Resonantly Enhanced Quantum Decay.

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Abstract. – We study the quantum decay out of the metastable state of a Josephson junction, via tunnelling, in the presence of an external sinusoidal force. We show that the Floquet picture, together with the complex scaling method, provides an adequate nonperturbative method to describe this process. The enhancement of the decay rate near the fundamental resonance assumes a Lorentzian line shape in agreement with recent experiments in the deep quantum regime. Moreover, our results exhibit novel features such as secondary resonances: at higher frequencies, corresponding roughly to a second harmonic induced by the nonlinear potential shape, and at a lower frequency, exactly at half the frequency of the first resonance, revealing a two-photon transition.

In this work we focus on the problem of the quantum decay out of a metastable state in the presence of external periodic driving. In recent years the decay theory of unstable states [1] has been extended to quantum systems interacting with an environment which includes dissipation [2, 3]. Our study has been motivated by recent experiments on r.f.-stimulated biased Josephson junctions both in the classical and the quantum regime [4]. In contrast to the classical regime where the effect is generally known as resonance activation [5], there exist no previous theoretical studies which address the deep quantum regime at zero temperature in a nonperturbative manner.

Here we present a full account of the decay rate enhancement of the ground state induced by an external sinusoidal force. In doing so, we shall cover the whole frequency regime extending from zero frequency to very high frequencies. The two limiting cases can both be solved analytically whereas the intermediate regime, except at resonances between the external frequency and internal level spacings, can be treated on a numerical basis only.

To obtain a detailed description of the decay process in the presence of an external periodic force, we refrain from the use of time-dependent perturbative or semi-classical methods alone. In these approaches [6], the periodicity of the Hamiltonian is not fully exploited. Moreover at strong forcing the perturbative approach no longer suffices. The concept which seems most appropriate for a rigorous treatment is the Floquet picture of quantum mechanics [7]. The starting point of our investigations is the driven Hamiltonian of a particle in a cubic metastable landscape

$$H(x,t) = -\frac{1}{2}\partial_x^2 + \frac{1}{2}x^2 - \frac{1}{2x_c}x^3 + xS\sin(\omega t). \tag{1}$$

Here we use dimensionless units. Time is measured in units of $1/\omega_0$, with ω_0 denoting the angular frequency of small oscillations at the bottom of the well. The external frequency ω and energies are thus measured in units of ω_0 and $\hbar\omega_0$. The dimensionless barrier height D is related to the exit point of the potential by $x_c = \sqrt{(27/2)D}$, see fig. 1 (the coordinate is measured in units of $\sqrt{\hbar/m\omega_0}$, with m being the mass of the particle). Furthermore the force strength is measured in units of $\sqrt{\hbar m\omega_0^3}$. The model in (1) is also used to approximately describe the decay of the phase in a current-biased Josephson junction. We will henceforth deal with systems containing only one or two unperturbed resonance states under the barrier, which corresponds to the standard experimental set-up [4]. Moreover we note that this situation clearly cannot be solved accurately within a semi-classical approximation.

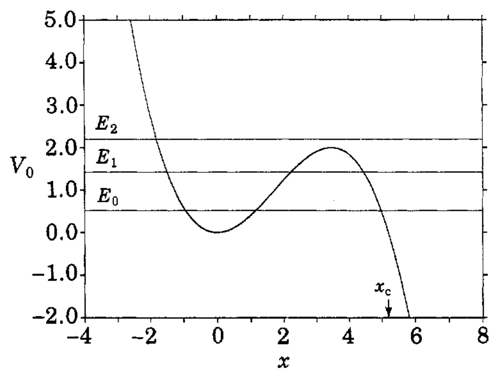

Fig. 1. – Unperturbed ($S = 0$) metastable potential $V_0(x)$ of the Hamiltonian in eq. (1) with $D = 2$, supporting two quasi-stationary states under the barrier. The horizontal lines indicate the positions of the real parts of the resonance energies E_0, E_1, E_2.

Given (1), solutions of the time-dependent Schrödinger equation can be cast in the form

$$\Psi_\varepsilon(x,t) = \Phi_\varepsilon(x,t)\exp[-i\varepsilon t], \tag{2}$$

$$\Phi_\varepsilon(x,t) = \Phi_\varepsilon(x,t+T), \tag{3}$$

i.e. the Floquet function $\Phi_\varepsilon(x,t)$ is periodic with period $T = 2\pi/\omega$ of the external force and the quasi-energies ε determine the long-time behaviour of the wave function.

Generally the decay of unstable states can be associated with complex-valued poles of the S-matrix. These correspond to simple poles on the unphysical sheet of the complex-valued energy Riemann surface and constitute the well-known resonance states. By use of the complex-scaling approach [8], *i.e.* if one rotates the coordinate $x \to x\exp[i\phi]$, one uncovers the resonance poles in the complex plane and ends up with square integrable eigenfunctions [9]. Here we apply this concept generalized to the quasi-energy formalism [7c, 10]. The decay rate Γ in the periodically driven case is thus given by the imaginary part of the Floquet resonances (complex-valued quasi-energies) as

$$\Gamma_\varepsilon = -2\,\mathrm{Im}\,\varepsilon \tag{4}$$

and is measured in units of ω_0. Treating the zero-temperature limit only, we restrict ourselves to the rate enhancement γ of the lowest unperturbed state, *i.e.*

$$\gamma(\omega, S) = \frac{\Gamma_{\varepsilon_0}(\omega, S) - \Gamma_0}{\Gamma_0}. \tag{5}$$

Here ε_0 denotes the lowest $(n = 0)$ class of quasi-energies, *i.e.* $\varepsilon_{0,k} \equiv \varepsilon_0 + k\hbar\omega$; $k = 0, \pm 1, \pm 2, \ldots$, which smoothly passes into the lowest resonance E_0, with imaginary part $\Gamma_0 = -2\,\mathrm{Im}\,E_0$, as the amplitude of the external force approaches zero.

Before focusing on the most interesting regime of resonant driving, $\omega \approx 1$, we deal with the two limiting cases of adiabatic ($\omega \ll \Gamma$) and fast ($\omega \gg 1$) driving. Following the reasoning put forward recently for an adiabatically driven symmetric double-well configuration [11], we obtain, by averaging over the slowly varying phase, for the adiabatic rate enhancement

$$\Gamma_{\varepsilon_0}^{\mathrm{ad}} = \Gamma_0 \frac{1}{2\pi} \int_0^{2\pi} \left(1 + 6\frac{S \sin\phi}{x_c}\right)^{3/4} \exp\left[-\frac{36}{5} D \left[\left(1 + 6\frac{S \sin\phi}{x_c}\right)^{3/2} - 1\right]\right] \mathrm{d}\phi. \tag{6}$$

In fig. 2*a*) we compare the semi-classical approximation (6) (dashed line) with the numerical exact results (circles). The excellent agreement for the absolute rate is due to the fact that in (6) we used the numerical value for Γ_0, and not its semi-classical estimate (which exceeds the numerical result by 36% for $D = 1$ [9]). The line shape of the enhancement is thus well described by its renormalized semi-classical estimate.

In the other limit of very high frequencies, the decay rate can be obtained again within a semi-classical approximation after a corresponding rotating-wave approximation [11]. In this way one finds the result

$$\Gamma_{\varepsilon_0}^{\mathrm{HF}} \approx \Gamma_0 \left[1 + \left(\frac{36}{10} - \frac{1}{4D}\right) \frac{S^2}{\omega^4}\right], \qquad \omega \gg 1. \tag{7}$$

In fig. 2*b*) we depict the rate enhancement defined in eq. (5). The semi-classical result (dashed line) is compared with the precise numerical values (solid line). Note that, for $\omega \to \infty$, the high-frequency limit approaches the unperturbed (zero force) value proportional

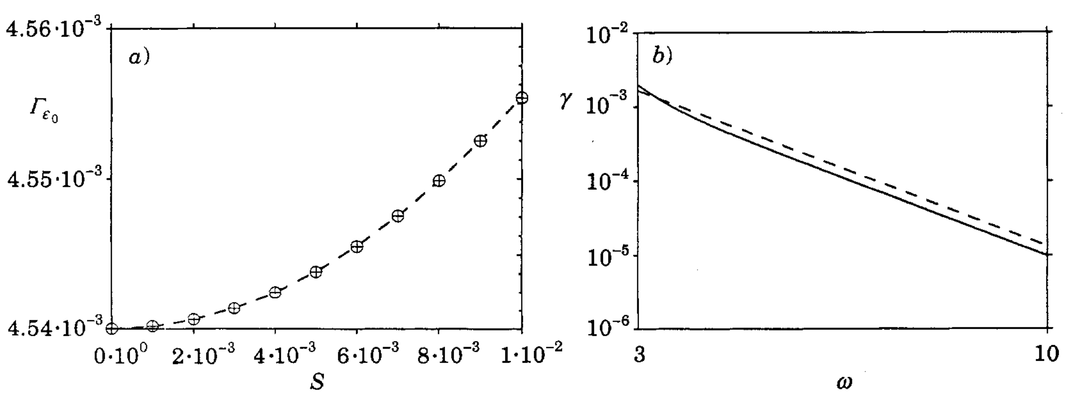

Fig. 2. – *a*) Adiabatic (dimensionless) decay rate Γ_{ε_0} of the driven metastable potential for the parameters $D = 1$, $w = 10^{-3}$ *vs.* the external field strength S. Dashed line: semi-classical result from eq. (6), circles: numerical result. *b*) Double-logarithmic plot of the high-frequency resonance enhancement γ, eq. (5), as a function of frequency for the parameters $D = 1$, $S = 2 \cdot 10^{-1}$. Dashed line: semi-classical result from eq. (7), full line: numerical result.

to ω^{-4}. Clearly, as $\omega \to \infty$, the system is no longer capable to respond to the external perturbation. The constant shift between the two curves in fig. 2*b*) is due to the fact that the prefactor of S^4/ω^4 in eq. (7) originates from an instanton formula for the decay rate that always overestimates the exact value [9].

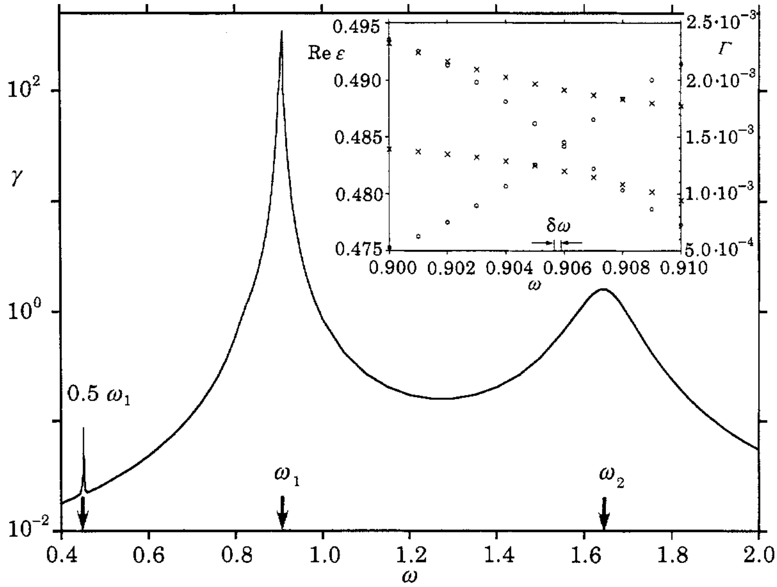

Fig. 3. – Resonance enhancement γ, eq. (5), as a function of frequency for the driven decay with the parameters $D = 2$, $S = 10^{-2}$. The inset depicts the behaviour of the real parts (crosses) and imaginary parts, $\Gamma_\varepsilon = -2\,\mathrm{Im}\,\varepsilon$, (circles) of the quasi-energies ε_0, $\varepsilon_1 - \omega$, in the vicinity of the first resonance frequency and the Bloch-Siegert shift $\delta\omega$.

Let us now consider the regime of frequencies near the resonance $\omega \approx \omega_1 \equiv \mathrm{Re}(E_1 - E_0) \approx$ ≈ 0.9057 (see fig. 1). For the numerical investigations we used a barrier height of $D = 2$ and an external dimensionless force $S = 10^{-2}$. For these parameters, a perturbative treatment is insufficient. The numerical values for the enhancement γ of the decay rate are depicted in fig. 3 *vs.* the external driving frequency. At the resonance frequency $\omega = \omega_1$ we find a dramatic enhancement of $\gamma \approx 245$ because of the one-photon stimulated decay (see also (1)). The line shape of the enhancement curve is almost perfectly symmetric around the resonance frequency. This driven-tunnelling–induced Lorentzian-like rate enhancement is clearly distinguished from the very asymmetric energy-diffusion–induced enhancement found in classical resonance activation [5]. This symmetric shape is due to the frequency behaviour of the real and imaginary parts of the corresponding complex-valued quasi-energies (see inset in fig. 3). At resonance, the corresponding real parts $\mathrm{Re}\varepsilon_0$, $\mathrm{Re}\,\varepsilon_1 - \omega$ of the quasi-energies exhibit an *avoided* crossing, while the imaginary parts *do cross* each other; both processes occur sym-

(1) For an estimate of the rate enhancement in the vicinity of the resonance at $\omega = \omega_1$, we treated the driven two-level system with time-decaying expansion coefficients. Using the semi-classical estimates for the decay rates out of two lowest states (see [9]) and employing the rotating-wave approximation, we arrive at the value $\gamma(\omega_1) = 322$, which is 31% higher than the exact one. Treating the two-level system numerically within Shirley's formalism [7*a*], and inserting the exact values for the decay rates, the deviation from the numerical value, *i.e.* $\gamma(\omega_1) = 245$ of the full problem, is reduced to 5%.

metrically around $\omega = \omega_1$. We emphasize that the particle decays out of the *Floquet state* with quasi-energy ε_0. Far away from the resonance, this state has the same shape as the lowest unperturbed wave function, which is approximately a Gaussian wave packet centred around the minimum of the well. In the vicinity of the avoided crossing, however, the Floquet state has admixtures from the first-excited resonance wave function. We mention that increasing the external force S further will only slightly alter the height of the main resonance peak but predominantly will broaden its width (see also [12] for a qualitative two-level treatment of this problem).

In addition to the main one-photon stimulated decay at $\omega \approx \omega_1$ one observes several additional features:

i) We find a second-harmonic–like transition near $\omega \approx \omega_2 \equiv \mathrm{Re}(E_2 - E_0) \approx 1.6459$ (see fig. 3). This second transition is forbidden within the harmonic approximation, but the anharmonicity of the cubic potential gives rise to a nonvanishing dipole matrix element $\langle 2 | x | 0 \rangle$ which determines the strength of this higher-order transition.

ii) We also found a small enhancement at the frequency $\omega_{1/2} = \omega_1/2 \approx 0.453$ (see fig. 3). This subharmonic transition can be viewed as a two-photon stimulated decay process.

iii) Finally we note that the main resonance undergoes a shift of the resonance frequency proportional to the square of the applied external amplitude frequency. This feature is analogous to Bloch-Siegert shift [13] in magnetic resonance, described by a sinusoidally driven two-level quantum dynamics. The shift is depicted in the inset in fig. 3 and amounts to $\delta\omega \approx 2 \cdot 10^{-4}$ for the parameters we chose.

At present, there exist no experimental data which allow for a detailed quantitative comparison with our results. The only existing data were obtained at low but finite temperatures $T > 0$, and, in addition, the strength of the amplitude (respectively, the power) of the external force has not been measured independently. Nevertheless, the data in ref. [4] (see fig. 18 and 20 in [4*a*]) typically exhibit the change of shape of the enhancement curve from a *very asymmetric* shape, with more weight located below the resonance frequency ω_1 than above, in the classical diffusive regime, towards the *tunnelling-induced symmetric* shape at temperatures $T < 30$ mK, below the crossover temperature. Additionally, the existing data are taken at a force strength (the rate enhancement is always smaller than $e = 2.718...$) that is much too low to quantitatively read off a Bloch-Siegert shift. Moreover, the regimes of $\omega = \omega_2$ and $\omega = \omega_1/2$ have presently not been covered in the extreme quantal limit by the existing experimental data. We thus hope that our predictions will motivate and guide future experimental efforts.

In conclusion, we have addressed the rate enhancement induced by an external periodic force in the deep quantum regime and for values of the external force that are too high for perturbative approaches to be valid. By use of the quasi-energy method, one finds a dramatic enhancement around the main first resonance with a characteristic *symmetric* line shape. In addition, we predict the existence of a resonance shift with increasing amplitude and characteristic additional enhancements in the driven decay rate around the first superharmonic $\omega = \omega_2$ and at the first subharmonic $\omega = \omega_1/2$. Away from these characteristic regimes the decay rate is still enhanced and can be described analytically by eqs. (6), (7) in the asymptotic regimes of low and high frequencies.

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