Localization and tunneling in periodically driven bistable systems

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The quantum dynamics of a quartic double well, subjected to a harmonically oscillating field, is studied in the framework of the Floquet formalism. The modifications of the familiar tunneling process due to the driving are investigated numerically and explained in terms of the local quasienergy spectrum. In particular, there is a one-dimensional manifold in the parameter space spanned by amplitude and frequency of the driving force, where tunneling is almost completely suppressed by the coherent driving. The influence of dissipation is also discussed.

1. Introduction

Bistable systems are abundant in physics, from the microscopic to the macroscopic realm. On the macroscopic level, bistability represents a basic concept in nonlinear dynamics. In quantum mechanics, on the other hand, bistable potentials are associated with a paradigmatic coherence effect: tunneling [1]. Accordingly, this class of systems represents a particularly promising field to study the interplay of classical nonlinearity and quantum coherence, and the way it is reflected in phase-space transport.

In the present work we investigate the influence of periodic driving on the quantal dynamics in a bistable potential. Being equivalent to adding one more degree of freedom, external driving is capable of qualitatively altering the dynamics: e.g., in the classical limit, it can render a bistable system chaotic [2,3]. Here, however, we concentrate on the alterations of the tunneling process, due to the driving, in the deep quantal regime: They take the form of mere quantitative changes of the tunnel splitting, from its complete vanishing up to its augmentation by orders of magnitude, as well as of qualitative changes as a consequence of the admixture of additional levels, beyond the ground-state doublet.

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Periodic driving is simple enough to still allow, by way of its discrete time-translational symmetry, for a systematic analytical treatment: The Floquet formalism provides a generalization of the notions of energy eigenvalues and eigenstates to periodically time-dependent systems [4–7]. Since its validity is not restricted to small amplitudes of the driving nor to large characteristic actions, we need not resort to perturbative or semiclassical methods. Furthermore, the Floquet formalism obviously enables to go beyond the two-state approximation commonly used in the context of tunneling.

In section 2, we present our working example, a harmonically driven quartic double well, and introduce some analytical concepts for later reference, such as the Floquet operator and the local spectrum. Section 3 contains our principal results. They form a survey of the coherence phenomena that replace tunneling in various regimes of the parameter space spanned by amplitude and frequency of the driving force. Aspects of the classical limit – the influence of chaotic classical dynamics and of incoherent processes induced by the environment – are briefly addressed in section 4. In section 5, we give a summary of our results.

This contribution is partially based on results originally published in earlier works by the present authors [8–11].

2. The periodically driven double well

The system we study is a quartic double-well potential driven by a monochromatic force. Its Hamiltonian reads, in dimensionless variables,

$$H_{\mathrm{DW}}(p, x; t) = H_0(p, x) + H_1(x; t) ,$$

$$H_0(p, x) = \frac{p^2}{2} - \frac{x^2}{4} + \frac{x^4}{64D} , \quad H_1(x; t) = Sx \cos(\omega t) , \tag{1}$$

where D denotes the barrier height, and S and ω are the amplitude and frequency of the driving force, respectively.

In systems with a discrete time-translational symmetry, a stroboscopic time evolution is generated by the *Floquet operator* [12–15], the propagator over a single period of the time-dependent force,

$$U = U(T, 0) = \mathcal{T} \exp\Big(-\mathrm{i} \int_0^T \mathrm{d}t\, H(t)\Big) , \tag{2}$$

where T denotes the time-ordering operator. Therefore, U may be called a *quantum map*. According to the Floquet theorem, its eigenstates take the form

$$|\psi_\alpha(t)\rangle = \exp(-\mathrm{i}\varepsilon_\alpha t)|\phi_\alpha(t)\rangle\,, \qquad \text{with } |\phi_\alpha(t+T)\rangle = |\phi_\alpha(t)\rangle\,. \tag{3}$$

The eigenvalues ε_α are called *quasienergies*. In fact, each of them is a representant of an infinite class of eigenvalues $\varepsilon_{\alpha,k} = \varepsilon_\alpha + k\omega$, $k = 0, \pm 1, \pm 2, \ldots$. The $\varepsilon_{\alpha,k}$ correspond to solutions equivalent to eq. (3), as is obvious if one defines $|\phi_{\alpha,k}\rangle = \exp(\mathrm{i}k\omega t)|\phi_\alpha\rangle$. In other words, the quasienergy spectrum is cyclic, i.e., defined mod ω, similar to the Brillouin-zone structure in the solid-state context.

Another, more special symmetry of the system described by the Hamiltonian (1) goes back to the inversion symmetry $x \to -x$, $p \to -p$, of phase space for the time-independent system $H_0(p, x)$. This symmetry is destroyed by an arbitrary periodic driving term, but for the harmonic time dependence chosen here, the relation $\cos(\omega t + \pi) = -\cos(\omega t)$ allows for invariance under the operation

$$\mathsf{P}\colon \quad x \to -x\,, \quad p \to -p\,, \quad t \to t + \tfrac{1}{2}T\,. \tag{4}$$

P forms a unitary symmetry and may be regarded as a *generalized parity*. As a consequence, the basis formed by the Floquet eigenstates can be divided into an even and an odd subset.

A quantity that provides some condensed information on the transport of probability between the two wells of the bistable potential, and that allows to relate this information directly to the relevant structures in the quasienergy spectrum, is the *probability to return* [16,17],

$$P_n^{\Psi(0)} = |\langle \Psi(nT)|\Psi(0)\rangle|^2 = |\langle \Psi(0)|U^n|\Psi(0)\rangle|^2\,, \tag{5}$$

defined with reference to some initial state $|\Psi(0)\rangle$, and with time restricted to a discrete series $t_n = nT$, $n = 0, \pm 1, \pm 2, \ldots$. The rôle of the quasienergies for this time evolution is made explicit by expanding eq. (5) in the Floquet basis,

$$P_n^{\Psi(0)} = \xi^{-1} + \sum_{\alpha \neq \beta} \exp[\mathrm{i}(\varepsilon_\alpha - \varepsilon_\beta)nT]\, |\langle \psi_\alpha|\Psi(0)\rangle|^2 |\langle \psi_\beta|\Psi(0)\rangle|^2\,. \tag{6}$$

Here, ξ^{-1}, the diagonal part excluded from the double sum in eq. (6), gives the long-time average of $P_n^{\Psi(0)}$. The spectral counterpart of $P_n^{\Psi(0)}$ is the two-point cluster function $P_2^{\Psi(0)}(\eta)$ of the *local* Floquet spectrum [16–19]. It is related to $P_n^{\Psi(0)}$ by Fourier transformation and thus contains all the frequencies involved in the time evolution of $P_n^{\Psi(0)}$, weighted according to their relative significance for the specific dynamics starting from $|\Psi(0)\rangle$.

3. Driven tunneling

In the present section, we discuss the modifications imposed on the familiar tunneling dynamics, due to periodic driving. That is, we concentrate on the time evolution, under the external force, of a state that is initially prepared as an approximation to a superposition of the two lowest unperturbed eigenstates, $|l, r\rangle = (|1\rangle \pm |2\rangle)/\sqrt{2}$, centered in one of the two wells. Accordingly, we trace the quasienergy doublet that corresponds to the unperturbed energies E_1 and E_2, through the parameter space spanned by amplitude S and frequency ω of the driving force. Thereby, we exclude dynamical complexity due to mere preparation effects from our investigation.

There are two regimes in the (S, ω) plane where tunneling is not qualitatively altered: Both in the limits of slow (adiabatic) and of fast driving, the separation of the time scales of inherent dynamics and external force effectively uncouples these two processes and is reflected in a mere renormalization of the tunneling rate $\Delta(S, \omega)$. Specifically, as both an analytical treatment and numerical experiments show [8], the driving always reduces the effective barrier height and thus augments the tunneling rate in the two limits at issue.

Qualitative changes in the tunneling behavior are expected as soon as the driving frequency becomes comparable with the internal frequencies of the double well, i.e., in particular, the tunnel splitting $\Delta = E_2 - E_1$ and the so-called resonances $E_3 - E_2$, $E_4 - E_1$, $E_5 - E_2, \ldots$. A physical understanding of the temporal complexity in this regime is obtained by relating it to the "landscape" of quasienergy planes $\varepsilon_{\alpha,k}(S, \omega)$ in parameter space. Features of particular significance are close encounters of quasienergies: Two quasienergies approaching each other form an exact crossing if they belong to different parity classes, otherwise the crossing will be avoided.

We discuss two specific instances of the quasienergy spectrum with the corresponding tunneling dynamics, one of them featuring an avoided crossing, the other an exact one. The "single-photon transition" at $\omega = E_3 - E_2$ is called the *fundamental resonance*. At $S = 0$, it corresponds to a crossing between the quasienergies $\varepsilon_{2,k}$ and $\varepsilon_{3,k-1}$ and, for $S > 0$, forms an avoided crossing, since the corresponding eigenstates have equal parity. Fig. 1a shows the time evolution of $P_n^{\Psi(0)}$, at the fundamental resonance ($D = 2$, $S = 2 \times 10^{-3}$, $\omega = 0.876$), for an initial state prepared as the ground state of a harmonic oscillator approximating one of the wells, i.e., a Gaussian approximation of $|l, r\rangle$. The monochromatic oscillation of $P_n^{\Psi(0)}$ characteristic of unperturbed tunneling has

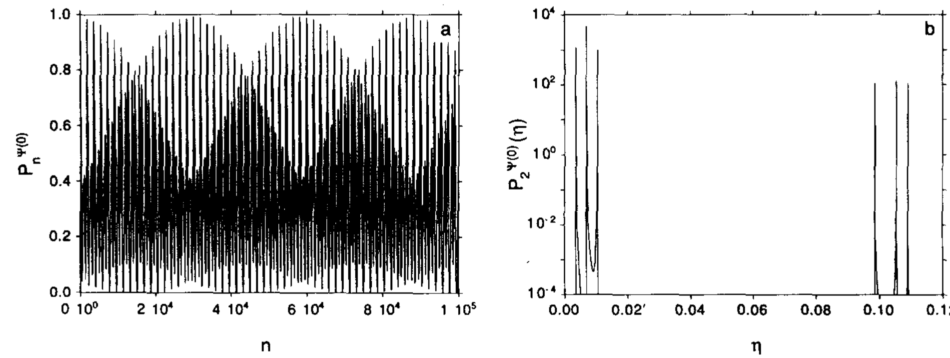

Fig. 1. Driven tunneling at the fundamental resonance, $\omega = E_3 - E_2$. (a) Time evolution of $P_n^{\Psi(0)}$ over the first 2×10^5 time steps; (b) corresponding local spectral two-point correlations. The parameter values are $D = 2$, $S = 2 \times 10^{-3}$ and $\omega = 0.876$.

given way to a more complex beat pattern. The two-point correlation $P_2^{\Psi(0)}(\eta)$ of the local spectrum reveals that these beats are mainly composed of two groups of three frequencies each (fig. 1b), which can be identified, in turn, as the quasienergy differences $\varepsilon_{3,-1} - \varepsilon_{2,0}$, $\varepsilon_{2,0} - \varepsilon_{1,0}$, $\varepsilon_{3,-1} - \varepsilon_{1,0}$, and $\varepsilon_{4,-1} - \varepsilon_{3,-1}$, $\varepsilon_{4,-1} - \varepsilon_{2,0}$, $\varepsilon_{4,-1} - \varepsilon_{1,0}$, at the avoided crossing.

In contrast, a two-photon transition that bridges the tunnel splitting Δ is "parity forbidden", and thus the quasienergies $\varepsilon_{1,k+1}$ and $\varepsilon_{2,k-1}$ give rise to an exact crossing. Eq. (6) indicates that a vanishing of the difference $\varepsilon_{2,-1} - \varepsilon_{1,1}$ will have a drastic consequence: For a state prepared as an exact superposition of the corresponding two quasienergy eigenstates only, $P_n^{\Psi(0)}$ and all other observables become constants, at least at discrete times nT, and thus it is possible that tunneling comes to a standstill! According to an argument going back to von Neumann and Wigner [20,21], exact crossings should occur along one-dimensional manifolds in the (S, ω) plane. Fig. 2a shows such a manifold for $\varepsilon_{2,-1} = \varepsilon_{1,1}$, as determined numerically: it is a closed curve, reflection-symmetric with respect to the line $S = 0$, with an approximately linear frequency dependence for $\Delta \lesssim \omega \lesssim E_3 - E_2$. A typical time evolution of $P_n^{\Psi(0)}$ for parameter values on the linear part of that manifold ($D = 2$, $S = 3.171 \times 10^{-3}$, $\omega = 0.01$) is presented in fig. 2b. With $P_n^{\Psi(0)}$ staying essentially constant around a value $\lesssim 1$, it clearly demonstrates the suppression of tunneling. Remaining oscillations of small amplitude can be ascribed to an admixture of higher-lying quasienergy states to the initial state. In addition, the time dependence synchronous with the driving frequency has not completely vanished, as is revealed by the evolution of $P^{\Psi(0)}(t)$, resolved within a single period of the driving force (fig. 2c). In fig. 2d, we compare $|\langle \Psi(t)|x\rangle|^2$, at a time ($t = 458T$) where the deviation of $P_n^{\Psi(0)}$ from unity is exceptionally large, with the initial

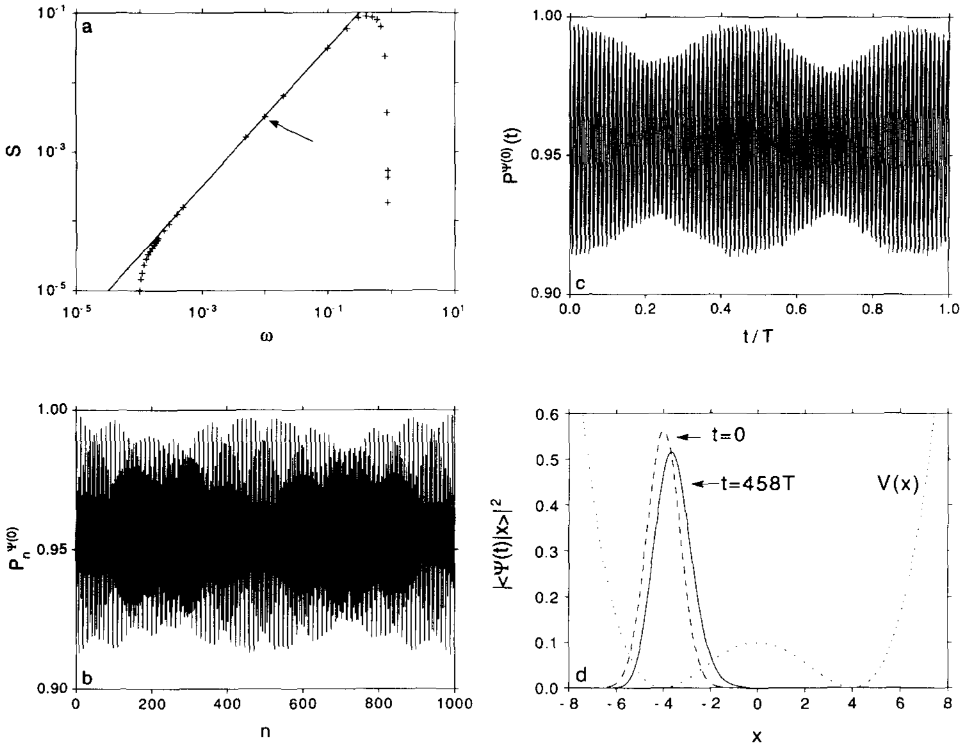

Fig. 2. Suppression of tunneling at an exact crossing, $\varepsilon_{2,-1} = \varepsilon_{1,1}$. (a) One of the manifolds in the (S, ω) plane where this crossing occurs (data obtained by diagonalization of the full Floquet operator for the driven double well are indicated by crosses, the full line has been derived from a two-state approximation, the arrow indicates the parameter pair for which parts (b)–(d) of this figure have been obtained); (b) time evolution of $P_n^{\Psi(0)}$ over the first 1000 time steps; (c) time evolution of $P^{\Psi(0)}(t)$ within the first period of the driving; (d) $|\langle \Psi(t)|x\rangle|^2$ at $t = 458\ T$ (full line), compared with the initial state (dashed line, the dotted line indicates the unperturbed potential). The parameter values are $D = 2$, $S = 3.171 \times 10^{-3}$ and $\omega = 0.01$, so that ω equals 52.77 times the unperturbed tunnel splitting.

state: This confirms that the leakage of probability into the initially empty, opposite well indeed remains extremely small. So the coherent suppression of tunneling truly amounts to a *localization* of the wave packet in one of the wells.

This phenomenon appears to be an elementary quantum-interference effect. In fact, much of it can be understood on basis of a two-state approximation. It is achieved by solving the equations of motion for the expansion coefficients of a localized initial state in the Hilbert space spanned by the unperturbed ground-state doublet $|1\rangle$, $|2\rangle$. The two-state approximation predicts an infinite number of manifolds where localization occurs and yields analytical expressions for them [11,22].

4. Influence of dissipation

The approach towards the macroscopic realm comprises, at least, two different aspects: the increase in characteristic phase-space scales allows the use of small-wavelength approximations and lets finer and finer details of the classical phase space flow show up in the quantum dynamics, while the growing rôle of the ambient degrees of freedom tends to reduce the complexity of the quantum dynamics of degrading coherence effects. We shall here restrict ourselves to the latter aspect (for semiclassical studies of driven tunneling, see refs. [23,24]) and present some preliminary results on the influence of dissipation on the quantum dynamics of the driven double well.

We incorporate dissipation by coupling the system at issue to a heat bath,

$$H(p, x; \{b_i, b_i^\dagger\}; t) = H_{\mathrm{DW}}(p, x; t) + H_{\mathrm{I}}(x; \{b_i, b_i^\dagger\}) + H_{\mathrm{R}}(\{b_i, b_i^\dagger\}) ,$$

$$H_{\mathrm{I}}(x; \{b_i, b_i^\dagger\}) = x \sum_i (g_i b_i + g_i^* b_i^\dagger) , \quad H_{\mathrm{R}}(\{b_i, b_i^\dagger\}) = \sum_i \omega_i (b_i^\dagger b_i + \tfrac{1}{2}) , \tag{7}$$

with frequency ω_i, second-quantization operators b_i, $b_i^\dagger$, and coupling constant g_i for the ith reservoir oscillator. Proceeding in a similar way as in ref. [25], we use the density operator in the Floquet basis, reduced to the double-well degree of freedom, as the basis of our description, and resort to the usual rotating-wave and Markov approximations. This allows to derive the equation of motion for the density matrix $\tilde{\sigma}$ (in the interaction picture) in the form of a master equation [25],

$$\dot{\tilde{\sigma}}_{\alpha\alpha} = \sum_\nu (W_{\alpha\nu} \tilde{\sigma}_{\nu\nu} - W_{\nu\alpha} \tilde{\sigma}_{\alpha\alpha}) , \quad \dot{\tilde{\sigma}}_{\alpha\beta} = -\frac{1}{2} \sum_\nu (W_{\nu\alpha} + W_{\nu\beta}) \tilde{\sigma}_{\alpha\beta} , \quad \alpha \neq \beta , \tag{8}$$

comprising a closed subset of equations for the approach of the diagonal elements towards a steady state, and another subset describing the decay of the non-diagonal elements. The coefficients $W_{\mu\nu}$ depend on the coupling constants and on the quasienergies and will not be given here. They determine the damping constant γ characterizing the motion in the classical limit of eq. (8). This master equation serves as a basis for numerical investigations of the dynamics. Within the approximations made, it amounts to a quantization of the driven Duffing oscillator [26].

Fig. 3a shows the time evolution of $P_n^{\sigma(0)} = \mathrm{tr}[\sigma(nT)\, \sigma(0)]$ with an initial state $\sigma(0) = |\Psi(0)\rangle \langle \Psi(0)|$ and parameters of H_{DW} as in fig. 1, but with a finite damping constant $\gamma = 4 \times 10^{-5}$, at zero temperature. The complex quantum

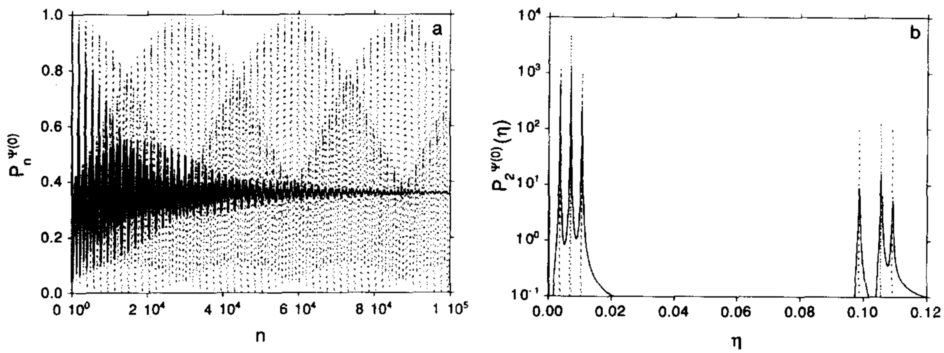

Fig. 3. Driven tunneling with dissipation. (a) Time evolution of $P_n^{\sigma(0)}$ over the first 2×10^5 time steps; (b) corresponding local spectral two-point correlations. The parameter values are as in the corresponding conservative case shown in fig. 1 (repeated here in dashed lines), but with a finite damping constant, $\gamma = 4 \times 10^{-5}$, at zero temperature.

beats characteristic of the corresponding conservative system (dashed line) die out and give way to a steady state with a finite constant value of $P_n^{\sigma(0)}$ (in a periodically driven system, the steady state may still possess a time dependence, with the period of the driving, which however is invisible in a stroboscopic plot like this). The broadening of the quasienergy levels, due to the incoherent transitions described by eq. (8), can be read off the Fourier transform of $P_n^{\sigma(0)}$, fig. 3b.

A phase-space distribution (specifically, the Husimi distribution [27]) for a steady state of the dissipative driven double well, at the parameter values $D = 2$, $S = 0.15$, $\omega = 0.876$, $\gamma = 10^{-3}$ and $T = 0$, is presented in fig. 4. A detailed study of the dissipative dynamics will follow [28].

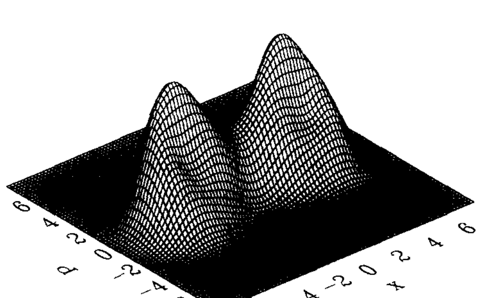

Fig. 4. A steady state of the driven double well with dissipation, represented as its Husimi phase-space distribution. The parameter values are $D = 2$, $S = 0.15$, $\omega = 0.876$, $\gamma = 10^{-3}$, at zero temperature.

5. Summary

The present work is intended to give an overview over various aspects of tunneling in a double well under the influence of periodic driving. The basic notions to discuss a periodically driven quantal dynamics are provided by Floquet theory, a time-domain analogue of Bloch theory: Quasienergies and quasienergy eigenstates replace the familiar concepts of energy eigenvalues and eigenstates, respectively. Consequently, driven tunneling is adequately analyzed in terms of the quasienergies that contribute to the time evolution of a state initially localized in one of the wells.

In the limits of slow and of fast driving, the familiar tunneling dynamics is merely accelerated. Qualitative modifications occur where the quasienergies corresponding to the ground-state doublet of the unperturbed double well interact, in parameter space, with quasienergies corresponding to higher-lying unperturbed eigenenergies. In particular, avoided crossings can lead to quite complex quantum beats, while at specific exact crossings, which form one-dimensional manifolds in parameter space, an almost complete suppression of tunneling occurs. It is essentially a two-quasienergy interference phenomenon, in fact much of it can be understood in terms of a two-state approximation of the double well.

Towards the classical limit, both diffusive transport due to classical chaos and incoherent processes induced by the environment become significant ingredients of the physics of the driven double well. A dissipative dynamics, introduced by coupling the double well to a heat bath, leads to a broadening of the quasienergy lines and to a corresponding decay of the coherence phenomena observed in the conservative case. The steady states approached by this system form the quantal analogues of the attractors of the Duffing oscillator.

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