nabla_\perp^2 \chi = 2e^\star v_F \mathscr{E}_x + \tilde{\rho}_1 \operatorname{Re}[\xi(\mathbf{r}) e^{i\chi}] \tag{A1}$$

where $\tilde{\rho}_1$ is proportional to the CDW amplitude, ρ_1, and given by

$$\tilde{\rho}_1 = \frac{2\rho_1}{m_F N(0)} = \frac{8\rho_1 \varepsilon_F}{m_F n_0} = \frac{4\Delta}{\lambda m_F} = \frac{\omega_Q^2}{\Delta} \tag{A2}$$

where ε_F and n_0 denote the Fermi energy and the equilibrium electron density, respectively. In (A1), $\xi(\mathbf{r})$ is a complex random Gaussian field, and describes the impurity potential for large $(\pm \mathbf{Q})$ momenta. The average of ξ is chosen equal to zero; furthermore, the correlations are given by $\langle \xi \xi \rangle = \langle \xi^\star \xi^\star \rangle = 0$, and

$$\langle \xi(\mathbf{r}) \xi^\star(\mathbf{r}') \rangle = [2\pi N(0) \tau_2]^{-1} \cdot \delta(\mathbf{r} - \mathbf{r}'). \tag{A3}$$

The equation of motion, (A1), is brought into a dimensionless form according to the following steps:

[8] A very limited selection of papers are Refs. 28, 22, 4, 29

(i) Scale the transverse coordinate: $\mathbf{r}_\perp \cdot (c/c_\perp) \to \mathbf{r}_\perp$.

(ii) Introduce dimensionless coordinates, $\mathbf{r}/L_0 \to \mathbf{r}$, and a dimensionless random field, $\bar{\xi} = K \cdot \xi$, such that

$$\langle \bar{\xi}(\mathbf{r})\, \bar{\xi}^\star(\mathbf{r}') \rangle = 8\pi \cdot \delta(\mathbf{r} - \mathbf{r}'). \tag{A4}$$

The requirement (A 4) leads to

$$K^2 = 16\pi^2 N(0)\, \tau_2 \cdot c_\perp^2\, L_0^3 / c^2. \tag{A5}$$

The characteristic length, L_0, usually called the Fukuyama-Lee-Rice length, is determined from the condition $\tilde{\rho}_1/K = c^2/L_0^2$. Of course, $\omega_0 = c/L_0$ can be identified as the impurity pinning frequency.

(iii) Define a dimensionless time according to $\omega_0^2\, t/\gamma \to t$ (which is most useful in the limit $\gamma \gg \omega_0$).

As a result the equation of motion has the following form:

$$M\ddot{\chi} + \dot{\chi} - \nabla^2 \chi = \mathscr{E}_x/\mathscr{E}_0 + \mathrm{Re}[\bar{\xi}(\mathbf{r})\, \mathrm{e}^{\mathrm{i}\chi}] \tag{A6}$$

where $M = (\gamma/\omega_0)^2$, and $\mathscr{E}_0 = \omega_0^2 / 2e^\star v_F$ is the characteristic field. One obtains the following result for the characteristic length:

$$\begin{aligned} L_0/v_F\, \tau_2 &= 16\pi^2 N(0)\, c_\perp^2 \cdot c^2/(v_F\, \tilde{\rho}_1^2) \\ &= 16\pi (v_F/d\, \tilde{\rho}_1)^2\, (c_\perp\, c/v_F^2)^2. \end{aligned} \tag{A7}$$

In obtaining the last equality, I used $N(0) = (\pi v_F\, d^2)^{-1}$, appropriate for weakly coupled chains, where d^2 is the area for one chain. Note that $L_0 \sim \tau_2 \sim (n_{\mathrm{imp}})^{-1}$, the standard weak pinning result. For an order of magnitude estimate, I use values which may be appropriate for $NbSe_3$ at low temperatures [31]: $N(0) \simeq 4.5 \times 10^{21}/(\mathrm{eV} \cdot \mathrm{cm}^3)$, $c \simeq 3 \times 10^6$ cm/s, $c_\perp^2 \simeq 0.1 c^2$, $\omega_Q \simeq 50$ K, $\Delta \simeq 100$ K; this gives $L_0 \sim 10 \cdot v_F\, \tau_2$.

Finally, I wish to comment briefly on some aspects of the static correlations of the phase. More precisely, let $\chi_0(\mathbf{r})$ be the solution of the *linearized* equation of motion, in the static limit and for $\mathscr{E}_x = 0$:

$$-\nabla^2 \chi_0 = \mathrm{Re}[\bar{\xi}(\mathbf{r})]. \tag{A8}$$

Clearly, $\chi_0(\mathbf{r})$ has a Gaussian distribution, with correlations given by

$$\begin{aligned} \Gamma_0(\mathbf{r}) &= \langle [\chi_0(\mathbf{r}) - \chi_0(0)]^2 \rangle \\ &= 8\pi \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{1 - \cos(\mathbf{q} \cdot \mathbf{r})}{q^4}. \end{aligned} \tag{A9}$$

Thus $\Gamma_0(\mathbf{r}) = |\mathbf{r}|$, which also motivates the above choice of the characteristic length. As shown by Efetov and Larkin [32], the same result also holds for the solution of the *non-linear* equation, $-\nabla^2 \chi = \mathrm{Re}[\bar{\xi}(\mathbf{r}) \exp(\mathrm{i}\chi)]$; this result can also be nicely demonstrated [33] by path integral ("supersymmetric") methods.[9] The latter method, however, clearly demonstrates that in performing the average over the random field, in fact *all* solutions of the equation of motion are taken into account with equal weight. On the other hand, it has been emphasized by Villain [35], that the correlation function of the *lowest energy solution* increases only logarithmically with distance, between two and four dimensions, implying a power law decrease of the order parameter correlations. The two methods are compared in detail for a simple example in [36].

Presently, I believe it is not clear whether the problem underlying this discrepancy is important, for example, for a calculation of the dynamic properties of CDW systems in three dimensions; it nevertheless indicates possible dangers in a perturbative treatment of disorder. Unfortunately, other methods [37–39] are applicable for the onedimensional case only.

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[9] Apparently, this has been known for some time: See the remark on p. 805 of [34]. Explicitly, the proof has been given in the appendix of [19]

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