Coherence and destruction of coherence in mesoscopic rings

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Recent investigations have shown that an important contribution to the magnetic response of an ensemble of mesoscopic normal metal rings at low temperatures is given by the coherent backscattering familiar from the theory of weak localization, here initiated by the electron-electron interaction. It is studied how this "collective" effect is weakened by the orbital effect of the magnetic field, by spin-orbit scattering, spin-flip scattering, and the Zeeman splitting. The renormalization of the interaction constant due to higher order contributions in the Cooper channel is discussed.

1. Introduction

Shortly after the first experimental observation of persistent currents in small normal metal rings threaded by a magnetic flux [1], it was pointed out [2] that there exists a contribution to this effect which relies on the coherent backscattering phenomenon familiar from the theory of weak localization [3–6], but here initiated by the electron-electron interaction. In agreement with experiment, the theory [2] predicts a persistent current which is periodic in the flux Φ, with period $\Phi_0 = h/2e$, that in order of magnitude $I \sim ev_{Fl}/L^2$, where l and L are the elastic mean free path and the circumference of the ring, respectively, and that the current decreases with temperature on the scale $\sim \hbar D/L^2$, where D is the diffusion constant.

In contrast, it has been discussed for some time [7–9] that, due to the descreteness of the single-electron levels (of a non-interacting system) and their dependence on the enclosed flux, there should be a contribution to the current at sufficiently low temperatures which is periodic in the flux Φ with period h/e. However, recent numerical investigations [10] have demonstrated that the first harmonic is suppressed in situations where an *ensemble* of rings is considered, and that only the second harmonic, with period Φ_0, survives ensemble averaging. In addition, the numerical results also show that the amplitude of the second harmonic is $I \sim (ev_F l/L^2)/M_{\text{eff}}^{1/2}$, where $M_{\text{eff}} = M \cdot l/L$; $M \sim (k_F L_\perp)^2$ denotes the number of transverse channels. Note that $M_{\text{eff}} \sim \xi/L \gg 1$, where in quasi-onedimensional situations, the disorder induced localization length is given by $\xi \sim M \cdot l$, and $\xi \gg L$ in the (metallic) regime of interest. In addition, however, the single-electron contribution was very recently studied by analytic methods [11], with the result $I \sim (ev_F/L)\, M^{-1}$, which is clearly at variance with the numerical answer.

Thus it appears that various questions remain concerning the single-electron effect. On the other hand, I emphasize that the interaction induced "collective" contribution is multiplied by an effective low frequency coupling constant, which is difficult to estimate. In comparison with experiment, it seems that all results obtained so far predict a persistent current which is about an order of magnitude too small (however, see below), and further investigations are necessary to completely clarify the origin of the experimental observation.

In this article, I discuss in more detail the collective contribution to the grand potential, Ω, and the persistent (equilibrium) current related to Ω by $I = -\partial\Omega/\partial\Phi$. Interaction effects of this type have been discussed earlier in the context of the theory of weak localization [12–16], e.g. weak localization corrections to the supercurrent in SNS junctions are studied in [14]. Here I use results familiar from those theories, and apply them to investigate the influence of the magnetic field (Zeeman splitting, orbital effect) and of spin scattering, for the quasi-onedimensional ring like geometry under consideration. For example, it is found, unlike the expectation formulated in [1a], that spin-orbit scattering *does not* affect (neither sign nor magnitude) the collective contribution. (This result was also mentioned in [17].)

In the next section (Sect. 2), I briefly summarize the results of [2] and estimate the effective interaction constant when higher order corrections to the grand potential are taken into account (see, e.g., [12–15]). In Sect. 3, I investigate the role of spin-orbit and spin-flip scattering, the Zeeman splitting and the orbital effect of the magnetic field. A discussion is given in the final Sect. 4.

2. Electron-electron interaction and coherent backscattering

Consider a normal metal ring of circumference L and transverse dimensions $L_\perp \ll L$ in a magnetic field. In addition, assume that the elastic mean free path, $l = v_F \tau$, is small compared to the circumference (typically $l \sim L_\perp$), such that

$$k_F^{-1} \ll l, \quad L_\perp \ll L \tag{1}$$

where k_F is the Fermi wavevector. In terms of energies, this reads

$$T_1 \equiv \frac{\hbar D}{L^2} \ll \frac{\hbar v_F}{L} \ll \frac{\hbar}{\tau} \ll \varepsilon_F \tag{2}$$

where T_1 is proportional to the well-known Thouless energy (temperature*), and $D = v_F l/3$ denotes the diffusion constant. The temperature regime of interest is $T \lesssim T_1$, such that the thermal diffusion length $l_T = (\hbar D/T)^{1/2}$ is comparable or larger than the circumference. In view of the conditions (2), I neglect the discreteness of the single-electron levels in the Green's functions appearing explicitly as lines in Fig. 1. (The lines denote Green's functions averaged with respect to impurities in the standard approximation [18]). The essential element of the contributions shown in Fig. 1 is the Cooperon or particle-particle impurity ladder (the hatched area in the figure), which in the presence of potential scattering only is of the standard diffusive form:

$$K = \frac{1}{2\pi N(0)\tau^2} \frac{1}{|2\varepsilon - \omega| + Dq^2} \tag{3}$$

in the frequency regime $\varepsilon(\varepsilon - \omega) > 0$. ε, ω denote Matsubara Fermi and Bose frequencies, respectively, and $N(0)$ is the (single spin) density of states at the Fermi surface. Also, the long wavelength low frequency limit is considered in (3).

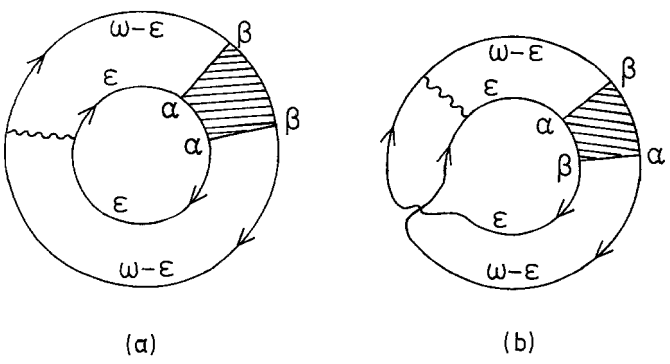

Fig. 1a, b. Feynman diagrams corresponding to **a** the Hartree and **b** the Fock contribution to the grand potential, in first order in screened Coulomb interaction (wavy line). The hatched area denotes the particle-particle impurity ladder (see also Fig. 3), and α and β the spin indices

* Boltzmann's constant is put equal to unity

Thus the contributions to Ω shown in Fig. 1 are easily found to be given by [2]

$$\Omega = 2\pi N(0)\, \bar{V} \hbar^{-1} T^2 \sum_{\varepsilon,\omega,q}{}' \frac{1}{|2\varepsilon - \omega| + Dq^2} \tag{4}$$

where the summation is restricted to $\varepsilon(\varepsilon - \omega) > 0$; $\bar{V}$ denotes the screened Coulomb interaction, averaged over the Fermi surface, in the static limit. Under the conditions given in (1), transverse dimensions of $\mathbf{q}$ are frozen out; also, note that the flux dependence enters (4) through

$$q = \frac{2\pi}{L}\left(n - \frac{\Phi}{\Phi_0}\right) \tag{5}$$

and $\Sigma_q \to \Sigma_n$. As a minor point, since I am interested in the flux dependence of Ω only, I remark that it is possible to overcome problems of convergence (partly) by taking the Φ-derivative of Ω. With the substitution $\omega' = 2\varepsilon - \omega$, contact is easily made with the results of [2]. Since Ω is even and periodic in Φ with period Φ_0, it has (up to an additive constant) the Fourier expansion

$$\Omega = 2 \sum_{m=1}^{\infty} \Omega_m \cos(2\pi m \Phi/\Phi_0) \tag{6}$$

where Ω_m is found to be given by ($\lambda_c \equiv N(0)\, \bar{V}$):

$$\Omega_m(T=0) = \frac{2\lambda_c}{\pi m^3} T_1 \tag{7}$$

and ($T_m \equiv T_1/m^2$)

$$\Omega_m(T \gg T_m) = \frac{\lambda_c}{2\pi m^3} T_1 \left(\frac{2\pi T}{T_m}\right)^{3/2} \exp\left[-\left(\frac{2\pi T}{T_m}\right)^{1/2}\right]. \tag{8}$$

A plot of the temperature dependence, $\Omega_m(T)$, is given in [2] where it was noted that for $T \lesssim 8\, T_m$, an excellent interpolation formula is given by $\Omega_m(T)/\Omega_m(0) \simeq \exp(-T/3\, T_m)$. In the zero temperature limit and taking $m = 1$ into account only, the persistent current is given by

$$I \simeq 8\lambda_c \frac{T_1}{\Phi_0} \sin\left(2\pi \frac{\Phi}{\Phi_0}\right), \quad T = 0 \tag{9}$$

which is in perfect agreement with experiment [1] (an estimate gives [2] $\lambda_c \simeq 0.3$; note also that the revised estimate [1b] of the mean free path is $l \sim 300$ Å, leading to $3\, T_1 \sim 75$ mK), except for the sign which experimentally was tentatively assigned to be negative (diamagnetic) [1a]. The theory predicts, for a repulsive interaction ($\lambda_c > 0$), a positive sign (paramagnetic current) for small flux.

However, the interaction constant λ_c is reduced by higher order contributions shown in Fig. 2 [12–15]. To take these into account, note that it is not possible to substitute $\omega' = 2\varepsilon - \omega$ as discussed below (5); instead, keeping in mind that the ε-summation is cut off at the Fermi energy, I use the formula

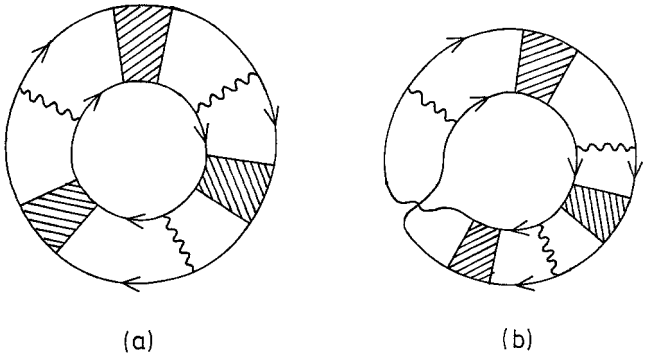

Fig. 2. Higher order Freynman diagrams which renormalize the effective interaction in the Cooper channel

$$2\pi\hbar^{-1}T\sum_{\varepsilon}{}' [|2\varepsilon-\omega|+Dq^2]^{-1}$$

$$=\ln\frac{\varepsilon_F}{T}+\psi\left(\frac{1}{2}\right)-\psi\left(\frac{1}{2}+\frac{|\omega|+Dq^2}{4\pi T/\hbar}\right) \tag{10}$$

where $\psi(\cdot)$ denotes Euler's psi function. Summing up the diagrams of the type shown in Fig. 2, I obtain the following expression (again omitting an additive constant):

$$\Omega=T\sum_{\omega,q}\ln\left[C_0+\psi\left(\frac{1}{2}\right)-\psi\left(\frac{1}{2}+\frac{|\omega|+Dq^2}{4\pi T/\hbar}\right)\right] \tag{11}$$

where

$$C_0=\ln(T_0/T), \qquad T_0\simeq\varepsilon_F\cdot\exp(1/\lambda_c). \tag{12}$$

Note that $C_0\gg 1$ since $T_0\sim 10^5$ K, $T\sim 0.1$ K. To make contact with [2], I consider first $T=0$; then, after a partial integration with respect to ω in (11), I obtain

$$\Omega=\hbar\sum_q\int\frac{d\omega}{2\pi}\frac{|\omega|}{|\omega|+Dq^2}\lambda_{\text{eff}}(\omega,q) \tag{13}$$

which differs from the corresponding expression (see Eq. (10) in [2]) by the fact that λ_c is replaced by an effective ω, q-dependent coupling constant [12]:

$$\lambda_c\to\lambda_{\text{eff}}=\left[\ln\frac{4\pi T_0/\hbar}{|\omega|+Dq^2}\right]^{-1}. \tag{14}$$

Thus the Fourier coefficients Ω_m are reduced, with the result that

$$\lambda_c\to[\ln(T_0/\pi T_m)]^{-1} \tag{15}$$

in the first order result given in (7). Numerically this means a reduction by a factor which I estimate to be about five, thereby worsening the agreement with experiment.

On the other hand, in the high temperature limit, $T\gg T_m$, it is sufficient to take into account the $\omega=0$ contribution in (11). Using the analytic properties of the psi function, I obtain ($C_0\gg 1$):

$$\Omega_m=\frac{1}{2\pi m^3}T_1\frac{2\pi T}{T_m}\exp\left(-\left(\frac{2\pi T}{T_m}\right)^{1/2}\right)$$
$$\cdot\left[1-\exp\left(-\frac{1}{C_0}\left(\frac{2\pi T}{T_m}\right)^{1/2}\right)\right]. \tag{16}$$

Thus Ω_m is reduced compared to (8) by

$$\lambda_c\to C_0^{-1}=[\ln(T_0/T)]^{-1} \tag{17}$$

in the regime $(2\pi T/T_m)^{1/2}\ll C_0$, while for $(2\pi T/T_m)^{1/2}\gg C_0$, the second term in the square brackets in (16) is negligible. Then, however, Ω_m is very small in any case.

In conclusion, the higher order corrections in the Cooper channel reduce the effective coupling constant compared to the first order result by a factor which is weakly temperature dependent, and estimated to be about four. However, it has to be kept in mind that the (attractive) electron-phonon interaction may give a further reduction of the effective coupling constant, which is difficult to estimate.

3. Destruction of phase coherence

In order to take into account spin scattering and the Zeeman splitting, it is necessary to determine the spin dependence of the Cooperon. Keeping in mind the discussion of higher order terms presented in the preceding section, I restrict myself here to the first order term (Fig. 1), which is given by (performing the substitution discussed below (5)):

$$\Omega=\lambda_c\pi N(0)\tau^2 T\sum_{\omega,q,\alpha,\beta}|\omega|[K_{\alpha\alpha\beta\beta}-K_{\alpha\beta\beta\alpha}] \tag{18}$$

where $K_{\alpha\alpha\beta\beta}$ and $K_{\alpha\beta\beta\alpha}$ (see Fig. 1a and 1b) correspond to the Hartree and Fock contribution, respectively, and α,β denote the spin indices. Note that the total scattering rate, τ^{-1}, which enters the electron Green's functions, is now given by

$$\frac{1}{\tau}=\frac{1}{\tau_0}+\frac{1}{\tau_{so}}+\frac{1}{\tau_s} \tag{19}$$

where τ_0, τ_{so}, τ_s are the scattering times for nonmagnetic impurities, spin-orbit scattering, and spin-flip scattering, respectively [19]; I assume $\tau_0\gg\tau_s$, τ_{so}, and also (see below) $\omega_s\tau_0\ll 1$, where $\hbar\omega_s=2\mu_B H$ is the Zeeman splitting. The lowest order expression for the Cooperon, K^0, i.e. the first term on the rhs of the equation shown in Fig. 3, is given by

$$K^0_{\alpha\beta\gamma\delta}=a\delta_{\alpha\beta}\delta_{\gamma\delta}+b\sigma^\kappa_{\alpha\beta}\sigma^\kappa_{\gamma\delta} \tag{20}$$

where σ^κ are the Pauli matrices (summation with respect to $\kappa=1, 2, 3$ is implied), and explicitly:

$$2\pi N(0)a=\tau_0^{-1} \tag{21}$$

$$2\pi N(0)b=\frac{1}{3}\left(\frac{1}{\tau_s}-\frac{1}{\tau_{so}}\right). \tag{22}$$

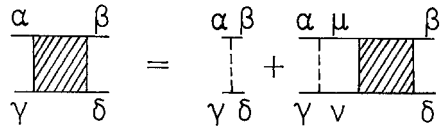

Fig. 3. Graphical representation of the equation for the Cooperon (hatched square). The dashed line is a standard impurity line, however, modified by spin-orbit and spin-flip scattering

The summation of the ladder diagrams is obtained as shown in Fig. 3, where it is necessary to take into account the Zeeman splitting in the Green's functions. Defining for $\alpha, \beta = \pm 1$ the quantity

$$\Pi_{\alpha\beta} = 2\pi N(0)\tau\left[1-\tau\left(|\omega|+Dq^2+i\frac{\alpha-\beta}{2}\omega_s\right)\right] \tag{23}$$

the Cooperon is determined by [19]

$$K_{\alpha\beta\gamma\delta} = K^0_{\alpha\beta\gamma\delta} + K^0_{\alpha\mu\gamma\nu}\,\Pi_{\mu\nu}\,K_{\mu\beta\nu\delta}. \tag{24}$$

Taking into account (20) and $\Pi_{++} = \Pi_{--}$, $\Pi_{+-} = \Pi^*_{-+}$, it follows from this equation that

$$\begin{aligned} &K_{++++} = K_{----}; \quad K_{++--} = K^*_{--++}; \\ &K_{+--+} = K^*_{-++-} \end{aligned} \tag{25}$$

while the other elements vanish. Then it is straightforward to derive the following results, which I present in terms of N_1, N_2 defined by

$$N_1(\omega, q) = |\omega| + Dq^2 + \frac{2}{3\tau_s} + \frac{4}{3\tau_{so}} \tag{26}$$

$$N_2(\omega, q) = |\omega| + Dq^2 + \frac{2}{\tau_s}. \tag{27}$$

Note that N_2 is independent of the spin-orbit scattering rate. The components of the Cooperon are found to be given by (compare [16, 19]):

$$2\pi N(0)\,\tau^2 \cdot K_{++++} = N_1^{-1} \tag{28}$$

$$2\pi N(0)\,\tau^2 \cdot K_{++--} = \frac{N_1 + N_2 - 2i\omega_s}{2(N_1 N_2 + \omega_s^2)} \tag{29}$$

$$2\pi N(0)\,\tau^2 \cdot K_{+--+} = \frac{-N_1 + N_2}{2(N_1 N_2 + \omega_s^2)}. \tag{30}$$

For the grand potential, (18), I need the combination

$$\sum_{\alpha\beta}[K_{\alpha\alpha\beta\beta} - K_{\alpha\beta\beta\alpha}] = 2[\mathrm{Re}\,K_{++--} - K_{+--+}] \tag{31}$$

and thus it follows immediately that

$$\Omega = \lambda_c T \sum_{\omega, q} \frac{|\omega|\, N_1}{N_1 N_2 + \omega_s^2} \tag{32}$$

which is the principal result of this section.

Restricting myself to the zero temperature limit, I discuss various special cases.

(i) $\omega_s = 0$

In the absence of Zeeman splitting, the grand potential (and thus the persistent current) is *independent* of the spin-orbit interaction, contrary to the discussion presented in [1a]. From the above results it follows immediately that, for strong spin-orbit scattering, the Fock term is multiplied by the familiar factor $(-1/2)$ known from the conductivity [3]; however, the Hartree term is multiplied by (1/4), leaving an overall independence of the spin-orbit interaction (for *all* values of the scattering rate). On the other hand, spin-flip scattering reduces the Fourier coefficients, according to

$$\Omega_m(\Gamma_s)/\Omega_m(0) = (1+\Gamma_s^{1/2})\exp(-\Gamma_s^{1/2}) \tag{33}$$

where I defined $\Gamma_s = 2\hbar/T_m\tau_s = 2(mL)^2/D\tau_s$, as to be expected.

(ii) Strong spin-orbit scattering

In this limit, N_1 can be replaced by a constant, namely $N_1 \simeq 4/3\tau_{so}$. Clearly, the result is then similar to (33), with the replacement

$$\frac{2}{\tau_s} \to \frac{2}{\tau_s} + 3\frac{\omega_s^2\,\tau_{so}}{4}. \tag{34}$$

Note that in terms of the parameters relevant for the experiment, I find

$$\hbar\omega_s = 2\mu_B H = \frac{12\pi^2}{k_F l}\frac{\Phi}{\Phi_0}\,T_1. \tag{35}$$

Thus, though typically $\hbar\omega_s \sim T_1$, (34) has nevertheless a small effect since $\hbar/\tau_{so} \gg T_1$.

(iii) $1/\tau_s,\ 1/\tau_{so} \ll T_1,\ \hbar\omega_s$

In this limit, the grand potential is given by $(T=0)$

$$\Omega = \lambda_c \hbar^{-1} \sum_q \int \frac{d\omega}{2\pi}\,\mathrm{Re}\,\frac{|\omega|}{|\omega| + Dq^2 + i\omega_s} \tag{36}$$

and thus, in accordance with (33)

$$\Omega_m(\omega_s)/\Omega_m(0) = e^{-z_s}[(1+z_s)\cos z_s + z_s \sin z_s] \tag{37}$$

with $z_s = (\hbar\omega_s/2T_m)^{1/2}$; using (35), I obtain

$$z_s^2 = 6\pi^2 m^2 (k_F l)^{-1}\frac{\Phi}{\Phi_0}. \tag{38}$$

Thus Zeeman splitting also leads to a reduction of quantum coherence, except when spin-orbit scattering is large (see *(ii)*). Note that for $z_s \to 0$:

$$\Omega_m(z_s)/\Omega_m(0) = 1 - z_s^3 + \ldots \tag{39}$$

Finally I remark that due to the finite transverse extension of the ring, $L_\perp$, the orbital effect of the magnetic

field also leads to a destruction of phase coherence [15, 6], which can be taken into account in the above expressions by the replacement $Dq^2 \to Dq^2 + 1/\tau_H$, where

$$\frac{1}{\tau_H} \simeq \frac{1}{3}\left(\frac{eHL_\perp}{\hbar}\right)^2 D. \tag{40}$$

The numerical factor in this expression depends weakly on the shape of the wire. For the present discussion, (40) is rewritten as follows:

$$\frac{\hbar}{\tau_H} = \frac{16\pi^4}{3}\left(\frac{\Phi}{\Phi_0}\right)^2 \left(\frac{L_\perp}{L}\right)^2 T_1. \tag{41}$$

For the experimental situation [1], taking $L_\perp \sim 10^{-2} L$ and $T_1 \sim 25$ mK, I arrive at the estimate $\hbar/\tau_H \simeq 1\,\text{mK} \cdot (\Phi/\Phi_0)^2$; thus the orbital effect is negligible.

4. Discussion

In this article, I presented a detailed investigation of the collective contribution to the persistent current in mesoscopic normal metal rings at low temperatures, which is an important contribution in experiments performed on an ensemble of rings. As in the theory of the magnetoconductance of long cylinders [20, 21], the fundamental harmonic for the flux dependence is given by $\Phi_0 = h/2e$, since the physics of these effects is closely related, namely due to the interference of a given path around the ring with its time-reversed counterpart. For the persistent current, which is an equilibrium property of the system, the effect is initiated by the electron-electron interaction.

As discussed above, higher order interaction terms lead to a reduction of the magnitude of the effect, estimated to be about a factor five compared to the first order result [2]. [Note that electron-electron interactions also lead to an additional phase breaking rate [3–6], usually called $1/\tau_\Phi$. However, this is negligible for an equilibrium quantity like the persistent current, since $\hbar/\tau_\Phi \ll T$ (unlike for the conductivity)]. Secondly, the diffusion pole type singularity of the Cooperon is weakened by the magnetic field (Zeeman splitting, orbital effect) and by spin scattering. In particular, I estimated that the orbital effect is very small for the experimental [1] situation. On the other hand, spin-flip scattering leads to a strong reduction of coherence for $\hbar/\tau_s > T_1$, as to be expected. Finally, the situation concerning Zeeman splitting and spin-orbit scattering is more delicate. In fact, for $\omega_s = 0$, I find the persistent current to be independent of the spin-orbit interaction, while in the absence of spin scattering, the Zeeman splitting becomes important. The experimental situation [1] seems to be $\hbar/\tau_{sO} > T_1$; in this case, the influence of Zeeman splitting is very small.

Finally, I compare in more detail the collective contribution with the results obtained numerically [10] and analytically [11] for the single-electron effect. Therefore, I write the amplitude of the first harmonic, (9), in the form

$$I^{(1)} = 8\lambda_c T_1/\Phi_0 = \lambda_c (8/3\pi)(l/L) \cdot I_0 \tag{42}$$

where $I_0 = ev_F/L$ is the amplitude of the persistent current in the clean limit. Note that the coupling constant, λ_c, is to be renormalized as discussed above. On the other hand, the single-electron contribution is numerically [10] estimated to be given by

$$I^{(1)}/I_0 \simeq 0.1 \cdot (l/LM)^{1/2} \tag{43}$$

where I used the relation $l \simeq 10^2/W^2$ to relate the elastic mean free path (in units of the lattice constant) to the parameter W characteristic for the strength of the disorder in the tight-binding model [9, 10]. Actually, the coefficient ($\sim 10^2$) in this relation depends weakly on the band filling, and is given by ~ 150 and ~ 60 for low filling and for a half-filled band, respectively. [Note that it is appropriate to consider the three-dimensional case.] I contrast, the analytical result for the single-electron contribution is found to be given by [11]

$$I^{(1)}/I_0 \simeq (1/\pi)\, M^{-1} \tag{44}$$

where, to be precise, $M = A k_F^2/4\pi$; A denotes the transverse area of the ring. Clearly, (44) is at variance with (43).

The parameters relevant for comparison with experiment [1] are $l \sim 300$ Å, $L \sim 2.2$ μm, and $A \sim 1.6 \cdot 10^{-11}$ cm^2, such that $L/l \sim 70$ and $M \sim 2.4 \cdot 10^4$. Thus from (42–44), I find $I^{(1)}/I_0$ to be given by $1.2 \cdot 10^{-2} \lambda_c$, $0.8 \cdot 10^{-4}$, and $0.7 \cdot 10^{-6}$, respectively (which should be compared to the experimental result of $3 \cdot 10^{-3}$). I conclude that the collective contribution dominates the single-electron effect, provided the renormalized coupling constant is not extremely small. An apparent – and unsolved – question is the discrepancy between the numerical and the analytical answer for the single-electron contribution.

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Note added in proof. As a cautionary remark, note that the arguments presented by Schmid [11] seem to imply that the discussion of the interaction constant and its renormalization by higher order terms (Sect. 2) is incomplete, in particular for the (long range) Coulomb interaction.