Nonlinear Diamagnetic Response in Mesoscopic Rings of Superconductors above T_c.

V. AMBEGAOKAR (*) ($) and U. ECKERN (**)

(*) *Institut für Theorie der Kondensierten Materie, Universität Karlsruhe Postfach 6980, D-7500 Karlsruhe 1*

(**) *Kernforschungszentrum Karlsruhe, Institut für Nukleare Festkörperphysik Postfach 3640, D-7500 Karlsruhe 1*

PACS. 72.15N – Collective modes; low-dimensional conductors.

PACS. 74.40 – Fluctuations and critical effects (inc. noise, chaos, nonequilibrium superconductivity, localisation, etc.)

Abstract. – We study the nonlinear diamagnetic response of mesoscopic rings of superconductors above the transition temperature. Based on the standard expression for the fluctuation correction to the grand potential, Ω, we investigate the flux and temperature dependence of the equilibrium current $I = -\partial\Omega/\partial\Phi$, for the cases where T_c is A) much smaller, and B) comparable to the coherence temperature $T_1 = \hbar D/L^2$ (D is the diffusion constant, and L the circumference of the ring). The results are compared with, and contrasted to, recent experimental and theoretical studies of mesoscopic normal metal rings. New experiments are suggested for the easily achievable situation $T_c > T_1$.

In a recent note [1], with a view to understanding an experiment [2, 3] on an ensemble of identically patterned small copper rings, we have studied interaction-induced flux periodic equilibrium currents in mesoscopic rings of normal metal, threaded by a magnetic field. Our calculation predicts a current with the following key features: i) periodicity in the flux Φ with period $\Phi_0 = h/2e$; ii) a decrease in magnitude with increasing temperature on the scale $\sim 3T_1$ (where $T_1 \equiv \hbar D/L^2$ is the correlation temperature, and D and L are the diffusion constant and the circumference of the ring, respectively); iii) a magnetic moment in the direction of a small applied field (*i.e.* a paramagnetic current), assuming a repulsive interaction between electrons, and iv) a magnitude $I^* \equiv 8T_1/\Phi_0$ times a dimensionless coupling constant.

As regards i) and ii) above, our theory agrees with experiment. In contrast to iii), in ref. [2] as printed the current is reported to be diamagnetic, although we understood and understand [3] that its direction is experimentally not yet definitely determined. As far as the magnitude iv) is concerned, multiple-scattering corrections [4] appear to make our theoretical prediction too small by a factor of about 5.

($) Permanent address: Laboratory for Atomic and Solid State Physics, Clark Hall, Cornell University, Ithaca, NY 14853, USA.

From a theoretical point of view, it is clear that the magnetic response of superconducting rings above their transition temperature must be diamagnetic, because the phonon-induced attraction between electrons is then dominant at the Debye energy. Here we study this case. We assume that the total coupling constant at the Debye energy is negative, denoted as usual by $-\lambda$, and related to T_c by $T_c \sim \hbar\omega_D \exp[-1/\lambda]$. We study two cases: *A*) $T_c \ll T_1$, and *B*) $T_c \gtrsim T_1$. Case *A*) relates to an interesting speculation [5]—about which we comment at the end of this note—that the currents observed in copper might be due to a very low-temperature superconducting transition.

In summary, we find that in case *A*), apart from the expected opposite sign, the current to a very good approximation is given by our first-order result [1], with the dimensionless coupling constant being replaced by $\sim [\ln(T_1/T_c)]^{-1}$. Case *B*) will be of interest for future experiments, since a strong enhancement, and an unusual signature—all harmonics in $(2\pi\Phi/\Phi_0)$ having the same amplitude—are predicted for the current as $T \to T_c$. Quantitative results are given below for $T_c = 10^{-3} T_1$ and $T_c = 3T_1$.

We start from an expression for the grand potential averaged over impurity positions obtained by summing ladder diagrams in the Cooper channel, or equivalently by doing a Gaussian functional integral [6-11]:

$$\Omega = T \sum_{\omega, q} \ln \mathcal{E}(\omega, q), \tag{1}$$

where the eigenvalues of the pair propagator $\mathcal{E}(\omega, q)$, in the regime $|\omega|$, $Dq^2 \ll \tau^{-1}$ (τ is the elastic scattering time) are given by

$$\mathcal{E}(\omega, q) = \ln \frac{T}{T_c} + \psi\left(\frac{1}{2} + \frac{\hbar(|\omega| + Dq^2)}{4\pi T}\right) - \psi\left(\frac{1}{2}\right). \tag{2}$$

$\psi(\cdot)$ denotes Euler's psi-function, ω the Matsubara (Bose) frequencies and, for a ring with transverse dimensions much smaller than L, the wave vector is one-dimensional, and in the presence of a magnetic field given by (¹)

$$q = \frac{2\pi}{L}\left(n - \frac{\Phi}{\Phi_0}\right) \tag{3}$$

with $n = 0, \pm 1, \pm 2, \ldots$. Clearly, Ω is even and periodic in the flux with period Φ_0, and thus has, up to an additive constant, the Fourier expansion

$$\Omega = 2 \sum_{m=1}^{\infty} \Omega_m \cos(2\pi m\Phi/\Phi_0). \tag{4}$$

The equilibrium «persistent» current is then given by

$$I = -\partial\Omega/\partial\Phi = \sum_{m=1}^{\infty} I_m \sin(2\pi m\Phi/\Phi_0) \tag{5}$$

with $I_m = 4\pi m\Omega_m/\Phi_0$. Using methods described in [1], the Fourier coefficients of the current

(¹) Note that the nonlocal effects of a magnetic field considered in [6] and [7] are not present for an Aharonov-Bohm flux.

are ($I^* = 8T_1/\Phi_0$)

$$\frac{I_m}{I^*} = i\frac{\pi}{2m^2}\sum_\omega \int_{-\infty}^{\infty} \mathrm{d}x \frac{x\exp[2\pi i x]}{\mathcal{E}_m(\omega, x)}\psi'\left(\frac{1}{2}+\frac{\hbar|\omega|}{4\pi T}+\frac{\pi T_m}{T}x^2\right), \tag{6}$$

where $T_m = T_1/m^2$ and $\mathcal{E}_m(\omega, x) = \mathcal{E}(\omega, q = 2\pi x/mL)$. We note in passing that even though the expression (2) is not accurate enough for large $\mathcal{E}$—where $\mathcal{E}$ must approach a constant corresponding to no attraction in the Cooper channel—to make (1) convergent, it suffices for a calculation of the flux-dependent current (5), since $x \sim 1$ controls the behaviour of (6).

Let us first consider case A): $T_c \ll T, T_1$. An examination of (6) suggests that it should be a good approximation to replace $\mathcal{E}_m$ in the denominator of (6) by a (possibly weakly temperature dependent) constant $\mathcal{E}^*_m$. The expression (6) then yields a current identical, apart from a factor $(-1/\mathcal{E}^*_m)$, to the first-order result given in [1]. In particular

$$I_m(T) = -I^* \cdot G(T/T_m)/m^2\,\mathcal{E}^*_m, \tag{7}$$

where $G(T/T_m)$, which equals $g_m(T)/g_m(0)$ as introduced in [1], is given by

$$G(T/T_m) = \frac{\pi}{4}\left(\frac{T}{T_m}\right)\sum_\omega\left(\frac{\hbar|\omega|}{T_m}\right)^{1/2}\exp\left[-\left(\frac{\hbar|\omega|}{T_m}\right)^{1/2}\right]. \tag{8}$$

A very good approximation, in the range $T \leqslant 8T_m$, is

$$G(T/T_m) \simeq \exp[-T/3T_m]. \tag{9}$$

In the low-temperature regime we estimate $\mathcal{E}^*_m = \ln(\alpha T_m/T_c)$ with $\alpha \sim 1$. The numerical results for $T_c/T_1 = 10^{-3}$ shown in fig. 1 can in fact be well fit over the whole temperature range by (7) with $\alpha \simeq 1.25$. In this figure, we show the first two harmonics obtained by calculating $I(\Phi) = -\partial\Omega/\partial\Phi$ directly from (1)-(3) for about ten independent values of the flux, and then Fourier analysing the result. (Note that it is sufficient—and convenient—to cut off

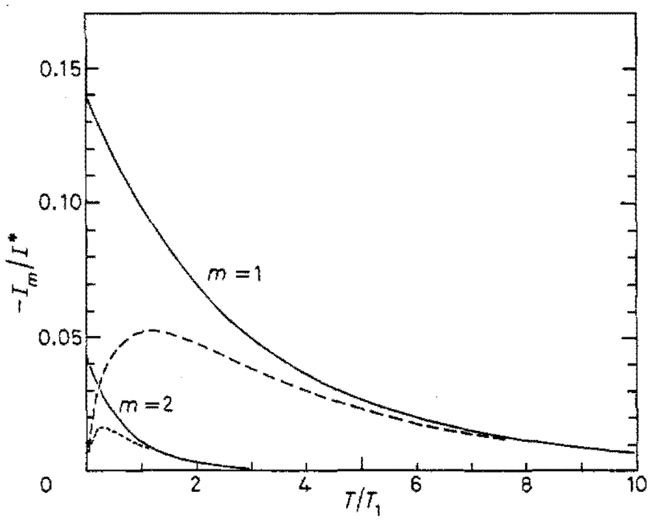

Fig. 1. – First and second harmonics of the current amplitude as defined in eq. (5) in units of $I^* \equiv 8T_1/\Phi_0$ for $T_c/T_1 = 10^{-3}$. Dashed lines: «classical» contribution as explained in the text.

the n-summation at a few hundred, provided the cut-off is smooth.) In addition, we show (as dashed lines) the asymptotic behaviour coming from the $\omega = 0$, or «classical», term in (6). These curves are reasonably well approximated by the corresponding part of (7), where $G(T/T_m)$ is replaced as in [1] by

$$[G(T/T_m)]^{\text{class}} = \frac{1}{4}\left(\frac{2\pi T}{T_m}\right)^{3/2} \exp\left[-\left(\frac{2\pi T}{T_m}\right)^{1/2}\right]. \tag{10}$$

In conclusion, for $T_c \ll T_1$, the persistent current is essentially given by the first-order expression [1], except that the coupling constant is to be replaced by $[-\ln(T_m/T_c)]^{-1}$, in the temperature range of interest.

Next we treat case B): $T_c \gtrsim T_1$. First consider the limit $T \to T_c$, in which case the classical long-wavelength fluctuations are dominant. This means that we take $\omega = 0$ and consider small x in (6), such that

$$\mathscr{E}(0, q) \simeq \varepsilon + (\pi D/8T_c)\, q^2 = \varepsilon[1 + (\xi q)^2], \tag{11}$$

where we identify the dirty limit coherence length, ξ, given by $\xi^2 = \pi D/8(T - T_c)$; $\varepsilon = (T - T_c)/T_c$. Then we find as in ref. [5]

$$T - T_c \ll T_c: \quad [I_m]^{\text{class}} = -\frac{4\pi T_c}{\Phi_0} \exp[-mL/\xi]. \tag{12}$$

In this limit the sum over harmonics in (5) can be done explicitly, yielding

$$I^{\text{class}} = -\frac{4\pi T_c}{\Phi_0} \frac{\exp[L/\xi]\sin\varphi}{(\exp[L/\xi] - \cos\varphi)^2 + \sin^2\varphi} \tag{13}$$

with $\varphi = 2\pi\Phi/\Phi_0$. All harmonics are thus important close to T_c ($\xi \gg L$), and in the limit $T = T_c^+$:

$$I^{\text{class}} = -\frac{2\pi T_c}{\Phi_0} \operatorname{ctg}\left(\frac{\pi\Phi}{\Phi_0}\right). \tag{14}$$

We remark that $(mL/\xi)^2 = 8T_c \cdot \varepsilon/\pi T_m$, so that $\xi > mL$ corresponds to $\varepsilon < T_m/T_c$; thus the anharmonic content is pronounced in the range $T \simeq T_c + O(T_m)$, which also applies to the case $T_c \gg T_1$, provided that other coherence destroying processes have not set in. Since for $T \gg T_1$ the current is in any case dominated by classical fluctuations, we conclude that for $T_c \gtrsim T_1$ these are the most important fluctuations at all temperatures, which we have explicitly confirmed numerically. Results are given in fig. 2 for $T_c = 3T_1$, where we show the first two harmonics *vs.* temperature. Note that at the critical temperature, every harmonic reaches the limit

$$[I_m/I^*]^{\text{class}} = -\pi T_c/2T_1. \tag{15}$$

As fig. 2 shows, there is a steep increase in the current (which will be even more pronounced for $T_c \gg T_1$) in the regime $T = T_c + O(T_1)$. Since the prefactor in (12) is *not* reduced by an effective coupling constant as in case A), there are simple and striking predictions here that cry out for experimental verification.

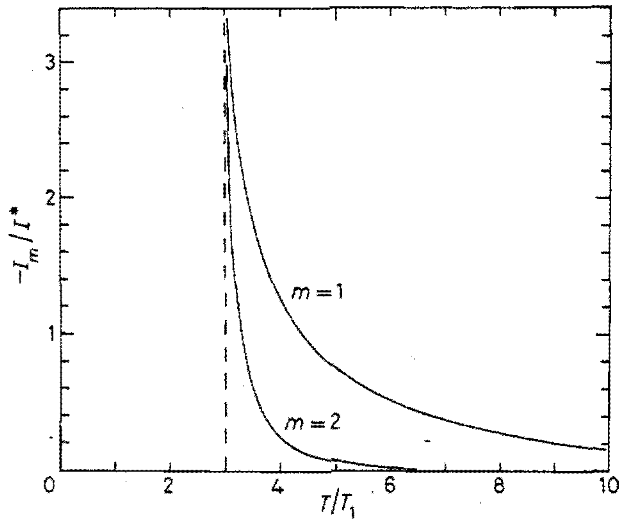

Fig. 2. – First and second harmonics as defined in eq. (5) for the case $T_c = 3T_1$. Note that both curves reach $3\pi/2 = 4.71$ as $T \to T_c^+$.

All of the results of case B) are implicit in ref. [5], which however considers the possibility that superconducting fluctuations explain the experiment on copper rings. Since copper is known not to be superconducting down to about $10\,\mu$K [12], the appropriate limit corresponds to our case A) (²). Now, the observed temperature scale of 80 mK [2] gives $T_1 \simeq 27$ mK. As a result the effective coupling constant which we found above, $[\ln(1.25T_1/T_c)]^{-1}$, is 0.12 for $T_c = 10\,\mu$K. This is too small to explain the experiment precisely, for which a value $\simeq 0.3$ is needed. On the other hand, no theory is presently doing much better, so that we cannot rule out the possibility that the battle between Coulomb repulsion and phonon-induced attraction in copper is just barely won by the latter. The experimental determination of the sign of the observed current would seem to resolve this question, particularly since spin-orbit coupling is found [4] (³) not to modify the results in [1]. The neglect of all but pairing fluctuations so far from T_c requires further study however.

Finally, we remark that we have here considered the interaction-induced contribution to the nonlinear magnetic response, in contrast to the so-called single-electron effect which relies on the discreteness of the single-electron levels in a finite system [13-17]. The latter, which has to be added to the interaction contribution, predicts for low temperatures and for experiments done on *single rings* an equilibrium current which is periodic in Φ with period $\Phi_0/2 = h/e$, and has an order of magnitude given by $\sim I^*$ [13, 14]. However, the sign of the current is expected to vary from ring to ring, depending on the actual impurity configuration; as a consequence, for an experiment on an *ensemble of rings*, only the «second» harmonic (period Φ_0) survives [13, 15, 16]. Its amplitude has been estimated in recent large-scale numerical simulations [16], with the result (see also [4])

$$I^{\text{s.e.}} \simeq 0.12(L/Ml)^{1/2} I^* \sin(2\pi\Phi/\Phi_0)\,, \tag{16}$$

where l and M denote the elastic mean free path and the number of transverse channels, respectively ($M = \mathcal{A}k_F^2/4\pi$, where $\mathcal{A}$ is the transverse area of the ring). Similar to case A)

(²) The «classical» approximation used in [5] which leads to the dashed curves in fig. 1 is thus incorrect.

(³) For a recent review of the single-electron effect and a comparison with the interaction contribution, see also [13].

above, the single-electron contribution is expected to decrease with temperature on a scale given by a few T_1.

Comparison with our results presented above leads to the following conclusions. i) For case B) ($T_c \gtrsim T_1$), where superconducting fluctuations are very strong, the single-electron effect is negligible for experiments *on single rings and on an ensemble of rings*. ii) For case A) ($T_c \ll T_1$) and *single-ring* experiments, the single-electron effect dominates. Its sign can be either diamagnetic or paramagnetic, and its period is $\Phi_0/2$. iii) For case A) and *ensemble* experiments, the interaction contribution dominates for typical parameters, provided the effective coupling constant is not extremely small. As an example, consider the parameters quoted for the copper rings [2, 3], which lead to $0.12(L/Ml)^{1/2} \simeq 0.7 \cdot 10^{-2}$. Finally we remark that recent analytical calculations [17] predict an amplitude for the single-electron effect which is clearly different from (16), and even smaller.

* * *

VA thanks the Alexander von Humboldt-Stiftung for a Senior U.S. Scientist award, and A. Schmid for the hospitality of his institute and for discussions. This work is supported in part by the U.S. National Science Foundation under grant No. DMR-88 15828.

REFERENCES

[1] Ambegaokar V. and Eckern U., *Phys. Rev. Lett.*, **65** (1990) 381.
[2] Levy L. P., Dolan G., Dunsmuir J. and Bouchiat H., *Phys. Rev. Lett.*, **64** (1990) 2074.
[3] Levy L. P., private communication and to be published.
[4] Eckern U., to be published.
[5] Fu Y. and Park C., to be published.
[6] Lee P. A. and Payne M. G., *Phys. Rev. B*, **5** (1972) 923.
[7] Kurkijärvi J., Ambegaokar V. and Eilenberger G., *Phys. Rev. B*, **5** (1972) 868.
[8] Altshuler B. L., Khmelnitskii D. E. and Spivak B. Z., *Solid State Commun.*, 48 (1983) 841.
[9] Altshuler B. L., Aronov A. G., Khmelnitskii D. E. and Larkin A. I., in *Quantum Theory of Solids*, edited by I. M. Lifshitz (MIR Publishers, Moscow) 1980, p. 130.
[10] Altshuler B. L. and Aronov A. G., in *Electron-Electron Interactions in Disordered Systems*, edited by A. L. Efros and M. Pollak (North-Holland, Amsterdam) 1985, p. 1.
[11] Eckern U. and Pelzer F., *J. Low Temp. Phys.*, **73** (1988) 433.
[12] Pobell F., private communication.
[13] Riedel E. K., to be published.
[14] Cheung H. F., Riedel E. K. and Gefen Y., *Phys. Rev. Lett.*, **62** (1989) 587.
[15] Bouchiat H. and Montambaux G., *J. Phys.* (*Paris*), **50** (1989) 2695.
[16] Montambaux G., Bouchiat H., Sigeti D. and Friesner R., to be published.
[17] Schmid A., to be published; Altshuler B. L., Gefen Y. and Imry Y., to be published.