Tunnel splittings and chaotic transport in periodically driven bistable systems

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We study the crossover from coherent transport by tunneling to diffusive transport through classically chaotic phase-space regions, and the corresponding changes in the spectrum. The harmonically driven quartic double well is used as a working model and treated numerically in the framework of the Floquet formalism. We find a transition from exponentially small tunnel splittings to level separations of the order of the mean level distance, when the corresponding classical dynamics becomes dominated by chaotic diffusion.

1. INTRODUCTION

In periodically driven bistable systems with a discrete symmetry, tunneling coexists with diffusive classical transport through chaotic phase-space areas [1–5]. In this work, we address the question how the tunnel splittings reflect the onset of chaotic motion in the corresponding classical system.

A rule-of-thumb known from quantum chaology says that eigenstates spread over classically chaotic phase-space areas show level repulsion, while those localized in regular regions have uncorrelated eigenvalues. Accordingly, we start from a simple working hypothesis: The tunnel splitting is correlated with the overlap of the associated pair of eigenstates with the globally chaotic part of the classical phase space. It grows from an exponentially small value to a size comparable with the mean level separation as the corresponding pair of tori dissolve in the chaotic sea.

2. THE MODEL

We consider a cosine-driven quartic double well, described by the Hamiltonian (in dimensionless units),

$$\begin{aligned} H(x,p;t) &= H_0(x,p) + xS\cos(\omega t), \\ H_0(x,p) &= \frac{p^2}{2} - \frac{1}{4}x^2 + \frac{1}{64D}x^4. \end{aligned} \tag{1}$$

Here, S and ω denote the amplitude and frequency, respectively, of the driving force. The barrier height D gives approximately the number of doublets below the barrier. For our numerical studies, we set $D = 8$ and $\omega = 0.95$ throughout.

The model (1) possesses two discrete symmetries, the periodicity of the driving force ($t \to t + 2\pi/\omega$) which allows to use the Floquet formalism, and a generalized parity ($x \to -x$, $p \to -p$, $t \to t + \pi/\omega$). It implies that the Floquet states are either even or odd, and gives rise to the tunnel doublets which form the subject of our study.

3. FLOQUET STATES AND CLASSICAL PHASE SPACE

As a representation of the Floquet states $|\psi_\alpha(t)\rangle = \exp(-i\varepsilon_\alpha t)\,|\phi_\alpha(t)\rangle$ in classical phase space, we use the Husimi distribution $h_\alpha(x,p;t)$ of $|\psi_\alpha(t)\rangle$, defined as a projection onto coherent states of the harmonic oscillator that approximates the motion on bottom of each well.

In particular, we consider the "low-lying" Floquet eigenstates whose time-averaged energies, for $S \to 0$, approach eigenvalues $E_{n,p}$ of H_0 below the barrier, where p denotes parity (*e*ven / *o*dd).

As a crude measure inhowfar a Floquet state belongs predominantly to the chaotic layer Λ around the separatrix, we calculate the overlap $\Gamma_n(t) = \int_\Lambda \mathrm{d}p\,\mathrm{d}x\, h_{n,e}(x,p;t)$ of the corresponding Husimi distribution with Λ.

4. RESULTS

In Fig.1, we compare the S-dependence of the tunnel splittings $\Delta_n = |\varepsilon_{n,e} - \varepsilon_{n,o}|$ (part a) with that of the overlaps (b) for the "lowest" seven doublets. It clearly shows a qualitative agreement: Both quantities exhibit a marked increase

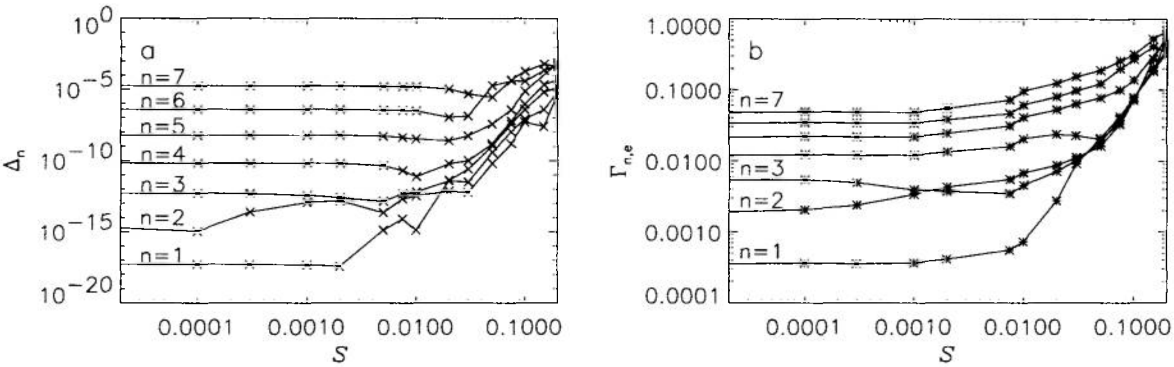

Figure 1. Tunnel splittings (part a) and overlaps with the chaotic layer (b) for the seven lowest tunnel doublets, as functions of the amplitude S of the driving force.

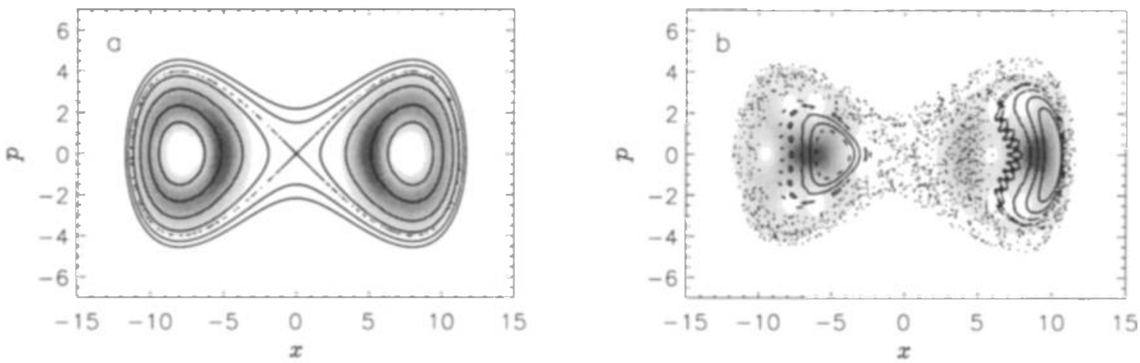

Figure 2. Husimi distribution (in gray-scale representation) for $|\psi_{4,e}(0)\rangle$, compared with the corresponding classical phase-space portraits, at $S = 10^{-5}$ (part a) and $S = 0.2$ (b).

as soon as chaotic behaviour begins to dominate the classical dynamics. This correlation is substantiated by comparing (Fig.2) $h_{4,e}(x,p;0)$, at an S-value below the transition (part a) and one above it (b), with the corresponding classical phase-space portraits. It demonstrates that the increase of the overlap Γ_n actually reflects both the growth of the classically chaotic phase-space region, and the change in position and shape of the individual eigenstates belonging to the tunnel doublet n. Insofar the naive picture sketched in the introduction is confirmed. However, in order to obtain a more quantitative understanding, a semiclassical description, in terms of path integrals, of the interplay of tunneling and chaotic diffusion is required.

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